Financial Engineering

Springer Semester 2025

Lecturer: Patrizia Semeraro, Assistant: Tommaso Vanzan

Problem set 4

Topics: Nonlinear derivatives

Exercise 1 A share of a stock currently sells at 235$. we buy a European call option with strike price 240$ for 12$.

1 What is the price at maturity such that there is no profit or loss (break-even price)?

2 If at maturity the share price is 238, what is the profit or loss?

3 Suppose that at maturity the share price is 261 and we decide to exercise the option. What is the overall cost of the operation?

4 Suppose that at maturity the share price is 261, we decide to exercise the option and sell the stock. What is the profit of the operation?

5 What is the maximum profit achievable by the option?

6 Draw a diagram illustrating how the profit from a long position in the option depends on the stock price at maturity.

Exercise 2 Suppose that a European call option to buy a share for 100$ costs 5$ and is held until maturity. Under what circumstances will the holder of the option make a profit? Under what circumstances the option may be exercised?

Exercise 3 The price of a share of a stock is currently 268$ and, since we expect to it to decrease, we buy a European put option with strike price 260$ for 9$.

1 What is the profit at maturity if the stock price is 230$?

2 What is the maximum profit achievable by the option?

3 Suppose that at maturity the share price is 264. What is the profit or loss?

4 What is the break-even price?

5 What is the maximum loss achievable?

Exercise 4 An investor buys a European put on a share for 3$. The stock price is 42$ and the strike price is 40$. Under what circumstances does the investor make a profit? Under what circumstances the option may be exercised? Draw a diagram illustrating how the profit from a long position in the option depends on the stock price at maturity.

Exercise 5 Suppose that a European put option to sell a share for 60$ costs 8$ and is held until maturity. Under what circumstances the seller of the option make a profit? Under what circumstances the option may be exercised? Draw a diagram illustrating how the profit from a short position in the option depends on the stock price at maturity.

Exercise 6 We hold in our portfolio a stock share bought at 220$. The share price has risen up to 272$, and we would like to protect our profit. To do so, we buy a put option with strike price 260$ for 3$.

1 What is the guaranteed profit by buying the put option?

2 If the share price rises to 280$, how much we gain by selling the share bought at 220?

3 Suppose that at maturity the share price is 261$. What is the put value?

Exercise 7 Trader A enters into a forward contract to buy an asset for 1000$ in one year. Trader B buys a call option to buy the asset for 1000$ in one year. The cost of the option is 100$. What is the difference between the positions of the traders? Show the profit for each trader as function of the price of the asset in one year.

Exercise 8 We believe that a stock price will stabilize around 272$, or increase by a little. We ths sell an European put option with strike price 260$ for 8$.

1 What is maximum profit of this strategy?

2 What is the maximum loss of this strategy?

3 What is the break-even price?

4 What is the profit/loss if the share price drops to 262$?

5 If at maturity, the share price is 250$, what is the profit/loss of the strategy?

Exercise 9 A portfolio is made by a long forward contract on an assset and a long European put option on the asset with the same maturity as the forward contract. The strike price is equal to the forward price of the asset at the time the portfolio is set up. Describe the profit from the portfolio.

Exercise 10 An investor sells a European call option with strike price K and maturity T and buys a put with the same strike price and maturity. Describe the investor's position.

Exercise 11 The price of a non-dividend-paying stock is 19$ and the price of a 3-month European call option on the stock with a strike price of 20$ is 1$. The risk-free rate is 4% annum. What is the price of a 3-month European put option with strike price 20$?

Exercise 12 The prices of an European call and put options on a non-dividend paying stock witn an expiration date in 12 months and strike price of 120$ are 20$ and 5$, respectively. The current stock price is 130$. What is the implied risk-free rate?

Exercise 13 European call and put options with strike price 24$ and exercise date in six months are trading at 5.09$ and 7.78$ respectively. The price of the underlying stock is 20.37%, and the interest rate is 7.48% per annum. Find an arbitrage opportunity.

