Impedance Control: An Approach to Manipulation:

Part III—Applications

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This three-part paper presents a unified approach to the control of a manipulator applicable to free motions, kinematically constrained motions, and dynamic interaction between the manipulator and its environment. In Part I the approach was developed from a consideration of the fundamental mechanics of manipulation. Part II presented techniques for implementing a desired manipulator impedance. In Part III a technique for choosing the impedance appropriate to a given application using optimization theory is presented. Based on a simplified analysis it is shown that if the task objective is to tradeoff interface forces and motion errors, the manipulator impedance should be proportional to the environmental admittance. An application of impedance control to unconstrained motion is presented. The superposition properties of nonlinear impedances are used to develop a real-time feedback control algorithm which permits a manipulator to avoid unpredictably moving objects without explicit path planning.

Introduction

The work presented in this three-part paper is an attempt to define an approach to manipulation which is sufficiently general to be applied both to the control of free motions and to the control of dynamic interaction between a manipulator and its environment. In Part I it was shown from a consideration of the mechanics of interaction that a general strategy is to control the motion of the manipulator and in addition control its dynamic behavior; controlling a vector quantity such as force or position alone is inadequate. To be compatible with the mechanics of an environment which in general will contain constrained inertial objects, the manipulator should exhibit the behavior of an impedance. It was also shown in Part I that for a broad class of nonlinear manipulators (basically those capable of positioning an unconstrained inertial object) the relation between the commanded motions and the commanded dynamic behavior could be represented by a generalized Norton equivalent network.

In Part II the implementation of a desired manipulator impedance either using a feedback strategy or using the intrinsic mechanics of the manipulator was discussed. We now turn to a consideration of a method for choosing an appropriate manipulator impedance. In this, the Norton equivalent network representation will prove to be of some value. We will also show how the superposition property of impedances leads to a simplification of a problem in manipulator control.

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Choosing an Appropriate Impedance

The manipulator impedance appropriate for a given situation depends on the task to be performed. In most manipulatory tasks there is a tradeoff to be made between allowable interface forces and allowable deviations from desired motions. Whether it has been rationally chosen or not, the manipulator impedance specifies a relation between interface forces and imposed motions. If the tradeoff implicit in the task is expressed as a performance index to be maximized or minimized which is a function of the interface forces and motions then the impedance appropriate for that task may be determined using optimization theory [10].

Because a general class of nonlinear manipulators can be represented by a generalized Norton equivalent network as shown in Fig. 1, considerable insight into manipulation can be gained by considering analogous (but simpler) systems with the same Norton network structure. Assume a manipulator interacts with a passive environment (no active energy source terms). For simplicity, consider a single degree-of-freedom and assume that both the manipulator impedance and the environmental admittance are simple linear dissipative elements. This simplified linear system has the same basic structure as a more general multiple degree-of-freedom nonlinear manipulator interacting with an environmental admittance. The following equations relate the port variables:

$$V = YF \tag{1}$$

$$F = Z(V_0 - V) \tag{2}$$

$$V = YZV_0/(1+YZ) \tag{3}$$

$$F = ZV_0/(1+YZ) \tag{4}$$

Now assume that one task is to minimize the transmission of power into the environmental admittance. Express this as an objective function to be optimized:

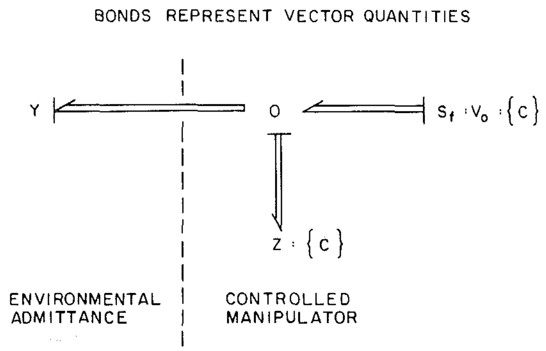

Fig. 1 A bond-graph equivalent network representation of an impedance-controlled manipulator interacting with an environmental admittance. Each bond represents a vector of power flows along multiple degrees of freedom. The bond graph for the manipulator is a generalized Norton equivalent network.

Objective: maximize $P=FV$ where $P=$ power transmitted

$$P=YZ^2V_0{}^2/(1+YZ)^2 \tag{5}$$

Maximizing the power transmitted requires the commanded motion V_0 to be maximized, or the commanded impedance Z to be maximized. Maximizing with respect to the admittance Y yields an equality condition:

$$ZY=1 \tag{6}$$

or

$$Z\text{manipulator}=Z\text{environment} \tag{7}$$

The first two conditions state essentially that the machine should operate on the boundaries of its performance envelope. The third condition states that (after the first two conditions have been satisfied) the machine and environment impedances should be matched. This is a familiar result and is a design rule of great versatility, applicable in any situation in which a source is to impart maximum power to a load. Its applicability to robotic transport tasks has recently been shown [19].

