Reverse information sharing: Reducing costs in supply chains with yield uncertainty

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Supply uncertainty in produce supply chains presents major challenges to retailers. Supply shortages create frequent disruptions in terms of promised delivery times, quantity and quality delivered. To alleviate these challenges, dual sourcing—a strategy in which buyers source a good from two different suppliers—is commonly employed by retailers in these supply chains. However, the benefits of dual sourcing cannot be fully realized when a lack of transparency exists between retailers and suppliers. In this case, perceived scarcity leads to over-ordering, further exacerbating the problem of supply unreliability in settings where multiple retailers compete for supply. This paper studies a supply chain for a perishable good consisting of N retailers who compete for supply and practice dual sourcing, but do not have transparency to the inventory distributions of their suppliers *a priori*. The paper develops an analytical model to capture the retailers' ordering dynamics over repeated iterations. When the retailers underestimate the suppliers' inventory, their orders converge to an equilibrium where all retailers drastically over-order. This results in higher retailer costs and supply chain waste, as well as higher costs to the suppliers for certain contract structures and parameters. The paper analyzes the impact of an information sharing scheme in which suppliers share inventory information downstream. This *reverse information sharing* counteracts perceptions of scarcity thereby reducing over-ordering.

Key words: Information sharing, yield uncertainty, ration gaming, blockchain

History:

1. Introduction

Uncertainty and lack of transparency are defining characteristics of fresh produce supply chains. Supply losses create uncertainty at every step of the supply chain, impacting promised delivery times, quantity and quality delivered. This make supply chain management extremely challenging for fresh produce retailers. In particular, this situation increases the likelihood of costly stockouts at retailers. Grocery stores in the U.S. are estimated to lose $75 billion in sales every year due to stockouts and unsaleables (Wells 2017).

In order to decrease the likelihood of stockouts, most grocery retailers diversify their supply sources, engaging in a strategy known as dual- or multi-sourcing. Grocery retailers source their products from an average of about 250 different suppliers (McLaughlin et al. 2015). In general, dual sourcing strategies are known to help reduce the likelihood of stockout in supply chains with supply uncertainty. However, the benefits of dual sourcing cannot be fully realized in supply chains suffering from lack of transparency—a common characteristic of produce supply chains (McLaughlin et al. 2015). One potential consequence of lack of transparency is *perceived scarcity*, which occurs when the retailers believe that supply is very limited. This paper demonstrates that employing dual sourcing strategies in settings with perceived scarcity is likely to lead to over-ordering, resulting in inflated costs to the retailers as well as substantial waste. Additionally, if the suppliers' contracts are such that they are sufficiently penalized for unfulfilled orders, retailer over-ordering also increases supplier costs.

This paper studies how *reverse information sharing*—namely, suppliers sharing inventory and order information downstream—enables better retailer decision-making when engaging in a dual-sourcing strategy. In many settings, this information sharing scheme is found to reduce costs for *both* retailers and suppliers, and reduces total supply chain waste. Reverse information sharing is always beneficial to retailers, however its benefits are the most substantial in settings where the

retailers perceives supply scarcity. Therefore, unlike classic dual sourcing literature, the assumption of the model is that retailers do *not* necessarily know the suppliers' inventory distributions *a priori*, but can learn information about the distributions over time through reverse information sharing. In practice, suppliers may be hesitant to share their inventory and order information with their retailers. Furthermore, a lack of trust may exist between the suppliers and retailers. Therefore, a specific implementation scheme is proposed where all computations are performed by a trusted third party (TTP). The TTP provides the retailers with enough information to enable them to optimally update their order decisions over time, while limiting the amount of knowledge they gain about the suppliers' private information.

This paper develops a model that captures a supply chain consisting of N grocery retailers who source a single good from two suppliers. The retailers compete with each other for supply, but not for demand. For example, the retailers might source from the same regional warehouse of a particular supplier, but are located far enough apart that they do not share customers (a competition model similar to that of the ration gaming literature, e.g., Bray et al. (2019), Cachon and Lariviere (1999), Cui and Shin (2018)). The model considers a discrete time infinite horizon. In each discrete time period (or iteration), all retailers allocate an order across the two suppliers, who then fulfill orders based on a pre-specified fulfillment mechanism. For example, suppose that in one iteration, a given retailer wishes to obtain 100 pounds of apples. The retailer must then decide how much to order from each supplier, knowing that supply uncertainty exists, in order to best meet this desired quantity (for example, by ordering 70 pounds from each supplier). The behavior of other retailers impacts the optimal ordering decision of any given retailer, and therefore ordering decisions can be thought of as a dynamic game among the retailers. The optimal ordering decision of each retailer also depends on the retailers' perceptions of the inventory distributions at the suppliers, which can be impacted through the proposed reverse information sharing scheme.

1.1. Results and contributions

The following are the main results and contributions of this paper:

1. The paper considers a dynamic setting where the retailers start with limited information about the suppliers' inventory. Through reverse information sharing, the retailers gain information over time about the suppliers' true inventory distributions, which allows them to update their ordering decisions. The analysis shows that, when the retailers are identical and their ordering decisions follow best response dynamics, the retailers' orders can converge to at most two Nash equilibria. In the first, each retailer orders its desired quantity from *both* suppliers (*Allocation* $2Q$). In the second, the orders placed to each supplier sum to the retailers desired quantity exactly (*Allocation* Q).

2. *Allocation* Q results in lower retailer costs and supply chain waste than *Allocation* $2Q$. Furthermore, when the contracts between the retailers and suppliers sufficiently penalize suppliers for unmet orders, *Allocation* Q also results in lower supplier costs. However, the dynamics will only converge to *Allocation* Q under certain conditions.

3. The analysis provides sufficient conditions for the dynamics to converge to either order allocation. Intuitively, when the retailers perceive supply scarcity, the dynamics will converge to *Allocation* $2Q$. When the retailers believe that there is ample supply, the ordering decisions will converge to *Allocation* Q.

4. When retailers falsely perceive supply scarcity, reverse information sharing can induce convergence to *Allocation* Q, resulting in a *drastic* decrease in order quantities, costs, and waste. Furthermore, by using a trusted third party, reverse information sharing can be accomplished in such a way that the retailers' ability to learn information about the upstream supply chain is limited. This is desirable when privacy is a concern to the suppliers.

Although this paper focuses on the use-case of a produce supply chain, yield uncertainty and lack of transparency are not unique to perishable food supply chains. The insights of this paper can be readily applied to other supply chain settings with yield uncertainty; in particular, to settings where dual sourcing strategies are employed and the retailers are not fully aware of the suppliers' inventory distributions.

2. Related literature

The topic of this paper overlaps with many areas of research, including: dual sourcing, yield uncertainty, ration gaming, information sharing in supply chains, and the impact of perceived supply. An overview of the most relevant literature in each area is provided.

2.1. Dual sourcing and yield uncertainty

Dual or multi-sourcing strategies have been explored extensively in the literature as a tool for mitigating the risks associated with yield uncertainty. When yield uncertainty is present, a retailer is not guaranteed to receive its entire order. Therefore, retailers are benefited by placing orders with multiple suppliers. The majority of work in this area focuses on single retailer or manufacturer who makes sourcing decisions in a single period (Dada et al. 2007, Anupindi and Akella 1993). This stream of work, although not the focus of this paper, has many interesting directions including studying the retailer's strategic choice of suppliers (Federgruen and Yang 2009, Gerchak and Parlar 1990, Niu et al. 2019), the suppliers' strategic choices of prices and other attributes (Demirel et al. 2018), and the effects of supply correlation and risk propagation between different tiers of suppliers (Ang et al. 2017, Bimpikis et al. 2018).

A handful of work extends these models to a setting with two or more retailers who compete for demand through Cournot competition (Tang and Kouvelis 2011, Wu et al. 2019). Both Tang and Kouvelis (2011) and Wu et al. (2019) assume that the retailers are faced with proportional random yield—meaning that the retailers receive a random portion of the amount ordered from each supplier. These random portions are uncorrelated, implying that the retailers do not compete for inventory. This paper, on the other hand, takes the opposite approach by assuming that the retailers do not compete for demand but instead compete for inventory. This is a more common assumption in the ration gaming and inventory competition models discussed in the next section.

Lastly, the dual sourcing literature generally assumes that the retailers are aware of the suppliers' yield uncertainties and can optimize their sourcing strategies using this information. Tomlin (2009) is unique in that it considers the case where retailers must learn the distribution of their suppliers'

yield. Like Tomlin (2009), this paper assumes that the retailers do *not* know the suppliers' yields *a priori*. However, the yield distributions can be learned over time. Although both Tomlin (2009) and this paper include "supply learning", the model considered in Tomlin (2009) focuses on the decisions of a single retailer, whereas this paper focuses on inventory competition among many retailers.

2.2. Ration gaming and inventory competition

Ration gaming (or capacity allocation) describes the situation when a supplier receives multiple orders, usually simultaneously, but does not have enough inventory to meet all orders. Therefore, the supplier must choose a fulfillment rule to distribute the inventory among the buyers. Knowing the fulfillment rule, and after forming some belief about the behaviors of the other buyers, each buyer can compute its optimal order quantity. Typically, the solution concept in ration gaming models is a Nash equilibrium order quantity for each buyer.

Cachon and Lariviere (1999) considers different fulfillment mechanisms and the Nash equilibria that they induce among the retailers, searching for mechanisms with desirable properties. For example, truth-telling mechanisms—those that incentivize the buyers to only order their true desired quantity— are often desirable since they mitigate over-ordering. Over-ordering generally has negative implications on the entire upstream supply chain. For example, Lee et al. (1997) and Bray et al. (2019) both study the impact of ration gaming on the bullwhip effect. Cachon and Lariviere (1999) show that, in their setting, although a truth-telling mechanism (i.e., one that induces the retailers to order exxactly their desired quantities) does exist, it does not maximize retailer profits nor supplier profits, and therefore the supply chain is better off with a different mechanism that is not truth-telling. Cui and Shin (2018) propose a behavioral model to study ration gaming under a proportional rationing rule, and explore the extent to which over-ordering is influenced by different characteristics of the supplier and other retailers.

Similar to ration gaming, another stream of literature studies inventory management decisions under supply competition (Bernstein and Federgruen 2005, Cachon 2001). Most related to this

paper, Cachon (2001) considers the optimal re-ordering point for N retailers who source from a single supplier. Similar to the ration gaming literature, the retailers compete for supply but not for customers. Like the model proposed in this paper, Cachon (2001) considers orders that take place over a time horizon.

The model proposed in this paper combines ideas from Cachon and Lariviere (1999) and Cachon (2001) with the yield uncertainty and dual sourcing literature. Like Cachon (2001) and Cachon and Lariviere (1999), we study the Nash equilibrium of the retailers' ordering decisions (in this paper, the equilibrium is found by considering the convergence of best response dynamics). However, instead of focusing on the retailers' underlying inventory management system, we instead focus on the decision of how to allocate each order across the two suppliers.

Furthermore, unlike the ration gaming and yield uncertainty literature which generally assume that the retailers are aware of the inventory distributions at the suppliers, we assume that the retailers only have a *perception* of the suppliers' inventory, which can change over time with reverse information sharing.

2.3. Information sharing and perceived supply

Information sharing in supply chains is a widely studied topic. Specifically, literature on upstream information sharing (e.g., from retailers to suppliers) is ubiquitous and arguably the most well-studied form of information sharing in supply chains (Shang et al. 2016, Li and Zhang 2008, Ha and Tong 2008, Cachon and Fisher 2000, Gaur et al. 2005, Lee et al. 2000, Cachon and Lariviere 2001, Li 2002). This type of information, while extremely valuable to supply chain management, is not the focus of this paper.

We refer to *reverse information sharing* as a supplier sharing information with its retailers. In the supply chain literature, a handful of studies consider information sharing schemes similar to reverse information sharing (Chen 2003). Choi et al. (2008) consider the value of a supplier sharing supply yields with its manufacturer in a serial supply chain, finding that information sharing is most valuable when supply yield has high variance and demand has low variance. Croson and

Donohue (2006) investigate the impact of inventory transparency on the bullwhip effect in a linear supply chain. Chen and Yu (2005) quantify the value of information about leadtimes for a retailer ordering goods from a single supplier. Jain and Moinzadeh (2005) consider a serial supply chain in which the manufacturer allows the retailer to observe its inventory levels. The optimal ordering policy for the retailer is computed, and the value of the information to the retailer is estimated through computational experiments. This paper considers a similar type of reverse information sharing as Jain and Moinzadeh (2005), but in a very different supply chain setting.

The idea of reverse information sharing has similarities to the concept of sellers sharing information with their customers, which has been studied in the marketing literature. In particular, the impact of a buyers' perception of supply, and how sellers can influence this perception, has been studied (Cui and Shin 2018, Gallino and Moreno 2014, Allon and Bassamboo 2011, Byun and Sternquist 2012). For example, Allon and Bassamboo (2011) consider how a firm can influence customer behavior through information about availability, and how these results depend on the customers' heterogeneity and the trustworthiness of the information.

Trust is a closely related topic to information sharing, and the importance of trust in supply chain information sharing schemes is a widely studied topic (Özer et al. 2011, 2014, Özer and Zheng 2017, Spiliotopoulou et al. 2016). Most relevant to this paper are settings that consider the extent to which a customer trusts information provided by a supplier. For example, in Allon and Bassamboo (2011), it is specifically noted that it is not realistic to assume that customers will blindly trust inventory information offered by a retailer. Özer et al. (2018) consider a setting where a seller assists its buyers in making their decision about which products or services to purchase. The assistance can come in the form of advice or information sharing, and the implications of different assistance schemes on trust is studied. Desai (2000) considers the problem of a manufacturer who must convince a retailer to order a new, high-demand product through signaling. Although the specific information that is shared in these settings is different from this paper, the notion of trust between the buyer and seller remains important.

In this paper, a specific information sharing implementation is proposed that takes place on a blockchain platform. Blockchain has emerged as a promising technology that can aid in real-time verifiable information sharing, and thus its usefulness in supply chains is becoming apparent (Gaur and Gaiha 2020, Cheung et al. 2018). For example, Chod et al. (2020) show that signaling through inventory transactions on a blockchain is an efficient mechanism for firms to signal their quality. This paper considers how suppliers' verifiable inventory and order data can be used to enable trustworthy reverse information sharing between a supplier and its retailers.

The remainder of the paper is organized as follows: Section 3 gives an overview of the model and discusses a retailer's optimal ordering decision in a single iteration when the behavior of other retailers is fixed. Section 4 considers a game-theoretic version of the model where all retailers are strategic, and discusses convergence of the best response dynamics. Section 5 presents an extensive numerical example to illustrate the various information sharing schemes proposed. Section 6 suggests a specific implementation of the reverse information sharing scheme that assures privacy. Section 7 summarizes the main findings and concludes.

3.   The Model

This section presents the details of the supply chain model studied in this paper. The notation that is introduced throughout this section is summarized in Table 1. Furthermore, Figure 1 illustrates the dynamis of the model.

