IMPEDANCE CONTROL: AN APPROACH TO MANIPULATION

Neville Hogan

Department of Mechanical Engineering
and
Laboratory of Manufacturing and Productivity
Massachusetts Institute of Technology
Cambridge, MA 02139

ABSTRACT

Manipulation fundamentally requires a manipulator to be mechanically coupled to the object being manipulated. A consideration of the physical constraints imposed by dynamic interaction shows that control of a vector quantity such as position or force is inadequate and that control of the manipulator impedance is also necessary. Techniques for control of manipulator behaviour are presented which result in a unified approach to kinematically constrained motion, dynamic interaction, target acquisition and obstacle avoidance.

INTRODUCTION

The work presented here is an attempt to define a unified approach to the control of mechanical manipulators. This approach encompasses and includes the simple positioning or transporting tasks typically performed by robots and/or prostheses. It also builds on this capability, extending it to facilitate the application of robots and/or prostheses to tasks involving static and dynamic interactions between the manipulator and its environment. It will be shown that the approach can lead to a simplification of manipulator control.

By any reasonable definition, manipulation fundamentally requires mechanical interaction with the object(s) being manipulated, and a useful classification of manipulatory tasks is by the magnitude of the mechanical work exchanged between the manipulator and its environment. In some cases the interaction forces are negligible, the instantaneous mechanical work done by the manipulator is negligible, ($dW = F.dX = 0$) and for control purposes the manipulator may be treated as an isolated system, with its output position or velocity as the controlled variable(s). Generally, the successful applications of industrial robots to date have been restricted to this case; examples are spray-painting and welding [28].

In other cases the manipulator encounters constraints in its environment and the interaction forces are not negligible; however, although the manipulator is kinematically coupled to its environment, dynamic interaction is still absent. Along the tangent to a pure kinematic constraint the interaction forces are zero ($F = 0$) whereas along the normal into the surface the motions are zero ($dX = 0$) and in all directions the instantaneous mechanical work done is again negligible ($dW = F.dX = 0$). In this case an appropriate control strategy is a combination of motion control along the tangent and force control along the normal. This control strategy is commonly termed "compliance" [19], more correctly called "accomodation" [33], and is the topic of a considerable body of laboratory research, although it has not yet seen widespread industrial application.

The most general case (which includes the previous two as special instances) is that in which the dynamic interaction is neither zero nor negligible ($dW \neq 0$). A large class of manufacturing operations fall into this category: examples include drilling, reaming, routing, counterboring, grinding, bending, chipping, fettling -- any task requiring work to be done on the environment. Many activities of daily living to be performed by an amputee using a prosthesis -- basically any task involving the use of a tool -- are also in this category. If the dynamic interaction is to be modulated, regulated or controlled, then strategies directed towards the control of a vector quantity such as position, velocity or force will be inadequate as they are insufficient to control the mechanical work exchanged between the manipulator and its environment.

A solution to this problem is to modulate and control the dynamic behaviour of the manipulator in addition to commanding its position or velocity. If the environment is regarded as a source of "disturbances" to the manipulator, then modulating the "disturbance response" of the manipulator will permit control of dynamic interactions [21]. One way to vary the dynamic behaviour of a manipulator would be to vary the parameters and/or structure of a feedback controller [25], but this is not the only way, nor always the best way. Exploiting the intrinsic properties of mechanical hardware can also provide a simple, effective and reliable way of dealing with mechanical interaction [7,8,20,34].

A unified framework for considering the action of both hardware and software in the control of dynamic behaviour can be obtained by making the reasonable postulate that no controller can make the manipulator appear to the environment as anything other than a physical system. Along each degree of freedom, instantaneous power flow between a physical system and its environment is always defined by the product of two conjugate variables, an effort (e.g. a force, a voltage) and a flow (e.g. a velocity, a current) [24]. Seen from the environment, physical systems come in only two types: admittances, which accept effort (e.g force) inputs and yield flow (e.g. motion) outputs, and impedances, which accept flow (e.g. motion) inputs and yield effort (e.g. force) outputs. The concepts of impedance and admittance are familiar to designers of electrical systems and are frequently regarded as equivalent and interchangable representations of the same system. For a linear sytem this usually is true, but for a nonlinear system it usually is not: The distinction between admittance and impedance is fundamental.

An important consequence of dynamic interaction between two physical systems such as a manipulator and its environment is that one must physically complement the other: Along any degree of freedom, if one is an impedance, the other must be an admittance and vice versa. Now, for almost all manipulatory tasks the environment at least contains inertias and kinematic constraints, physical systems which accept force inputs and which determine their motion in response. However, while a constrained inertial object can always be pushed upon, it doesn't always move; in those cases the describing equations cannot be written in impedance form (motion in, force out). In contrast, they can always be written in admittance form (force in, motion out). When a manipulator is mechanically coupled to such an environment, to ensure physical compatibility with the environmental admittance, something has to give, and the manipulator should assume the behaviour of an impedance.

Thus a very general strategy for controlling a manipulator is to control its motion (as in conventional robot control) and in addition give it a "disturbance response" for deviations from that motion which has the form of an impedance. The dynamic interaction between manipulator and environment and environment may then be modulated, regulated and controlled by changing that impedance, and hence the approach described in this paper has been termed "impedance control" [12,13].

