Deep Reinforcement Learning for an NP-Hard Scheduling Problem

Elisabeth Bankl, BSc

FH Technikum Wien

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Supervisor: Dipl.-Ing. Dipl.-Ing. Dr. techn. Christoph Redl, BSc

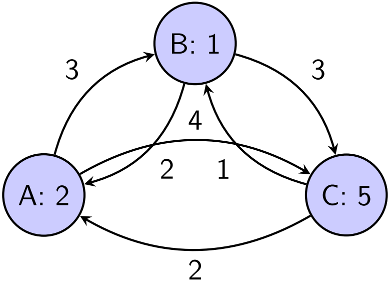
Research Questions

- How effectively can a scheduling problem be solved using Deep Reinforcement Learning?
- How close to optimal are the solutions the RL agent discovers, in percentage, depending on problem size?
- Can the RL agent compete with state-of-the-art solutions in terms of execution time and achieved optimality?

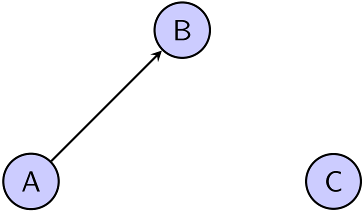
Problem Statement

- Tasks need to be completed sequentially
- Some tasks must be finished before starting others (precedence constraints)
- The execution time of a task depends on the task before it (transition time)
- Goal: Order the tasks in a way that minimizes the total execution time and follows all precedence constraints
- Sequential Ordering Problem (SOP) / Asymmetric Travelling Salesperson Problem with Precedence Constraints

Graph Representation



(a) Distances



(b) Precedence constraints

Formal Problem Definition (1/2)

Adapted from Schoen (2024).
Let $V = \{1, 2, \ldots, n\}$ be the set of nodes.
c_i: cost for visiting node i first
c_{ij}: cost for moving from node i to node j
Precedence constraints: if $p_{ij} = 1$, node i must be visited before node j.
Binary variable y_i for $i \in V$ as:

$$y_i = \begin{cases} 1 & \text{if node } i \text{ is the first node in the tour} \\ 0 & \text{otherwise} \end{cases}$$

The sum of these variables should be 1:

$$\sum_{i \in V} y_i = 1$$

Formal Problem Definition (2/2)

Binary variable y_{ij} for $i \in V$ and $j \in V \setminus \{i\}$ as:

$$y_{ij} = \begin{cases} 1 & \text{if the tour includes the edge}(i,j) \\ 0 & \text{otherwise} \end{cases}$$

$$0 \leq \sum_{j \in V \setminus \{i\}} y_{ij} \leq 1$$

$$\sum_{i \in V \setminus \{j\}} y_{ij} + y_j = 1$$

Objective: Minimize the total cost of the tour:

$$\min \sum_{i \in V} c_i y_i + \sum_{i \in V} \sum_{j \in V \setminus \{i\}} c_{ij} y_{ij}$$

Existing algorithms

- **Greedy / Nearest Neighbor Algorithm**
  Iteratively go to the nearest unvisited node Cirasella et al. (2001)
- **LKH-3**
  A variable-depth local search heuristic Helsgaun (2017).
- **Branch and Bound**
  Uses the dynamic Hungarian algorithm to find lower bounds and a local-search domination technique to prune suboptimal branches Jamal et al. (2017)
- **Ant Colony Optimization**
  A metaheuristic where artificial ants deposit pheromones on selected edges to guide optimization Skinderowicz (2013)

Markov Decision Process

- **State Space**:
  - **Distance Matrix (D)**: Distances between nodes
  - **Precedence Matrix (P)**: Which nodes must be visited before others
  - **Cost Matrix (C)**: Cost of choosing a node as the first node in the tour, later cost for choosing a node next
  - **Visited Nodes Matrix (V)**: A binary matrix indicating which nodes have been visited.
- **Action Space**: The actions are the nodes that the agent can visit next.
  - Nodes with unmet precedence constraints are not allowed.
  - Previously visited nodes are not allowed.