Consider a supply chain consisting of two suppliers who supply a single perishable good to N grocery retailers. Because of the nature of the fresh produce supply chain, inventory at the suppliers is stochastic. Fluctuations in supply can be caused by many factors such as weather, pests, and defects during transportation, among others. Therefore, shortages can take place. As in many supply chains with yield uncertainty, retailers are benefited by employing a dual-sourcing strategy. Under a dual sourcing strategy, the retailers order goods from both suppliers in order to mitigate the risk of shortages. However, due to a lack of transparency between the retailers and suppliers, the retailers do not know the suppliers' inventory distributions. Instead, the retailers have a *perception* about the suppliers' inventory. This perception impacts their ordering decisions.

Parameters

N	Number of retailers
Q_i	Quantity desired by Retailer i each iteration
c_{h_i}	Holding cost for Retailer i
c_{s_i}	Stockout cost for Retailer i

Random variables

X^j	Random variable representing Supplier j's inventory realizations, distributed as $X^j \sim f_{X^j}$. In iteration k, Supplier j receives X_k^j units of inventory, a realization of the random variable X^j.
$A_i^j(\mathbf{q}_{-i})$	Random variable representing the amount of inventory that will be available from Supplier j for Retailer i in a given iteration, given the order quantities $\mathbf{q}_{-i}$ to Supplier j from all other retailers. The perceived CDF of $A_i^j(\mathbf{q}_{-i})$ in iteration k is denoted by $F_{A_i^j(\mathbf{q}_{-i}),k}$. For brevity, $A_i^j(\mathbf{q}_{-i})$ may also be written as A_i^j or simply as A^j when the retailers are identical.

Decision variables

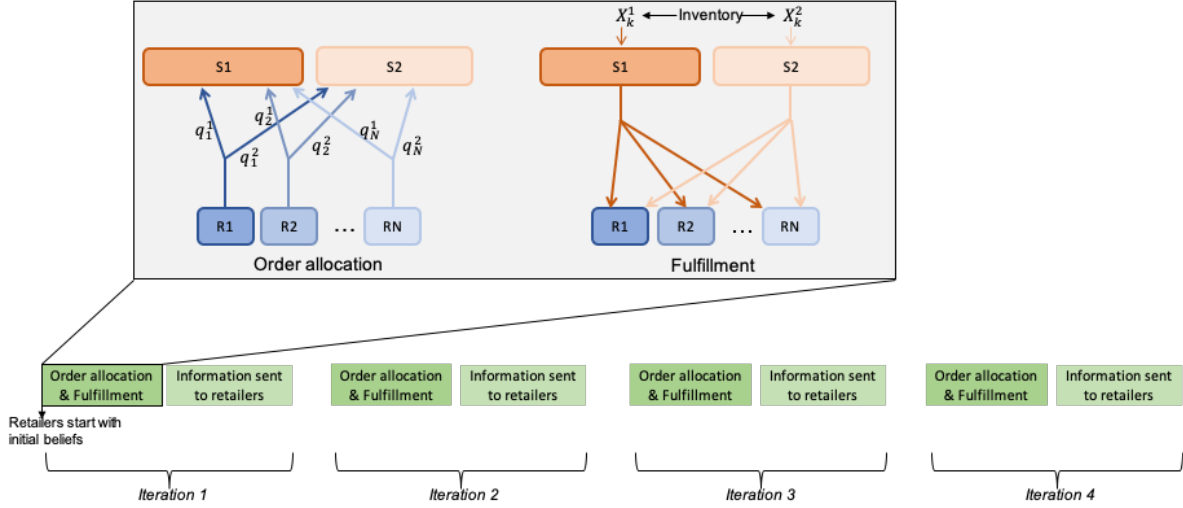|	
$q_{i,k}^j$	Amount of inventory that Retailer i orders from Supplier j during iteration k

Table 1   Model notation

Figure 1   **Supply chain dynamics with two suppliers and N retailers (S1 = Supplier 1, and R1 = Retailer 1, etc.). Each iteration consists of three steps: 1) orders from retailers are allocated across both suppliers, 2) suppliers fulfill orders using their available inventory, 3) information is transmitted.**

In this paper, all N retailers source from the same two suppliers, and hence they compete with each other for inventory. However, the assumption is that their physical retail locations are far enough apart that they do not compete for demand. This is similar to the setting in the ration gaming literature (Cachon and Lariviere 1999). Orders take place over an infinite, discrete time horizon. Each discrete time period is referred to as one *iteration*. In each iteration, the suppliers each receive a stochastic amount of inventory (described in Section 3.1), the retailers simultaneously

allocate orders to both suppliers (described in Section 3.2), and the suppliers then fulfill orders (described in Section 3.3). In this sense, the dynamics of the model can be thought of as a repeated ration game.

Each iteration, the retailers have a desired total quantity of inventory that they would like to receive, and must decide how much inventory to order from each supplier in order to achieve this goal (Section 3.4 describes their problem in more detail). The optimal ordering decision depends on the retailers' current perception of the suppliers' inventory distributions, as well as the number and behavior of other retailers. Through reverse information sharing, information about these unknown quantities is gained over time, allowing the retailers to update their order decisions. The information sharing schemes considered in this paper are discussed in more detail in Section 3.6.

3.1. Inventory

In iteration k, Supplier j receives a random quantity of inventory X_k^j that is drawn i.i.d. from a supplier-specific distribution with density f_{X^j}. Prior to the first iteration, the suppliers are aware of the number of retailers, N, and their desired quantity per iteration, Q_i for $i \in \{1, ..., N\}$. Supplier j is *not* aware of the other supplier's inventory distribution. Knowing this information, each supplier decides on and fixes their own sourcing strategy (i.e., fixes f_{X^j}) prior to the first iteration. These inventory distributions are assumed to remain fixed throughout the time horizon. The supplier's decision is not the focus of this paper, and f_{X^j}, $j = 1, 2$, will be considered given.

Although there are markets for less fresh produce, typically grocery stores have strict freshness requirements. Therefore, it is unlikely that a supplier would sell their old produce to grocery retailers. The supplier may instead decide to donate, discard, or salvage the old produce. Therefore, the following natural assumption is introduced:

ASSUMPTION 1. *At the start of a new iteration, the remaining inventory (if any) from the previous iteration is no longer utilized.*

This assumption is also similar to assuming that the product has a fixed lifetime, which is commonly employed in the perishable inventory management literature (Schmidt and Nahmias 1985, Chen et al. 2014).

3.2. Order allocation

Each iteration, all retailers place an order. Retailer i wishes to receive a total of Q_i units of inventory, and must decide how much to order from each supplier. It is assumed that Q_i is fixed over the horizon for each retailer. For example, suppose that Retailer i wishes to receive 10 units of inventory each iteration ($Q_i = 10$). In a given iteration, based on the retailer's information about the supply chain, Retailer i may decide to order 7 units from Supplier 1 and 6 units from Supplier 2. Note that even though the retailer desires 10 units, she chose to order a total of 13 units in order to hedge against inventory uncertainty. This order allocation decision is the primary decision of interest in this paper. The decision variables $q_{i,k}^1$ and $q_{i,k}^2$ denote the amount that Retailer i orders from Supplier 1 and 2, respectively, during iteration k. Because of the inventory uncertainty, it is possible that $q_{i,k}^1 + q_{i,k}^2 > Q_i$, as in the example above. When this is the case, the retailer is said to have *over-ordered*.

3.3. Fulfillment

In each iteration, once the orders from all retailers are placed, the suppliers must fulfill orders. Suppose that Supplier j receives X_k^j units of inventory in iteration k, and receives orders $\{q_{1,k}^j, q_{2,k}^j, ..., q_{N,k}^j\}$. If $\sum_i q_{i,k}^j \leq X_k^j$, each retailer will receive their entire order. If $\sum_i q_{i,k}^j > X_k^j$, Supplier j will distribute inventory according to a specified fulfillment mechanism. This paper focuses on a *random lottery mechanism* (also called the *lexicographic rule*, as in Cachon and Lariviere (1999)), which is used in practice (Bray et al. 2019). Under this mechanism, retailers are placed in a random order and orders are sequentially fulfilled until supply runs out. In a single supplier setting, this mechanism is truth inducing, meaning that all retailers will order exactly their desired quantity (Cachon and Lariviere 1999). However, under a dual-sourcing strategy, retailers may be benefited by over-ordering.

3.4. Retailers' costs and optimal order allocation

Recall that Q_i denotes Retailer i's desired quantity in each iteration. In each iteration, if the retailer receives more or less inventory than Q_i, she incurs marginal costs. Because the retailers practice

dual sourcing and thus may over-order, it is possible for a retailer to receive more than Q_i in a given iteration. Furthermore, because of the supply uncertainty at the suppliers, it is also possible for a retailer to receive less than Q_i in a given iteration. For every unit that is received over Q_i, the retailer faces additional holding cost. If the retailer receives less than Q_i, she faces stockout costs. When determining the order allocation $q_{i,k}^1$ and $q_{i,k}^2$ for iteration k, Retailer i's objective is to minimize the following marginal cost function:

$$\min_{\mathbf{q}=(q^1,q^2)} C(\mathbf{q}) := \mathbb{E}[c_{h_i}(R(\mathbf{q})-Q_i)^+ + c_{s_i}(Q_i-R(\mathbf{q}))^+] \qquad \text{(Problem } C_{ret})$$

where $R(\mathbf{q})$ is the total amount of inventory received from both suppliers. Additionally, c_{h_i} and c_{s_i} are the holding cost and stockout cost for Retailer i, respectively. The random variable $R(\mathbf{q})$ can be written as

$$R(\mathbf{q}) = \min\{q^1, A_i^1(\mathbf{q}_{-i,k}^j)\} + \min\{q^2, A_i^2(\mathbf{q}_{-i,k}^j)\} \qquad (1)$$

where $A_i^j(\mathbf{q}_{-i,k}^j)$ denotes the amount of inventory at Supplier j that is available for Retailer i, given the order quantities placed by the other retailers to Supplier j in iteration k, denoted by $\mathbf{q}_{-i,k}^j$. The random function $A_i^j(\mathbf{q}_{-i,k}^j)$ can be written as

$$A_i^j(\mathbf{q}_{-i,k}^j) = \left(X_k^j - \sum_{l\in\{1,\dots,N\}\setminus i} \mathbb{1}_{o(l)<o(i)}\mathbf{q}_{l,k}^j \right)^+ \qquad (2)$$

where $o(l)$ denotes the position of retailer l in the random lottery of iteration k. Notice that because of Assumption 1, $A_i^j(\mathbf{q}_{-i,k}^j)$ depends only on the inventory received during iteration k. Because the orders from other retailers are fixed for now, $A_i^j(\mathbf{q}_{-i,k}^j)$ can be written simply as A_i^j.

As an example, suppose that during iteration k, $X_k^1 = 20$, and 10 retailers each order 5 units of inventory from Supplier 1. Recall that each supplier uses the random lottery mechanism. Then, the distribution of A_i^1 is the following:

$$A_i^1 = \begin{cases} 20 & w.p.\ 1/10 \\ 15 & w.p.\ 1/10 \\ 10 & w.p.\ 1/10 \\ 5 & w.p.\ 1/10 \\ 0 & w.p.\ 6/10 \end{cases} \qquad (3)$$

Notice that, because of the lottery mechanism and because the retailers are all identical in terms of the quantity ordered, the distribution of A_i^1 is identical across all retailers in this example, and therefore can be denoted simply by A^1. Because Retailer i does not know the suppliers' inventory or the number of other retailers (or the other retailers' order quantities, in the case where the retailers are asymmetric), the optimal order allocation decision depends on Retailer i's *perception* of the distributions of A^1 and A^2. This perception is formed, in part, based on the chosen information sharing scheme (discussed in Section 3.6).

To gain intuition about the retailers' optimal ordering decisions, first take the perspective of a single retailer (Retailer 1), and consider fixing the ordering decisions of other retailers (Retailers 2 through N). Section 4 further builds on this model by considering the best response dynamics of all N retailers acting strategically.

For brevity, $A_1^j(\mathbf{q}_{-i,k})$ will be denoted simply by A^j. During iteration k, the retailer's perceived distribution of A^j is denoted by its perceived cumulative distribution function (CDF) $F_{A^j,k}$. The retailer will choose the order allocation that solves Problem Problem C_{ret}. The solution to Problem C_{ret} is denoted by $\mathbf{q}_{1,k}^*$, where the subscript 1 denotes Retailer 1. The following proposition characterizes the retailer's optimal solution to Problem C_{ret}. The proof of Proposition 1 shows that, on the domain $\mathcal{D} := \{q^1, q^2 : q^1 + q^2 > Q, q^1 \leq Q, q^2 \leq Q\}$, the cost function in Problem C_{ret} has a unique minimizer. Therefore, the optimal solution either lies at a unique point in the interior of this domain (given by $\tilde{q}^1, \tilde{q}^2$ in the Proposition below), or lies on the boundary of the domain.

PROPOSITION 1. *When* $\tilde{q}^1 + \tilde{q}^2 > Q$, $\mathbf{q}_{1,k}^* = \{\tilde{q}^1, \tilde{q}^2\}$ *where*

$$\tilde{q}^1 := Q - F_{A^2,k}^{c^{-1}}\left(\frac{c_s}{c_s + c_h}\right)$$

and

$$\tilde{q}^2 := Q - F_{A^1,k}^{c^{-1}}\left(\frac{c_s}{c_s + c_h}\right)$$
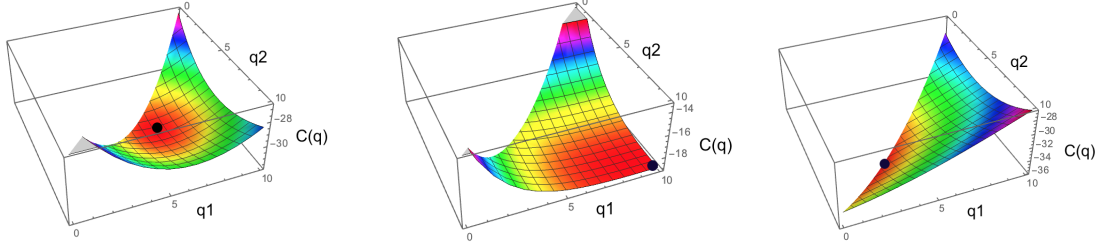
Figure 2   Illustration of three different optimality cases for Retailer 1, with $Q = 10$. In the leftmost figure, the optimal solution occurs at $(\tilde{q}^1, \tilde{q}^2)$. In the middle figure, the optimal solution occurs at (Q, Q). In the rightmost figure, the optimal solution occurs along the boundary $q^1 + q^2 = Q$.

where $F_{A^j,k}^c$ denotes the complementary CDF of the perceived distribution of A^j during cycle k (i.e., $F_{A^j,k}^c(x) = 1 - F_{A^j,k}(x)$). Notice that $F_{A^j,k}^{c^{-1}}\left(\frac{c_s}{c_s + c_h}\right) \geq 0$ since A^j is non-negative. If $\tilde{q}^1 + \tilde{q}^2 \leq Q$, $\mathbf{q}_{1,k}^*$ is the unique solution to

$$
\begin{cases}
q^1 = Q - q^2 \\
\\
F_{A^1,k}^c(q^1) = F_{A^2,k}^c(q^2)
\end{cases}
\tag{4}
$$

The proof of Proposition 1 is in Appendix EC.1. Figure 2 illustrates the possible optimal solutions for Retailer 1.

