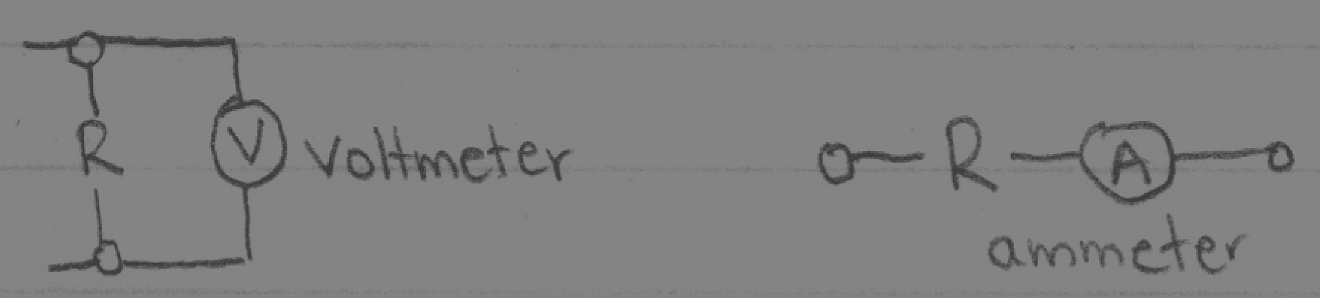
Nairn and Chan: Feedback Systems

Sampling and Mixing

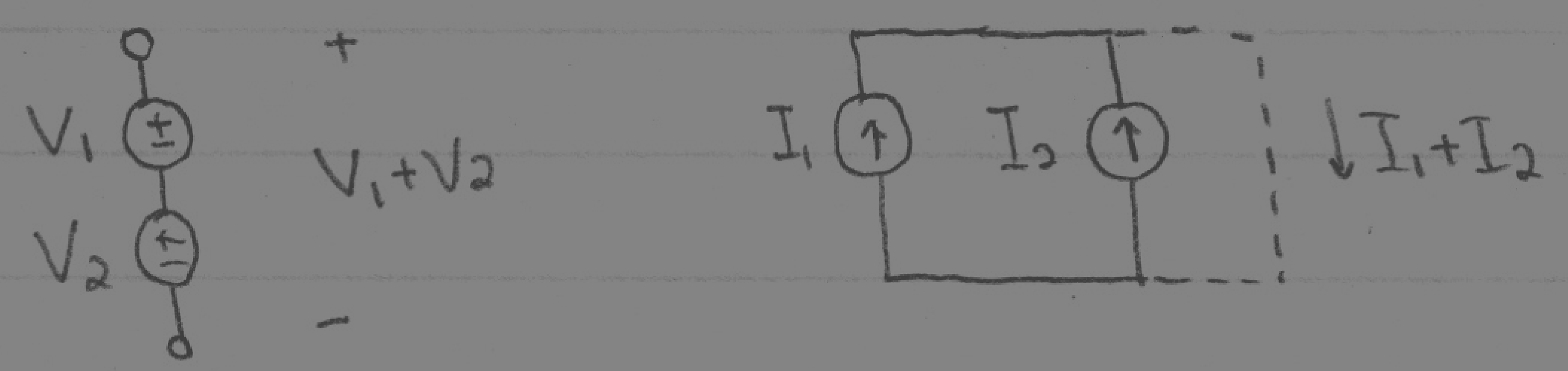
Sampling and mixing are two synonyms for measuring an output and providing an input.

If we recall from way back: SAMPLING



Measuring a voltage is done in "shunt", a fancy word for parallel. Measuring a current is done in series. Remember that these are determined by the number of paths current can travel (multiple = shunt, single = series).

If we recall: MIXING



Combining voltages requires us to put them in series. Combining currents requires us to put them in shunt.

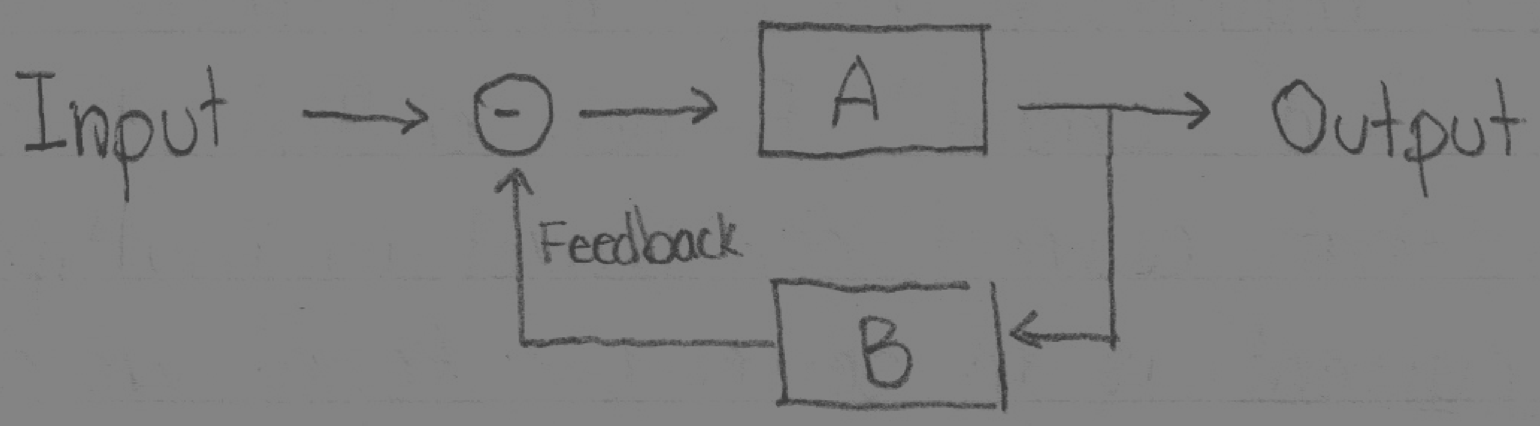
Input ↗ mix series (voltage)
↘ mix shunt (current)

Output ↗ sample series (current)
↘ sample shunt (voltage)

An Introduction to Feedback

At its core, feedback is a control system. There exists positive and negative feedback, though we'll only focus on negative in this course.

Algorithmically, negative feedback works by
1) SAMPLING the output of interest
2) MIXING a percentage of the sample with our original input.



The 'A' system is our regular amplifier system. The 'B' system samples some aspect of the output, and mixes it with our input. ALL AMPLIFIERS THAT USE FEEDBACK CAN BE MODELLED AS THIS SYSTEM.

Taking I = Input
F = Feedback
O = Output
A = forward gain
B = feedback factor

$$O = (I - F)A$$
$$O = (I - BO)A$$
$$O = AI - ABO$$
$$O + AB(O) = A(I)$$
$$(1 + AB)\,O = (A)\,I$$
$$\frac{O}{I} = \frac{A}{1+AB}$$

And as such the gain of the system can be represented by $\frac{A}{1+AB}$, where $A \gg 1$, usually from active components
$B < 1$, usually from passive components

Note that as $A \to \infty$, the gain approaches $1/B$

$$\left.\frac{O}{I}\right|_{A\to\infty} = \frac{\infty}{1+\infty B} \div \frac{\infty}{\infty B} = \frac{1}{B}$$

So theoretically, we can create a system with some gain X by constructing the feedback network such that

$$X = \frac{1}{B}$$

Feedback on Gain Variation

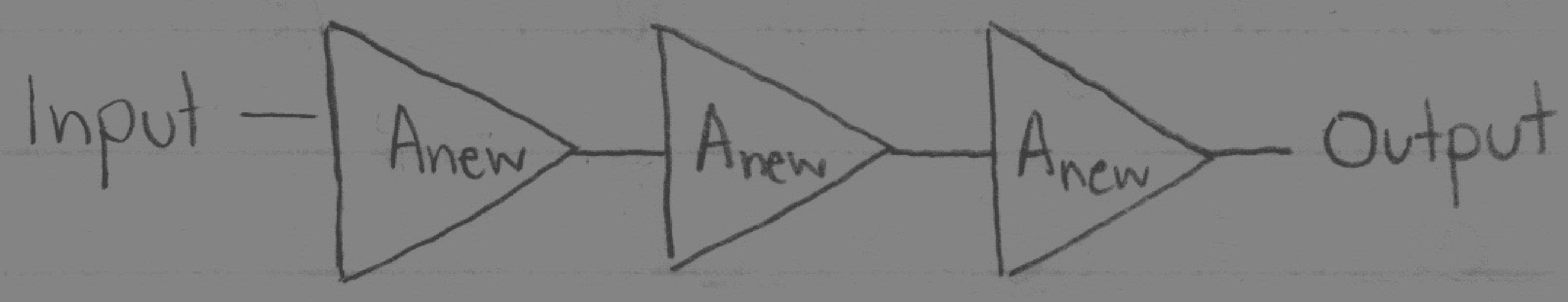
Say we wanted to make an amplifier with a gain of 1000, but in some mistake in production, we're left with amplifiers with gain [100, 1000]. This leaves us with amplifiers that have a 90% error, worst case.

