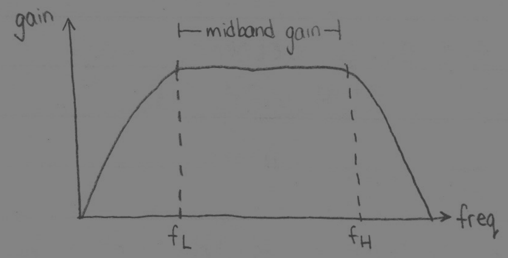
Nairn and Chan: Frequency Response

Quick Review of Frequency Response Concepts



So far all of our analysis has been done in the midband region of an amplifier's frequency response.

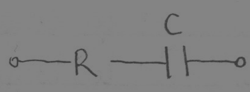
f_H is the upper cutoff frequency, determined by parasitic capacitance in the circuit. f_L is the lower cutoff frequency, determined by coupling capacitors.

Both of these are DEFINED as the point where the gain is 3 dB less than the maximum gain; where the gain is $1/\sqrt{2}$, or $\approx 70.71\%$ of its original value.

The bandwidth, or the width of the region of "regular" operation is given by

$$f_H - f_L$$

Series Impedances: these are simply added together, like series resistances



, where $Z_R = R$
$Z_C = 1/j\omega C$

$$\therefore Z_{series} = R + 1/j\omega C = \frac{1 + j\omega RC}{j\omega C}$$

As it's a complex number, Z_{series} has both a magnitude and a phase.

Let's look at the magnitude, the phase is inconsequential for now.

$$|Z_{series}| = \sqrt{\frac{(1+j\omega RC)^2}{(j\omega C)^2}} = \frac{\sqrt{(1)^2 + (\omega RC)^2}}{\sqrt{(-1)(\omega^2)(C^2)}} = \frac{\sqrt{1+(\omega RC)^2}}{\omega C}$$

Note that as ω becomes small

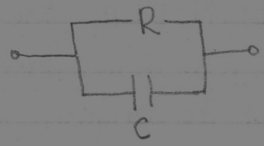
$$|Z| \approx \frac{1}{\omega C}$$

and as ω becomes large

$$|Z| \approx \frac{\cancel{\omega} R \cancel{C}}{\cancel{\omega} \cancel{C}} = R$$

So at large frequencies, the series impedance is dominated by the resistor, while at low frequencies it as dominated by the capacitor.

Parallel Impedance: Same as always.



$$Z_{para} = \left[R^{-1} + (1/j\omega C)^{-1}\right]^{-1} = \left[\frac{1}{R} + j\omega C\right]^{-1} = \left[\frac{1+j\omega CR}{R}\right]^{-1} = \frac{R}{1+j\omega RC}$$

And consequently,

$$|Z_{paral}| = \frac{R}{\sqrt{1+(\omega RC)^2}}$$

$\nearrow$ ω small: $|Z| \approx R$

$\searrow$ ω large: $|Z| \approx \frac{1}{\omega C}$

Notice that $|Z|$'s crossover point occurs at

$$R = \frac{1}{\omega C}$$

$$\omega = \frac{1}{RC}$$

Bode Plot Review

Woo, just what I wanted re-introduced. Looks like we're stuck having to use this concept again.

Let's say we have some transfer function (which we have toiled long and hard to solve for):

$$A(j\omega) = \frac{\underbrace{10^4}_{\text{midband gain}} \overbrace{(1 + j\omega/10^4)}^{\text{zeros}}}{\underbrace{(1 + j\omega/10^5)(1 + j\omega/10^6)(1 + j\omega/10^7)}_{\text{poles}}}$$

There are 4 important points on the ω axis:

zeros: 10^4 (numerator)

poles: $10^5, 10^6, 10^7$ (denominator)

Let's begin with the magnitude plot, because it's easier. All we have to do is follow some pretty simple steps.

① Start at the maximum gain, in dB ($20\log(x)$)

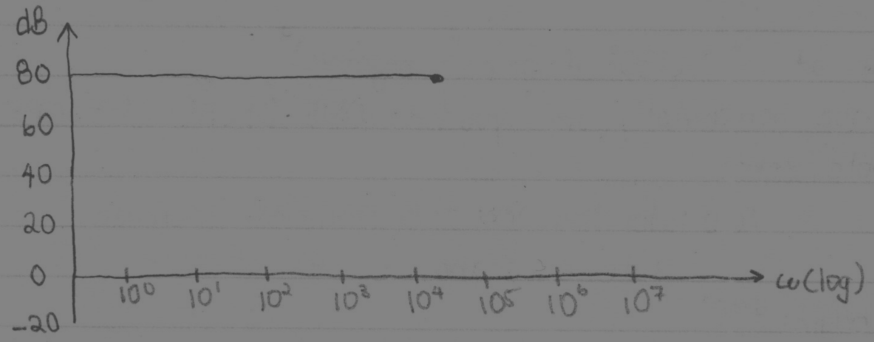
② Continue horizontally until you hit a pole/zero

  a) Zero: add 20 dB/decade to the current rate of change

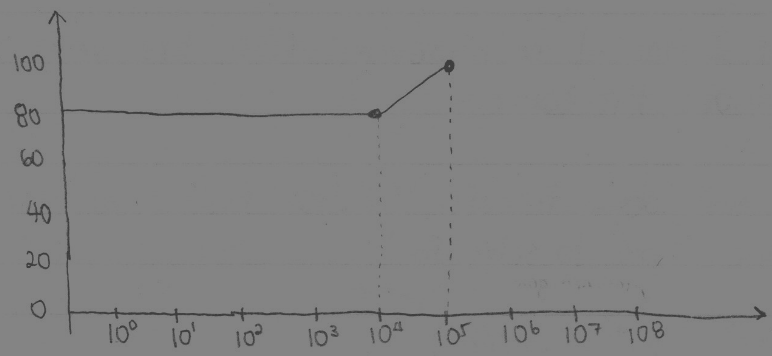
  b) pole: decrease current rate of change by 20 dB/decade

③ Continue until all poles/zeros are exhausted

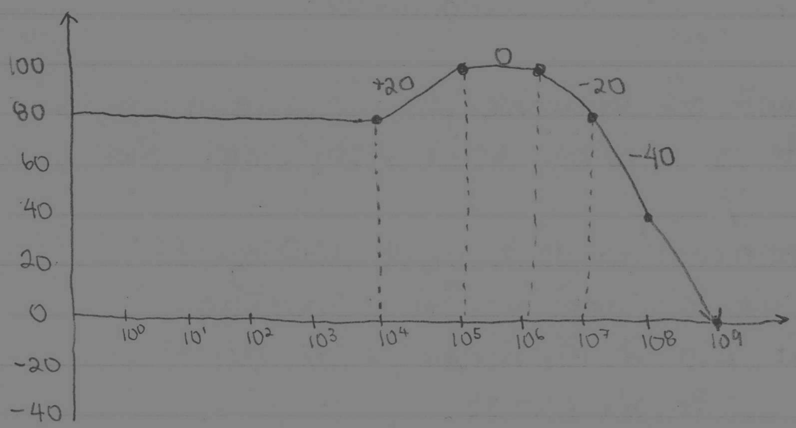
① $20\log(10^4) = 80$ dB



② Hit 10^4, increase by 20 dB/decade



③ Hit 10^5, decrease by 20 dB/decade
10^6, decrease by 20 dB/decade
10^7, decrease by 20 dB/decade



That's our magnitude plot! Not too bad, eh? The phase is somewhat more complex, though. Each pole/zero affects the plot one decade before and after itself.