3.5.   Suppliers' costs and contracts

Each supplier/retailer pair has a contract in place, which determines how the supplier is compensated. The contract also incentivizes the supplier to meet the retailer's orders by penalizing unfulfilled orders. Throughout the paper it is assumed that every supplier/retailer pair has an identical contract in place. For each order, the supplier is compensated per unit delivered and is penalized if the retailer's order is not met in full. For example, if a retailer orders 5 units and only receives 3 units, the supplier is paid for the 3 units that are delivered and is penalized for not meeting the entire order. Therefore, the suppliers' profit is a combination of their per unit payments and their penalties. Supplier j's cost per iteration is given by:

$$
\sum_{i=1}^{N} (p \cdot r_{i,k}^j - penalty(q_{i,k}^j, r_{i,k}^j))
\tag{5}
$$

Where p is the unit price received, $q_{i,k}^j$ is the amount that was ordered by Retailer i during iteration k, and $r_{i,k}^j$ is the amount that was delivered to Retailer i. The function $penalty(q_{i,k}^j, r_{i,k}^j)$ governs the cost that the supplier incurs when they do not fulfill an order in its entirety. The penalty function depends on the contract between the suppliers and retailers, and can have many different structures. For example, several major retailers use a penalty function known as the "On-Time In-Full" (OTIF) policy, which takes the following form:

$$penalty(q, r) = \gamma(q - r)\mathbf{1}_{r/q \leq \tau} \tag{6}$$

where τ is an OTIF threshold, and γ is a penalty amount (Health 2017, Cosgrove 2019). Smith and Nassauer (2019) report that Walmart's OTIF policy is set to have $\tau = .87$ and $\gamma = .03$. Although penalty functions can vary, they should be non-decreasing in r—the amount delivered to the retailer. The OTIF penalty function described above clearly satisfies this assumption, since the penalty is linearly increasing in $q - r$ if $r/q \leq \tau$.

3.6. Information sharing

As the analysis in Section 3.4 elucidates, the optimal ordering decision of a retailer in iteration k depends in part on their belief about the distribution of $A_i^j(\mathbf{q}_{-i,k}^j)$. Notice, from Equation 2, that $A_i^j(\mathbf{q}_{-i,k}^j)$ depends on the inventory that Supplier j receives, the orders from *other* retailers, as well as the number of other retailers. In order for the retailers to update their orders optimally, they must either be told $\mathbf{q}_{i,k}^*$ directly, or receive an estimate of $F_{A_i^j(\mathbf{q}_{-i,k}^j)}$ each iteration. The three information sharing schemes given below describe three different possibilities regarding *what* information is shared between the suppliers and retailers. A detailed discussion of *how* the information should be shared is deferred to Section 6. For simplicity of exposition, up until Section 6 it will be assumed that the suppliers share information directly with the retailers. This decision does not impact the analysis of the retailers' ordering dynamics, which is the main focus of the paper.

This paper compares three information sharing schemes:

1. **Base information (BI) sharing**: Retailer i has an initial belief about F_{X^j} for $j \in \{1, 2\}$ and $\mathbf{q}_{-i}$. The retailers do not directly receive any new information over the horizon. (Section 6 discusses how the retailers can still indirectly derive information.)

2. **Full information (FI) sharing**: The retailers know the true distribution F_{X^j} for $j \in \{1, 2\}$. Additionally, the retailers observe $\mathbf{q}_{-i,k}$—the quantities ordered by the other retailers—*after* each iteration k.

3. **Reverse information sharing (RI)**: To begin, the retailers start as in the BI sharing scheme above. Each iteration, their estimate of F_{X^j} is updated based on all historical inventory realizations for Supplier j. Additionally, the retailers observe $\mathbf{q}_{-i,k}$ after each iteration k.

The difference between the FI and RI schemes is the estimate of the inventory distributions. In the FI scheme, the distribution of X^j is known for $j = 1, 2$. The FI scheme is considered to be an *unrealistic* best-case scenario for the following reasons: 1) In real-world settings, the suppliers likely do not know the true distribution of X^j, and 2) if a lack of trust exists between the suppliers and retailers, the retailers may not trust an estimate of F_{X^j} provided directly by Supplier j. On the other hand, an empirical estimate of F_{X^j} based on verifiable historical inventory data promotes trust and is implementable in practice. Therefore, the RI scheme is considered to be a realistic alternative to the FI scheme.

In the BI scheme, although the retailers do not *directly* receive updated estimates of the suppliers' inventory distributions or the orders from other retailers, it is still possible for the retailers to learn about this distribution based on the inventory that they receive each iteration. However, as discussed in Section 6, learning in this fashion happens at a very slow rate (and will occur regardless of an information sharing scheme). Therefore, in the BI scheme, it will be assumed that $F_{A_i^j(\mathbf{q}_{-i}),k}(x) = F_{A_i^j(\mathbf{q}_{-i}),0}(x)$ for all k, $x \geq 0$, and all $\mathbf{q}_{-i} \in \Pi_{i \in \{1,\ldots,N\} \setminus i}[0, Q_i]$.

4. N identical retailers

In this section, the analysis of Section 3.4 is extended to the case where all N retailers strategically update their orders. This updating is assumed to follow best response dynamics (Fudenberg et al. 1998). Namely, in iteration k, the Retailer i will update its order allocation to be the best response to the perceived distribution of $A_i^j(\mathbf{q}_{-i,k-1})$ in iteration k, where $\mathbf{q}_{-i,k-1}$ is a vector of the chosen order quantities of the other retailers in the previous iteration. This section studies the convergence behavior of these dynamics.

When all N retailers are identical, they share a common objective function and will thus choose the same order quantities each iteration. Therefore, $A_i^j(\mathbf{q}_{-i,k-1})$ is written simply as $A^j(q_{k-1}^j)$ in this section, where q_{k-1}^j denotes the order placed to Supplier j by *all* retailers in iteration $k-1$. Let $\mathbf{q}_k^* = (q_k^{1^*}, q_k^{2^*})$ denote the retailers' optimal order allocation during iteration k. The optimal order allocation is a direct extension of the characterization given in Proposition 1 for the single retailer case. When a retailer places an order in iteration k, they will choose the order allocation that solves Problem C_{ret}, computed using the distributions $F_{A^1(q_{k-1}^{1^*}),k}$ and $F_{A^2(q_{k-1}^{2^*}),k}$.

From Proposition 1, we know that $\mathbf{q}_k^* = \{\tilde{q}^1, \tilde{q}^2\}$ when $\tilde{q}^1 + \tilde{q}^2 > Q$, where $\tilde{q}^1$, $\tilde{q}^2$ satisfy

$$\tilde{q}^1 := Q - F_{A^2(q_{k-1}^{2^*}),k}^{c^{-1}}\left(\frac{c_s}{c_s + c_h}\right) \tag{7}$$

and

$$\tilde{q}^2 := Q - F_{A^1(q_{k-1}^{1^*}),k}^{c^{-1}}\left(\frac{c_s}{c_s + c_h}\right) \tag{8}$$

If $\tilde{q}^1 + \tilde{q}^2 \leq Q$, $\mathbf{q}_k^*$ is the unique solution to

$$\begin{cases} q^1 = Q - q^2 \\ \\ F_{A^1(q_{k-1}^{1^*}),k}^c(q^1) = F_{A^2(q_{k-1}^{2^*}),k}^c(q^2) \end{cases} \tag{9}$$

In this setting, it is natural to study the long-run behavior of the retailers, and ask the following questions: *Do the retailers' order allocations converge? If they do converge, what do they converge to?* Before answering these questions, convergence is first defined in this setting, as well as the notion of *temporary convergence*.

DEFINITION 1 (CONVERGENCE). The retailers' dynamics are said to *converge* to $\mathbf{q}^*$ if, for every $\epsilon > 0$, there exists a K such that for all $k > K$, $||\mathbf{q}_k^* - \mathbf{q}^*|| < \epsilon$. Here, $|| \cdot ||$ is taken to be the L^1 norm.

The definition of convergence above is a standard definition. In what follows, the notion of *temporary convergence* is introduced. Intuitively, the order allocation dynamics are said to temporarily converge if they converge for a finite number of iterations. This may occur in the RI scheme if the retailers' initial belief about one or both of the suppliers' inventory distributions is far from the true distribution. For example, suppose that the retailers begin by strongly believing that the suppliers have limited inventory. However, in reality, the suppliers have ample inventory. In this case, it is possible that the order dynamics will temporarily converge to an equilibrium where all retailers over-order. However, once the retailers have observed enough inventory realizations so that their estimate of the inventory distributions have sufficiently changed, their orders may shift to a new equilibrium where no over-ordering occurs. This phenomenon is observed empirically in Section 5.

DEFINITION 2 (TEMPORARY CONVERGENCE). The retailers' dynamics are said to *temporarily converge* to $\mathbf{q}^*$ from time K to $K + m$ if, for every $\epsilon > 0$ and $k \in \{K, ..., K + m\}$, $||\mathbf{q}_k^* - \mathbf{q}^*|| < \epsilon$.

The convergence of the order allocations depends heavily on the notion of Nash equilibrium. The definition of Nash Equilibrium, below, says that the chosen allocation must simultaneously be a best response for all retailers.

DEFINITION 3 (NASH EQUILIBRIUM). An order allocation $(\mathbf{q}^{1^{NE}}, \mathbf{q}^{2^{NE}})$ is a Nash equilibrium if and only if

$$\mathbf{q}^{j^{NE}} = \operatorname{argmin}_{\mathbf{q}} C(\mathbf{q}|F_{A^1(\mathbf{q}^{1NE})}, F_{A^2(\mathbf{q}^{2NE})})$$

for $j = 1, 2$ where $C(\mathbf{q}|F_{A^1(\mathbf{q}^1)}, F_{A^2(\mathbf{q}^2)})$ denotes the cost from Problem C_{ret} computed with respect to the distributions $F_{A^1(\mathbf{q}^1)}$ and $F_{A^2(\mathbf{q}^2)}$.

Proposition 2 (below) gives us insight into the long-run behavior of the best response dynamics under any information scheme where the perceived distribution of $A^j(q)$ converges pointwise to a function $F_{A^j(q),\infty}(x)$ on the domain $x \in [0, Q]$ as $k \to \infty$, for all fixed $q \in [0, Q]$ and $j = 1, 2$. Notice that this is equivalent to convergence of the estimated inventory distributions of both suppliers.

In order to understand the long-term convergence behavior of the best response dynamics, it suffices to consider the Nash equilibria of the dynamics under $F_{A^j(q^j),\infty}(x)$. The following proposition is an extension of the classic theorem of best response dynamics, which says that if the

best response dynamics of a game with a static payoff function converge to a strategy profile, the strategy profile must be a Nash equilibrium of the game (Fudenberg et al. 1998).

PROPOSITION 2. *If $F_{A^j(q^j),k}(x)$ converges pointwise to $F_{A^j(q^j),\infty}(x)$ on the domain $x \in [0,Q]$ for all fixed $q^j \in [0,Q]$ for $j = 1,2$, and the retailers' dynamics converge, they must converge to a Nash equilibrium under the distributions $F_{A^1(\cdot),\infty}(x)$ and $F_{A^2(\cdot),\infty}(x)$.*

Let $F_{A^j(q)}(x)$ denote the true distribution of $A^j(q)$ for all $q \in [0,Q]$, meaning that it is computed using the true inventory distribution of Supplier j. Assuming that $F_{A^j(q),k}(x)$ converges pointwise to $F_{A^j(q)}(x)$ for all $q \in [0,Q]$ under the RI scheme, in order to understand the long-run behavior of the retailers it suffices to consider the Nash equilibrium of the best response dynamics using the true distribution of $A^j(q)$. This is equivalent to considering the convergence behavior of the best response dynamics under the FI scheme.

We now study the long-run behavior of the retailers' order allocation. Proposition 3 characterizes all possible convergent order allocations. Under Condition 1, there are at most three possible Nash equilibria. However, the best response dynamics can only converge to two of the three equilibria. When Condition 1 does not hold, it is possible that more than three equilibria exist.

PROPOSITION 3. *Assume that $F_{A^j(q^j),k}(x)$ converges pointwise to $F_{A^j(q^j)}(x)$ as $k \to \infty$, for all $q \in [0,Q]$. Under Condition 1 (given below), there are at most three Nash equilibria order allocations with respect to the distributions $F_{A^j(q^j)}(x)$, $j = 1,2$, characterized by:*

Equilibrium NE_Q. *Each retailer orders exactly Q units of inventory in total ($q_1 + q_2 = Q$).*

Equilibrium NE_{2Q}. *Each retailer orders exactly Q units of inventory from at least one of the suppliers (either $q^1 = Q$ and/or $q^2 = Q$).*

Equilibrium NE'. *Each retailer orders (q^1, q^2) where $q^1 < Q, q^2 < Q$ and $q^1 + q^2 > Q$.*

Furthermore, the best response dynamics can only converge to NE_Q and NE_{2Q}. When Condition 1 (below) does not hold, NE_Q and NE_{2Q} remain Nash equilibria, and there could be additional equilibria that satisfy the conditions of Equilibrium NE' (above), meaning that there could be multiple equilibria that lie in the domain $\mathcal{D} = \{q^1, q^2 : q^1 + q^2 > Q, q^1 < Q, q^2 < Q\}$.

CONDITION 1. The distributions of X^1 and X^2 (the suppliers' inventory distributions) satisfy

$$f_{X^j}(Q-q) < \sum_{m=2}^{N} f_{X^j}(Q+(m-1)q)$$

for all $q \in [0, Q]$.

Condition 1 results in a regularity condition on the best response dynamics given by Equations 7 and 8. A Nash Equilibrium can only occur in the domain $\mathcal{D}$ if Equations 7 and 8 are satisfied simultaneously for some choice of q^1 and q^2. In other words, a Nash equilibrium occurs at some allocation $(q^{1^{NE}}, q^{2^{NE}})$ in $\mathcal{D}$ if and only if

$$q^{1^{NE}} = Q - F^{c^{-1}}_{A^2(q^{2^{NE}}),k}\left(\frac{c_s}{c_s + c_h}\right) \tag{10a}$$

$$q^{2^{NE}} = Q - F^{c^{-1}}_{A^1(q^{1^{NE}}),k}\left(\frac{c_s}{c_s + c_h}\right) \tag{10b}$$

$$(q^{1^{NE}}, q^{2^{NE}}) \subset \mathcal{D} \tag{10c}$$

are all satisfied. In the $q^1 - q^2$ plane, the intersection points of the functions $q^{2^{NE}}(q^1)$ and $q^{1^{NE}}(q^2)$ (defined by Equations 10a and 10b above) determine the Nash equilibria on the domain $\mathcal{D}$. When Condition 1 is met, these functions can intersect at most once on domain $\mathcal{D}$. Condition 1 will hold as long as the suppliers' inventory distributions are "reasonable," meaning that the suppliers do not have unrealistically small quantities of inventory. Since there are N retailers, each desiring a total quantity of Q every iteration, under reasonable inventory distributions $f_{X^j}(Q-q)$ should be quite small, and $f_{X^j}(Q+(m-1)q)$ should be larger than $f_{X^j}(Q-q)$ for some $m \geq 2$. Therefore, in most realistic settings Condition 1 will hold.