IMPLEMENTATION OF IMPEDANCE CONTROL

A distinction between impedance control and the more conventional approaches to manipulator control is that the controller attempts to implement a dynamic relation between manipulator variables such as end-point position and force rather than just control these variables alone. This change in perspective results in a simplification of several control problems.

Most of our work to date [1-3,5,9-14] has focused on controlling the impedance of a manipulator as seen at its "port of interaction" with the environment, its end effector. Following the lead from the prior work on path control, [18,22,29,31,32] we have investigated ways of presenting the environment with a dynamic behaviour which is simple when expressed in workspace (e.g. Cartesian) coordinates. The lowest-order term in any impedance is the static relation between output force and input displacement, a stiffness. If, in common with most current work on robot control, we assume actuators capable of generating commanded forces (or torques), $\underline{T}$, sensors capable of observing actuator position (or angle), $\underline{\theta}$, and a kinematic relation between actuator position and end-point position, $\underline{X} = L(\underline{\theta})$, it is straightforward to design a feedback control law to implement in actuator coordinates a desired relation between end-point force, $\underline{F}$, and position, $\underline{X}$. Defining the desired equilibrium position for the end-point in the absence of environmental forces as $\underline{X}o$, a general form for the desired force-position relation is:

$$\underline{F} = K(\underline{X}o - \underline{X}) \qquad (1)$$

Compute the Jacobian, $J(\theta)$:

$$d\underline{X} = J(\underline{\theta})\, d\underline{\theta} \qquad (2)$$

From the principal of virtual work:

$$\underline{T} = J^{t}(\underline{\theta})\, \underline{F} \qquad (3)$$

The required relation in actuator coordinates is:

$$\underline{T} = J^{t}(\underline{\theta})\, K(\underline{X}o - L(\underline{\theta})) \qquad (4)$$

No restriction of linearity has been placed on the relation $K(\underline{X}o - \underline{X})$. Note that if $K(\underline{X}o - \underline{X})$ is chosen so as to make the end-point stiff, then this relation will accomplish Cartesian end-point position control and the "inverse kinematics problem" [23] has been completely eliminated. Only the forward kinematic equations for the manipulator need be computed. This may be important for those manipulators for which no closed-form solution to the inverse kinematic problem exists. Note also that in this relation the inverse Jacobian is not required.

The next important term in the manipulator impedance is the relation between force and velocity. Again, given the above assumptions, it is straightforward to define a feedback law to implement in actuator coordinates a desired relation between end-point force and end-point velocity such as:

$$\underline{F} = B(\underline{v}) \qquad (5)$$

From the manipulator kinematics:

$$\underline{v} = J(\underline{\theta})\ \underline{w} \quad (6)$$

The required relation in actuator coordinates is:

$$\underline{T} = J^{t}(\underline{\theta})\ B(J(\underline{\theta})\ \underline{w}) \quad (7)$$

Again note that inversion of the Jacobian is not required.

The dynamic behaviour to be imposed on the manipulator should be as simple as possible, but no simpler. The foregoing control laws for velocity- and position-dependent behaviour take no account of the inertial, frictional or gravitational dynamics of the manipulator. Under some circumstances this is reasonable, but in many situations these effects cannot be neglected. One approach we have taken to dealing with inertial manipulator behaviour is to "mask" the true dynamics of the manipulator and impose simpler dynamics. No physically realisable strategy can eliminate the inertial effects of a manipulator but the apparent inertia seen at the end effector can be modified, and elsewhere we have presented a derivation of a feedback control law which makes the end-point inertia appear to be that of rigid body with an inertia tensor which is invariant under translation and rotation [5,12-14].

IMPEDANCE CONTROL WITHOUT FEEDBACK

However, feedback control is not the only way to modulate the dynamic behaviour of a manipulator. One alternative we have explored is to use kinematic redundancies to provide a measure of control over the inertial component of the end-point dynamics. Remember that inertial behaviour is properly described as an admittance and the fundamental form of the constitutive equation for a generalised inertial system is a relation determining generalised velocity, $\underline{w}$, (e.g the velocities of the manipulator joints) as a function of generalised momentum, $\underline{h}$:

$$\underline{w} = Y(\underline{\theta})\ \underline{h} \quad (8)$$

$Y(\underline{\theta})$ is the inverse of the more commonly used inertia tensor, and is termed the mobility tensor. The elements of the mobility tensor in general will depend on the manipulator configuration. At any given configuration, the generalised momenta in joint coordinates and actuator coordinates are related by the Jacobian:

$$\underline{h} = J^{t}(\underline{\theta})\ \underline{p} \quad (9)$$

The mobility tensor in end-point coordinates $W(\underline{\theta})$ is related to the mobility in joint coordinates $Y(\underline{\theta})$ as follows:

$$W(\underline{\theta}) = J(\underline{\theta})\ Y(\underline{\theta})\ J^{t}(\underline{\theta}) \quad (10)$$

The physical meaning of the end-point mobility tensor is that if the system is at rest (zero velocity) then a force vector applied to the end-point causes an acceleration vector (not necessarily co-linear with the applied force) which is obtained by premultiplying the force vector by the mobility tensor (see appendix).

Note that the Jacobian in the above equation need not be square, and that the end-point mobility is configuration dependent. As a result, redundant degrees of freedom can be used to modulate the end-point mobility. This effect can be represented by the ellipsoid corresponding to the mobility tensor. The locus of deviations of the generalised momentum from zero for which the kinetic energy is constant is an ellipsoid, the "ellipsoid of gyration" [30]. As shown in the appendix, the eigenvalues and eigenvectors of the symmetric tensor $W(\underline{\theta})$ define the size, shape and orientation of the ellipsoid of gyration in end-point coordinates (see figure 1a).

