Forward contract

Definition (Forward Contract)

A forward contract is an agreement to buy or sell an asset on a fixed date in the future, called delivery time, for a price specified in advance, called forward price.

Terminology (I): (Short & Long position)

- The party to the contract who agrees to sell the asset is said to be taking a short forward position. The other party, obliged to buy the asset at delivery, is said to have a long forward position.

Scenario:

The principal reason for entering into a forward contract is to become independent of the unknown future price of a risky asset. A forward contract is a direct agreement between two parties, it is typically settled by physical delivery of the asset on the agreed date.

Terminology (II): (Forward Price)

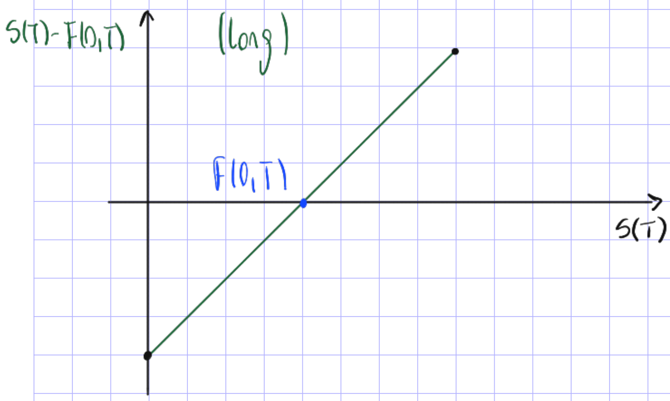
- Let us denote the time when the forward contract is settled (exchanged) by 0, the delivery time by T, and the forward price by $F(0,T)$. The time t market price of the underlying asset will be denoted as $S(t)$.

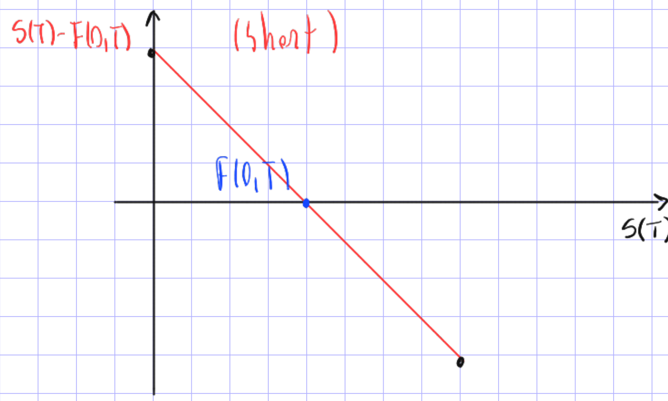
Dynamic of the security:

- No payment is made by either party at time 0, when the forward contract is exchanged.
- At delivery the party with a long forward position will benefit if $F(0,T) < S(T)$, since he buys the asset for $F(0,T)$ and sells it for the market price $S(T)$, making an instant profit of $S(T) - F(0,T)$. Meanwhile, the party holding a short forward position will suffer a loss of $S(T) - F(0,T)$ because he sells below the market price. If $F(0,T) > S(T)$ the situation is symmetrically reversed.

Payoff (Forward Contract):

The payoffs at delivery are $S(T) - F(0,T)$ for a long forward position and $F(0,T) - S(T)$ for a short forward position.





Pricing:

(I) Stock Paying No Dividends:

We start by assuming r (the risk-free rate) constant throughout the period of interest, with continuous compounding.

- To find the price we can build an analogue strategy to end up with the asset, then check if the price implied by that strategy can be adopted to the one we want to price. -

An alternative to taking a long forward position in stock with delivery at time T and forward price $F(0,T)$ is to borrow $S(0)$ dollars to buy the stock at time 0 and keep it until time T. The amount $S(0)e^{rT}$ to be paid to settle the loan with interest at time T is a natural candidate for the forward price $F(0,T)$.

Theorem:

For a stock paying no dividends the forward price is: $F(0,T) = S(0)\,e^{rT}$, where r is a constant risk-free interest rate under continuous compounding. $\left(\hookrightarrow F(t,T) = S(t)\,e^{r(T-t)}\right)$

Proof.

