

Hypothesis Testing for Population Proportion

Requirements:

Random sample

$n \cdot p \geq 10$; $n \cdot q \geq 10$

n is sample size

Traditional Method (Non-Bayesian Testing):

Step 1: Define Claim and Opposite, H_0 (contains equal sign) and H_1

Step 2: Define significance level (alpha)

Step 3: Calculate Z-test statistic

$$Z_t = \frac{\hat{p} - p}{\sqrt{(p \cdot q) / n}}$$

\hat{p} - sample proportion of success

p and q are hypothetical values (success and failure)

$$q = 1 - p$$

Step 4*: Draw a picture: according to H_1 it is left-tail, right-tail or two-tail

On the picture put Z-critical value with corresponding alpha from Z-table

Step 5: Interpret results:

If Z-test statistic is in Rejection Region \Rightarrow Reject H_0 and accept H_1

If the Z-test statistic is in the Fail to Rejection Region \Rightarrow We know nothing! There is not enough evidence to accept H_1

P-value method (Bayesian Testing):

Step 1: Define Claim and Opposite, H_0 (contains equal sign) and H_1

Step 2: Define significance level (alpha)

Step 3: Calculate Z-test statistic

$$Z_t = \frac{\hat{p} - p}{\sqrt{(p \cdot q) / n}}$$

\hat{p} - sample proportion of success

p and q are hypothetical values (success and failure)

Step 4*: Draw a picture: according to H_1 it is left-tail, right-tail or two-tail

On the picture put Z-test statistic

P-value is : 1 - Area (Fail to Rejection Region); P-value is the area in the Tail;

If we have two tails we need to multiply by 2

Step 5: Interpret results:

If P-value \leq alpha \Rightarrow Reject H_0 and accept H_1

If P-value $>$ alpha \Rightarrow Fail to Reject H_0 ; We know nothing! There is not enough evidence to accept H_1

Hypothesis Testing with Two Proportions:

Traditional Method (Non-Bayesian Testing)

Step 1: Define Claim and Opposite, H_0 and H_1

Step 2: Define significance level (alpha)

Step 3: Calculate Z-test statistic:

$$Z_t = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}_{\text{total}} \cdot (1 - \hat{p}_{\text{total}}) \cdot \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$p_1 - p_2$ is hypothetical difference
 \hat{p}_{total} is $(X_1 + X_2) / (n_1 + n_2)$

Step 4*: Draw a picture: according to H_1 it is left-tail, right-tail or two-tail

On the picture put Z-critical value with corresponding alpha from Z-table

Step 5: Interpret results:

If Z-test statistic is in Rejection Region \Rightarrow Reject H_0 and accept H_1

If the Z-test statistic is in the Fail to Rejection Region \Rightarrow We know nothing! There is not enough evidence to accept H_1

Hypothesis Testing for Population Mean. Population standard deviation is known

Requirements:

Random sample

Population standard deviation is known

$n > 30$ or Population is normally distributed

n is sample size

(If $n < 30$ we use T distribution)

Traditional Method (Non-Bayesian Testing):

Step 1: Define Claim and Opposite, H_0 (contains equal sign) and H_1

Step 2: Define significance level (alpha)

Step 3: Calculate Z-test statistic:

$$Z_t = \bar{X} - \mu / \sigma / \sqrt{n}$$

\bar{X} - sample mean

μ - population hypothetical mean

σ - population standard deviation

n - sample size

Step 4*: Draw a picture: according to H_1 it is left-tail, right-tail or two-tail

On the picture put Z-critical value with corresponding alpha from Z-table

Step 5: Interpret results:

If Z-test statistic is in Rejection Region \Rightarrow Reject H_0 and accept H_1

If the Z-test statistic is in the Fail to Rejection Region \Rightarrow We know nothing! There is not enough evidence to accept H_1

P-value method (Bayesian Testing):

Step 1: Define Claim and Opposite, H_0 (contains equal sign) and H_1

Step 2: Define significance level (alpha)

Step 3: Calculate Z-test statistic

$$Z_t = \bar{X} - \mu_0 / \sigma / \sqrt{n}$$

\bar{X} - sample mean

μ_0 - population hypothetical mean

σ - population standard deviation

n - sample size

Step 4*: Draw a picture: according to H_1 it is left-tail, right-tail or two-tail

On the picture put Z-test statistic

P-value is : $1 - \text{Area (Fail to Rejection Region)}$; P-value is the area in the Tail;

If we have two tails we need to multiply by 2

Step 5: Interpret results:

If $P\text{-value} \leq \alpha \Rightarrow$ Reject H_0 and accept H_1

If $P\text{-value} > \alpha \Rightarrow$ Fail to Reject H_0 ; We know nothing! There is not enough evidence to accept H_1

Hypothesis Testing with Two Means.

