Financial Engineering

Springer Semester 2025

Lecturer: Patrizia Semeraro, Assistant: Tommaso Vanzan

Problem set 3

Topics: Forward and futures contracts

Exercise 1 An investor enters into a short forward contract to sell 10^5£ for US dollars at an exchange rate of 1.3 USD per pound. How much does the investor gain or lose if the exchange rate at the end of the contract is either 1.29 or 1.32?

Exercise 2 A trader enters into a short cotton forward contract for the delivery of 50000 pounds at 50 cents per pound. How much does the trader gain or lose if the cotton price at the end of the contract is 48.20 cents per pound? What about if the final price is 51.30 cents per pound?

Exercise 3 A trader enters into a short forward contract on 100 million Japanese yen. The forward exchange rate is 0.009 USD per yen. How much does the trader gain or lose if the exchange rate at the end of the contract is either 0.0084 yen or 0.0101 USD per yen?

Exercise 4 The price of gold is currently (December 2024) 2650$ per ounce. The forward price for delivery in 1 year is 2800$ per ounce. An arbitrageur can borrow money at 3% per annum. What should the arbitrageur do? Assume that there are no storing costs, and gold does not provide a recurrent income.

Exercise 5 Suppose that you enter into a short futures contract to sell silver for 32$ per ounce in July 2025. The size of the contract is 5000 ounces. The initial margin is 4000$ and the maintenance margin is 3000$. What change in the futures price will lead to a margin call? What happens if you do not meet the margin call?

Exercise 6 A trader buys two July futures contracts on orange juice. Each contract is for the delivery of 15000 pound, the current future price is 160 cents per pound, the initial margin is 6000$ per contract, and the maintenance margin is 4500$ per contract. What price change would lead to a margin call? Under what circumstances could 2000$ be withdrawn from the margin account?

Exercise 7 A company enters into a short futures contract to sell 5000 bushels of wheat for 750 cents per bushel. The initial margin is 3000$ and the maintenance margin is 2000$. What price change would lead to a margin call? Under what circumstances could 1500$ be withdrawn from the margin account?

Exercise 8 Suppose that in September 2021 a company takes a long position in a contract on May 2022 crude oil futures. It closes out its position in March 2022. The futures price (per barrel) is 48.3$ when it enters into the contract, 50.5$ when it closes out its position, and 49.1$ at the end of December 2021. One contract is for the delivery of 1000 barrels. What is company's total profi? When is it realized? How is it taxed if it is a hedger? And if it is a speculator? Assume that the company has a December 31 year end.

Exercise 9 Suppose that on October 24, 2022, a company sells one April 2023 live cattle futures contract. It closes out its position on January 21, 2023. The futures price (per pound) is 121.20 $ when it enters into the contract, 118.3$ when it closes out its position, and 118.8$ at the end of December 2022. One contract is for the delivery of 40000 pounds of cattle. What is the total profit? How is it taxed if the company is a hedger? And if it is a speculator? Assume that the company has a December 31 year end.

Exercise 10 Suppose that you enter into a 6-month forward contract on a non-dividend-paying stock when the stock price is 30$ and the risk-free interest rate (with continuous compounding) is 5% per annum. What is the forward price?

Exercise 11 A 1-year long forward contract on a non-dividen-paying stock is entered into when the stock price is 40$ and the risk-free rate of interest is 5% per annum with continuous compounding.

a) What are the forward price and the initial value of the forward contract?

b) Six months later, the price of the stock is 45$ and the risk-free interest rate is still 5%. What are the forward price and the value of the forward contract?

Exercise 12 The spot price of silver is 32$ per ounce. The storage cost are 0.24$ per ounce per year payable quarterly in advance. Assuming that the interest rates are 5% per annum for all maturities, calculate the futures price of silver for delivery in 9 months.

Exercise 13 Suppose that the price of a stock on 1 April 2000 turns out to be 10% lower than it was on 1 January 2000. Assuming that the risk-free rate is constant at 6%, what is the percentange drop of the forward price on 1 April 2000 as compared to that on 1 January 2000 for a forward contract with delivery on 1 October 2000?

Exercise 14 Consider a stock whose price on 1 January 2000 is 120$ and which will pay a dividend of 1$ on 1 July 2000 and 2$ on 1 October 2000. The interest rate is 12%. Is there an arbitrage opportunity if on 1 January 2000 the forward price for delivery of the stock on 1 November 2000 is 131$? If so, compute the arbitrage profit.

Exercise 15 Suppose that the price of a stock is 45$ at the beginning of the year, the risk-free rate is 6%, and a 2$ dividend is to be paid after half a year. For a long forward position with delivery in one year, find its value after 9 months if the stock price at that time turns out to be 49$, and if turns out to be 51$.

