The Heterogeneous Fleet Vehicle Routing Problem with Draft Limits

Mathematical Optimization - A.A. 2024-2025

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Problem Description

Context: In modern maritime transport, naval gigantism has led to increasingly large vessels with greater drafts

Operational issue: Ports have draft limits-ships that are too heavily loaded cannot enter them, which affects the sequence of port visits.

Objective: Minimize the total network cost (port access + sailing) by deciding how many and which ships (of different sizes) to use, and in what sequence to visit the ports.

Novelty:

- Incorporates load-dependent draft limits into routing decisions.
- Considers a heterogeneous fleet (ships with different capacities, costs, and drafts).
- Integrates fleet sizing and routing under draft constraints.

Formulation

SETS

- $I = [1, I_{max}]$ $\rightarrow$ set of ports
- $I0 = [0, I_{max}]$ $\rightarrow$ set of ports including the depot
- $S = [1, S_{max}]$ $\rightarrow$ set of ships

PARAMETERS

- Q_s $\rightarrow$ Capacity (tons) of the ship s
- q_i $\rightarrow$ Demand (tons) of the port i
- L_{is} $\rightarrow$ Maximum loading for ship s to access port i (tons)
- t_{ij} $\rightarrow$ Sailing time between port i and port j (h)
- c_s $\rightarrow$ Hourly sailing cost for ship s (€/h)
- r_{is} $\rightarrow$ Access cost for ship s entering port i (€)

VARIABLES

- l_{is} $\rightarrow$ Loading of ship s entering port i
- $u_i \in N^+ \quad \forall i \in I$ $\rightarrow$ Position of port i in the sequence of visited ports
- $p_s = \sum_{\{i \in I\}} q_i Y_{is} \ \forall s \in S$ $\rightarrow$ Total load for ship s

DECISION VARIABLES

- $X_{ijs} \in \{0, 1\} \quad \forall i \in I0 \quad \forall j \in I0 \quad \forall s \in S$ Takes value 1 if the arc (i, j) is traversed by ship s
- $Y_{is} \in \{0,1\} \quad \forall i \in I \quad \forall s \in S$ Takes value 1 if port i is served by ship s

OBJECTIVE FUNCTION

The goal is to minimize the total network cost

$$min \sum_{i \in I0} \sum_{j \in I0} \sum_{s \in S} c_s t_{ij} X_{ijs} + \sum_{i \in I} \sum_{s \in S} r_{is} Y_{is}$$

where the first term

$$\sum_{i \in I0} \sum_{j \in I0} \sum_{s \in S} c_s t_{ij} X_{ijs}$$

represents the sailing cost, and the second term

$$\sum_{i \in I} \sum_{s \in S} r_{is} Y_{is}$$

represents the sum of the fixed costs to access ports

CONSTRAINTS

1)

$$\sum_{s \in S} Y_{is} = 1 \quad \forall i \in I$$

imply that each port is assigned to a ship

2)

$$\sum_{i \in I} q_i Y_{is} \leq Q_s \quad \forall s \in S$$

ensure that the maximum load capacity of a ship is never exceeded

3)

$$\sum_{i \in I0} X_{ijs} = Y_{is} \quad \forall j \in I \quad \forall s \in S$$

if ship s serves port j, it must have previously visited another port including the depot

4)

$$\sum_{i \in I0} X_{ijs} = \sum_{i \in I0} X_{jis} \quad \forall j \in I \quad \forall s \in S$$

for each port and ship, the number of incoming arcs to j equals the number of outgoing arcs from j, ensuring flow conservation

5)

$$X_{0js} \leq \sum_{j \in I} Y_{js} \quad \forall s \in S$$

a ship may depart from the depot only if it serves at least one port

6)

$$X_{0js} \geq \sum_{j \in I} \frac{Y_{js}}{I_{max}} \quad \forall s \in S$$

if a ship serves any port, it must depart from the depot

7)

$$u_j \geq u_i + 1 - I_{max}(1 - \sum_{s \in S} X_{ijs}) \quad \forall i \in I \quad \forall j \in I0$$

if ship s traverses arc (i, j), then port j must appear after port i in the visit sequence

8)

$$l_{js} \geq l_{is} - q_i - Q_s\left(1 - X_{ijs}\right) \quad \forall i \in I \quad \forall j \in I0 \quad \forall s \in S$$

if ship s travels from port i to port j, then the load at j must be at least the load at i minus the demand at i.