① Begin at 0° (180° if gain is negative)*
② Continue horizontally until you hit ONE DECADE BEFORE a pole/zero:
a) zero: for the next two decades, increase rate of change by 45°/decade

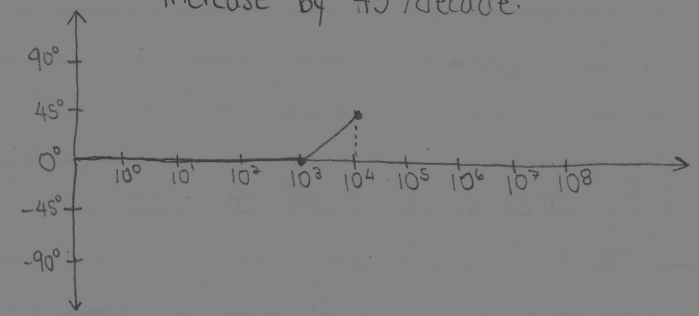
* zero @ origin: +90°
pole @ origin: -90°

b) pole: for the next two decades, decrease rate of change by 45°/decade

③ Repeat until all poles/zeros are exhausted

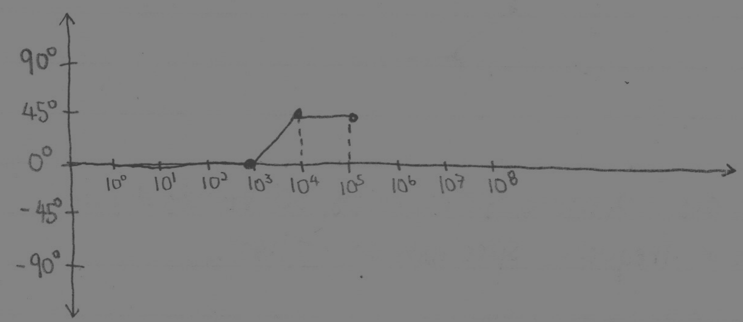
Enough theory, let's begin.

① Begin at 0°, hit 10^3, one decade before the zero; 10^4 increase by 45°/decade.



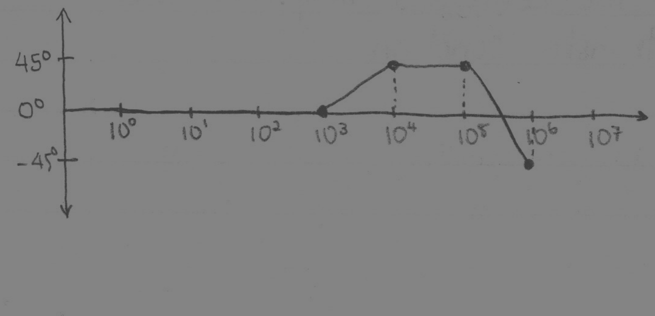
② We're at 10^4, one decade before a pole, 10^5.

Change due to zero: +45°/decade
Change due to pole: -45°/decade
} total: 0°

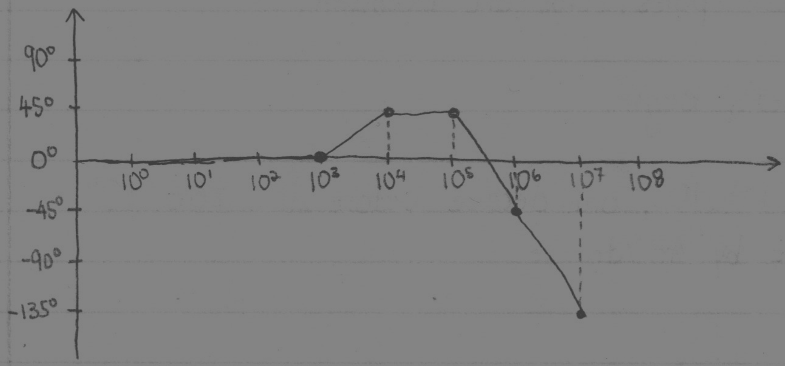


③ Now we're affected by the 10^5 and 10^6 pole.

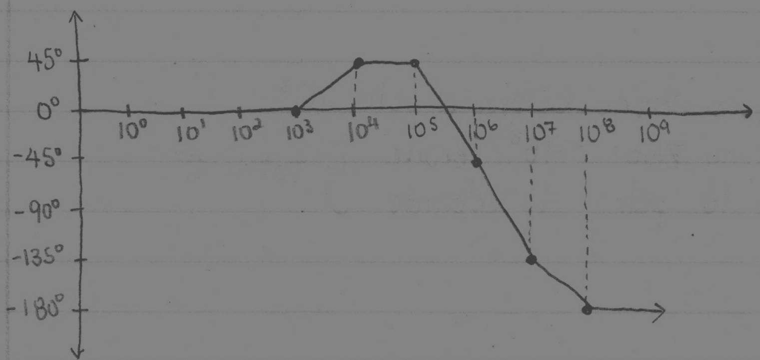
10^5 pole: -45°/d
10^6 pole: -45°/d
} total: -90°/d



