✓ **Exercise 1** (10 points)
A portfolio consists of a 4-year zero-coupon bond with face value of 10000 $ and a 3-year coupon bond that pays a coupon on 2% ona annual basis with face value of 3000 $. The current yield on all bonds is 5% per annum.
(*a*) Compute the duration of the portfolio (using continuous compounding).
(*b*) Compute the percentage change in the value of the portfolio in the case of a 0.2% per annum decrease in yields.
(*c*) Compare the result obtained in (*b*) with the 1*st* order and 2*nd* order approximations based on the use of duration and of convexity, and comment the results. Specifically explain what duration and convexity are.

(a) DURATION of the PORTFOLIO

The yield is r

4-year ZCB: $B(0,T)_1 = F \cdot e^{-r \cdot T} = 10'000 \cdot e^{-0.05 \cdot 4} = 8187.3075$

$$\text{Duration}_1 = \frac{4 \cdot 10'000 \cdot e^{-0.05 \cdot 4}}{10'000 \cdot e^{-0.05 \cdot 4}} = 4 \longrightarrow$$ The duration of a ZCB is simply its maturity

3-year CB: $B(0,T)_2 = \sum_{i=1}^{n-1} c_i e^{-r t_i} + (c_n + F) e^{-r t_n} = 60 \cdot e^{-0.05 \cdot 1} + 60 \cdot e^{-0.05 \cdot 2} + (60 + 3000) e^{-0.05 \cdot 3} = 2745.1304$

annual coupon $c = 0.02 \cdot 3'000 = 60$

$$\text{Duration}_2 = \frac{1 \cdot 60 e^{-0.05} + 2 \cdot 60 e^{-0.05 \cdot 2} + 3 \cdot 3060 e^{-0.05 \cdot 3}}{2745.1304} = 2.9386 \longrightarrow D(y) = \frac{\sum_{i=1}^{n} t_i \cdot c_i \cdot e^{-y t_i}}{\sum_{i=1}^{n} c_i \cdot e^{-y t_i}}$$

The duration of the portfolio is the wheited mean of D_1 and D_2 with market value wheigh $\frac{B(0,T)_1}{B(0,T)_{tot}}$ & $\frac{B(0,T)_2}{B(0,T)_{tot}}$

$B(0,T)_{tot} = B(0,T)_1 + B(0,T)_2 = 10932.4379$

$$\text{Duration}_{tot} = \frac{8187.3075}{B(0,T)_{tot}} \cdot 4 + \frac{2745.1304}{B(0,T)_{tot}} \cdot 2.9386 = 3.7335$$ ✓

(b) PERCENTAGE CHANGE

$B(0,T)_1' = 10'000 \cdot e^{-0.048 \cdot 4} = 8253.0687$

$(5\cdot3 - 0.2\cdot3)\%$

$B(0,T)_2' = 60 \cdot e^{-0.048} + 60 e^{-0.096} + 3060 e^{-0.144} = 2761.3124$

$B(0,T)_{tot}' = 11014.3811$

$$\frac{\Delta B}{B} = \frac{B(0,T)_{tot}' - B(0,T)_{tot}}{B(0,T)_{tot}} = 0.7495\%$$ ✓

(c) FIRST & SECOND ORDER APPROXIMATION

$$B(y) = B(y_0) - D(y_0) B(y_0) \Delta y + \frac{1}{2} C(y_0) B(y_0) \Delta y^2$$

$$D(y) = \frac{\sum_{i=1}^{n} t_i c_i e^{-y t_i}}{\sum_{i=1}^{n} c_i e^{-y t_i}} \qquad C(y) = \frac{\sum_{i=1}^{n} t_i^2 c_i e^{-y t_i}}{\sum_{i=1}^{n} c_i e^{-y t_i}}$$

$$\frac{\Delta B(0,T)}{B(0,T)} \simeq -D(y) \cdot \Delta y$$

$\Delta y = -0.2\% = -0.002$ $D(y) = 3.7335$ $\frac{\Delta B(0,T)}{B(0,T)} \simeq 0.7471\%$ ✓

$$\frac{\Delta B(0,T)}{B(0,T)} \simeq -D(y) \Delta y + \frac{1}{2} C(y) \Delta y^2$$

ZCB: $C_1(y) = \frac{4^2 \cdot 10'000 \cdot e^{-0.05 \cdot 4}}{10'000 \cdot e^{-0.05 \cdot 4}} = 16$

