

# PROJECT 1: MUON COLLIDER

## FYS5555 - RESEARCH-BASED PARTICLE PHYSICS

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### 1. TRANSITION AMPLITUDES

The contributions to the process  $\mu^+\mu^- \rightarrow b\bar{b}$  includes the propagator for QED ( $\gamma$ ) as well as the propagators for the electroweak theory ( $Z$ ) and Higgs ( $H$ ). The Feynman diagrams for each of these processes can be seen in figure 1. We know that the total amplitude  $\langle |M_{tot}|^2 \rangle$  is a sum of the amplitudes for each individual process including interference terms, and can be expressed as

$$(1) \quad \langle |M_{tot}^2| \rangle = \langle |M_\gamma|^2 \rangle + \langle |M_Z|^2 \rangle + \langle |M_H|^2 \rangle + 2\langle M_\gamma^\dagger M_Z \rangle + 2\langle M_\gamma^\dagger M_H \rangle + 2\langle M_Z^\dagger M_H \rangle$$

From figure 1 we can derive the transition amplitude for each process:

$$(2) \quad \begin{aligned} -iM_\gamma &= -i\frac{e^2}{3s} [\bar{v}^{s'}(p_2)\gamma^\mu u^s(p_1)] [\bar{u}^r(p_3)\gamma_\mu v^{r'}(p_4)] \\ -iM_Z &= i\frac{g_Z^2}{s - m_Z^2 + im_Z\Gamma_Z} [\bar{v}^{s'}(p_2)\frac{1}{2}\gamma^\mu (c_V - c_A\gamma^5)u^s(p_1)] \\ &\quad \times [\bar{u}^r(p_3)\frac{1}{2}\gamma_\mu (\tilde{c}_V - \tilde{c}_A\gamma^5)v^{r'}(p_4)] \\ -iM_H &= -i\frac{m_\mu m_b g_W^2}{2m_W(s - M_H^2 + im_H\Gamma_H)} [\bar{v}^{s'}(p_2)u^s(p_1)] [\bar{u}^r(p_3)v^{r'}(p_4)] \end{aligned}$$

where  $s = (p_1 + p_2)^2$ ,  $e^2 = 4\pi\alpha$  and  $\Gamma_i$  being the decay width's of an unstable particle  $i = Z, H$ .  $c_V, c_A, \tilde{c}_V, \tilde{c}_A$  are the vector and axial vector couplings of respectively  $\mu^-$  and  $b$ .<sup>1</sup>

### 2. DIFFERENTIAL CROSS SECTION

The spin-averaged square amplitude can be expressed as

$$(3) \quad \langle |M_{tot}|^2 \rangle = \frac{1}{4} \sum_{\text{spins}} |M_{tot}|^2$$

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<sup>1</sup>All measured variables used in this report such as the coupling constants, decay widths and masses of the particles can be found at <http://pdg.lbl.gov/> [1]

