# Homework 5

$$\begin{array}{rcl}
\mathtt{I} & = & \lambda x.x \\
\mathtt{K} & = & \lambda xy.x \\
\mathtt{S} & = & \lambda xyz.xz(yz) \\
\mathtt{B} & = & \lambda xyz.x(yz) \\
\mathtt{C} & = & \lambda xyz.xzy \\
\mathtt{1} & = & \lambda xy.xy \\
\mathtt{W} & = & \lambda xy.xyy \\
[M,N] & = & \lambda z.zMN \qquad \text{where } z \notin MN \\
\mathtt{P} & = & \lambda xyz.zxy \\
\mathtt{U}^n_k & = & \lambda x_0 \ldots x_n.x_k \\
\omega & = & \lambda x.xx \\
\Omega & = & (\lambda x.xx)(\lambda x.xx) \\
\mathtt{Y} & = & \lambda f.(\lambda x.f(xx))(\lambda x.f(xx)) \\
\mathtt{c}_0 & = & \lambda fz.z \\
\mathtt{c}_1 & = & \lambda fz.fz \\
\mathtt{c}_2 & = & \lambda fz.f(fz) \\
\mathtt{c}_n & = & \lambda fz.f^n(z) = \lambda fz.\underbrace{f(f(f(\ldots(fz))))}_{n\ f\text{s}} \\
S^+ & = & \lambda nfz.f(nfz) \\
\mathtt{c}_+ & = & \lambda mnfz.mf(nfz) \\
\mathtt{c}_\times & = & \lambda mnf.m(nf) \quad (= \mathtt{B}) \\
\mathtt{c}_\wedge & = & \lambda mn.nm
\end{array}$$

Simplify the following lambda terms until no further beta redexes are present, or until a reduction cycle is detected. (*Hint.* When possible, use equations of combinatory logic to take shortcuts and perform several steps in one.)

1. $\mathtt{SCSIK}\Omega$
2. $\mathtt{CS}(\Omega\mathtt{I})\mathtt{P}(\omega)\mathtt{KI}$
3. $\mathtt{B}\omega(\mathtt{P}xy)(\mathtt{KI})$
4. $\mathtt{K}(\mathtt{K}(\mathtt{K}x))\Omega\Omega\Omega$
5. $\mathtt{SII}(\mathtt{BK}(\omega))abcd$
6. $(\lambda x.x(\lambda y.yx))\mathtt{I}(\lambda y.yy)$
7. $\mathtt{S}(\lambda pq.qp)(\mathtt{KI})\mathtt{I}$
8. $\mathtt{B}(\lambda xy.xyy)(\lambda xy.xyy)(\lambda z.z\mathtt{KI})(\mathtt{KI})$
9. $\omega(\lambda wx.ww(x\mathtt{I}))$
10. $\omega(\lambda wx.x(ww(x\mathtt{I})))\mathtt{I}$