1) $F(0,T) > S(0)e^{rT}$

$t=0$

- Borrow $S(0)$ $(+S(0))$
- Buy $S(0)$ $(-S(0))$
- Take a short forward position, i.e. agree to sell one share for $F(0,T)$ at time T $(+0)$

$V(0) = 0$

(No cashflow)

$t=T$

- Clear the loan at $S(0)e^{rT}$ $(-S(0)e^{rT})$
- Sell $S(T)$ at the forward price $(+F(0,T))$

$V(T) = F(0,T) - S(0)e^{rT} > 0$

(# Arbitrage)

2) $F(0,T) < S(0)e^{rT}$

$t=0$

- Short sell $S(0)$ $(+S(0))$
- Long forward 0
- I invest $S(0)$ $(-S(0))$

$V(0) = 0$

$t=T$

- Return $S(T)$ to the owner by buying at $F(0,T)$ $(-F(0,T))$
- Cash the investment $(+S(0)e^{rT})$

$V(T) = S(0)e^{rT} - F(0,T) > 0$

(# Arbitrage)

$\boxtimes$

Example 1:

1) $S(0) = 40\$$

2) $F(0,3) = 43\$$ (3 months)

3) $r = 0.05$ annual, with continuous compounding.

Is there an arbitrage strategy?

- Solution -

It is enough to check whether $S(0)e^{rT} = F(0,T)$ or not.

$S(0)e^{rT} = 40 \cdot e^{0.05 \cdot \frac{3}{12}} = 40.50\$$

$F(0,3) = 43\$$

The equation does not hold, thus there exists an arbitrage strategy as follows:

$t = 0$

- Borrowing $S(0)$ $(+S(0) = 40\$)$
- Buy the asset $(-S(0)) = -40\$$
- Take a short forward position $(+0)$

$V(0) = 0$

$t = 3/12$

- Selling the stock at $F(0,T)$ $(+43\$)$
- Pay back the loan $(S(0) \cdot e^{rT} = -40.5\$)$

$V\left(\frac{3}{12}\right) = 43 - 40.5 = 2.5\$$

Example 2:

1) $S(0) = 40\ \$$

2) $F(0,T) = 39\ \$$

3) $r = 0.05$

Is there an arbitrage opportunity?

- Solution -

It is enough to check whether $S(0)e^{rT} = F(0,T)$ or not.

$S(0)e^{rT} = 40 \cdot e^{0.05 \cdot \frac{3}{12}} = 40.50\ \$$

$F(0,3) = 39\ \$$

The equation does not hold, thus there exists an arbitrage strategy as follows:

$t = 0$	$t = 3/12$
•) borrow to sell short : $(+S(0) = 40\$)$	•) Take back the investment : $(+40 \cdot e^{r\frac{3}{12}} = 40.5\$)$
•) invest : $(-S(0) = -40\$)$	•) Buy the stock due to the long position : $(-39\ \$)$
•) long forward : 0	•) Give back the stock to the owner (-0)
$V(0) = 0$	$V(\frac{3}{12}) = 40.5 - 39 = 1.5$

(II) Stock Paying Dividends:

Theorem:

The forward price of a stock $S(t)$ that pays dividends d_t at time t, is given by:

$$F(0,T) = \left(S(0) - e^{-rt} d_t\right) e^{rT}$$

↓ Idea

$t=0$ $\underbrace{+S(0)}_{\text{A T pagherò } S(0)e^{rT}}$, $t=t$ ricevo $\underbrace{d_t}_{\text{investo, al tempo T ricevo } d_t e^{(T-t)r}}$ tempo T $\boxed{\left(+d_t e^{-rt} - S(0)\right) e^{rT}}$

Proof.

1) If $F(0,T) > \left(S(0) - e^{-rt} d_t\right) e^{rT}$

time 0		time t	time T	
·) borrow	$+ \$ S(0)$	·) I'm the owner, and therefore I cash the dividents $(+d_t)$	·) Close the loan	$(-S(0)e^{rT})$
·) buy	$- \$ S(0)$		·) Sell the asset	$(+F(0,T))$
·) short forward	0		·) collect d_t	$(+d_t e^{r(T-t)})$
$V(0) = 0$	0	$V(t) = d_t - d_t = 0$	$V(T) = F(0,T) + d_t e^{r(T-t)} - S(0)e^{rT}$	

$$V(T) = F(0,T) + \left(d_t e^{-rt} - S(0)\right) e^{rT}$$

$$V(T) > 0$$

(# Arbitrage)