In what follows we address the following question: *When do the dynamics converge to NE_Q versus NE_{2Q}?* Corollary 1 provides sufficient conditions for the dynamics to converge to either NE_Q or NE_{2Q}, based on the distribution of X^j for $j = 1, 2$.

COROLLARY 1. *Let let (q_U^1, q_U^2) be the solution to the system of Equations 11 and let (q_L^1, q_L^2) be the solution to the system of Equations 12, given by:*

$$\begin{cases} \mathbb{E}[X^2 \mathbb{1}_{X^2 \in [Q-q^2, Q-q^2+Nq^1]}] = q^1 N \dfrac{c_s}{c_s + c_h} \\[2mm] \mathbb{E}[X^1 \mathbb{1}_{X^1 \in [Q-q^1, Q-q^1+Nq^2]}] = q^2 N \dfrac{c_s}{c_s + c_h} \end{cases} \tag{11}$$

and

$$\begin{cases} \mathbb{E}[X^2 \mathbb{1}_{X^2 \in [Q-q^2, Q-q^2+(N-1)q^1]}] = q^1 N \dfrac{c_s}{c_s + c_h} \\[2mm] \mathbb{E}[X^1 \mathbb{1}_{X^1 \in [Q-q^1, Q-q^1+(N-1)q^2]}] = q^2 N \dfrac{c_s}{c_s + c_h} \end{cases} \tag{12}$$

If $(q_L^1, q_L^2) \in \mathcal{D}$ and $(q_U^1, q_U^2) \in \mathcal{D}$, the best response dynamics can converge to either NE_Q or NE_{2Q}, depending on the initial ordering decisions of the retailers. If $(q_L^1, q_L^2) \in \mathcal{D}'$ where $\mathcal{D}' = \{q^1, q^2 : q^1 \geq Q \text{ or } q^2 \geq Q\}$, then the dynamics will converge to NE_Q. Finally, if $(q_U^1, q_U^2) \in \mathcal{D}''$ where $\mathcal{D}'' = \{q^1, q^2 : q^1 + q^2 \leq Q\}$, then the dynamics will converge to NE_{2Q}.

Furthermore, if Condition 13 (below) holds for $j = 1, 2$, then the order allocation corresponding to equilibrium NE_{2Q} is the allocation (Q, Q), and the dynamics could converge to the this equilibrium.

$$\mathbb{E}[X^j \mathbb{1}_{X^j \in [0, (N-1)Q]}] \leq QN \dfrac{c_s}{c_s + c_h} \tag{13}$$

Corollary 1 gives a practical method for determining the convergence behavior of the ordering dynamics, based only on the suppliers' inventory distributions. Recall that equilibrium NE_{2Q} is characterized by an order allocation (q^1, q^2) such that *at least one q^j is equal to Q*. The final statement of Corollary 1 provides a condition such that the order allocation of NE_{2Q} will be equal to (Q, Q). Intuitively, this condition will hold as long as *both* suppliers have limited inventory (in the sense of Equation 13 holding).

Corollary 1 is demonstrated pictorially in Figure 3. Assuming that the inventory at both suppliers follows a normal distribution, Figure 3 shows ranges of the means and variances of the inventory distributions that would result in the retailers converging to either NE_Q or NE_{2Q} (or the indeterminate case where the dynamics can converge to either equilibrium). These images were created using the conditions of Corollary 1, and solving the systems of Equations 11 and 12 for

various parameters of the inventory distributions. When the coefficient of variation is high for both distributions (bottom right of Figure 3), the dynamics will always converge to NE_{2Q} for the range of means explored. In this case, there is a large amount of risk associated with the inventory, as it can fluctuate widely from iteration to iteration at both suppliers. Therefore, the retailers will always over-order to hedge against this risk.

When the coefficient of variation is relatively small for both suppliers (top left of Figure 3), the retailers' dynamics will converge to NE_{2Q} when the means are small, and will converge to NE_Q when both means are large enough. There is also a middle ground where the dynamics can converge to either equilibrium. Finally, when the inventory distributions are asymmetric in terms of their variability, the convergence behavior depends almost entirely on the mean of the inventory distribution of the supplier with *more* variability.

It should also be noted that in all scenarios considered in Figure 3, NE_{2Q} always corresponds to the allocation order (Q, Q). In other words, the last condition of Corollary 1 is always met for the ranges of parameters explored.

The following proposition concerns the convergence rate of the best response dynamics when the suppliers' inventory distributions are fixed. Let L^j be the largest integer such that the following inequality holds:

$$\sum_{m \geq L^j + 1}^{N} f_{X^j}(Q + (m-1)q) \geq \sum_{m=0}^{L^j - 1} (L^j - m) f_{X^j}(Q + (m-1)q) \tag{14}$$

for all $q \in [0, Q]$. Notice that the expression above is equivalent to the expression in Condition 1 when $L^j = 1$. The integers L^1 and L^2 impact the convergence rate of the best response dynamics, described below in Proposition 4.

PROPOSITION 4. *Suppose the best response dynamics start at (Q, Q). If (Q, Q) is a Nash equilibrium under the true distribution $F_{A^j(q)}(x)$, the retailers' best response dynamics converge immediately. If (Q, Q) is not Nash equilibrium under the true distribution $F_{A^j(q)}(x)$, let $L = \min\{L^1, L^2\}$ (where each L^j is defined by Equation 14). Furthermore, let $b = Q -$*
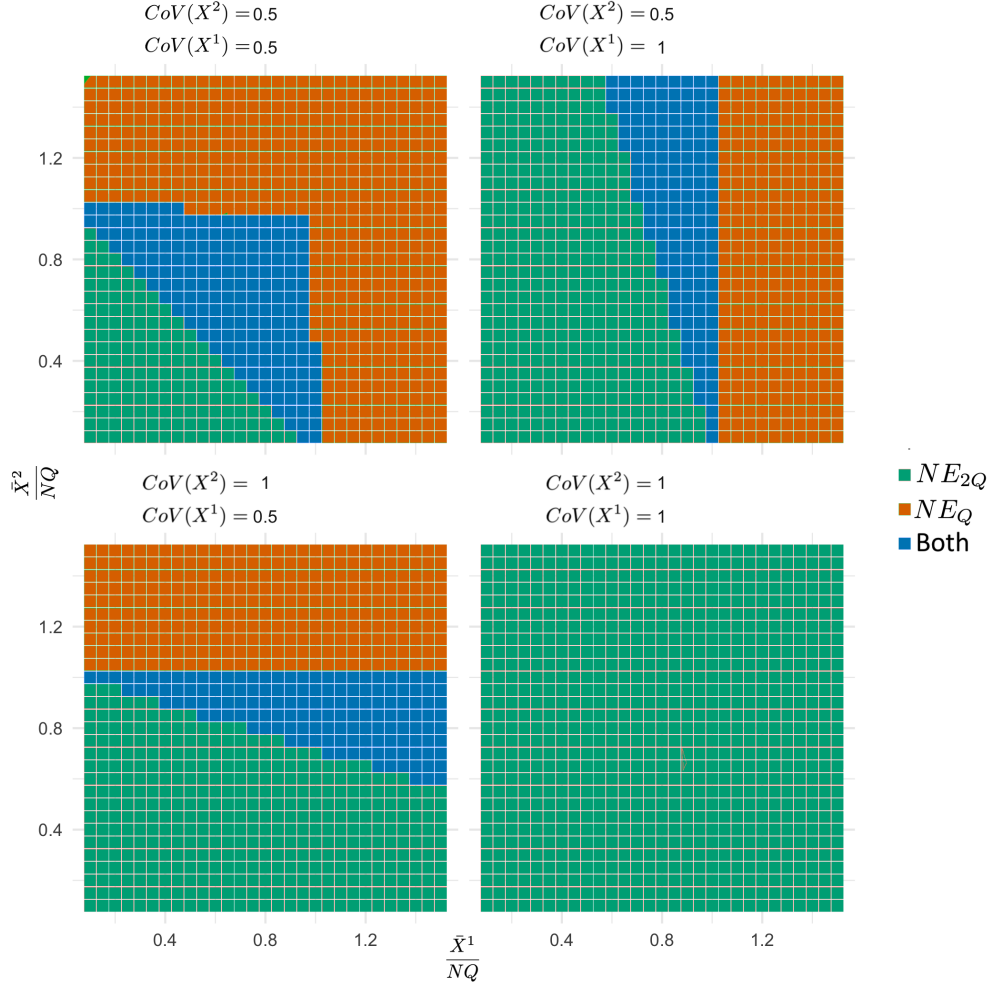
Figure 3   Illustration of Corollary 1 when $X^j \sim N(\bar{X}^j, (\bar{X}^j \cdot CoV(X^j))^2)$, $Q = 10$, $N = 50$, and $\frac{c_s}{c_s + c_h} = .8$. **Note**

that $CoV(X^j)$ **is the coefficient of variation of** X^j**, defined as** $\frac{\sigma_j}{\bar{X}^j}$ **where** σ_j **is the standard deviation of** X^j**.**

$\min\{F^{c^{-1}}_{\mathcal{A}^1(Q)}\left(\frac{c_s}{c_s + c_h}\right), F^{c^{-1}}_{\mathcal{A}^2(Q)}\left(\frac{c_s}{c_s + c_h}\right)\}$*, where* $\mathcal{A}^j$ *is the uncensored version of* A^j *(i.e., the operation*

$(\cdot)^+$ *is removed in Equation 2). The retailers' order dynamics will converge to* NE_Q *after at most*

n *iterations, where*

$$n = \frac{\log\left(\frac{-2b + (1+L)Q}{2(Q-b)}\right)}{\log(L)} \tag{15}$$

for $L > 1$ *and* $0 < b < Q$.

First, notice that when $b \in [0, Q/2]$, $n \leq 1$ and the dynamics converge after only one iteration. This occurs when the suppliers have ample inventory. Specifically, this occurs when $\mathbb{P}[A^j(Q) \geq Q/2] \geq \frac{c_s}{c_s + c_h}$ for $j = 1, 2$. When $b \in [Q/2, Q]$, n is decreasing in L. In this case, it always holds that

$n \leq \frac{Q}{2(Q-b)}$, which is the limit of Equation 15 when $L \to 1$. This provides an upper bound on the number of iterations required to reach convergence, independent of L (as long as $L \geq 1$, which can be enforced by Condition 1).

4.1. Supply chain costs

This section considers the cost of each equilibrium order allocation in terms of 1) supplier costs, 2) retailer costs, and 3) supply chain waste.

Suppose the best response dynamics converge to an order allocation **q**. We know from Proposition 3 that **q** either corresponds to NE_Q or to NE_{2Q}. If **q** corresponds to NE_Q, then every retailer orders exactly Q units of inventory total. If **q** corresponds to NE_{2Q}, each retailer orders Q units of inventory from at least one of the suppliers. As Proposition 3 and Corollary 1 state, there are cases where the dynamics can converge to either NE_Q or NE_{2Q}. The question that this section seeks to answer is: *Which Nash equilibrium is better for each of the supply chain metrics listed above?*

First consider the cost to the suppliers. Recall that, for one iteration, Supplier j's cost is given by

$$C_{Sj} = \sum_{i=1}^{N} (-p \cdot r_{i,k}^{j} + penalty(q_{i,k}^{j}, r_{i,k}^{j})) \tag{16}$$

where p is the unit price, $q_{i,k}^{j}$ is the amount that was ordered, and $r_{i,k}^{j}$ is the amount that was actually delivered to Retailer i in the given iteration. The term $penalty(q_{i,k}^{j}, r_{i,k}^{j})$ is the penalty cost.

In iteration k, total waste is given by

$$\text{Waste} = X_k^1 + X_k^2 - \sum_i \min\{R_{i,k}, Q\}$$

where $R_{i,k}$ denotes the total amount received by Retailer i in iteration k.

The following proposition says that the order allocation NE_Q is best for the retailers' cost and supply chain waste under very realistic assumptions. Furthermore, it is also best for the suppliers as long as the suppliers are sufficiently penalized for unmet orders. When the suppliers are not penalized for unmet orders, they prefer allocation NE_{2Q}.

PROPOSITION 5. *If penalty$(q,x) = 0$ for all x, and $C_{Sj}(\mathbf{q})$ denotes the supplier's expected cost per iteration under order allocation $\mathbf{q}$:*

$$C_{Sj}(NE_{2Q}) \leq C_{Sj}(NE_Q)$$

Furthermore, there exists an $H > 0$ such that when the partial derivative of the penalty function with respect to the amount delivered, $\frac{\partial penality(q,r)}{\partial r}$, is larger than H for all q,

$$C_{Sj}(NE_{2Q}) \geq C_{Sj}(NE_Q)$$

as long as $F_{Xj}(QN) > 0$.

Let $C_R(\mathbf{q}; X_k^1, X_k^2)$ denote the retailers' expected cost in iteration k, given order allocation $\mathbf{q}$ and inventory realizations X_k^1 and X_k^2. Suppose that the order allocation at NE_{2Q} is equal to (Q, Q). Then,

$$C_R(NE_Q; X_k^1, X_k^2) \leq C_R(NE_{2Q}; X_k^1, X_k^2)$$

when $NQ \leq 5X_k^j$ for $j = 1, 2$.

Let $W(\mathbf{q}; X_k^1, X_k^2)$ denote the supply chain waste during iteration k given order allocation $\mathbf{q}$ and inventory realizations X_k^1, X_k^2. Then,

$$W(NE_Q; X_k^1, X_k^2) \leq W(NE_{2Q}; X_k^1, X_k^2).$$

The proof of Proposition 5 can be found in Appendix EC.1.

To provide intuition about the proposition, first consider the suppliers' costs. If there were no penalty costs, the suppliers would prefer larger orders from the retailers. This would ensure that the most inventory possible is sold for a unit price of p. In this case, the supplier does not care how many orders can be fulfilled (or how many orders are not fulfilled)—they only care about delivering the largest quantity possible. Therefore, when penalty cost is zero, NE_{2Q} is the most preferred equilibrium to the suppliers, because it results in the largest order quantities for both suppliers.

When $p = 0$ and penalty costs exist, the suppliers' costs are minimized when they are able to meet every order, which occurs when orders are small. Therefore, in this situation, NE_Q is the most preferred equilibrium to the suppliers, because it results in the smallest order quantities.