So we run around for a while, and manage to scrounge up parts to build perfect feedback networks of exactly $B = 1/10$. Attaching them to our amplifiers:

$$\left.\frac{O}{I}\right|_{best} = \frac{1000}{1+(1000)(1/10)} = 9.9$$

$$\left.\frac{O}{I}\right|_{worst} = \frac{100}{1+100(1/10)} = 9.1$$

Which now means our amplifiers have an error of 1% - 9%, which is much better than our original amplifiers. We've lost out on the high gain, though. To rectify that, we can attach them in cascade.



$$\left.\frac{O}{I}\right|_{best} = 9.9 \times 9.9 \times 9.9 = 970 \rightarrow 3\% \text{ error}$$

$$\left.\frac{O}{I}\right|_{worst} = 9.1 \times 9.1 \times 9.1 = 754 \longrightarrow 24.6\% \text{ error}$$

Which gives us a much better error range than we had at the start, at the cost of using three amplifiers and the feedback networks.

Feedback Configurations and Amplifier Types

Obviously, since we only have 2 methods of sampling / mixing, there are a total of 4 feedback amplifier types:

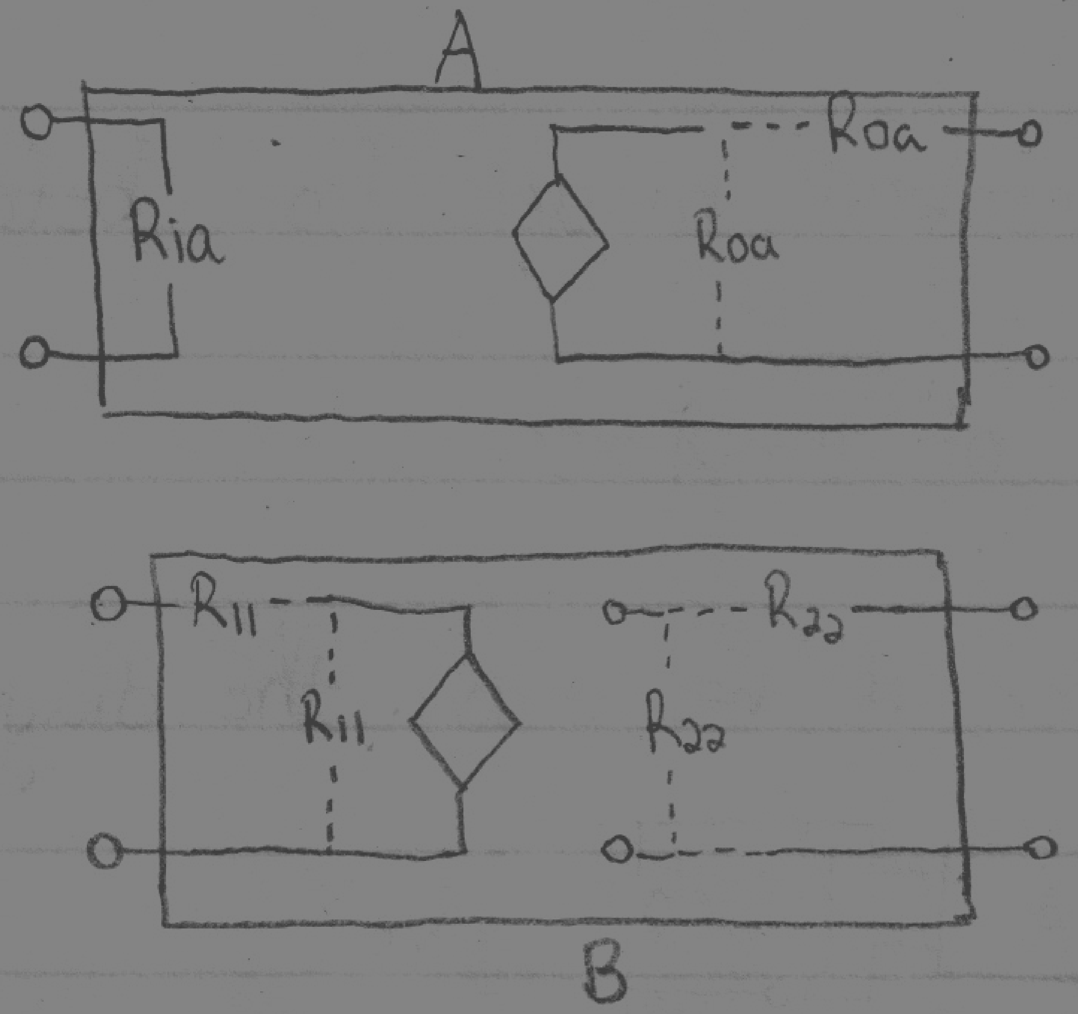
1) $A = V_o/V_i$, the voltage amplifier [Series-Shunt]
   - mixes in series (V)
   - samples in shunt (V)

2) $G = I_o/V_i$, the transconductance amplifier [Series-Series]
   - mixes in series (V)
   - samples in series (I)

3) $A = I_o/I_i$, the current amplifier [Shunt-Series]
   - mixes in shunt (I)
   - samples in series (I)

4) $R = V_o/I_i$, the transresistance amplifier [Shunt-Shunt]
- mixes in shunt (I)
- samples in shunt (V)

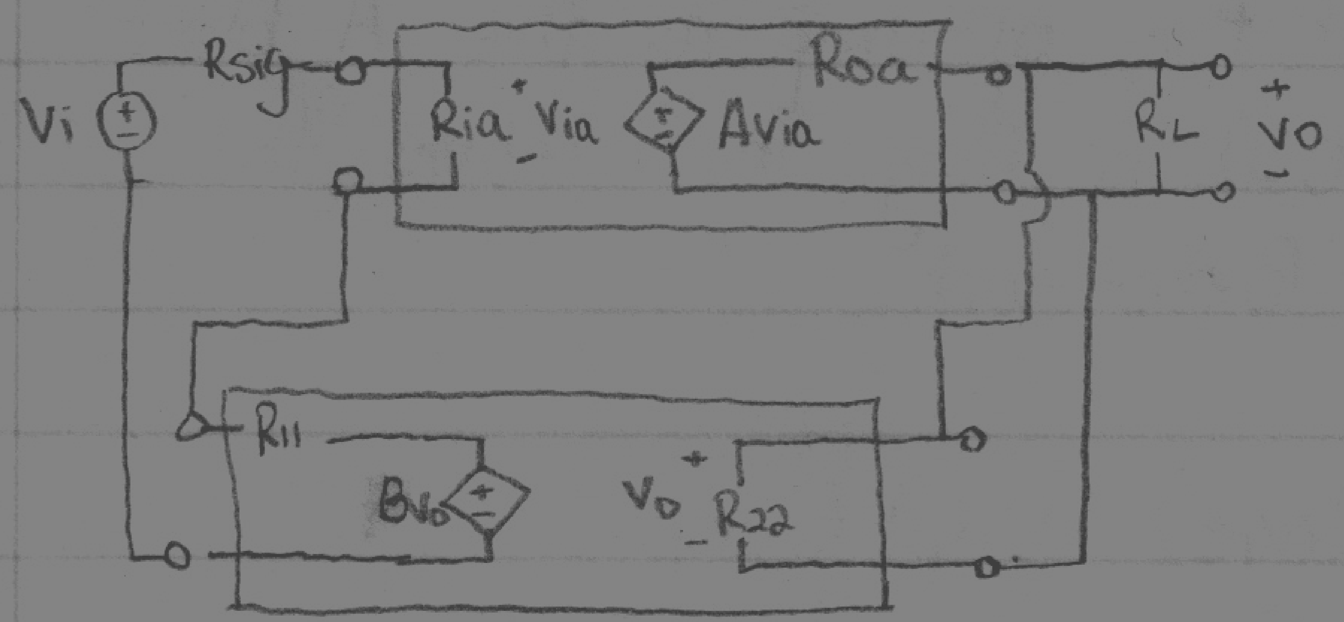
And all four can be modelled as:



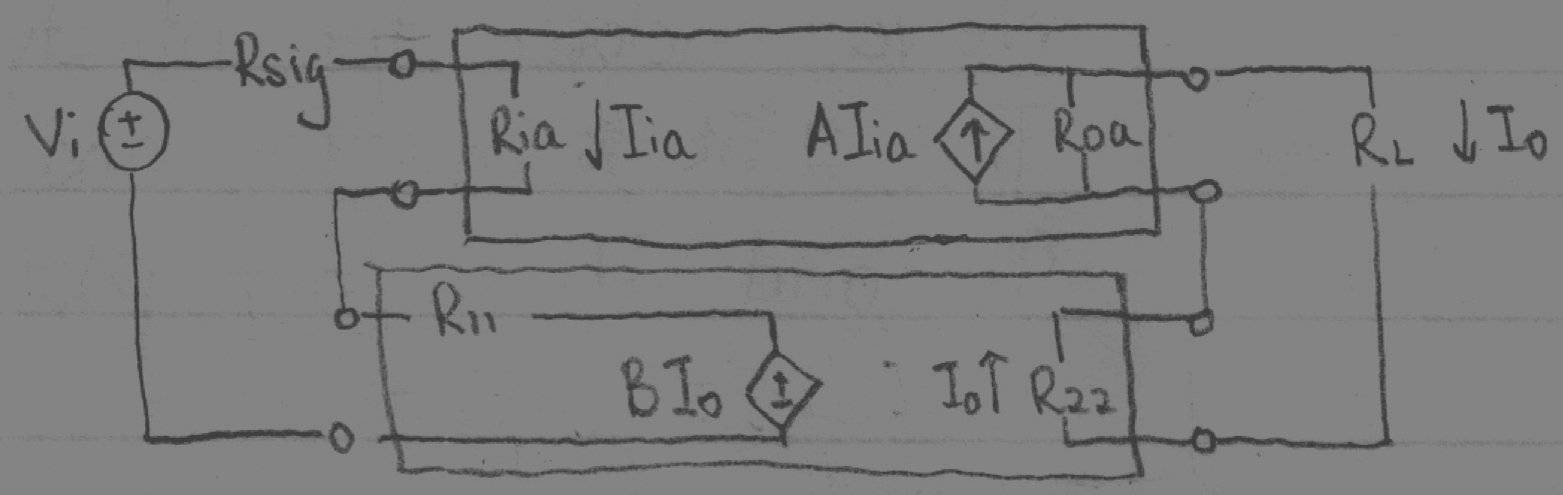
a 2-part 2-port system. Note that both A and B blocks are Thevenin/Norton equivalents of their respective parts.