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Problem set 3

Topics: Forward and futures contracts

Solution 1 The payoff - and also the profit - from a short position in a forward contract on one unit of an asset is:

$$F(0,T) - S(T) \quad \text{or} \quad K - S_T$$

In this case:

$$F(0,T) = 1.30\,\$ \text{ per pound}$$

then if the exchange rate at the end of the contract is:

$$S(T) = 1.29\,\$ \text{ per pound}$$

the investor is obligated to sell pounds at 1.30 \$ when they are worth 1.29 \$ and he makes a gain of:

$$[F(0,T) - S(T)] \cdot 100\,000 = (1.30 - 1.29) \cdot 100\,000 = 1\,000\,\$$$

while if the exchange rate at the end of the contract is:

$$S(T) = 1.32\,\$ \text{ per pound}$$

the investor is obligated to sell pounds at 1.30 \$ when they are worth 1.32 \$ and he makes a negative gain, i.e. a loss, of:

$$[F(0,T) - S(T)] \cdot 100\,000 = (1.30 - 1.32) \cdot 100\,000 = -2\,000\,\$$$

Solution 2 The payoff - and also the profit - from a short position in a forward contract on one unit of an asset is:

$$F(0,T) - S(T) \quad \text{or} \quad K - S_T$$

In this case:

$$F(0,T) = 50 \text{ cents per pound}$$

then if the cotton price at the end of the contract is:

$$S(T) = 48.20 \text{ cents per pound}$$

the trader is obligated to sell at 50 cents per pound cotton that is worth 48.20 cents per pound and he makes a gain of:

$$[F(0,T) - S(T)] \cdot 50\,000 = (0.50 - 0.4820) \cdot 50\,000 = 900\,\$$$

while if the cotton price at the end of the contract is:

$$S(T) = 51.30 \text{ cents per pound}$$

the trader is obligated to sell at 50 cents per pound cotton that is worth 51.30 cents per pound and he makes a negative gain, i.e. a loss, of:

$$[F(0,T) - S(T)] \cdot 50\,000 = (0.50 - 0.5130) \cdot 50\,000 = -650\,\$$$

Solution 3 The payoff - and also the profit - from a short position in a forward contract on one unit of an asset is:

$$F(0,T) - S(T) \quad \text{or} \quad K - S_T$$

In this case:

$$F(0,T) = 0.0090\,\$ \text{ per yen}$$

then if the exchange rate at the end of the contract is:

$$S(T) = 0.0084\,\$ \text{ per yen}$$

the trader is obligated to sell yens at 0.0090 \$ per yen when they are worth 0.0084 \$ per yen and he makes a gain of:

$$[F(0,T) - S(T)] \cdot 100\,000 = (0.0090 - 0.0084) \cdot 100\,000\,000 = 60\,000\,\$$$

while if the exchange rate at the end of the contract is:

$$S(T) = 0.0101\,\$ \text{ per yen}$$

the trader is obligated to sell yens at 0.0090 \$ per yen when they are worth 0.0101 \$ per yen and he makes a negative gain, i.e. a loss, of:

$$[F(0,T) - S(T)] \cdot 100\,000 = (0.0090 - 0.0101) \cdot 100\,0001,000 = -110\,000\,\$$$

Solution 4 In this case the theoretical forward price is:

$$S(0)e^{rT} = 2650\,e^{0.03 \cdot 1} \approx 2730\,\$$$

and since his value is different from the forward price on the market - that is 2800 \$ - there is possibility of arbitrage. In particular, since $F(0,1) = 2800 > 2730$ an arbitrageur can:

- at time $t = 0$:

borrow 2600\$	+1200
purchase gold at spot price	-2600
enter into a short forward contract with delivery 1 year	0
	0

- at time $t = 1$:

clear the loan	$-2600\,e^{0.03 \cdot 1} = -2730$
sell the gold forward	+2800
	70

realizing a net profit of 70 \$ for each once of gold.

Solution 5 In this case there will be a margin call if there is a loss from the margin account equal to:

$$4000 - 3000 = 1000\,\$$$

and this happens when the price of silver increases by:

$$\frac{1000}{5000} = 0.20\,\$ \text{ per ounce}$$

Hence when it rises to:

$$32 + 0.20 = 32.2\,\$ \text{ per ounce}$$

if the margin call is not met the position is closed out.

Solution 6 In this case there will be a margin call if there is a loss on one contract equal to:

$$6000 - 4500 = 1500\,\$$$

and this happens when the futures price of orange juice decreases by 1500 \$, i.e. by:

$$\frac{1500}{15000} = 0.10\,\$ \text{ per pound}$$

hence when it falls to:

$$160 - 10 = 150\,\$ \text{ per pound}$$

The sum 2000 \$ can be withdraw from the margin account if there is a gain of:

$$\frac{2000}{2} = 1000\,\$$$

on one contract. This happens when the futures price of orange juice increases by 1000 \$, i.e. by:

$$\frac{1000}{15000} = 0.0667\,\$ \text{ per pound}$$

hence when it rises to:

$$160 + 6.67 = 166.67 \text{ cents per pound}$$

Solution 7 In this case there will be a margin call if there is a loss on one contract equal to:

$$3000 - 2000 = 1000\,\$$$

and this happens when the futures price of wheat futures increases by 1000 \$, i.e. by:

$$\frac{1000}{5000} = 0.20\,\$ \text{ per bushel}$$

hence when it rises to:

$$7.50 + 0.20 = 7.70\,\$ \text{ per bushel}$$

The sum 1500 \$ can be withdraw from the margin account if there is a gain of 500 \$ on contract and this happens when price of wheat futures decrease by 500 \$, i.e. by:

$$\frac{500}{5000} = 0.1\,\$ \text{ per bushel}$$

hence when it falls to:

$$7.50 - 0.1 = 7.40 \text{ cents per bushel}$$

Solution 8 The company's total profit is:

$$(50.50 - 48.30) \cdot 1000 = 2200\,\$$$

of which:

$$(49.10 - 48.30) \cdot 1000 = 800\,\$ \text{ realized in 2021}$$
$$(50.50 - 49.10) \cdot 1000 = 1400\,\$ \text{ realized in 2022}$$

If the company is a hedger it is taxed on the whole profit of 2200 \$ in 2022, while if it is a speculator it is taxed on 800 \$ in 2021 and on 1400 \$ in 2022.

Solution 9 The company's total profit is:

$$(1.2120 - 1.1830) \cdot 40000 = 1160\,\$$$

of which:

$$\begin{aligned}(1.2120 - 1.1880) \cdot 40000 &= 960\,\$ \text{ realized in 2022}\\ (1.1880 - 1.1830) \cdot 40000 &= 200\,\$ \text{ realized in 2023}\end{aligned}$$

If the company is a hedger it is taxed on the whole profit of 1160 $ in 2023, while if it is a speculator it is taxed on 960 $ in 2022 and on 200 $ in 2023.

Solution 10 The forward price in the case of a non dividend paying stock is:

$$F(0,T) = S(0)e^{rt} \quad \text{or} \quad F_0 = S_0 e^{rt}$$

and in this example:

$$F\left(0, \frac{6}{12}\right) = 30e^{0.05\frac{6}{12}} = 30.76$$

Solution 11

a) The forward price in the case of a non dividend paying stock is:

$$F(0,T) = S(0)e^{rt} \quad \text{or} \quad F_0 = S_0 e^{rt}$$

and in this example:

$$F(0,1) = 40e^{0.05 \cdot 1} = 42.05$$

while the initial value of the forward contract is always $V(0) = 0$.

b) After six months the forward price is:

$$F(t,T) = S(t)e^{r(T-t)}$$

that is:

$$F\left(\frac{6}{12}, 1\right) = S\left(\frac{6}{12}\right)e^{0.05\left(1-\frac{6}{12}\right)} = 45e^{0.05\frac{6}{12}} = 46.14$$

while the value of the forward contract is:

$$V(t) = \left[F(t,T) - F(0,T)\right]e^{-r(T-t)}$$

that is:

$$V\left(\frac{6}{12}\right) = \left[F\left(\frac{6}{12}, 1\right) - F(0,1)\right]e^{-0.05\left(1-\frac{6}{12}\right)} = (46.14 - 42.05)e^{-0.05\left(\frac{6}{12}\right)} = 3.99$$

Solution 12 The future price in the case of an investment asset with storage costs is:

$$F(0,T) = (S(0) + U)\,e^{rT}$$

where U is the present value of the storage costs. In this case we have:

$$U = 0.06 + 0.06e^{-0.05\frac{3}{12}} + 0.06e^{-0.05\frac{6}{12}} = 0.178$$

hence the futures price of silver is:

$$F\left(0, \frac{9}{12}\right)(32 + 0.178)e^{0.05\frac{9}{12}} = 33.4\,\$ \text{ per ounce}$$

Solution 13 Denoting by $S(0)$ the price of the stock on 1st January 2000, the forward price on this date for a forward contract with delivery on 1st October 2000 is:

$$F(0,T) = S(0)e^{rT}$$

that is:

$$F\left(0, \frac{9}{12}\right) = S(0)e^{0.06\frac{9}{12}}$$

while the forward price on 1st April 2000 for a forward contract with delivery on 1st October 2000 is:

$$F(t,T) = S(t)e^{r(T-t)}$$

that is:

$$F\left(\frac{3}{12}, \frac{9}{12}\right) = S\left(\frac{3}{12}\right)e^{0.06\left(\frac{9}{12}-\frac{3}{12}\right)} = 0.9S(0)e^{0.06\frac{6}{12}}$$

Therefore the percentage drop of the forward price is:

$$\frac{F\left(0,\frac{9}{12}\right) - F\left(\frac{3}{12},\frac{9}{12}\right)}{F\left(0,\frac{9}{12}\right)} = \frac{S(0)e^{0.06\frac{9}{12}} - 0.9S(0)e^{0.06\frac{6}{12}}}{S(0)e^{0.06\frac{9}{12}}} = 0.1134 = 11.34\%$$