such that the spin-averaged amplitudes for the individual theories are

$$\begin{aligned}
(4) \quad \langle |M_H|^2 \rangle &= \frac{m_b^2 m_\mu^2 g_W^4}{4[(s-m_H^2)^2 + m_H^2 \Gamma_H^2]} [(p_1 \cdot p_2)(p_3 \cdot p_4) - m_b^2(p_1 \cdot p_2) - m_\mu^2(p_3 \cdot p_4) + m_\mu^2 m_b^2] \\
(5) \quad \langle |M_\gamma|^2 \rangle &= \frac{8e^4}{9s^2} [(p_1 \cdot p_4)(p_2 \cdot p_3) + (p_1 \cdot p_3)(p_2 \cdot p_4) + m_b^2(p_1 \cdot p_2) + m_\mu^2(p_3 \cdot p_4) + 2m_b^2 m_\mu^2] \\
(6) \quad \langle |M_Z|^2 \rangle &= \frac{g_Z^4}{2[(s-m_Z^2)^2 + m_Z^2 \Gamma_Z^2]} \{ (c_V^2 + c_A^2)(\tilde{c}_V^2 + \tilde{c}_A^2)[(p_1 \cdot p_4)(p_2 \cdot p_3) + (p_1 \cdot p_3)(p_2 \cdot p_4)] \\
&\quad + m_b^2(c_V^2 + c_A^2)(\tilde{c}_V^2 - \tilde{c}_A^2)(p_1 \cdot p_2) \\
&\quad + m_\mu^2(c_V^2 - c_A^2)(\tilde{c}_V^2 + \tilde{c}_A^2)(p_3 \cdot p_4) \\
&\quad - 4c_V c_A \tilde{c}_V \tilde{c}_A [(p_1 \cdot p_4)(p_2 \cdot p_3) - (p_1 \cdot p_3)(p_2 \cdot p_4)] \\
&\quad + 2m_\mu^2 m_b^2 (c_V^2 - c_A^2)(\tilde{c}_V^2 - \tilde{c}_A^2) \}
\end{aligned}$$

where the derivation for each individual spin-averaged amplitude can be seen separately in the attachments.

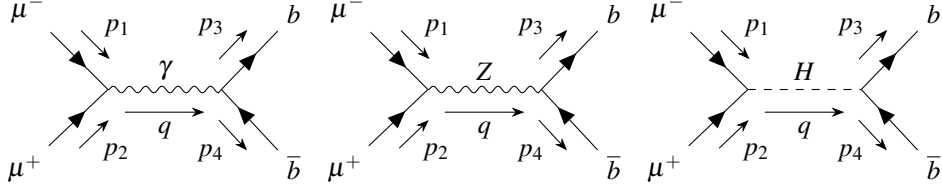


FIGURE 1. Feynman diagrams for the process  $\mu^+ \mu^- \rightarrow b \bar{b}$

The spin-averaged amplitudes can be used to calculate the differential cross section in the center-of-mass frame, which can be expressed as

$$(7) \quad \frac{d\sigma}{d(\cos \theta)} = 3 \times \frac{1}{32\pi s} \frac{p_f^*}{p_i^*} \langle |M|^2 \rangle$$

where  $p_f^*$  and  $p_i^*$  are respectively the magnitude of the final and initial particle states in the center-of-mass frame. A factor of 3 is included to describe the additional three colour charges  $r\bar{r}$ ,  $b\bar{b}$  and  $g\bar{g}$  for the quarks in the final state. Thus, the differential cross sections for each individual propagator including  $H$ ,  $\gamma$  and  $Z$  (excluding the interference terms) at a center-of-mass energy  $\sqrt{s} = 150$  GeV can be seen in figure 3. Here, we see that the most dominating contributions are that of QED and  $Z$ , whereas the process including Higgs as the propagator is nearly zero. Thus, we choose to neglect the Higgs process in the rest of the calculations of the total amplitude squared, allowing us to rewrite eq. (1) as