For any fixed p, as the penalty cost grows, the supplier will prefer smaller and smaller order quantities. Therefore, at some point, NE_Q will become the most preferred equilibrium to the suppliers. Notice that the OTIF penalty function, discussed in Section 3.5 is *not* able satisfy Proposition 5. Under an OTIF contract, $\frac{\partial penality(q,r)}{\partial r} = 0$ when $r/q \leq \tau$. Therefore, $\frac{\partial penality(q,r)}{\partial r}$ cannot be made arbitrarily large, as the proposition calls for. Under an OTIF contract, the suppliers may always prefer allocation NE_{2Q} regardless of the magnitude of the penalty (the parameter γ in Equation 6). This misalignment of incentives is most likely to occur when the OTIF threshold τ (from Equation 6) is too low. In this case, the suppliers may be able to stay below the OTIF threshold even when retailers engage in over-ordering, thereby always incurring zero penalty costs even when orders are consistently not met in full. In this case, the suppliers will prefer that the retailers over-order.

From the retailers' perspective, the intuition behind Proposition 5 is that NE_Q most often results in the highest probability of each retailer receiving a positive amount of inventory. At equilibrium NE_{2Q}, since every retailer is over-ordering, many retailers end up without any inventory. Furthermore, some of those that do receive a shipment will receive $2Q$—double their desired quantity.

For the waste component, the argument is similar to that of the retailers. Since the suppliers' inventory distributions are fixed throughout the horizon, the only way to decrease supply chain waste is by increasing the service level at the retailers. Therefore, similar to the argument above for the retailers, waste is minimized at NE_Q.

4.2. Non-identical retailers

This section briefly considers the case when the retailers are not identical (in terms of their holding and stockout costs, desired order quantities, and contracts). Problem C_{ret} can easily be extended to a retailer-specific objective function. In iteration k, Retailer i's optimization problem, following best response dynamics, is given by:

$$\min_{\mathbf{q}_i} \; C_i(\mathbf{q}_i, |\{F_{A^j(\mathbf{q}_{-i,k-1}),k}\}_{j=1,2}) := \mathbb{E}[c_{h_i}(R(\mathbf{q}_i) - Q_i)^+ + c_{s_i}(Q_i - R(\mathbf{q}_i))^+ |\{F_{A^j(\mathbf{q}_{-i,k-1}),k}\}_{j=1,2}] \tag{17}$$

where $\mathbf{q}_{-i,k-1}$ denotes the orders from other retailers during iteration $k-1$. Furthermore, the notion of convergence and Nash equilibria can also be easily extended. The characterization of all Nash equilibria, however, is much more complex. From Section 4.1 it is clear that avoiding NE_{2Q} is desirable in terms of many supply chain metrics. In general, over-ordering is undesirable for retailer costs and supply chain waste, as well as supplier costs as long as the suppliers are sufficiently penalized when orders are not met in full. Corollary 2, which is an extension of Corollary 1 for the case of non-identical retailers, characterizes circumstances under which $\mathbf{q}_i = (Q_i, Q_i)$ for all $i \in \{1, ..., N\}$ is a possible equilibrium allocation.

COROLLARY 2. *If*

$$\mathbb{E}[X^j \mathbb{1}_{X^j \in [0,(N-1)Q_{min}]}] \leq Q_{min} N \frac{c_s}{c_s + c_h}$$

for $j = 1, 2$, the best response dynamics can converge to $\mathbf{q}_i = (Q, Q)$ for all i. Here, $Q_{min} := \min_i Q_i$.

5. Numerical experiments

This section presents a detailed example scenario and the resulting dynamics under all three information sharing schemes. In this scenario there are 100 identical retailers with $Q = 10$, $c_h = 4$, and $c_s = 8$. The initial order allocation for all retailers is either $(5,5)$ or $(10,10)$. The dynamics are simulated over 50 iterations.

Note that if all retailers order Q from both suppliers, the amount of inventory that Supplier j would need each iteration is $max\_inv := QN = 1,000$. Inventory realizations at supplier j are distributed as $X^j \sim \text{LogNormal}(\mu^j, {\sigma^j}^2)$. The mean and variance of X^j will be expressed in terms of max_inv as $\beta^j max\_inv$ and $(\rho \beta^j max\_inv)^2$, respectively. In the BI scheme, and to start in the RI scheme, the retailers will initially assume that inventory is distributed as $X^j \sim \text{LogNormal}(\alpha\mu^j, (\alpha\sigma^j)^2)$. Therefore, the parameter α represents the retailers' misperception about the suppliers' inventory. When $\alpha = 1$, the retailers know the inventory distribution exactly. However, when $\alpha < 1$, they perceive inventory to be scarcer than it is. In this scenario, β^j, ρ, and α are given by:

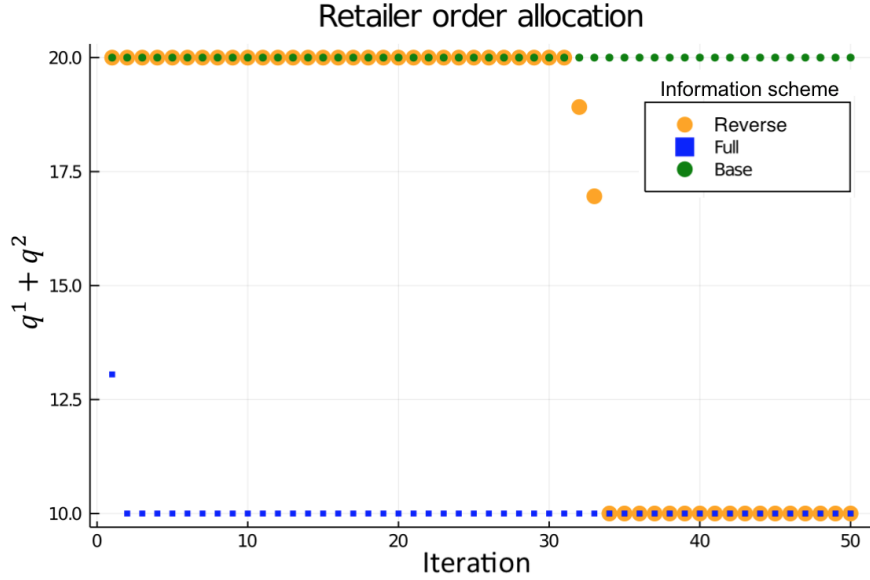$$\beta^1 = 0.8, \ \beta^2 = 0.7, \ \rho = 0.5, \ \alpha = 0.6$$

Figure 4   Evolution of retailers' order allocation for each information sharing scheme. The starting allocation in all cases is $\mathbf{q} = (10, 10)$.

The inventory distributions are intentionally constructed so that, in the BI scheme, the order dynamics could converge to either NE_Q or NE_{2Q}. The convergent allocation depends on the starting allocation. In the BI scheme, when the retailers start at $\mathbf{q} = (5, 5)$, the best response dynamics converge to $\mathbf{q} = (5, 5)$. However, when the retailers start at $\mathbf{q} = (10, 10)$, the best response dynamics converge to $\mathbf{q} = (10, 10)$. This demonstrates the existence of both Nash equilibria NE_Q and NE_{2Q}. However, in the full information scheme, the dynamics will converge to NE_Q, which happens to be the order allocation $(5.5, 4.5)$, regardless of the starting allocation.

In the RI scheme, the order allocation dynamics, starting at $(10, 10)$, temporarily converge to $(10, 10)$. However, after some time, the retailers learn more about the true distribution of the available inventory, and the retailers' order allocations eventually converge to NE_Q. This convergence happens very quickly over only three iterations. Figure 4 shows the evolution of the order allocations under each information sharing scheme, starting at the point $\mathbf{q} = (10, 10)$.

Now consider the performance of each information sharing scheme in terms of the supply chain metrics. Let the total waste over the 50 iterations under the FI scheme be denoted by W^*, and the retailer cost be denoted by C_R^*. The BI scheme, starting at $\mathbf{q} = (5, 5)$, yields total waste equal to

$1.16W^*$ and retailer cost equal to $5.57C_R^*$. The RI scheme, starting at $\mathbf{q} = (5,5)$, yields waste equal to $1.06W^*$ and retailer cost equal to $3.37C_R^*$. Thus, the RI scheme results in 14% less waste and 34% less cost to the retailers than the BI scheme. Although, under both schemes, the retailers choose an allocation such that $q^1 + q^2 = Q$, the allocation chosen under the BI scheme is not optimal. In the RI scheme, the retailers' eventually converge to the optimal allocation, resulting in lower costs and waste.

The cost to the suppliers depends on their contracts and parameters of the cost function given in Equation 5. We will assume that each supplier/retailer pair uses a *marginal penalty function*, given by $penalty(q,r) = \gamma \cdot (q - r)$. Notice that this penalty function is able to satisfy the conditions of Proposition 5. The marginal price received by the supplier per unit delivered is set to $p = 4$.

Figure 5 shows Supplier 1's profits under different values of γ. Supplier 2's profits follow a similar pattern. Profits are shown for the RI and BI schemes, starting from initial allocation $\mathbf{q} = (10, 10)$. The profits are scaled by the profit obtained by starting at allocation NE_Q under the FI scheme. A y-value greater than 1 indicates that the supplier's profits under the given information scheme and associated value of γ, starting from an order allocation of $\mathbf{q} = (10, 10)$, achieve higher profit than allocation NE_Q in the FI scheme.

When γ is low, the BI scheme achieves the highest profits for Supplier 1 (the y-axis exceeds a value of 1 and outperforms the RI scheme) because Supplier 1 is better off with larger order allocations. Thus, the BI scheme is preferred to the suppliers since the retailers order 10 units from both suppliers for the entire horizon. When γ is large enough, NE_{2Q} becomes less desirable to the supplier until, eventually, the supplier achieves higher profits when the retailers choose NE_Q (corresponding to a y-value less than 1).

6. Enabling privacy through a trusted third party

This section discusses implementations of the reverse information (RI) and full information (FI) sharing scheme and its consequences regarding privacy. The retailers' private information is considered to be their stockout and holding costs, and the suppliers' private information is their inventory
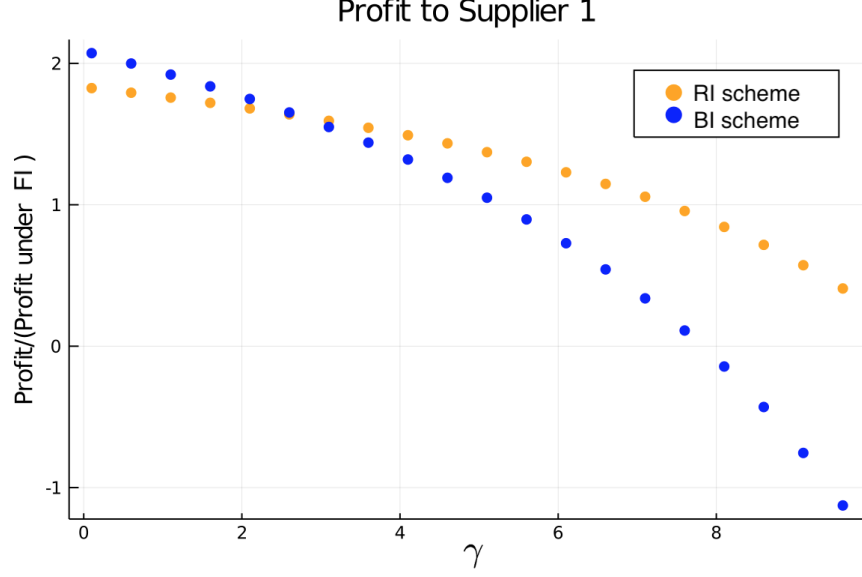
Figure 5 Supplier 1's costs for various values of γ

distribution, F_{X^j}, and N, the number of retailers in the supply chain. In this section, an implementation of the RI and FI sharing schemes is proposed that does not allow the suppliers to infer the retailers' private information, and allows the retailers to gain limited new information about the suppliers' private information.

For simplicity, this section is written assuming that all retailers are identical. However, the implementation described can also be utilized when the retailers are not identical. Note that a sufficient quantity for the suppliers to keep private, in order to ensure that N and F_{X^j} remain private, is the distribution of $A^j(q)$ for all $q \in [0, Q]$, which is a function of F_{X^j} and N. The implementation described in this section ensures that the retailers' marginal information gain (relative to no information sharing) is exactly a single point that lies on the true CDF of $A^j(q)$ for a single $q \in [0, Q]$, for $j = 1, 2$. Suppose that the two dimensional function $F_{A^j(q)}(x)$ can be parametrized by an m-dimensional vector, which the retailers wish to learn and Supplier j wishes to keep hidden. The retailers' marginal information gain in the proposed implementation reduces the retailers' learning task by exactly one dimension. Instead of learning an m-dimensional vector, retailers must learn an $m - 1$-dimensional vector.

Before discussing the retailers' learning problem, first consider the suppliers' ability to learn the retailers' private information. In any information sharing scheme, the only information that the

suppliers observe about the retailers is their order quantities. Recall from Proposition 1 that the optimal order quantity sent to Supplier j depends on the retailers' perceived distribution of A^{-j}—the distribution of available inventory at the *other* supplier—as well as the ratio $\frac{c_s}{c_s+c_h}$. Because Supplier j does not know the retailers' perceived distribution of A^{-j}, the ratio $\frac{c_s}{c_s+c_h}$ cannot be readily identified.

Now consider the retailers' ability to learn the suppliers' private information under the BI scheme, where no information sharing occurs. Each iteration, the retailers observe the inventory that they receive from each supplier, which is typically a censored observation of $A^j(q_k^{j^*})$. Neither inventory realizations, nor the number of other retailers, is every directly observed. Recall that $A^j(q_k^j) = (X_k^j - M q_k^j)^+$, where M is a random variable representing the number of retailers of higher priority than a given retailer, and is therefore a discrete uniform random variable taking integer values from 0 to N. Therefore, any new observation of $A^j(q)$, for any q, helps to estimate the distribution of X^j as well as N.

Every iteration, each retailer is likely to receive either zero or $q_k^{j^*}$ units of inventory from Supplier j. When a retailer receives $q_k^{j^*}$ units of inventory, this is a censored observation since the amount of inventory received by Retailer i is equal to $\min\{A_i^j, q_k^{j^*}\}$. Because of the random lottery fulfillment mechanism, only one retailer each iteration receives an amount of inventory in the open interval $(0, q_k^{j^*})$. Therefore, after K iterations, the expected number of times that a retailer receives either zero or $q_k^{j^*}$ units is $K(N-1)/N$. Notice that this information is always gained regardless of the presence of an information sharing scheme. In what follows, we discuss a specific implementation of the FI and RI schemes that allows the retailers to learn very little *additional* information about the distribution of A^j. Namely, the implementation proposed ensures that the additional information learned by the retailers consists of only one point that lies on the two dimensional function $F_{A^j(q)}(x)$.