CB: $C_2(y) = \frac{60 \cdot e^{-0.05 \cdot 1} + 4 \cdot 60 \cdot e^{-0.05 \cdot 2} + 9 \cdot 3060 e^{-0.05 \cdot 3}}{2745.1304} = 8.7348$

$C_{tot}(y) = \frac{8187.3075}{10932.4379} \cdot 16 + \frac{2745.1304}{10932.4379} \cdot 8.7348 = 14.1757$

$\frac{\Delta B(0,T)}{B(0,T)} \simeq -D(y) \Delta y + \frac{1}{2} C(y) \Delta y^2 \simeq 0.7495\%$ ✓

Since duration provides a first-order approximation of the price sensitivity to changes in the yield, while convexity captures the second-order effect, the second-order approximation yields a price variation that is closer to the exact repricing of the portfolio.

Exercise 3 (10 points)

The price of a stock on 1^{st} January is 130\$, and it will pay a dividend of 3\$ on 1^{st} June and a dividend of 2\$ on 1^{st} September. The interest rate is 10% per annum. On 1^{st} January the forward price for delivery of the stock on 1^{st} October is 142\$.

3 1. Find if there is an arbitrage opportunity, explaining why.

4 2. If so, illustrate the arbitrage opportunity and compute the arbitrage profit.

3 3. If the stock does not pay dividends and forward price for delivery of the stock on 1^{st} October is still 142\$, is there an arbitrage oppurtunity? Why? Find the value of the interest rate that makes the market arbitrage free.

1. $F(0,T) = (S(0) - e^{-rt} \cdot d_t) \cdot e^{rT} = (130 - 3 \cdot e^{-0.1 \cdot \frac{5}{12}} - 2e^{-0.1 \cdot \frac{8}{12}}) \cdot e^{0.1 \cdot \frac{9}{12}} = 135 < 142$ => ARBITRAGE OPPORTUNITY

↳ c'è sempre possibilità arbitraggio se sono ≠

2. on January 1^{ST}

borrow	+ 130
buy stock	− 130
	0

on June 1^{ST}

collect dividend	+3
invest	−3
	0

on September 1^{st}

collect dividend	+2
invest	−2
	0

on October 1^{ST}

pay the back (give back borrow)	$-130\, e^{0.1 \cdot \frac{9}{12}} = 140.1249$
sell the stock	+ 142
collect resut dividends	$+3e^{0.1 \cdot \frac{4}{12}} + 2e^{0.1 \frac{1}{12}} = 5.1184$
	6.9935 ✓

3. $F(0,T) = S(0)\, e^{rT} = 130 \cdot e^{0.1 \cdot \frac{9}{12}} = 140.1249 < 142$ => ARBITRAGE OPPORTUNITY

$F(0,T) = 142 = S(0)\, e^{rT} = 130\, e^{r \cdot \frac{9}{12}}$ $\qquad r = \ln\left(\frac{142}{130}\right) \frac{12}{9} = 11.77\ \%$

TIPOLOGIA 3 - BLACK-SCHOLES PRICING 07/21/2023

Exercise 1

We take a long position in a European Call option on a stock S with strike $K = 80$ euros and maturity $T = 1$ year. The price of the underlying follows a geometric Brownian motion with $S_0 = 80$ euros, $\mu = 0.2$ and $\sigma = 0.4$ per year. The risk-free interest rate available on the market is $r = 0.04\%$ per year continuously compunded.

1. Find the price of the Call option.

1.Compute the Delta Δ of the Call option and, accordingly, establish how many shares of stock S we need to buy/sell in order to make our long position Delta-neutral.

3. Compute the Delta Δ of a put option with the same underlying, maturity and strike and, accordingly, establish how many shares of stock S we need to buy/sell in order to make our short position Delta-neutral.

1. $C(t) = S(t)\mathcal{N}(d_1) - Ke^{-r(T-t)}\mathcal{N}(d_2)$

$d_1 = \dfrac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \dfrac{\ln\left(\frac{80}{80}\right) + \left(0.04 + \frac{1}{2}0.4^2\right)\cdot 1}{0.4} = 0.3 \qquad \mathcal{N}(d_1) = 0.6179$

$d_2 = \dfrac{\ln\left(\frac{S(0)}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \dfrac{\ln\left(\frac{80}{80}\right) + \left(0.04 - \frac{1}{2}0.4^2\right)\cdot 1}{0.4} = -0.1 \qquad \mathcal{N}(d_2) = 1-\mathcal{N}(0.1) = 0.4602$

