$(x_1, x_2, \dots, x_M)$: training samples

$N = 2$, y_0, y_1, S_0, S_1

$$S_0 = \left\{ x : \sum_{j=0}^{1} P(j \mid 0)(x - y_j)^2 \le \sum_{j=0}^{1} P(j \mid 1)(x - y_j)^2 \right\}$$

Enc: $E(x) = \begin{cases} 0 & \text{if } x \in S_0 \\ 1 & \text{if } x \in S_1 \end{cases}$

$S_1 = \mathbb{R} \setminus S_0$ (Gen. NNC)

Gen CC

$x_1, x_2, \dots, \underbrace{x_{10{,}000}}_{M}$

$$y_j = \frac{\sum_{i=0}^{1} P(j \mid i) \int_{S_i} x f(x)\,dx}{\sum_{i=0}^{1} P(j \mid i) \int_{S_i} f(x)\,dx}$$

$$P(i \mid j) = \begin{cases} \varepsilon & \text{if } i \neq j \\ 1-\varepsilon & \text{if } i = j \end{cases}$$

$$= \boxed{\frac{\sum_{i=0}^{1} P(j \mid i) \frac{1}{M} \sum_{x_\ell \in S_i} x_\ell}{\sum_{i=0}^{1} P(j \mid i) |S_i| / M}}$$

$i, j \in \{0, 1\}$

Noiseless case, $N=1$

Minimize $\quad E[(X-y)^2]$

the single ($N=1$) output point

$y = E[X]$ is the opt. y

$E[(X-E(X))^2] = var(X) = 1$

the opt. <u>distortion</u>

$$D = \frac{1}{M}\sum_{\ell=1}^{M}(x_\ell - Q(x_\ell))^2 \quad \text{(noiseless case)}$$

$$D = \frac{1}{M}\sum_{\ell=1}^{M}\sum_{j=0}^{1} p(j \mid i_{x_\ell})(x_\ell - y_j)^2$$

$$i_{x_\ell} = \begin{cases} 0 & \text{if } x_\ell \in S_0 \\ 1 & \text{if } x_\ell \in S_1 \end{cases}$$

E.g. if $x_\ell \in S_0$, then

$$\underbrace{p(0 \mid 0)}_{1-\varepsilon}(x_\ell - y_0)^2 + \underbrace{p(1 \mid 0)}_{\varepsilon}(x_\ell - y_1)^2$$

GLM : COSQ Alg. $X \sim N(0,1)$

Checking your program

$\varepsilon = 0.05$
$\varepsilon = 0.1$

1) Lloyd-Max quantizer $N=2$ vs GLM with $N=2$ and noise $\varepsilon > 0$ — D_1, D_2

2) GLM with $N=2$, $\varepsilon = 0$ should provide the same output points and intervals as Lloyd-Max

3) Design Q with Lloyd-Max $N=2$, use it on noisy channel, $\varepsilon > 0$, and compare distortion to GLM.

$D_1 < D_2$ for all $\varepsilon > 0$.

$D_1 = D_2$ for $\varepsilon = 0$