In order for either the FI or RI sharing scheme to be effective, the retailers must either obtain an estimate of F_{A^j} each iteration, or receive $\mathbf{q}_k^*$ directly. However, there are many possible implementations for sharing this information, and many factors to consider when choosing the best implementation. This section focuses on an implementation that promotes privacy and trust.

Suppose that suppliers share their inventory and order data with a trusted third party (TTP), and the TTP continually updates its estimate of the function $F_{A^j(q)}(x)$. In the FI scheme, it can be assumed that the TTP knows the function $F_{A^j(q)}(x)$ exactly for all $x > 0$, $q \in [0, Q]$. Additionally, retailers share their holding and stockout costs with the TTP. Using this information, the TTP computes and transmits $\mathbf{q}_k^{NE}$ to the retailers each iteration, where $\mathbf{q}_k^{NE}$ corresponds to a Nash equilibrium order quantity under the current estimate of $F_{A^j(q)}(x)$. Notice that the TTP does *not* follow best response dynamics. Instead, the TTP directly transmits an (estimated) equilibrium order allocation every iteration.

Before discussing the benefits of this implementation strategy, we first discuss the TTP itself. In practice, the TTP does *not* need to be a physical entity, such as a third-party firm (although it could be). Instead, it could be a platform such as blockchain that is capable of verifying and auditing data and performing computations automatically. The proposed implementation relies on the suppliers communicating truthful inventory and order information to the TTP. Even when the incentives of the suppliers and retailers align (i.e., when they both prefer allocation NE_Q), there could still be reason for the suppliers to transmit false information. For example, they may try to secure a larger order allocation than the other supplier by lying about their inventory. To that end, blockchain technology can play a critical role. The benefits of blockchain in supply chain settings are beginning to emerge. Some of the key advantages to using blockchain are the ability to readily trace, verify, and audit data (Gaur and Gaiha 2020, Cheung et al. 2018). Because blockchain enables items to be fully traced in supply chains, and prevents tampering with existing data, it is extremely difficult for individual entities to enter false information into the platform. Therefore, the supply chain entities should prefer the use of blockchain to more standard platforms.

To understand the benefits of the proposed implementation, first consider the FI sharing scheme. Under this scheme, the TTP directly transmits $\mathbf{q}^{NE}$ to all retailers in the very first iteration, where $\mathbf{q}^{NE}$ are the orders corresponding to one of the Nash equilibria of the system. If both NE_Q and NE_{2Q} exist, the TTP will choose to transmit orders corresponding to the equilibrium that produces

the lowest total supply chain cost. Note that if the TTP were to follow the best response dynamics, instead of transmitting the Nash equilibrium order quantities immediately, the orders would end up at a Nash equilibrium eventually. Thus, by transmitting the Nash equilibrium order allocation immediately, the TTP effectively speeds up the convergence process.

This speed up has two important benefits. First, it ensures that the orders converge to the Nash equilibrium that produces the smallest total supply chain cost, in the case where two equilibria exist. If the TTP were to follow best response dynamics, allowing the retailers each to arbitrarily choose their initial order allocation in the first iteration, the TTP would not have control over which equilibrium the dynamics would converge to. Second, it minimizes the retailers' ability to learn the suppliers private information. To understand this point, consider the following. The true distribution of A^j, denoted $F_{A^j(q)}(x)$, can again be thought of as a two-dimensional function in $x \geq 0$ and $q \in [0, Q]$. Any best response order quantity derived using the true distribution of A^j will give the retailers new information regarding the function $F_{A^j(q)}(x)$. Knowing that $q_k^{j^*}$ was derived using best response dynamics and is the solution to either Equations 7 and 8 or system of Equations 9, the retailers are able to learn a point that lies on the function $F_{A^j(q)}(x)$ for every new order allocation quantity $q_k^{j^*}$. Namely, the retailers will be able to infer that the function $F_{A^j(q_{k-1}^*)}(x)$ must go through the point $(Q - q_k^{-j^*}, \frac{c_s}{c_s + c_h})$ or $(q_k^{-j^*}, F^c_{A^j(q_{k-1}^*),k}(q_k^j))$. In other words, *variation in the order quantities under the best response dynamics enables the retailers to learn the suppliers' private information*. Therefore, under the FI sharing scheme, it is beneficial to transmit the order allocations corresponding to the Nash equilibrium of the system immediately. By directly transmitting a Nash equilibrium order allocation, the retailers are only able to infer one point that lies on the function $F_{A^j(q)}(x)$. Let the Nash equilibrium order quantity be denoted $(q^{1^{NE}}, q^{2^{NE}})$. If $q^{1^{NE}} + q^{2^{NE}} = Q$ the retailers can infer that the function $F_{A^j(q^{j^{NE}})}(x)$ must go through the point $(q^{-j^{NE}}, F^c_{A^j(q^{j^{NE}})}(q^{j^{NE}}))$. If $q^{1^{NE}} \geq Q$ or $q^{2^{NE}} \geq Q$, the retailers can infer the function $F_{A^j(q^{j^{NE}})}(x)$ must go through the point $(Q - q^{-j^{NE}}, \frac{c_s}{c_s + c_h})$. Because the equilibrium order quantities were transmitted immediately, the retailers do not learn any additional points.

In the RI scheme, a similar implementation can be followed. Namely, each iteration, instead of transmitting each retailer's best response, the TTP can transmit a Nash equilibrium order allocation based on the current estimated function $F_{A^j(q),k}(x)$. (Similar to above, if two Nash equilibria exist, the TTP will choose the one with the smallest total supply chain cost). Under this implementation, the retailers cannot learn more than they could under the FI scheme. Thus, the retailers can again learn at most one point that lies on the true function $F_{A^j(q)}(x)$. This reduces the dimensionality of the retailers' learning problem by exactly one dimension.

7. Conclusions

This paper considers how reverse information sharing can improve supply chain performance in settings with yield uncertainty and lack of transparency. In particular, if the suppliers are willing to share inventory and order information with a trusted third party, the third party can transmit sufficient information to the retailers that allows them to update their order allocations. This transmission can be done in a way that limits the retailers' ability to learn new information about the supplier's "private information" (such as their inventory distribution or the number of retailers that they sell to). This information sharing scheme is particularly beneficial in situations where the retailers perceive supply scarcity at the suppliers. In these settings, without any information sharing, the retailers are incentivized to over-order. However, with appropriate reverse information sharing, over-ordering can be mitigated, benefiting the entire supply chain.

The reverse information sharing scheme proposed in this paper could be implemented in practice through the use of blockchain. Not only does blockchain enable end-to-end transparency and visibility, thereby making it unlikely that supply chain entities could enter false information, but secure computations can also be automatically performed on blockchain platforms. Besides the reverse information sharing scheme proposed in this paper, there are many other potential uses for blockchain in supply chains, and specifically in perishable food supply chains. Having visibility into the freshness levels of inventory at various points in the supply chain, or aggregate statistics about these freshness levels, could have consequences for inventory management. For example, if

the retailers had visibility into freshness levels at their suppliers, prices and shipping times could be modified according to the freshness levels. In addition, contracts could have an objective freshness component. Currently, quality is largely subjective and retailers can reject shipments if they perceive the quality to be too low. Objective measures of freshness, enabled through full traceability, could increase fairness and objectiveness in perishable supply chains.

Acknowledgments

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EC.1.   Proofs

Proof of Proposition 1

First, we write $C(\mathbf{q})$ in an equivalent form as

$$C(\mathbf{q}) = Qc_s + c_h \int_Q^\infty 1 - F_R(x)dx - c_s \int_0^Q 1 - F_R(x)dx$$

$$= Qc_s + (c_h + c_s) \int_Q^\infty (1 - F_R(x))dx - c_s\mathbb{E}[R]$$

We can write $\mathbb{E}[R]$ as

$$\mathbb{E}[R] = \int_0^{q^1} 1 - F_{A^1}(x)dx + \int_0^{q^2} 1 - F_{A^2}(x)dx$$

To analyze the function $C(\mathbf{q})$ we will consider separate domains, over which $\int_Q^\infty (1 - F_R(x))dx$ is smooth and differentiable.

Let $\mathcal{D}_1 = \{q^1, q^2 : q^1 + q^2 > Q, q^1 \leq Q, q^2 \leq Q\}$. On domain $\mathcal{D}_1$,

$$\int_Q^\infty (1 - F_R(x))dx = \int_{x=Q}^{q^1+q^2} \left(\int_{b=x-q^2}^{q^1} f_{A^1}(b)F_{A^2}^c(x-b)db \right) + F_{A^1}^c(q^1)F_{A^2}^c(x-q^1)dx$$

the derivative of the objective function with respect to q_j, $j \in \{1,2\}$, is

$$(-c_s + (c_h + c_s)F_{A_{-j}}^c(Q - q_j))F_{A_j}^c(q_j)$$

The unique value of q^j that satisfies first order condition for optimality is

$$q^{j^*} = Q - F_{A_{-j}}^{c^{-1}} \left(\frac{c_s}{c_s + c_h} \right)$$

The uniqueness of q_j^* comes from noting that $F_{A^j}^c(x)$ is a complementary CDF and is therefore always positive and monotone decreasing.

The second derivative is positive at q^{j^*} and therefore it is indeed a local minimum. The optimal solution over this domain is thus given by

$$q^{1^*} := Q - F_{A^2}^{c^{-1}} \left(\frac{c_s}{c_s + c_h} \right)$$

and

$$q^{2^*} := Q - F_{A^1}^{c^{-1}} \left(\frac{c_s}{c_s + c_h} \right)$$

When $q^{1^*}, q^{2^*} \notin \mathcal{D}_1$, we will show that the optimal solution lies on the boundary. Namely, either satisfies $q^1 + q^2 = Q$, $q^1 = Q$, or $q^2 = Q$. To show this, we will first consider the function $C(\mathbf{q})$ over the domain $\mathcal{D}_2 = \{q^1, q^2 : q^1 + q^2 \leq Q\}$. On domain $\mathcal{D}_2$, $1 - F_R(x) = 0$ for $x \in [0, Q)$. For $x = Q$, $1 - F_R(Q) = \mathbb{P}(R = Q) = \mathbb{P}(A^1(l) \geq q^1)\mathbb{P}(A^2(l) \geq q^2)$ when $q^1 + q^2 = Q$. When $q^1 + q^2 = Q$, the function $C(\mathbf{q})$ can be written as

$$C(\mathbf{q}) = Qc_s - c_s \left(\int_0^{q^1} 1 - F_{A^1}(x)dx + \int_0^{q^2} 1 - F_{A^2}(x)dx \right)$$

When $q^1 + q^2 < Q$, the derivative of $C(\mathbf{q})$ with respect to both q^1 and q^2 is negative, so we know that the optimal solution must satisfy $q^1 + q^2 = Q$. On this domain, a point satisfies the first-order conditions for optimality if and only if it satisfies the following set of equations:

$$\begin{cases} q^1 = Q - q^2 \\ F_{A^1}^c(q^1) = F_{A^2}^c(q^2) \end{cases} \tag{EC.1}$$

Now consider the domain $\mathcal{D}_3 := \{q^1, q^2 : q^2 \geq Q, q^1 \leq Q\}$. When $q^2 \geq Q$, $\frac{\partial C(\mathbf{q})}{\partial q^2} > 0$. Therefore, the lowest cost is obtained on the boundary by setting $q^2 = Q$. If $\tilde{q}^1 := Q - F_{A^2}^{c^{-1}}\left(\frac{c_s}{c_s + c_h}\right) \leq Q$, then $(\tilde{q}^1, Q)$ is optimal. By symmetry, the same holds on domain $\mathcal{D}_4 := \{q^1, q^2 : q^1 \geq Q, q^2 \leq Q\}$. Namely, $(Q, \tilde{q}^2)$ is optimal if $\tilde{q}^2 \leq Q$.

Finally, on the domain $\mathcal{D}_5 := \{q^1, q^2 : q^2 \geq Q, q^1 \geq Q\}$, the optimal point is (Q, Q).

Proof of Proposition 2

Suppose that the dynamics, starting from $\mathbf{q}_0$, converge to some order allocation strategy $\tilde{\mathbf{q}}$. In other words, for any ϵ there exists an N such that for all $n > N$,

$$|q_j^{*^{(n)}}(\mathbf{q}_0, F_{A^j,1}, ..., F_{A^j,n}) - \tilde{q}_j| < \epsilon$$

where $q_j^{*^{(n)}}(\mathbf{q}_0, F_{A^j,1}, ..., F_{A^j,n})$ denotes the order allocation to supplier j after n iterations of the best response dynamics using perceived distributions $F_{A^j,1}, ..., F_{A^j,n}$, starting from allocation $\mathbf{q}_0$.

Our goal is to show that, in this case, there exists a Nash equilibrium of the dynamics under $F_{A^j,\infty}$ such that for any $\epsilon > 0$, there exists an N such that for all $n > N$,

$$|q_j^{*^{(n)}}(\mathbf{q}_0, F_{A^j,1}, ..., F_{A^j,n}) - q_j^{NE}| < \epsilon$$

Since $F_{A^j(q),k}(x)$ converges pointwise to $F_{A^j(q),\infty}(x)$ on the domain $x \in [0, Q]$ for all $q \in [0, Q]$, for any $\epsilon > 0$, there exists a K such that for all $k > K$ and all $n \geq 1$,

$$|q_j^{*(n)}(\mathbf{q}_k, F_{A^j,\infty}) - q_j^{*(n)}(\mathbf{q}_k, F_{A^j,k+1}, ..., F_{A^j,k+n})| < \epsilon$$

where $\mathbf{q}_k = q_j^{*(k)}(\mathbf{q}_0, F_{A^j,1}, ..., F_{A^j,k})$ for $j \in \{1, 2\}$. Therefore, by the triangle inequality, we know that $q_j^{*(n)}(\mathbf{q}_k, F_{A^j,\infty})$ converges to $\tilde{q}_j$ as $n \to \infty$. However, from the theory of best response dynamics and fictitious play (Fudenberg et al. 1998), we know that if the best response dynamics under $q_j^{*(n)}(\mathbf{q}_k, F_{A^j,\infty})$ converge, they must converge to a Nash equilibrium under $q_j^{*(n)}(\mathbf{q}_k, F_{A^j,\infty})$. Therefore, $\tilde{\mathbf{q}} = \mathbf{q}^{NE}$ where $\mathbf{q}^{NE}$ is a Nash equilibrium under $q_j^{*(n)}(\mathbf{q}_k, F_{A^j,\infty})$.

Proof of Proposition 3

Let $F_{A^j(q)}$ be the "fixed" distribution of $A^j(q)$, for $j = 1, 2$, meaning that it is computed with respect to fixed inventory distributions. Let the random variable $\mathcal{A}^j(q^j)$ denote the expected available inventory at Supplier j including backorders. Therefore, $\mathcal{A}^j(q^j)$ can take negative values and is the uncensored version of the random variable $A^j(q^j)$. Specifically, when the retailers are identical, $A^j(q^j)$ can be written as $A^j(q) = (X^j - Mq^j)^+$, where M is a random variable denoting the number of retailers ranked above a given retailer in the lottery mechanism. Because of the lottery mechanism, M follows discrete uniform distribution, taking values from 0 through $N - 1$, each with probability $1/N$. The random variable $\mathcal{A}^j(q^j)$ can be written as $\mathcal{A}^j(q^j) = X^j - Mq^j$. On the domain $(0, \infty]$, $F_{\mathcal{A}^j(q^j)}(x)$ and $F_{A^j(q^j)}(x)$ are identical.