$C(0) = 80 \cdot 0.6179 - 80e^{-0.04} \cdot 0.4602 = 14.0596$

2. $\Delta = \mathcal{N}(d_1) = 0.6179$

Posizione Δ-neutra $\Rightarrow \Delta = 0$

Sei long call $\Rightarrow$ $V = C + xS$ $\quad x =$ n° di azioni $\quad x>0 =$ long $\quad x<0$ shor

$\Delta V = \Delta C + x\Delta S$

$0 = 0.6179 + x \Rightarrow x = -0.6179 \Rightarrow$ shorT position

3. $P = Ke^{-r(T-t)}\mathcal{N}(-d_2) - S(t)\mathcal{N}(-d_1)$

$\Delta_{put} = \mathcal{N}(d_1) - 1 = -0.3821$

$P = xS - P \quad \Rightarrow \quad \Delta P = 0 = x - P \quad \Rightarrow \quad x = -0.3821 \Rightarrow$ shor position

Exercise 1 (10 points)

An option is written on a stock without dividends and with current price of 50 euros. At the end of each one of next two semesters the stock price can rise by 25% or fall by 20%, the risk-free (nominal annual) interest rate is 4% convertible 2 times a year and the strike of the option is 60 euros. 1. Find the current price of a European Put option written on this underlying and with maturity T = 1 year.

2. Is it optimal to exercise the corresponding (i.e. with the same parameters) American option before maturity? Why? What is its fair value? Comment on the results.

3. Discuss whether there exists a probability measure P under which the price process has costant mean.

1.

t=0	t=6 mounth	Payoff	t=T=1year	Payoff $(K-S(T))^+$	K=60
			78.125	0	
	62.5	5.1 ↑ non esercito			
			50	10	
50					
			50	10	
	40	20 ↑ esercito			
			32	28	

t = 6 mounth

$r_y = r_{y/2} / 2 \Rightarrow r_{y/2} = 0.02$

$q_u = \frac{(1+r)-d}{u-d}$ $\quad q_d = 1-q_u \quad u = \frac{S_u}{S_0} = 1.25 \quad d = \frac{S_d}{S_0} = 0.8 \quad q_u = 0.49 \quad q_d = 0.51$

Prezzo di option usando continue value $\longrightarrow V_t = \frac{1}{1+r}\left(q_u V^u_{t+1} + q_d V^d_{t+1}\right) \Rightarrow V_u = \frac{1}{1+0.02}\left(0.49\cdot 0 + 0.51\cdot 10\right) = 5$

$V_d = \frac{1}{1+0.02}\left(0.49\cdot 10 + 0.51\cdot 28\right) = 18.8$

$P = \frac{1}{1+0.02}\left(0.49\cdot 5 + 0.51\cdot 18.8\right) = 11.80$

2. $V_d = 60-40 = 20 \checkmark > 18.8 \Rightarrow$ esercito

$V_u = 0 < 5 \Rightarrow$ non esercito

$P = \frac{1}{1+0.02}\left(0.49\cdot 5 + 0.51\cdot 20\right) = 12.40 \rightarrow$ higher because early exercise

3. $\mathbb{E}(S(1)) = p\cdot 62.5 + (1-p)\,40 = 50 \Rightarrow p = 0.44$

Per trovare p t.c. il processo abbia media costante $\mathbb{E}(S(1)) = p\cdot S_u + (1-p) S_d = S_0 \checkmark$

$\mathbb{E}(S(2)) = p^2\, 78.125 + 0.44\,(1-0.44)\cdot 50\cdot 2 + (1-0.44)^2\cdot 32 = 50 \checkmark$

TIPOLOGIA 5 - REPLICATING / ARBITRAGGIO 06/19/2025

Exercise 2(10 points)

A stock (indexed by A) is available on the market at the current price $S_A(0) = 8$ euros. In one year, the price may increase by 25% or decrease by 25% or stay unchanged. Another stock (indexed by B) is also available. Its current price is $S_B(0) = 12$ The possiblescenarios for B are

$$\begin{cases} S_B^+ \text{ increase by} 25\%, & \text{if A goes down,} \\ S_B^0 \text{decrease by} 25\%, & \text{if A unchanged,} \\ S_B^- \text{ decrease by} 25\%, & \text{if A goes up.} \end{cases}$$

Another stock (indexed by B) is also available. Its current price is $S_B(0) = 12$ euros that, in one year, may increase by 25% (when the price of stock A is increased or unchanged) or decrease by 25% (when the price of stock A is increased). The risk-free interest rate on the market is 4% per year (simple compounding).

