POD-NN

POD-NN (usefull when you do not have affinity, so you cannot exploite offline-online phase) is a strategy that allows not to rely on affinity on the online stage: the projection stage is not performed and thus the speedup is guaranteed, yet having accurate solutions.

The POD-NN algorithm relies on two stages:

1. a POD,
2. a training of a Feed-forward Neural Network that predicts the entries of the reduced vector u_{rb}.

As usual, we need **a lot of FOM simulations**. Let us import gedim!

In [1]:
```python
import sys
sys.path.append('../../CppToPython')
```

In [2]:
```python
import numpy as np
import GeDiM4Py as gedim
```

In [3]:
```python
lib = gedim.ImportLibrary("../../CppToPython/release/GeDiM4Py.so")

config = { 'GeometricTolerance': 1.0e-8 }
gedim.Initialize(config, lib)
```

The parametric version of the heat conductivity equation

Solving the following equation on square $\bar{\Omega} = [-1, +1] \times [-1, +1]$, studying a parametric diffiusion coefficient

$$\begin{cases} \nabla \cdot (k_\mu \nabla u) = 0 & \text{in } \Omega \\ k_\mu \nabla u \cdot n_1 = \mu_2 & \text{in } \Gamma_{down} \\ u = \sin(\mu_3 \pi x) & \text{in } \Gamma_{up} \text{ - here we can see that we cannot use the affinity: no separation of variable} \\ k_\mu \nabla u \cdot n_2 = 0 & \text{otherwise } \text{ Omogeneuson Neumann} \end{cases}$$

where $k = \mu_1$ if $x^2 + y^2 \leq R^2$ and $k = 1$ otherwise. The parametric space is $\mathcal{P} = [0.1, 10] \times [0, 1] \times [-1, 1]$.

The problem is *standard*. However, we note a nonlinear dependency of the Dirichlet boundary term over Γ_{up} --> here is the problem, the sin!

In [4]:
```python
def Heat_R():
        return 0.5 # Take the radius

def Domain(numPoints, points): # Put 1 allover the points to consider all the domain
        matPoints = gedim.make_nd_matrix(points, (3, numPoints), np.double)
        values = np.ones(numPoints)
        return values.ctypes.data

############## DIRICHLET VARYING WRT mu_3 ####################
def Dirichlet_Term(numPoints, points): # We do not put here another depencency (mu_3) because Gedim does not support this
    # So we will define mu_3 and after call this function
        matPoints = gedim.make_nd_matrix(points, (3, numPoints), np.double)
        values = np.ones(numPoints)
        for p in range(0, numPoints):
        # The values on the Dirichlet boundary I set the value sin(mu_3 * pi * x)
                values[p] = np.sin(mu_3*np.pi*matPoints[0,p])  ### mu_3 is not defined, but not a problem
        return values.ctypes.data
###################

def Circle(numPoints, points): # In the circle
        matPoints = gedim.make_nd_matrix(points, (3, numPoints), np.double)
        values = np.ones(numPoints)
        for p in range(0, numPoints):
                if (matPoints[0,p] * matPoints[0,p] + matPoints[1,p] * matPoints[1,p]) > (Heat_R() * Heat_R() + 1.0e-16):
                        values[p] = 0.
        return values.ctypes.data

def NotCircle(numPoints, points): # Out of the circle
        matPoints = gedim.make_nd_matrix(points, (3, numPoints), np.double)
        values = np.ones(numPoints)
        for p in range(0, numPoints):
                if (matPoints[0,p] * matPoints[0,p] + matPoints[1,p] * matPoints[1,p]) <= (Heat_R() * Heat_R() + 1.0e-16):
                        values[p] = 0.
        return values.ctypes.data

def Heat_weakTerm_down(numPoints, points):
        values = np.ones(numPoints)
        return values.ctypes.data
```

Let us define the High Fidelity Simulation Parameters and import the mesh.