Consider the functions

$$\hat{q}^1(q^2) = Q - F_{\mathcal{A}^2(q^2)}^{c^{-1}}\left(\frac{c_s}{c_s + c_h}\right)$$

and

$$\hat{q}^2(q^1) = Q - F_{\mathcal{A}^1(q^1)}^{c^{-1}}\left(\frac{c_s}{c_s + c_h}\right)$$

Notice that $\tilde{q}^j$ (from the solution to Problem Problem C_{ret}) can equivalently be written as

$$\tilde{q}^j := \max\{\hat{q}^j(q_{k-1}^{-j*}), Q\} = \max\{Q - F_{\mathcal{A}^{-j}(q^{-j})}^{c^{-1}}\left(\frac{c_s}{c_s + c_h}\right), Q\} = Q - F_{\mathcal{A}^{-j}(q^{-j})}^{c^{-1}}\left(\frac{c_s}{c_s + c_h}\right).$$

First, it will be shown that $\hat{q}^1(q^2)$ is increasing in q^2. This is clear by noting that $F_{\mathcal{A}^2(q^2)}^{c^{-1}}\left(\frac{c_s}{c_s + c_h}\right)$ is equal to the value of x that solves $\mathbb{P}(X^2 - Mq^2] \geq x) = \left(\frac{c_s}{c_s + c_h}\right)$. For

$q^{2''} > q^{2'}$, $\mathbb{P}(X^2 - Mq^{2''}] \geq y) < \mathbb{P}(X^2 - Mq^{2'}] \geq y)$ for all $y > 0$. Therefore, $F^{c^{-1}}_{\mathcal{A}^2(q^2)}\left(\frac{c_s}{c_s + c_h}\right)$ is decreasing in q^2 and thus $\hat{q}^1(q^2)$ is increasing in q^2. Similarly, $\hat{q}^2(q^1)$ is increasing in q^1.

Let

$$x(q) := F^{c^{-1}}_{\mathcal{A}^j(q)}\left(\frac{c_s}{c_s + c_h}\right)$$

Our goal is to lower bound the derivative $x'(q)$. Notice that $x(q)$ satisfies

$$\mathbb{P}(\bar{X}^j - Mq \geq x(q)) = \frac{c_s}{c_s + c_h}$$

By the independence of X^j and M,

$$\mathbb{E}_M[F^c_{X^j}(x(q) + Mq)] = \frac{c_s}{c_s + c_h} \tag{EC.2}$$

Taking the implicit derivative of Equation EC.2, we find that

$$x'(q) = -\frac{\mathbb{E}_M[MF^{c'}_{X^j}(x(q) + Mq)]}{\mathbb{E}_M[F^{c'}_{X^j}(x(q) + Mq)]} = -\frac{\mathbb{E}_M[Mf_{X^j}(x(q) + Mq)]}{\mathbb{E}_M[f_{X^j}(x(q) + Mq)]}$$

Let $T_0 := f_{X^j}(x(q))$ and $T_1 := f_{X^j}(x(q) + q)$. By factoring out $\mathbb{P}[M = m]$ from every term, we can write

$$|x'(q)| = \frac{T_1 + \sum_{m \geq 2} m f_{X^j}(x(q) + mq)}{T_0 + T_1 + \sum_{m \geq 2} f_{X^j}(x(q) + mq)}$$

$$\geq \frac{T_1 + 2\sum_{m \geq 2} f_{X^j}(x(q) + mq)}{T_0 + T_1 + \sum_{m \geq 2} f_{X^j}(x(q) + mq)}$$

$$\geq \frac{T_1 + 2A}{T_0 + T_1 + A}$$

where $A = \sum_{m \geq 2} f_{X^j}(x(q) + mq)$. Notice then that $x'(q) < -1$ when $T_0 < A$ for all q. In other words, $x'(q) < -1$ when

$$f_{X^j}(x(q)) < \sum_{m=2}^{N} f_{X^j}(x(q) + mq)$$

for all q. Finally, noting that $Q - x(q) = q^*$, this condition can be written as

$$f_{X^j}(Q - q) < \sum_{m=2}^{N} f_{X^j}(Q + (m-1)q) \tag{EC.3}$$

for all $q \in [0, Q]$

We are now able to prove that there are at most three Nash equilibrium. A Nash equilibrium must be a best response for every retailer. Therefore, for all i, $\mathbf{q}_i^*$ must satisfy one of the optimality conditions listed in Proposition 1. Since all retailers are identical, for simplicity let $(q^{1^{NE}}, q^{2^{NE}})$ denote a Nash equilibrium order allocation to Supplier 1 and 2, respectively. By Proposition 1, $(q^{1^{NE}}, q^{2^{NE}})$ must satisfy either

$$\begin{cases} q^{1^{NE}} = \max\{Q - F^{c^{-1}}_{\mathcal{A}^2(q^{2^{NE}}),k}\left(\frac{c_s}{c_s+c_h}\right), Q\} \\ q^{2^{NE}} = \max\{Q - F^{c^{-1}}_{\mathcal{A}^1(q^{1^{NE}}),k}\left(\frac{c_s}{c_s+c_h}\right), Q\} \\ q^{1^{NE}} + q^{2^{NE}} > Q \end{cases} \qquad \text{(EC.4)}$$

$$\begin{cases} q^{1^{NE}} = Q - q^{2^{NE}} \\ F^{c}_{\mathcal{A}^1(q^{1^{NE}}),k}(q^{1^{NE}}) = F^{c}_{\mathcal{A}^2(q^{2^{NE}}),k}(q^{2^{NE}}) \end{cases} \qquad \text{(EC.5)}$$

Notice that $F^{c}_{\mathcal{A}^1(q),k}(q)$ is decreasing in q. Therefore, System EC.5 has at most one solution. Furthermore, the functions $\hat{q}^1(q^2) = Q - F^{c^{-1}}_{\mathcal{A}^2(q^2),k}\left(\frac{c_s}{c_s+c_h}\right)$ and $\hat{q}^2(q^1) = Q - F^{c^{-1}}_{\mathcal{A}^1(q^1),k}\left(\frac{c_s}{c_s+c_h}\right)$ can intersect at most once if $\hat{q}^{2'}(q^1) > 1$ and $\hat{q}^{1'}(q^2) > 1$. Condition EC.3 (which is also the condition of the Proposition) ensures that $\hat{q}^{2'}(q^1) > 1$ and $\hat{q}^{1'}(q^2) > 1$. Therefore EC.4 has at most one solution where $q^1_{NE} < Q$ and $q^2_{NE} < Q$, which satisfies $\hat{q}^2(q^1) = \hat{q}^1(q^2)$. Additionally, the functions $\max\{Q - F^{c^{-1}}_{\mathcal{A}^2(q^2_{NE}),k}\left(\frac{c_s}{c_s+c_h}\right), Q\}$ and $\max\{Q - F^{c^{-1}}_{\mathcal{A}^1(q^1_{NE}),k}\left(\frac{c_s}{c_s+c_h}\right), Q\}$ can intersect at most once on the boundary where $q^1_{NE} = Q$ or $q^2_{NE} = Q$, and $q^1_{NE} + q^2_{NE} > Q$. A Figure showing these potential Nash equilibria is shown in Figure EC.1

Therefore, System EC.5 has at most two intersection points: One where $Q - F^{c^{-1}}_{\mathcal{A}^2(q^2_{NE}),k}\left(\frac{c_s}{c_s+c_h}\right) < Q$ and $Q - F^{c^{-1}}_{\mathcal{A}^1(q^1_{NE}),k}\left(\frac{c_s}{c_s+c_h}\right) < Q$, and another where either $q^{NE}_1 = Q$ or $q^{NE}_2 = Q$ (or both).

Finally, we prove that the best response dynamics cannot converge to the point corresponding to Nash equilibrium NE'. Let $\bar{q}^2(q^1)$ be equivalent to the function $\hat{q}^1(q^2)$ written in terms of q^1 (in other words, the graph of the function $\bar{q}^2(q^1)$ is equivalent to the set of points $\{(\hat{q}^1(q^2), q^2)\}$ for $q^2 \in [0, Q])$. The allocation NE' occurs at the unique point where $\bar{q}^2(q^1) = \hat{q}^1(q^1)$. Figure EC.2 gives an intuitive "proof by picture".

Consider the best response dynamics, starting at point $\mathbf{q}_0$. The point $\mathbf{q}_k$ is used to denote the optimal order allocation after k iterations. Notice that $q^2_{k+1} = \hat{q}^2(q^1_k)$ and $q^1_{k+1} = \hat{q}^1(q^2_k)$. Let R^1, R^2, R^3, and R^4 correspond to the domains as shown in Figure EC.2. In R^1, after
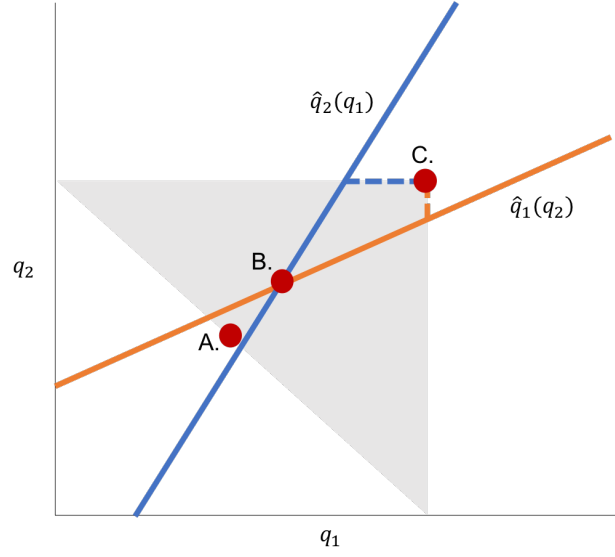
Figure EC.1   Picture of the solutions to Systems EC.4 and EC.5. Point A is a solution to System EC.5, point B
is a solution to $\hat{q}^2(q^1) = \hat{q}^1(q^2)$ (i.e. satisfies System EC.4), and point C also satisfies System EC.4 but has
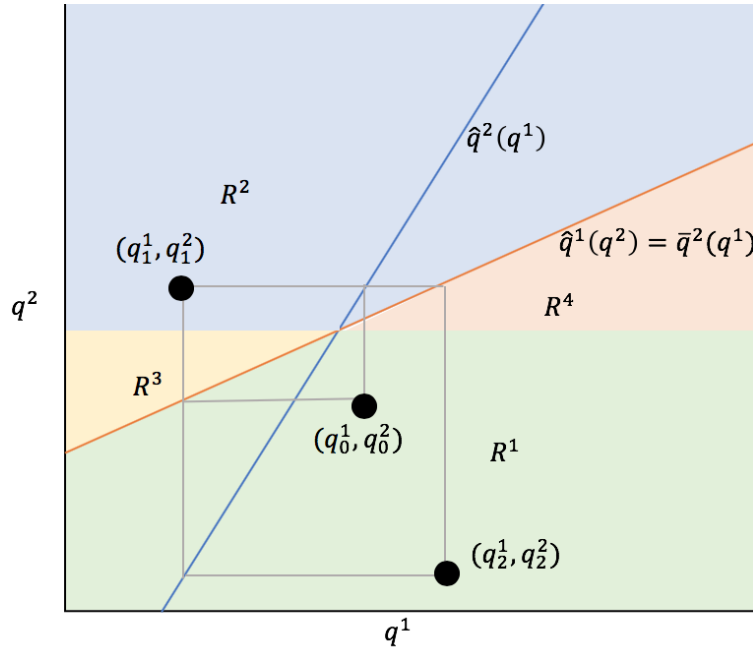$$q^1 = q^2 = Q.$$

Figure EC.2   Illustration of the regions considered, for the proof that the best response dynamics cannot
converge to NE'. The point NE' is given by the intersection of the orange and blue lines. Three updates,
starting from $\mathbf{q}_0$, are also shown.

one iteration, $q_1^1 < q_0^1$, and therefore, after two iterations $q_2^2 < q_0^2$ since $q_2^2 = \hat{q}^2(q_1^1)$ and $\hat{q}^2(q^1)$
is an increasing function that lies below $\bar{q}^2(q^1)$ in this region. Furthermore, notice that if
$(q_k^1, q_k^2) \in R^1$, (q_{k+2}^1, q_{k+2}^2) is necessarily also in R^2. Therefore, since $q_2^2 < q_0^2 < q_{NE'}^2$ for all

$\mathbf{q}_0$ starting in R^1, the dynamics cannot converge to $\mathbf{q}_{NE'}$ after starting in R^1. A similar argument holds for allocation dyanmics starting in R^2. In R^2, the q^2 components increase every two iterations, and thus since $q_0^2 > q_{NE'}^2$ to begin with, the dynamics cannot converge to $q_{NE'}^2$.

If the dynamics begin in either R^3 or R^4, after one iteration the allocation ends up in either R^1 or R^2. Then, the arguments above apply. Therefore, there is no starting point such that the best response dynamics can converge to $\mathbf{q}_{NE'}$.

Proof of Corollary 1

The proof of Proposition 3 demonstates that the convergence of the retailers' order dynamics depend on the intersection of the functions $\hat{q}^2(q^1)$ and $\hat{q}^2(q^2)$ in the $q^1 - q^2$ plane. When the point of intersection lies in domain $\mathcal{D} = \{q^1, q^1 > 0, q^2 > 0 q^2 : q^1 + q^2 \geq Q, q^1 \leq Q, q^2 \leq Q\}$, the order dynamics can converge to either NE_Q or NE_{2Q}. Corollary 2 approximates the intersection point by using an upper and lower bound on Equations 10a and 10b .

Equations 10a and 10b can be re-written as

$$\sum_{m=0}^{N-1} \mathbb{P}[X^2 \geq Q - q^1 + mq^2] = N \frac{c_s}{c_s + c_h} \tag{EC.6a}$$

$$\sum_{m=0}^{N-1} \mathbb{P}[X^1 \geq Q - q^2 + mq^1] = N \frac{c_s}{c_s + c_h} \tag{EC.6b}$$

Our goal is to come up with a very tight lower and upper approximation of the (q^1, q^2) that satisfy Equations EC.6a and EC.6b. Consider the following string of inequalities:

$$\int_{m=0}^{N} \mathbb{P}[X^j \geq Q - q^{-j} + mq^j]dm \leq \sum_{m=0}^{N-1} \mathbb{P}[X^j \geq Q - q^{-j} + mq^j] \leq \int_{m=0}^{N} \mathbb{P}[X^j \geq Q - q^{-j} + mq^j - q^j]dm \tag{EC.7}$$

Because X^j is a non-negative random variable,

$$\int_{m=0}^{N} \mathbb{P}[X^j \geq Q - a + mb]dm = \frac{1}{b} \int_{x=Q-a}^{Q-a+Nb} \mathbb{P}[X^j \geq x]dx = \frac{1}{b}\mathbb{E}[X^j \mathbb{1}_{X^j \in [Q-a, Q-a+Nb]}]$$

Therefore, a lower approximation to the solution of System EC.6 is given by solving System 11 and an upper approximation is given by solving System 12.