To illustrate the modulation of the end-point mobility using linkage redundancy, consider a planar three-link mecahnism. Assuming the links are rods of uniform density with lengths in the ratio of 1 : 2 : 3 , figures 1b through 1d show the effect on the ellipsoid of gyration of changes in linkage configuration for a fixed position of the end point.

An alternative representation of inertial behaviour is via the ellipsoid of inertia [30]. Asada [4] has suggested its use as a tool for designing robot mechanisms. However, the ellipsoid of gyration is the more fundamental representation; it is readily obtained even when the Jacobian of the linkage is non-invertible. Also, while the matrix $Y(\underline{\theta})$ may never have zero eigenvalues, (assuming real links with non-zero mass) the matrix $W(\underline{\theta})$ may, because of the kinematics of the linkage. Thus the end-point inertia tensor, $M(\underline{\theta})$, the inverse of the mobility tensor, does not exist for some linkage configurations; If the inertial behaviour of the tip is expressed in the conventional (impedance) form as $M(\underline{\theta})$ there exist locations in the workspace for which the eigenvalues of the tensor $M(\underline{\theta})$ become infinite. On the other hand the worst the eigenvalues of $W(\underline{\theta})$ will do is go to zero, which is easier to deal with computationally. Again, a reminder of the fact that the difference between impedance and admittance is fundamental.

SUPERPOSITION OF IMPEDANCES

One useful consequence of the assumptions underlying impedance control is that if the dynamic behaviour of the manipulator is dissected into a set of components, these may be reassembled by simple addition even when any or all of the components is nonlinear. This is a direct consequence of the assumption that the environment is an admittance, containing at least an inertia. That admittance acts to sum both forces applied to it and impedances coupled to it.

When the manipulator is decoupled from its environment the terms in the dynamic equations due to the environmental admittance disappear and in principle the manipulator alone need exhibit no inertial behaviour. In practice the uncoupled manipulator still has inertia (albeit nonlinear and configuration-dependent). Because of the inevitable inertial dynamics of the isolated manipulator the superposition of impedances holds even when the manipulator is uncoupled from its environment as there is always an inertial load to sum forces and impedances.

This simple observation has many important consequences. One which is immediately useful is that different controller actions aimed at satisfying different task requirements may readily be superimposed. For example, suppose that a desired end-point position- and velocity-dependent behaviour is to be implemented on a manipulator using a feedback control strategy as outlined above in equations (4) and (7). At the same time kinematic redundancies in the manipulator are to be used to modulate the end-point mobility. At any given end-point position, $\underline{X}$, (which is always determinable from the configuration, $\underline{\theta}$) the manipulator configuration may be chosen to best approximate a desired inertial behavour (for example, the mobility normal to a kinematic constraint surface may be maximised). This configuration may then be used in the feedback law which implements the position- and velocity-dependent behaviour. As the equations never require inversion of the Jacobian, they can be applied to a manipulator with kinematic redundancies. Note that this approach to end point control in the presence of kinematic redundancies is significantly different from the use of a generalised pseudoinverse [32].

OBSTACLE AVOIDANCE

The additive property of impedances permits complicated tasks to be dealt with one piece at a time and all of the pieces combined by simple addition. We have taken advantage of this to implement a real-time feedback control law which drives the manipulator end-point to a target location while simultaneously preventing unwanted collision with unpredictably moving objects in the manipulator's workspace [2,3].

Obstacle avoidance is generally regarded as a problem in position control, specifically that of planning a collision-free path [17]. The approach we have taken is not to plan a path, but to specify an impedance which produces the desired behaviour without explicit path planning. In the following example, recall that although the need for the manipulator to have the behaviour of an impedance arose from considerations of the mechanical interaction between a manipulator and its environment, cases in which the mechanical work exchanged is negligible (e.g. free motions) may be treated as special (or degenerate) instances.

The primary difference between impedance control and the more conventional approaches is that the controller attempts to implement a dynamic relation between manipulator variables such as end-point position and force rather than just control these variables alone. That entire relation becomes the command to the manipulator which may be updated as often as practical considerations (such as speed of computation) dictate. In this sense, impedance control is an augmentation of conventional position control Each command to the manipulator specifies a position and in addition specifies a relation determining the accelerating force to be applied to the total mechanical admittance in response to deviations of the actual position from the commanded position.

If the position- and velocity-dependent terms in the commanded impedance are each assumed to to satisfy the requirements for the existence of a potential function (the vector force fields which they define have no curl) then the manipulator behaviour is simplified. It may be thought of as analogous to that of a sticky marble rolling on a continuously deformable surface. Varying the impedance varies the shape of the surface and the stickiness of the marble. Target acquisition and obstacle avoidance may now be dealt with separately as follows.

Successive target locations may be specified by means of a (time-varying) depression in the surface. Each single command has a position-dependent component which specifies a potential function which is a "valley" with its bottom at the target. This "valley" is depicted by a map of isopotential contours in Figure 2a.