$$\begin{aligned}
(8) \quad \langle |M|^2 \rangle &= \langle |M_\gamma|^2 \rangle + \langle |M_Z|^2 \rangle + 2\langle M_\gamma^\dagger M_Z \rangle \\
&= \frac{8e^4}{9s^2} [(p_1 \cdot p_4)(p_2 \cdot p_3) + (p_1 \cdot p_3)(p_2 \cdot p_4) + m_b^2(p_1 \cdot p_2) + m_\mu^2(p_3 \cdot p_4) + 2m_b^2 m_\mu^2] \\
&\quad + \frac{g_Z^4}{2[(s-m_Z^2)^2 + m_Z^2 \Gamma_Z^2]} \{ (c_V^2 + c_A^2)(\tilde{c}_V^2 + \tilde{c}_A^2)[(p_1 \cdot p_4)(p_2 \cdot p_3) + (p_1 \cdot p_3)(p_2 \cdot p_4)] \\
&\quad + m_b^2(c_V^2 + c_A^2)(\tilde{c}_V^2 - \tilde{c}_A^2)(p_1 \cdot p_2) + m_\mu^2(c_V^2 - c_A^2)(\tilde{c}_V^2 + \tilde{c}_A^2)(p_3 \cdot p_4) \\
&\quad - 4c_V c_A \tilde{c}_V \tilde{c}_A [(p_1 \cdot p_4)(p_2 \cdot p_3) - (p_1 \cdot p_3)(p_2 \cdot p_4)] + 2m_\mu^2 m_b^2 (c_V^2 - c_A^2)(\tilde{c}_V^2 - \tilde{c}_A^2) \} \\
&\quad + 2 \times \frac{4e^2 g_Z^2}{3s(s-m_Z^2 + im_Z \Gamma_Z)} \{ c_V \tilde{c}_V [(p_1 \cdot p_4)(p_2 \cdot p_3) - (p_1 \cdot p_3)(p_2 \cdot p_4)] \\
&\quad - c_A \tilde{c}_A [(p_1 \cdot p_3)(p_2 \cdot p_4) - (p_1 \cdot p_4)(p_2 \cdot p_3)] \\
&\quad + c_V c_V m_b^2(p_1 \cdot p_2) - c_V \tilde{c}_V m_\mu^2(p_3 \cdot p_4) + 2c_V \tilde{c}_V m_\mu^2 m_b^2 \}
\end{aligned}$$

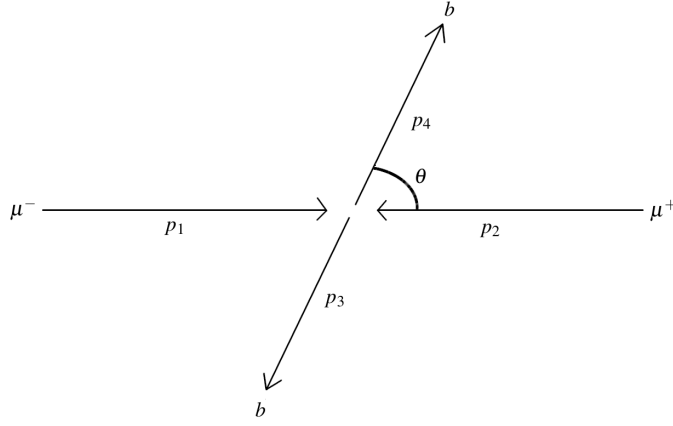


FIGURE 2. Collision in the center-of-mass frame

The momentas in the center-of-mass frame, as seen in figure 2, can be expressed as

$$(9) \quad p_1 = (E, 0, 0, p), \quad p_3 = (E, -p' \sin \theta, 0, -p' \cos \theta)$$

$$(10) \quad p_2 = (E, 0, 0, -p), \quad p_4 = (E, p \sin \theta, 0, p' \cos \theta)$$

$$(11) \quad s = (p_1 + p_2)^2 = 4E^2$$

$$(12) \quad p_1 \cdot p_2 = E^2 + p^2, \quad p_1 \cdot p_3 = p_2 \cdot p_4 = E^2 + pp' \cos \theta$$

$$(13) \quad p_3 \cdot p_4 = E^2 + p'^2, \quad p_1 \cdot p_4 = p_2 \cdot p_3 = E^2 - pp' \cos \theta$$

where  $p = \sqrt{E^2 - m_\mu^2}$  and  $p' = \sqrt{E^2 - m_b^2}$ .

The differential cross-section, when plugging the momentas defined above into the results from eq. (8), can be seen in figure 4. Here, there is almost a complete overlap between the analytical calculations and the numerical calculations as derived from CompHep. We also see that the interference term along with the pure QED term clearly also has an affect at  $\langle |M_{tot}|^2 \rangle$ , although their contributions are smaller than that of pure Z.

