

Politecnico di Torino
Financial Engineering-Exam Simulation 22-23
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Student number

All the answers must be clearly motivated, the numerical results are not sufficient.

Answers written with a pencil are null.

Exercise 1 (10 points)

You hold a portfolio of (vanilla, European-style) options written on the same stock share, whose price follows a geometric Brownian motion with drift 9% and volatility 25%. At present, the stock price is 30 dollars, and the risk-free rate, with continuous compounding, is 3%. The portfolio consists of:

1. A short position in 1000 put options, strike 27, maturing in three months
2. A long position in 500 call options, strike 30, maturing in four months
3. A short position in 1500 call options, strike 28, maturing in two months

- (a) How many stock shares do you need to make the portfolio delta-neutral?
(b) Can you also make the portfolio gamma-neutral by using stock shares? If so, explain how. Otherwise, how should you change the position in the last call to make the portfolio gamma neutral?

Exercise 2 (10 points) You hold 100 units of an asset with current price $S(0) = 100$. In order to hedge the change of this price in the next 3 months, you use two future contracts, with current futures price $F_A(0) = 60$ and $F_B(0) = 40$. We neglect issues related with marking-to-market of futures contracts. We know that the volatilities of the changes in asset prices over the three months are $\sigma_S = 4$, $\sigma_A = 3$, $\sigma_B = 5$, respectively (Note: by this we mean that $V(F_B(0.25) - F_B(0)) = 25$, etc.). The correlations are $\rho_{S,A} = 0.6$, $\rho_{S,B} = .0.7$, and $\rho_{A,B} = 0$ (again, they are correlations between changes in prices). Find the minimum variance hedge (assume infinite asset divisibility, i.e., you may take a non-integer position in any future contract).

Exercise 3 (12 points)

Consider a European call option and a European put option written on the same stock. The price of the stock today is 100 \$ and at the end of 3 months it can either move up by a factor of 1.1 or move down by a factor of 0.8. The risk-free interest rate is 10% per annum with continuous compounding. The strike price is 105 \$ both for the call and for the put option.

- (a) Find the price of the call option using the risk-neutral valuation method and the price of the put option using the replicating portfolio method.
- (b) Verify that the prices found in (a) satisfy the put-call parity.
- (c) Consider the straddle that can be constructed using the two options, draw a table that shows the profit that can be obtained and determine for what range of stock prices the straddle leads to a gain.