A's dependent source is based on the SAMPLING method, while B's dependent source is based on the MIXING method.

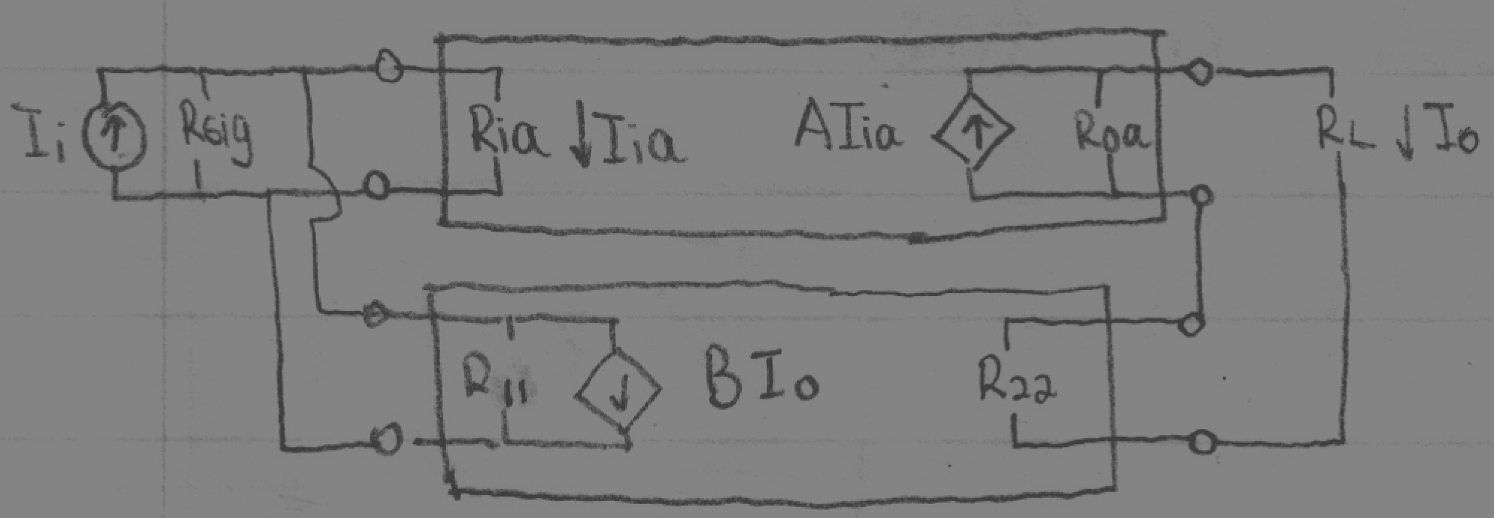
Series-Shunt



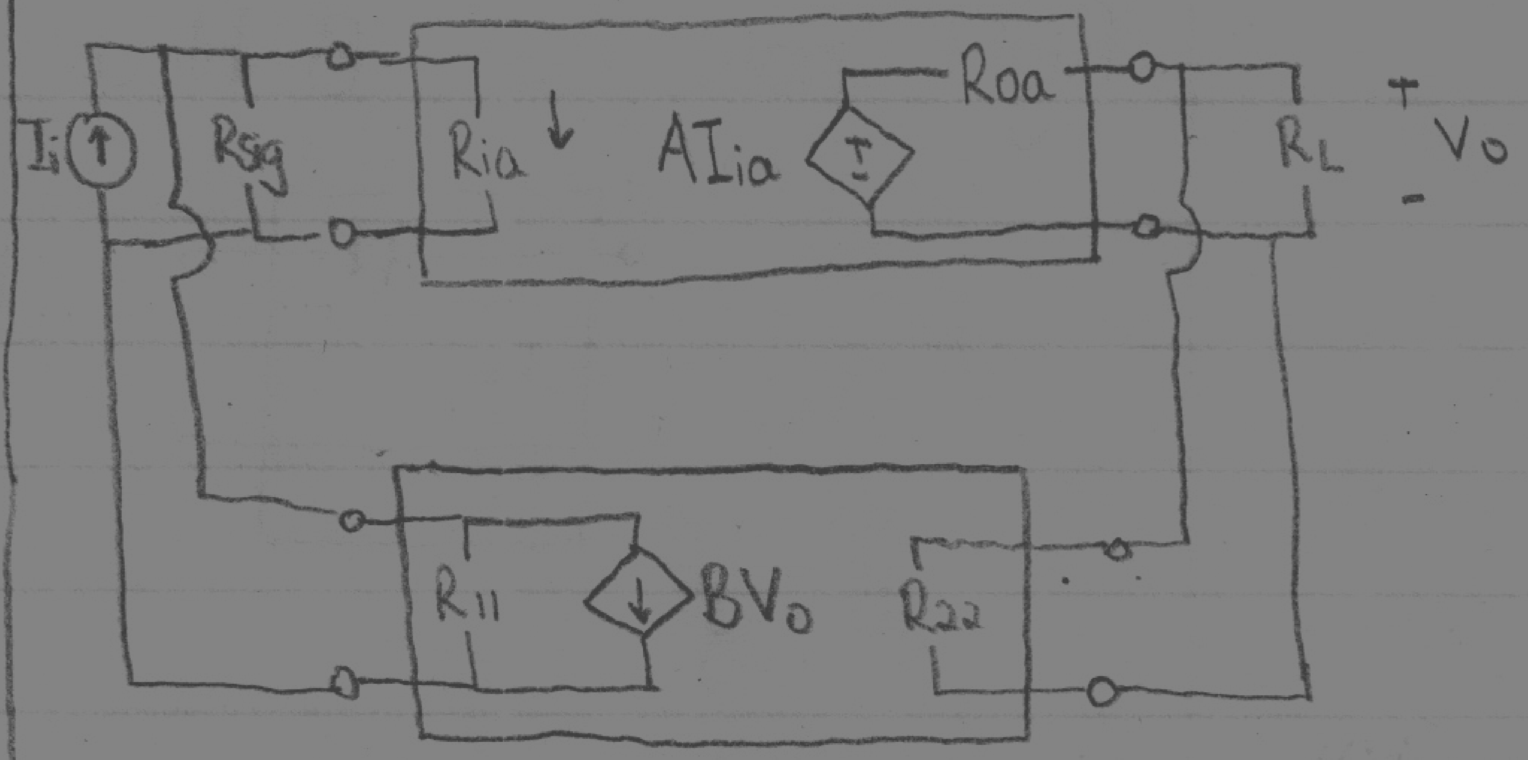
Series-Series



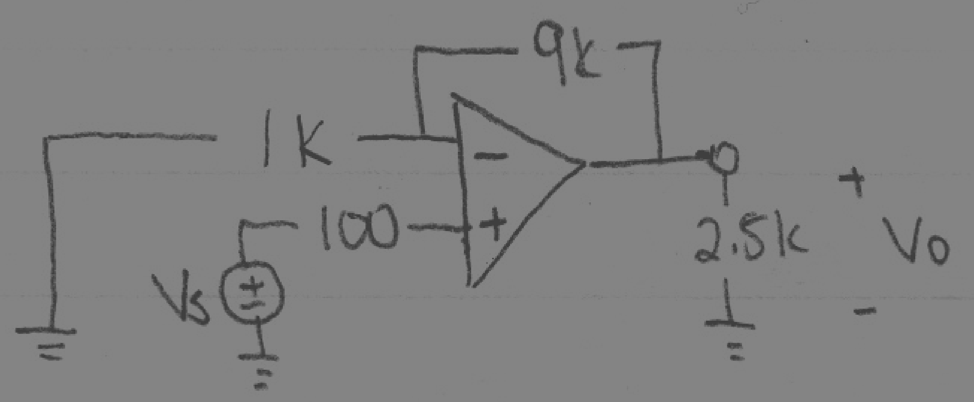
Shunt-Series



Shunt-Shunt



Let's do a simple example. There's a fairly simple step-by-step process for feedback analysis.