### 3. FORWARD-BACKWARD ASYMMETRY

Forwards-backwards asymmetry can be expressed as

$$(14) \quad \sigma_{FB} = \frac{N_{\theta > \pi/2} - N_{\theta < \pi/2}}{N_{\theta > \pi/2} + N_{\theta < \pi/2}} = \frac{\sigma_{\theta > \pi/2} - \sigma_{\theta < \pi/2}}{\sigma_{\theta > \pi/2} + \sigma_{\theta < \pi/2}}$$

where  $N_{\theta > \pi/2}$  and  $N_{\theta < \pi/2}$  stands for the amount of events scattering respectively at angles  $\theta > \pi/2$  and  $\theta < \pi/2$ . Integrating eq. (8) with respect to  $\cos \theta \in [-1, 1]$  using the trapezoidal rule, as provided by Python, we obtain the asymmetries of the different processes as shown in figure 5a. Here, we see that the asymmetry of the pure QED alone is nearly negligible, meaning that the angular distribution of the final-state leptons is fairly symmetric, as can also be seen in figure 3b. However, the asymmetry of the pure Z-channel is more prominent, and a bit more skewed as can also be seen in figure 3c. Whereas the contribution from Higgs is zero due to remaining constant throughout the different scattering angles, which we can also see from the angular distribution

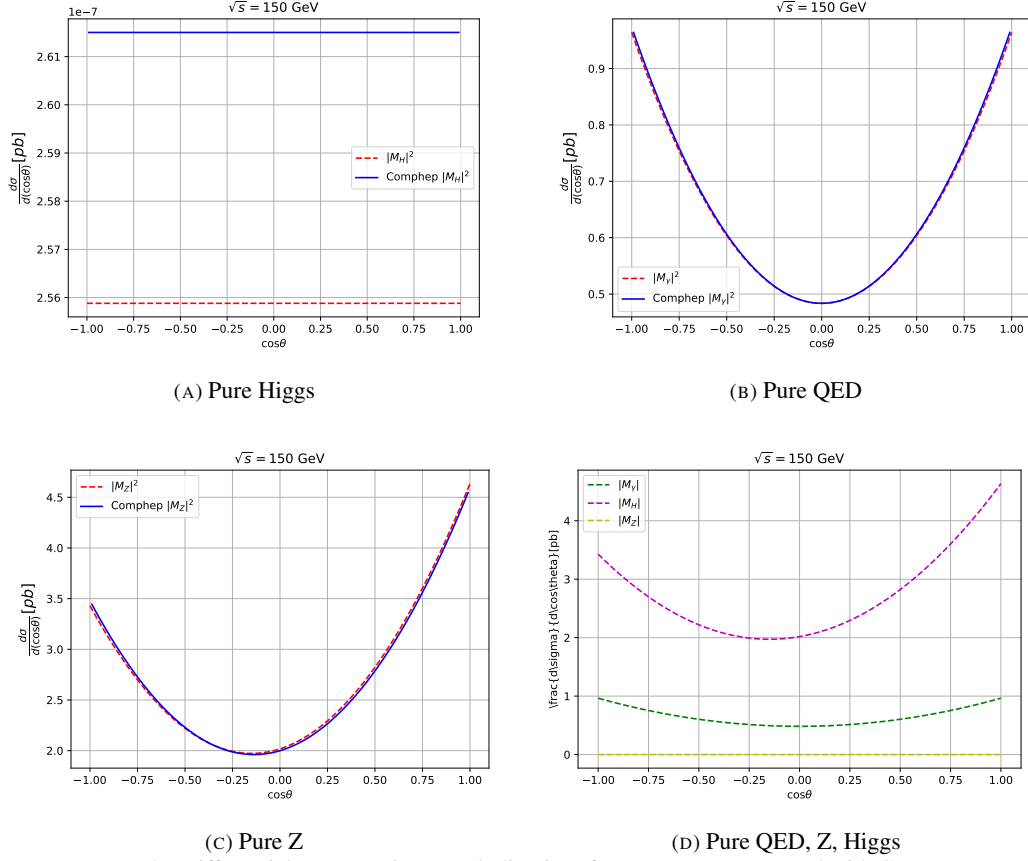


FIGURE 3. Differential cross-sections, excluding interference terms, compared with CompHep

