# Final Project

**Team:** Agostini Chiara, Grieco Addolorata, Simonelli Elisa

18 January 2022

## 1 Design a Tchebyshev array factor of five elements with a main lobe / side lobe ratio of R=120. Optimize the inter-element spacing to minimize the beamwidth

The project is aimed to design and optimization of a 5 elements Tchebyshev array at 2.3 GHz frequency, where the ratio between main lobe and side lobe is 120 and the current distribution is uniform. The main goal is minimize width of the main lobe beam. The activity was performed by *MATLAB* with the toolbox *Antenna Designer*.

### 1.1 Determine the current coefficients

Let be $N_{tot}$=2N+1 the elements number of the array. Considering the uniform distribution of the current, it can be assumed that the coefficient of the supply current is $C_n$=$C_{-n}$ con $n=1,2,3,..$, so from the previous formulation, the polynomial degree $N$ which well approximates the array factor is:

$$\frac{N_{tot}-1}{2} = 2$$

Array factor is a 2-degree Tchebyshev polynomial

$$F(u) = T_2(x) = 2x^2 - 1$$

where

$$x = a + bcos(u) \tag{1}$$

$$u = k_0 dcos(\psi) \tag{2}$$

In order to satisfy the ration between main lobe and side lobe, it can be imposed $T_n(x_1)$=R, where $x_1$ is the coordinate that identify the main lobe.

$$x_1 = cosh\left[\frac{1}{N}cosh(R)\right] = 7.78$$

Moreover, the visibility window is:

$$-1 \leq x \leq x_1 \tag{3}$$

### 1.2 Evaluate the beamwidth of the array factor and compare with a uniform array

Aimed to minimize the width of the main lobe beam in broadside configuration, is necessary to enlarge the visibility window to include more side lobes and only one main lobe. Therefore, the optimal spacing between elements was assessed as follows:

$$d_{opt} = max\left[\frac{\lambda_0}{2} < d < \lambda_0\right] = max[65.2mm < d < 130.4mm]$$

Coefficients $a$ and $b$ are calculated as:

$$a = \frac{x_1 - 1}{2} = 3.39 \quad (4)$$
$$b = \frac{x_1 + 1}{2} = 4.39 \quad (5)$$

And the optimal spacing is:

$$d_{opt} = \frac{2\pi n \pm acos\left[\frac{1-a}{b}\right]}{k_0}, n = 1, 2, 3, ... \quad (6)$$
$$(7)$$

Implementing in *MATLAB*, the result is $d_{opt}$=*85.9mm*. Now, is possible to define the visibility window:

$$-k_0 d_{opt} \leq u \leq k_0 d_{opt}$$

Subsequently, current coefficients were calculated:

$$T_2(x) = (2a^2 + b^2 - 1) + 4abcosu + b^2 cos2u = C_0 + 2C_1 cosu + 2C_2 cos2u$$

where

```
C0 = 2*a^2 + b^2 - 1;
C1 = 4*a*b/2;
C2 = (b^2)/2;

Csum = C0 + 2*C1 + 2*C2;
```

Figure 1: Current's coefficients

Since coefficients $a$ and $b$ are already calculated, it is possible to obtain the current coefficients:

$$C_0 = 41.23, C_1 = 29.75, C_2 = 9.63, C_{sum} = 120$$

### 1.3 Evaluate the array tapering efficiency

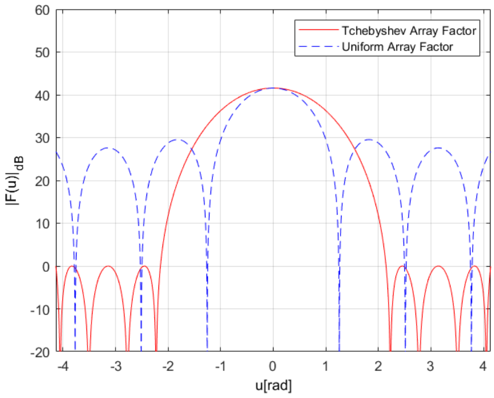
During the next phase, is helpful to evaluate the Tchebyshev array performances, by calculating the tapering efficiency, defined by:

$$\eta_t = \frac{1}{N_{tot}} \frac{|\Sigma C_n|^2}{|\Sigma C_n^2|} = 0.48$$

In order to obtain an higher ration between main lobe and side lobe, is needed a lower efficiency.

### 1.4 Plot the array pattern (rectangular and polar diagrams)

It was compared the Tchebyshev array factor with respect an uniform array in rectangular coordinates. From *Figure 2*, it can be notice an higher beam width of Tchebyshev array factor with respect an uniform array. This is achieved in order to satisfy the request for the value of R. This, implies that the array is less directive.