③ Now we're affected by the 10^6 and 10^7 poles: $-90°/d$



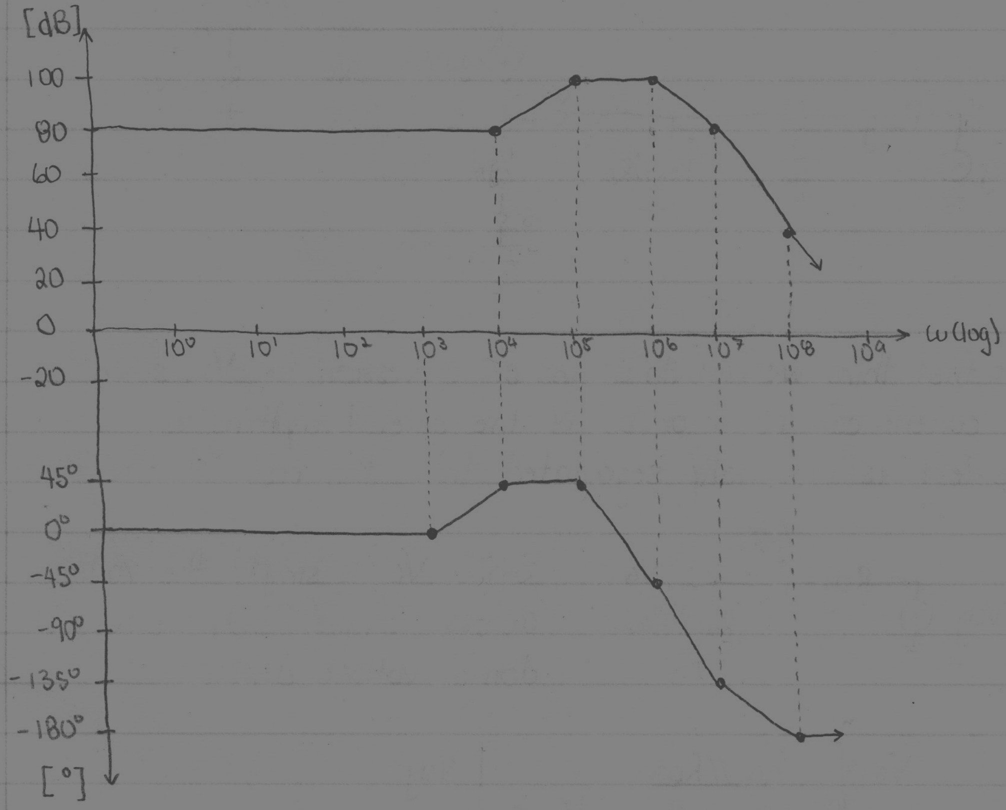
③ Finally we've escaped the range of 10^6's influence, and we just have 10^7's effects.



After that, there are no more poles/zeros, so we continue for the rest of the frequency spectrum at $-180°$.

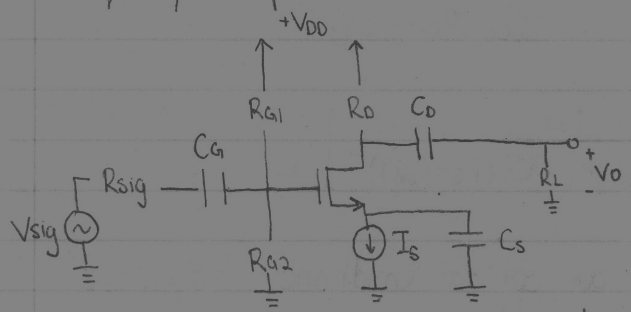
Normally, though, both of these plots are drawn on the graph. This will become important later on as we explore stability.

On the next page is the combined magnitude and phase plot of our transfer function.



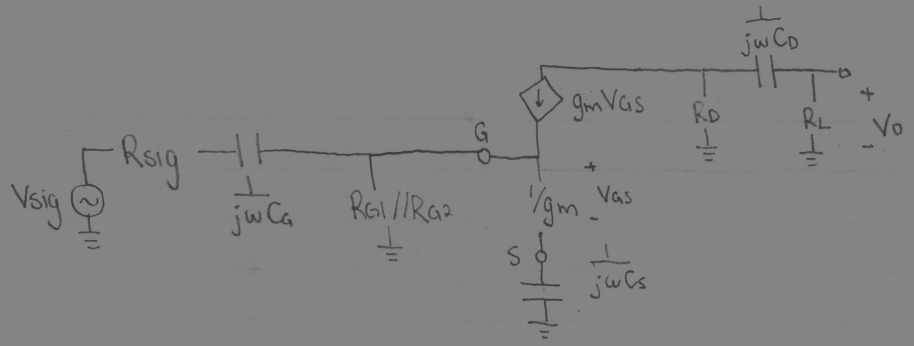
Alright, we've got basically all the knowledge we need to start frequency response.

Frequency Response of the Common-Source Amplifier

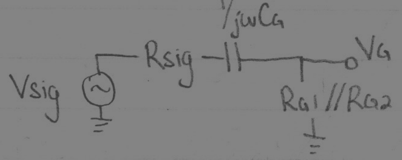


Find the transfer function $\frac{V_o}{V_{sig}}(j\omega)$ which essentially means find me the gain when we can be at sub-optimal frequencies.

As with all "find the gain"-esque questions, we need to convert this to small signal. But now, we need to leave the capacitors, as we can't simply assume they're shorts anymore.



The first thing we can do is find an expression for V_G, since no current can go in or out of the gate. Everything on the left is essentially segregated from the rest of the circuit



Since V_G is simply the voltage across $(R_{G1}//R_{G2})$, we're just doing voltage division

$$V_G = \left[\frac{R_{G1}//R_{G2}}{R_{G1}//R_{G2} + R_{sig} + 1/j\omega C_G}\right] V_{sig}$$

From here, we'll let $R_G = R_{G1}//R_{G2}$ for simplicity's sake.

$$\frac{V_G}{V_{sig}} = \frac{R_G}{R_G + R_{sig}}\left[\frac{1}{1 + \frac{1}{j\omega C_G}(R_G + R_{sig})^{-1}}\right]$$

$$= \frac{R_G}{R_G + R_{sig}}\left[\frac{j\omega}{j\omega + (C_G(R_G + R_{sig}))^{-1}}\right]$$

Notice that we've extracted our optimal (midband) frequency's results: the resistor-only divider. But now with extra terms, we can create the poles/zeros.

$$\frac{j\omega}{j\omega + 1/C_G(R_G + R_{sig})} = \frac{S}{S + \frac{1}{C_G(R_G + R_{sig})}}$$

This form is a little strange. Is it a pole or a zero?

