Paints-R-Us Term Project

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Model Overview

This project will consider a linear product mix optimization problem for a fictional paint company, Paints-R-Us. Paints-R-Us is a wholesale paint manufacturer located in the Pacific Northwest. The Global Production Manager, Steve Brush, has been tasked with maximizing Paints-R-Us's profit in the upcoming quarter. Steve Brush oversees the global production plan, and in collaboration with the production planners will develop a production plan which optimizes the profits that Paints-R-Us can create in the quarter accounting for the following criteria:

- Demand in the given quarter for each of the 5 paint types that Paints-R-Us produces.
- The warehousing storage capacity for the production facility.
- The throughput rate of the 5 paint types that Paints-R-Us manufactures on the 5 operating production lines that Paints-R-Us operates.
- The delivery requirements of specific paints on specific days per pre planned customer orders.

The output that Steve must produce is a production schedule for each production line during the quarter which maximizes the profit that the business can achieve. The production plan is decided by the upcoming demand. Steve believes more profit can be achieved through a production schedule that is derived through linear programming. However, the challenge is mathematically representing the production process and constraints. Steve has brought together his production planners to develop the model that will generate the production schedule which accommodates for committed demand and maximizes profit.

The Northwest facility has booked orders for the next 90-days, all of which must be fulfilled by the specified delivery date. Additionally, there is additional market demand that is filled through on-demand sales. Paints-R-Us must have a production plan which meets their booked order requirements and doesn't exceed the forecasted total on-demand sales projections for any of their 5 product lines.

Each of the 5 production lines have a unique production rate for the number of barrels of different paints they can produce per day. Some production lines are restricted on the types of paint that they can produce, and some production lines can create certain types of paint at a faster rate than other paints. The supply chain can provide the required raw materials to meet the maximum paint production rate for the quarter. The production plan feeds supply chain planning and thus is not a constraint for production planning purposes.

Another factor that Steve must consider is for any production line to switch to a different paint, the production line must be shutdown for a day. This shutdown is so that the production line can be cleaned and set up for the production of the new paint type.

Finally, Paints-R-Us has warehousing constraints. Their production plan must account for the amount of expected sales per day and ensure that paint production beyond the daily paint demand doesn't exceed Paints-R-Us's warehousing capacity.

Steve and his production planning team have partnered with local PSU Graduate students to build a model which can product the optimized production schedule and can also be easily adapted for future quarters or for longer forecast periods.

Variables

Let

i = the product code, $i = 1, 2..., I$
j = the index of periods, $j = 1, 2..., J$
k = the production line used, $k = 1, 2..., K$
P_i = profit per unit sold of product i
D_i = forecasted maximum number of units sold of product i in all of period j
$D_{i,j}$ = committed sales in units of product i on period j
W = maximum storage capacity of units within the warehouse on any period j
$R_{i,k}$ = unit throughput of product i
on production line k per period j

The decision variables are

$S_{i,j}$ = number of units sold of product i on period j
$T_{i,j}$ = number of units produced of product i on period j
$X_{i,j,k} \in \{0, 1\}$ do we produce product i on period j on production line k

Linear Program

The linear programming formulation is

$$\max \quad \sum_i \sum_j P_i \cdot S_{i,j}$$

The formula looks to maximize profit. It does this by measuring the profit per barrel for each paint type Paints-R-Us sells on the market. It then multiplies that profit by the total barrels sold throughout the period.

Constraints

Demand

We must have enough paint products in stock to meet our demands. The way we implement this in the model is by setting up two demand constraints. The first constraint grabs the sum of paint barrels sold for the day $\sum_j S_{i,j}$, and sets this value less than or equal to the demand of paint products $\leq D_i$, for all paint products $\forall i$. The second constraint tells the model the amount of barrels sold for paint products during the day $S_{i,j}$ must equal the demand of paint products for the day $D_{i,j}$.

We expect the paint product to fluxate between five scenarios:

1. High Profit Margin P, High Demand D

2. High Profit Margin P, Low Demand D

3. Average Profit Margin P, Average Demand D

4. Average Profit Margin P, High Demand D

5. Low Profit Margin P, Average Demand D.

$$\sum_j S_{i,j} \leq D_i \quad \forall i$$
$$S_{i,j} = D_{i,j}$$

Storage

Our facility has a finite amount of storage, but if we don't store the products then we cannot sell the products. We address these constraints by keeping a running total.