9)

$$l_{is} \leq L_{is} \quad \forall i \in I \quad \forall s \in S$$

the load of ship s upon entering port i must not exceed the port-specific loading limit L_{is}

10)

$$l_{0s} = \sum_{i \in I} q_i Y_{is} \quad \forall s \in S$$

the initial load of ship s equals the total demand of the ports it serves

VALID INEQUALITIES

These inequalities help the solver reduce solution time by eliminating infeasible or suboptimal routes early.

VI1)

$$X_{0js} \leq 1 - \frac{1}{TOT_q}\left(\sum_{i \in I} q_i Y_{is} - L_{js}\right) \quad \forall j \in I \quad \forall s \in S$$

for each port j, if the total load of the ship s, to which it has been assigned, is greater than the maximum allowed load for s to enter j, then j cannot be the first port visited in the route

VI2)

$$X_{ijs} = 0 \quad \forall i \in I \quad \forall j \in J \quad \forall s \in S \mid q_i + q_j > L_{is}$$

for each ship s and each pair of ports i and j, if the sum of their demand, q_i and q_j, is greater than the maximum allowed load for $s$to enter i, then j cannot be served immediately after i by ship s

VI3)

$$p_s - (u_i - 1)q_{big} \leq L_{is} + Q_{is}(1 - Y_{is}) \quad \forall i \in I \quad \forall s \in S$$

$$q_{big} = \max_{i \in I} q_i$$

allows to identify the earliest position a port i can occupy in the visiting sequence, without violating draft limit constraints, given the ship s to which it has been assigned and the set of ports assigned to it

VI4)

$$u_i \leq I^* \quad \forall i \in I$$

that the latest position a port can assume in the visiting sequence is equal to the maximum number of ports that can be assigned simultaneously to the same ship, I^*.

I^* is computed by sorting ports in non-decreasing order of demand and counting how many can be assigned to the largest ship before exceeding its capacity.

Matheuristic

OVERVIEW

- The MIP model efficiently solves only small instances (≤15 ports)
- For larger networks, the computational time grows exponentially
- To handle larger instances, two **matheuristics** are proposed:
    - **Large Neighborhood Search (LNS)**
    - **Iterated Local Search (ILS)**
- Both methods combine mathematical programming with local search principles

LARGE NEIGHBORHOOD SEARCH (LNS)

Main idea:

- Use a randomized operator to partially destroy the solution and exploit the mathematical model to optimally rebuild a feasible solution starting from the partial solution obtained

Parameters:

- m: number of removed ports (best value = 5)
- α: proximity factor for selecting nearby ports (best value = 1.5)

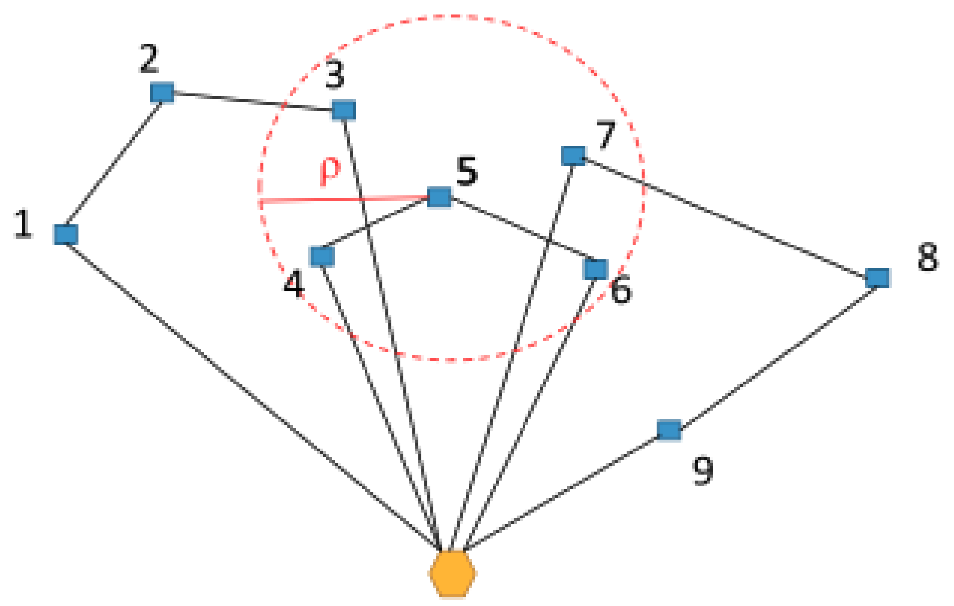
Key advantage:

- A very large neighborhood can be efficiently explored at each itaration allowing to quickly move toward strongly better solution