This turns out to be a little bit of a trick question.

$$\frac{s}{s+\frac{1}{C_G(R_G+R_{sig})}} = (s)\left(\frac{1}{s+\frac{1}{C_G(R_G+R_{sig})}}\right)$$

$$= (s)\left[\frac{C_G(R_G+R_{sig})}{sC_G(R_G+R_{sig})+1}\right]$$

$$= [C_G(R_{sig}+R_G)](s)\left[\frac{1}{1+sC_G(R_G+R_{sig})}\right]$$

$$= [C_G(R_G+R_{sig})](s)\left[\frac{1}{1+s/[C_G(R_G+R_{sig})]^{-1}}\right]$$

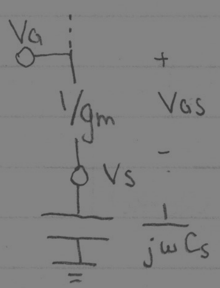
So we've done some mathematical gymnastics to bring this into a form we can recognize. This isn't a pole or a zero. It's both:

$C_G(R_G+R_{sig})$: constant term

s : a zero at the origin

$\frac{1}{1+s/[C_G(R_G+R_{sig})]^{-1}}$: a pole at $\frac{1}{C_G(R_G+R_{sig})}$.

So what happens is that we get a slope of +20 dB/decade from 0 to $[C_G(R_G+R_{sig})]^{-1}$, at which point the slope becomes 0. Now that we've sorted that out, let's continue: we'll find V_{GS} as a ratio of V_G.



V_{GS} is the voltage across $1/g_m$, which is a divider across the series impedances of the resistor and the cap.

$$V_{GS} = \left[\frac{1/g_m}{1/g_m + 1/j\omega C_S}\right] V_G$$

$$\frac{V_{GS}}{V_G} = \frac{j\omega}{j\omega + g_m/C_S}$$

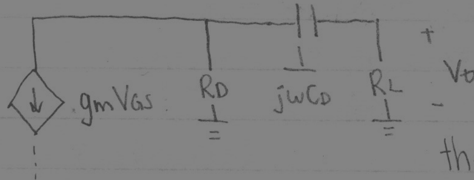
Once again, this is a combination of a pole and a zero.

$$\frac{s}{s + g_m/C_s} = s\left[\frac{1}{s + g_m/C_s}\right]$$
$$= (s)\left(C_s/g_m\right)\left[\frac{1}{1 + sC_s/g_m}\right]$$
$$= (s)\left(C_s/g_m\right)\left[\frac{1}{1 + s/[g_m/C_s]}\right]$$

C_s/g_m : another constant
s : zero at the origin
$(1 + s/(g_m/C_s))^{-1}$: pole at g_m/C_s

So once again, this is +20 dB/decade until g_m/C_s, where it flattens out.

Lastly, we'll see how V_{GS} gets turned into V_o.



We can see that R_D is in parallel with the series impedance of the cap and R_L, and the dependent source's current will distribute accordingly.

$$R_L\text{'s current} = -\left(\frac{R_D}{R_L + 1/j\omega C_D + R_D}\right) g_m V_{GS}$$
$$V_o = (R_L\text{'s current})(R_L)$$
$$\frac{V_o}{V_{GS}} = -\left(\frac{R_D}{R_L + 1/j\omega C_D + R_D}\right) R_L g_m$$
$$= -g_m R_D\left[\frac{R_L}{R_L + R_D}\right]\left[\frac{s}{s + 1/C_D(R_D + R_L)}\right]$$

We've skipped a few steps, but this is exactly like the above cases. Again, we have a zero at the origin and a pole at $(C_D(R_L + R_D))^{-1}$.

Finally, we can put this disgusting transfer function together.

$$\frac{V_O}{V_{sig}} = \frac{V_G}{V_{sig}} \frac{V_{GS}}{V_G} \frac{V_O}{V_{GS}}$$

$$= \left[\frac{R_G}{R_G + R_{sig}}\right]\left[\frac{s}{s + \frac{1}{C_G(R_G+R_{sig})}}\right]\left[\frac{s}{s + g_m/C_S}\right] -g_m R_D \left[\frac{R_L}{R_L + R_D}\right]$$

$$\left[\frac{s}{s + \frac{1}{C_D(R_D+R_L)}}\right]$$

Which is great and all, but let's substitute some numbers in so we can plot this.

$R_{sig} = 200k$ $\quad C_G = 0.1\mu$ $\quad g_m = 4m$

$R_G = 1.8M$ $\quad C_S = 1.0\mu$

$R_D = 20k$ $\quad C_D = 0.1\mu$

$R_L = 30k$

$$\therefore \frac{V_O}{V_{sig}} = (0.9)\left(\frac{s}{s+0.2}\right)\left(\frac{s}{s+4000}\right)(-80)(0.6)\left(\frac{s}{s+5\times10^{-3}}\right)$$

$$= -43.2\left(\frac{s}{s+5m}\right)\left(\frac{s}{s+0.2}\right)\left(\frac{s}{s+4000}\right)$$

Zeros: 3, at the origin

Poles: 5×10^{-3}, 2×10^{-1}, 4×10^{3}

Start: $20\log(43.2) = 32.7$ dB

The gain is negative: we can ignore this on the magnitude plot, but this appears as us starting at $\pm180°$ in the phase plot, as opposed to $0°$.

Since actually drawing the plots are a giant pain in the ass, let's just build piecewise functions for the slopes of the magnitude and phase graphs.

$$\text{Magnitude:} \begin{cases} +60\ \text{dB/d} & 0 \le \omega \le 5\times10^{-3} \\ +40\ \text{dB/d} & 5\times10^{-3} \le \omega \le 2\times10^{-1} \\ +20\ \text{dB/d} & 2\times10^{-1} \le \omega \le 4\times10^{3} \\ 0\ \text{dB/d} & \omega \ge 4\times10^{3} \end{cases}$$

Phase is pretty gross. Let's look at each pole/zero individually.

Origin: 3, each causing a $+90°$ shift

$$180° + (90°)3 = 450° \equiv 90°$$

Pole @ 5×10^{-3} : $-45°/d$ for $[5\times10^{-4}, 5\times10^{-2}]$

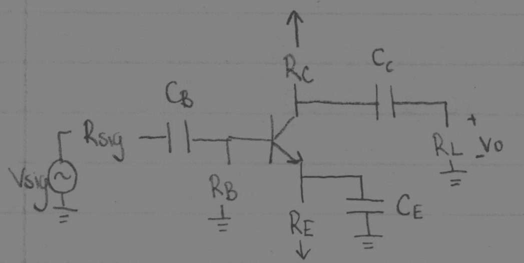
Pole @ 2×10^{-1} : $-45°/d$ for $[2\times10^{-2}, 2\times10^{0}]$

Pole @ 4×10^{3} : $-45°/d$ for $[4\times10^{2}, 4\times10^{4}]$

I'd merge them together, but in the interest of time, let's not. Having only looked at the coupling capacitors, we can only determine the f_L of this system

$$f_L = \max(\text{poles}) = \frac{4000\ \text{rad/s}}{2\pi} = 636.6\ \text{Hz}$$

<u>Frequency Response of Common Emitter Amplifier</u>



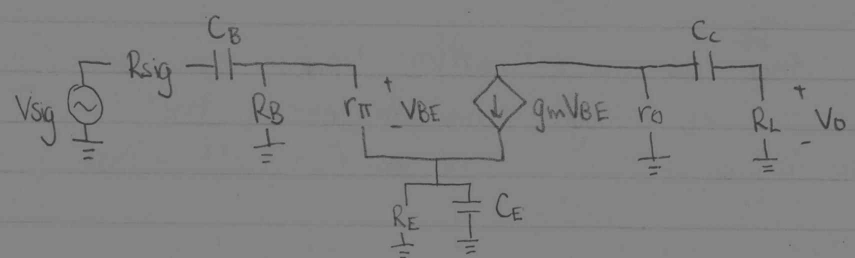
Let's face it. The time it takes to find Vo/Vsig is atrociously long just to determine where the poles are.