1. Consider a European put option with maturity of one year, with strike of 8 euros and written on stock A. Verify if it is possible to replicate such an option only by means of stock A and of cash invested or borrowed at risk free rate. If yes explain the replicating strategy, if no explain why.

2. Verify if it is possible to replicate the European put option above investing in stock A, stock B and cash. If yes, find a replicating portfolio and compute the cost of the replicating strategy. Comment on the result.

3. Discuss whether the market (A, B, C), where C is the risk-free asset is complete. If possible find a risk neutral measure.

$$S_A = 8 \begin{array}{ll} 10 & w_1 \\ 8 & w_2 \\ 6 & w_3 \end{array} \qquad S_B = 12 \begin{array}{ll} 9 & \\ 9 & \\ 15 & w_1 \end{array}$$

a. EUROPEAN PUT (A)

$(K - S(T))^+ \quad K = 8$

$\Phi(A) \begin{cases} 0 \\ 0 \\ 2 \end{cases}$ Replicating portfolio: (h_A, h_C)

$t = 1 \quad V^h(1) = h_A S_A(1) + h_C C(1) \qquad C = C(0)(1+r) = 1 \cdot (1 + 0.04) = 1.04$

$$\begin{cases} 6 \cdot h_A + 1.04 \cdot h_C = 2 \\ 8 \cdot h_A + 1.04 h_C = 0 \\ 10 \cdot h_A + 1.04 h_C = 0 \end{cases} \qquad A^c = \left(\begin{array}{cc|c} 6 & 1.04 & 2 \\ 8 & 1.04 & 0 \\ 10 & 1.04 & 0 \end{array} \right)$$

$h_A = 0 \Rightarrow h_C = \frac{2}{1.04} \neq 0 = h \qquad \Rightarrow$ no sol

b. European put option stock A, stock B, cash

$$V^h(1) = h_A S_A(1) + h_B S_B(1) + h C(1) \qquad \begin{cases} 6 \cdot h_A + 15 h_B + 1.04 h_C = 2 \\ 8 h_A + 9 h_B + 1.04 h_C = 0 \\ 10 h_A + 9 h_B + 1.04 h_C = 0 \end{cases} \qquad \begin{array}{l} h_B = \frac{1}{3} \\ h_B = -h_C \frac{1.04}{9} \rightarrow h_C = -2.88 \\ h_A = 0 \end{array}$$

$V(0) = 12 \cdot \frac{1}{3} - 2.88 \cdot 1 = 1.11$

c. Matrice dei Payoff

$$\text{Payoff} = \begin{array}{c} \text{stati} \\ \begin{pmatrix} 10 & 8 & 6 \\ 9 & 9 & 15 \\ 1.04 & 1.04 & 1.04 \end{pmatrix} \end{array} \text{asset}$$

$\text{Rank}(\text{Payoff}) = 3 \Rightarrow$ puoi replicare quello che vuoi

$$Q = (q_1, q_2, q_3) \rightarrow \begin{cases} S_0 = \frac{1}{1+r} \mathbb{E}^Q(S_1) \iff (1+r) S_0 = \sum_{i=1}^{3} q_i S_1(w_i) \\ q_1 + q_2 + q_3 = 1 \end{cases}$$

$$\begin{array}{l} 10 q_1 + 8 q_2 + 6 q_3 = 1.04 \cdot 8 \\ 9 q_1 + 9 q_2 + 15 q_3 = 1.04 \cdot 12 \\ 1.04 q_1 + 1.04 q_2 + 1.04 q_3 = 1.04 \cdot 1 \rightarrow q_1 + q_2 + q_3 = 1 \checkmark \end{array}$$

risolvi per Q

Ma! $q_2 < 0 \Rightarrow$ non esiste misura martingala equivalente $\Rightarrow$ il mercato NON è arbitrage free

TIPOLOGIA 6 - DELTA HEDGING & GREEKS 07/10/2025

Exercise 3 (10 points) Let $dS = 0.06Sdt + 0.12SdW$ and risk-free interest rate is 2% per annum (all rates are continuously compounded).
(a) Price a European call option with strike price 60, maturing in 2 years, written on the stock $S(t)$ (no dividends) whose current price $S(0)$ is 57.
(b) If an investor holds 100 such calls and wishes a portfolio that is not sensitive to variations of the underlying asset (on the short term), how many stock shares should she hold?
(c) Find the probability that the option is exercised.