2) If $F(0,T) < \left(S(0) - e^{-rt} d_t\right) e^{rT}$

time 0:		time t:		time T:	
·) short sell $S(0)$	$(+S(0))$	·) Borrow d_t	$(+d_t)$	·) Return the amount we borrowed	$(-F(0,T))$
·) long forward	0			·) Cash the investment	$(+e^{rT}S(0))$
·) invest $S(0)$	$(-S(0))$	·) and pay dividents	$(-d_t)$	·) Cash the loan paid	$(-d_t e^{r(T-t)})$
$V(0) = 0$		$V(t) = d_t - d_t = 0$		$V(T) = -F(0,T) + e^{rT}\left(S(0) - d_t e^{-rt}\right) > 0$	

(# Arbitrage)

∎

Dividend Yield:

Scenario:

Dividends are often paid continuously at a specified rate, rather than at discrete time instants. For example, in a case of highly diversified portfolio of stocks it is natural to assume that dividends are paid continuously rather than to take into account frequent payments scattered throughout the year.

Another example is foreign currency, attracting interest at the corresponding rate.

- We will firstly derive a formula for the forward price in the case of foreign currency -

Pricing: (Foreign currency)

i.e. I have to pay $P(t)$ dollars to obtain a pound.

Let the price of one British pound in New York be $P(t)$ dollars, and let the risk-free interest rate for investment in British pounds and US dollars be r_{GBP} and r_{USD}.

Let us compare the following strategies. (At time 0 I have $P(0)$ dollars)

A: Invest $P(0)$ dollars at rate r_{USD} for time T $\left(+\ P(0)\cdot e^{r_{USD}\cdot T}\right)$

B: Buy 1 pound at $P(0)$ dollars, invest it for time T at rate r_{GBP}. (At time T I will have $e^{r_{GBP}T}$ pounds)

So I will take a short forward position at time 0 where I have to sell $e^{r_{GBP}T}$ pounds for $F(0,T)$ dollars each.

At time T I'll have a positive cash flow of $e^{r_{GBP}T}\cdot F(0,T)$ dollars.

Both strategies must give back the same results, thus $P(0)\,e^{r_{USD}\cdot T} = e^{r_{GBP}\cdot T}\cdot F(0,T)$

which leads to: $F(0,T) = P(0)\cdot e^{(r_{USD}-r_{GBP})\cdot T}$

(Price of a pound at time $t=0$)

OBS. $P(0)$: price of the pound (stock) at $t=0$

r_{GBP} : Dividend's rate of the pound (stock)

r_{USD} : Rate at which I am subjected when I borrow $P(0)$ dollars to buy the pound (stock)

Link with stocks:

Next suppose that a stock pays dividends continuously at rate $r_{div}>0$ called the (continuous) dividend yield. If the dividends are reinvested in the stock then an investment in one share held at time 0 will increase to become $e^{r_{div}}$ shares at time T.

Theorem :

The forward price for stock paying dividends continuously at rate r_{div} is:

$$F(0,T) = S(0)\, e^{(r - r_{div})T}$$

OBS.
$S(0)$: price of the stock at time $t=0$
r_{div} : Dividend's rate of the stock
r : Rate at which I am subjected when I borrow $S(0)$ dollars to buy the stock

Proof.

(Prove it by yourself,
she didn't do it)

Value of a forward Contract:

Every forward contract has value zero when initiated. As time goes by, the price of the underlying asset may change. Along with it, the value of the forward contract will vary and will no longer be zero, in general. (Just think at the scenario in which the value of the asset vertically increases, if so the long position at $F(0,T)$ fixed gains value).

In particular, the value of a long forward contract will be $S(T) - F(0,T)$ at delivery, which may turn out to be positive, zero or negative.

Scenario for Evaluating the forward contract:

Suppose that the forward price $F(t,T)$ for a forward initiated at time t, where $0 < t < T$, is higher than $F(0,T)$. This is a good news for an investor with a long forward position initiated at time 0. At time T such an investor will gain $F(t,T) - F(0,T)$ as compared to an investor entering into a new long forward contract at time t with the same delivery date T.