There's another method that's faster, called "finding the short-circuit time constants", which gives us pretty good estimates for the poles in the system, without having to solve for the transfer function.

It goes like this:

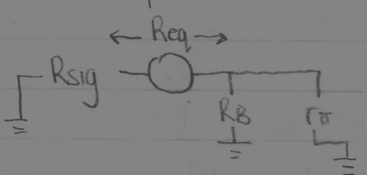
1. Convert to small signal.
2. For each COUPLING (not parasitic; these are open circuit at the frequency where coupling caps are in play) capacitor:
   - set independent sources to 0
   - replace OTHER capacitors with short circuits
   - replace capacitor in question with a test source and find R_{eq}
3. The pole is located at $1/R_{eq}C$

Let's try it out.

1. 

   We've used the π model here.

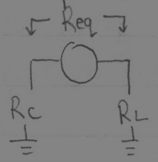
2. Find the pole from C_B.



On one side there's R_{sig}. On the other, there's the parallel equivalent of R_B and r_π.

$$\therefore R_{eq} = R_{sig} + R_B // r_\pi \rightarrow \omega_p = \frac{1}{C_B(R_{sig} + R_B // r_\pi)}$$

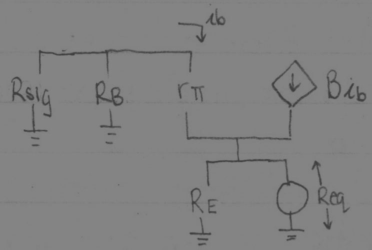
② Find the pole due to C_C.



This is simply R_C and R_L in series

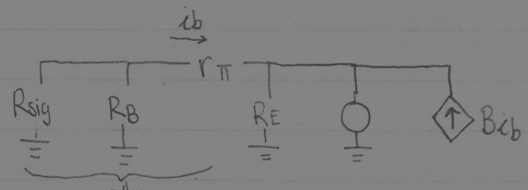
$$\therefore \omega_{p2} = \frac{1}{C_C(R_C+R_L)}$$

② Find the pole due to C_E



Remember, because this is a BJT, the source provides $B i_b$ current.

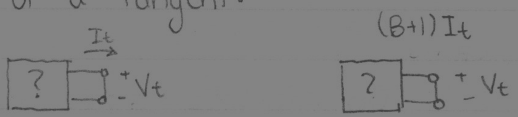
However, it's kind of hard to see what the equivalent resistance is, so let's re-draw this.



We have R_E // some other stuff.

That "other" is $r_\pi + R_{sig}//R_B$. However, since the "other" section has i_b through it, and the emitter has $(B+1)\, i_b$, so the "other" section is seen as $(B+1)$ LESS than it actually is.

This concept is a little unintuitive, but if we go off on a bit of a tangent:



$$Req_1 = \frac{V_t}{I_t} \qquad Req_2 = \frac{V_t}{(B+1)I_t} = \frac{Req_1}{B+1}$$

It becomes a little clearer why exactly that the other section's resistance is smaller.

$$\therefore R_{eq} = R_E // \left[\frac{r_\pi + R_B//R_{sig}}{B+1}\right] \rightarrow \omega_{p3} = \frac{1}{C_E R_{Eq}}$$

Typically, ω_L comes from the capacitor whose Req is the lowest.

However, the max(poles) = ω_L only works if the poles are spread out enough not to affect each other.
We usually say they're "far enough" if each pole is at least a factor of 4 or 5 from the next.

Let's say we had: $\omega_{p1} = 2\times10^{-3}$
$\omega_{p2} = 5\times10^{-2}$
$\omega_{p3} = 6\times10^{-2}$

ω_{p1} is less than $\frac{1}{5}\omega_{p2}$. ω_{p2} is more than $\frac{1}{5}\omega_{p3}$. We can calculate ω_L through the summation of all poles that violate these conditions

$$\therefore \omega_L = \sum \omega_{pi}, \quad \text{where} \quad \frac{\omega_{pi}}{\omega_{p(i+1)}} < 5$$

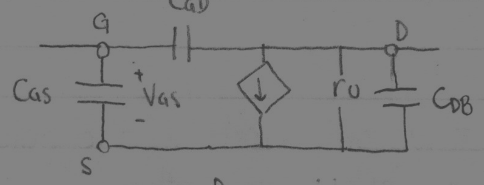
$$= 5\times10^{-2} + 6\times10^{-2}$$
$$= 1.1\times10^{-1} \text{ rads/s}$$

High Frequency Models of the BJT and MOSFET

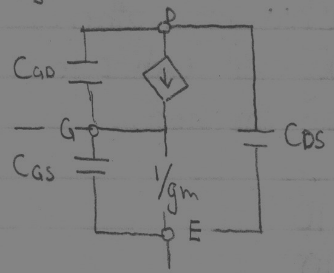
So far we've just looked at coupling capacitors and their low frequency behaviour. What about parasitic capacitors and high frequency?

We'll introduce a new model (which is really just a variant of the old ones) for high frequency analysis.

MOSFET:



(π model)



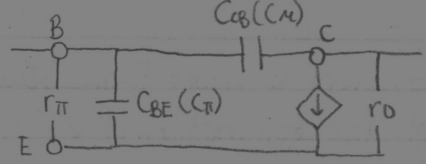
(T-model)

$C_{GS} = 1\text{ fF} \rightarrow 10\text{pF}$

$C_{GD} \doteq 1/10\, C_{GS}$

$C_{GD} \leq C_{DB} \leq C_{GS}$

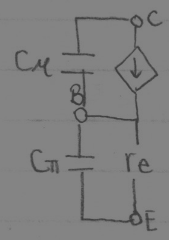
BJT:



(π model)

$C_\pi = 0.1\text{fF} \rightarrow 1\text{pF}$

$C_\mu = C_\pi / 10$



(T model)

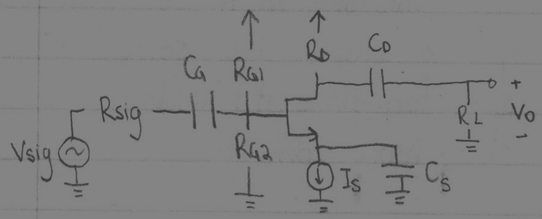
For the mosfet, we have three parasitic capacitors that connect all nodes. C_{DB} is $C_{\text{drain to bulk}}$, where bulk is essentially (for our purposes) just another name for source.

