ECON 402 Discussion: Week 3 (problems)

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Announcements

- Homework 1: solutions are posted on Canvas, check them out.
- Homework 2: due March 21st at 11:59pm, as one (small) .pdf doc!
- Office hours: Fridays at 4pm on Zoom, or by appointment.

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- Topics today
  1. Introduction
  2. Labor supply
  3. OLG model
  4. HW1 solutions

Overlapping Generations Model

Definition: The building blocks of the OLG growth model are

- Time is discrete and infinite: $t = \{0, 1, 2, ...\}$.
- Aggregate production is Cobb Douglas: $Y_t = K_t^\alpha L_t^{1-\alpha}$.

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- Households start life with no financial assets and leave no bequests. They have $\ell_1 = 1$ and $\ell_2 = 0$ units of labor, supplied inelastically.

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- Individual consumption is given by
    1. c_{1t}, consumption of young person in t
    2. c_{2t+1}, consumption of (that same) old person in $t+1$

  while aggregate consumption is

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- Equilibrium market clearing conditions:
    1. In the goods/product market, must have

$$Y_t = C_t + I_t$$

    2. Since physical capital is the only savings vehicle, we have

$$K_{t+1} = s_t L_t$$

Overlapping Generations Model

- Example: Find the steady state of the OLG model!
- Step 1: Solve the consumer optimization problem

$$\max_{c_{1t}, c_{2t+1}} \quad \ln c_{1t} + \beta \ln c_{2,t+1}$$

$$\text{s.t.} \quad c_{1t} + s_t = w_t$$

$$c_{2t+1} = (1 + r_{t+1})s_t$$

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See last week's discussion and homework solutions to derive

$$c_{1t}^* = \frac{w_t}{1 + \beta}$$

$$c_{2t+1}^* = \frac{\beta}{1 + \beta}(1 + r_{t+1})w_t$$

$$s_t^* = \frac{\beta}{1 + \beta}w_t$$

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$$
\begin{aligned}
R_t &= \frac{\partial}{\partial K_t} Y_t \qquad \text{by perfect competition} \\
&= f'(k_t) \qquad \text{see homework 1} \\
&= \alpha k_t^{\alpha-1}
\end{aligned}
$$

$$
\begin{aligned}
w_t &= \frac{\partial}{\partial L_t} Y_t \\
&= f(k_t) - k_t f'(k_t) \\
&= (1-\alpha) k_t^\alpha
\end{aligned}
$$

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K_{t+1} &= s_t^* L_t \\
\frac{K_{t+1}}{L_{t+1}} &= \frac{s_t^* L_t}{L_{t+1}} \\
\Rightarrow k_{t+1} &= \frac{\frac{\beta}{1+\beta} w_t L_t}{(1+n)L_t} \qquad \text{since pop growth is } n \\
&= \frac{\frac{\beta}{1+\beta}(1-\alpha)k_t^\alpha}{(1+n)} \qquad \text{by step 2} \\
&= \frac{\beta(1-\alpha)}{(1+\beta)(1+n)} k_t^\alpha
\end{aligned}
$$

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- Note: We say that the economy is dynamically inefficient if $k_* > k_{gold}$. Since we have *too much* capital, saving less could actually increase consumption for everyone in the economy.

- This is pareto improving, and hence the inefficiency.

- What can be done about this? The government can provide another savings vehicle to crowd out private saving... government debt!

HW1 Solutions

- Let's take a look!