For manipulation, another common task is to minimize deviations from desired motions while simultaneously minimizing interface forces. Assume this objective may be expressed as follows:

$$\text{Objective: minimize } Q=p(V_0-V)^2+F^2 \tag{8}$$

p is a weighting coefficient specifying an allowable tradeoff between interface forces and motion errors. Rewriting the objective using equations (3) and (4):

$$Q=(p+Z^2)V_0{}^2/(1+YZ)^2 \tag{9}$$

Minimizing this objective requires the commanded motion V_0 to be minimized or the environmental admittance Y to be maximized, two physically reasonable conditions. Minimizing with respect to the commanded impedance yields the following equality condition:

$$Z-pY=0 \tag{10}$$

or

$$Z\text{manipulator}=pY\text{environment} \tag{11}$$

This condition may be considered as a designer's "rule of thumb" for manipulation, analogous to the impedance matching rule applicable to power transmission: "Make the *manipulator impedance proportional to the environmental admittance.*" If the environment is unyielding (low admittance), the manipulator should accommodate the environment (low impedance); if the environment offers little resistance (high admittance), the manipulator may impose motion upon it (high impedance).

Although these results were obtained using an extreme simplification of the mechanics of manipulation, this simple static analysis captures the essence of the interaction between manipulator and environment, and yields an intuitively satisfying result: that manipulation (at least insofar as it is modeled by the cost function of equation (8)) and power transmission are fundamentally conflicting task requirements. In view of the fact that a manipulator must be versatile – it may be called upon to transmit power in one phase of a working cycle (e.g., transport a workpiece as fast as possible) and manipulate at another (e.g., assemble the workpiece to another) – a controllable mechanical impedance is imperative.

The simple analysis presented above demonstrates that the tradeoff implicit in the specification of most manipulatory tasks may be mapped directly onto a statement about the manipulator impedance. That analysis was purely static: algebraic equations related the port variables, not differential equations. In the following a method is presented for determining an appropriate impedance in a simple dynamic case.

Assume that the end-point inertial behavior of the manipulator has been modified to be that of a rigid body using (for example) the technique outlined in Part II. The nodic (noninertial) interface forces can be represented by a generalized Norton equivalent network as shown in Fig. 1 and are assumed to depend only on the displacement (and its rate of change) from a commanded time-varying (virtual) position, with the displacement- and velocity-dependent terms assumed to be separable. The dynamic equations for the interaction port behavior are:

$$\mathbf{F}\text{int}=K[\mathbf{X}_0-\mathbf{X}]+B[\mathbf{V}_0-\mathbf{V}]-Md\mathbf{V}/dt \tag{12}$$

The environment will be assumed to be a rigid workpiece

Nomenclature

- Y = admittance
- Z = impedance
- Sf = flow source
- Se = effort source
- $:\{c\}$ = modulation by command set
- $\mathbf{V}_0$ = commanded (virtual) velocity
- $\mathbf{V}$ = velocity
- $\mathbf{X}_0$ = commanded (virtual) position
- $\mathbf{X}$ = position
- $\mathbf{F}$ = force
- P = power transmitted
- Q = objective function
- p = weighting coefficient
- **F**int = interface force
- **F**ext = external force
- $K[\bullet]$ = force/displacement relation
- $B[\bullet]$ = force/velocity relation
- M = inertia tensor in end-point coordinates
- Me = environmental inertia tensor
- Ftol = force tolerance
- Xtol = position tolerance
- t = time
- k = stiffness
- b = viscosity
- m = mass
- m_m = manipulator mass
- m_e = environmental mass
- S = strength of Gaussian random process
- δ = Dirac delta function
- H = Pontryagin function
- $\lambda_1\ \lambda_2\ \lambda_3$ = LaGrange multipliers
- $E[\bullet]$ = expectation operator

