HW9

Since

$$\sigma^2 \sim GAMMA(\alpha, \beta)$$

,

$$\pi(\sigma^2) \sim \frac{1}{\sigma^2} \implies \pi(\sigma^2) \sim INVGAM(\alpha, \beta)$$
$$\pi(\mu, \sigma^2|Y_1...Y_n) \propto \pi(\mu, \sigma^2, Y_1...Y_n) = f(Y_1...Y_n|\mu, \sigma^2)\pi(\sigma^2)\pi(\mu)$$
$$p(\mu|Y_1...Y_n, \sigma^2) \propto f(Y_1...Y_n|\mu, \sigma^2)\pi(\mu)\pi(\sigma^2)$$

Given that these priors are independent,

$$p(\mu|Y_1...Y_n, \sigma^2) \propto f(Y_1...Y_n|\mu, \sigma^2)\pi(\mu)$$

$$\propto (e^- \frac{\sum(y_i - \mu)^2}{2\sigma^2})(e^- \frac{(\mu - \mu_0)^2}{2\sigma_0^2})$$
$$\propto e^(\frac{2n\bar{y}\mu}{2\sigma^2} - \frac{\mu^2 - 2\mu_0\mu}{2\sigma_0^2})$$
$$\propto e^(\mu(\frac{n\bar{y}}{\sigma^2} + \frac{\mu_0}{\sigma_0^2}) - \frac{\mu^2}{2}(\frac{n}{\sigma^2} + \frac{1}{\sigma_0^2}))$$

Therefore,

$$p(\mu|Y_1...Y_n, \sigma^2) \sim N(\sigma_0^2 * (\frac{n\bar{y}}{\sigma^2} + \frac{\mu_0}{\sigma_0^2}, \frac{1}{\frac{n}{\sigma^2} + \frac{1}{\sigma_0^2}})$$

Given this, we can see that

$$\sigma_0^2 = \frac{1}{\frac{n}{\sigma^2} + \frac{1}{\sigma_0^2}}; \mu_0 = \sigma_0^2 * (\frac{n\bar{y}}{\sigma^2} + \frac{\mu_0}{\sigma_0^2})$$
$$p(\sigma^2|Y_1...Y_n, \mu) \propto \sigma^{-n} e^(- \frac{\sum(y_i - \mu)^2}{2\sigma^2})\sigma^{-2\sigma-2} e^{\frac{-\beta}{\sigma^2}}$$
$$= \sigma^{-2(\alpha + \frac{n}{2})-2} e^{-\frac{\sum(y_i-\mu)^2+\beta}{2\sigma^2}}$$

From this, we can see that

$$p(\sigma^2|Y_1...Y_n, \mu) \sim INVGAM(\alpha + \frac{n}{2}, \beta + \sum(Y_i - \mu)^2)$$

```r
data("ChickWeight")

begin = which(ChickWeight$Time %in% c(0))
missing = c()

for (i in 1:(length(begin)-1)){
```

```r
  if (begin[i] != (begin[i+1]-12)){
    missing = append(missing, c(begin[i]:(begin[i+1]-1)))
  }

}
```

```r
new_weight = ChickWeight$weight[-missing]
new_diet = ChickWeight$Diet[-missing]
diffs = c()
one_diffs = c()
two_diffs = c()
three_diffs = c()
four_diffs = c()
diets = c()

for (i in 1:length(new_weight)){

  if (i %% 12 == 0 & i > 0){

    diffs = append(diffs, (new_weight[i]-new_weight[i-11]))

    if (new_diet[i] == 1){
      one_diffs = append(one_diffs, (new_weight[i]-new_weight[i-11]))
      diets = append(diets, 1)
    }

    if (new_diet[i] == 2){
      two_diffs = append(two_diffs, (new_weight[i]-new_weight[i-11]))
      diets = append(diets, 2)
    }

    if (new_diet[i] == 3){
      three_diffs = append(three_diffs, (new_weight[i]-new_weight[i-11]))
      diets = append(diets, 3)
    }

    if (new_diet[i] == 4){
      four_diffs = append(four_diffs, (new_weight[i]-new_weight[i-11]))
      diets = append(diets, 4)
    }

  }

}
```

```r
library(invgamma)

mc_mat = matrix(nrow=10000,ncol=7)

theta_c = c(0,0,0,0)
mu_c = 0
sig2_c = 1
tau2_c = 1
```

```r
a1 = 2.00001
a2 = 2.00001
b1 = a1-1
b2 = a2-1
mu0 = 0
sig2mu = 100
mc_mat[1,] = c(theta_c,mu_c,sig2_c,tau2_c)
X = diffs

Xbar<-c(mean(one_diffs), mean(two_diffs), mean(three_diffs), mean(four_diffs))
n = length(diffs)
for(i in 2:10000){

  # Theta
  mean_tmp = sig2_c*mu_c/(sig2_c+n*tau2_c) + n*tau2_c*Xbar/(sig2_c+n*tau2_c)
  var_tmp = sig2_c*tau2_c/(sig2_c+n*tau2_c)
  theta_c = rnorm(4,mean=mean_tmp,sd=sqrt(var_tmp))

  # mu
  mean_tmp = tau2_c*mu0/(tau2_c+2*sig2mu) + 2*sig2mu*mean(theta_c)/(tau2_c + 2*sig2mu)
  var_tmp = sig2mu*tau2_c/(tau2_c + 2*sig2mu)
  mu_c = rnorm(1,mean=mean_tmp,sd=sqrt(var_tmp))

  # sigma2
  a_tmp = a1 +(n/2)
  b_tmp = b1 + sum((X-rep(theta_c,each=45))^2)
  sig2_c = rinvgamma(1,shape=a_tmp,scale=b_tmp)

  # tau2
  a_tmp = a2 + (n/2)
  b_tmp = b2 + sum((theta_c-mu_c)^2)
  tau2_c = rinvgamma(1,shape=a_tmp,scale=b_tmp)

  mc_mat[i,] = c(theta_c,mu_c,sig2_c,tau2_c)
}

mc_mat<-mc_mat[-c(1:1000),]
```

