

Politecnico di Torino
Financial Engineering-Exam 02-01-2024
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Student number

Exercise 1 (10 points)

Consider a one-period (annual) model formed by a bond (paying a risk-free rate of 5% per year) and by two stocks with prices S^1 and S^2 evolving as follows:

$$S_0^1 = 10;$$

$$S_T^1(w) = \begin{cases} 12; & \text{if } w = w_1 \\ 10; & \text{if } w = w_2 \\ 6; & \text{if } w = w_3 \end{cases} \quad (0.1)$$

and

$$S_0^2 = 10;$$

$$S_T^2(w) = \begin{cases} 15; & \text{if } w = w_1 \\ 8; & \text{if } w \in \{w_2, w_3\} \end{cases} \quad (0.2)$$

with $P(w_1), P(w_2), P(w_3) > 0$ and $P(w_1) + P(w_2) + P(w_3) = 1$.

1. Establish if the market is free of arbitrage and complete.
2. Consider the derivative A with maturity T of one year and with

$$\Phi_A = \left(\frac{S_T^1 + S_T^2}{2} - 8 \right)^+ \quad (0.3)$$

and find its price.

3. Discuss whether the market formed only by the bond and by stock S^2 would remain free of arbitrage and complete.

Exercise 2 (10 points)

A portfolio consists of a 3-year zero-coupon bond with face value of 3000 \$ and a 5-year zero-coupon bond with face value of 6000 \$. The current yield on all bonds is 6% per annum.

- (a) Compute the duration of the portfolio (using continuous compounding).
- (b) Compute the percentage change in the value of the portfolio in the case of a 0.2% per annum decrease in yields.
- (c) Compare the result obtained in (b) with the *1st* order and *2nd* order approximations based on the use of duration and of convexity, and comment the results.

Exercise 3 (10 points)

Let $dS = \mu_t dt + \sigma_t dW$ and risk-free interest rate is 4% per annum (all rates are continuously compounded). (a) When $\sigma_t = 0.01$ for $t \in [0, 4]$ and

$$\mu_t = \begin{cases} 0.02t; & 0 \leq t \leq 2 \\ 0.01(10 - t); & 2 < t \leq 4 \end{cases} \quad (0.4)$$

compute the probability of having at $t = 4$ a profit greater or equal than 0.4.

(b) When $\mu_t = 0.01$ for $t \in [0, 4]$ and $\sigma_t = 0.2$ for $t \in [0, 4]$ compute the probability of having at $t = 4$ a profit greater than or equal to 0.4.

(c) Assuming $\mu_t = 0.03S_t$ and $\sigma_t = 0.01S_t$ for $t \in [0, 1]$, in years, i.e.

$$dS = 0.03S_t dt + 0.01S_t dW$$

and that the risk free rate is 2% per annum, price an European call option with strike price 100, maturing in one year, written on the stock $S(t)$ (no dividends) whose current price $S(0)$ is 98.