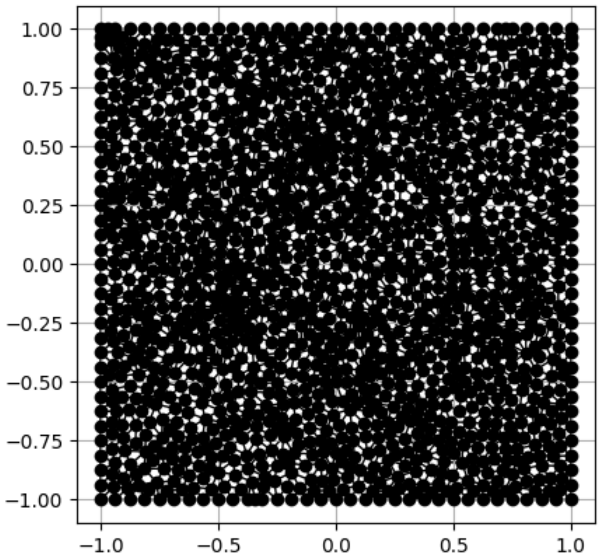
In [5]:
```python
order = 1
```

In [6]:
```python
%%writefile ImportMesh.csv
InputFolderPath
../../CppToPython/Meshes/Mesh3
```

```
Overwriting ImportMesh.csv
```

In [7]:
```
[meshInfo, mesh] = gedim.ImportDomainMesh2D(lib)
```

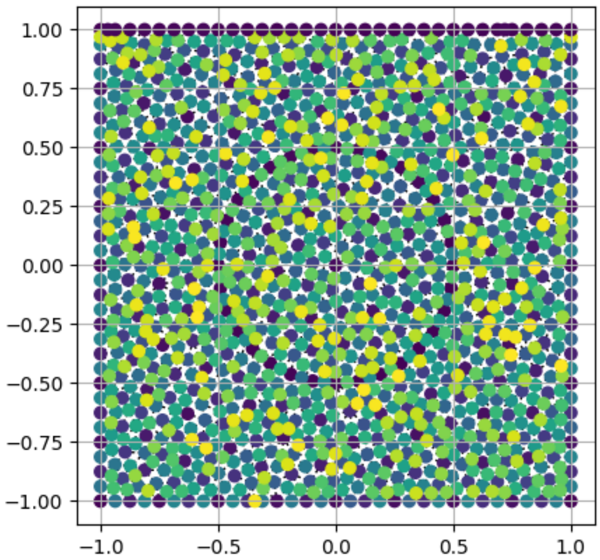
In [8]:
```
gedim.PlotMesh(mesh)
```



Let us create the space

In [9]:
```
discreteSpace = { 'Order': order, 'Type': 1, 'BoundaryConditionsType': [1, 3, 3, 2] }
[problemData, dofs, strongs] = gedim.Discretize(discreteSpace, lib)
```

In [10]:
```
gedim.PlotDofs(mesh, dofs, strongs)
```



Assemble the system

We can assemble only the parts that are $\mu-$ independent (together with the inner product matrix!). Namely, the Dirichlet term needs to be assembled later on. It is non-affine and nonlinear w.r.t to the parameter $\boldsymbol{\mu}$!

In [11]:
```
[stiffness1, stiffnessStrong1] = gedim.AssembleStiffnessMatrix(NotCircle, problemData, lib)

# Moltiply to the inner prodocond (as lab 2), to deal with non omogenoeus condition
[stiffness2, stiffnessStrong2] = gedim.AssembleStiffnessMatrix(Circle, problemData, lib)

weakTerm_down1 = gedim.AssembleWeakTerm(Heat_weakTerm_down, 1, problemData, lib)

#### inner product
# ||grad(u)||^2
inner_product = stiffness1 + stiffness2 # Inner product matrix (same notation of the theory stiffness_matrix = A)

######## DIRICHLET CANNOT BE ASSEMBLED NOW #########################
# We want to performe simulation for different parameters --> so every time that we performe this operation,
# before to doing that, we have to define the Dirichlet condition, compute everything, and definte it again
```

Let us define the training set for the POD

In [12]:
```
### define the training set

snapshot_num = 300
mu1_range = [0.1, 10.]
mu2_range = [-1., 1.]
mu3_range = [-1., 1.]
P = np.array([mu1_range, mu2_range, mu3_range])
```

```python
training_set = np.random.uniform(low=P[:, 0], high=P[:, 1], size=(snapshot_num, P.shape[0]))
```

We can now proceed with the snapshot matrix creation. However, we need to be careful: the problem is not affine in the parameters and we need to assemble the Dirichlet term for each parametric instance.