Reinforcement Learning Algorithm

- **Reward**:
  - Negative cost of selecting the first node in the path
  - Negative distance between nodes
- **Algorithm**: Maskable Proximal Policy Optimization (PPO) Schulman et al. (2017)
- **Network Architecture**: MLP

Dataset

- Available datasets for SOP:
  - Not enough instances for training
  - Used for benchmarking
- Randomly generate problem instances during training.
  - Distances between nodes are sampled from a uniform distribution
  - Precedence constraints are sampled from binary distribution with probability p
  - Node costs from a uniform distribution
- Distances between the nodes follow the triangle inequality $c_{ij} + c_{jk} \geq c_{ik}$
- Size: 25 nodes

Performance Evaluation

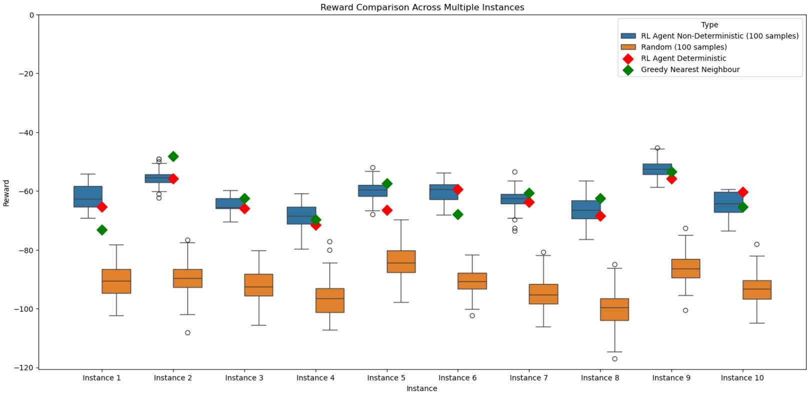


Figure: Comparing of the average reward of the agent with the simple greedy heuristic and random paths ($p = 0.1$).

Performance Evaluation depending on the precedence constraints

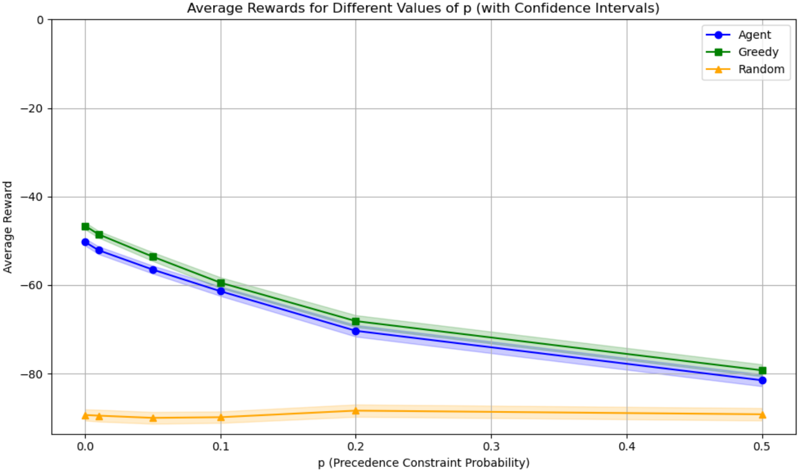


Figure: Comparing the average reward achieved by the agent, the simple greedy heuristic and random paths.

Current Limitations & Next Steps

- Maximum problem size: 25 nodes
- The RL agent's performance is still worse, on average, than the greedy heuristic.
- Change the problem instances
  - Make instance generator more deterministic
  - Clustered instances
  - Use existing instance generators
- Experiment with different RL algorithms
  - Deep Q-Learning and its variants
- Reward Shaping: use $\frac{1}{r}$ or $-r^2$ instead of $-r$
- Modify network Architecture:
  - GNN
  - Attention

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