Overbar also denotes expectation

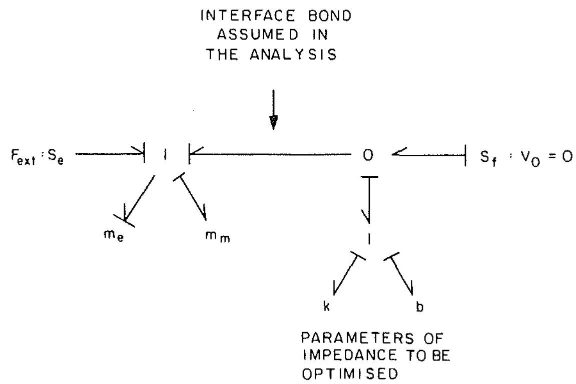

Fig. 2 A bond graph equivalent network showing the interface bond assumed in the derivation of the optimal dynamic impedance

acted on by unpredictable (or merely unpredicted) forces. Its dynamic equations are:

$$Me\, d\mathbf{V}/dt = \mathbf{F}\text{ext} + \mathbf{F}\text{int} \tag{13}$$

Both the isolated manipulator (**F**int = 0) and the coupled system have the behavior of a mass driven by motion-dependent forces. The dynamic equations of the coupled system are:

$$(Me + M)d\mathbf{V}/dt = K[\mathbf{X}_0 - \mathbf{X}] + B[\mathbf{V}_0 - \mathbf{V}] + \mathbf{F}\text{ext} \tag{14}$$

A further simplification is to assume that the position-dependent terms are curl-free[1]. A potential function is then definable which is analogous to stored elastic energy. A similar set of assumptions permit the velocity-dependent terms to be described as a dissipative potential field. Finally, the elastic and viscous terms will be assumed linear.

The combined inertia tensor, $Me + M$, for the manipulator and the workpiece will not in general be diagonal. However, it is symmetric and thus can be diagonalized by rotating the coordinate axes in which the task is described. The stiffness and viscosity tensors are to be chosen to suit the task. It will be assumed that the eigenvectors of the symmetric stiffness and viscosity tensors are colinear with those of the inertia tensor. Given this assumption, the general six degree-of-freedom problem decomposes into six single degree-of-freedom problems. Consequently, each degree of freedom may be dealt with separately as follows.

The task considered will be that of maintaining a fixed position in the face of perturbations from the environment. (These might be due to excitation forces from a power tool or due to the process of using the tool.) To reflect the paucity of a-priori information about the perturbations from the environment they will be modeled as a zero-mean, Gaussian, purely-random process of strength S. The tradeoff implicit in this task will be modeled as before (equation (8)) as the minimization of interface forces and position errors. For simplicity, the interface is assumed to be between the total inertia (controlled manipulator plus environment) and the elastic and viscous elements as shown in the equivalent network of Fig. 2. The inertial behavior of the manipulator has essentially been lumped with the admittance of the environment.

The objective function to be minimized is:

$$Q = \int_0^\infty \{(F/F\text{tol})^2 + [(X_0 - X)/X\text{tol}]^2\}\, dt \tag{15}$$

Writing the equations for a single degree of freedom in phase variable form:

$$d/dt \begin{bmatrix} X \\ V \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix} \begin{bmatrix} X \\ V \end{bmatrix} + \begin{bmatrix} 0 \\ k/m \end{bmatrix} X_0 + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} F\text{ext} \tag{16}$$

$$F = [k \quad m] \begin{bmatrix} X \\ V \end{bmatrix} \tag{17}$$

In these equations m refers to the combined apparent mass of manipulator and workpiece along this degree of freedom. Because of the random forcing term the objective function (equation (15)) is a random variable and the optimum impedance is obtained by minimizing its expectation with respect to the parameters k and b of the manipulator impedance, subject to the dynamic constraints imposed by the system (equations (16) and (17)). The final simplifying assumption is to consider only steady state conditions (the method is readily generalized to the transient case using standard numerical techniques). The analysis is presented in Appendix I. Summarizing the results:

$$k_{\text{opt}} = F\text{tol}/X\text{tol} \tag{18}$$

$$b_{\text{opt}} = \sqrt{2(k_{\text{opt}} m)} \tag{19}$$

In this simple case the optimum stiffness is equal to the ratio of force tolerance, Ftol, to position tolerance, Xtol. With no penalty on velocity errors, the optimum damping is such as to yield a damping ratio of 0.707. A nonzero penalty on velocity errors would yield a more heavily damped system.

Viewed simply as an optimization problem, these results are the well-known solution to the second-order feedback regulator problem [13]. Their importance in this context is twofold: First, they demonstrate that a tradeoff modeled by an objective function such as equation (15) can be used to derive a specification of the appropriate manipulator impedance. Because of the assumptions permitting decoupling of the end-point behavior along each degree of freedom, these results can be applied to each degree of freedom in turn. Furthermore, the analytical technique can be applied to nonlinear systems [6, 9].