In [13]:
```python
#### snapshot matrix creation
thetaA1 = 1
snapshot_matrix = []

tol = 1. - 1e-7
N_max = 10

for mu in training_set: # All the parameters that I have to use
  thetaA2 = mu[0]
  thetaf1 = mu[1]
  mu_3 = mu[2]

  #### the problem is not affine: I have to assemble in this stage!! ###
  ## label --> in that way assemble the Dirichlet term non-omogeneous
  Dirichlet_top = gedim.AssembleStrongSolution(Dirichlet_Term, 3, problemData, lib)
  f1_D = stiffnessStrong1 @ Dirichlet_top # Change the value of the forcing term considering the Dirichlet condition
  f2_D = stiffnessStrong2 @ Dirichlet_top

  stiffness = thetaA1*stiffness1 + thetaA2*stiffness2
  weakTerm_down = thetaf1*weakTerm_down1 # Forcing to the Neumann boundary condition
  Dirichlet_contribution = thetaA1*f1_D + thetaA2*f2_D # Contribution on the Dirichlet boundary condition

  f = weakTerm_down - Dirichlet_contribution

  snapshot = gedim.LUSolver(stiffness, f, lib)

  # if you do not want to plot uncomment
  # gedim.PlotSolution(mesh, dofs, strongs, snapshot, Dirichlet_top)
  snapshot_matrix.append(np.copy(snapshot))

snapshot_matrix = np.array(snapshot_matrix)
```

Let us build and analyze the covariance matrix.

In [14]:
```python
### covariance matrix

C = snapshot_matrix @ inner_product @ np.transpose(snapshot_matrix) # Build covariance wrt the inner product

# VM, L, VMt = np.linalg.svd((C))

# Look for eigenvalue (as to be real and non-negative) and eigenvector
L_e, VM_e = np.linalg.eig(C)
eigenvalues = []
eigenvectors = []


#### check
for i in range(len(L_e)):
  eig_real = L_e[i].real
  eig_complex = L_e[i].imag
  assert np.isclose(eig_complex, 0.) # Check if the eigenvalue are non-negative
  eigenvalues.append(eig_real)
  eigenvectors.append(VM_e[i].real)


total_energy = sum(eigenvalues)
retained_energy_vector = np.cumsum(eigenvalues)
relative_retained_energy = retained_energy_vector/total_energy # To check the torelance (as always)


if all(flag==False for flag in relative_retained_energy>= tol):
  N = N_max
else:
  N = np.argmax(relative_retained_energy >= tol) + 1

print(N)
print(relative_retained_energy) # In 9 basis function we reach a good approximation
# --> here that we have non-linearity, the number of basis function that we have to use
# is increased (in the other lab was only 3)
```

```
9
[0.92034006 0.97085465 0.99625438 0.99909937 0.99997566 0.99999823
 0.99999961 0.99999985 1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
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 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.        ]
```

And now let us build the basis functions and $\mathbb{B}$.

In [15]:
```python
# Create the basis function matrix
basis_functions = []
for n in range(N):
  eigenvector =  eigenvectors[n]
  basis = np.transpose(snapshot_matrix)@eigenvector
  norm = np.sqrt(np.transpose(basis) @ inner_product @ basis) ## inner product
  basis /= norm # To have more stability on the bases --> Good operation that we have to do!
  basis_functions.append(np.copy(basis))

basis_functions = np.transpose(np.array(basis_functions))

# FINISH THE OFFLINE PHASE, now consider another parameter and compute the online phase
```

If we want to perform standard ROMs we still need to assemble the system.

During offline phase you have to perform

- POD
- Project: $Au = f$, where f is the inner condition and compute

$$\sum_{q_A} \Theta_A^{q_A}(\mu) A^{q_A} = \sum_{q_f} \Theta_f^{q_f}(\mu) f^{q_f}$$

- Store $A_N^{q_A}$ & $f_N^{q_f}$ so that during the online phase you have only to compute the $\mu :=$ the weight

Can we asseble it?

In [16]:
```python
########## ASSEMBLE WHAT I CAN ##### STILL OFFLINE
# I can assemble the reduced stiffness matrix
# Online I'll assemble the Dirichlet term
reduced_stiff1 = np.transpose(basis_functions) @ stiffness1 @ basis_functions
reduced_stiff2 = np.transpose(basis_functions) @ stiffness2 @ basis_functions
reduced_w =  np.transpose(basis_functions) @ weakTerm_down1
```

For each new parameter I have to assemble the Dirichlet term, once again.