For the upper bound, we only have so much storage available. Therefore, we calculate the number of barrels produced of paint i for time period j. We then subtract the number of barrels sold of paint i for the same time period j and ensure that figure is less than our warehouse capacity W, for all time periods up to j.

For the lower bound, we can sell all the paint in the warehouse, but we cannot sell what we don't have in the warehouse. Therefore, we calculate the number of barrels produced of paint i for time period j and subtract the number of barrels sold of paint i for the same time period j and ensure that figure is greater than or equal to zero, for all time periods up to j.

Upper Bound - We only have so much storage available

$$\sum_i \sum_{j=1}^{j} T_{i,j} - S_{i,j} \leq W \quad \forall j$$

Lower Bound - We cannot sell what we do not have in storage

$$\sum_{j=1}^{j} T_{i,j} - S_{i,j} \geq 0 \quad \forall i,j$$

Throughput

The amount of total barrels of each of the 5 types of paint produced each day must be accounted for so that it can be moved to storage. To account for this constraint it is specified that the rate of paint i producted on machine k on machines where that paint was produced on that day on that machine will be equal to the amount of barrels produced variable. This will be done for each machine for all paints and on all days.

$$\sum_k R_{i,k} \cdot X_{i,j,k} = T_{i,j} \quad \forall i,j$$

Production

There are 2 production constraints that must be accounted for in the model:

1. Only one type of paint can be produced on a production line in a given day. To account for this it is specified that for all days on all production lines the the sum of product types produced must equal 1:

$$\sum_i X_{i,j,k} \leq 1 \quad \forall j,k$$

2. When a production line is changed to produce a different product i the production line k must shut down for one day to conduct setup for the production of the new product:

$$X_{i_1,j,k} + X_{i_2,j+1,k} \leq 1 \quad \forall\, i_1 = 1...I,\ i_2 = 1...I,\ i_1 \neq i_2, j = 1...J_{j-1}, k$$

Logic Constraints

1. The amount of barrels of paint sold for paint i on day j cannot be negative:

$$T_{i,j} \geq 0$$

2. The amount of barrels produced of paint i on day j cannot be negative:

$$S_{i,j} \geq 0$$

3. The choice to produce paint i on day j on product line k is a binary decision:

$$X_{i,j,k} \in \{0,1\}$$

Model

Let

i = the product code, $i = 1, 2..., I$
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The decision variables are

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$T_{i,j}$ = number of units produced of product i on period j
$X_{i,j,k} \in \{0,1\}$ do we produce product i on period j on production line k

The linear programming formulation is

$$\max \quad \sum_i \sum_j P_i \cdot S_{i,j}$$

$$\text{s.t} \quad \sum_j S_{i,j} \leq D_i \qquad \forall i$$

$$S_{i,j} \geq D_{i,j}$$

$$\sum_i \sum_{j=1}^{j} T_{i,j} - S_{i,j} \leq W \quad \forall j$$

$$\sum_{j=1}^{j} T_{i,j} - S_{i,j} \geq 0 \qquad \forall i,j$$

$$\sum_k R_{i,k} \cdot X_{i,j,k} = T_{i,j} \quad \forall i,j$$

$$\sum_i X_{i,j,k} \leq 1 \qquad \forall j,k$$

$$X_{i_1,j,k} + X_{i_2,j+1,k} \leq 1 \quad \forall i_1 = 1...I, I_2 = 1...I, i_1 \neq i_2, j = 1...J_{j-1}, k$$

$$T_{i,j}, S_{i,j} \geq 0$$

$$X_{i,j,k} \in \{0,1\}$$

Define Variables

Total Warehousing Capacity

Total Warehousing, W, capacity at any given time for the northwest facility is 100,000 barrels of paint.

$$W = 100,000 \; Barrels$$

Minimum Order Requirements

Paints-R-Us largest customer has standing orders for the products that Paints-R-Us sells throughout the quarter. The amount of paint produced and sold must meet the defined constraints on the specified day:

$$D_{1,15} = 1000 \; Barrels$$
$$D_{2,19} = 2500 \; Barrels$$
$$D_{3,45} = 7500 \; Barrels$$

Maximum Demand Constraint

Within the forecast period each product has a maximum forecasted demand, Paints-R-Us forecasted sales cannot exceed this demand for each product type:

$$D_1 = 20000 \; Barrels$$
$$D_2 = 20000 \; Barrels$$
$$D_3 = 36000 \; Barrels$$
$$D_4 = 75000 \; Barrels$$
$$D_5 = 100000 \; Barrels$$

