

Homework2

Problem 1.

1. $(P \longrightarrow (Q \vee R))$ given
2. $(R \longrightarrow S)$ given
3. $(\neg S \longrightarrow \neg Q)$ given
4. (P) given
5. $(\neg P \vee (Q \vee R))$ line 1: implies rule
6. $(\neg R \vee S)$ line 2: implies rule
7. $(Q \vee R)$ line 1, line 4: modus ponus
8. $(S \vee Q)$ line 6, line 7: resolution
9. $(S \vee \neg Q)$ line 3: implies rule and negation
10. (S) line 8, line 9: disjunctive syllogism

Problem 2.

1. $(\neg P(a) \longrightarrow Q(a))$ given
2. $(P(a) \longrightarrow Q(a))$ given
3. $(\forall x, Q(x) \longrightarrow S(x))$ given
4. $(Q(a) \longrightarrow S(x))$ line 3: universal instantiation
5. $(P(a) \longrightarrow S(a))$ line 1, 2: hypothetical syllogism
6. $(\neg P(a) \vee Q(a))$ line 2: implies rule
7. $(P(a) \vee Q(a))$ line 1: implies rule and double negation
8. $(Q(a))$ line 6, 7: disjunctive syllogism (note I think I have the wrong name for this)
9. $(S(a))$ line 4, 9: modus ponens

Problem 3.

1. prove $(\neg \forall x P(x) \longrightarrow \forall y (Q(y) \wedge R(y)))$ is equal to $(\exists x \neg (P(x) \longrightarrow \forall y (Q(y) \wedge R(y))))$
2. $(\neg (P(a) \longrightarrow \forall y (Q(y) \wedge R(y))))$ existential instantiation
3. $(\neg (P(a) \longrightarrow (Q(a) \wedge R(a))))$ universal instantiation
4. $(\neg (\neg P(a) \vee (Q(a) \wedge R(a))))$ implies rule
5. $(\neg (P(a) \longrightarrow (Q(a) \wedge R(a))))$ implies rule and double negation
6. $(\neg (P(a) \longrightarrow \text{forall } y (Q(y) \wedge R(y))))$ universal generalization
7. $(\neg (\forall x P(x) \longrightarrow \text{forall } y (Q(y) \wedge R(y))))$ universal generalization