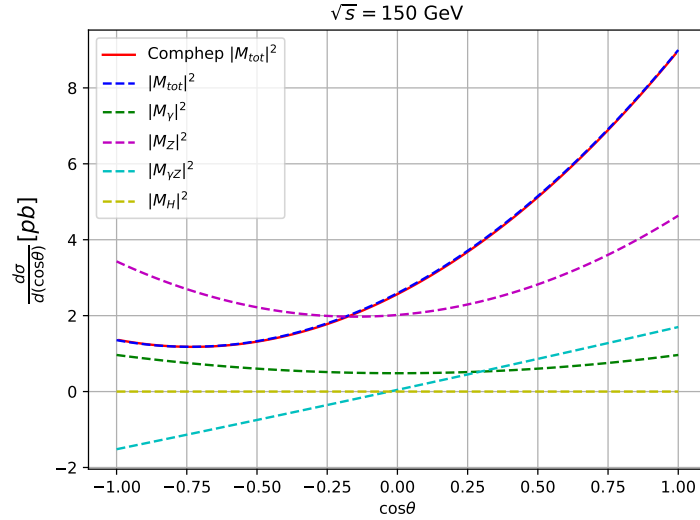
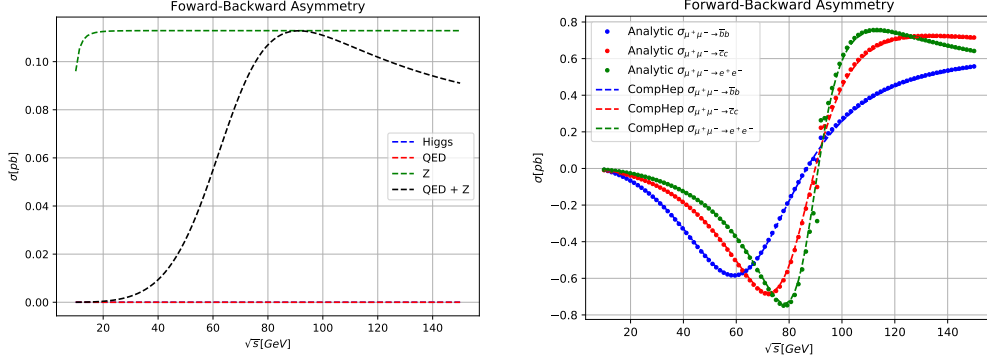


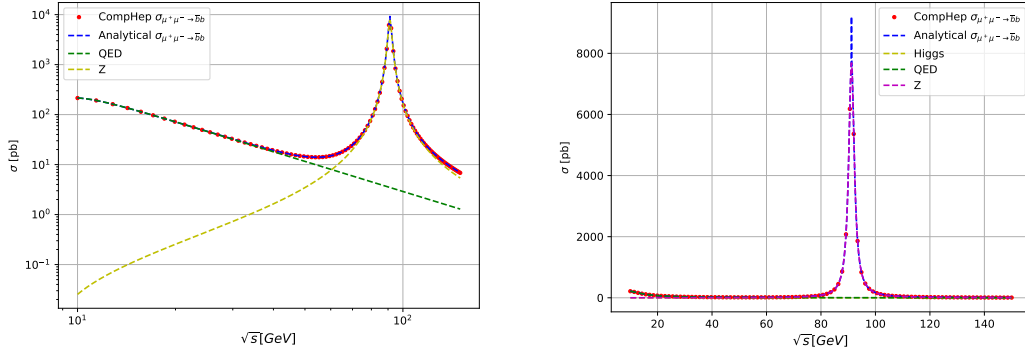
FIGURE 4. Differential cross section compared with CompHep