Solution 14 The theoretical forward price for delivery of the stock on 1st November 2000 is:

$$F(0,T) = (S(0) - I)e^{rT}$$

where I is the present value of the dividends, that is:

$$I = 1 \cdot e^{-0.12\frac{6}{12}} + 2 \cdot e^{-0.12\frac{9}{12}} = 2.77$$

therefore:

$$F\left(0, \frac{10}{12}\right) = (120 - 2.77)e^{0.12\frac{10}{12}} = 129.56$$

and since this value is less than the quoted forward price of 131 \$ there is an arbitrage opportunity that can be realised as follows:

- on 1st January 2000:

borrow 120\$	+120
buy one stock	-120
enter into a short forward position	0
	0

- on 1st July 2000:

collect the first dividend	+1
invest the dividend risk free	-1
	0

- on 1st October 2000:

collect the second dividend	+2
invest the dividend risk free	-2
	0

- on 1st November 2000:

clear the loan	$-120e^{0.12\frac{10}{12}} = -132.62$
sell the stock forward	$+131$
collect the result of the investment of the dividends	$1 \cdot e^{0.12\frac{4}{12}} + 2 \cdot e^{0.12\frac{1}{12}} = 3.06$
	1.44

Solution 15 The initial forward price is:

$$F(0,T) = (S(0) - I)e^{rT}$$

that is:

$$F(0,1) = \left(45 - 2e^{-0.06\frac{1}{2}}\right) e^{0.06} = 45.72$$

then if $S\left(\frac{9}{12}\right) = 49$ we have that the forward price at $t = \frac{9}{12}$ is:

$$F(t,T) = S(t)e^{r(T-t)}$$

that is:

$$F\left(\frac{9}{12}, 1\right) = S\left(\frac{9}{12}\right) e^{0.06\left(1-\frac{9}{12}\right)} = 49e^{0.06\frac{3}{12}} = 49.74$$

and the value of the forward contract is:

$$V(t) = \left[F(t,T) - F(0,T)\right] e^{-r(T-t)}$$

that is:

$$V\left(\frac{9}{12}\right) = \left[F\left(\frac{9}{12}, 1\right) - F(0,1)\right] e^{-0.06\left(1-\frac{9}{12}\right)} = (49.74 - 45.72)e^{-0.06\left(\frac{3}{12}\right)} = 3.96$$

Instead, if $S\left(\frac{9}{12}\right) = 51$, we have that the forward price at $t = \frac{9}{12}$ is:

$$F\left(\frac{9}{12}, 1\right) = S\left(\frac{9}{12}\right) e^{0.06\left(1-\frac{9}{12}\right)} = 51e^{0.06\frac{3}{12}} = 51.77$$

and the value of the forward contract at $t = \frac{9}{12}$ is:

$$V\left(\frac{9}{12}\right) = \left[F\left(\frac{9}{12}, 1\right) - F(0,1)\right] e^{-0.06\left(1-\frac{9}{12}\right)} = (51.77 - 45.72)e^{-0.06\left(\frac{3}{12}\right)} = 5.96$$

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Topics: Forward and futures contracts

Exercise 1 An investor enters into a short forward contract to sell $10^5\,£$ for US dollars at an exchange rate of 1.3 USD per pound. How much does the investor gain or lose if the exchange rate at the end of the contract is either 1.29 or 1.32?

Solution 1 The payoff - and also the profit - from a short position in a forward contract on one unit of an asset is:

$$F(0,T) - S(T) \quad \text{or} \quad K - S_T$$

In this case:

$$F(0,T) = 1.30\,\$ \text{ per pound}$$

then if the exchange rate at the end of the contract is:

$$S(T) = 1.29\,\$ \text{ per pound}$$

the investor is obligated to sell pounds at 1.30 \$ when they are worth 1.29 \$ and he makes a gain of:

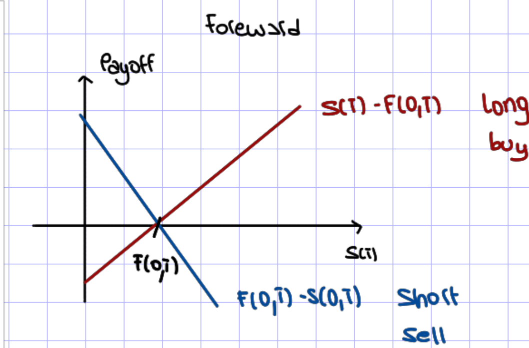
$$[F(0,T) - S(T)] \cdot 100\,000 = (1.30 - 1.29) \cdot 100\,000 = 1\,000\,\$$$

while if the exchange rate at the end of the contract is:

$$S(T) = 1.32\,\$ \text{ per pound}$$

the investor is obligated to sell pounds at 1.30 \$ when they are worth 1.32 \$ and he makes a negative gain, i.e. a loss, of:

$$[F(0,T) - S(T)] \cdot 100\,000 = (1.30 - 1.32) \cdot 100\,000 = -2\,000\,\$$$



Exercise 2 A trader enters into a short cotton forward contract for the delivery of 50000 pounds at 50 cents per pound. How much does the trader gain or lose if the cotton price at the end of the contract is 48.20 cents per pound? What about if the final price is 51.30 cents per pound?