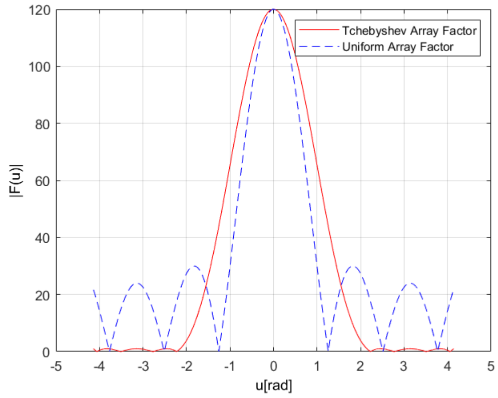



Figure 2: Array factor comparison in dB scale *(left)* and in linear scale *(right)*

Indeed, in *Figure 3*, it's shown the polar plot Tchebyshev array factor:

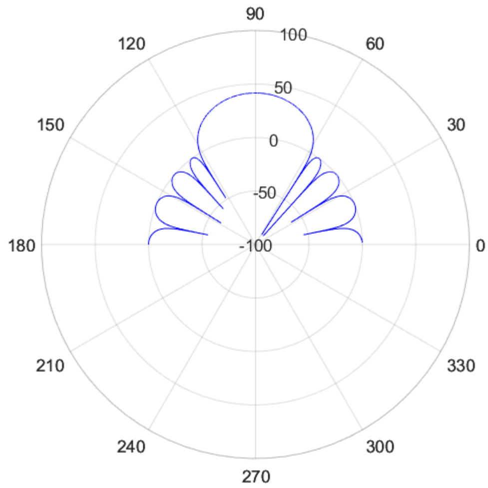


Figure 3: Array factor in polar coordinates

The amplitudes of both the first-null beam and the half-power beam in the uniform and non-uniform configuration were calculated. The beam amplitude is evaluated to quantitatively determine the performance of the array.
For the uniform case, it results:

$$FNBW_{uniforme} = \frac{2\lambda_0}{(N_{tot}+1)d_{opt}} = 29^\circ \tag{8}$$

$$HPBW_{uniforme} = \frac{2.65\lambda_0}{\pi(N_{tot}+1)d_{opt}} = 12.23^\circ \tag{9}$$

While, for non-uniform case, it was used the Toolbox *Antenna Designer* and it was obtained:

$$FNBW_{non-uniform} = 65.31^\circ \tag{10}$$

$$HPBW_{non-uniform} = 21.32^\circ \tag{11}$$

Ratio *(R)* between main lobe and side lobe is 41.45 dB which in linear scale is $\approx$ 120, as requested.

# 2 Design a rectangular folded $\lambda/4$ patch fed by a coaxial cable for 2.3 GHz , whose size is compatible with the inter-antenna distance derived above

It was designed a folded patch at $\lambda/4$ with frequency of 2.3 GHz. Geometry, position of feed, gain and materials were evaluated in order to get good performances to the patch antenna.

## 2.1 Use the FR-4 Substrate (available thickness: 0.8, 1.0, 1.6 mm)

The first step is about the materials: FR4 will be used as substrate, varying its thickness as required, while copper, which is often used for antenna design due to its excellent conductive properties, has been chosen as the conductor ($\sigma = 5.96e7$).

## 2.2 Evaluate by equations the size (L, W), directivity, BW, position of the feed to match 50 ohm

The required parameters were calculated analytically, taking into account up to the second decimal value.

$$W = \frac{\lambda_0}{2}\sqrt{\frac{2}{\epsilon_r + 1}} \approx 38.95mm$$

Since W/h>1 effective permectivity is:

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\left(1 + \frac{12h}{W}\right)^{-0.5} \approx 4.41$$

$$\Delta L = 0.412h\frac{(\epsilon_{eff} + 0.3)\left(\frac{W}{h} + 0.264\right)}{(\epsilon_{eff} - 0.258)\left(\frac{W}{h} + 0.8\right)} \approx 0.37mm$$

Therefore, the length of patch:

$$L = \frac{c}{4f_0\sqrt{\epsilon_{eff}}} - \Delta L \approx 30.28mm$$

The position of feeding, in order to match the requested impedance, 50 $\Omega$ , depends on the input resistance at the edge of the patch, which depends on the slot radiation conductance:

$$R_{in}(0) = \frac{1}{G_s} = 2\frac{60\lambda_0}{W}\left[1 - \frac{1}{24}(k_0 h)^2\right]^{-1} \approx 401.62\Omega$$

Therefore, the position of the feed is:

$$x_0 = \frac{\lambda_0}{2\pi\sqrt{\epsilon_{eff}}} acos\left(\sqrt{\frac{R_{in}}{R_{in}(0)}}\right) \approx 11.95mm$$

Once that the geometrical parameters have been derived it's useful to evaluate the radiating characteristics of the patch. The maximun theoretical directivity on the broadside is:

$$D = \frac{1}{2}\left[\frac{2}{15}\left(\frac{W}{\lambda_0}\right)^2\right] R_{in}(0) \approx 3.78dB$$