of figure 3a. Looking at the total asymmetry from fig. 5b of the final-state quarks  $b\bar{b}$ ,  $\sigma_{FB}$  is approximately zero for  $\sqrt{s} < 20\text{GeV}$ , which is due to the QED term being the most dominating at small energies as well as symmetric in the angular distribution. As we move a bit higher up on the energy scale,  $\sigma_{BF}$  becomes negative, meaning that the cross-sections for scattering at  $\theta < \pi/2$  is much more likely than scattering at  $\theta > \pi/2$  when  $\sqrt{s} \in (20, 90)\text{GeV}$ . If we move to higher



(A) Asymmetry for individual processes for final state  $b\bar{b}$  (B) Asymmetry for different final states  $b\bar{b}$ ,  $c\bar{c}$  and  $e^+e^-$

FIGURE 5. Forward-Backward asymmetry



(A) Logarithmic scale

(B) Linear scale

FIGURE 6. Total cross section

energies, scattering at  $\theta > \pi/2$  starts to dominate due to the contribution from the Z term. Close to  $m_Z$  the asymmetry is about zero. The angular distribution of the final-state  $b\bar{b}$  compared to the final states  $c\bar{c}$  and  $e^+e^-$  appears to be more shifted than that of the two latter cases. This may be due to the vector coupling  $c_V$  being greater for  $b\bar{b}$  than  $e^+e^-$  and  $c\bar{c}$ .

#### 4. TOTAL CROSS SECTION

Inserting the momentas defined in eqs. (9)-(13) into the total amplitude of eq. (8), and integrating over  $\cos \theta \in [-1, 1]$  using the trapezoidal rule, we get the total cross sections as seen in figure 6. Looking at fig. 6a the total cross section is dominated by QED at lower energy scales due to the fact that the center-of-mass energy  $\sqrt{s}$  is not yet high enough to produce a Z as a propagator. As  $\sqrt{s}$  increases, the production of a Z-boson becomes more and more likely until the pure Z-channel starts to dominate over  $\gamma$  which leaves us with a maximum when  $\sqrt{s}$  approaches  $m_Z$ . This can also be seen in the linear plot of figure 6b where there is a resonant peak at  $\sqrt{s} \approx m_Z$ .

#### 5. NEW PHYSICS

We can introduce a new gauge boson  $Z'$  to CompHep by assuming the same interactions as Z in the Electroweak theory, only at a substantially higher mass. It is postulated [1] that the mass of the  $Z'$ -boson has a lower bound of 4.5 TeV in the Standard Model, with CL = 95%, through a pp direct search. Thus, we can set the mass of our hypothetical gauge boson at say  $m_{Z'} = 5.0$  TeV.

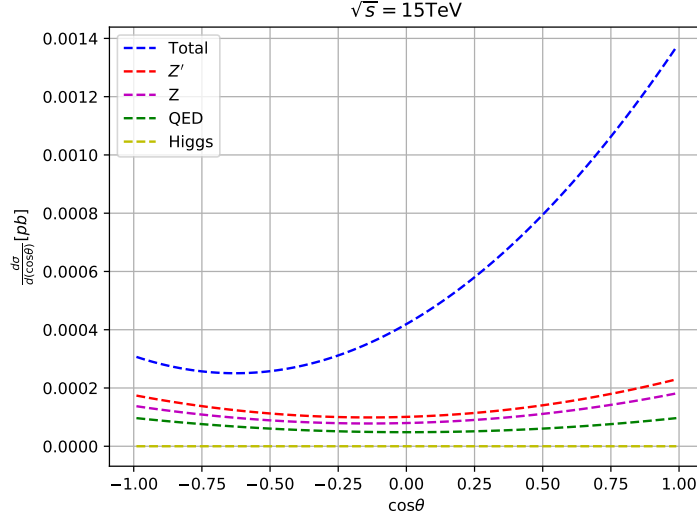


FIGURE 7. Differential cross section when including SM  $Z'$  for the process  $\mu^+\mu^- \rightarrow b\bar{b}$  using CompHep