Exercise 14 [Covered call writing] In our portfolio we hold a share whose price is currently 260, and we believe the market will be stationary. To increment our profit, we sell an European call option with strike price 280$ for 10$.

1 What is the profit if the share price at maturity is 268?

2 What is the share price such that the loss due to the exercise of the option is cancelled by the premium got?

3 If at maturity the share price is 284$, by how much we have increased our profit?

4 If at maturity the share price is 260$, which advantage do we get by the option?

Exercise 15 [Naked call writing] Since we believe that a share price will stay around 272$, we sell a call option with strike price 280$, receiving a premium of 16$.

1 What is the maximum profit of this strategy?

2 What is the maximum loss of this strategy?

3 What is the break-even price at maturity?

4 If at maturity the share price is 286$, what is the profit/loss?

4 If at maturity the share price is 248$, what is the profit/loss?

Financial Engineering

Springer Semester 2025

Lecturer: Patrizia Semeraro, Assistant: Tommaso Vanzan

Problem set 4

Topics: Nonlinear derivatives

Exercise 1

1 The break even price is the price at which the profit is zero. The profit is given by:

$$\pi = \underbrace{\max\{S_T - 240,\, 0\}}_{\text{Payoff}} - \underbrace{12}_{\text{costo}}$$

The break-even price is given by:

$$\pi = 0 \implies \max\{S_T - 240,\, 0\} = 12$$

hence we have:

$$S_T - 240 = 12 \implies S_T = 252$$

$S_T < 252$ → perdi
$S_T = 252$ → break-even
$S_T > 252$ → guadagni

2 Considering the price $S_T = 238\$$, we have:

$$\pi = \max\{238 - 240,\, 0\} - 12 = 0 - 12 = -12$$

Then we have a loss of 12\$.

3 In general, whenever a European call option is exercised the overall cost of the operation is given by the sum of the price of the option and the strike price. In this case we have:

$$240 + 12 = 252$$

4 Considering the price $S_T = 261\$$, we have:

$$\pi = \max\{261 - 240,\, 0\} - 12 = 21 - 12 = 9$$

The profit of the operation is 9\$.

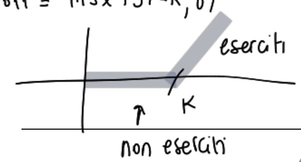
5 The maximum profit is given by the case in which the stock price at maturity is very high, i.e. $S_T \to +\infty$. In this case we have:

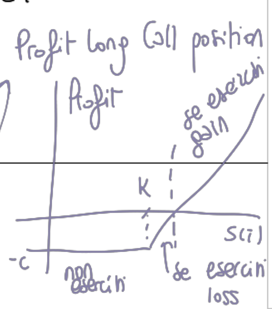
$$\pi = \max\{S_T - 240,\, 0\} - 12 \to +\infty$$

6 The profit of the long position in the option is given by:

*) Call Europea: diritto di acquistare a scadenza a valore K

*) Payoff = $\max\{S_T - K, 0\}$





Exercise 1 A share of a stock currently sells at 235\$. we buy a European call option with strike price 240\$ for 12\$.

S(0)

↘ K := per quanto compriamo

1 What is the price at maturity such that there is no profit or loss (break-even price)?

2 If at maturity the share price is 238, what is the profit or loss?

↳ prezzo di mercato dell'azione

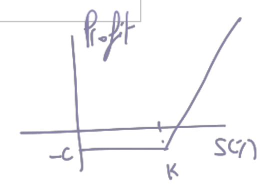
3 Suppose that at maturity the share price is 261 and we decide to exercise the option. What is the overall cost of the operation?

4 Suppose that at maturity the share price is 261, we decide to exercise the option and sell the stock. What is the profit of the operation?