In [17]:
```
########### I CANNOT DO THAT ################ STILL ONLINE??? WE NEED THE PARAMETER
thetaA2 = 2.
thetaf1 = 0.8
mu_3 = 1.
```

In [18]:
```
#### the problem is not affine: I have to assemble in this stage!! ###

Dirichlet_top = gedim.AssembleStrongSolution(Dirichlet_Term, 3, problemData, lib) ## label
f1_D = stiffnessStrong1 @ Dirichlet_top
f2_D = stiffnessStrong2 @ Dirichlet_top
r_f1_D = np.transpose(basis_functions) @ (stiffnessStrong1 @ Dirichlet_top)
r_f2_D = np.transpose(basis_functions) @ (stiffnessStrong2 @ Dirichlet_top)
```

Solve linear system for a new μ

In [19]:
```
reduced_rhs = thetaA1*reduced_stiff1 + thetaA2*reduced_stiff2
reduced_lhs = thetaf1*reduced_w - (thetaA1*r_f1_D + thetaA2*r_f2_D)
```

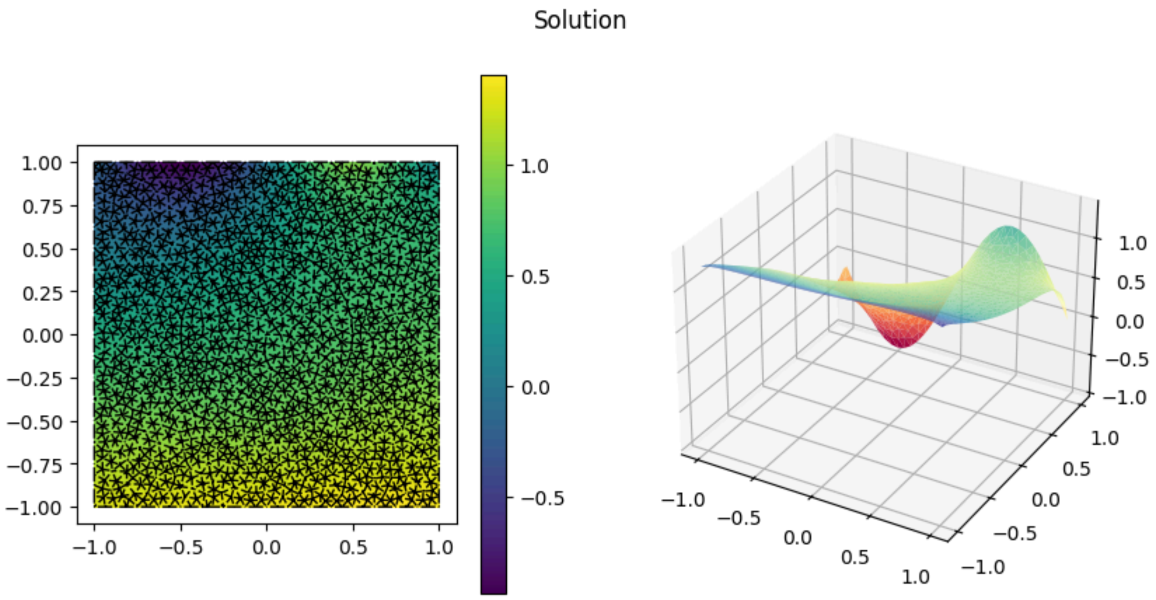
In [20]:
```
#####solve
reduced_solution = np.linalg.solve(reduced_rhs, reduced_lhs)
print(reduced_solution)
```

```
[ -6.03581072   0.71357335  -2.71814498  23.38612387 -22.62647494
   0.54477816  -0.73222505  15.39373308   1.74032697]
```