Daily Production Rate Constraint

Each production line can produce barrels of paint per day rate as defined:

Production line 1

$$R_{1,1} = 75 \; Barrels$$
$$R_{1,2} = 100 \; Barrels$$
$$R_{1,3} = 150 \; Barrels$$
$$R_{1,4} = 100 \; Barrels$$
$$R_{1,5} = 200 \; Barrels$$

Production line 2

$$R_{2,1} = 100 \; Barrels$$
$$R_{2,2} = 160 \; Barrels$$
$$R_{2,3} = 250 \; Barrels$$
$$R_{2,4} = 0 \; Barrels$$
$$R_{2,5} = 0 \; Barrels$$

Production line 3

$$R_{3,1} = 0 \; Barrels$$
$$R_{3,2} = 0 \; Barrels$$
$$R_{3,3} = 180 \; Barrels$$
$$R_{3,4} = 200 \; Barrels$$
$$R_{3,5} = 225 \; Barrels$$

Production line 4

$$R_{4,1} = 100 \; Barrels$$
$$R_{4,2} = 100 \; Barrels$$
$$R_{4,3} = 175 \; Barrels$$
$$R_{4,4} = 175 \; Barrels$$
$$R_{4,5} = 175 \; Barrels$$

Production line 5

$$R_{5,1} = 125 \; Barrels$$
$$R_{5,2} = 200 \; Barrels$$
$$R_{5,3} = 0 \; Barrels$$
$$R_{5,4} = 175 \; Barrels$$
$$R_{5,5} = 180 \; Barrels$$

Profit per barrel of paint

Each product line produces profit as defined:

$$P_1 = 170 \; Per \; Barrel$$
$$P_2 = 135 \; Per \; Barrel$$
$$P_3 = 75 \; Per \; Barrel$$
$$P_4 = 25 \; Per \; Barrel$$
$$P_5 = 15 \; Per \; Barrel$$

R Code

```
### ---------------- Load R Packages --------------------------------
library(ggplot2, quietly = TRUE)          # Used for graphs
library(magrittr, quietly = TRUE)         # Provides the piping command %>%
library(pander, quietly = TRUE)           # Used for nicely formatted tables
library(ompr, quietly = TRUE)             # Allows specifying model algebraically
library(ompr.roi, quietly = TRUE)         # Glue for ompr to solve with ROI
library(ROI, quietly = TRUE)              # R Optimization Interface
```

```
## ROI: R Optimization Infrastructure

## Registered solver plugins: nlminb, glpk.

## Default solver: auto.
```

```r
library(ROI.plugin.glpk, quietly = TRUE)    # Plugin for solving
library(writexl)                            # Write to Excel
library(candela, quietly = TRUE)            # Use Candela to create a Gnatt Chart

#' Parameters
NPaint <- 5        #' Product Code for paint
NDay   <- 60       #' Index of Time Period
NProd  <- 5        #' Production Line
whse   <- 100000   #' Max capacity of warehouse

#' Create a matrix with names Paint 1... Paint 5
Paint <- lapply(list(rep("Paint ",NPaint)),paste0,1:NPaint)

#' Create a matrix with names Production Line 1... Production Line 5
ProdLine <- lapply(list(rep("Production Line ",NProd)),paste0,1:NProd)

#' Create a matrix of Production Rate (# of barrels to make for R(i,k)
ProductionRate <- matrix(c(75,100,150,100,200    #' Production Line 1
                          ,100,160,250,0,0        #' Production Line 2
                          ,0,0,180,200,225        #' Production Line 3
                          ,100,100,175,175,175    #' Production Line 4
                          ,125,200,0,175,180),    #' Production Line 5
                    ncol=NPaint,nrow=NProd, dimnames=c(Paint, ProdLine))

#' Show data from production line #1
ProductionRate[,1]
```

```
## Paint 1 Paint 2 Paint 3 Paint 4 Paint 5
##      75     100     150     100     200
```

```r
#' Display the Matrix in a nicely formatted table
pander(ProductionRate, caption = attr(ProductionRate,"Number of Barrels to make for R{i,k}"))
```

Table 1: Table continues below

	Production Line 1	Production Line 2	Production Line 3
Paint 1	75	100	0
Paint 2	100	160	0
Paint 3	150	250	180
Paint 4	100	0	200
Paint 5	200	0	225