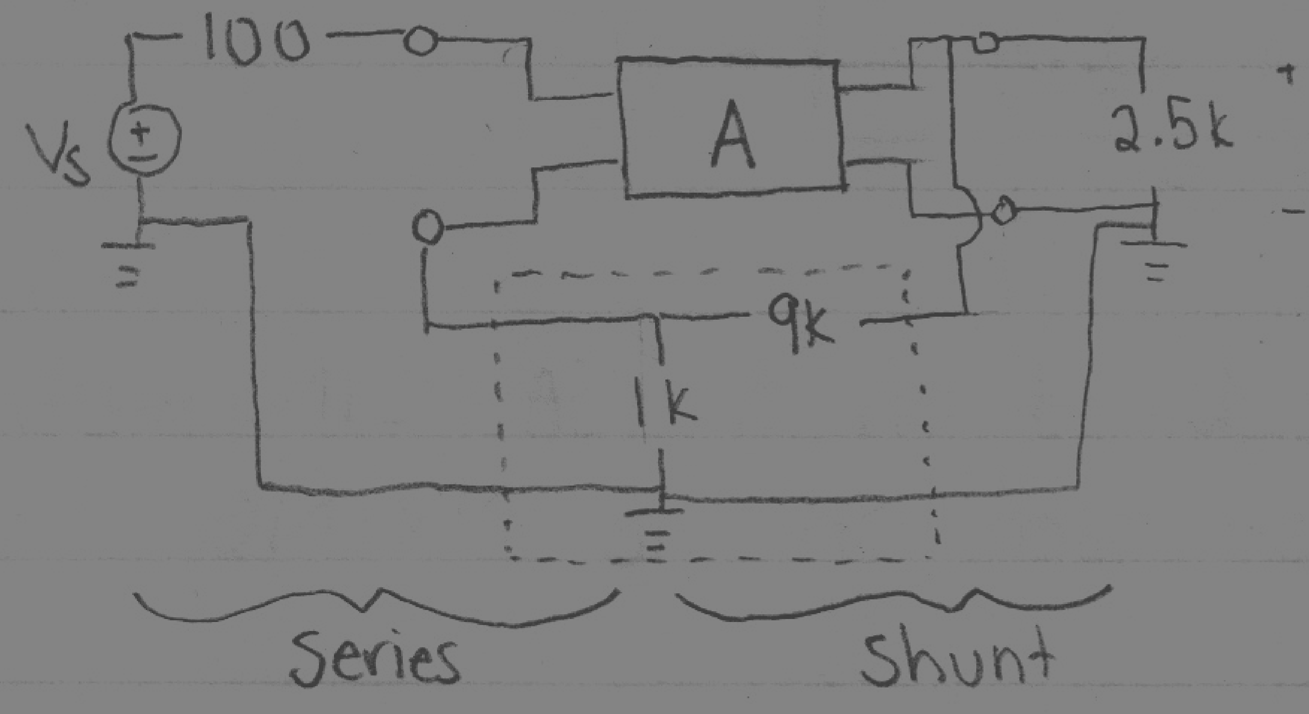
ex. 

We are given that:
$R_{ia} = 4k$ $A = 100\,000$
$R_{oa} = 2k$

Model this using the feedback models from before.

1) Determine the type of amplifier.

This is a little clearer if we draw the diagram differently.

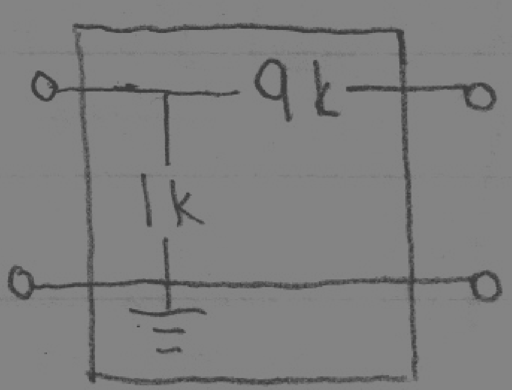


We've enclosed the entirety of the op-amp as the A block. The 2.5k is our R_L, and $100\,\Omega$ is R_{sig}. That leaves the 9k and 1k as our passive β network.

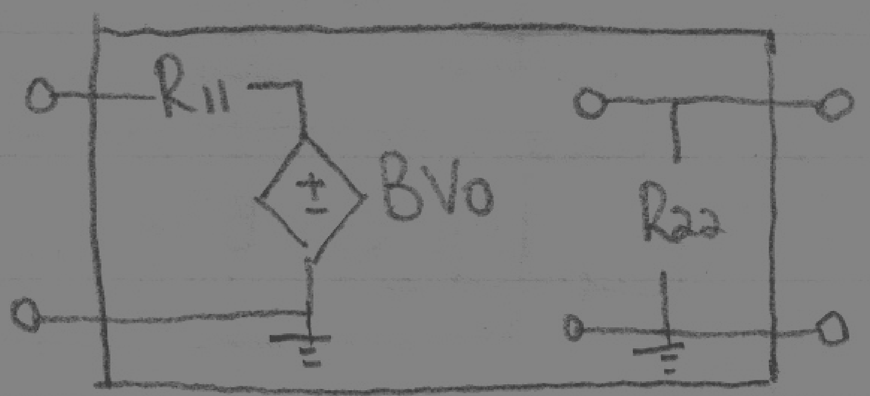
This is a series-shunt (voltage) amplifier.

2) Construct the β block.

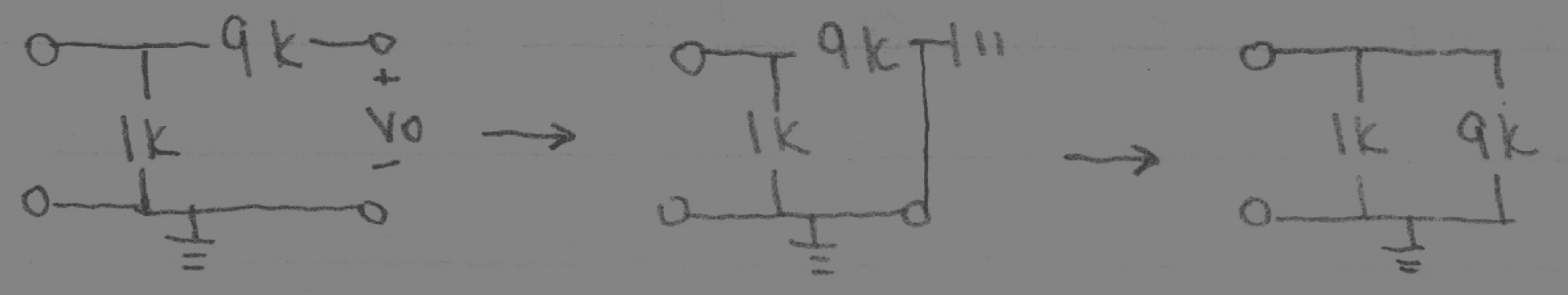
Right now, our β block looks like this.



but we want

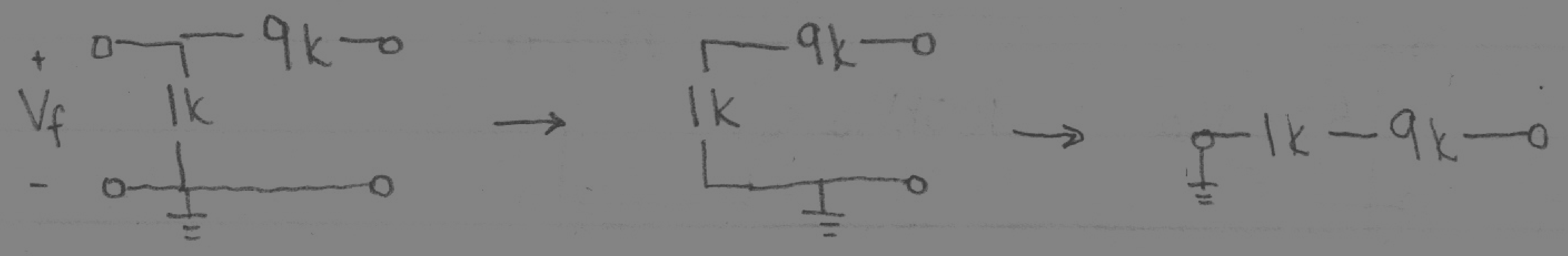


For R_{11}: we set [what's being sampled] to 0, then find the equivalent resistance looking into the terminals on the left.



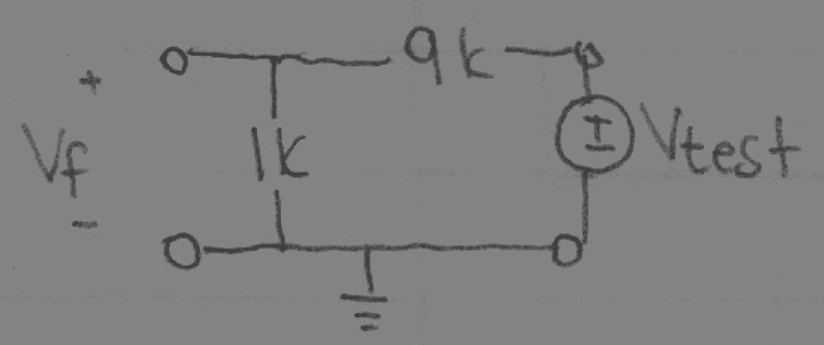
$\therefore R_{11} = 1k \,//\, 9k = 900$

For R_{22}: we set [what's being mixed] to ∞ or maximum, then find the equivalent resistance looking into the terminals on the right.