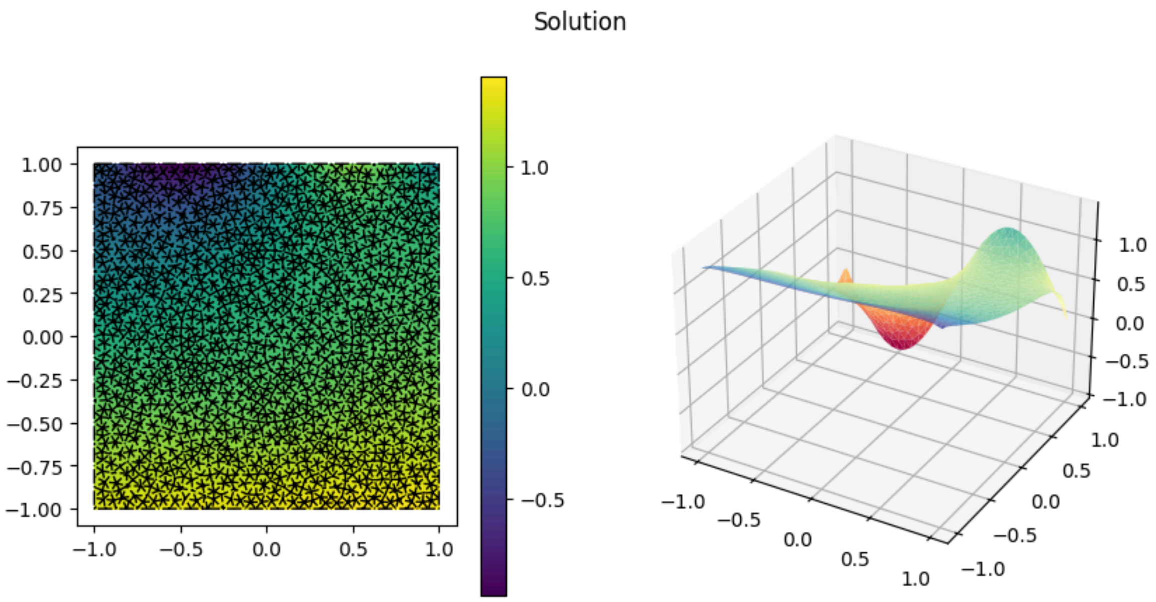
In [21]:
```
###### plot #######
proj_reduced_solution = basis_functions @ reduced_solution
gedim.PlotSolution(mesh, dofs, strongs, proj_reduced_solution, Dirichlet_top)

stiffness = thetaA1*stiffness1 + thetaA2*stiffness2
weakTerm_down = thetaf1*weakTerm_down1
f = weakTerm_down - (thetaA1*stiffnessStrong1 + thetaA2*stiffnessStrong2) @ Dirichlet_top

full_solution = gedim.LUSolver(stiffness, f, lib)
```



In [22]:
```
gedim.PlotSolution(mesh, dofs, strongs, full_solution, Dirichlet_top)
```



Let us comment a bit on the error analysis and the *speed up*.

In [23]:
```
### compute error
import time

abs_err = []
rel_err = []
testing_set = np.random.uniform(low=P[:, 0], high=P[:, 1], size=(100, P.shape[0]))
speed_up = []

print("Computing error and speedup analysis") # Same block that we already see to compute the error

for mu in testing_set:
  thetaA2 = mu[0]
  thetaf1 = mu[1]
  mu_3 = mu[2]

  #### the problem is not affine: I have to assemble in this stage!! ###
  start_assembling = time.time()
  Dirichlet_top = gedim.AssembleStrongSolution(Dirichlet_Term, 3, problemData, lib) ## label
  f1_D = stiffnessStrong1 @ Dirichlet_top
  f2_D = stiffnessStrong2 @ Dirichlet_top
  r_f1_D = np.transpose(basis_functions) @ (stiffnessStrong1 @ Dirichlet_top)
  r_f2_D = np.transpose(basis_functions) @ (stiffnessStrong2 @ Dirichlet_top)
  time_assembling =  time.time() - start_assembling # Time of assemble the part of the problem -> everytime you have to assemble the problem

  ##### full #####
  stiffness = thetaA1*stiffness1 + thetaA2*stiffness2
  weakTerm_down = thetaf1*weakTerm_down1
  f = weakTerm_down - (thetaA1*stiffnessStrong1 + thetaA2*stiffnessStrong2) @ Dirichlet_top


  start_fom = time.time()
  full_solution = gedim.LUSolver(stiffness, f, lib)
  time_fom = time.time() - start_fom # Here not adding the time for assembling

  #### reduced #####
  reduced_rhs = thetaA1*reduced_stiff1 + thetaA2*reduced_stiff2
  reduced_lhs = thetaf1*reduced_w - (thetaA1*r_f1_D + thetaA2*r_f2_D)

  start_rom = time.time()
  reduced_solution = np.linalg.solve(reduced_rhs, reduced_lhs)
  time_rom = time.time() - start_rom

  speed_up.append(time_fom/(time_rom + time_assembling)) # Time assembling is for ROM and FOM so, NOT like here,
                                                        # you have to take in account in all the two part

  proj_reduced_solution = basis_functions@reduced_solution

  ### computing error
  error_function = full_solution - proj_reduced_solution
  error_norm_squared_component = np.transpose(error_function) @ inner_product @ error_function
  absolute_error = np.sqrt(abs(error_norm_squared_component))
  abs_err.append(absolute_error)

  full_solution_norm_squared_component = np.transpose(full_solution) @  inner_product @ full_solution
  relative_error = absolute_error/np.sqrt(abs(full_solution_norm_squared_component))
  rel_err.append(relative_error)
```

```
Computing error and speedup analysis
```

In [24]:
```
print("avarege relative error = ", np.mean(rel_err) )
print("avarege absolute error = ", np.mean(abs_err) )
print("avarege speed_up = ", np.mean(speed_up) )
```

```
avarege relative error =  0.00045595117863909046
avarege absolute error =  0.0016646749673025001
avarege speed_up =  12.673577305539716
```

The speed up is quite small for a linear problem. Let understand the role of POD-NN in this setting.