Therefore to find the value at time t, of the forward contract exchanged at time 0, we just have to discount this gain $F(t,T) - F(0,T)$ back to time t.

Theorem:

For any t s.t. $0 \leq t \leq T$ the time t value of a long forward contract with forward price $F(0,T)$ is given by:

$$V(t) = [F(t,T) - F(0,T)] e^{-r(T-t)}$$

Proof:

1) $V(t) < [F(t,T) - F(0,T)]\, e^{-r(T-t)}$

time t		time T	
·) borrow $V(t)$	$(+V(t))$	·) pay back the loan :	$-V(t)\, e^{r(T-t)}$
·) buy long forward $F(0,T)$	$(-V(t))$	·) buy $S(T)$:	$-F(0,T)$
·) short forward with $F(t,T)$	$(+0)$	·) sell $S(T)$:	$+F(t,T)$
$V(0) = 0$		$V(T) = F(t,T) - F(0,T) - V(t)\, e^{r(T-t)} > 0$	

(∄ Arbitrage)

2) $V(t) > [F(t,T) - F(0,T)]\, e^{-r(T-t)}$

$\square$

Specific case: (Value of a forward contract at time t with an underlying asset that pays no dividends)

So:
$$V(0) = 0$$
$$V(t) = [F(t,T) - F(0,T)]\, e^{r(T-t)}$$
$$V(T) = S(T) - F(0,T)$$

For a non-paying dividend asset:
$$V(t) = [S(t)\, e^{r(T-t)} - S(0)\, e^{rT}]\, e^{-r(T-t)}$$
$$= S(t) - S(0)\, e^{rt}$$

Futures:

Purpose:

One of the two parties to a forward contract will be losing money. There is always a risk of default the party suffering a loss may not pay the debt. Futures contracts are designed to eliminate such risk.

Structure:

Just like a forward contract, a futures contract involves an underlying asset, a stock with price $S(t)$ and a specified time delivery T.

(Integer, it might be written as n)

In addition to the usual stock prices, the market dictates the so called future prices $f(t,T), 0 \le t \le T$, these prices are unknown at time 0, except for $f(0,T)$, and we shall treat them as random variables.

As in the case of a forward contract, it costs nothing to initiate a futures position. The difference lies in the cashflow during the lifetime of the contract.
A long forward contract involves just a single payment $S(T) - F(0,T)$ at delivery. A futures contract involves a random cash-flow known as marking to market. Namely at each time step $(t-1, t)$ the holder of a long futures position will receive the amount: $f(t,T) - f(t-1,T)$ if positive otherwise he has to pay the same amount. The opposite cash-flow applies for a short forward position.

The following two conditions need to be impose:

1. The futures price at delivery is $f(T,T) = S(T)$ (This is thanks to 2.)
2. At each time step t the value of a futures position is zero.

(It means that it costs nothing to close, open or alter a futures position at any time step between 0 and T.)

Dynamic & Terminology:

(obblighi veri e propri)

To ensure that the obligations involved in a futures position are fulfilled, certain practical regulations are enforced. Each investor entering into a futures contract has to pay a deposit, called the initial margin, which is kept by the clearing house as collateral (collaterale ≡ deposito di garanzia).

In the case of a long futures position the amount $f(n,T) - f(n-1,T)$ is added to the deposit if >0, or subtracted if <0, at each time step n, typically once a day. (The opposite with the same amount applies to the short futures position).

Any excess that builds up above the initial margin can be withdrawn (ritirato) by the investor.

On the other hand, if the deposit drops below a certain level, called maintenance margin, the clearing house will issue a margin call, requesting the investor to make a payment and restore the deposit to the level of the initial margin. A futures position can be closed at any time, in which case the deposit will be returned to the investor. In particular, the futures position will be closed immediately by the clearing house if the investor fails to respond to the margin call. As a result, the risk of default is eliminated.

↳ il rischio che una delle due parti non rispetti gli accordi stipulati.

Example 6.1

Suppose that the initial margin is set at 10% and the maintenance margin at 5% of the futures price. The table below shows a scenario with futures prices $f(n,T)$. The columns labelled 'margin 1' and 'margin 2' show the deposit at the beginning and at the end of each day, respectively. The 'payment' column contains the amounts paid to top up the deposit (negative numbers) or withdrawn (positive numbers).