PROCEDURE

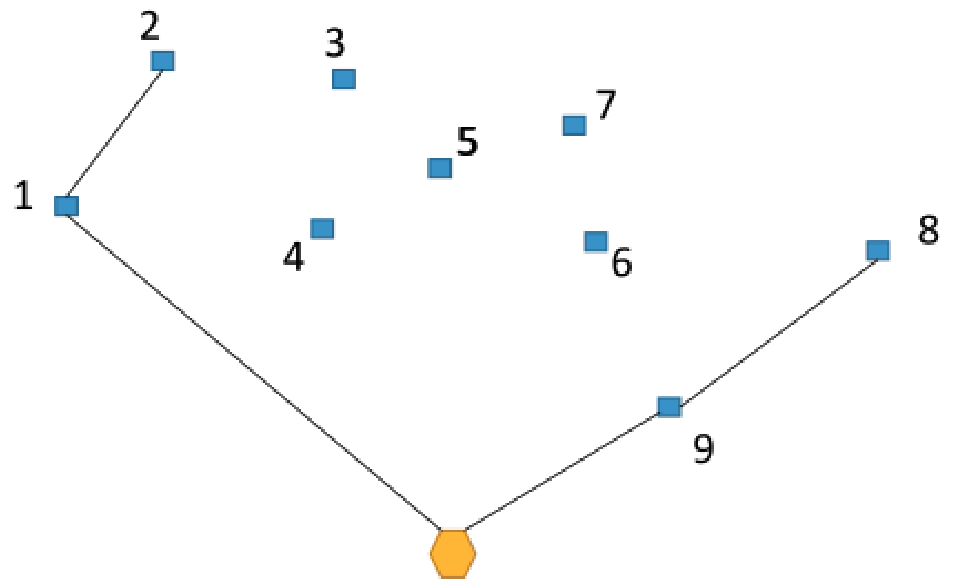
1. Run the mathematical model with a short time limit (15s) and keep the solution obtrained so far
2. At each iteration are selected m random ports and removed from the solution
3. The destroy operator removes all nodes within a certain radius from the node

$$\rho_i = \alpha v_i$$



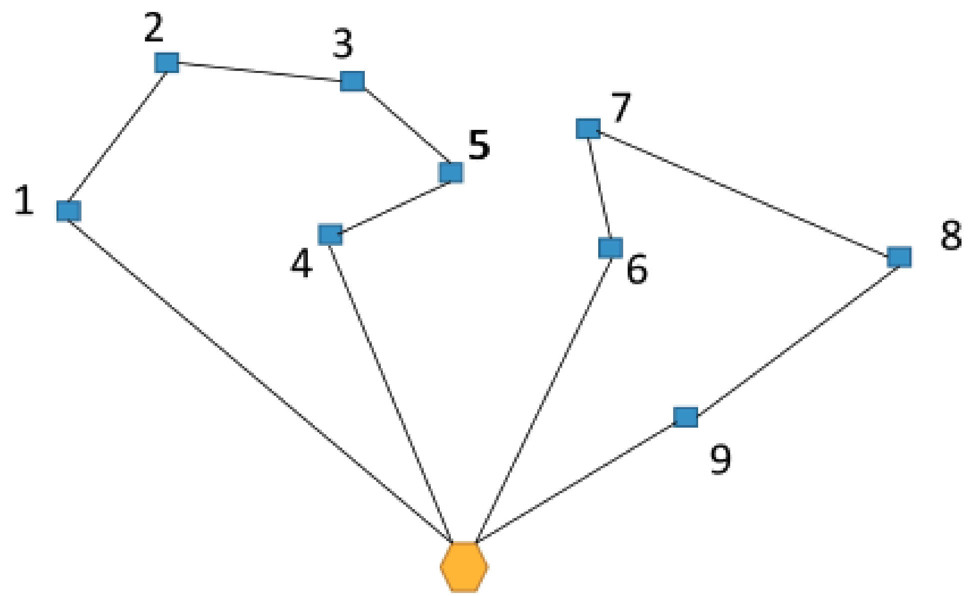
PROCEDURE

4. The over-constrained mathematical model is run fixing to 1 the variables corresponding to the arcs selected



PROCEDURE

5. If the obained solution is bettere than the actual best solution, it is kept as current best, otherwise is discarded



ITERATED LOCAL SEARCH (ILS)

Main idea:

- Combine a deterministic **Local Search (LS)** phase with a randomized **Diversification (DIV)** phase.