$\therefore R_{22} = 9k + 1k = 10k$

For B: Apply a test source for what we'd sample. Determine the output of the mixed end.

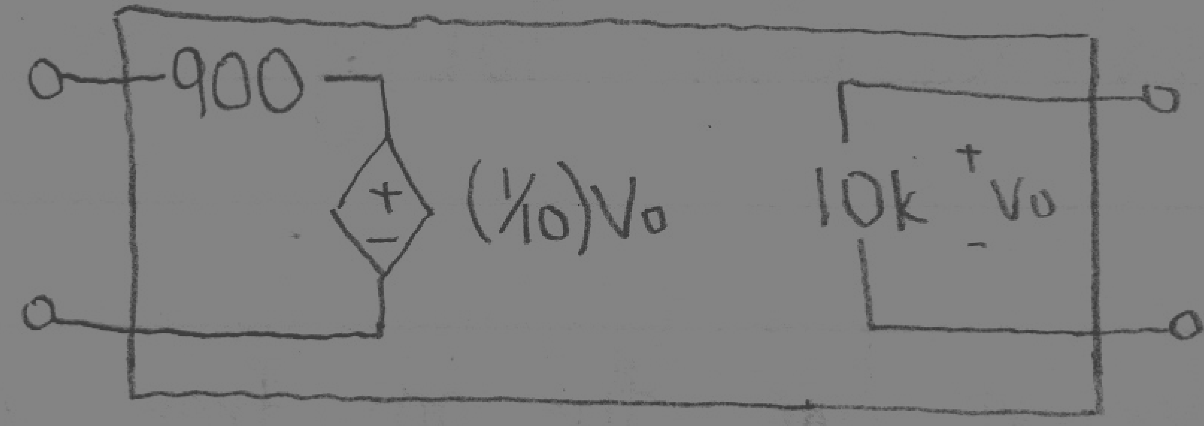


$$V_f = \left(\frac{1k}{1k+9k}\right) V_o$$

$$\frac{V_f}{V_o} = \frac{1}{10}$$

$\therefore B = 1/10$

Now we have our whole B block:



3) Construct the A block.

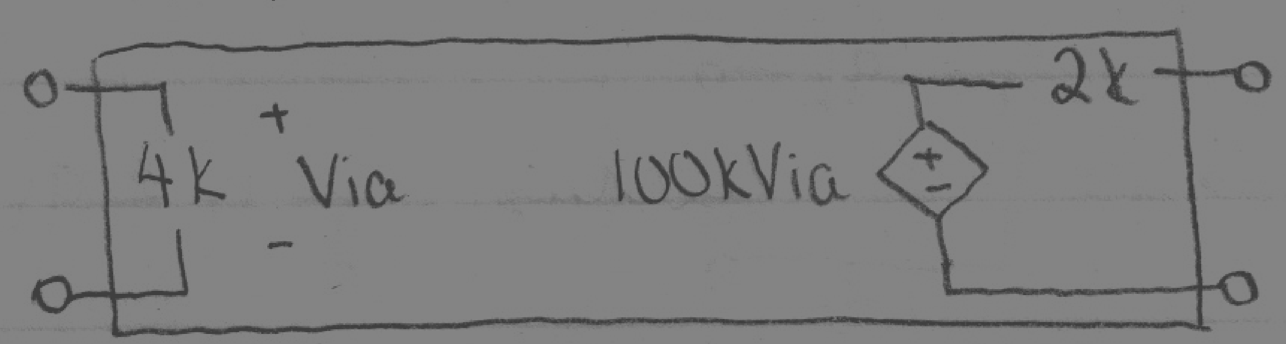
In this particular question it's somewhat simpler because the gain, input resistance, and output resistance has already been provided.

If it hadn't been, we would

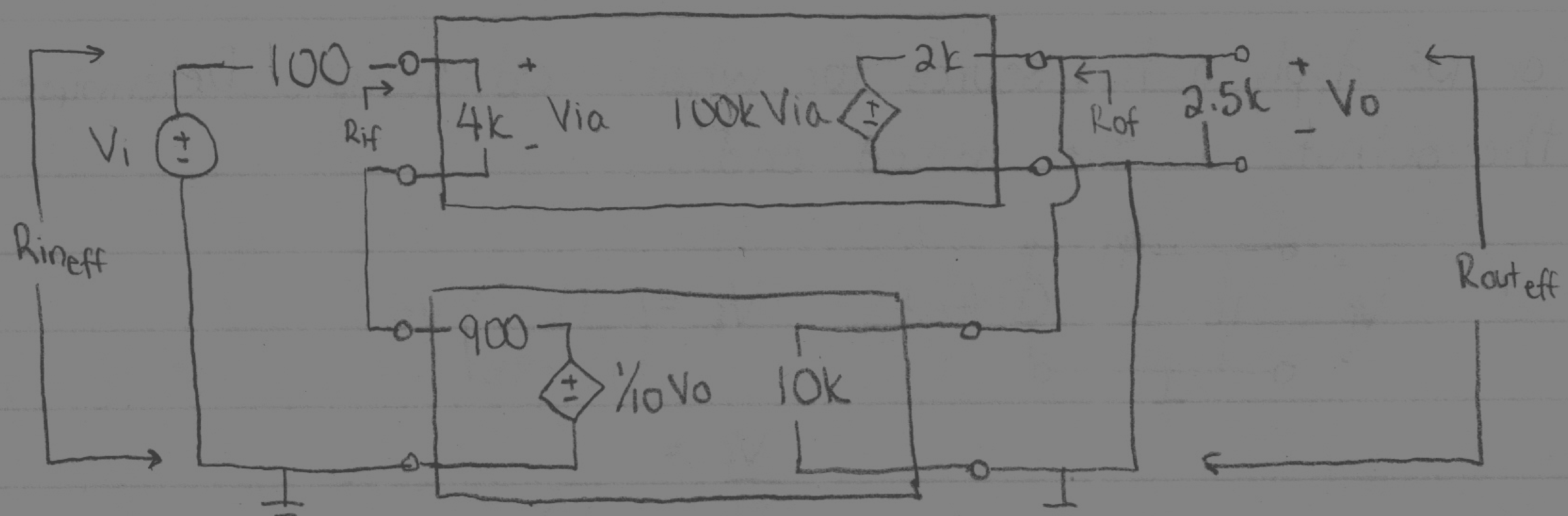
a) disconnect the feedback network (B) from the circuit

b) Solve for A, R_{ia} R_{oa} normally

But we use what we have, so A:



4) Build entire feedback model.



5) Determine new gain (A_f), new input resistance (R_{if}) and new output resistance (R_{of}) taking into account the feedback.

These are calculated using A_{eff}, the gain from V_i to V_o with the feedback model, R_{ieff}, the input resistance INCLUDING R_{sig}, and R_{oeff}, the output resistance INCLUDING R_L.

A_{eff} : For $V_i \rightarrow V_{ia}$, it is multiplied by the input divider. This is then gained by the amplifier's intrinsic gain A. Finally, the output voltage is given as another voltage divider.

$$A_{eff} = \underbrace{\left(\frac{4k}{100+900+4k}\right)}_{\text{input divider}} 100k \underbrace{\left(\frac{2.5k // 10k}{2.5k // 10k + 2k}\right)}_{\text{output divider}}$$

$$= 40\,000 \text{ V/V}$$

R_{ieff} : The resistances are in series, so we just add them up.

$$R_{ieff} = 100 + 4k + 900$$
$$= 5k$$

R_{oeff} : Cutting independent sources means $V_i = 0 \rightarrow V_{ia} = 0 \rightarrow$ the dependent source provides 0V, or is just a wire. As such the resistances are in parallel.

$$R_{oeff} = 2k // 2.5k // 10k$$
$$= 1k$$

A_f : This is simply the gain formula used at the start.

$$A_f = \frac{A_{eff}}{1 + A_{eff} B}$$
$$= \frac{40000}{1 + 40000(1/10)}$$
$$= 9.9975 \text{ V/V}$$

R_{if} and R_{of} : When put into the feedback configuration, R_{if}/R_{of} follow the same formula to determine the entire system's resistance.

$$R_{(i/o)f} = R_{(i/o)eff}\,(1 + A_{eff}B)^{\pm 1}$$

Whether we multiply or divide by the 1+AB factor is dependent on the configuration of our feedback.

R_{if} : Since R_{ieff}'s resistances are in series, this INCREASED our input resistance.

removing something from a series connection

$$R_{if} = R_{ieff}(1 + A_{eff}B) - R_{sig}$$
$$= (5k)(4001) - 100$$
$$= 20.0049\ M$$

R_{of} : Since R_{oeff}'s resistances are in parallel, this DECREASED our output resistance

$$R_{of\,(intermediate)} = \frac{R_{oeff}}{1 + A_{eff}B}$$
$$= \frac{1k}{4001}$$
$$= 0.25\,\Omega$$

removing something from a parallel connection

$$R_{of} = \frac{R_{of(inter)}\,R_L}{R_L - R_{of(inter)}}$$
$$= \frac{(0.25)(2.5k)}{(2.5k) - (0.25)}$$
$$\approx 0.25\,\Omega$$

So now we can determine exactly what happens to an amplifiers A, Rin, Rout when adding a known feedback network in a known configuration.