Solution 2 The payoff - and also the profit - from a short position in a forward contract on one unit of an asset is:

$$F(0,T) - S(T) \quad \text{or} \quad K - S_T$$

In this case:

$$F(0,T) = 50 \text{ cents per pound}$$

then if the cotton price at the end of the contract is:

$$S(T) = 48.20 \text{ cents per pound}$$

the trader is obligated to sell at 50 cents per pound cotton that is worth 48.20 cents per pound and he makes a gain of:

$$[F(0,T) - S(T)] \cdot 50\,000 = (0.50 - 0.4820) \cdot 50\,000 = 900\,\$$$

while if the cotton price at the end of the contract is:

$$S(T) = 51.30 \text{ cents per pound}$$

the trader is obligated to sell at 50 cents per pound cotton that is worth 51.30 cents per pound and he makes a negative gain, i.e. a loss, of:

$$[F(0,T) - S(T)] \cdot 50\,000 = (0.50 - 0.5130) \cdot 50\,000 = -650\,\$$$

$F(0,T)$ $\quad (F(0,T) - S(T)) \cdot 100 \cdot 10^6$

Exercise 3 A trader enters into a short forward contract on 100 million Japanese yen. The forward exchange rate is 0.009 USD per yen. How much does the trader gain or lose if the exchange rate at the end of the contract is either 0.0084 yen or 0.0101 USD per yen?

S_1 $\quad S_2$

Solution 3 The payoff - and also the profit - from a short position in a forward contract on one unit of an asset is:

$$F(0,T) - S(T) \quad \text{or} \quad K - S_T$$

In this case:

$$F(0,T) = 0.0090\,\$ \text{ per yen}$$

then if the exchange rate at the end of the contract is:

$$S(T) = 0.0084\,\$ \text{ per yen}$$

the trader is obligated to sell yens at 0.0090 $ per yen when they are worth 0.0084 $ per yen and he makes a gain of:

$$[F(0,T) - S(T)] \cdot 100\,000 = (0.0090 - 0.0084) \cdot 100\,000\,000 = 60\,000\,\$ \checkmark$$

while if the exchange rate at the end of the contract is:

$$S(T) = 0.0101\,\$ \text{ per yen}$$

the trader is obligated to sell yens at 0.0090 $ per yen when they are worth 0.0101 $ per yen and he makes a negative gain, i.e. a loss, of:

$\nearrow 10^6$

$$[F(0,T) - S(T)] \cdot 100\,000 = (0.0090 - 0.0101) \cdot 100\,0001,000 = -110\,000\,\$$$

Exercise 4 The price of gold is currently (December 2024) 2650$ per ounce. The forward price for delivery in 1 year is 2800$ per ounce. An arbitrageur can borrow money at 3% per annum. What should the arbitrageur do? Assume that there are no storing costs, and gold does not provide a recurrent income.

Solution 4 In this case the theoretical forward price is:

$$(*)\, S(0)e^{rT} = 2650\, e^{0.03 \cdot 1} \approx 2730\,\$ \checkmark \quad \text{Prezzo che impedisce arbitrag.}$$

and since his value is different from the forward price on the market - that is 2800 $ - there is possibility of arbitrage. In particular, since $F(0,1) = 2800 > 2730$ an arbitrageur can:

- at time $t = 0$: Confronto con il mercato

borrow ~~2600$~~ 2650: presi in prestito	~~+1200~~ +2650
purchase gold at spot price compri 1 ounce	~~−2600~~ −2650
enter into a short forward contract with delivery 1 year	0
	0

- at time $t = 1$:

clear the loan	$-2600\, e^{0.03 \cdot 1} = -2730$ → rimborso del prestito (*)
sell the gold forward	+2800 → vendo ounce comprata t=0
	70

realizing a net profit of 70 $ for each once of gold.

Cosa dovrebbe fare l'arbitraggio?

Devi confrontare il forward price teorico (no-arbitrage) con quello di mercato, se sono diversi costruire un arbitraggio

Forward teorico $F(0,T) = S(0)e^{rT}$

Idea dell'arbitraggio è:
- compro ora a poco
- vendo dopo a tanto

Sell $F(0,T) - S(T)$ — prezzo iniziale

Exercise 5 Suppose that you enter into a short futures contract to sell silver for 32\$ per ounce in July 2025. The size of the contract is 5000 ounces. The initial margin is 4000\$ and the maintenance margin is 3000\$. What change in the futures price will lead to a margin call? What happens if you do not meet the margin call?

Size contract: quante ne prendi

garanzia o deposito o cauzione

Solution 5 In this case there will be a margin call if there is a loss from the margin account equal to:

$$4000 - 3000 = 1000\,\$$$ ⇒ perdita max prima della margin call

and this happens when the price of silver increases by:

$$\frac{1000}{5000} = 0.20\,\$ \text{ per ounce}$$ ⇒ $\Delta F = \frac{PnL}{a}$

Hence when it rises to:

$$32 + 0.20 = 32.2\,\$ \text{ per ounce}$$ ⇒ prezzo iniziale + ΔF allora hai una margin call

quando si arriva sotto a 3000 si verifica una margin call: il broker ti dice che devi mettere altri soldi sul conto

if the margin call is not met the position is closed out.