a. $C(t) = S(t)\mathcal{N}(d_1) - Ke^{-r(T-t)}\mathcal{N}(d_2)$

$$d_1 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{57}{60}\right) + \left(0.02 + \frac{1}{2}0.12^2\right)\cdot 2}{0.12\cdot\sqrt{2}} = 0.018 \qquad \mathcal{N}(d_1) \simeq 0.508$$

$$d_2 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{57}{60}\right) + \left(0.02 - \frac{1}{2}0.12^2\right)\cdot 2}{0.12\cdot\sqrt{2}} = -0.151 \qquad \mathcal{N}(d_2) \simeq 0.44$$

$$C_0 = 57\cdot 0.58 - 60\cdot e^{-0.02\cdot 2}\cdot 0.44 = 356$$

b. $\Delta = 0$ $\qquad \Delta_{call} = \mathcal{N}(d_1) \simeq 0.508$

$$V = xS + C\cdot 100$$
$$0 = \Delta V = x + \Delta_C \cdot 100$$
$$x = 50.7 \qquad \text{short position}$$

c. Call $\Rightarrow$ esercito post K $\qquad \mathbb{P}(S(T) > K) = \mathbb{P}(S(T) > 60) = \mathbb{P}(\ln(S(T)) > \ln(60))$

$$\ln(S(T)) \sim \mathcal{N}\left(\ln(S_0) + \left(\mu - \frac{1}{2}\sigma^2\right)\cdot T,\ \sigma^2 T\right) \sim \mathcal{N}(4.14,\ 0.03)$$

$$\mathbb{P}\left(Z > \frac{\ln(60) - 4.14}{\sqrt{0.03}}\right) = \mathbb{P}(Z > -0.32) = \mathbb{P}(Z < 0.32) = 0.6255$$

$$\Phi(-a) = 1 - \Phi(a)$$
$$\mathbb{P}(Z > -a) = \mathbb{P}(Z < a)$$
$$\mathbb{P}(Z < -a) = 1 - \Phi(a)$$
$$\mathbb{P}(Z > a) = 1 - \Phi(a)$$

Exercise 2

Your current wealth is 100,000 euro, which you invest in a financial portfolio S whose value process follows a geometric Brownian motion with drift coefficient 7% and volatility coefficient 25%. The risk-free rate (with continuous compounding) is 3%.

1. Your target is to double your wealth in 10 years. Find the shortfall probability, i.e., the probability you will not achieve your target.
2. Define the loss of a financial portfolio (use logreturns) its value at risk.
3. Find the value at risk of the portfolio loss after 10 years at level 0.95.

1. $V_T = V_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2 \right) T + \sigma W_T \right)$ $W_T \sim \mathcal{N}(0,T)$ $\mu = 0.07$ $\sigma = 0.25$

Target $V_T = 2V_0$

Short fall Probability: $\mathbb{P}(V_T < 2V_0) = \mathbb{P}\left(V_0 e^{(0.07 - \frac{1}{2}\cdot 0.25^2)\cdot 10 + 0.25 W_T} < 2V_0 \right) = \mathbb{P}(0.38 + 0.25 W_t < \ln(2)) = \mathbb{P}(W_t < 1.25)$

$W_T \sim \mathcal{N}(0,10)$ $\mathbb{P}\left(Z < \frac{1.25 - 0}{\sqrt{10}} \right) = \mathbb{P}(Z < 0.3953) = 0.6517$

2. LogReturn $R_T = \ln\left(\frac{V_T}{V_0}\right)$ $\Rightarrow$ $L = -\ln\left(\frac{V_T}{V_0}\right) \sim \mathcal{N}\left(\left(\mu - \frac{\sigma^2}{2}\right) T, \sigma^2 T \right) = \mathcal{N}(-0.38, 0.625)$

$VaR_\alpha = \inf\{ y \in \mathbb{R} : \mathbb{P}(L \le y) \geq \alpha \} = \inf\left\{ y \in \mathbb{R} : \mathbb{P}\left(Z \le \frac{y - 0.38}{\sqrt{0.625}} \right) \geq \alpha \right\}$

3. $T = 10$ $\mathbb{P}(L(10) \le y) = 0.95$ $\Rightarrow$ $\text{quantile}_{0.95} = \mu + \sigma z_{0.95} = -0.38 + 0.625 \cdot z_{0.95}$ $z_{0.95} : \Phi(z_{0.95}) = 0.95$, $z_{0.95} = 1.65$

$= 0.916 = y$

$L(10) = -\ln\left(\frac{V(10)}{V(0)}\right)$ $\mathbb{P}(L(10) \le 0.916) = 0.95$

$VaR_{0.95} = V_0 L(10) = 0.95 \cdot 10^6$

Exercise 3 (10 points)

Consider a market model where it is possible to trade on a stock with current price $S(0) = 20$ euro. Assume that the stock price follows a geometric Brownian motion with $\mu = 0.03$, $\sigma = 0.1$ and the risk free-interest rate is $r = 0.05$.