5 What is the maximum profit achievable by the option?

6 Draw a diagram illustrating how the profit from a long position in the option depends on the stock price at maturity.

Exercise 2 Suppose that a European call option to buy a share for 100$ costs 5$ and is held until maturity. Under what circumstances will the holder of the option make a profit? Under what circumstances the option may be exercised?



Exercise 2 In general the payoff of a long position in a European call option is given by:

$$\max\{S_T - K, 0\}$$

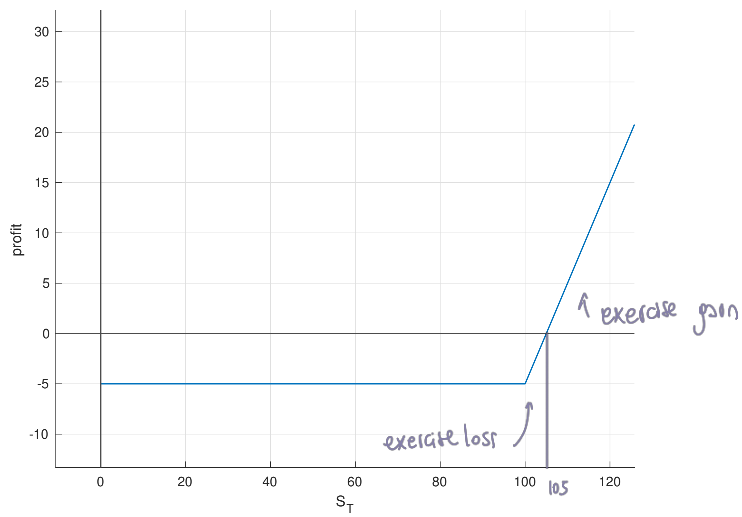
where S_T is the stock price at maturity and K is the strike price. The option is exercised if $S_T > 100$ while the option is not exercised if $S_T \leq 100$. In this case we have:

$$\max\{S_T - 100, 0\} = \begin{cases} 0 & \text{if } S_T \leq 100 \\ S_T - 100 & \text{if } S_T > 100 \end{cases}$$

Moreover, the profit is given by:

$$\pi = \max\{S_T - 100, 0\} - 5$$

Graphically the profit of the long position in the option is given by:



The holder exercised the option if $S_T > 100$ but makes a profit only if $S_T > 105$, then if $100 \leq S_T \leq 105$ the option is exercised, but the holder makes a loss.

Exercise 3

1 Considering the price $S_T = 230\$$, we have:

$$\pi = \max\{\underset{K}{260} - \underset{S(T)}{230}, 0\} - \underset{P}{9} = 30 - 9 = 21$$

The profit at maturity is 21$.

2 The maximum profit is given by the case in which the stock price at maturity is very low, i.e. $S_T \to 0$. In this case we have:

$$\pi = \max\{260 - 0, 0\} - 9 = 260 - 9 = 251$$

3 Considering the price $S_T = 264\$$, we have:

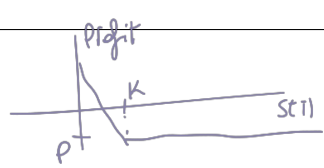
$$\pi = \max\{260 - 264, 0\} - 9 = 0 - 9 = -9$$

The lass at maturity is 9$.

Long put position

Pay off: $\max\{K - S(T), 0\}$

Profit: $\max\{K - S(T), 0\} - P$



Exercise 3 The price of a share of a stock is currently 268$ and, since we expect to it to decrease, we buy a European put option with strike price 260$ for 9$.