Furthermore, if Condition 13 (below) holds for $j = 1, 2$, then the order allocation corresponding to equilibrium NE_{2Q} is allocation (Q, Q). f $Q \leq \hat{q}^2(q^1)$ and $Q \leq \hat{q}^1(q^2)$ (as in Figure EC.2), the order allocation corresponding to equilibrium NE_{2Q} is the allocation (Q, Q). This should be clear by considering Figure EC.2. Using the integral lower bound

from Expression EC.7, this statement is equivalent to Condition 13 in the statement of the corollary.

Proof of Proposition 4

As in the proof of Proposition 3, notice that the construction of L^j ensures that $\frac{\partial \hat{q}^{-j}(q^j)}{\partial q^j} \geq L^j$, where $\hat{q}^{-j}(q^j)$ is defined in the proof of Proposition 3. Also recall, as in the proof of Proposition 3, that the best response dynamics are governed by functions $\hat{q}^1(q^2)$ and $\hat{q}^2(q^1)$ in the $q^1 - q^2$ plane. In other words, given a starting allocation (q_0^1, q_0^2), the allocation in the next iteration is given by $(q_1^1, q_1^2) = (\hat{q}^1(q_0^2), \hat{q}^2(q_0^1))$ as long as $(\hat{q}^1(q_0^2), \hat{q}^2(q_0^1)) \in \{q^1, q^2 : q^1 + q^2 \geq Q, q^1 \leq Q, q^2 \leq Q\}$.

Let $b^j := \hat{q}^j(Q)$. Notice that $\hat{q}^j(q^{-j}) \leq y^j(q^{-j})$ where $y^j(q^{-j})$ is the linear function with slope L^j going through the point (Q, b^j). Let (q_n^1, q_n^2) be the order allocation after n iterations, starting from (Q, Q), of the true best response dynamics (using the function $\hat{q}^j(q^j)$). Let (y_n^1, y_n^2) be the order allocation after n, starting from (Q, Q), iterations of the best response dynamics using the linear functions $y^j(\cdot)$. By construction, $y_n^j \geq q_n^1$. See Figure EC.3 for reference. Therefore, if (y_n^1, y_n^2) converges to NE_Q after n iterations (meaning that $y_n^1 + y_n^2 \leq Q$), (q_n^1, q_n^2) must also satisfy $q_n^1 +_n^2 \leq Q$. Thus, it suffices to consider the convergence of (y_n^1, y_n^2).

First consider the case when the suppliers are symmetric, so $b^1 = b^2 = b$ and $L^1 = L^2 = L$. Then, starting from (Q, Q), $y_n^1 = y_n^2 = (Q - b)\frac{L^n - 1}{L - 1} + Q$. It follows that, when $n \geq \frac{\log\left(\frac{-2b + (L+1)Q}{2(Q-b)}\right)}{\log(Q)}$, $y_n^j \leq Q/2$, which implies that $y_n^1 + y_n^2 \leq Q$.

When the suppliers are not symmetric, the same inequality holds by setting $b = \max\{b^1, b^2\}$ and $L = \min\{L^1, L^2\}$.

Proof of Proposition 5

First we will consider the cost to Supplier j in an arbitrary iteration k. Let q^j be the amount that each retailer orders from Supplier j in this iteration. When $penalty(q, r) = 0$, the supplier's revenue is $p \min\{Nq^j, X_k^j\}$. Therefore, the supplier is benefited by larger orders. Thus, $C_{S^j}(NE_{2Q}) < C_{S^j}(NE_Q)$.

Now consider the case when the penalty function is not identically zero. Let $l := \lfloor X_k^j / q^j \rfloor$ be the index of the lowest priority retailer (according to the random lottery mechanism) that still receives their entire order, and let $\hat{r}$ be the remaining inventory that the $l + 1$st retailer receives. The supplier's cost is given by

$$-p \min\{Nq^j, X_k^j\} + \left(penalty(q^j, \hat{r}) + (N - (l+1))penalty(q^j, 0)\right) \mathbb{1}_{X_k^j < q^j N}$$
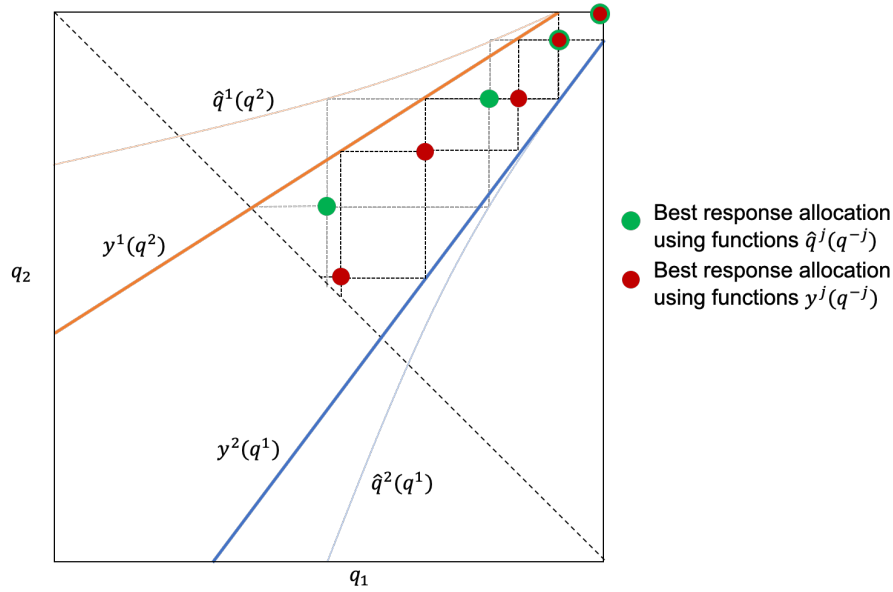
Figure EC.3   Illustration of the functions $y^j(q^{-j})$ and $\hat{q}^j(q^{-j})$ as well as the best response dynamics induced by each set of functions, starting from allocation (Q, Q).

Notice that the supplier's expected cost depends on $\mathbb{P}(X_k^j < q^j N)$. As long as this probability is positive, there is always a chance of the supplier incurring penalty costs. Furthermore, notice that $\mathbb{P}(X_k^j < q^j N) < \mathbb{P}(X_k^j < QN)$ for $q^j < Q$. Therefore, there is always a penalty cost high enough such that the suppliers will prefer smaller orders.

Now consider the expected cost to the retailers. For simplicity, we will prove the Proposition for the case that X^j is a multiple of q^j. When X^j is not a multiple of q^j, the same logic holds however the exposition is much more tedious. Let p_b be the probability that a given retailer receives its order from both suppliers, p_j be the probability that it only receives its order from Supplier j, and p_0 be the probability that it does not receive anything. The retailer's expected cost can be written as

$$c_h(q^1 + q^2 - Q)p_b + c_s(Q - q^2)p_1 + c_s(Q - q^1)p_2 + c_s Q p_0 \tag{EC.8}$$

The probabilities can be written as

$$p_b = \frac{X_k^1}{Nq^1}\frac{X_k^2}{Nq^2} \tag{EC.9}$$

$$p_1 = \frac{X_k^1}{Nq^1}\left(1 - \frac{X_k^2}{Nq^2}\right) \tag{EC.10}$$

$$p_2 = \frac{X_k^1}{Nq^1}\left(1 - \frac{X_k^2}{Nq^2}\right) \tag{EC.11}$$

$$p_0 = (1 - \frac{X_k^1}{Nq^1})(1 - \frac{X_k^2}{Nq^2}) \tag{EC.12}$$

We wish to compare Equation EC.8 under NE_Q to NE_{2Q}. Let (q^1, q^2) be the orders under NE_Q, so that $q^1 + q^2 = Q$. For now, let (Q, Q) be the orders under NE_Q. A sufficient condition for the cost under NE_Q to be less than the cost under NE_{2Q} is:

$$\max\{q^1, q^2\}(p_1(NE_Q) + p_2(NE_Q)) + Qp_0(NE_Q) < Qp_0(NE_{2Q})$$

Notice that the maximum cost incurred by any one retailer is $c_s Q$. Therefore, when X^j is not a multiple of q^j, there can be at most two retailers each iteration that receive an "in-between" amount of inventory (i.e., an amount between 0 and q^j). Therefore, a sufficient condition for the inequality above to hold is that

$$\max\{q^1, q^2\}N(p_1(NE_Q) + p_2(NE_Q)) + QNp_0(NE_Q) + 2Q < QNp_0(NE_{2Q}) - 2Q$$

Notice that this cost is now the expected cost of all retailers. To account for the costs incurred by the retailers that recieve an "in-between" amount, we have added $2Q$ to the right-hand side and subtracted $2Q$ from the left hand side.

Without loss of generality assume that $q^1 \geq q^2$. Then the inequality above holds if:

$$N \leq \frac{((q_1)^2 + q_1 q_2 - (q_2)^2)X_k^2 X_k^1}{(q_2 Q(4q_1^2 + 4q_1 q_2 + X_k^1(q^1 - q^2)))} \tag{EC.13}$$

To avoid confusion with exponents, q_j in the expression above is the order allocation to Supplier j (formerly written as q^j). Notice that if $q_1 = q_2$, Condition is met EC.13. Therefore, the suppliers are symmetric, the retailers always prefer NE_Q to NE_{2Q}. Substituting $q_2 = Q - q_1$, Condition EC.13 can be re-written as

$$N \leq X_k^2 \frac{Q^2 - 3Qq_1 + (q_1)^2}{Q^3 - 3Q^2 q_1 + qQ(q_1)^2}$$

On the domain $Q/2 \leq q_1 \leq Q$, the RHS above has a unique minimum at $q^1 = \frac{2}{3}Q$, at which the RHS is equal to $X_k^2 \frac{5}{Q}$. Therefore, as long as

$$NQ \leq 5X_k^j$$

for $j = 1, 2$, the retailers will prefer NE_Q.

This condition is extremely reasonable and can be interpreted as saying that neither supplier has *extremely* limited supply.

Now consider supply chain waste. We will prove this in the case that the suppliers are symmetric, however analogous (but messier) logic can be used to prove the statement in the case that the suppliers are not symmetric. Because the suppliers are symmetric, the retailers will order the same quantity from both suppliers, denoted by q. Also for simplicity assume that X_k^j is a multiple of q for $j = 1, 2$. For a single iteration with orders (q, q), waste is given by

$$(X_k^1 - Nq)^+ + (X_k^2 - Nq)^+ + n_2(2q - Q) + n_1(q - Q)^+$$

where n_2 is the number of retailers who receive their entire order from both suppliers, and n_1 is the number who receive their entire order from only one supplier.

First consider the order allocation NE_Q. Since the suppliers are symmetric, $NE_Q = (Q/2, Q/2)$. Notice that at equilibrium NE_Q there is never waste at the retailers. Waste at the suppliers is given by $(X_k^1 - NQ/2)^+ + (X_k^2 - NQ/2)^+$.

Now consider the order allocation $NE_{2Q} = (Q, Q)$. Waste is given by

$$(X_k^1 - NQ)^+ + (X_k^2 - NQ)^+ + n_2 Q$$

The expected value of n_2 is $N \cdot \max\{1, \frac{X_k^1}{NQ}\} \cdot \max\{1, \frac{X_k^2}{NQ}\}$. Consider diving the domain of possible inventory realizations into three segments: $[0, NQ/2]$, $(NQ/2, NQ)$, and $[NQ, \infty)$. By considering the nine possible combinations of X_k^1 and X_k^2 falling into each of these segments, it is straightforward to see that the expected waste under NE_Q is smaller than the expected waste under NE_{2Q}. For example, consider the case when both X_k^1 and X_k^2 lie in $[0, NQ/2]$. In this case, there is no waste at NE_Q, so automatically we have that $W(NE_Q) \leq W(NE_{2Q})$. Now consider the case when $X_k^1 \in [0, NQ/2]$ and $X_k^2 \in (NQ/2, NQ)$. Waste under NE_Q is given by $X_k^2 - NQ/2$. Waste under NE_{2Q} is given by

$$N \frac{X_k^1}{NQ} \frac{X_k^2}{NQ} Q = X_k^1 X_k^2 \frac{1}{NQ}$$

Since $X_k^1 \leq NQ/2$, $X_k^1 X_k^2 \frac{1}{NQ} \leq X_k^2$. Therefore, $W(NE_Q) \leq W(NE_{2Q})$. This analysis can be repeated for all nine cases. Similar analysis can be done for the case when the supplier's are not symmetric (and thus the allocation at NE_Q is not symmetric).

Proof of Corollary 2

The proof of Corollary 2 is very similar to the proof of the last statement of Corollary 1. If Q_i is the best response to all other retailers ordering Q_l, for $l \in \{1, ..., N\} \setminus i$, then it

is true that (Q_i, Q_i) for all i is a Nash equilibrium. Notice that Q_i is the best response to all other retailers ordering Q_l, for $l \in \{1, ..., N\} \setminus i$ when $Q_i - F^{c^{-1}}_{\mathcal{A}^j_i(\mathbf{Q}_{-i})}\left(\frac{c_s}{c_s + c_h}\right) \geq Q$, where $\mathcal{A}^j_i$ is the uncensored version of A^j_i as stated in the Corollary. This is equivalent to

$$F^{c^{-1}}_{\mathcal{A}^j_i(\mathbf{Q}_{-i})}\left(\frac{c_s}{c_s + c_h}\right) \leq 0$$

or

$$\mathbb{P}[\mathcal{A}^j_i(\mathbf{Q}_{-i}) \geq 0] \leq \frac{c_s}{c_s + c_h}.$$

This can be written as

$$\mathbb{P}\left[X^j - \sum_{l \in \{1,...,N\} \setminus i} \mathbb{1}_{o(l) < o(i)} Q_l \geq 0\right] \leq \frac{c_s}{c_s + c_h}.$$

A sufficient condition for the inequality above to hold is that

$$\mathbb{P}\left[X^j - \sum_{l \in \{1,...,N\} \setminus i} \mathbb{1}_{o(l) < o(i)} Q_{min} \geq 0\right] \leq \frac{c_s}{c_s + c_h},$$

where $Q_{min} = \min_i Q_i$. Following the same technique as in the proof of Corollary 2, a sufficient condition for the inequality above to hold is that

$$\mathbb{E}[X^j \mathbb{1}_{X^j \in [0, (N-1)Q_{min}]}] \leq Q_{min} N \frac{c_s}{c_s + c_h}$$

for $j = 1, 2$.