An example of primal equilibrium

We consider a fisheries management problem borrowed from (Kall, Wallace, 1994).

Let z_t the biomass of fish stock available (state variable) and $x_t \in [0, 1]$ be the fraction of stock caught (control variable).

A possible state transition function is:

$$z_{t+1} = z_t - x_t z_t + \rho z_t \left(1 - \frac{z_t}{K}\right)$$

where ρ is a growth rate and K is the carrying capacity of the environment.

The objective is to maximize

$$\sum_{t=0}^{\infty} \beta^t z_t x_t$$

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If ρ is random, we may consider scenarios ξ_t^s for that parameter and make the problem stochastic.

To keep problem size under control, we need to limit the planning horizon T. However, we do not want $X_T = 1$. So, we look for a suitable terminal value function $Q(Z_{T+1})$.

Let us consider an average growth rate $\bar{\xi}$ and assume that, after T, we catch a fraction such that the population is stable:

$$x_t = \bar{\xi}\left(1 - \frac{z_{T+1}}{K}\right), \qquad t \geq T+1$$

Then, using the basic property of the geometric series we find:

$$Q(Z_{T+1}) = \sum_{t=T+1}^{\infty} \beta^{t-T-1} x_t z_t = \frac{\bar{\xi} z_{T+1}(1 - z_{T+1}/K)}{1-\beta}$$

By adding this term to the finite horizon cost, we obtain a nonlinear stochastic programming model that can be tackled, e.g., by progressive hedging.