1 What is the profit at maturity if the stock price is 230$?

2 What is the maximum profit achievable by the option?

3 Suppose that at maturity the share price is 264. What is the profit or loss?

4 What is the break-even price?

5 What is the maximum loss achievable?

4 The break even price is the price at which the profit is zero, in this case it is given by:

$$\pi = 0 \implies \max\{260 - S_T, 0\} = 9$$

hence we have:

$$260 - S_T = 9 \implies S_T = 251$$

5 The maximum loss is given by the case in which the option is not exercised. In this case we have:

$$\pi = \max\{260 - S_T, 0\} - 9 = 0 - 9 = -9$$ → Non eserciti, hai come loss P

Exercise 4 In general the payoff of a long position in a European put option is given by:

$$\max\{K - S_T, 0\}$$

→ 42 → 40

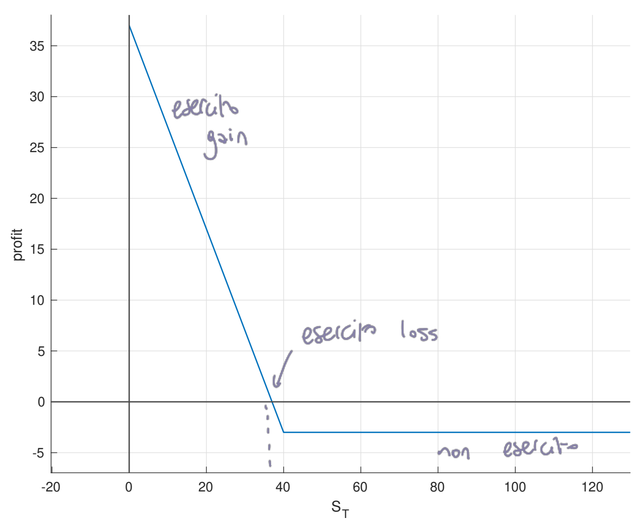
where S_T is the stock price at maturity and K is the strike price. The option is exercised if $S_T < 40$ while the option is not exercised if $S_T \geq 40$. In this case we have:

$$\max\{40 - S_T, 0\} = \begin{cases} 40 - S_T & \text{if } S_T < 40 \\ 0 & \text{if } S_T \geq 100 \end{cases}$$

Moreover, the profit is given by:

$$\pi = \max\{40 - S_T, 0\} - 3$$

Graphically the profit of the long position in the option is given by:



The holder exercised the option if $S_T < 40$ but makes a profit only if $S_T < 37$, then if $37 \leq S_T \leq 40$ the option is exercised, but the holder makes a loss.

→ Long Put Position

Exercise 4 An investor buys a European put on a share for 3$. The stock price is 42$ and the strike price is 40$. Under what circumstances does the investor make a profit? Under what circumstances the option may be exercised? Draw a diagram illustrating how the profit from a long position in the option depends on the stock price at maturity.

Exercise 5 Suppose that a European put option to sell a share for 60$ costs 8$ and is held until maturity. Under what circumstances the seller of the option make a profit? Under what circumstances the option may be exercised? Draw a diagram illustrating how the profit from a short position in the option depends on the stock price at maturity.

Exercise 5 In general the payoff of a short position in a European put option is given by:

$$-\max\{K - S_T, 0\}$$

Sell = Short

where S_T is the stock price at maturity and K is the strike price. The part that has a long position decides if exercised the put, then the option exercised if $S_T < K$ while the option is not exercised if $S_T \geq K$. In this case, the payoff of the short position is given by:

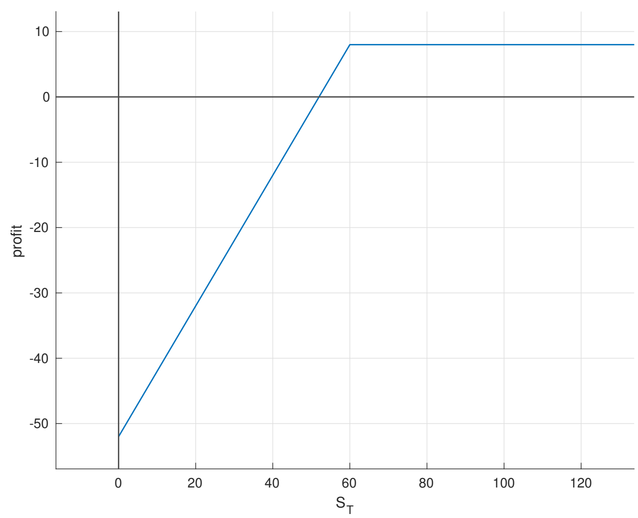
Short Put

$$-\max\{60 - S_T, 0\} = \begin{cases} -60 + S_T & \text{if } S_T < 60 \\ 0 & \text{if } S_T \geq 60 \end{cases}$$

Moreover, the profit of the short position is given by:

$$\pi = 8 - \max\{60 - S_T, 0\}$$

Graphically the profit of the short position in the option is given by:



The seller of the option makes a profit if $S_T > 52$ while the option is exercised if $S_T < 60$, then if $52 \leq S_T \leq 60$ the seller makes a gain that is smaller than 8$.

Exercise 6

1 The guaranteed profit is given by the difference between the price at which the share was bought and the strike price of the put option, minus the cost paid for the option:

$$260 - 220 - 3 = 37$$

2 If the share price rises to 280$, we do not exercise the put option, even if we have bought it, and we sell the share bought at 220$ for 280$, hence we have:

$$280 - 220 - 3 = 57$$

3 If the share price is 261$, the payoff of the put option is given by:

$$\max\{260 - 261, 0\} = 0$$

The profit of the put option is given by:

$$\pi = 0 - 3 = -3$$

Exercise 6 We hold in our portfolio a stock share bought at 220$. The share price has risen up to 272$, and we would like to protect our profit. To do so, we buy a put option with strike price 260$ for 3$.

1 What is the guaranteed profit by buying the put option?

2 If the share price rises to 280$, how much we gain by selling the share bought at 220?

3 Suppose that at maturity the share price is 261$. What is the put value?

Exercise 7 Trader A enters into a forward contract to buy an asset for 1000$ in one year. Trader B buys a call option to buy the asset for 1000$ in one year. The cost of the option is 100$. What is the difference between the positions of the traders? Show the profit for each trader as function of the price of the asset in one year.

Long Call Position

max{ S(T)-K,0}

Exercise 7 In this case the profit of trader A is given by:

$$\pi_A = S_T - 1000$$

where S_T is the price of the asset at maturity. The profit of trader B is given by:

$$\pi_B = \max\{S_T - 1000, 0\} - 100$$

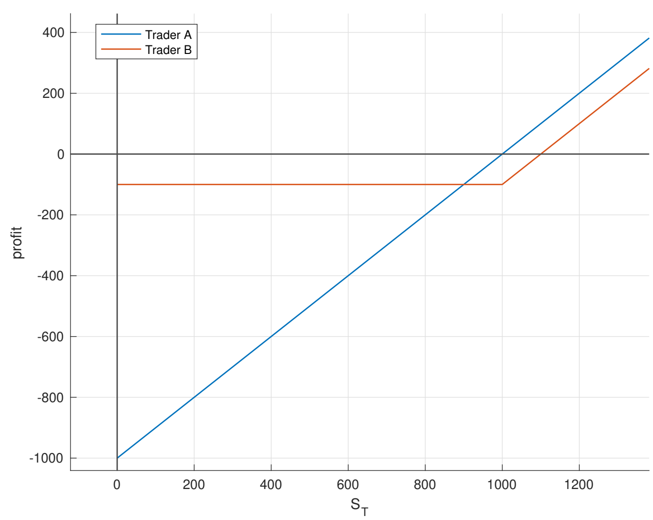
where S_T is the price of the asset at maturity. The trader A does better than trader B, i.e. $\pi_A > \pi_B$, if:

$$S_T - 1000 > \max\{S_T - 1000, 0\} - 100 \implies S_T - 900 > \max\{S_T - 1000, 0\}$$