For the BJT, it's very similar. There exists a $C_{CE} \approx C_\pi/100$, but usually this is so small that there's no point in including it.

There's a small section in the notes that finds the "unity short-circuit current gain frequency" aka when I_o/I_i, but it's a pretty standard solve, so I'll just write the results:

MOS: $f_T = \dfrac{g_m}{2\pi(C_{GS} + C_{GD})}$ BJT: $f_T = \dfrac{g_m}{2\pi\,(C_\pi + C_\mu)}$

High Frequency Analysis of the Common Source Amplifier



Much like low-frequency analysis, there exists a shortcut to find the poles due to parasitic capacitors. Unimaginatively, it is called the "open-circuit time constant" method, which we can use to find f_H.

① Convert to high frequency small-signal model (coupling caps are short circuit)

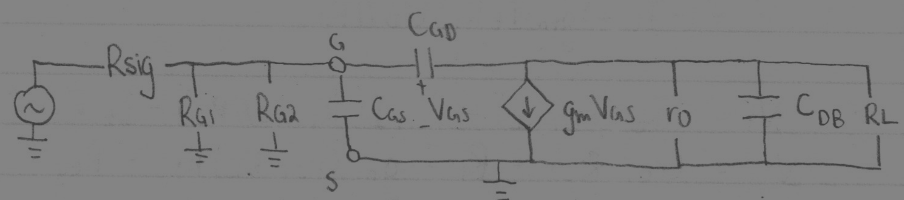
② For each parasitic capacitor

- set independent sources to 0
- replace OTHER parasitic caps with open circuits
- solve for R_{eq}

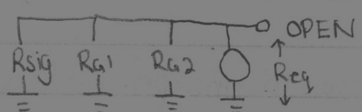
③ $\omega_p = \dfrac{1}{C\,R_{eq}}$

Let's give it a shot.

①



② Pole due to C_{GS}:

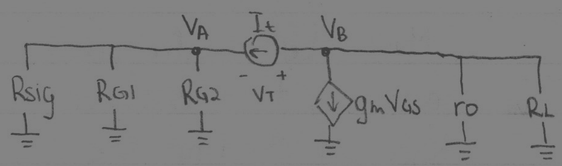


Simply everything in parallel.

$$R_{eq} = R_{sig} // R_{G1} // R_{G2}$$

$$\therefore \omega_{p1} = \frac{1}{C_{GD}(R_{sig} // R_{G1} // R_{G2})}$$

② Pole due to C_{GD}



This one we'll do a proper Req solve for, since it's a bit more complex.

$$V_T = V_B - V_A$$
$$= V_B - V_{GS}$$
$$= V_B - I_t(R_{sig} // R_{G1} // R_{G2})$$

V_{GS} is caused by I_t through the parallel equivalent of the three resistors. Let's let $R_1 = R_{sig} // R_{G1} // R_{G2}$ for simplicity's sake.

$$= V_B - I_t R_1$$

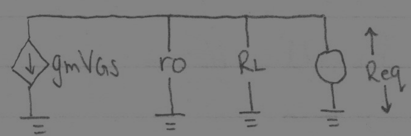
For V_B, we have current leaving the drain through both sources, so it must be coming through r_o and R_L.

$$\therefore\ V_B = -(I_t + g_m V_{GS})(r_o // R_L)$$
$$= (-I_t - g_m I_t R_1)(r_o // R_L)$$

$$\Rightarrow \quad V_T = -I_t(r_o // R_L) - g_m I_t R_1 (r_o // R_L) - I_t R_1$$
$$-\frac{V_T}{I_t} = (r_o // R_L) + R_1 + g_m (r_o // R_L) R_1 = R_{eq}$$

which is the cascode resistance, which is fairly neat.

③ Pole due to C_{DB}



Since the independent sources got cut, $V_{GS} = 0V$, so $R_{eq} = r_o // R_L$

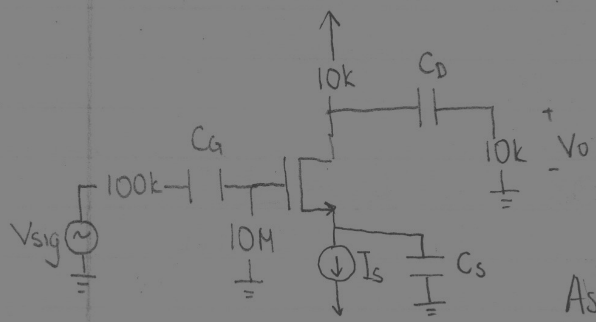
and $\omega_{p3} = \dfrac{1}{C_{DB}(r_o // R_L)}$

For f_H, we can apply the same method:

$$f_H = \frac{\sum_i \omega_{pi}}{2\pi} \qquad \forall \omega_{pi} \text{ where } \frac{\omega_{pi}}{\omega_{p(i+1)}} < 5$$

Let's do an example!

Low Frequency Design: An Example



Given: $g_m = 2 mA/V^2$

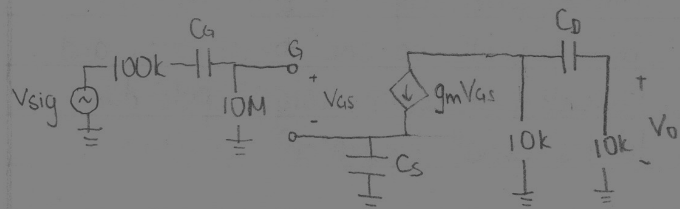
$C_G + C_S + C_D = 3 \mu F$

Poles must be separated by at least a factor of 5.

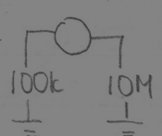
Ignore r_o.

Assign values to C_G, C_D, and C_S such that f_L is the lowest it can possibly be.

① Let's draw the small signal model.