	Production Line 4	Production Line 5
Paint 1	100	125
Paint 2	100	200
Paint 3	175	0
Paint 4	175	175
Paint 5	175	180

```r
#' Create matrix of profit per barrel [Pi]
Profit <- matrix(c(170, 135, 75, 25, 15),
                        ncol=1,dimnames=c(Paint,"Profit Per Barrel"))
#' Column 1 row 3
Profit[3]
```

```
## [1] 75
```

```r
#' Display the Matrix in a nicely formatted table
pander(Profit, caption = attr(Profit,"Profit per Barrel"))
```

	Profit Per Barrel
Paint 1	170
Paint 2	135
Paint 3	75
Paint 4	25
Paint 5	15

```r
#' Max Demand Constraints [Di]
MaxDemand <- matrix(c(20000, 20000, 36000, 75000, 100000),
                          ncol=1,dimnames=c(Paint,"Max Barrels Per Paint"))
#MaxDemand[4]

#' Display the Matrix in a nicely formatted table
pander(MaxDemand, caption = attr(MaxDemand,"Max Demand Per Paint Type"))
```

	Max Barrels Per Paint
Paint 1	20000
Paint 2	20000
Paint 3	36000
Paint 4	75000
Paint 5	1e+05

```r
#' The Model
Model_PaintsRUs <- MIPModel() %>%
  #' Variables
  add_variable(X[i,j,k], i=1:NPaint, j=1:NDay, k=1:NProd, type="binary") %>%    #' X{i,j,k}
  add_variable(S[i,j],   i=1:NPaint, j=1:NDay,             type="integer",   lb = 0) %>%    #' S{i,j}
  add_variable(D[i,j],   i=1:NPaint, j=1:NDay,             type="integer",   lb = 0) %>%    #' D{i,j}
  add_variable(T[i,j],   i=1:NPaint, j=1:NDay,             type="integer",   lb = 0) %>%    #' T{i,j}

  #' Set Objective Function
  set_objective(sum_expr(Profit[i] * S[i,j], i=1:NPaint, j=1:NDay),"max") %>%

  #' Constraints
  add_constraint(sum_expr(ProductionRate[i,k] * X[i,j,k] , k=1:NProd)
              == T[i,j], i=1:NPaint,j=1:NDay) %>%                                  #' R{i,k}•X{i,
  add_constraint(sum_expr(T[i,j] - S[i,j], i=1:NPaint,j=1:j) <= whse, j=1:NDay) %>%          #' T{i,j} -S{i
  add_constraint(sum_expr(T[i,j] - S[i,j], j=1:j) >= 0, i=1:NPaint, j=1:NDay) %>%            #' T{i,j} -S{i
  add_constraint(sum_expr(S[i,j], j=1:NDay) <= MaxDemand[i], i=1:NPaint) %>%                 #' S{i,j} <= D
```

```r
  #' D{i,j} constraints
  add_constraint(D[1, 15] == 1000) %>%
  add_constraint(D[2, 19] == 2500) %>%
  add_constraint(D[3, 45] == 7500) %>%
  # add_constraint(D[4, 60] == 8000) %>%
  # add_constraint(D[5, 75] == 15000) %>%

  add_constraint(S[i,j] == D[i,j],i=1:NPaint, j = 1:NDay)  %>%                         #' S{i,j} = D{
  add_constraint(sum_expr(X[i,j,k], i=1:NPaint) <= 1, j=1:NDay,   k=1:NProd) %>%        #' X{i,j,k} <=


#' Solve Model
  solve_model(with_ROI(solver = "glpk", verbose = TRUE))
```

```
## <SOLVER MSG>  ----
## GLPK Simplex Optimizer, v4.57
## 1268 rows, 2400 columns, 40503 non-zeros
##      0: obj =  -0.000000000e+00 inf =   1.100e+04 (3)
##    165: obj =   1.070000000e+06 inf =   0.000e+00 (0)
## *  500: obj =   3.452125000e+06 inf =   0.000e+00 (448)
## * 1000: obj =   4.502500000e+06 inf =   0.000e+00 (348) 1
## * 1500: obj =   5.473400000e+06 inf =   0.000e+00 (207) 2
## * 1705: obj =   5.482500000e+06 inf =   2.220e-16 (0) 1
## OPTIMAL LP SOLUTION FOUND
## GLPK Integer Optimizer, v4.57
## 1268 rows, 2400 columns, 40503 non-zeros
## 2400 integer variables, 1500 of which are binary
## Integer optimization begins...
## + 1705: mip =     not found yet <=                +inf        (1; 0)
## + 1712: >>>>>   5.482500000e+06 <=   5.482500000e+06   0.0% (8; 0)
## + 1712: mip =   5.482500000e+06 <=     tree is empty   0.0% (0; 15)
## INTEGER OPTIMAL SOLUTION FOUND
## <!SOLVER MSG> ----
```