(margine dopo l'opening) (margine dopo il payment)

10% (140)

n	$f(n,T)$	cash flow	margin 1	payment	margin 2
0	140	opening:	0	−14	14
1	138	− 2	(14-2) 12	0	12
2	130	− 8	(12-8) 4	− 9	13
3	140	+10	23	incasso + 9	14
4	150	+10	24	+ 9	15
		closing:	15	+15	0
			total:	10	

$>5\%(138) = 6.9 \Rightarrow$ no payments.

$4 < 6.5 \Rightarrow$ payments to reach back the initial margin.
Payment $= 10\%(130) - 4 = 13 - 4 = 9$

(incasso 9 e ritorno all'initial margin)

(non serve più il margine 2, tutto il margine 1 viene incassato.)

On day 0 a futures position is opened and a 10% deposit paid. On day 1 the futures price drops by \$2, which is subtracted from the deposit. On day 2 the futures price drops further by \$8, triggering a margin call because the deposit falls below 5%. The investor has to pay \$9 to restore the deposit to the 10% level. On day 3 the forward price increases and \$9 is withdrawn, leaving a 10% margin. On day 4 the forward price goes up again, allowing the investor to withdraw another \$9. At the end of the day the investor decides to close the position, collecting the balance of the deposit. The total of all payments is \$10, the increase in the futures price between day 0 and 4.

Stesso spiegazione.

Liquidity:

In finance liquidity represent how easy an asset, or any security in general, can be converted in money without causing an important variation of its value. We say that an asset is liquid if it is easily convertible in money without losing its value.

Liquidity and Futures:

An important feature of futures market is liquidity. This is due to standardisation and the presence of the clearing house.

Pricing of futures:

Theorem:

If the rate is constant, then $f(0,T) = F(0,T)$

OBS. This construction cannot be performed if the interest rate changes unpredictably. However if interest rates changes are known in advance, the argument can be suitably modified and the equality between the futures and the forwards remains valid.

In an economy with constant interest rates r we obtain a simple structure of future prices: $f(t,T) = S(t)\,e^{r(T-t)}$

$$\left(f(0,T) = S(0)\,e^{rT} \right)$$

No Arbitrage Pricing of forward contracts with replicating portfolios.

Definition: (Portfolio)

A portfolio is a vector $h = (x, y)$ where:

x = is the number of asset X

y = is the number of asset Y

Value of the portfolio:

$$V_h(t) = x\, V_X(t) + y\, V_Y(t)$$

Example:

Buy a share and short forward, $h = (x_S, x_F) \Rightarrow h = (1, -1)$

$V_h(0) = S_0 - V_F(0) = S(0)$, Cash position: $-S(0)$

Replicating portfolio for a forward contract: (long forward position)

Is a portfolio h s.t.

·) $V_h(T) = V_F(T) \Rightarrow V_h(T) = S(T) - F(0,T)$

Principle:

If a claim (derivative) can be replicated by a portfolio, then the price of the claim at 0 must be equal to the value of the portfolio in 0.

$\Rightarrow V_h(0) = 0$ since the value of a forward contract in 0 is 0.

A possible replicating forward portfolio is:

Buy $S(0)$ and borrow, in $[0,T]$, $F(0,T)e^{-rT}$ \$: $h = (\underset{\text{(cash)}}{-F(0,T)e^{-rT}}, \underset{\text{(asset)}}{1})$

- $V_h(0) = -F(0,T)e^{-rT} + S(0)$
- $V_h(T) = -F(0,T)e^{-rT}e^{rT} + S(T) = S(T) - F(0,T) \equiv$ payoff of the forward contract

Since it must hold that: $V_h(0) = V_f(0) = 0$ (it is a replicating portfolio)

Then:

$$-F(0,T)e^{-rT} + S(0) = 0 \Rightarrow F(0,T) = S(0)e^{rT}$$

Scheme:

Replicating strategy

time 0	time T
borrow : $+F(0,T)e^{-rT}$	payback : $-F(0,T)$
buy S_0 : $-S(0)$	sell : $S(T)$
$F(0,T)e^{-rT} - S(0)$	$S(T) - F(0,T)$

Hedging Strategy:

	$t=0$	$t=T$
borrow	\$ $F(0,T)e^{-rT}$	$-F(0,T)$
buy	$-S(0)$	$S(T)$
short forward	0	$F(0,T) - S(T)$
	0	0

$F(0,T)$ is the no-arbitrage price.