Conversely, given an observation of the relative location (with respect to the end-point) of an obstacle (or any other region in the workspace to be avoided) that object may be avoided by specifying a (time-varying) bump in the deformable surface. Now each single command also contains a position-dependent component which specifies a potential field with an unstable equilibrium point at the location of the object to be avoided. The potential function is a "hill" centered over the obstacle (See Figure 2b)

The target-acquisition command and the obstacle-avoidance command could be combined in a number of ways, but remember that the admittance sums the impedances. The inevitable inertial behaviour of the end-point guarantees the superposition of the components of the impedance-controller action independent of the linearity of the components. It is always possible to command obstacle-avoidance and target-acquisition (or any other aspect of the complete task) independently and then combine all commands by simply adding the impedances, in this case the corresponding potential fields (see Figure 2c) [10,11]. It is important to note that this combined potential field represents a <u>single command</u> to the manipulator. Of course, <u>neither targets</u> nor obstacles need stay fixed in the workspace and a typical task will require multiple impedance commands (just as locating the spot welds on an automobile requires multiple position commands to a conventional robot controller) and by updating the impedance commands repeatedly this approach may be used to make a manipulator avoid "invaders", objects which may move about the workspace in an unpredictable (or merely unpredicted) manner.

The use of potential functions as commands to a robot is similar to the approach used by Khatib [16] to navigate a manipulator through a complicated environment. The distinguishing feature (and advantage) of impedance control is that the same controller used to deal with free motions can also be used to deal with real mechanical interaction. The success of impedance control as a unifying framework for dealing with both kinematically constrained manipulations and free motions (including avoiding moving "invaders") has been demonstrated by performing both of these tasks in real time using a spherical coordinate manipulator [1,2,3]. The same controller was used for both tasks and the algorithm was simple enough to be implemented using 8-bit 2 MHz microprocessors (Z-80, one for each axis) for the real-time controller. One example of the obstacle-avoidance behaviour achieved is shown in figure 3.

As an aside, note that to be of practical value, the "repulsive" force fields used to implement collision avoidance must be nonlinear; the repulsive force must drop to zero for sufficiently large separations between the end-effector and objects in the environment (see figure 2b). This is precisely the type of non-invertible, nonlinear force/displacement behaviour for which no inverse compliance form exists. The concept of tuning the end-point stiffness and damping of a manipulator has been discussed in the literature under the general heading of "compliance", "compliant motion control", "fine motion control", or "force control" [15,19,23,25,26,33]. In most of this prior work, the manipulator has been given the behaviour of a compliance, (or more correctly, an admittance). The control strategy presented here is considerably more general; If the end-point dynamic behaviour is expressed as an impedance, the above obstacle-avoidance behaviour is included as a special case; If it were expressed as a compliance this useful behaviour would be excluded. In addition, the superposition property of impedances coupled to an admittance would not be preserved.

SUMMARY AND CONCLUSION

This paper has presented a unified approach to manipulation termed "impedance control". Because by its nature manipulation requires mechanical interaction between sytems, the focus of the approach is on the characterisation and control of interaction. By assuming that no control system may make a physical system behave like anything other than a physical system several simple but fundamental observations may be made: Command and control of a vector such as position or force is not enough to control dynamic interaction between systems; the controller must also command and control a relation between port variables; in the most common case in which the environment is an admittance (e.g. a mass, possibly kinematically constrained) that relation should be an impedance, a function, possibly nonlinear, dynamic, or even discontinuous, specifying the force produced in response to a motion imposed by the environment. Even more important, if the environment is an admittance, the total impedance is expressible as a sum of component impedances, even when the components are nonlinear.

An alternative approach to manipulator control in the presence of significant dynamic interaction is to change the structure and/or parameters of a feedback controller as the conditions imposed by the environment change. This would require the controller to monitor the environment continuously, identify changes, and adapt its own behaviour accordingly -- a far-from-trivial task. Changes in the structure and parameters of the environment may take place very rapidly (consider the transition from free motion to constrained motion as an object comes in contact with a surface) and there may not be sufficient time for the lengthy process of system identification. On the other hand, if the controller is structured so that the manipulator always impresses a force on the environment in relation to its motion (that is, it behaves as an impedance) there are no practical situations in which its behaviour is inappropriate, no practical task has been excluded, and the need to identify the structure of the environment has been reduced.

Of course, impedance control does not preclude the application of adaptive strategies, and indeed the two approaches may complement each other, controlled impedance taking care of the transitions and allowing time for identification and adaptation to optimise performance. Strictly speaking, impedance control as outlined above is a subset of parameter-adaptive control; the primary distinctions are that the parameters to be modulated are expressed in terms of a physically meaningful quantity, mechanical impedance, and no assumption is made that the implementation of the impedance will be through feedback control strategies. An impedance may be implemented in a number of ways, using to advantage the resources of a specific manipulator. Simple feedback control laws for imposing position- and velocity-dependent components of cartesian end-point impedance were presented above. Because care was taken to express the desired behaviour as an impedance, compatible with the fundamental mechanics of manipulation, solving the inverse kinematics problem proves to be unnecessary. It was also shown that a possible alternative to feedback control strategies is to use the intrinsic mechanics of the manipulator such as "redundant" degrees of freedom to modulate its dynamic behaviour.