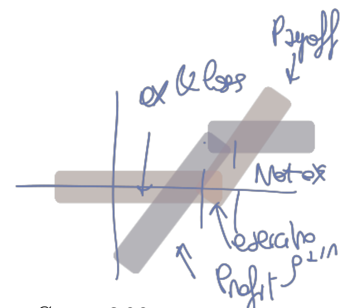
that is:

$$\begin{array}{ll} \text{if } S_T \leq 1000 & S_T - 900 > 0 \implies S_T > 900 \\ \text{if } S_T > 1000 & S_T - 900 > S_T - 1000 \implies \text{always satisfied} \end{array}$$

hence the trader A does better than trader B if $S_T > 900$. Graphically the profit of the traders are:



Exercise 8



1 In this case we enter a short position in the put option, hence we have:

$$\pi = 8 - \max\{260 - S_T, 0\}$$

The maximum profit is given by the case in which the option is not exercised, i.e. $S_T \geq 260$, hence we have:

$$\pi = 8 - 0 = 8$$

2 The maximum loss is given by the case in which the option is exercised, i.e. $S_T < 260$, and in particular when $S_T = 0$ we have:

$$\pi = 8 - (260 - S_T) = S_T - 252 = -252$$

Short Put Position

P- max{ K-S(T),0}

Exercise 8 We believe that a stock price will stabilize around 272$, or increase by a little. We ths sell an European put option with strike price 260$ for 8$.

1 What is maximum profit of this strategy?

2 What is the maximum loss of this strategy?

3 What is the break-even price?

4 What is the profit/loss if the share price drops to 262$?

5 If at maturity, the share price is 250$, what is the profit/loss of the strategy?

3 The break-even price is the price at which the profit is zero, in this case it is given by:

$$\pi = 0 \implies 8 - \max\{260 - S_T, 0\} = 0$$

hence we have:

$$8 = \max\{260 - S_T, 0\}$$

that is:

$$260 - S_T = 8 \implies S_T = 252$$

4 Considering the price $S_T = 262\$$, we have:

$$\pi = 8 - \max\{260 - 262, 0\} = 8 - 0 = 8$$

5 Considering the price $S_T = 250\$$, we have:

$$\pi = 8 - \max\{260 - 250, 0\} = 8 - 10 = -2$$

The profit of the strategy is -2$.

Exercise 9 A portfolio is made by a long forward contract on an assset and a long European put option on the asset with the same maturity as the forward contract. The strike price is equal to the forward price of the asset at the time the portfolio is set up. Describe the profit from the portfolio.

Exercise 9 The final value of the long forward contract is given by:

$$V_T = S_T - F_0$$

where S_T is the price of the asset at maturity and F_0 is the forward price at time 0. The payoff of the long put option is given by:

$$\max\{F_0 - S_T, 0\}$$

where F_0 is the strike price. The final value of the portfolio is given by:

$$V_T + \max\{F_0 - S_T, 0\} = S_T - F_0 + \max\{F_0 - S_T, 0\}$$

that is:

$$V_T + \max\{F_0 - S_T, 0\} = \begin{cases} S_T - F_0 + F_0 - S_T = 0 & \text{if } S_T < F_0 \\ S_T - F_0 + 0 = S_T - F_0 & \text{if } S_T \geq F_0 \end{cases}$$

hence:

$$\max\{S_T - F_0, 0\}$$

that is the same as the final value of a European call option on the asset with the same maturity as the forward contract and strike price equal to the forward price at time 0. The profit equals the final value of the call option lass the amount paid for the put while it does not cost anything to enter into the forward contract.