## h=0.8 mm

```
c=physconst('LightSpeed');
f0=2.3e9;
w=2*f0*pi;
lambda0=c/f0;
h_mm=0.8;
h=h_mm/1000%metri altezza substrato
epsilon_r=4.6; %permettività relativa FR-4
W=(lambda0/2)*(sqrt(2/(epsilon_r+1))) %metri
W_mm=W*1000
epsilon_eff=((epsilon_r+1)/2)+((epsilon_r-1)/2)*(1+(12*h)/W)^(-0.5);
delta_L=((0.412*h)*(epsilon_eff+0.3)*((W/h)+(0.264)))/((epsilon_eff-0.258)*((W/h)+(0.8)));
L=c/(2*f0*(sqrt(epsilon_eff)))-(2*delta_L) %metri
L_mm=L*1000
```

```
h = 8.0000e-04

W = 0.0389
W_mm = 38.9479

L = 0.0303
L_mm = 30.2869
```

### Evaluate Feed Position

```
R_in= 50 %Ohm
beta=((2*pi)*(sqrt(epsilon_eff)))/lambda0
x0=(1/beta)*acos(sqrt(R_in/R_R))
x0_mm=x0*1000
```

### Evaluate Directivity

```
D0=2/15*(W/lambda0)^(2)*R_R
D0_dB=10*log10(D0)
```

### Fattore qualità e BW

```
tandelta=0.030;
h_c=2e-2; %spessore conduttore scelta: foglio di rame
mu=1;
sigma=5.8e7; %conducibilità conduttore
Q_d=1/tandelta;  %fattore qualità dielettrico
Q_r=((w*epsilon_r)/(2*h))*(L)*(w*R_in); %fattore qualità radiazione
Q_c=sqrt(h_c*pi*f0*mu*sigma);
Q_tot=Q_d+Q_c+Q_r
BW=1\Q_tot
L_feed=1/beta*acos(sqrt(R_0/R_in))
```

```
Q_tot = 3.6273e+24
BW = 3.6273e+24
L_feed = 0.0122
```

Figure 4: Matlab code to define parameters

Same calculations were done for *h(1)=1mm* e *h(1.6)=1.6mm*.

$$W_1 \approx 38.97mm \tag{12}$$
$$W_{1.6} \approx 38.97mm \tag{13}$$
$$\epsilon_{eff_1} \approx 4.37 \tag{14}$$
$$\epsilon_{eff_{1.6}} \approx 4.27 \tag{15}$$
$$\Delta L_1 \approx 0.46mm \tag{16}$$
$$\Delta L_{1.6} \approx 0.73mm \tag{17}$$
$$L_1 \approx 30.26mm \tag{18}$$
$$L_{1.6} \approx 30.08mm \tag{19}$$
$$R_{in_1} \approx 401.63\Omega \tag{20}$$
$$R_{in_{1.6}} \approx 401.69\Omega \tag{21}$$
$$x_{0_1} \approx 12mm \tag{22}$$
$$x_{0_{1.6}} \approx 12.14mm \tag{23}$$
$$D_1 \approx 3.78dB \tag{24}$$
$$D_{1.6} \approx 3.78dB \tag{25}$$
$$\tag{26}$$

## 2.3 Refinement with Matlab MoM. Plot currents, impedances, reflection coefficient and gain patterns

### 2.3.1 Patch refinement

Starting from the theoretical derived parameters, an optimization procedure is needed in order to achieve better results. The optimization goal of our project is to minimize the magnitude of the reflection coefficient centering it at. The parameters to be optimized are the patch width and feeding point. The procedure consists in designing, a folded patch starting from the theorical values, then, perform several simulations by changing the couple, as shown in the following paragraphs, and evaluate for each of them the minimum value of the reflection coefficent, up to find an optimal value. The refinement procedure is briefly resumed in the flowchart of *Figure 5*

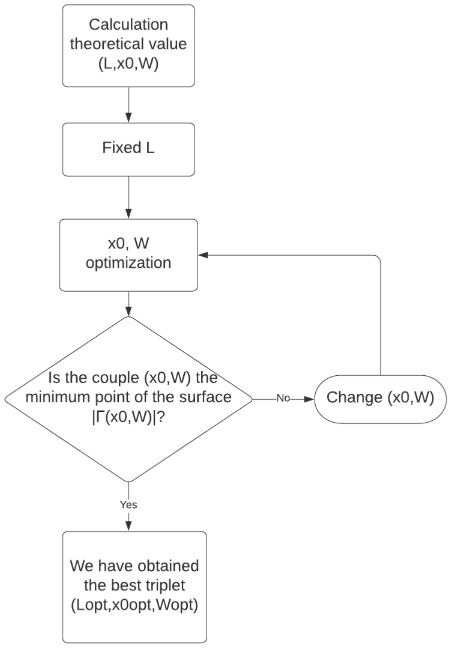



Figure 5: Patch refinement flowchart

#### 2.3.2 Mesh Density

Before of proceeding with the patch optimization, it's important to check mesh density in *Matlab*. An high mesh density allows to reach very precise results but the price to pay is the high computational complexity, indeed a low mesh density produces the results in a shorter time but with less accuracy: for this reason a trade-off is needed. *Figure 6* shows mesh density about folded patch antenna.