Algorithm outline:

1. Start from an initial feasible solution (MIP-based)
2. Local Search
3. Diversification
4. Repeat LS and DIV until the stopping condition is reached

- **Key advantage:**
  More robust than LNS — better solution quality with similar computational time

LOCAL SEARCH (LS)

Procedure:

1. Every port is given a score $\sigma_i = \frac{\lambda_i}{v_i}$
2. The ports with the highest score are removed
3. Use the MIP model to optimally rebuild the partial destroyed one
4. Accept the new solution if it improves the objective function
5. Repeat until no further improvement is found

DIVERSIFICATION (DIV)

Procedure:

1. Randomly select m ports to remove.
2. Include also nearby ports within a distance $\rho_i = \alpha v_i$
3. Rebuild the solution using the MIP model
4. Even if the new solution is worse, keep it to promote exploration

Dataset

- Parameters
  - Number of ships $s \in [3, 5, 6, 10]$
  - Number of ports $\mathrm{i} \in [15, 25, 50]$
  - 2 scenarios for Draft Restriction (DR):
    - $LOW_{DR} = 30\%$ of ports affected by draft limits
    - $HIGH_{DR} = 70\%$ of ports affected by draft limits
  - 2 scenarios for Capacity Tightness ($CT = \frac{TOT_q}{TOT_Q}$)
    - $LOW_{CT} = 30\%$ of ship capacity corresponds to the total demand
    - $HIGH_{CT} = 70\%$ of ship capacity corresponds to the total demand

SMALL INSTANCES

- 40 small instaces divided in 4 sets of 10 instances each
  - **Set 1**:
    - 15 ports
    - 3 ships
    - $(LOW_{DR} - LOW_{CT})$
  - **Set 2**:
    - 15 ports
    - 3 ships
    - $(HIGH_{DR} - LOW_{CT})$
  - **Set 3**:
    - 15 ports
    - 3 ships
    - $(LOW_{DR} - HIGH_{CT})$
  - **Set 4**:
    - 15 ports
    - 3 ships
    - $(HIGH_{DR} - HIGH_{CT})$

MEDIUM & LARGE INSTANCES

- 22 medium instances divided in 2 sets of 11 instances each
  - **Set 5**:
    - 25 ports
    - 5 ships
    - $(HIGH_{DR} - HIGH_{CT})$
  - **Set 6**:
    - 25 ports
    - 6 ships
    - $(HIGH_{DR} - HIGH_{CT})$
- 10 large instances in 1 set
  - **Set 7**:
    - 50 ports
    - 10 ships
    - $(HIGH_{DR} - HIGH_{CT})$

Mathematical model – Small instances

			COMPUTATIONAL TIME (s)								
SET	OF	MIP GAP	NO VI	VI1	VI2	VI3	VI4	VI1+VI4	VI2+VI4	VI3+VI4	ALL VI
1	306,995	0,004	26,639	23,212	26,605	46,984	18,354	63,357	20,229	41,095	48,738
2	306,995	0,004	29,152	52,730	36,653	28,255	20,592	20,056	19,376	96,213	53,668
3	442,242	379,151	352,766	379,151	347,327	352,058	352,184	280,978	338,016	403,024	351,720
4	289,879	0,005	4,24	2,916	4,202	3,23	1,701	1,746	1,67	2,949	1,78

Mathematical model – Medium instances

			COMPUTATIONAL TIME (s)								
SET	OF	MIP GAP	NO VI	1	2	3	4	1+4	2+4	3+4	ALL VI
5	323,21	0,01	27,356	23,303	26,467	25,437	17,769	14,885	17,810	41,249	48,854
6	307,063	0,01	15,288	29,916	15,79	19,023	17,912	15,741	17,909	22,854	27,141

Mathematical model – Large instances

			COMPUTATIONAL TIME (s)								
SET	OF	MIP GAP	NO VI	1	2	3	4	1+4	2+4	3+4	ALL VI
7	323,21	0,01	27,356	23,303	26,467	25,437	17,769	14,885	17,810	41,249	48,854

Matheuristic

	LNS		ILS	
SET	MIP GAP	TIME	MIP GAP	TIME
SMALL	323,21	0,01	27,356	23,303
MEDIUM	307,063	0,01	15,288	29,916
LARGE	300,292	0.004	3.243	4.904

Matheuristic VS Mathematical Model

	MATHEURISTIC		MATHEMATICAL MODEL	
SET	MIP GAP	TIME	MIP GAP	TIME
SMALL	323,21	0,01	27,356	23,303
MEDIUM	307,063	0,01	15,288	29,916
LARGE	300,292	0.004	3.243	4.904