**5.1. Differential cross section.** When including the  $Z'$ -boson into our theory, our total amplitude squared now becomes

$$(15) \quad \langle |M_{tot}|^2 \rangle = \langle |M_\gamma|^2 \rangle + \langle |M_Z|^2 \rangle + \langle |M_{Z'}|^2 \rangle + 2\langle M_\gamma^\dagger M_Z \rangle + 2\langle M_\gamma^\dagger M_{Z'} \rangle + 2\langle M_Z^\dagger M_{Z'} \rangle$$

Calculating the differential cross-section at  $\sqrt{s} = 15$  TeV and plotting using CompHep, we get the plot as shown in figure 7, where  $d\sigma/d(\cos\theta)$  is plotted with respect to all processes. We see that at higher energies  $Z$  and  $\gamma$  becomes more and more suppressed in comparison to  $Z'$ . However, the total differential cross section is a factor 2 smaller than that of the differential cross section from figure 4. This is due to the high energies involved as can be seen from the amplitudes in (2), where each process is inversely proportional to  $s$ .

**5.2. Total cross section.** Integrating the differential cross section over  $\cos\theta \in [-1, 1]$  we get the total differential cross section as seen in figure 8. Here, we clearly see resonant peak at  $m_{Z'}$  as we would expect.

**5.3. Forward-backward asymmetry.** The forward-backward asymmetry for the process  $\mu^+\mu^- \rightarrow b\bar{b}$  when including  $Z'$  can be seen in figure 9. Here, it seems as though scattering at  $\theta > \pi/2$  is dominant at energies below roughly 1.4 TeV and above 1.9 TeV, while scattering at  $\sqrt{s} \in (1.4, 1.9)$  TeV mostly happens for  $\theta < \pi/2$ .

## 6. ATLAS DATA ANALYSIS

Using the 8TeV dataset as published by ATLAS OpenData, we can plot the invariant mass  $m_{ll}$  for possible processes leading to dilepton final states. The OpenData considers the process

$$pp \rightarrow l^+ l^-,$$

where we will be looking at processes with lepton pairs of  $e^+e^-$  and  $\mu^+\mu^-$  in the final state.

In doing so, we end up with the stacked histograms as shown in figure 10. For the individual processes we see that the expected contribution for Higgs (through Monte Carlo simulations) is fairly low, whereas the process including  $Z$  are the most dominating ones above  $\sim 25$  GeV. We also see that Drell-Yan processes, i.e.  $q\bar{q} \rightarrow l^+ l^-$ , are mostly present at energies below 25 GeV. We also see a resonant peak about the  $Z$ -mass  $m_Z = 91.2$  GeV, as one would expect from the theory.

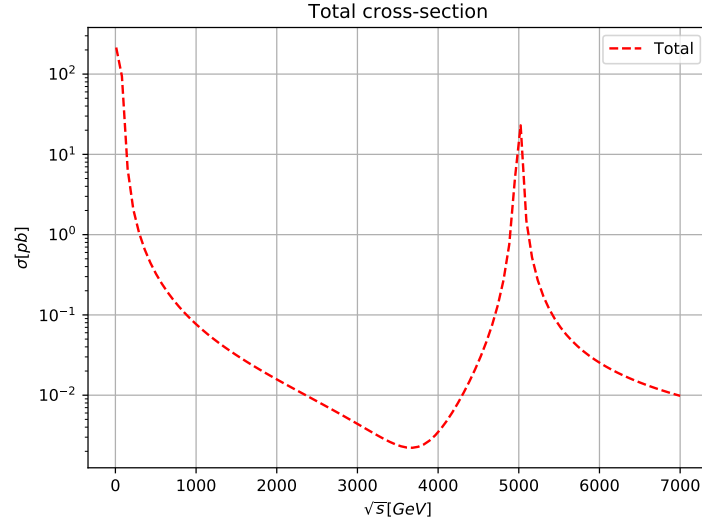


FIGURE 8. Total cross section when including SM  $Z'$  for the process  $\mu^+\mu^- \rightarrow b\bar{b}$  using CompHep