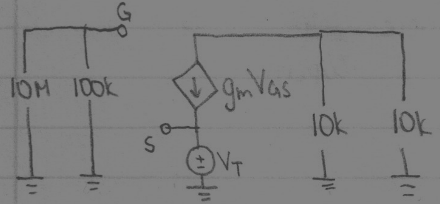


② Pole due to C_G:



$R_{eq} = 10.1 M \qquad \omega_{p1} = \frac{1}{(10.1 M) C_G}$

② Pole due to C_S:

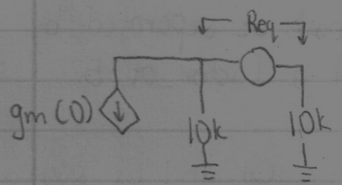


$g_m V_{GS} = g_m (0 - V_T)$

$\therefore$ current source provides $-g_m V_T$ current

$$\frac{V_T}{-g_m V_T} = R_{eq} = \frac{1}{g_m} \qquad \therefore \omega_{p2} = \frac{1}{(500)\,C_S}$$

② Pole due to C_D:



$R_{eq} = 10k + 10k$
$= 20k$

$$\therefore \omega_{p3} = \frac{1}{(20k)\,C_D}$$

So now, we've got 3 Reqs:

$R_{eq1} = 10.1\text{ M}$
$R_{eq2} = 500$
$R_{eq3} = 20k$

Almost obviously, the pole created by C_G will be the lowest one, and C_S will produce the largest pole due to its small equivalent resistance.

If we let $R = 500\,\Omega \rightarrow R_{eq1} = 20200R$
$R_{eq3} = 40R$

If we then let $RC_s = \omega_{p1} \rightarrow R_{eq1} C_G = 20200RC_G = 25\,\omega_{p1}$
$R_{eq2} C_D = 40RC_D = 5\omega_{p1}$

$20200RC_G = 25RC_S$
$C_G = (1.24\times10^{-3})C_S$

$40RC_D = 5RC_S$
$C_D = (0.125)C_S$

$C_S + 0.125C_S + (1.24\times10^{-3})C_S = 3\mu$
$C_S = 2.66\mu F$

Consequently: $C_G = 3.30\ nF$ $\quad$ $C_D = 332.5\ nF$

$\therefore\ f_{cs} = f_L = 119.7\ Hz$

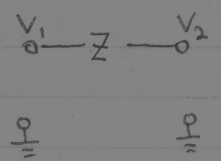
$f_{cD} = 23.9\ Hz$

$f_{cG} = 4.78\ Hz$

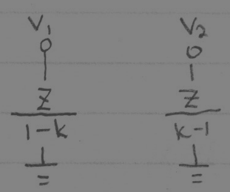
The Miller Effect and Miller Theorem

Since we're dealing so much with impedances, let's introduce the Miller theorem:

If you have an impedance between two non-ground nodes:

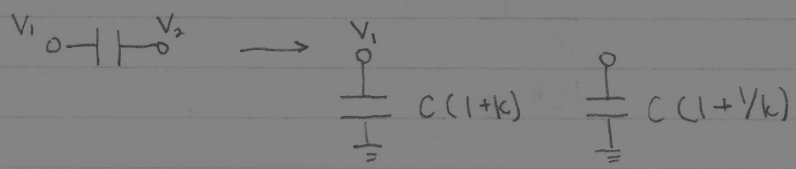


We can split this up into two separate branches



where $k = V_2/V_1$.

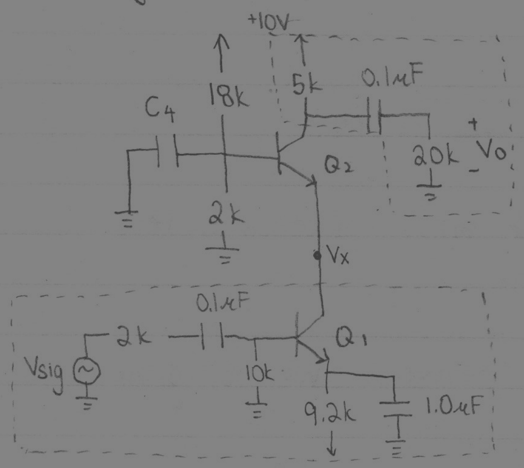
Specifically, for capacitors:



This can simplify analysis as we can now look at voltages with respect to ground.

Cascode's Effect on Bandwidth

Placing transistors in cascode can increase f_H, thereby increasing the bandwidth of the amplifier.



Assume the Vx and Vo nodes each have a parasitic capacitance of 1pF.

Note that the non-cascode version (in the dotted boxes) has $A_m = 80$ V/V, $f_L = 600$ Hz and $f_H = 9$ MHz. $V_A = 50$ V, $I_c = 1$ mA, $g_m = 40$ mA/V, $r_\pi = 2.5$k.

$C_\pi = 1.0$pF and $C_\mu = 0.1$ pF for Q_1 and Q_2.

1) Determine the value of C_4 such that its pole doesn't the other poles.
2) Find A_m.
3) Find F_H.

1) We need to be a factor of 5 away,

$$\therefore \omega_{pnew} = \tfrac{1}{5}\,\omega_{Lold} \quad \longrightarrow \quad 754 = \frac{1}{C_4 R_{eq}}$$

$$= \frac{2\pi}{5} f_{Lold}$$

$$= 754 \text{ rad/s}$$

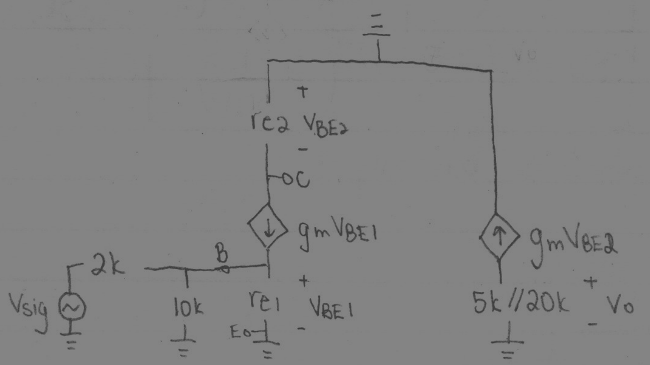
But what is R_{eq}? Obviously there's the 18k//2k. But then we have to think about the resistance looking into the base.

Current flows from B2 → E2 → C1 → E1.

