ECON 251

Discussion Section

Week 4 **Solutions**

1. Go over my slides on causal inference and determining gains to migration.

2. State and discuss simple linear regression assumptions (SLR1-SLR5).

SLR1 (linearity)	$Y = \beta_0 + \beta_1 X + U$
SLR2 (random sampling)	$\{(X_1, Y_1), \ldots, (X_N, Y_N)\}$ **is a random sample**
SLR3 (variation in treatment)	$\widehat{\mathrm{Var}}(X_i) := \sum_{i=1}^{N} (X_i - \overline{X})^2 > 0$
SLR4 (mean independence)	$E[U \mid X] = 0$
SLR5 (homoskedasticity)	$Var(U \mid X) = \sigma^2$

3. If $\log Y$ denotes log earnings, S is years of schooling and U is the error term in the linear population regression function (PRF) $\log Y = \alpha + \beta \cdot S + U$, then β is the Mincer (1972) returns to an additional year of education. Answer the following questions.

   a) What is the least squares estimator $\hat{\beta}^{OLS}$ of the returns parameter β?

$$\hat{\beta}^{OLS} = \frac{\widehat{\mathrm{Cov}}(S_i, \log Y_i)}{\widehat{\mathrm{Var}}(S_i)} = \frac{\sum (S_i - \overline{S})(\log Y_i - \overline{\log Y})}{\sum (S_i - \overline{S})^2}$$

   b) Under what assumptions is $\hat{\beta}^{OLS}$ unbiased for β? **SLR1 + 2 + 3 + 4**

   c) Under what assumptions is $\hat{\beta}^{OLS}$ consistent for β? **SLR1 + 2 + 3 +** $\mathrm{Cov}(S, U) = 0$

d) Variation in S is endogenous whenever it is related to U. Differentiate log earnings with respect to schooling and show that it does not equal to β if S is endogenous.

We write $\log Y = \alpha + \beta S + U(S)$ to indicate that schooling is endogenous, then

$$\frac{\partial}{\partial S}\log Y = \beta + \frac{\partial}{\partial S}U(S)$$

where $\frac{\partial}{\partial S}U(S)$ is the part of the relationship reflecting selection bias (or why not all the observed correlation we try to estimate is causal).

4. Prove that conditional independence $E(U|X) = 0$ implies exogeneity $\mathrm{Cov}(U, X) = 0$ using the law of iterated expectations $E[Z] = E_A\big[E[Z|A]\big]$.

$$\begin{aligned}
\mathrm{Cov}(U, X) &= E[UX] - E[U]E[X] \\
&= E[UX] - 0 \cdot E[X] \\
&= E_X\big[E[UX|X]\big] \\
&= E_X\big[X \cdot E[U|X]\big] \\
&= E_X[X \cdot 0] \\
&= 0
\end{aligned}$$