We want to use a feed-forward NN. Let us define the Class Net with pytorch.

POD NN

- Apply POD
- Train $\Pi^{NN}(\mu, \mu \rightarrow u(\mu)$ where $\mu \in \mathcal{R}^3$ and $u(\mu) \in \mathcal{R}^N$, that in this case $N = 9$, chosen by the POD (tol that we select before)

The dimension N is fixed before with the value of the tollerance.

NB More parameters, more basis function, the number of the basis function is related to the dinamics complete of the parameters, where they are, which dimension have

In [25]:
```
# Define the Net
import torch
import torch.nn as nn
import torch.nn.functional as F

mu_dim = P.shape[0]
basis_dim = N

# I need this dimension
input_dim = mu_dim
output_dim = basis_dim
```

```
nodes = 30

class Net(nn.Module):
    def __init__(self):
        super(Net, self).__init__()
        # Starting layer
        self.fc1 = nn.Linear(input_dim, nodes)

        # Hidden layer
        self.fc2 = nn.Linear(nodes, nodes)
        self.fc3 = nn.Linear(nodes, nodes)
        self.fc4 = nn.Linear(nodes, nodes)
        # Finish hidden layer

        # Last layer
        self.fc5 = nn.Linear(nodes, output_dim)

        self.tanh = nn.Tanh()
        # self.apply(self._init_weights)


    def forward(self, x):  ### Forward law ----> prediction
        # Activation function
        x = self.tanh(self.fc1(x)) # Iperbolic tangent [-1, 1]
        x = self.tanh(self.fc2(x))
        x = self.tanh(self.fc3(x))
        x = self.tanh(self.fc4(x))

        x = self.fc5(x) # This is the output, we do not change the range of the solution
        # It is better to leave the final solution in the space where it is to do not modify it

        return x
```

In [26]:
```
seed_num = 31 # You can change this, different seed == different initialization of weight --> maybe better performance

torch.manual_seed(seed_num)
net = Net()
torch.set_default_dtype(torch.float32)

my_loss = nn.MSELoss() # MSE loss
optimizer = torch.optim.Adam(net.parameters(), lr=0.001)

epoch_max = 500000
epoch = 0
tol = 1e-5
loss = 1. # To start the optimization
```

We need to prepare the outputs to train the NN. Indeed, our goal is to define

$$\boldsymbol{\pi}(\boldsymbol{\mu}) = \underline{u}_{\text{rb}}^{NN}(\boldsymbol{\mu}).$$

Namely, our inputs are the parameters of the training set and the output is the Galerkin projection of the snapshots of the training set. The output is of the form $\underline{u}_{\text{rb}}$ where:

$$\mathbb{B}\underline{u}_{\text{rb}}(\boldsymbol{\mu}) = \mathbb{P}^{\boldsymbol{\mu}} u_\delta(\boldsymbol{\mu}), \qquad (1)$$

where $\mathbb{P}^{\boldsymbol{\mu}} = \mathbb{B}\mathbb{X}_N^{-1}\mathbb{B}^T\mathbb{X}_{N_\delta}$ (direct projection, projecjtion matrix along the line in which I project, this is not a change of variable) is the reduced vector related to the Galrkin projector, i.e. the best approximation of u_δ in V_N w.r.t. the inner-product defined by the matrix X_δ.

Instead of computing the inverse of $\mathbb{X}_N = \mathbb{B}^T\mathbb{X}_\delta\mathbb{B}$ we solve the following system:

$$\mathbb{B}^T\mathbb{X}_\delta\mathbb{B}\underline{u}_{\text{rb}}(\boldsymbol{\mu}) = \mathbb{B}^T\mathbb{X}_\delta u_\delta(\boldsymbol{\mu})$$

to find $u_{\text{rb}}(\boldsymbol{\mu})$ for each snapshot.

In this way we are taking the vector of the reduced solution related to the parameter $\boldsymbol{\mu}$ **without solving the reduced system**, thanks to the relation (1). This element is the closest element (the best choice) to u_δ in the norm of the problem.

Computation performe during the theory (check lesson 10 or 11)

$$\mathbb{B}^T\mathbb{X}_\delta\mathbb{B}\underline{u}_{\text{rb}} = \mathbb{B}^T\mathbb{X}_\delta u_\delta(\boldsymbol{\mu})$$

$$\underline{u}_{\text{rb}} = \mathbb{X}_N^{-1}\mathbb{B}^T\mathbb{X}_\delta u_\delta$$

Inverte the matrix is NOT a good idea, so we can solve the linear sisteme to take the solution.