First, we encounter $r_{e2} = V_T/I_c = 25\Omega$. Then, from C1 → E1 we have $r_{o1} = V_A/I_c = 50k\Omega$, but this is multiplied by $(\beta+1)$ as it has that much more current.

$$\therefore R_{eq} = 18k // 2k // (25 + (\beta+1)50k)$$
$$= 1.8k$$
$$\therefore C_4 = 737 \text{ nF}$$

2) A_m: coupling = short
parasitic = open

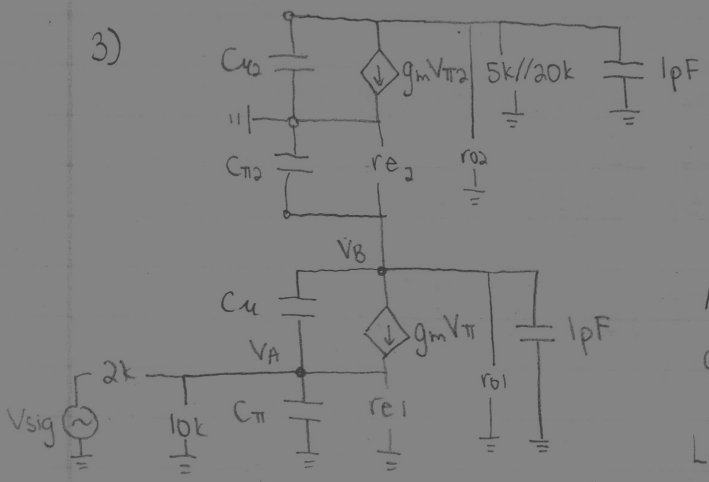


$$V_{BE1} = \left(\frac{10k // (\beta+1)r_{e1}}{2k + 10k // (\beta+1)r_{e1}}\right) V_{sig}$$
$$= \left(\frac{2016}{2k + 2016}\right) V_{sig}$$
$$= 0.5 V_{sig}$$

$$\therefore g_m V_{BE1} = 20.1m\, V_{sig}$$
$$V_{BE2} = (20.1m) V_{sig} (r_{e2})$$
$$= 0.502 V_{sig}$$
$$\therefore g_m V_{BE2} = 0.02 V_{sig}$$
$$V_o = 80 \text{ V/V, ignoring } r_o$$

So we can conclude that things are essentially the same: gain changes are negligible.

3)



This is gross, not going to lie. HOWEVER! By the all powerful Miller theorem, we can separate C_μ.

Also, note that $C_{\mu2}$ is // with Vo's 1pF and $C_{\pi2}$ is // with V_B's 1pF.

Let's combine them.

What is k? Notice how $V_A = V_\pi$. So it's just a matter of expressing V_B in terms of V_π.

V_B is $g_{m1}V_{\pi1}$, the current, through the INPUT RESISTANCE OF A CASCODE AMPLIFIER, in parallel with r_{o1}.

$$V_B = -g_{m1}V_{\pi1}\left[r_\pi \,//\, \left(\frac{r_o + R_L}{1+g_m r_o}\right)\right] // r_{o1}$$

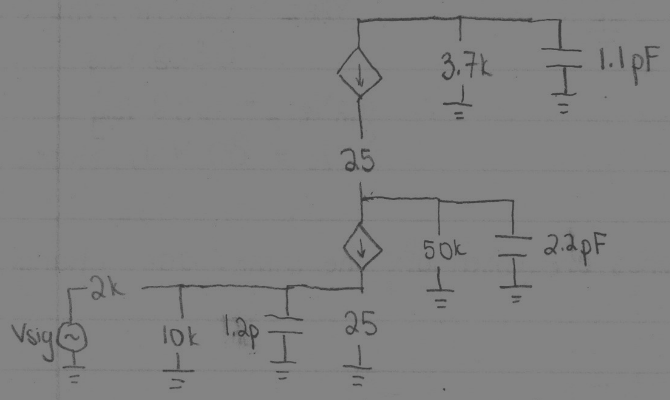
(by the equation sheet)

$$\frac{V_B}{V_\pi} = -(40m)\left[50k \,//\, \left(2.5k \,//\, \left(\frac{50k + (5k//20k)}{1+(40m)(50k)}\right)\right)\right]$$

$$= -(40m)\left[50k \,//\, \left(2.5k \,//\, \left(\frac{54k}{2001}\right)\right)\right]$$

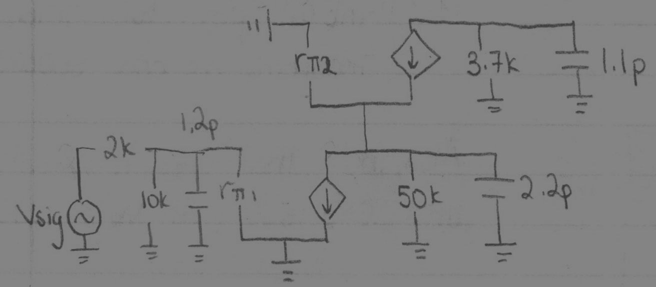
$= -1.07$, which is basically 1.

Let's redraw with our values.



And now we can finally run open-circuit time constant analysis.

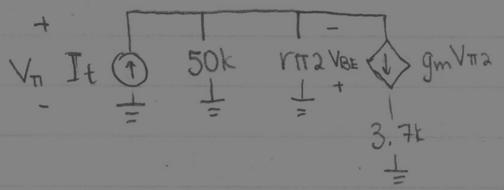
But looking at it, T-model might be difficult here.

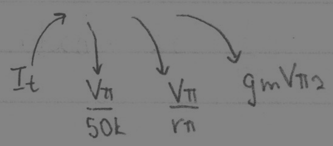


$1.1pF$: $R_{eq} = 3.7k \,//\, R_{outcascode}$
$= 3.7k \,//\, \text{large}$
$= 3.7k$

$$\omega_{1.1pF} = \frac{1}{(3.7k)(1.1pF)} = 2.46 \times 10^{8} \text{ rads/s}$$

$2.2pF$: $V_{sig} = 0 \;\therefore\; g_m V_{BE1} = 0$





$$I_t = \frac{V_\pi}{50k} + \frac{V_\pi}{2.5k} + g_m V_\pi$$

$$I_t = V_T \left(50k + 2.5k + \frac{1}{g_m}\right)^{-1}$$

$$R_{eq} = 50k \,//\, 2.5k \,//\, 25 \longrightarrow \omega_{2.2pF} = \frac{1}{(25)(2.2p)} = 1.82 \times 10^{10} \text{ rad/s}$$
$= 25$

$1.2pF$: $R_{eq} = r_\pi \,//\, 2k \,//\, 10k \longrightarrow \omega_{1.2pF} = \frac{1}{(1k)(1.2p)} = 8.33 \times 10^{8} \text{ rad/s.}$
$= 1000$

Since $\frac{\omega_{1.2}}{\omega_{1.1}} < 5$, $\omega_H = 10.79 \times 10^{8} \text{ rad/s} = \omega_{1.2} + \omega_{1.1}$
$f_H = 1.72 \times 10^{8}$ Hz