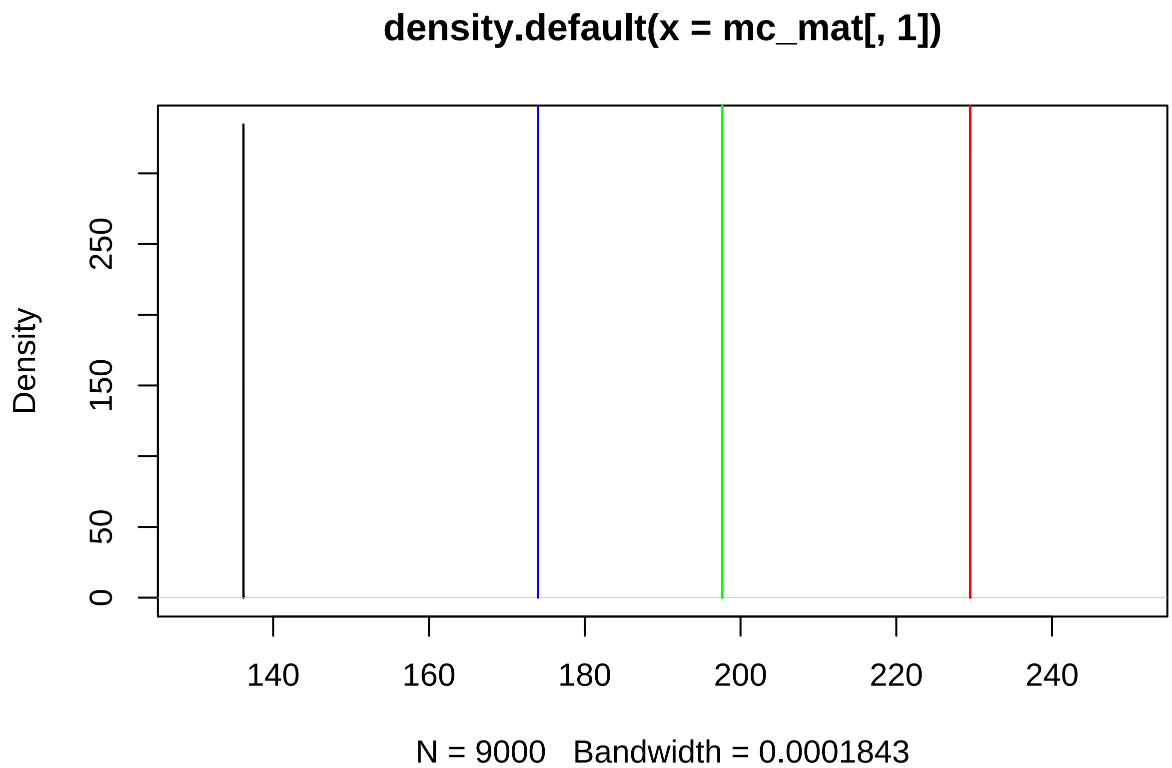
```r
par(mfrow=c(1,1))

plot(density(mc_mat[,1]), xlim = c(130, 250))

dtmp<-density(mc_mat[,2])
points(dtmp$x,dtmp$y,col="blue",type="l")

dtmp<-density(mc_mat[,3])
points(dtmp$x,dtmp$y,col="red",type="l")

dtmp<-density(mc_mat[,4])
points(dtmp$x,dtmp$y,col="green",type="l")
```



From the distributions, it is clear that there is a difference in the change of weights in the chicks for each diet group. Group three has the highest change, followed by groups 4, 2 and then 1.