In [27]:
```
####### training set ########
reduced_inner_product = np.transpose(basis_functions) @ inner_product @ basis_functions
x_train = torch.tensor(np.float32(training_set)) # The input are all the parameters that I have
y_train = []


for i in range(snapshot_matrix.shape[0]):
  snapshot_to_project = snapshot_matrix[i]

  projected_snapshot = np.linalg.solve(reduced_inner_product, np.transpose(basis_functions)@inner_product@snapshot_to_project)
    # == B X snapshots
    # We have to solve X_N u_rb = BT X_delta u_delta
```

```
  y_train.append(projected_snapshot) # On dimension N = 9

y_train = np.float32(y_train)
y_train = torch.tensor(y_train) # To np object to tensor
```

Let us train our neural network!

The loss is

$$\sum_{i=1}^{300} = \frac{1}{300}||\pi(\mu)^{NN} - u_\delta^P||_{l_2}^2$$

where u_δ^P is the projected u_δ.

During the epochs we

- Change tha learning rate
- The loss fluctuated

In [28]:
```
while loss >= tol and epoch < epoch_max:
  epoch = epoch + 1
  optimizer.zero_grad()

  ## compute output
  output = net(x_train) # Net apply to my parameter --> to each parameter I have a solution in dimension 9 that are the projection

  loss = my_loss(output, y_train) # Computing the loss

  if epoch >= 20000:
    optimizer.param_groups[0]['lr'] = 0.0001 # To change the learning rate during the epoch with a dictionary

  #compute the gradients
  loss.backward()

  # optimizer update
  optimizer.step()

  if epoch % 600 == 199:
    print("epoch", epoch, 'loss', loss.item(), 'lr', optimizer.param_groups[0]['lr'] )
```

```
epoch 199 loss 22.54425621032715 lr 0.001
epoch 799 loss 7.93619441986084 lr 0.001
epoch 1399 loss 5.095813751220703 lr 0.001
epoch 1999 loss 3.3643243312835693 lr 0.001
epoch 2599 loss 2.026888370513916 lr 0.001
epoch 3199 loss 1.184842586517334 lr 0.001
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epoch 445399 loss 0.0003960751637350768 lr 0.0001
epoch 445999 loss 0.00039562489837408066 lr 0.0001
epoch 446599 loss 0.0017447342397645116 lr 0.0001
epoch 447199 loss 0.00039482314605265856 lr 0.0001
epoch 447799 loss 0.0008484896970912814 lr 0.0001
epoch 448399 loss 0.00039645933429710567 lr 0.0001
epoch 448999 loss 0.0003932363470084965 lr 0.0001
epoch 449599 loss 0.0003927050274796784 lr 0.0001
epoch 450199 loss 0.0003930026141460985 lr 0.0001
epoch 450799 loss 0.00039174433914013207 lr 0.0001
```

```
epoch 451399 loss 0.00039132602978497744 lr 0.0001
epoch 451999 loss 0.0008857931825332344 lr 0.0001
epoch 452599 loss 0.00039078042027540505 lr 0.0001
epoch 453199 loss 0.0003915097040589899 lr 0.0001
epoch 453799 loss 0.00040018983418121934 lr 0.0001
epoch 454399 loss 0.0003889090148732066 lr 0.0001
epoch 454999 loss 0.00038847618270665407 lr 0.0001
epoch 455599 loss 0.00038805865915492177 lr 0.0001
epoch 456199 loss 0.00038754669367335737 lr 0.0001
epoch 456799 loss 0.00038709197542630136 lr 0.0001
epoch 457399 loss 0.0037755174562335014 lr 0.0001
epoch 457999 loss 0.00038616242818534374 lr 0.0001
epoch 458599 loss 0.00043257736251689494 lr 0.0001
epoch 459199 loss 0.000402679055696353 lr 0.0001
epoch 459799 loss 0.00038481064257211983 lr 0.0001
epoch 460399 loss 0.00038436095928773284 lr 0.0001
epoch 460999 loss 0.00038419532938860357 lr 0.0001
epoch 461599 loss 0.0003834779199678451 lr 0.0001
epoch 462199 loss 0.00041074634646065533 lr 0.0001