Quick Summary of Feedback Analysis

① Determine the type of feedback

- series - shunt
- series - series
- shunt - series
- shunt - shunt

② Construct the B block

- identify feedback network
- find R_{11}
  - set [what we're sampling] to 0
  - find Req looking into left
- find R_{22}
  - set [what we're mixing] to ∞/maximum
  - find Req looking into right
- find B
  - apply test source of [what we're sampling] on the right, find resultant of [what we're mixing] on the left

③ Construct the A Block

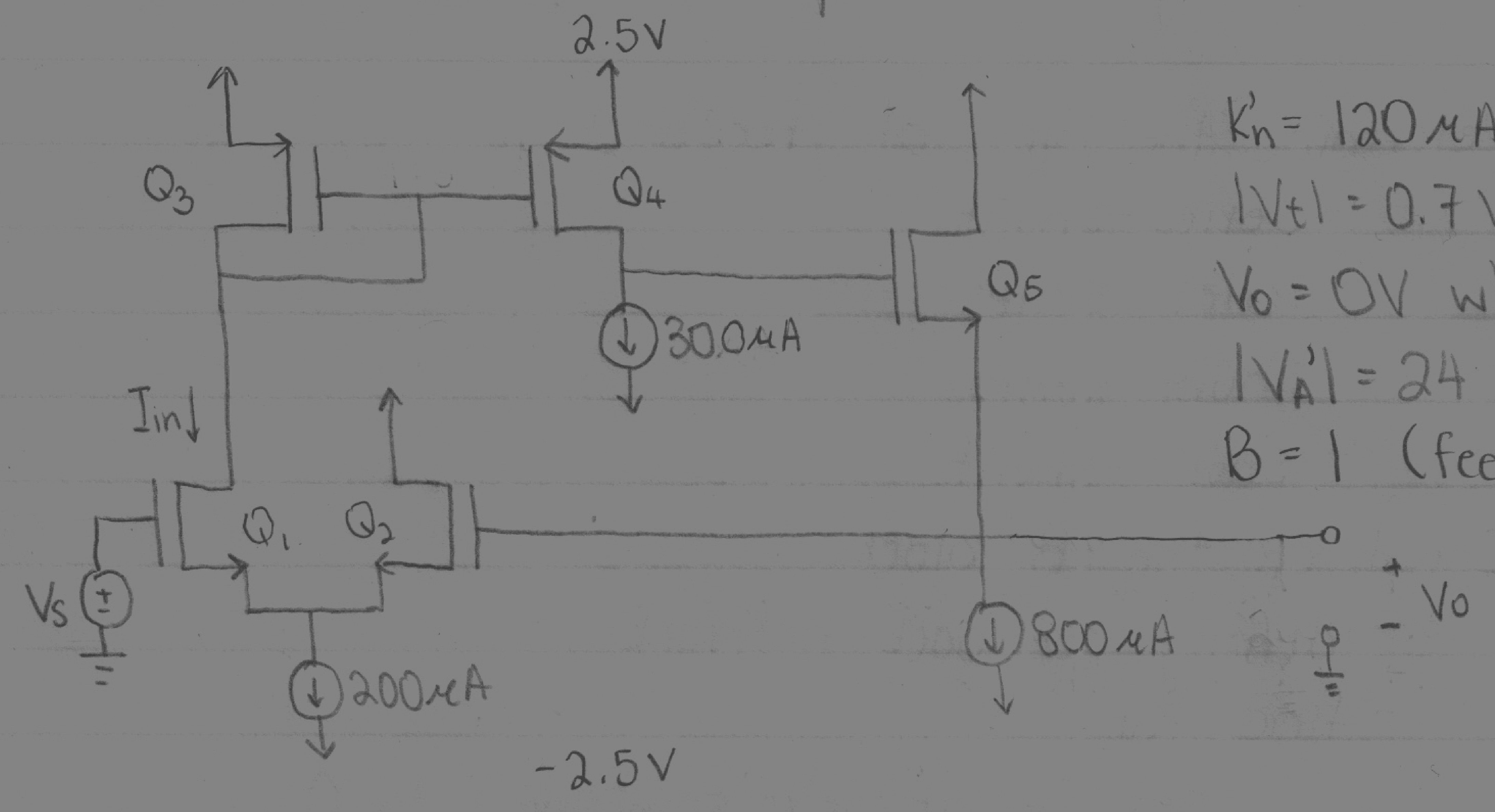
- find A, Rin, Rout of amplifier w/o feedback
- find R_{ieff} and R_{oeff} based on amplifier type
- find A_{eff} using input and output dividers
- $A_f = \dfrac{A_{eff}}{1 + A_{eff} B}$
- find $R_{if\,(inter)}$ / $R_{of\,(inter)}$ through multiplying/dividing by $(1 + A_{eff} B)$
- remove R_{sig} from $R_{if\,(inter)}$ and R_L from $R_{of\,(inter)}$

Series: $R_{inter} - R_x$

Parallel: $\dfrac{R_{inter} R_x}{R_x - R_{inter}}$

Some questions may ask for AB, the loop gain.

A More Difficult Feedback Example



$K_n' = 120\,\mu A/V^2 = 2K_p'$
$|V_t| = 0.7\,V$
$V_O = 0V$ when $V_S = 0V$
$|V_A'| = 24\ V/\mu m$
$B = 1$ (feedback factor)

Let's run an entire feedback analysis on this. Additionally, we are given that the (W/L) are:

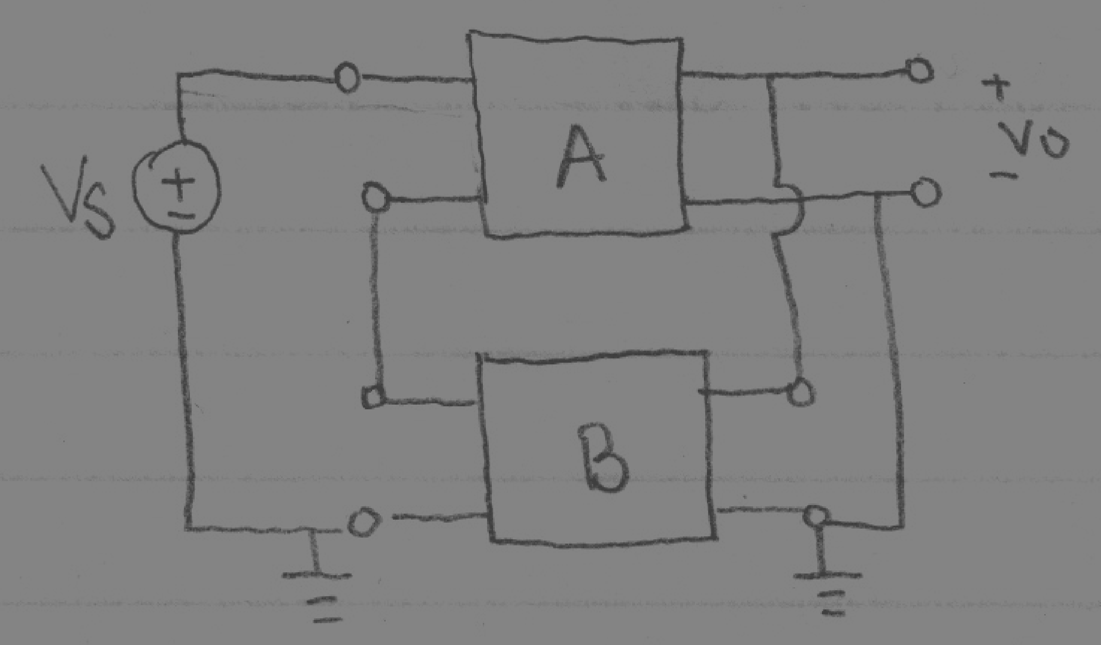
$Q_1 = Q_2 = Q_5 = 20$ $\qquad Q_3 = 40$ $\qquad Q_4 = 120$

Before we go into any quantitative analysis, let's take a look at the big picture. Legolas! What do your elf-eyes see?

- differential input biased at $200\,\mu A$ (Q_1, Q_2)
  - this means each side gets $100\,\mu A$ if voltage is balanced
- current mirror that takes that multiplies current by 3 (Q_3, Q_4)
- voltage follower (common-drain configuration) (Q_5)
- output voltage is being sampled (as no current goes into the gate)
- Since I_{in}, the current through Q_1/Q_3, is affected by the mismatched voltage coming from V_O, we are also mixing voltages ($\therefore$ mixing in series)

1) Determine amplifier type.

This is a series-shunt amplifier.

