Sampling & Interpolation

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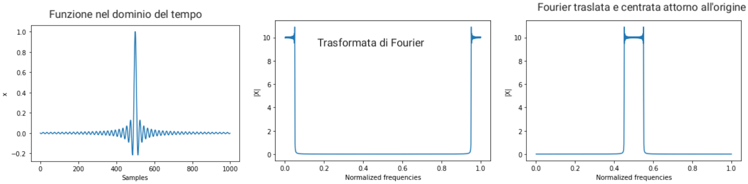
Analisi tempo-frequenza e multiscala – 01RMQNG

Symmetry of spectra

Trattiamo segnali simmetrici rispetto all'origine

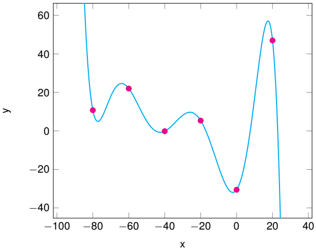
- Functions that are symmetric around the origin, (e.g. DFT of real signals) are represented with the symmetry around the middle index of the samples
- Use `scipy.fftpack.fftshift()` to center around the origin --> Utile per applicare dopo i filtri nella maniera corretta
- Normalized frequency : DFT spectrum with domain $1/N, \ldots, (N-1)/N, 1$.

$$x(t) = \text{sinc}(t) = \begin{cases} \frac{\sin(\pi t)}{\pi t} & t \neq 0 \\ 1 & t = 0 \end{cases}$$



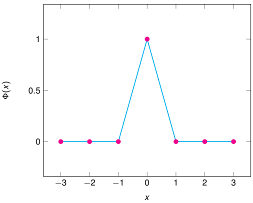
Method of getting values at positions in between the data points :

- samples $\{(x_i, y_i)\}_{i=1}^{m}$
- $\Delta = x_{i+1} - x_i$ for all $i \in \{1, \dots, m\}$
- function $\Phi(x) : \Phi(0) = 1, \Phi(k\Delta) = 0 \; \forall k \in \mathbb{N}$ and $k \neq 0$
- $y(x) = \sum_{k=-\infty}^{+\infty} y_k \Phi(x - k\Delta)$ <-- Interpolazione

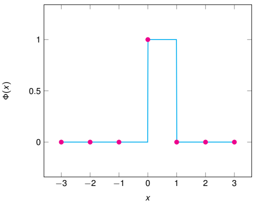


Varie Phi(x) che possiamo definire:

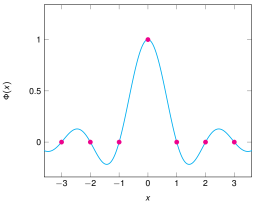
Linear interpolation



Zero-Order Hold



Sinc



Interpolation with sinc

Proprietà della funzione sinc

Sinc function

- $\Phi(x) = \text{sinc}(x/\Delta)$
- $\Phi(0) = 1$
- $\Phi(k\Delta) = \text{sinc(k)} = 0$ with $k \neq 0$
- Bandlimitedness :

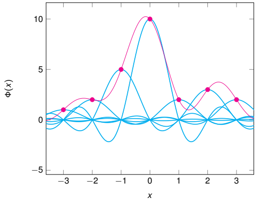
$$\text{sinc}(x) = \frac{1}{\pi}\int_0^{\pi} \cos(\omega x) \mathrm{d}\omega = \frac{\sin(\omega x)}{\pi x}\bigg|_0^{\pi} = \frac{\sin(\pi x)}{\pi x}$$

- Sinc is an infinite sum of cosine functions with frequencies in the range $\omega \in [0, \pi]$

Interpolation with sinc

Vogliamo applicare il teorema di campionamento

$$y(x) = \sum_{k=-\infty}^{+\infty} y_k \Phi(x - k\Delta)$$



Sampling & recovery

Sampling : process of converting a signal into a sequence of values

- Analog signal : $x(t)$
- Sample : $x_k = x(k\Delta)$ (or $x_k = \Delta x(k\Delta)$ which preserves energy)
- Interpolate : $\widehat{x}(t) = \sum_{k=-\infty}^{\infty} x_k \Phi(t - k\Delta)$

Recovery :

Il teorema di campionamento cerca di dare una risposta a questa domanda

- Can we perfectly recover an analog signal from its samples ? $\widehat{x}(t) = x(t)$?
- What rate of sampling ?

Enunciato del teorema di campionamento

"If a function $x(t)$ contains no frequencies higher than B hertz, it is completely determined by giving its ordinates at a series of points spaced $1/(2B)$ seconds apart."

Oggi vedremo l'applicazione del teorema e vedremo cosa succede se le ipotesi del teorema non vengono rispettate