Consumption Assets:

These assets are goods that need to be stored.

Futures and Forwards on these assets are actually needed by companies.

- If S is an investment asset then the forward price would be: $F(0,T) = S(0)\, e^{rT}$
- If S is a consumption asset then the forward price would be: $F(0,T) = (S(0) + U)\, e^{rT}$

Consumption assets have storage cost that must be paid by the one who has the asset. This storage cost might be thought as cost to keep the good in the warehouse.
They can be seen as negative income during the life of the contract.

$U :=$ present value of storage costs.

Proof. 1) $F(0,T) > (S(0) + U)\, e^{rT}$

$t=0$	$t=T$
•) borrow : $(+S(0)+U)$	•) pay back the loan : $-(S(0)+U)\, e^{rT}$
•) buy asset : $(-S(0))$	•) sell the asset as imposed by the short forward position : $(+F(0,T))$
•) pay storage : $(-U)$	
•) short forward : 0	
$V(0) = 0$	$V(T) = F(0,T) - (S(0)+U)\, e^{rT} > 0$

2) $F(0,T) < (S(0) + U)\, e^{rT}$

$t=0$	$t=T$
•) Short selling : $(+S(0))$	
(Save Storage) •) Borrow money : $(+U)$	
•) Invest : $(-S(0) - U)$	•) Take back the investment : $(S(0)+U)\, e^{rT}$
•) Long forward : 0	•) Buy the asset : $-F(0,T)$
$V(0) = 0$	$V(T) = (S(0)+U)\, e^{rT} - F(0,T) > 0$

5 Recap Lezione 5 – Forward and Futures Contracts

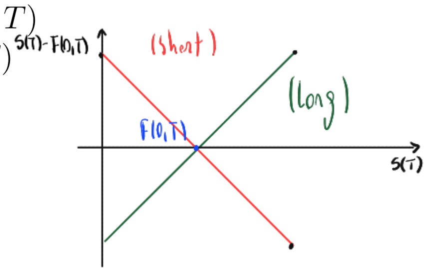
Idea generale della lezione La lezione introduce in modo formale i **forward contracts** e i **futures contracts**, analizzandone **payoff**, **pricing per assenza di arbitraggio**, **valore nel tempo** e differenze strutturali. L'attenzione è posta sulla costruzione di strategie di arbitraggio e di **replicating portfolios**, sul ruolo dei **dividends**, del **dividend yield**, dei **storage costs** e sul funzionamento dei mercati futures con **marking to market** e **margin system**.

Forward contract Un **forward contract** è un accordo tra due parti per acquistare o vendere un asset a una data futura T, detta **delivery time**, a un prezzo fissato oggi, detto **forward price**. Alla stipula non avviene alcun pagamento iniziale.

Forward price Il **forward price** $F(0,T)$ è il prezzo che rende nullo il valore del forward contract al tempo iniziale. Esso è determinato in modo univoco dall'assenza di arbitraggio e dipende dal prezzo spot dell'asset, dal tasso di interesse e da eventuali flussi intermedi.

Long e short forward position

- **Long forward position** Obbligo di acquistare l'asset a T al prezzo $F(0,T)$
- **Short forward position** Obbligo di vendere l'asset a T al prezzo $F(0,T)$



Payoff del forward Alla data di consegna T il payoff è

$$\text{Long} : S(T) - F(0,T), \qquad \text{Short} : F(0,T) - S(T).$$

Il forward è uno **zero-sum game**.

Teorema: Pricing del forward su stock senza dividendi Si assume un tasso risk-free costante r con continuous compounding. Il forward price su uno stock che non paga dividendi è

$$F(0,T) = S(0)e^{rT}.$$

La relazione deriva dall'assenza di arbitraggio e dalla possibilità di replicare il payoff del forward tramite una strategia di trading dinamicamente coerente.