An advantage of impedance control is that it permits a unified treatment of many aspects of manipulator control. Real mechanical interaction may be treated in the same framework as free (unconstrained) motions. Impedance control has been used to develop a feedback control law for avoiding unpredictably moving objects. By taking advantage of the superposition of impedances, target acquisition and obstacle avoidance could be considered separately and implemented as different components of a total commanded impedance which were combined by simple addition. This approach does not require explicit path planning and the control law was simple enough to be implemented using 8-bit microprocessors. Furthermore, the same controller was capable of coping with kinematically constrained motions.

However, the applicability of impedance control extends beyond the workless conditions imposed by free motions or pure kinematic constraints to include the control of energetic interactions. It promises to be particularly useful for understanding, controlling and coordinating the actions of mutually interacting manipulators, such as the fingers of a hand, the hand and the arm, or two arms. Using this approach each subsystem presents a simple behavior to the other subsystems. As a result, prediction and control of the combined behavior of the entire system is simplified. The ultimate goal of this effort is to understand the subtleties of adaptive tool-using, one of the distinguishing features of primate behaviour. Impedance control may provide the basis of an understanding of tool-using behaviour and permit its practical implementation on an amputee's artificial limb or on an industrial robot.

ACKNOWLEDGEMENTS

Portions of the work reported in this paper were supported by:
NSF Grant No. PFR 7917348
NIHR Grant No. G00 820 0048, Department of Education
The Whitaker Health Sciences Fund
Polaroid Corporation
The John and Fannie Hertz Foundation
The Ralph E. Cross (1933) Fund
American Can Company
The TRW Foundation Faculty Fellowship
This support is gratefully acknowledged. Portions of the work were performed in the Eric P. and Evelyn E. Newman Laboratory for Biomechanics and Human Rehabilitation, the Acoustics, Vibrations and Machine Dynamics Laboratory, and the Laboratory for Manufacturing and Productivity.

REFERENCES

1. Andrews, J. R., "Douloi Systems: Design and Construction of a Manipulator and Controller for Research in Impedance Control", S.B. Thesis, Department of Mechanical Engieering, MIT, May 1981.

2. Andrews, J. R., "Impedance Control as a Framework for Implementing Obstacle Avoidance in a Manipulator", S.M. Thesis, Department of Mechanical Engineering, MIT, February, 1983.

3. Andrews, J. R. and Hogan, N., "Impedance Control as a Framework for Implementing Obstacle Avoidance in a Manipulator", pp. 243-251 in: Control of Manufacturing Processes and Robotic Systems, Eds. Hardt, D. E. and Book, W. J., American Society of Mechanical Engineers, New York, 1983.

4. Asada, H., "A Goemetrical Representation of Manipulator Dynamics and its Application to Arm Design", pp. 1-8 in: Robotics Research and Advanced Applications, Ed. Book, W. J., American Society of Mechanical Engineers, New York, 1982.

5. Cotter, S. L., "Nonlinear Feedback Control of Manipulator Endpoint Impedance", S.M. Thesis, Department of Mechanical Engineering, MIT, September, 1982.

6. Crandall, S. H., Karnopp, D. C., Kurtz, E. F. Jr. and Pridmore-Brown, D. C., "Dynamics of Mechanical and Electromechanical Systems", McGraw-Hill, New York, 1968.

7. Cutkosky, M. R. and Wright, P. K., "Position Sensing Wrists for Industrial Manipulators", Proceedings, 12th International Symposium on Industrial Robots", Paris, France, June 1982.

8. Drake, S., "Using Compliance in Lieu of Sensory Feedback for Automatic Assembly", Charles Stark Draper Laboratory Report T-657, September 1977.

9. Hogan, N., "Mechanical Impedance Control in Assistive Devices and Manipulators", Proceedings of the Joint Automatic Controls Conference Vol. 1, San Francisco, August 1980.

10. Hogan, N., "Impedance Control of a Robotic Manipulator", Presented at the Winter Annual Meeting of the American Society of Mechanical Engineers, Washington, D.C., 1981.

11. Hogan, N., "Programmable Impedance Control of Industrial Manipulators", Proceedings of the Conference on CAD/CAM in Mechanical Engineering, M.I.T., March, 1982.

12. Hogan, N., "Impedance Control: An approach to Manipulation. Part I: Control of Mechanical Interaction. Part II: Control of End-Point Impedance", submitted to ASME Journal of Dynamic Systems, Measurement and Control, June 1983.

13. Hogan, N., "Impedance Control of Industrial Robots", accepted for publication in Robotics and Computer Aided Manufacturing, Vol. 1, 1984.

14. Hogan, N. and Cotter, S. L., "Cartesian Impedance Control of a Nonlinear Manipulator", pp. 121-128 in: Robotics Research and Advanced Applications, Ed. Book, W. J., American Society of Mechanical Engineers, New York, 1982.

15. Ishida, T., "Force Control in Coordination of Manipulator Fine Motions", Fifth International conference on Artificial Intelligence, 1977.

16. Khatib, O., Le Maitre, J. F., "Dynamic Control of Manipulators Operating in a Complex Environment", Third Symposium on Theory and Practice of Robots and Manipulators, Udine, Italy, pp. 267-282, 1978.

17. Lozano-Perez, T., "Automatic Planning of Manipulator Transfer Movements", IEEE Transactions on Systems, Man and Cybernetics, Vol. SMC-11, No. 10, pp. 681-698, 1981.

18. Luh, J. Y. S., Walker, M. W. and Paul, R. P., "Resolved Acceleration Control of Mechanical Manipulators", IEEE Transactions on Automatic Control, vol AC-25, pp.468-474, 1980.