Second, and more important, the results are expressed in terms of the mechanical behavior of the end-point regardless of how that behavior is achieved. Although a large number of (gratuitous) assumptions were made in the derivation, none of them are impractical and the result expresses the required impedance command to the manipulator in terms of readily available mechanical quantities associated with the task. The optimal impedance may be implemented by any means, feedback or otherwise, permitted by a given manipulator design. As outlined in Part II, the primate neuromuscular system has the capacity to change its mechanical impedance by simultaneous activation of opposing muscles [6, 9, 14] and the above analytical technique has been used to derive a prediction of antagonist coactivation which has been shown to be consistent with experimental observation [6, 9].

In this simple analysis the external forces were almost completely unmodelled. The assumption of a purely random process is tantamount to an assumption of complete unpredictability. The analysis demonstrates that even with extremely little information about the environment, the interaction between manipulator and environment may be controlled so as to meet task specifications. Naturally, the

[1] For each component of the vector force field defined by $K[\cdot]$ and each component of **X**, the crossed partial derivatives are identical.

more information about the environment that is available, the better one would expect the system performance to be. However, this suggests the tantalizing possibility that the impedance may be chosen to tradeoff performance against need for information about the environment. This is a topic for further research.

Obstacle Avoidance Using Superposition of Impedances

One useful and important consequence of the assumptions underlying impedance control is that if the dynamic behavior of the manipulator is dissected into a set of component impedances, these may be reassembled by simple addition even when the behavior of any or all of the components is nonlinear. This is a direct consequence of the assumption that the environment is an admittance, containing at least an inertia. That inertia acts to sum both forces applied to it and impedances coupled to it.

The additive property of impedances permits complicated tasks to be dealt with one piece at a time and all of the pieces combined by simple addition. We have taken advantage of this to implement a real-time feedback control law which drives the manipulator end-point to a target location while simultaneously preventing unwanted collision with unpredictably moving objects in the manipulator's workspace [1-3, 7, 8].

Obstacle avoidance is generally regarded as a problem in position control, specifically that of planning a collision-free path [15]. The approach we have taken is not to plan a path, but to specify an impedance which produces the desired behavior without explicit path planning. In the following example, recall that although the need for the manipulator to have the behavior of an impedance arose from considerations of the mechanical interaction between a manipulator and its environment, cases in which the mechanical work exchanged is negligible (e.g., free motions) may be treated as special (or degenerate) instances.

The primary difference between impedance control and the more conventional approaches is that the controller attempts to implement a dynamic relation between manipulator variables such as end-point position and force rather than just control these variables alone. That entire relation becomes the command to the manipulator which may be updated as often as practical considerations (such as speed of computation) dictate. In this sense, impedance control is an augmentation of conventional position control. Each command to the manipulator specifies a position (as in conventional control) and in addition specifies a relation determining the accelerating force to be applied to the total mechanical admittance in response to deviations of the actual position from the commanded position.

If the position- and velocity-dependent terms in the commanded impedance are each assumed to satisfy the requirements for the existence of a potential function then the manipulator behavior is simplified. It may be thought of as analogous to that of a sticky marble rolling on a continuously deformable surface. Varying the impedance varies the shape of the surface and the stickiness of the marble. Target acquisition and obstacle avoidance may now be dealt with separately as follows.

Successive target locations may be specified by means of a (time-varying) depression in the surface. Each single command has a position-dependent component which specifies a potential function which is a "valley" with its bottom at the target. This "valley" is depicted by a map of isopotential contours in Fig. 3(*a*).

Conversely, given an observation of the relative location (with respect to the end-point) of an obstacle (or any other region in the workspace to be avoided) that object may be avoided by specifying a (time-varying) bump in the deformable surface. Now each single command also contains a position-dependent component which specifies a potential field with an unstable equilibrium point at the location of the object to be avoided. The potential function is a "hill" centered over the obstacle (see Fig. 3(*b*)).