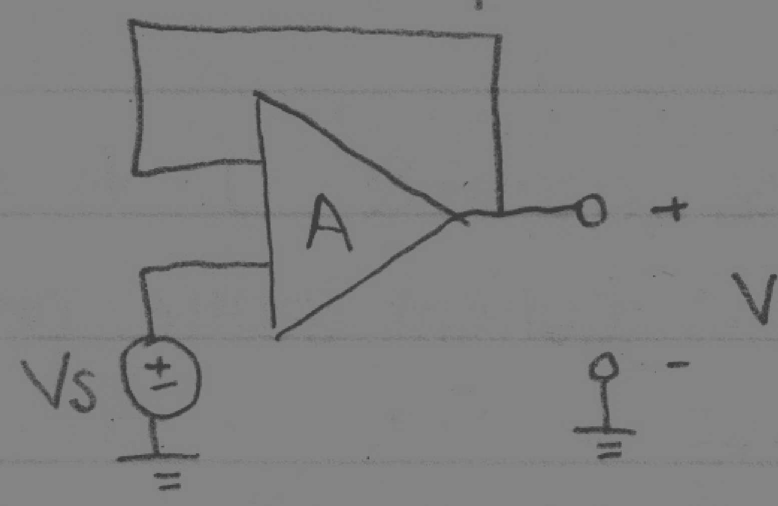


This is our tentative feedback model. Note that this time there is no R_{sig} or R_L.

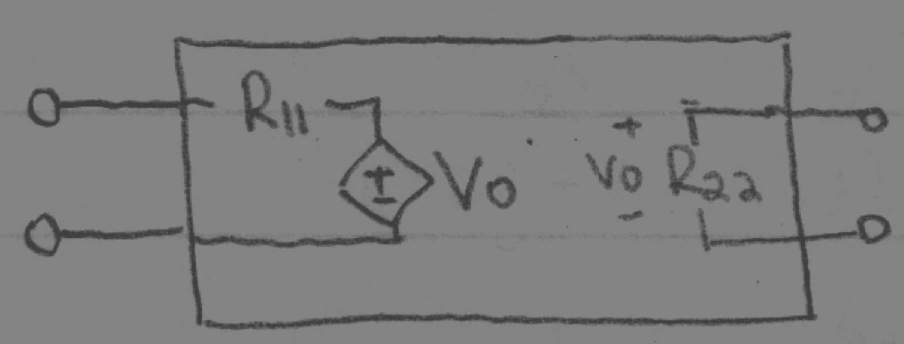
2) Construct B block.

This is where things get more difficult. We know $B=1$, but it's fairly difficult to see exactly what the feedback network is. In these scenarios, it is rather useful to abstract the entire amplifier itself as an unknown block.

We have: 2 inputs
1 output, which is connected directly to one of the inputs.

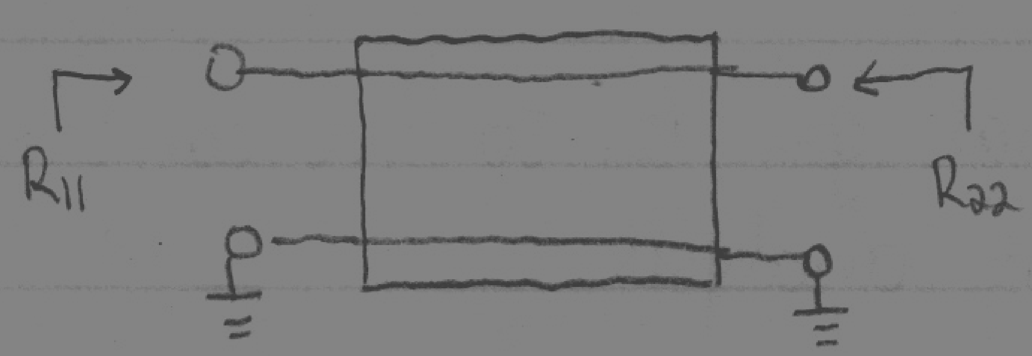


Oddly enough, there isn't actually a B-block. But perhaps that's not quite the right description. We have a B block that is simply a voltage follower of the output.



So what are R_{11} and R_{22}?

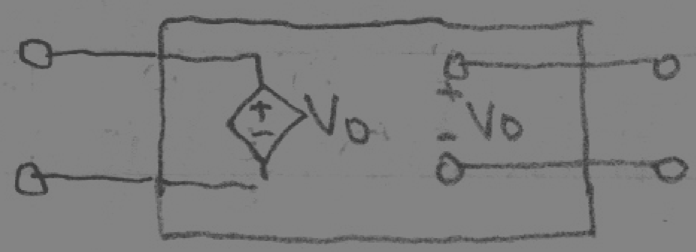
If we follow our normal method:



R_{11}: set V_o to 0V, $R_{eq} = 0\,\Omega$

R_{22}: set left to maximum (open), $R_{eq} = \infty$

and as such, our B block is now simply



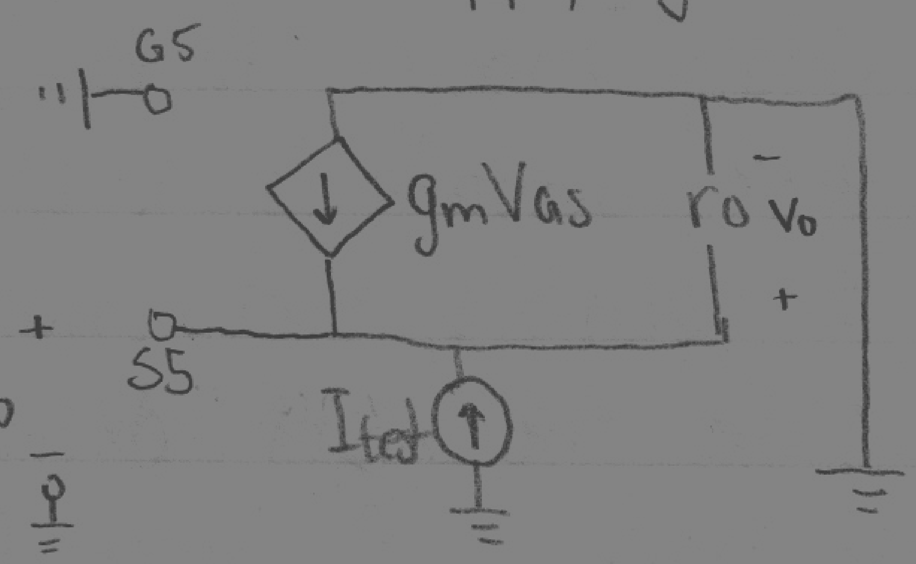
3) Construct A block.

$R_{in} = R_{ia} = \infty$, as current can't enter the gate.

For R_{out}, we have three paths:

- into gate of Q_2 $[R = \infty]$
- up through the source of Q_5

Remember that going down through the supply isn't a valid path, because in small signal, our test source injects current directly into the source node, REPLACING where the supply ground would have been.