19. Mason, M. T., "Compliance and Force Control for Computer Controlled Manipulators", IEEE Trans. Sys., Man, Cybern., Vol. SMC-11, No. 6, 1981.

20. Nevins, J.L. and Whitney, D.E., "Computer-Controlled Assembly", Scientific American, Vol. 238, No. 2, pp. 62-74, Feb. 1978.

21. Paul, F.W., Gettys, T.K. and Thomas, J.D., "Defining of Iron Castings Using a Robotic Positioned Chipper", In: Robotics Research and Advanced Applications, Wayne J. Book, Ed. A.S.M.E., New York, 1982.

22. Paul, R. P., "Manipulator Cartesian Path Control",IEEE Transactions on Systems, Man, and Cybernetics, vol. SMC-9, pp.702-711, 1979.

23. Paul, R. P., "Robot Manipulators: Mathematics, Programming, and Control", M.I.T. Press, Cambridge, Massachusetts, 1981.

24. Paynter, H. M., "Analysis and Design of Engineering Systems", M.I.T. Press, Cambridge, Massachusetts, 1961.

25. Salisbury, J. K., "Active Stiffness Control of a Manipulator in Cartesian Coordinates", IEEE Conference on Decision and Control, New Mexico, 1980.

26. Shimano, B. E., "Force Control", Exploratory Study of Computer Integrated Assembly Systems, Binford, T. O. et.al., Eds., Stanford Artificial Intelligence Laboratory Memo 285.4, June, 1977.

27. Stiefel, E. L. and Scheifele, G., "Linear and Regular Celestial Mechanics", Springer-Verlag, New York, 1971.

28. Tanner, W. R., (Ed.) "Industrial Robots. Volume 2: Applications", Society of Manufacturing Engineers, Michigan, 1981.

29. Taylor, R. H., "Planning and Execution of Straight Line Manipulator Trajectories", IBM Journal of Research and Development, vol. 23, pp.424-436, 1979.

30. Whitaker, E. T., "A Treatise on the Analytical Dynamics of Particles and Rigid Bodies", Cambridge University Press, 1904 and Dover, New York, 1944.

31. Whitney, D.E., "Resolved Motion Rate Control of Manipulators and Human Prostheses", IEEE Transactions on Man-Machine Systems, Vol. MMS-10, No. 2, June 1969.

32. Whitney, D. E., "The Mathematics of Coordinated Control of Prosthetic Arms and Manipulators", ASME Journal of Dynamic Systems, Measurement, and Control, pp. 303-309, December 1972.

33. Whitney, D. E., "Force Feedback Control of Manipulator Fine Motions", ASME Journal of Dynamic Systems Measurement and Control, pp. 91-97, June 1977.

34. Whitney, D.E. and Nevins, J.L., "What is Remote Center Compliance and What Can It Do?", Proceedings, 9th ISIR, Washington, D.C., March, 1979.

APPENDIX

Generalised Inertial Systems and the Mobility Tensor

Any mechanical linkage is a generalised inertial system. The defining property of an inertial system is its ability to store kinetic energy, defined as the integral of (generalised) velocity with respect to (generalised) momentum [6]. At any configuration defined by the generalised coordinates the kinetic energy is a quadratic form in (generalised) momentum.

$$Ek = 1/2\ \underline{h}^{t}\ Y(\underline{\theta})\ \underline{h}$$

From Hamilton's equations [27], the (generalised) velocity is the momentum gradient of the kinetic energy.

$$H(\underline{h},\underline{\theta}) = Ek(\underline{h},\underline{\theta})$$

$$d\underline{\theta}/dt = \underline{w} = \nabla_{\underline{h}} H = Y(\underline{\theta})\ \underline{h}$$

Kinetic energy is commonly confused with kinetic co-energy. The two are not identical and are related by a Legendre transform [6].

$$Ek^{*} = \underline{w}^{t}\ \underline{h} - Ek = \underline{w}^{t}\ Y^{-1}\ \underline{w} - 1/2\ \underline{w}^{t}\ Y^{-1}\ Y\ Y^{-1}\ \underline{w}$$

$$Ek^{*} = 1/2\ \underline{w}^{t}\ Y^{-1}(\underline{\theta})\ \underline{w} = 1/2\ \underline{w}^{t}\ I(\underline{\theta})\ \underline{w}$$

At any configuration kinetic co-energy is a quadratic form in (generalised) velocity and its velocity gradient is the (generalised) momentum [6].

$$\underline{h} = I(\underline{\theta})\ \underline{w}$$

For a generalised inertial system, Y is a symmetric, twice-contravariant tensor. To distinguish it from its inverse, the inertia tensor I, (symmetric, twice-covariant) Y will be termed the mobility tensor [12]. A knowledge of the geometric relation between coordinate frames is sufficient to transform any tensor from one frame to another. As the joint angles are a set of generalised coordinates, for any configuration of the linkage of figure 1 the end-point coordinates are related to the joint angles via the kinematic transformations.

$$\underline{X} = L(\underline{\theta})$$

Differentiating these transformations yields the relation between velocities (at any given configuration).

$$d\underline{X}/dt = \underline{v} = J(\underline{\theta})\,\underline{w}$$

$J(\underline{\theta})$ in these equations is the configuration-dependent Jacobian. As the coordinate transformation does not store, dissipate or generate energy, incremental changes in energy are the same in all coordinate frames. This yields the relation between forces in each coordinate frame.