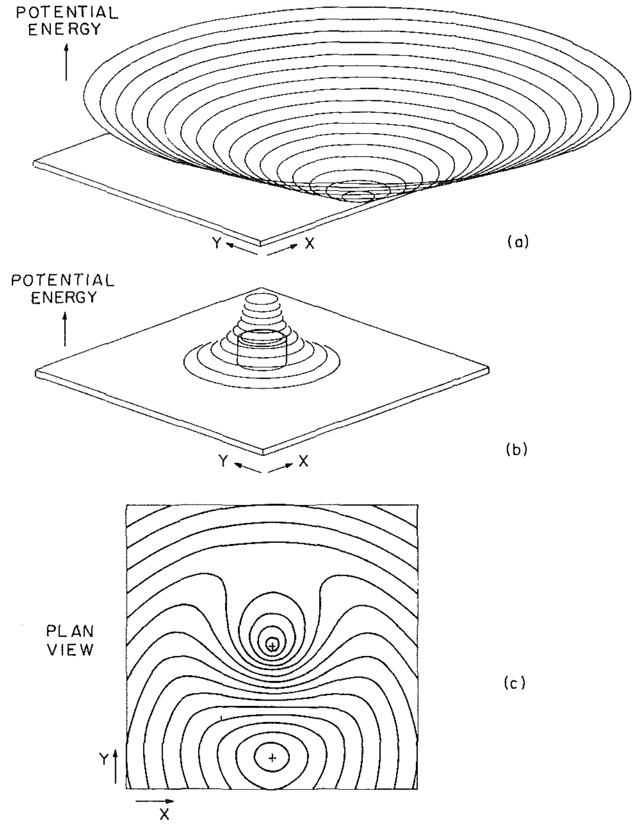

Fig. 3 A diagram of the potential functions corresponding to the static component of the commanded impedances which may be used for (*a*) target acquisition (*b*) obstacle avoidance and (*c*) simultaneous target acquisition and obstacle avoidance. A plan view of the isopotential contours are shown in (*a*) and (*b*).

The target-acquisition command and the obstacle-avoidance command could be combined in a number of ways, but remember that the admittance sums the impedances. The inevitable inertial behavior of the end-point guarantees the superposition of the components of the impedance-controller action independent of the linearity of the components. It is always possible to command obstacle-avoidance and target-acquisition (or any other aspect of the complete task) independently and then combine all commands by simply adding the impedances, in this case the corresponding potential fields (see Fig. 3(*c*)) [7, 8]. Furthermore, a number of obstacles and a target may be specified simultaneously. Each task component may be represented as a generalized Norton equivalent network and the combination of all the task components represented by the equivalent network of Fig. 4.

It is important to note that the combined potential field of Fig. 3(*c*) represents a *single command* to the manipulator. Of course, neither targets nor obstacles need stay fixed in the workspace and a typical task will require multiple impedance commands (just as locating the spot welds on an automobile requires multiple position commands to a conventional robot controller) and by updating the impedance commands repeatedly this approach may be used to make a manipulator avoid "invaders," objects which may move about the workspace in an unpredictable (or merely unpredicted) manner [2, 3].

The use of potential functions as commands to a robot is

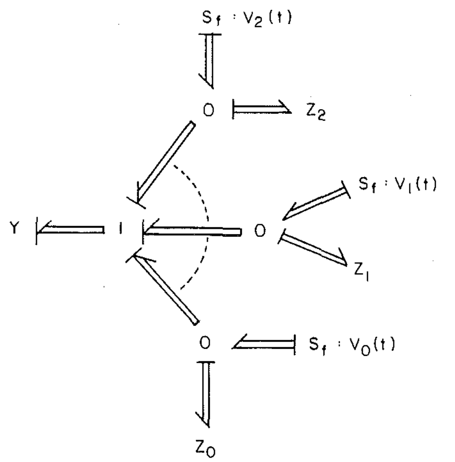

Fig. 4 A bond graph equivalent network representation of commands to an impedance-controlled manipulator specifying simultaneous target acquisition and avoidance of multiple obstacles. Each task component is represented by a generalized Norton equivalent network.

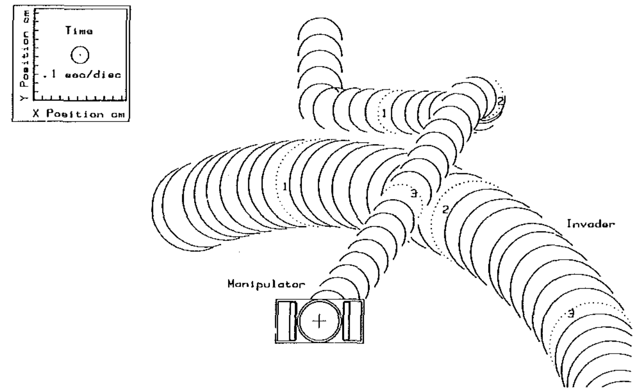

Fig. 5 Avoidance of an unpredictably moving "invader" by a spherical-coordinate manipulator controlled by 8-bit, 2MHz, Z80 microprocessors. The half circles show successive positions of the manipulator end-effector and the invader in the vertical plane at 100 millisecond intervals. All of the behavior shown here is the robot's response to a single impedance command from the supervising computer, a PDP 11/44.

similar to the approach used by Khatib and LeMaitre [12] to navigate a manipulator through a complicated environment. The distinguishing feature (and advantage) of impedance control is that the same controller used to deal with free motions can also be used to deal with real mechanical interaction. The success of impedance control as a unifying framework for dealing with both kinematically constrained manipulations and free motions (including avoiding moving "invaders") has been demonstrated by performing both of these tasks in real time using a spherical coordinate manipulator [1, 2]. The same controller was used for both tasks and the algorithm was simple enough to be implemented using 8-bit 2 MHz microprocessors (Z-80, one for each axis) for the real-time controller. One example of the obstacle-avoidance behavior achieved is shown in Fig. 5.