Nel futures ogni variazione di prezzo genera profit/loss giornaliero

Short futures $PnL = -a \cdot \Delta F$ (variazione di valore; size contract)

$\Delta F > 0$: perdi

$\Delta F < 0$: guadagni

$\Delta F = F_{nuovo} - F_{vecchio}$: variazione del prezzo x ounce

+ realizzi la perdita maturata fino a quel momento

LONG

Exercise 6 A trader buys two July futures contracts on orange juice. Each contract is for the delivery of 15000 pound, the current future price is 160 cents per pound, the initial margin is 6000\$ per contract, and the maintenance margin is 4500\$ per contract. What price change would lead to a margin call? Under what circumstances could 2000\$ be withdrawn from the margin account?

Solution 6 In this case there will be a margin call if there is a loss on one contract equal to:

$$6000 - 4500 = 1500\,\$$$

and this happens when the futures price of orange juice decreases by 1500 \$, i.e. by:

$$\frac{1500}{15000} = 0.10\,\$ \text{ per pound}$$

hence when it falls to:

$$160 - 10 = 150\,\$ \text{ per pound}$$ ✓ → prezzo che fa scattare margin call

The sum 2000\$ can be withdraw from the margin account if there is a gain of:

$$\frac{2000}{2} = 1000\,\$$$ → Ho 2 contratti!

on one contract. This happens when the futures price of orange juice increases by 1000\$, i.e. by:

$$\frac{1000}{15000} = 0.0667\,\$ \text{ per pound}$$ → se sale di 0.0667 posso prelevare

Posso prelevare se il prezzo sale

hence when it rises to:

$$160 + 6.67 = 166.67 \text{ cents per pound}$$

$a :=$ size contract $= 15000$

$\Delta F = F_{nuovo} - F_{vecchio} = -160$

$PnL = 6000 - 4500 = 1500$ max perdita pre margin call

$\Delta F = \frac{PnL}{a} = 0.1$ $16 - 0.1 = F_{nuovo} = 1.5$

$\Delta F < 0$ perchè perdo

NB la margin call avviene se il prezzo scende sell   q   futuro

Exercise 7 A company enters into a short futures contract to sell 5000 bushels of wheat for 750 cents per bushel. The initial margin is 3000$ and the maintenance margin is 2000$. What price change would lead to a margin call? Under what circumstances could 1500$ be withdrawn from the margin account?

Solution 7 In this case there will be a margin call if there is a loss on one contract equal to:

$$3000 - 2000 = 1000\,\$ \quad ✓$$

and this happens when the futures price of wheat futures increases by 1000 \$, i.e. by:

$$\frac{1000}{5000} = 0.20\,\$ \text{ per bushel} \quad ✓$$

hence when it rises to:

$$7.50 + 0.20 = 7.70\,\$ \text{ per bushel} \quad ✓$$

The sum 1500 \$ can be withdraw from the margin account if there is a gain of 500 \$ on contract and this happens when price of wheat futures decrease by 500 \$, i.e. by:

$$\frac{500}{5000} = 0.1\,\$ \text{ per bushel}$$

hence when it falls to:

$$7.50 - 0.1 = 7.40 \text{ cents per bushel}$$

$3000 - 2000 = 1000$ margin call

$\frac{PnL}{q} = \Delta F = \frac{1000}{5000} = 0.2$

$F_{nuovo} = 7.7$

SHORT: guadagni se il prezzo scende

Primo livello prelevabile +500

→ Di quanto scende x guadagnare

Cosa fa RIC? $f(0,T) :=$ valore contratto a $t=0$ $\Rightarrow 5000 \cdot 750 = 37500$

$f(1,T) :=$ valore contratto a $t=1$

SHORT $PnL = -(f(1,T) - f(0,T))$

•) Margin call: $3000 - (f(1,T) - f(0,T)) < 2000$

$3000 - f(1,T) + f(0,T) - 2000 > f(1,T) = 38500$

$F_{nuovo} = \frac{f(1,T)}{q} = 7.7$

•) 500: $3000 - (f(1,T) - f(0,T)) > 4500 = (3000 + 1500) \Rightarrow F_{nuovo} = 7.2$

buy

Exercise 8 Suppose that in September 2021 a company takes a long position in a contract on May 2022 crude oil futures. It closes out its position in March 2022. The futures price (per barrel) is 48.3$ when it enters into the contract, 50.5$ when it closes out its position, and 49.1$ at the end of December 2021. One contract is for the delivery of 1000 barrels. What is company's total profi? When is it realized? How is it taxed if it is a hedger? And if it is a speculator? Assume that the company has a December 31 year end.

Solution 8 The company's total profit is:

$$(50.50 - 48.30) \cdot 1000 = 2200\,\$ \quad ✓$$

prezzo uscita - prezzo ingresso

of which:

$$\left.\begin{aligned} (49.10 - 48.30) \cdot 1000 &= 800\,\$ \text{ realized in 2021} \\ (50.50 - 49.10) \cdot 1000 &= 1400\,\$ \text{ realized in 2022} \end{aligned}\right\}$$

Come ha realizzato il profitto

If the company is a hedger it is taxed on the whole profit of 2200 \$ in 2022, while if it is a speculator it is taxed on 800 \$ in 2021 and on 1400 \$ in 2022.