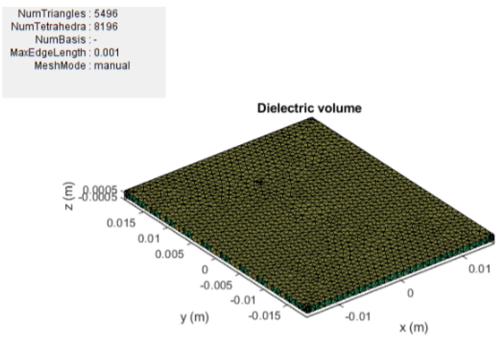



Figure 6: Antenna mesh

#### 2.3.3 Optimization Process

The first step consists by fixing the legth L=30.30mm, theoretically calculated in the previous paragraph. Empirically was denoted the L does not influence the resonance.
The second step consists by varying $l_{feed}$ and W having the minimum reflection coefficient at the working frequency. Through *Matlab* function *counturf*, optimal antenna values were evaluated, $l_{feed}$*=10mm* e *W=37.85mm*, visible in *Figure 7*:

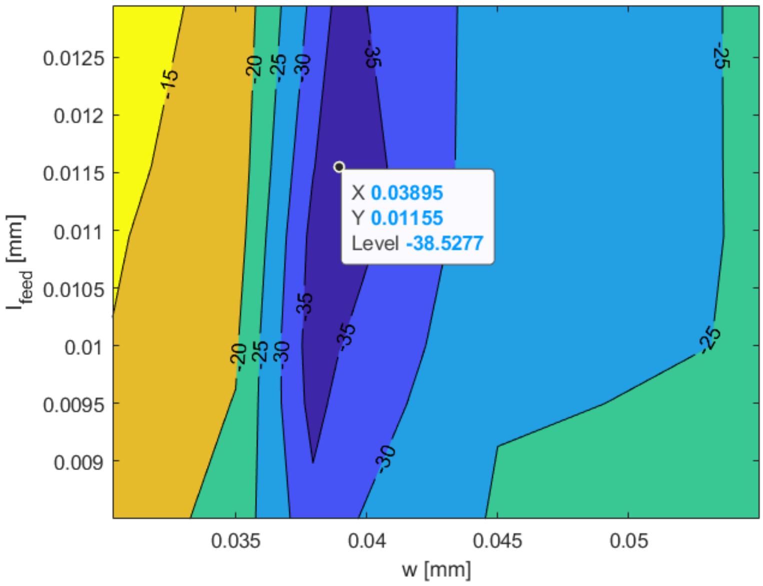



Figure 7: Countur

Is reported, in *Figure 8* the *Matlab* code for countur creation:

```
clear, clc, close all
%contour plot

l_feed=xlsread('nuovo.xlsx','A2:A7');
w=xlsread('nuovo.xlsx','B1:G1');

gamma=xlsread('nuovo.xlsx','B2:G7');


V=[-39 -35 -30 -25 -20 -15 -10 -5 0];

figure,contourf(w,l_feed,gamma,[V],'ShowText','on');
xlabel('w [mm]'), ylabel('l_{feed} [mm]');
```

Figure 8: Matlab code

Considering the optimal values of $L_{feed}$ and $W$, the plots about reflection coefficient, impedence, current distribution and gain pattern are:

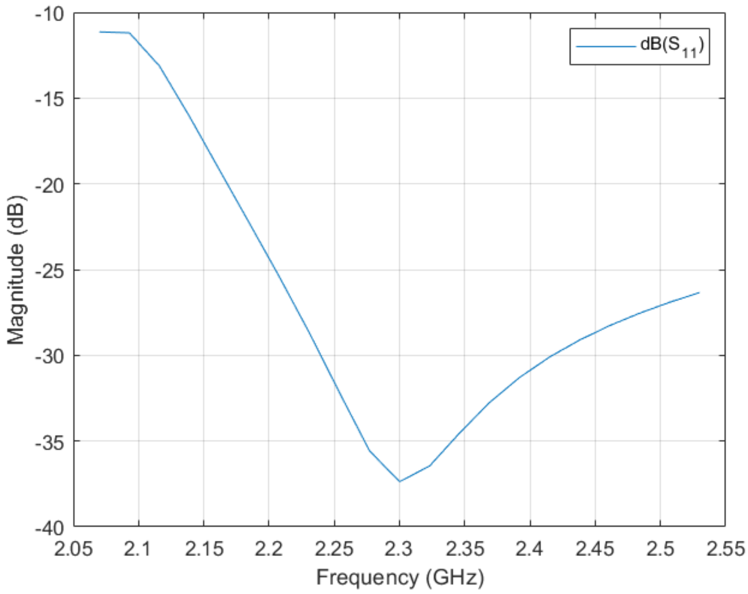



Figure 9: Reflection coefficient

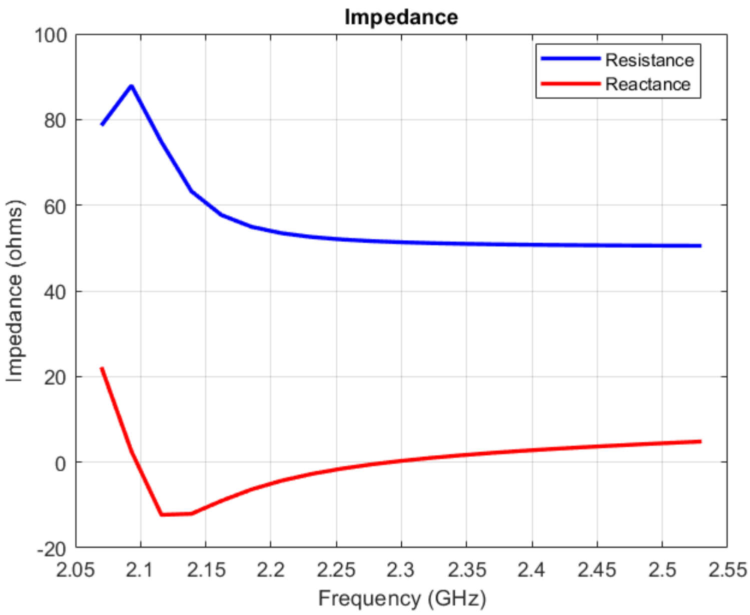



Figure 10: Impedence

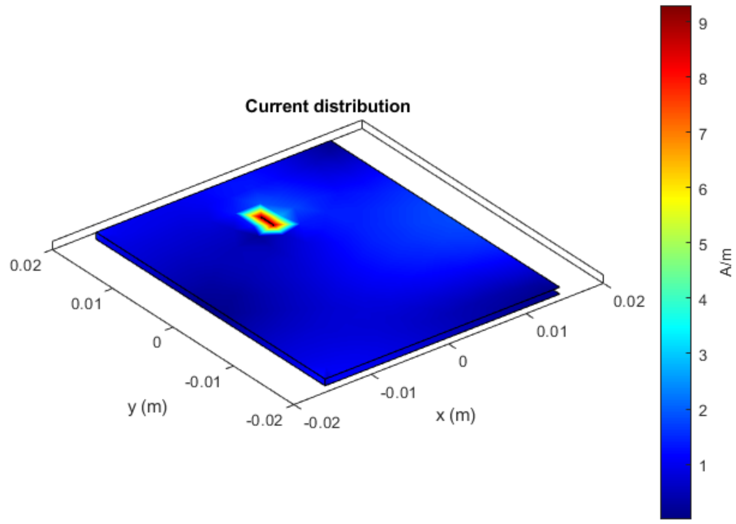



Figure 11: Current distribution

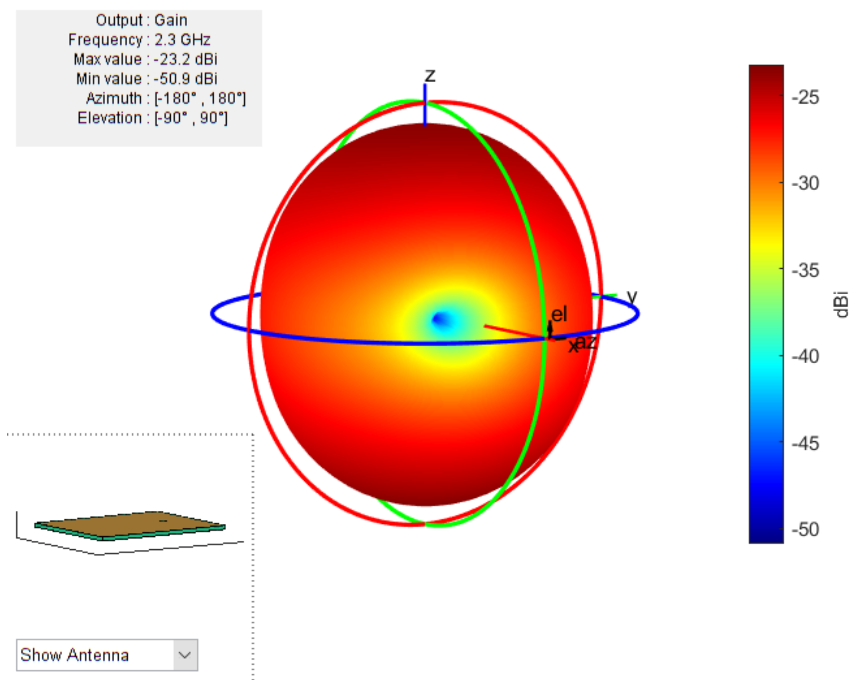



Figure 12: Gain pattern

Substrate thicknesses of *h=1mm* and *h=1.6mm* were considered; then applying the same optimisation as for the antenna with a substrate thickness of *h=0.8mm*, the optimum values were found, such that the antenna resonates. In *Figure 13*, The diagrams of the reflection coefficient considering the two different thicknesses are shown. Leaving *L=30.30mm* and $l_{feed}$=*10mm*, there is resonance for the following values of $W$:

$$W(1) = 37.4mm \tag{27}$$

$$W(1.6) = 37mm \tag{28}$$

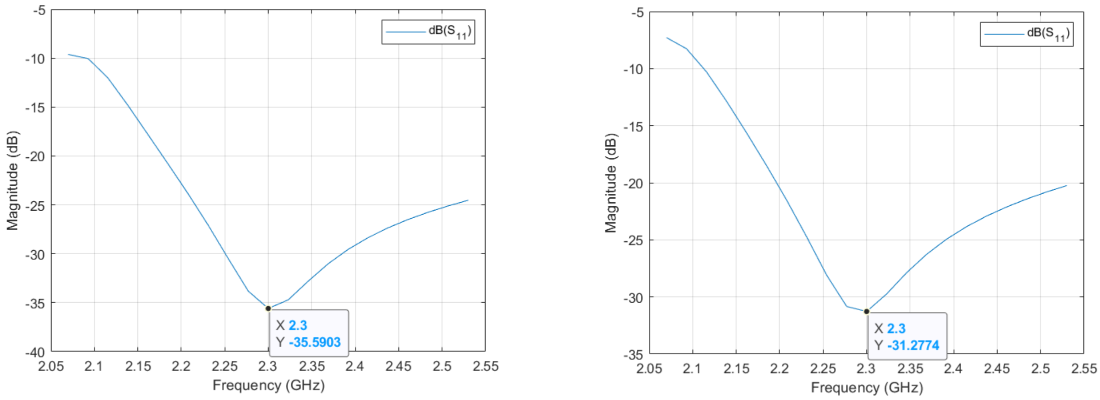



Figure 13: Reflection coefficient with h=1mm *(left)* reflection coefficient with h=1.6mm *(right)*

# 3 Evaluate the performance of the overall array of patches in both broadside case and 45° off the boresight direction (PSI= 90°-45°)

After design and optimization of the single antenna, a 5 patch linear array was builded in *Matlab*. The array elements were aligned so that the radiating slot was parallel to the array axis. The performance of the broadside configuration was analysed and then compared to an array for which the direction of maximum gain is 45° to the pointing direction.

## 3.1 Identify phase coefficients for beam-steering

In order to modify the inclination of the main lobe (45° off the boresight direction) a different phase must be applied to each element of the array:

$$u + u_0 = 0 -> k_0 d_{opt} cos(\Psi_0) = -\alpha d_{opt} -> k_0 d_{opt} cos(45°) = -\alpha d_{opt} = 85.9°$$

Therefore, centering the reference system on the central patch of the array, the antennas must be feeded in the following way:

| n | $\lvert C_n \rvert$ | $\alpha d_{opt} n$ |
|---|---|---|
| -2 | 9.63 | -171.8° |
| -1 | 29.75 | -85.9° |
| 0 | 41.23 | 0° |
| 1 | 29.75 | 85.9° |
| 2 | 9.63 | 171.8° |

In *Figure 14* is showed the polar plot of phased configuration, obtained in *MATLAB* through Toolboox *Sensor Array Analyzer*, by the pattern multiplication theorem:

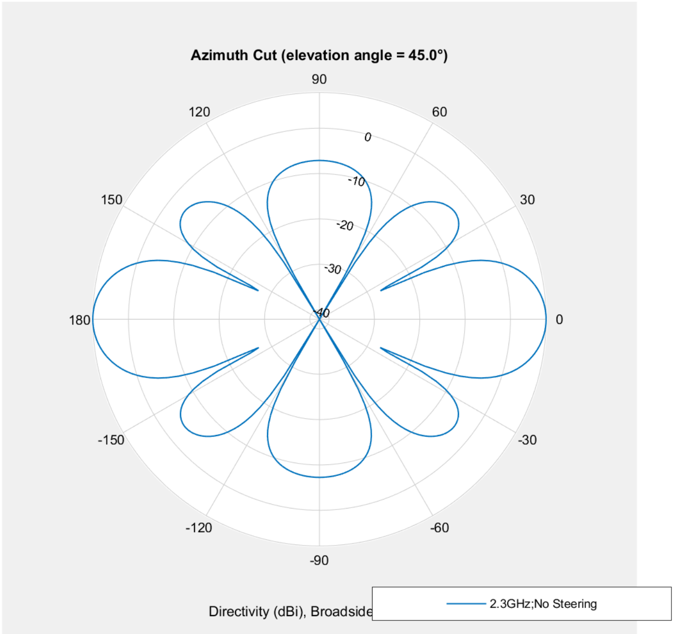



Figure 14: Polar plot of the phased array in dBi scale

## 3.2 Compute and plot the total array gain by using the pattern multiplication principle

Starting from the obtained results for the single patch, the global gain of the array was evaluated using the pattern multiplication theorem in *MATLAB*:

$$G_T(\Theta, \Phi) = G_0(\Theta, \Phi) G_F(\Theta, \Phi)$$

In *Figure 15* is showed the gain of the array. First of all the antenna was created with the Toolboox *PCB AntennaDesigner*, and then was imported in *Sensor Array Analyzer* where the array was builded, finally was calculated the global gain.