As an aside, note that to be of practical value, the "repulsive" force fields used to implement collision avoidance must be nonlinear; the repulsive force must drop to zero for sufficiently large separations between the end-effector and objects in the environment (see Fig. 3(*b*)). This is precisely the type of noninvertible, nonlinear force/displacement behavior for which no inverse compliance form exists. The concept of tuning the end-point stiffness and damping of a manipulator has been discussed in the literature under the general heading of "compliance," "compliant motion control," "fine motion control," or "force control" [5, 11, 17, 18, 21–24, 28]. In most of this prior work, the manipulator has been given the behavior of a linear compliance (a special case of an admittance). The control strategy presented here is considerably more general; If the end-point dynamic behavior is expressed as an impedance, the above obstacle-avoidance behavior is included as a special case; If it were expressed as a compliance this useful behavior would be excluded. In addition, the superposition property of impedances coupled to an admittance would not be preserved.

Summary and Conclusion

This paper has presented a method for controlling a manipulator which may interact dynamically with its environment. The approach is solidly based on the mechanics of interaction and was developed in Part I from some reasonable physical assumptions about manipulation: that the controlled manipulator may be represented as an equivalent physical system; that manipulation is a fundamentally nonlinear problem (therefore impedance and admittance must be distinguished); and that the environment contains kinematically constrained inertial objects and is an admittance (therefore the manipulator must have the causality of an impedance). Two theoretical consequences of these assumptions are that a broad class of nonlinear manipulators may be represented by a generalization of the familiar Norton equivalent network, and that impedances may be superimposed even when they are nonlinear.

Impedance control is an extension of conventional position control strategies. A time-varying position (the virtual position) is commanded; in addition an impedance is commanded, a relation (possibly dynamic, nonlinear, discontinuous and time-varying) between interface forces and displacements from that position. This simple strategy of commanding a relation rather than just a position (or a velocity) has a profound impact on the problems of manipulator control. In Part II it was shown that it leads to the elimination of the "inverse kinematic problem" [21] (that of determining a joint trajectory from an end-point trajectory).

Impedance control focuses on the interaction port and describes the required behavior in terms of the mechanical properties of the manipulator (e.g., its impedance) independent of the way this behavior is to be achieved. This sets the stage for considering alternatives to feedback control. These are important for high-speed manipulation; at sufficiently high frequencies the behavior of any controlled system is dominated by its open loop behavior. In Part II it was shown that multiple actuators and "excess" linkage degrees of freedom may be used to modulate end-point impedance. It is suggested that the primate central nervous system uses these non-feedback strategies and that the apparent redundancies in the primate musculoskeletal system may in fact play an essential functional role in controlling interactive behavior.

In this third part of the paper it was shown that in general, the impedance appropriate to a given task may be deduced from the task objective, and a method which uses optimization theory to do this was presented. Although the examples presented were extremely simple, they retained the structure of the basic manipulation problem, represented by the generalized Norton equivalent network coupled to an admittance. The static example led to an instructive result: while power transmission requires machine impedance to match environmental impedance, manipulation (trading off movement errors against interface forces) requires a machine impedance proportional to environmental admittance; power transmission and manipulation are, in a sense, "orthogonal" tasks. The dynamic example showed that the appropriate impedance can be expressed in terms of force and motion tolerances independent of the way the impedance is implemented e.g., without assuming feedback control. The method used is general and has been applied to a nonlinear system.

The concept of tuning the dynamic behavior of a manipulator has been explored by a number of researchers. However, most of this prior work considered only linear dynamic behavior and implemented it as an admittance (force in, motion out). The restriction to linearity is not necessary and as shown in the collision-avoidance example, nonlinear behavior has its uses. The restriction to admittance causality is not consistent with the physical constraints of interacting with a (possibly constrained) inertial environment. That approach might be justified by arguing that the environment could be modelled as an impedance, (e.g., a spring [18, 28]); Unfortunately, admittances coupled to an impedance at a common point (the end-effector of the robot) do not enjoy the superposition properties of impedances coupled to an admittance at a common point. Impedance control offers a significant advantage over this alternative.