Argomento di non arbitraggio Se $F(0,T) > S(0)e^{rT}$ oppure $F(0,T) < S(0)e^{rT}$, è possibile costruire una strategia con valore nullo a $t = 0$ e payoff positivo certo a T, generando arbitraggio. L'assenza di arbitraggio impone l'uguaglianza.

Teorema: Pricing del forward su stock con dividendi discreti Se lo stock paga un dividendo discreto d_t a una data intermedia t con $0 < t < T$, il forward price è

$$F(0,T) = \big(S(0) - e^{-rt}d_t\big)e^{rT}.$$

La dimostrazione utilizza un portafoglio che replica il payoff del forward includendo l'incasso dei dividendi intermedi.

Dividend yield continuo In molti contesti i dividendi sono modellati come pagati continuamente a un tasso costante δ, detto **dividend yield**. Una unità di stock detenuta da 0 a T genera un flusso continuo proporzionale al prezzo dell'asset.

Forward su asset con dividend yield Per uno stock che paga dividendi continuamente a tasso δ, il forward price è

$$F(0,T) = S(0)e^{(r-\delta)T}.$$

Valore di un forward contract Alla stipula il valore del forward è nullo. Nel tempo, il valore può diventare positivo o negativo in funzione dell'evoluzione del forward price.

Valore del forward a una data intermedia Per $0 \le t \le T$, il valore al tempo t di un long forward stipulato a $t = 0$ è

$$V(t) = \big[F(t,T) - F(0,T)\big]e^{-r(T-t)}.$$

Caso specifico senza dividendi Se l'asset non paga dividendi

$$V(t) = S(t) - S(0)e^{rt}.$$

Futures contracts Un **futures contract** è simile a un forward ma è scambiato su un exchange ed è standardizzato. Il rischio di default è eliminato tramite il sistema di **marking to market** e di margini.

Marking to market

- Regolazione quotidiana di profitti e perdite
- Valore nullo della posizione futures a ogni istante

Margin system

- **Initial margin** Deposito iniziale a garanzia
- **Maintenance margin** Livello minimo del deposito
- **Margin call** Richiesta di reintegro del margine

Il clearing house chiude la posizione se la margin call non viene soddisfatta, eliminando il rischio di default.

Liquidità nei futures markets Un'importante caratteristica dei futures markets è l'elevata **liquidity**, dovuta a

- Standardizzazione dei contratti che rende gli strumenti omogenei e facilmente scambiabili
- Presenza del clearing house che garantisce l'adempimento delle obbligazioni

Pricing dei futures Se il tasso di interesse è costante

$$f(0,T) = F(0,T), \qquad f(t,T) = S(t)e^{r(T-t)}.$$

Se i tassi sono deterministici ma variabili nel tempo, la relazione rimane valida con opportune modifiche.

Portfolio Un **portfolio** è un vettore $h = (x, y)$ che rappresenta le quantità detenute di due asset. Il valore del portafoglio è

$$V_h(t) = xV_X(t) + yV_Y(t).$$

Replicating portfolio Un portafoglio è detto **replicating portfolio** se riproduce esattamente il payoff di un derivato a scadenza.

Portafoglio replicante di un forward Un possibile portafoglio replicante per un long forward è

$$h = \big(-F(0,T)e^{-rT}, 1\big).$$

Il valore del portafoglio è

$$V_h(0) = -F(0,T)e^{-rT} + S(0), \qquad V_h(T) = S(T) - F(0,T).$$

Principio di no-arbitrage pricing Se un claim è replicabile tramite un portafoglio, allora il suo prezzo iniziale deve coincidere con il valore iniziale del portafoglio. Per un forward

$$V_f(0) = 0.$$

Consumption assets Un **consumption asset** è un bene fisico che deve essere stoccato. I forward e futures su tali asset sono rilevanti per imprese industriali.

Investment asset Se l'asset è un investment asset e non genera costi o benefici intermedi

$$F(0,T) = S(0)e^{rT}.$$

Storage costs I consumption assets comportano costi di stoccaggio che possono essere interpretati come flussi negativi durante la vita del contratto. Indicando con U il valore attuale dei storage costs.

Consumption asset Se l'asset è un consumption asset

$$F(0,T) = (S(0) + U)e^{rT}.$$

La formula è ottenuta tramite argomenti di arbitraggio analoghi al caso senza costi.