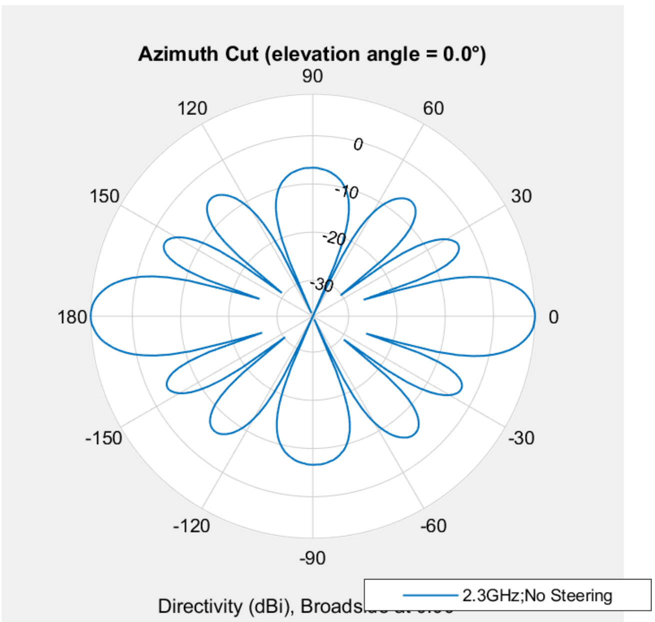



Figure 15: Polar diagram of the total array gain in dBi scale

## 3.3 Compute and plot the total array gain by mean of a fullwave model of the array with Matlab MoM

Using the Toolbox *Sensor Array Analyzer*, the total gain was evaluated using the fullwave model. In *Figure 16*, the 3D-radiation pattern of 5dB is shown.

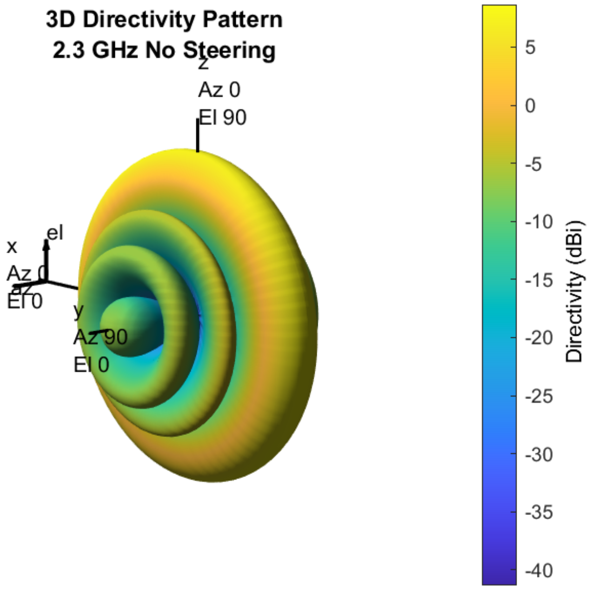



Figure 16: Radiation pattern in dBi scale

## 3.4 Plot of the near field just over the array plane to analyze the fringing field from the edges and observe possible non-uniformity due to the inter-antenna coupling

After building the patch antenna model on *PCB Antenna* Toolbox, it was imported inside the *MATLAB* workspace, where, using the function **[X, Y, Z] = sphere(20)**, the points of the sphere on which the electric field E and the magnetic field H of the individual element are analysed, were defined.

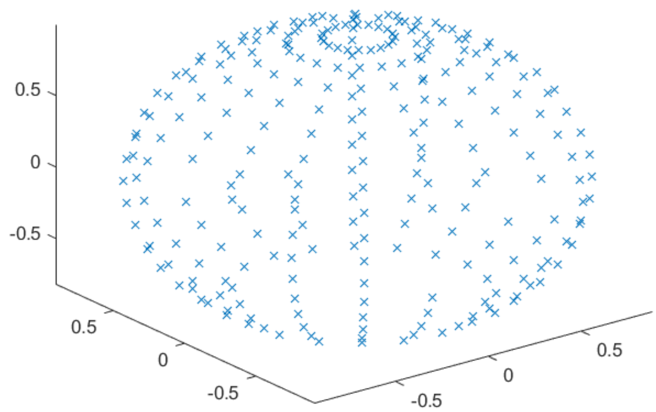



Figure 17: Benchmarks of E and H fields

After that, in *Figure 18*, the E and H fields of the individual element were plotted.