Exercise 10 In this case the short call provides a payoff of:

$$-\max\{S_T - K, 0\} = \begin{cases} 0 & \text{if } S_T \leq K \\ K - S_T & \text{if } S_T > K \end{cases}$$

while the long put provides a payoff of:

$$\max\{K - S_T, 0\} = \begin{cases} K - S_T & \text{if } S_T < K \\ 0 & \text{if } S_T \geq K \end{cases}$$

Short call

Exercise 10 An investor sells a European call option with strike price K and maturity T and buys a put with the same strike price and maturity. Describe the investor's position.

Long Put

The payoff of the portfolio is given by:

$$-\max\{S_T - K, 0\} + \max\{K - S_T, 0\} = \begin{cases} K - S_T & \text{if } S_T < K \\ 0 & \text{if } S_T = K \\ K - S_T & \text{if } S_T > K \end{cases}$$

and in any case the payoff is equal to $K - S_T$ that is the same as a short position in a forward contract with delivery price K.

Exercise 11 The price of a non-dividend-paying stock is 19$ and the price of a 3-month European call option on the stock with a strike price of 20$ is 1$. The risk-free rate is 4% annum. What i the price of a 3-month European put option with strike price 20$?

Exercise 11 From put call parity we have:

$$c + Ke^{-rT} = p + S_0 \quad \Rightarrow c - p = S_0 - Ke^{-rT} \checkmark$$

where c is the call price, p is the put price, K is the strike price, S_0 is the stock price, r is the risk-free rate and T is the time to maturity. Rearranging gives:

$$p = c + Ke^{-rT} - S_0$$

Substituting the values we have:

$$\begin{aligned} p &= 1 + 20e^{-0.04\cdot\frac{3}{12}} - 19 \\ &= 1 + 20e^{-0.01} - 19 \\ &= 1 + 19.80 - 19 \\ &= 1.80 \end{aligned}$$

Thus, the price of the put option is 1.80$.

Exercise 12 The prices of an European call and put options on a non-dividend paying stock with an expiration date in 12 months and strike price of 120$ are 20$ and 5$, respectively. The current stock price is 130$. What is the implied risk-free rate?

Exercise 12 From the put-call parity we have:

$$c + Ke^{-rT} = p + S_0$$

⇒ ricavare da questo r

where c is the call price, p is the put price, K is the strike price, S_0 is the stock price, r is the risk-free rate and T is the time to maturity. Rearranging gives:

$$\begin{aligned} Ke^{-rT} &= p + S_0 - c \\ e^{-rT} &= \frac{p + S_0 - c}{K} \\ -rT &= \ln\left(\frac{p + S_0 - c}{K}\right) \\ r &= -\frac{1}{T}\ln\left(\frac{p + S_0 - c}{K}\right) \end{aligned}$$

In this case, substituting the given values we obtain:

$$\begin{aligned} r &= -\frac{1}{1}\ln\left(\frac{5 + 130 - 20}{120}\right) \\ &= -\ln\left(\frac{115}{120}\right) \\ &= -\ln(0.9583) \\ &= 0.0426 \end{aligned}$$

Thus, the implied risk-free rate is 4.26% per annum.

Exercise 13 European call and put options with strike price 24$ and exercise date in six months are trading at 5.09$ and 7.78$ respectively. The price of the underlying stock is 20.37%, and the interest rate is 7.48% per annum. Find an arbitrage opportunity.

Exercise 13 From the put-call parity we have:

$$c + Ke^{-rT} = p + S_0$$

where c is the call price, p is the put price, K is the strike price, S_0 is the stock price, r is the risk-free rate and T is the time to maturity. In this case we have:

$$5.09 + 24e^{-0.0748 \cdot \frac{6}{12}} > 7.78 + 20.37$$
$$5.09 + 24e^{-0.0374} > 28.15$$
$$29.21 > 28.15$$

This means that the left-hand side is greater than the right-hand side, which indicates an arbitrage opportunity. To exploit this opportunity, we can buy the put option and sell the call option as follows:

COME IMPOSTARE ARBITRAGGI?