The practical value of the additive property of nonlinear impedances was shown in this third part of the paper by using it to develop a feedback control law for avoiding unpredictably moving objects. By taking advantage of the superposition of impedances, target acquisition and obstacle avoidance could be considered separately and implemented as different components of a total commanded impedance which were combined by simple addition. This approach does not require explicit path planning and the control law was simple enough to be implemented using 8-bit MHz microprocessors. Note, however, that impedance control does not preclude a preplanning or navigational approach and the two methods may usefully complement one another; path-planning is appropriate for the predictable aspects of the environment, impedance control offers a method for dealing with its less predictable aspects.

The choice of a realistic but appropriately simple form for the impedance to be imposed leads to a dramatic simplification of the problems of controlling the complete system (manipulator and environment). Restricting attention to impedances with exact differentials (force fields with zero curl) permits the definition of potential functions for the position- and velocity-dependent behavior. Because of the simple form of the imposed dynamic equations the (elastic) potential function and the external forces are sufficient to define static stability. Asada [4] has shown how elastic fields may be used as the basis of an approach to planning stable grasp. Stable equilibrium configurations of end-effector and workpiece are defined by finding minima of the potential energy function. Gravitational forces are readily included by expressing them as a potential function and combining it with the potential function of the manipulator by simple addition. Note, however, that the dynamic stability of the end-effector is not guaranteed (that is, in principle, sustained oscillations are possible). To ensure dynamic stability the dissipative field must be chosen appropriately; the complete impedance must be controlled, not just the elastic behavior.

The use of potential functions in effect maps the end-point dynamics into a set of static functions and the visualization, prediction and planning of the behavior of the complete system is simplified. For example, in the absence of external active sources the total energy of the system, kinetic plus potential, may never increase. This permits easy prediction of the maximum velocities which may result from a given set of commands without computing the detailed trajectories. Conversely, as the potential energy function is one of the commands, it is readily chosen so that a desired maximum velocity is never exceeded. If the impedance command is given when the system is at zero velocity (e.g., a workpiece has just been grasped) then it is not even necessary to know the mass of the grasped object.

A feature of impedance control is that it permits a unified treatment of many aspects of manipulator control. The actions of both controller software and manipulator hardware may be described through an equivalent physical system. As a result powerful methods (such as bond graphs) for network analysis of nonlinear systems may profitably be applied. Real mechanical interaction may be treated in the same framework as free (unconstrained) motions. The impedance controller used to avoid unpredictably moving objects was also capable of coping with kinematically constrained motions [1, 2]. Targets to be acquired are treated in the same way as obstacles to be avoided as different components of a total task, where each component is described by a generalized Norton equivalent network. Path control [20, 25], rate control [26, 27], and acceleration control [16], could be considered in a single framework as important special cases of impedance control (e.g., position control: maximize impedance; rate control: no static impedance component). Pure force control [11] (force commanded as a function of time only) could also be considered in the same framework by regarding it as a special case in which the impedance is purely elastic. A potential function with a constant gradient defines the magnitude of the commanded force, and the virtual position (which may go outside the workspace) defines the direction of the commanded force. The hybrid combination of force and position control in orthogonal directions [17, 23] proposed for dealing with pure kinematic constraints is also included under impedance control.

Most important, the applicability of impedance control extends beyond the workless conditions imposed by free motions or pure kinematic constraints to include the control of energetic interactions such as are encountered when using a power tool. It promises to be particularly useful for understanding, controlling and coordinating the actions of mutually interacting manipulators, such as the fingers of a hand, the hand and the arm, or two arms. Using this approach each subsystem presents a simple behavior to the other subsystems; This will facilitate the prediction and control of the combined behavior of the entire system.

An alternative approach to manipulator control in the presence of significant dynamic interaction is to change the structure and/or parameters of a feedback controller as the conditions imposed by the environment change. This would require the controller to monitor the environment continuously, identify changes, and adapt its own behavior accordingly – a far-from-trivial task. Changes in the structure and parameters of the environment may take place very rapidly (consider the transition from free motion to constrained motion as an object comes in contact with a surface) and there may not be sufficient time for the usually lengthy process of system identification. On the other hand, if the controller is structured so that the manipulator always impresses a force on the environment in relation to its motion (that is, it behaves as an impedance) there are no practical situations in which its behavior is inappropriate, no practical task has been excluded, and the need to identify the structure of the environment has been reduced.

Of course, impedance control does not preclude the application of adaptive strategies, and indeed the two approaches may complement each other, controlled impedance taking care of the transitions and allowing time for identification and adaptation to optimize performance. Strictly speaking, impedance control is a subset of parameter-adaptive control; the primary distinctions are that the parameters to be modulated are expressed in terms of a physically meaningful quantity, mechanical impedance, and unlike other work on parameter adaptation, no assumption is made that the implementation of the impedance will be through feedback control strategies. An impedance may be implemented in a number of ways, using to advantage the resources of a specific manipulator.