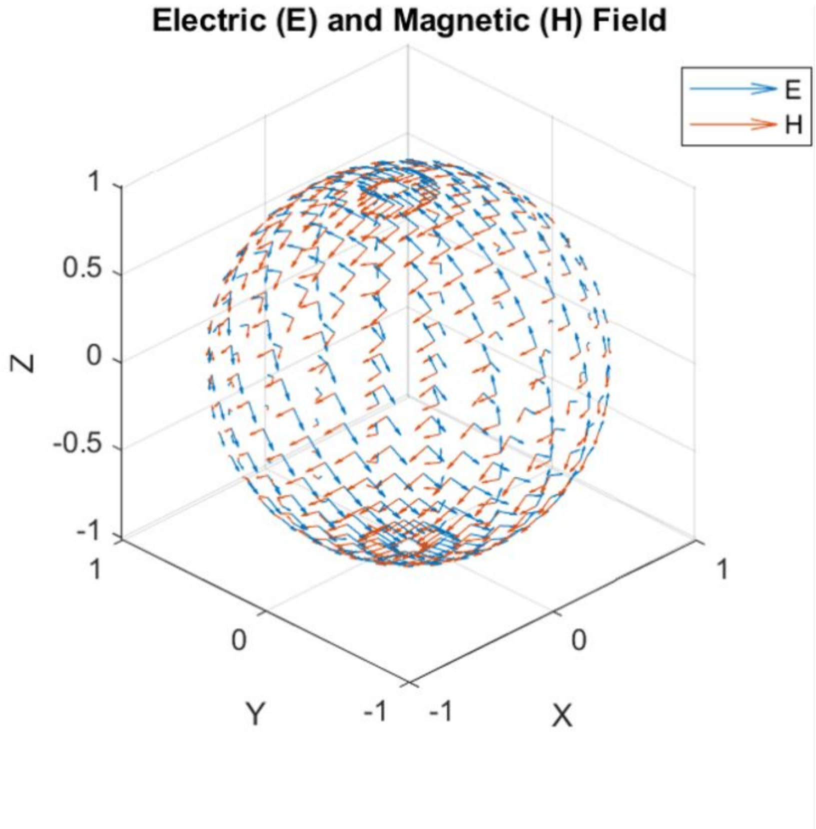



Figure 18: E and H fields of single element

The same work was conducted using the 5-elements array. Reference points in the sphere were increased using the same function **[X, Y, Z] = sphere(50)**, visible in *Figure 19*.

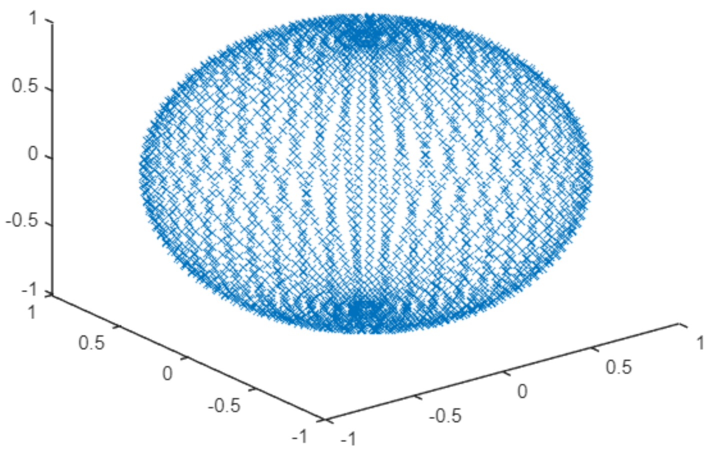



Figure 19: Benchmarks of E and H fields

Then, in *Figure 20* the E and H fields of the array have been plotted.

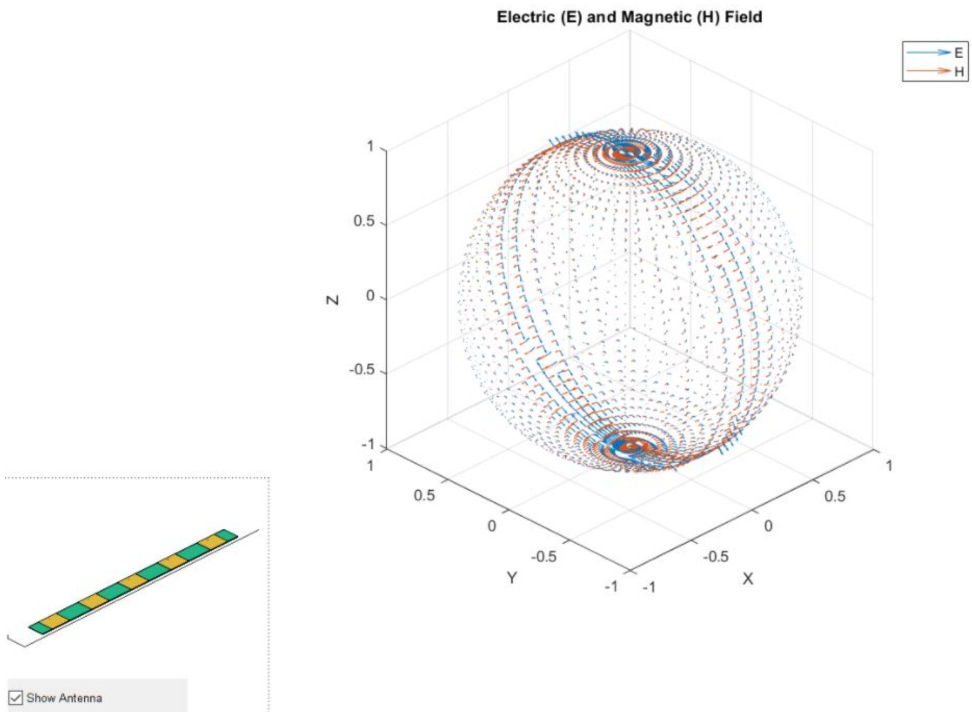



Figure 20: E and H fields of array