V_{GS} becomes $0 - V_o = -V_o$

$\therefore$ current source provides $-g_m V_o$

$$V_o = (I_{test} + (-g_m V_o)) r_o$$

$$V_o = I_{test} r_o - g_m V_o r_o$$

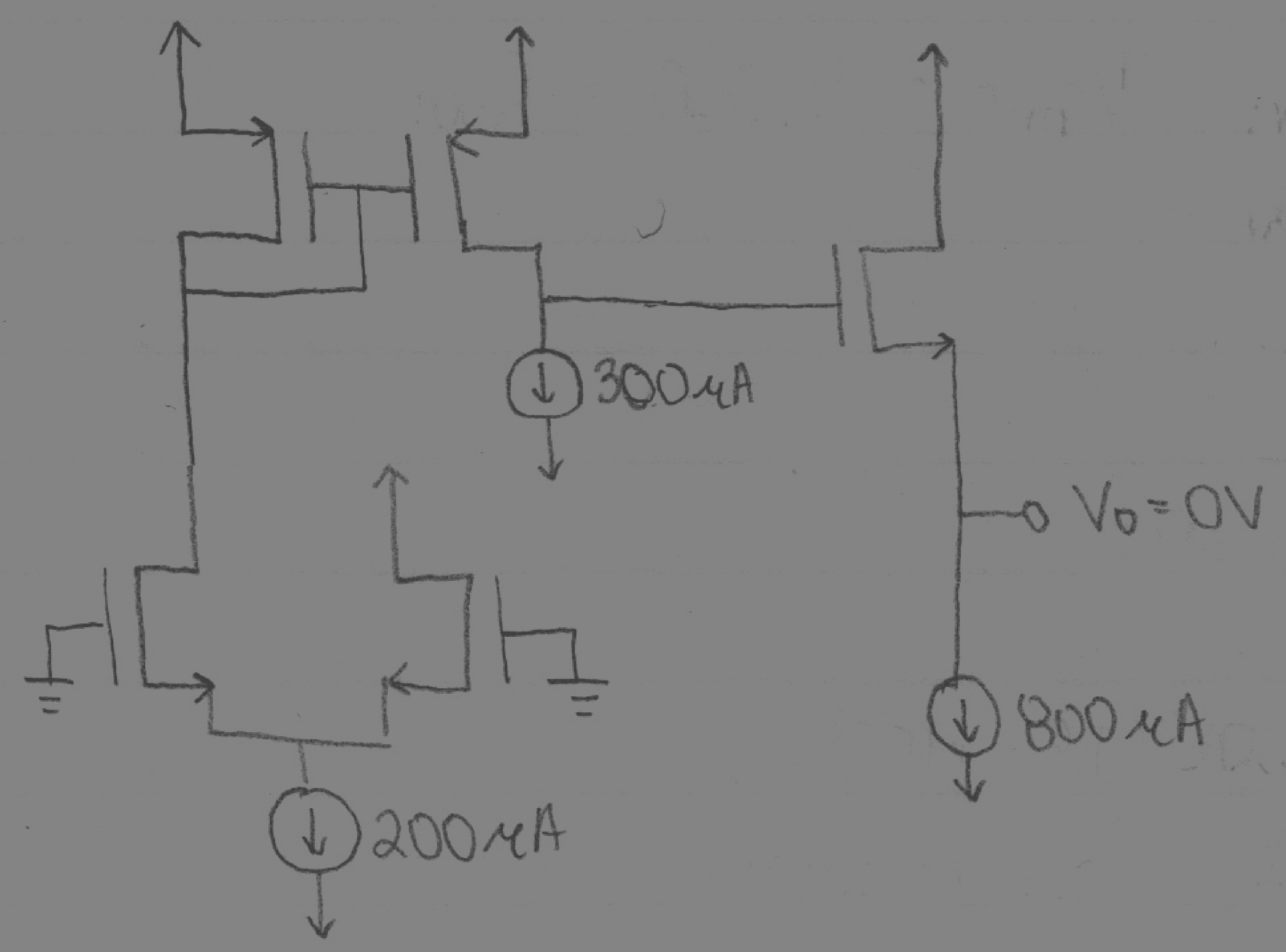
$$V_o(1 + g_m r_o) = I_{test} r_o$$

$$\frac{V_o}{I_{test}} = \frac{r_o}{1 + g_m r_o}$$

$$R_{out} = \frac{1}{1/r_o + g_m} \approx \frac{1}{g_m}$$

But we don't actually know what g_m is, so we can find that from a DC operating point analysis.

We know when $V_s = 0\,V \rightarrow V_o = 0V$.



Current splits equally across Q_1 and Q_2 because they're balanced.

Q_4 gets 3x Q_3's current, incidentally, 300 µA.

Q_5 is biased by the 800µA source.

If we assume the circuit has been built in such a way that all MOSFETs are in saturation, we can easily determine their bias points.

$$I_{DS} = (1/2)(k'_{np})(W/L)\,V_{ov}^2$$

$$\sqrt{\frac{2I_{DS}}{(k'_{np})(W/L)}} = V_{ov} \qquad r_o = \frac{V_A}{I_D}$$

$$\therefore\ V_{ov(1,2)} = \sqrt{\frac{2(100\mu)}{(120\mu)(20)}} = 0.289\text{ V} \qquad r_{o1,2} = \frac{24}{100\mu} = 240\text{k}$$

$$V_{ov(3)} = \sqrt{\frac{2(100\mu)}{(60\mu)(40)}} = 0.289\text{ V} \qquad r_{o3} = \frac{24}{100\mu} = 240\text{k}$$

$$V_{ov(4)} = \sqrt{\frac{2(300\mu)}{(60\mu)(120)}} = 0.289\text{ V} \qquad r_{o4} = \frac{24}{300\mu} = 80\text{k}$$

$$V_{ov(5)} = \sqrt{\frac{2(800\mu)}{(120\mu)(20)}} = 0.816\text{ V} \qquad r_{o5} = \frac{24}{800\mu} = 30\text{k}$$

We can then calculate $g_m = \frac{2I_D}{V_{ov}}$

$$g_{m1,2,3} = \frac{2(100\mu)}{0.289} = 6.92\times10^{-4} \qquad g_{m4} = \frac{2(300\mu)}{0.289} = 2.08\times10^{-3} \qquad g_{m5} = \frac{2(800\mu)}{0.816} = 1.96\times10^{-3}$$

So now we finally have $1/g_{m5} = 510\,\Omega = R_{out}$.
If we wanted to be exact,

$$R_{out} = \frac{r_{o5}}{1 + g_{m5} r_{o5}}$$
$$= \frac{30k}{1 + (30k)(1.96\times10^{-3})}$$
$$= 502\,\Omega$$

Now, to finally find A. In the amplifier without feedback, applying all the voltage on one side causes the shared source connection to be at half that.

$\therefore$ Q1's dependent source creates $g_m V_{GS} = g_m (V_s - \frac{1}{2} V_s)$
$= \frac{g_m V_s}{2}$ current

This gets mirrored to 3x at Q4, so Q4 now has

$\frac{3 g_{m1} V_s}{2}$ current through it,

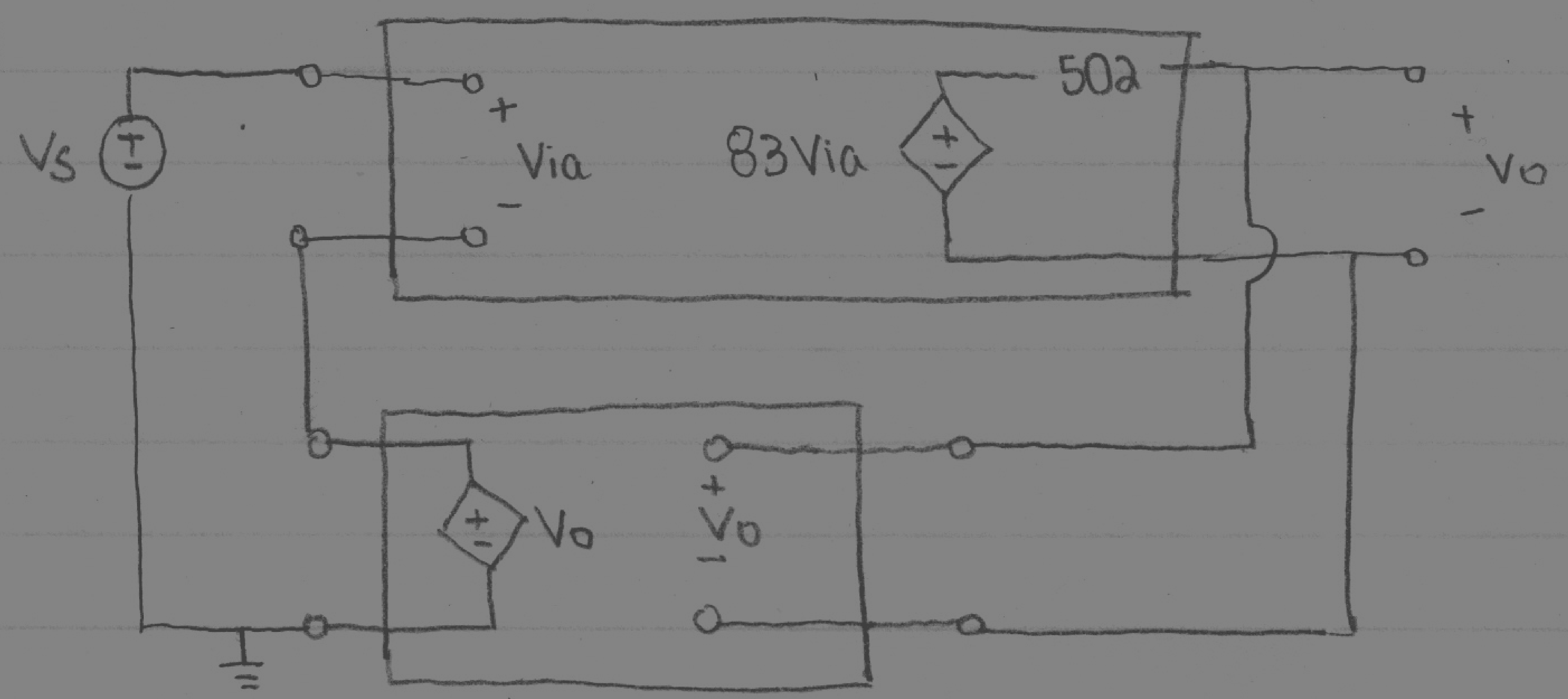
causing a drop of $\left(\frac{3 g_{m1} V_s}{2}\right) r_{o4}$ across r_{o4}.

That consequently means that Q4's drain is at that same voltage as that across r_{o4}. Finally, this gets reflected at a 1:1 ratio to the output due to Q5, the voltage follower.

$$\therefore \quad A = \frac{V_o}{V_s} = \frac{3 g_{m1} r_{o4}}{2}$$
$$= \frac{3(6.92\times10^{-4})(80k)}{2}$$
$$= 83\ V/V$$

Finally, we can put it all together.

4)



$$A_{eff} = (1)\,A\left(\frac{502}{502}\right) = 83\ \text{V/V}$$

$$\therefore\ A_f = \frac{A_{eff}}{1 + A_{eff}B} = \frac{83}{1+83} = 0.989\ \text{V/V}$$

$$R_{if} = R_{in}(1 + A_{eff}B) = \infty\,(84) = \infty$$

$$R_{of} = \frac{R_{out}}{1 + A_{eff}B} = \frac{502}{84} = 5.98\ \Omega$$

Note that there's no R_{sig} or R_L so we don't have to do any intermediate steps or remove them from our final resistances.