Instructional Module

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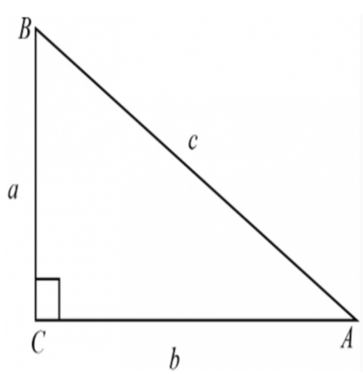
Derivatives of Trigonometric Functions

Introduction

This module will briefly discuss Derivatives of Transcendental Functions particularly Trigonometric Functions. The principles for Trigonometric Functions rely heavily on the principles of Pythagorean Theorem. In the right triangle $\triangle ABC$ shown below,$\angle BCA$ or simply $\angle C$ measures 90° (hence making it a right triangle). The legs of the triangle ABC are the line segments b and a, the lines on either side of $\angle C$, also called *legs.* Meanwhile, the *hypotenuse* of the triangle is the line segment c which is opposite $\angle C$.



The Pythagorean theorem states that the sum of the squares of the measures of the legs a and b of $\triangle ABC$ is equal to the square of the measure of the hypotenuse c that is $a^2 + b^2 = c^2$. By laying the given right triangle $\triangle ABC$ in a unit circle as shown below, the formula $a^2 + b^2 = c^2$ becomes $a^2 + b^2 = 1$. Recall that the radius of a unit circle is equal to 1 unit, which in the triangle shown below, corresponds to the hypotenuse c.

Trigonometric functions, also known as Circular Functions, can be defined as the functions of an angle of a triangle. Trig functions identify the relationships between the angles and sides of a triangle. The basic trigonometric functions are sine, cosine, tangent, cotangent, secant and cosecant.

The Trigonometric Functions

Solving for the values of the legs a and b of the triangle above leads us to the use of the SOHCAHTOA formulas, which stemmed from the three trigonometric functions sine (sin), cosine (cos), and tangent (tan).

The Sine Function

The *sine* of an angle is the ratio between the length of the opposite side to the length of the hypotenuse. From the above diagram, the value of the sine of $\angle BAC$ will be:

- **Sin A** $= \frac{Opposite}{Hypotenuse} = \frac{CB}{BA}$

The Cosine Function

The *cosine* of an angle is the ratio of the length of the adjacent side to the length of the hypotenuse. From the above diagram, the cos function will be derived as follows:

- **Cos A** $= \frac{Adjacent}{Hypotenuse} = \frac{CA}{BA}$

The Tangent Function

The *tangent function* is the ratio of the length of the opposite side to that of the adjacent side. From the diagram taken above, the tan function will be the following:

- **Tan A** $= \frac{Opposite}{Adjacent} = \frac{CB}{CA}$

The tangent can also be expressed in terms of sin and cos as follows:

- **Tan A** $= \frac{SinA}{CosA} = \frac{CB}{CA}$

Other Trigonometric Formulas

Aside from the initial three functions, there are other trigonometric formulas as shown in the following:

- **Sec A** $= \frac{1}{CosA} = \frac{Hypotenuse}{Adjacent} = \frac{BA}{CA}$
- **Cosec A** $= \frac{1}{SinA} = \frac{Hypotenuse}{Opposite} = \frac{BA}{CB}$
- **cot a** $= \frac{1}{TanA} = \frac{Adjacent}{Opposite} = \frac{CA}{CB}$