Hedger: il profitto e' tassato quando l'operazione e' conclusa ⇒ nel 2022

Spector: il profitto e' tassato quando matura 800 in 2021 e 1400 in 2022

Exercise 9 Suppose that on October 24, 2022, a company sells one April 2023 live cattle futures contract. It closes out its position on January 21, 2023. The futures price (per pound) is 121.20 \$ when it enters into the contract, 118.3\$ when it closes out its position, and 118.8\$ at the end of December 2022. One contract is for the delivery of 40000 pounds of cattle. What is the total profit? How is it taxed if the company is a hedger? And if it is a speculator? Assume that the company has a December 31 year end.

Solution 9 The company's total profit is:

$$(1.2120 - 1.1830) \cdot 40000 = 1160\,\$ \checkmark$$

of which:

$$(1.2120 - 1.1880) \cdot 40000 = 960\,\$ \text{ realized in 2022}$$
$$(1.1880 - 1.1830) \cdot 40000 = 200\,\$ \text{ realized in 2023}$$

If the company is a hedger it is taxed on the whole profit of 1160 \$ in 2023, while if it is a speculator it is taxed on 960 \$ in 2022 and on 200 \$ in 2023.

$(121.20 - 118.3) \cdot 40000$ ✓

2022 $(121.20 - 118.8) \cdot 40000$ ✓
2023 $(118.8 - 118.3) \cdot 40000$ ✓

Hedger 2023 Spector 2022 e 2023

Exercise 10 Suppose that you enter into a 6-month forward contract on a non-dividend-paying stock when the stock price is 30\$ and the risk-free interest rate (with continuous compounding) is 5% per annum. What is the forward price?

Solution 10 The forward price in the case of a non dividend paying stock is:

$$F(0,T) = S(0)e^{rt} \quad \text{or} \quad F_0 = S_0 e^{rt}$$

and in this example:

$$F\left(0, \frac{6}{12}\right) = 30e^{0.05\frac{6}{12}} = 30.76 \checkmark$$

Exercise 11 A 1-year long forward contract on a non-dividen-paying stock is entered into when the stock price is 40\$ and the risk-free rate of interest is 5% per annum with continuous compounding.

a) What are the forward price and the initial value of the forward contract?

b) Six months later, the price of the stock is 45\$ and the risk-free interest rate is still 5%. What are the forward price and the value of the forward contract?

Solution 11

a) The forward price in the case of a non dividend paying stock is:

$$F(0,T) = S(0)e^{rt} \quad \text{or} \quad F_0 = S_0 e^{rt}$$

and in this example:

$$F(0,1) = 40e^{0.05 \cdot 1} = 42.05 \checkmark$$

while the initial value of the forward contract is always $V(0) = 0$.

Il forward e' costruito in modo da non richiedere nessun pagamento iniziale

b) After six months the forward price is:

$$F(t,T) = S(t)e^{r(T-t)}$$

that is:

$$F\left(\frac{6}{12}, 1\right) = S\left(\frac{6}{12}\right) e^{0.05\left(1-\frac{6}{12}\right)} = 45e^{0.05\frac{6}{12}} = 46.14$$

while the value of the forward contract is:

$$V(t) = [F(t,T) - F(0,T)]\, e^{-r(T-t)}$$

that is:

$$V\left(\frac{6}{12}\right) = \left[F\left(\frac{6}{12}, 1\right) - F(0,1)\right] e^{-0.05\left(1-\frac{6}{12}\right)} = (46.14 - 42.05)e^{-0.05\left(\frac{6}{12}\right)} = 3.99$$

Exercise 12 The spot price of silver is 32\$ per ounce. The storage cost are 0.24\$ per ounce per year payable quarterly in advance. Assuming that the interest rates are 5% per annum for all maturities, calculate the futures price of silver for delivery in 9 months.

Solution 12 The future price in the case of an investment asset with storage costs is:

$$F(0,T) = (S(0) + U)\,e^{rT}$$

where U is the present value of the storage costs. In this case we have:

$$U = 0.06 + 0.06e^{-0.05\frac{3}{12}} + 0.06e^{-0.05\frac{6}{12}} = 0.178$$

hence the futures price of silver is:

$$F\left(0, \frac{9}{12}\right)(32 + 0.178)e^{0.05\frac{9}{12}} = 33.4\,\$ \text{ per ounce}$$

$$F\left(0, \frac{9}{12}\right) = S(0)\,e^{-0.05\frac{9}{12}}$$

$S_0 + U$

32

Pagato ogni 3 mesi:

$$\frac{0.24}{4} = 0.06$$

Exercise 13 Suppose that the price of a stock on 1 April 2000 turns out to be 10% lower than it was on 1 January 2000. Assuming that the risk-free rate is constant at 6%, what is the percentange drop of the forward price on 1 April 2000 as compared to that on 1 January 2000 for a forward contract with delivery on 1 October 2000?