Populations are independent and standard deviations σ_1, σ_2 are known

Requirements:

Random sample

Population standard deviation is known

$n > 30$ or Population is normally distributed

n is sample size

(If $n < 30$ we use T distribution)

Traditional Method (Non-Bayesian Testing):

Step 1: Define Claim and Opposite, H_0 and H_1

Step 2: Define significance level (α)

Step 3: Calculate Z-test statistic:

$$Z_t = (\bar{X}_1 - \bar{X}_2) - D_0 / \sqrt{((\sigma_1^2 / n_1) + (\sigma_2^2 / n_2))}$$

D_0 - Hypothetical difference

Step 4*: Draw a picture: according to H_1 it is left-tail, right-tail or two-tail

On the picture put Z-critical value with corresponding α from Z-table

Step 5: Interpret results:

If Z-test statistic is in Rejection Region \Rightarrow Reject H_0 and accept H_1

If the Z-test statistic is in the Fail to Rejection Region \Rightarrow We know nothing! There is not enough evidence to accept H_1

Hypothesis Testing with Two Means.

Populations are independent and standard deviations σ_1, σ_2 are unknown

a) First approach - assumption that (σ_1^2 / n_1) is not equal to (σ_2^2 / n_2)

Traditional Method (Non-Bayesian Testing)

Step 1: Define Claim and Opposite, H_0 and H_1

Step 2: Define significance level (α)

Step 3: Calculate T-test statistic:

$$T_t = (X1_bar - X2_bar) - D_0 / \sqrt{((s1 * s1) / n1) + ((s2 * s2) / n2)}$$

D_0 - Hypothetical difference

$s1$ and $s2$ - sample standard deviation

Step 4*: Draw a picture: according to H_1 it is left-tail, right-tail or two-tail

On the picture put T-critical value with corresponding alpha from T-table and DF

Degree of Freedom = $\min(n1 - 1; n2 - 1)$; take the minimum of these two

Step 5: Interpret results:

If T-test statistic is in Rejection Region => Reject H_0 and accept H_1

If the T-test statistic is in the Fail to Rejection Region => We know nothing! There is not enough evidence to accept H_1

b) Second approach - assumption that $(\sigma_1 * \sigma_1)$ is equal to $(\sigma_2 * \sigma_2)$

Traditional Method (Non-Bayesian Testing)

Step 1: Define Claim and Opposite, H_0 and H_1

Step 2: Define significance level (alpha)

Step 3: Calculate T-test statistic:

$$T_t = (X1_bar - X2_bar) - D_0 / \sqrt{(((n1 - 1) (s1 * s1)) + ((n2 - 1) (s2 * s2))) / (n1 + n2 - 2)) * \sqrt{(1 / n1) + (1 / n2))}$$

D_0 - Hypothetical difference

$s1$ and $s2$ - sample standard deviation

Step 4*: Draw a picture: according to H_1 it is left-tail, right-tail or two-tail

On the picture put T-critical value with corresponding alpha from T-table and DF

Degree of Freedom = $n1 + n2 - 2$

Step 5: Interpret results:

If T-test statistic is in Rejection Region => Reject H_0 and accept H_1

If the T-test statistic is in the Fail to Rejection Region => We know nothing! There is not enough evidence to accept H_1

Hypothesis Testing with Two Means.

Paired data

Traditional Method (Non-Bayesian Testing):

Step 1: Define Claim and Opposite, H_0 and H_1

Step 2: Define significance level (alpha)

Step 3: Calculate T-test statistic:

$$T_t = (X1_bar - X2_bar) - D_0 / s_d / \sqrt{n}$$

D_0 - Hypothetical difference

s_d - standard deviation difference; $S1 - S2$ or $\sigma_1 - \sigma_2$ (and then we have Z-test statistic)

Step 4*: Draw a picture: according to H_1 it is left-tail, right-tail or two-tail

On the picture put T-critical value with corresponding alpha from T-table and DF

Degree of Freedom = $n_1 + n_2 - 2$

Step 5: Interpret results:

If T-test statistic is in Rejection Region \Rightarrow Reject H_0 and accept H_1

If the T-test statistic is in the Fail to Rejection Region \Rightarrow We know nothing! There is not enough evidence to accept H_1

Hypothesis Testing for Population Mean. Population Standard Deviation is unknown

Requirements:

Random sample

Population standard deviation is unknown

$n > 30$ or Population is normally distributed

n is sample size

(If $n < 30$ we use T distribution too)

Traditional Method (Non-Bayesian Testing):

Step 1: Define Claim and Opposite, H_0 (contains equal sign) and H_1

Step 2: Define significance level (alpha)

Step 3: Calculate T-test statistic:

$$T_t = \frac{\bar{X} - \mu}{s / \sqrt{n}}$$

\bar{X} - sample mean

μ - population hypothetical mean

s - sample standard deviation

n - sample size

Step 4*: Draw a picture: according to H_1 it is left-tail, right-tail or two-tail

On the picture put T-critical value with corresponding alpha from T-table and DF

Step 5: Interpret results:

If T-test statistic is in Rejection Region \Rightarrow Reject H_0 and accept H_1

If the T-test statistic is in the Fail to Rejection Region \Rightarrow We know nothing! There is not enough evidence to accept H_1

Hypothesis Testing for Variance and Standard Deviation Chi-squared distribution

Traditional Method:

Step 1: Define Claim and Opposite, H_0 (contains equal sign) and H_1

Step 2: Define significance level (alpha)

Step 3: Calculate Test statistic:

$$\chi^2 = (n - 1) (s * s) / (\sigma * \sigma)$$

Step 4*: Draw a picture: according to H_1 it is left-tail, right-tail or two-tail; Keep in mind that distribution starts from Zero and it's only Right-Skewed

On the picture put Chi-squared critical value with corresponding alpha and DF

Be careful if it is left-skewed (1 - alpha) and after that look in the table

Step 5: Interpret results:

If Test statistic is in Rejection Region => Reject H_0 and accept H_1

If the Test statistic is in the Fail to Rejection Region => We know nothing! There is not enough evidence to accept H_1

Hypothesis Testing for Comparing Two Variances - Two independent Samples F- distribution

Step 1: Define Claim and Opposite, H_0 (contains equal sign) and H_1

Step 2: Define significance level (alpha)

Step 3: Calculate Test statistic:

$$F = \text{variance}(\text{larger}) / \text{variance}(\text{smaller})$$

Step 4*: Draw a picture: it is always upper-tailed

On the picture put F-critical value which is based on DF of these two samples and alpha, which is calculated with calculator (or in Python)

Step 5: Interpret results:

If Test statistic is in Rejection Region => Reject H_0 and accept H_1

If the Test statistic is in the Fail to Rejection Region => We know nothing! There is not enough evidence to accept H_1

Hypothesis Testing: Chi-Square Test

It helps us to understand the relationship between two categorical variables:

grade level, sex, age group, year. Chi-Square test involve the frequency of events; the count; Expected Vs Observed categorical distribution

Example - determine if this die is a fair or not with 95% certainty; 600 trials for the next 6 days;

Example - a school principal expected that students will be absent equally during the 5-day school week;

Step 1: Construct two tables: Observed vs Expected value

Step 2: Define Claim and Opposite, H_0 (contains equal sign) and H_1

Step 3: Define significance level (alpha)

Step 4: Calculate Test statistic:

$$\text{Chi-Squared} = \sum ((\text{Observed} - \text{Expected})^2 / \text{Expected})$$

Step 5: Draw a picture:

Keep in mind that distribution starts from Zero and it's only Right-Skewed

On the picture put Chi-Squared critical value which is based on $DF(n - 1)$ and α , which is calculated with calculator (or with Python)

Step 6: Interpret results:

If Test statistic is in Rejection Region \Rightarrow Reject H_0 and accept H_1

if the Test statistic is in the Fail to Rejection Region \Rightarrow We know nothing! There is not enough evidence to accept H_1

Test of Independence Using Chi-Square Distribution

(watch the video again in your playlist)

Example: Is the average number of studying hours depend on the type of student;

Step 1: Construct two tables: Observed vs Expected value ($E.V = \text{Row}_{\text{total}} * \text{Col}_{\text{total}} / N$)

Step 2: Define Claim and Opposite, H_0 (contains equal sign) and H_1

Step 3: Define significance level (α)

Step 4: Calculate Test statistic:

$$\text{Chi-Squared} = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

Step 5: Draw a picture:

Keep in mind that distribution starts from Zero and it's only Right-Skewed

On the picture put Chi-Squared critical value which is based on $DF(n - 1)$ and α , which is calculated with calculator (or with Python)

Step 6: Interpret results:

If Test statistic is in Rejection Region \Rightarrow Reject H_0 and accept H_1

if the Test statistic is in the Fail to Rejection Region \Rightarrow We know nothing! There is not enough evidence to accept H_1