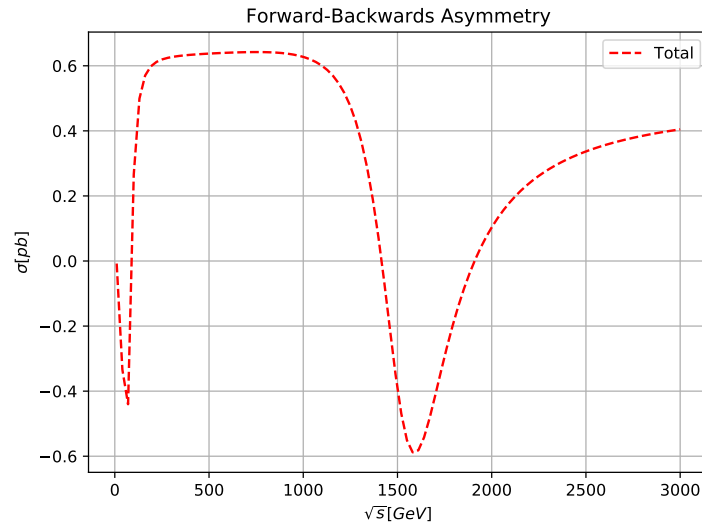


FIGURE 9. Forward-backward asymmetry when including SM  $Z'$  for the process  $\mu^+\mu^- \rightarrow b\bar{b}$  using CompHep

The Github repository for this project can be found at <https://github.com/elisabethchr/FYS5555> which includes the code and data used for the analysis of the invariant mass of dileptons.

#### REFERENCES

- [1] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D98, 030001 (2018) and 2019 update
- [2] Thomson, M., 2013, *Modern Particle Physics*, Cambridge University Press, Cambridge

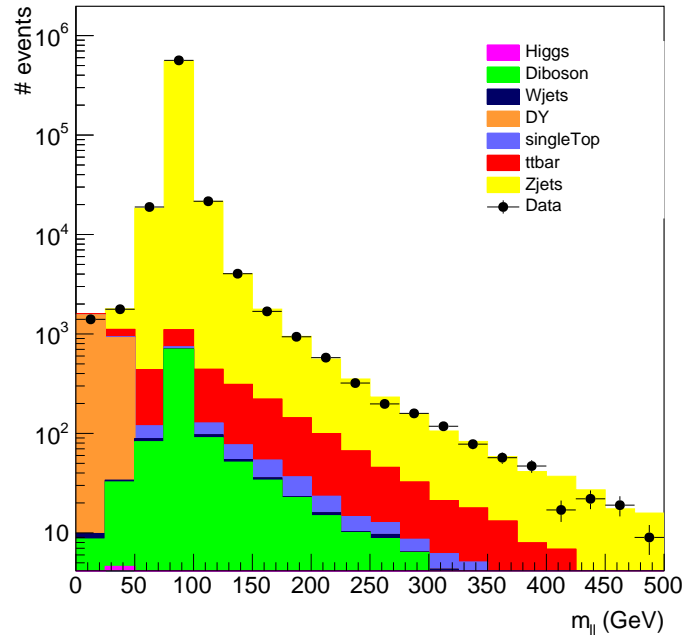


FIGURE 10. Invariant mass distributions for dileptons using the 8 TeV dataset provided by ATLAS OpenData