Essentially, impedance control is an attempt to combine the control of "transport" tasks (which are the philosophical underpinning of conventional robot control) with the control of "interactive" tasks such as the use of a tools. The ultimate goal of this work is to understand the subleties of adaptive tool-use, one of the distinguishing features of primate behavior. Impedance control may provide the basis for understanding tool-using behavior in primates, restoring this capability to an amputee using an artificial limb, and implementing it on an industrial robot.

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APPENDIX I

Optimal Impedance for a One-Dimensional Dynamic System

The system equations in phase variable form are:

$$\begin{bmatrix} \dot{X} \\ \dot{V} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix} \begin{bmatrix} X \\ V \end{bmatrix} + \begin{bmatrix} 0 \\ k/m \end{bmatrix} X_0 + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} F\mathrm{ext}$$

The interface force is: $F = k(X_0 - X) + bV$

The objective function to be minimized is:

$$Q = \int_0^\infty \{F/F\mathrm{tol})^2 + [(X_0 - X)/X\mathrm{tol}]^2\}\,dt$$

The external force Fext is a zero-mean, Gaussian, purely random process of strength S. Thus:

$$E[F\mathrm{ext}(t)] = 0 \quad E[F\mathrm{ext}(t)F\mathrm{ext}(t+\tau)] = S\delta(\tau)$$

In steady state $\bar{X} = X_0$, $\bar{V} = 0$ thus without loss of generality assume $X_0 = 0$. The covariance propagation equations are:

$$\begin{aligned} \dot{\overline{X^2}} &= 2\overline{XV} \\ \dot{\overline{XV}} &= \overline{V^2} - \frac{b}{m}\overline{XV} - \frac{k}{m}\overline{X^2} \\ \dot{\overline{V^2}} &= \frac{S}{m^2} - 2\frac{b}{m}\overline{V^2} - 2\frac{k}{m}\overline{XV} \end{aligned}$$

Because of the random forcing, the optimum impedance is obtained by minimizing the expectation of the objective function subject to the constraints imposed by the covariance propagation equations. Writing $p^2 = F\mathrm{tol}/X\mathrm{tol}$

$$E[Q] = \frac{1}{F\mathrm{tol}^2}\int_0^\infty \{b^2\overline{V^2} + 2k\,b\overline{XV} + (k^2 + p^2)\overline{X^2}\}\,dt$$

The Pontryagin function is:

$$\begin{aligned} H &= b^2\overline{V^2} + 2kb\overline{XV} + (k^2+p^2)\overline{X^2} + 2\lambda_1\overline{XV} \\ &+ \lambda_2\left(\overline{V^2} - \frac{b}{m}\overline{XV} - \frac{k}{m}\overline{X^2}\right) \\ &+ \lambda_3\left(\frac{s}{m^2} - 2\frac{b}{m}\overline{V^2} - 2\frac{k}{m}\overline{XV}\right) \end{aligned}$$

The minimizing conditions are:

$$\frac{\partial H}{\partial k} = 0 = 2b\overline{XV} + 2k\overline{X^2} - \frac{\lambda_2}{m}\overline{X^2} - \frac{2\lambda_3}{m}\overline{XV}$$

$$\frac{\partial H}{\partial b} = 0 = 2b\overline{V^2} + 2k\overline{XV} - \frac{\lambda_2}{m}\overline{XV} - \frac{2\lambda_3}{m}\overline{V^2}$$

The LaGrange multipliers are determined from the costate equations:

$$\frac{\partial H}{\partial \overline{XV}} = -\dot{\lambda}_1 = (k^2+p^2) - \lambda_2\frac{k}{m}$$

$$\frac{\partial H}{\partial \overline{XV}} = -\dot{\lambda}_2 = 2kb + 2\lambda_1 - \lambda_2\frac{b}{m} - 2\lambda_2\frac{k}{m}$$

$$\frac{\partial H}{\partial \overline{V^2}} = -\dot{\lambda}_3 = b^2 + \lambda_2 - 2\lambda_3\frac{b}{m}$$

Assuming a steady-state solution exists, it may be obtained by setting all rates of change to zero. Manipulating the resulting equations yields:

$$\overline{XV} = 0 \qquad \overline{V^2} = \frac{S}{2bm} \qquad \overline{X^2} = \frac{S}{2bk}$$

$$k_{\text{opt}}^2 = p^2 \qquad k_{\text{opt}} = F\text{tol}/X\text{tol}$$

$$b_{\text{opt}}^2 = 2k_{\text{opt}}m \qquad b_{\text{opt}} = \sqrt{2k_{opt}m}$$