Solution 13 Denoting by $S(0)$ the price of the stock on 1st January 2000, the forward price on this date for a forward contract with delivery on 1st October 2000 is:

$$F(0,T) = S(0)e^{rT}$$

that is:

$$F\left(0, \frac{9}{12}\right) = S(0)e^{0.06\frac{9}{12}}$$

while the forward price on 1st April 2000 for a forward contract with delivery on 1st October 2000 is:

$$F(t,T) = S(t)e^{r(T-t)}$$

that is:

$$F\left(\frac{3}{12}, \frac{9}{12}\right) = S\left(\frac{3}{12}\right)e^{0.06\left(\frac{9}{12}-\frac{3}{12}\right)} = 0.9S(0)e^{0.06\frac{6}{12}}$$

Therefore the percentage drop of the forward price is:

$$\frac{F\left(0, \frac{9}{12}\right) - F\left(\frac{3}{12}, \frac{9}{12}\right)}{F\left(0, \frac{9}{12}\right)} = \frac{S(0)e^{0.06\frac{9}{12}} - 0.9S(0)e^{0.06\frac{6}{12}}}{S(0)e^{0.06\frac{9}{12}}} = 0.1134 = 11.34\%$$

Exercise 14 Consider a stock whose price on 1 January 2000 is 120\$ and which will pay a dividend of 1\$ on 1 July 2000 and 2\$ on 1 October 2000. The interest rate is 12%. Is there an arbitrage opportunity if on 1 January 2000 the forward price for delivery of the stock on 1 November 2000 is 131\$? If so, compute the arbitrage profit.

Solution 14 The theoretical forward price for delivery of the stock on 1st November 2000 is:

$$F(0,T) = (S(0) - I)e^{rT}$$

where I is the present value of the dividends, that is:

$$I = 1 \cdot e^{-0.12\frac{6}{12}} + 2 \cdot e^{-0.12\frac{9}{12}} = 2.77$$

therefore:

$$F\left(0, \frac{10}{12}\right) = (120 - 2.77)e^{0.12\frac{10}{12}} = 129.56$$

and since this value is less than the quoted forward price of 131 \$ there is an arbitrage opportunity that can be realised as follows:

- on 1st January 2000:

borrow 120\$	+120
buy one stock	-120
enter into a short forward position	0
	0

- on 1st July 2000:

collect the first dividend	+1
invest the dividend risk free	-1
	0

- on 1st October 2000:

collect the second dividend	+2
invest the dividend risk free	-2
	0

- on 1st November 2000:

clear the loan	$-120e^{0.12\frac{10}{12}} = -132.62$
sell the stock forward	+131
collect the result of the investment of the dividends	$1 \cdot e^{0.12\frac{4}{12}} + 2 \cdot e^{0.12\frac{1}{12}} = 3.06$
	1.44

Exercise 15 Suppose that the price of a stock is 45$ at the beginning of the year, the risk-free rate is 6%, and a 2$ dividend is to be paid after half a year. For a long forward position with delivery in one year, find its value after 9 months if the stock price at that time turns out to be 49$, and if turns out to be 51$.

Solution 15 The initial forward price is:

$$F(0,T) = (S(0) - I)e^{rT}$$

that is:

$$F(0,1) = \left(45 - 2e^{-0.06\frac{1}{2}}\right)e^{0.06} = 45.72$$

then if $S\left(\frac{9}{12}\right) = 49$ we have that the forward price at $t = \frac{9}{12}$ is:

$$F(t,T) = S(t)e^{r(T-t)}$$

that is:

$$F\left(\frac{9}{12},1\right) = S\left(\frac{9}{12}\right)e^{0.06\left(1-\frac{9}{12}\right)} = 49e^{0.06\frac{3}{12}} = 49.74$$

and the value of the forward contract is:

$$V(t) = [F(t,T) - F(0,T)]\,e^{-r(T-t)}$$

that is:

$$V\left(\frac{9}{12}\right) = \left[F\left(\frac{9}{12},1\right) - F(0,1)\right]e^{-0.06\left(1-\frac{9}{12}\right)} = (49.74 - 45.72)e^{-0.06\left(\frac{3}{12}\right)} = 3.96$$

Instead, if $S\left(\frac{9}{12}\right) = 51$, we have that the forward price at $t = \frac{9}{12}$ is:

$$F\left(\frac{9}{12},1\right) = S\left(\frac{9}{12}\right)e^{0.06\left(1-\frac{9}{12}\right)} = 51e^{0.06\frac{3}{12}} = 51.77$$

and the value of the forward contract at $t = \frac{9}{12}$ is:

$$V\left(\frac{9}{12}\right) = \left[F\left(\frac{9}{12},1\right) - F(0,1)\right]e^{-0.06\left(1-\frac{9}{12}\right)} = (51.77 - 45.72)e^{-0.06\left(\frac{3}{12}\right)} = 5.96$$