```r
#' Display the model solution
Model_PaintsRUs
```

```
## Status: optimal
## Objective value: 5482500
```

```r
#Extract constraint X_{i,j,k}
Xijk <- get_solution(Model_PaintsRUs,X[i,j,k])%>%
  dplyr::filter(value > 0)

#Extract constraint S_{i,j}
Sij <- get_solution(Model_PaintsRUs,S[i,j])%>%
  dplyr::filter(value > 0)

#Extract constraint D_{i,j}
Dij <- get_solution(Model_PaintsRUs,D[i,j])%>%
  dplyr::filter(value > 0)

#Extract constraint T_{i,j,k}
Tij <- get_solution(Model_PaintsRUs,T[i,j])%>%
  dplyr::filter(value > 0)
```
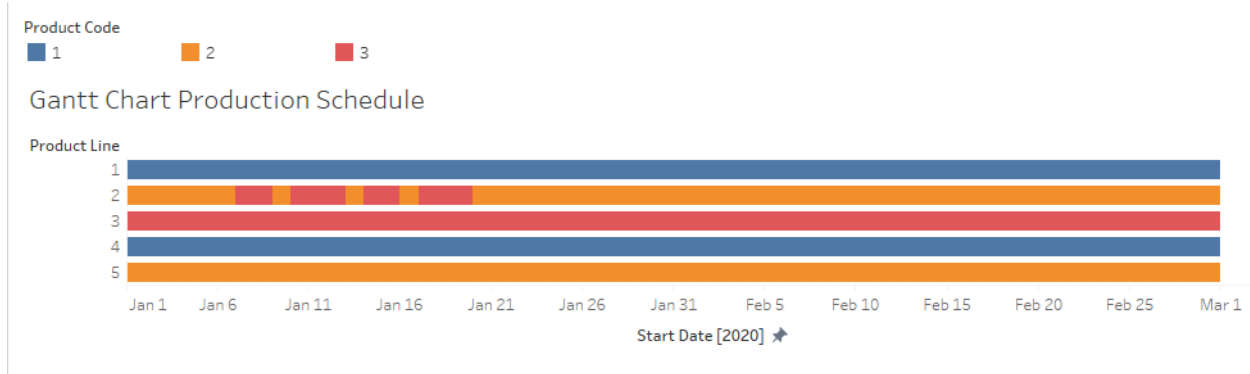
Figure 1: Gantt Chart

```r
#put solution into a data frame
rduals1 <-get_solution(Model_PaintsRUs,X[i,j,k], type = "primal")

#Export data to an excel file
write_xlsx(x = rduals1, path = "Solution.xlsx", col_names = TRUE)
write_xlsx(x = Xijk, path = "GanttChart.xlsx", col_names = TRUE)
```

Conclusions w/ Visualizations

The model is capable of informing a business, with the computring power to run it, which products on which days a production line should produce different products in order to maximize profit. That information can then be fed into a visualization tool such as Tableau or excel and produce a gantt chart as shown below to visualize which products should be produced on which production lines in order to hit the profit target that the business unit aims to achieve.

Due to the lack of computing power available for this project the model had to amended to only account for 60 days of production, but with more computing power a company could amend this program to forecast a 90 day or even a 365 day production cycle.

With a 60 day constraint the model returned that an optimal profit value for this plant is $5,482,500 during the period. The plant is capable of producing 5 product types, but the recommended production plan recommended that only the 3 highest profit margin product be produced; products 1,2 and 3. Each production line with the exception of produciton line 2 will produce only one product during the forecast period. Production line 2 will create product 2 and product 3 during the forecast period, as displayed in the Gantt Chart Below.

This model is based off of fictionalized data but the objective was to create a model which could be adapted to any business that produces and/or manufactures products. This model successfuly does that; by adding and updating constraints, adjusting profit margins, or changing maximum demand, the model can produce completely new results that are specific to that use case. More complex situations with more standing orders, or supply input constraints could be added to this model to further add complexity and business value to this model.