$$dEp = \underline{T}^t\, d\underline{\theta} = \underline{F}^t\, d\underline{X} = \underline{F}^t\, J(\underline{\theta})\, d\underline{\theta}$$

At any given configuration

$$\underline{T} = J^t(\underline{\theta})\,\underline{F}$$

The same approach yields the relation between the momenta in each coordinate frame.

$$dEk = d\underline{h}^t\,\underline{w} = d\underline{p}^t\,\underline{v} = d\underline{p}^t\, J(\underline{\theta})\,\underline{w}$$

At any given configuration

$$\underline{h} = J^t(\underline{\theta})\,\underline{p}$$

These relations may be used to express the mobility in end-point coordinates.

$$\underline{v} = J\,\underline{w} = J\,Y\,\underline{h} = J\,Y\,J^t\,\underline{p}$$

Denoting the end-point mobility by $W(\underline{\theta})$

$$W(\underline{\theta}) = J\,Y\,J^t$$

$$\underline{v} = W(\underline{\theta})\,\underline{p}$$

The physical meaning of the mobility tensor is that if the system is at rest an applied force will produce an acceleration equal to the force vector premultiplied by the mobility tensor. At rest, $d\underline{\theta}/dt = 0$ and hence:

$$d\underline{v}/dt = J\, d\underline{w}/dt$$

$$d\underline{w}/dt = Y\, d\underline{h}/dt$$

From the generalised Hamiltonian [27]:

$$d\underline{h}/dt = T - \nabla_{\underline{\theta}} H$$

At rest, $\underline{h} = 0$ hence $H(\underline{h},\underline{\theta}) = Ek = 0$ and $\nabla_{\underline{\theta}} H = 0$. Thus:

$$d\underline{h}/dt = \underline{T}$$

$$d\underline{v}/dt = J\,Y\,J^t\,\underline{F} = W\,\underline{F}$$

As the mobility tensor is symmetric it may be diagonalised by rotating the coordinate axes to coincide with its eigenvectors. A force applied in the direction of an eigenvector (when the system is at rest) results in an acceleration in the same direction equal to the applied force multiplied by the corresponding eigenvalue. The eigenvalues represent the inverse of the apparent mass or inertia seen by the applied force or torque.

Because the kinetic energy is a quadratic form in momentum, it may be represented graphically by an ellipsoid (see figure 1), the ellipsoid of gyration [30]. This may be thought of as the set of all momenta which produce the same kinetic energy (an isokinetic contour in momentum space). The lengths of the principle axes of the ellipsoid of gyration are inversely proportional to the square roots of the eigenvalues, proportional to the square roots of the associated apparent mass or inertia. The long direction of the ellipsoid of figure 1 is the direction of the greatest apparent inertia.

In the general case when the system is not at rest the relation between applied force and resulting motion is (in general) nonlinear and must be written in terms of a complete set of state equations for the inertial system. A convenient set of state variables are the Hamiltonian states, generalised position (e.g. $\underline{\theta}$) and generalised momentum ($\underline{h}$). The state and output equations are in the form of a generalised admittance [12] as follows.

State equations:

$$d\underline{h}/dt = -\nabla_{\underline{\theta}}\,[1/2\;\underline{h}^t\; Y(\underline{\theta})\;\underline{h}] + J^t(\underline{\theta})\,\underline{F}$$

$$d\theta/dt = \nabla_{\underline{h}}\,[1/2\;\underline{h}^t\; Y(\underline{\theta})\;\underline{h}] = Y(\underline{\theta})\,\underline{h}$$

Output equations (position and velocity):

$$\underline{X} = L(\underline{\theta})$$

$$\underline{v} = J(\underline{\theta})Y(\underline{\theta})\,\underline{h}$$

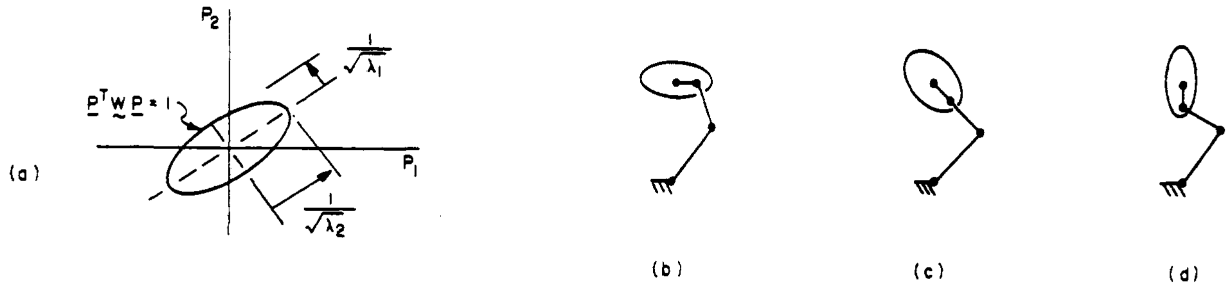

Figure 1: A schematic representation of the influence of kinematic redundancies on the mobility (inverse effective mass) of the end-point of a planar linkage. The ellipsoid of gyration associated with the mobility tensor is shown in (a). The eigenvalues of the mobility tensor are inversely proportional to the effective mass in the direction of the corresponding eigenvectors and the square root of their ratio determines the ratio of the major and minor axes of the ellipsoid, which are co-linear with the eigenvectors. For a planar, three-member linkage with links of uniform density and cross section and lengths in the ratio 1 : 2 : 3 the effect on the ellipsoid of gyration of changing the linkage configuration for a fixed position of the end-point is shown in (b),(c) and (d).