Quando trovi una violazione di parità:

1. scrivi la relazione teorica
2. confronta i due lati
3. **short il lato maggiore**
4. **long il lato minore**
5. decomponi i portafogli
6. verifica payoff costante a scadenza

Se fai questi passi **nell'ordine**, non sbagli mai.

- at time $t = 0$:

buy a share	-20.37
buy a put option	-7.78
write and sell a call option	5.09
borrow the needed amount	+20.37
	0

- at time $t = \frac{6}{12}$:

pay the loan back	$-20.37 \cdot e^{0.0748 \cdot \frac{6}{12}} = -23.94$
sell the share	S_T
put option payoff	$\max\{24 - S_T, 0\}$
call option payoff	$-\max\{S_T - 24, 0\}$

$$S_T - 23.94 + \max\{24 - S_T, 0\} + -\max\{S_T - 24, 0\} = 0.06$$

Exercise 14

1 We enter a short position in the call option, hence we have:

$$\pi = 10 - \max\{268 - 280, 0\} = 10 - 0 = 10$$

The profit at maturity is 10$.

→ PREZZO di MERCATO

2 The share price such that the loss due to the exercise of the option is cancelled by the premium got is given by:

$$\pi = 0 \implies 10 - \max\{S_T - 280, 0\} = 0$$

hence we have:

$$10 = \max\{S_T - 280, 0\}$$

that is:

$$S_T - 280 = 10 \implies S_T = 290$$

Exercise 14 [Covered call writing] In our portfolio we hold a share whose price is currently 260, and we believe the market will be stationary. To increment our profit, we sell an European call option with strike price 280$ for 10$.

1 What is the profit if the share price at maturity is 268?

2 What is the share price such that the loss due to the exercise of the option is cancelled by the premium got?

3 If at maturity the share price is 284$, by how much we have increased our profit?

4 If at maturity the share price is 260$, which advantage do we get by the option?

3 Considering the price $S_T = 284\$$, we have:

$$\pi = 10 - \max\{284 - 280, 0\} = 10 - 4 = 6$$

Then we have increased our profit of 6$.

4 Considering the price $S_T = 260\$$, we have:

$$\pi = 10 - \max\{260 - 280, 0\} = 10 - 0 = 10$$

The profit of the strategy is 10$. The advantage is that by selling the call option, we still own the share (since the option's owner does not exercise given the price at maturity), but in the meantime we got a premium of 10$.

Exercise 15

1 The maximum profit is given by the case in which the option is not exercised, i.e. $S_T \leq 280$, hence we have:

$$\pi = 16 - \max\{S_T - 280, 0\} = 16 - 0 = 16$$

2 The maximum loss is given by the case in which the option is exercised, i.e. $S_T > 280$, and in particular when $S_T \to +\infty$ we have:

$$\pi = 16 - (S_T - 280) = -S_T - 264 \to -\infty$$

3 The break-even price is the price at which the profit is zero, in this case it is given by:

$$\pi = 0 \implies 16 - \max\{S_T - 280, 0\} = 0$$

hence we have:

$$16 = \max\{S_T - 280, 0\}$$

that is:

$$S_T - 280 = 16 \implies S_T = 296$$

4 Considering the price $S_T = 286\$$, we have:

$$\pi = 16 - \max\{286 - 280, 0\} = 16 - 6 = 10$$

The profit of the strategy is 10$.

5 Considering the price $S_T = 248\$$, we have:

$$\pi = 16 - \max\{248 - 280, 0\} = 16 - 0 = 16$$

The profit of the strategy is 16$.

Exercise 15 [Naked call writing] Since we believe that a share price will stay around 272$, we sell a call option with strike price 280$, receiving a premium of 16$.

1 What is the maximum profit of this strategy?

2 What is the maximum loss of this strategy?

3 What is the break-even price at maturity?

4 If at maturity the share price is 286$, what is the profit/loss?

4 If at maturity the share price is 248$, what is the profit/loss?