1. Consider a European call option with strike $K = 20$ euro and maturity in one year. Find its price.

2. Establish how many shares of the stock we need to buy/sell in order to make our short position Delta-neutral.

3. Consider now two European call options written on a stock that follows a geometric Brownian motion with undefined parameters μ and σ. The strikes are $K_1 = 20$ euro and $K_2 = 40$ euro, and the call prices are 6 and 2 euro, respectively. Both options have maturity in one year. Establish under which condition on μ and $\sigma > 0$ the profit of a bear spread from these two call options is positive with a probability of at least 50%.

3.

3) Recall that a bear spread is obtained by:

- selling the CALL option with lower strike. (A)
- buying the " " with Higher strike. (B)

We have the following scheme:

	A	B	TOTAL PAYOFF	TOTAL PROFIT
$S_T \leq K_1$	0	0	0	$C_1 - C_2$ 4
$K_1 < S_T < K_2$	$-(S_T - K_1)$	0	$K_1 - S_T$	$K_1 - S_T + C_1 - C_2 = 24 - S_T$
$S_T \geq K_2$	$-(S_T - K_1)$	$S_T - K_2$	$K_1 - K_2$	$K_1 - K_2 + C_1 - C_2 = -16$

TIPOLOGIA 3- WIENER & ITO 07/12/2023

Exercise 3

Let $W(t)$ be a standard Wiener process.

1. Show that
$$cov(W(t), W(s)) = min\{s,t\} \tag{0.1}$$
and that
$$\rho(W(t), W(s)) = \sqrt{\frac{min\{s,t\}}{max\{s,t\}}}$$

2. Write down the SDE for the process
$$X(t) = W(t)^4$$

3. Show that
$$Var(Y(t)) = \frac{t^3}{3},$$
where
$$Y(t) = \int_0^t (t-s)\ dW(s)$$

1\. $W(t) \sim \mathcal{N}(0,t) \Rightarrow cov(W(t),W(s)) = \mathbb{E}(W(t)W(s)) = \mathbb{E}\left(W(s)\left(W(t)-W(s) + W^2(s)\right)\right) = \overset{0}{\overset{\shortparallel}{\mathbb{E}(W(s)(W(t)-W(s))}} + \mathbb{E}(W^2(s)) = Var(s) = min(s,t)$

$$\rho = \frac{cov(W(t),W(s))}{\sqrt{Var(W(t))}\sqrt{Var(W(s))}} = \frac{min(s,t)}{\sqrt{t}\cdot\sqrt{s}} = \frac{min(s,t)}{\sqrt{t\cdot s}} \underset{t<s}{=} \frac{t}{\sqrt{t}\cdot\sqrt{s}} = \frac{\sqrt{t}\cdot\sqrt{t}}{\sqrt{t}\cdot\sqrt{s}} = \sqrt{\frac{t}{s}}$$

$$\underset{s<t}{=} \frac{s}{\sqrt{t}\sqrt{s}} = \sqrt{\frac{s}{t}}$$

$$\Rightarrow \sqrt{\frac{min}{max}}$$

2\. $f(t,x) = x^4 \quad \partial_t f = 0 \quad \partial_x f = 4x^3 \quad \partial_{xx} f = 12x^2$

$$\partial f(t,W(t)) = f_t(t,W(t))dt + f_x(t,W(t))\,dW(t) + \frac{1}{2} f_{xx}(t,W(t))\,dt = 4x^3 dW(t) + 6x^2 dt$$

3\. $\mathbb{E}(Y^2) = \mathbb{E}\left(\left(\int_0^t (t-s)\,dW_s\right)^2\right) = \int_0^t (t-s)^2 ds = \frac{t^3}{3}$ $\Rightarrow Var = \mathbb{E}(Y^2) = \frac{t^3}{3}$

$\mathbb{E}(Y) = \mathbb{E}\left(\int_0^t (t-s)\,dW\right) = 0$