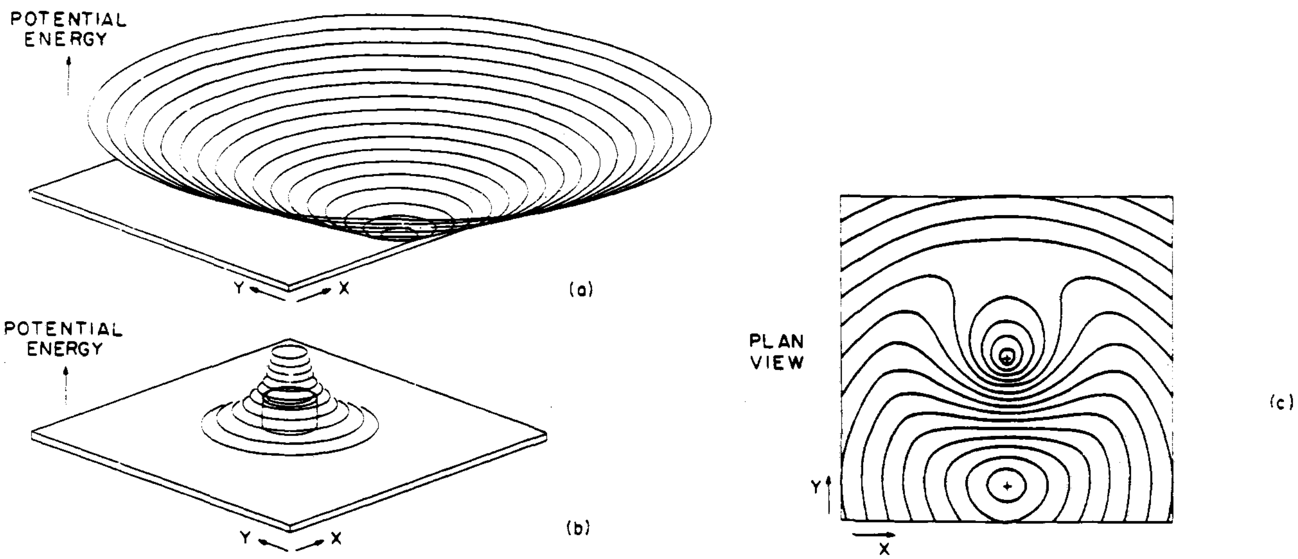

Figure 2: Graphical representation of the use of impedance control for real-time collision avoidance. Each figure represents a position-dependent component of a single impedance command (one of a time-sequence). An isopotential contour map of the component used for target acquisition independent of obstacles is shown in (a). The component used for obstacle avoidance independent of target is shown in (b). Because of the additive property of impedances, simultaneous target acquisition and obstacle avoidance may be achieved by simply superimposing these two components as in (c).

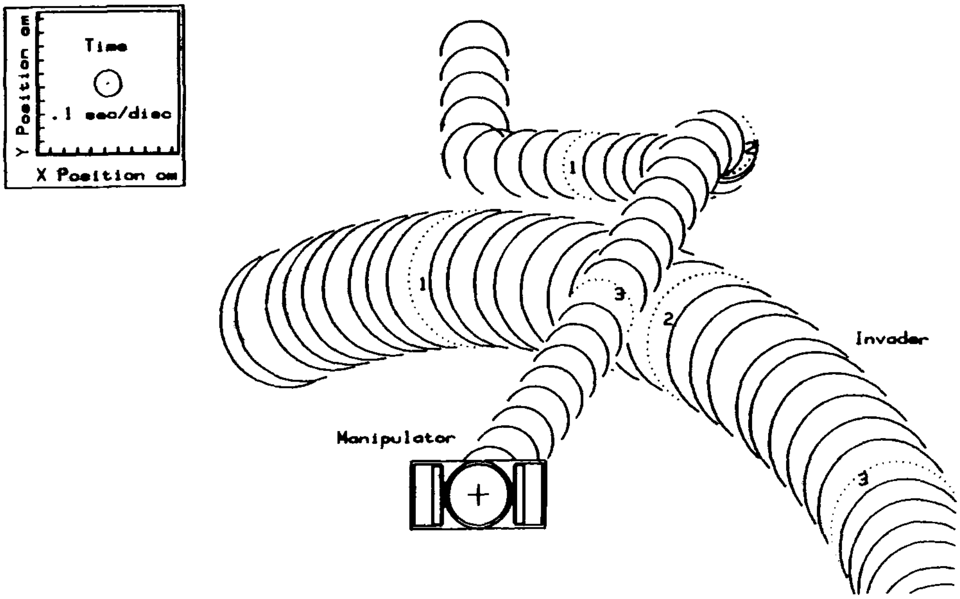

Figure 3: Avoidance of an unpredictably moving "invader" by an a spherical-coordinate manipulator controlled by 8-bit, 2MHz microprocessors. Successive positions of the manipulator end-effector and the invader in the vertical plane at 100 millisecond intervals are shown. All of the behaviour shown here is the robot's response to a single impedance command from the supervising computer, a PDP 11/44.