epoch 462799 loss 0.00038282075547613204 lr 0.0001
epoch 463399 loss 0.00038206944009289145 lr 0.0001
epoch 463999 loss 0.00038165124715305865 lr 0.0001
epoch 464599 loss 0.00038123628473840654 lr 0.0001
epoch 465199 loss 0.0003879536525346339 lr 0.0001
epoch 465799 loss 0.00038028915878385305 lr 0.0001
epoch 466399 loss 0.0003798605757765472 lr 0.0001
epoch 466999 loss 0.00039400163223035634 lr 0.0001
epoch 467599 loss 0.00037896280991844833 lr 0.0001
epoch 468199 loss 0.00037850733497180045 lr 0.0001
epoch 468799 loss 0.0003780836414080113 lr 0.0001
epoch 469399 loss 0.0003776949888560921 lr 0.0001
epoch 469999 loss 0.0003771970805246383 lr 0.0001
epoch 470599 loss 0.00037679701927118003 lr 0.0001
epoch 471199 loss 0.00037635944318026304 lr 0.0001
epoch 471799 loss 0.00037592955050058663 lr 0.0001
epoch 472399 loss 0.0003754817880690098 lr 0.0001
epoch 472999 loss 0.0003789659822359681 lr 0.0001
epoch 473599 loss 0.0003746206348296255 lr 0.0001
epoch 474199 loss 0.0003741947002708912 lr 0.0001
epoch 474799 loss 0.0003737795923370868 lr 0.0001
epoch 475399 loss 0.00040575143066234887 lr 0.0001
epoch 475999 loss 0.0003729123855009675 lr 0.0001
epoch 476599 loss 0.0003725052811205387 lr 0.0001
epoch 477199 loss 0.0003720730892382562 lr 0.0001
epoch 477799 loss 0.00040165442624129355 lr 0.0001
epoch 478399 loss 0.00037124776281416416 lr 0.0001
epoch 478999 loss 0.0003708541044034064 lr 0.0001
epoch 479599 loss 0.0004295867111068219 lr 0.0001
epoch 480199 loss 0.00036997924325987697 lr 0.0001
epoch 480799 loss 0.00036960048601031303 lr 0.0001
epoch 481399 loss 0.00037589360726997256 lr 0.0001
epoch 481999 loss 0.000377755262888968 lr 0.0001
epoch 482599 loss 0.00039587021456100047 lr 0.0001
epoch 483199 loss 0.00036831331090070307 lr 0.0001
epoch 483799 loss 0.001055592903867364 lr 0.0001
epoch 484399 loss 0.00038367771776393056 lr 0.0001
epoch 484999 loss 0.0016684618312865496 lr 0.0001
epoch 485599 loss 0.00036626963992603123 lr 0.0001
epoch 486199 loss 0.0003658573841676116 lr 0.0001
epoch 486799 loss 0.00036906119203194976 lr 0.0001
epoch 487399 loss 0.0005374337779358029 lr 0.0001
epoch 487999 loss 0.00036463019205257297 lr 0.0001
epoch 488599 loss 0.000856159022077918 lr 0.0001
epoch 489199 loss 0.0003638640628196299 lr 0.0001
epoch 489799 loss 0.00036347529385238886 lr 0.0001
epoch 490399 loss 0.00036997318966314197 lr 0.0001
epoch 490999 loss 0.0003626379475463182 lr 0.0001
epoch 491599 loss 0.0003622284275479615 lr 0.0001
epoch 492199 loss 0.0005978976842015982 lr 0.0001
epoch 492799 loss 0.00036145211197435856 lr 0.0001
epoch 493399 loss 0.0004438463947735727 lr 0.0001
epoch 493999 loss 0.0003610891290009022 lr 0.0001
epoch 494599 loss 0.0003635162429418415 lr 0.0001
epoch 495199 loss 0.0003598766925279051 lr 0.0001
epoch 495799 loss 0.0003594899899326265 lr 0.0001
epoch 496399 loss 0.00035907907295040786 lr 0.0001
epoch 496999 loss 0.00036486584576778114 lr 0.0001
epoch 497599 loss 0.00035831218701787293 lr 0.0001
epoch 498199 loss 0.0003579370677471161 lr 0.0001
epoch 498799 loss 0.001111219055019319 lr 0.0001
epoch 499399 loss 0.00035717658465728164 lr 0.0001
epoch 499999 loss 0.0005764150409959257 lr 0.0001
```

Perfomr all the epochs, even small problem can be very difficult to solve.

Let us compute a specific instance of the problem! Namely we compute $\boldsymbol{\pi}(\boldsymbol{\mu}_{test})$.

What is the output?

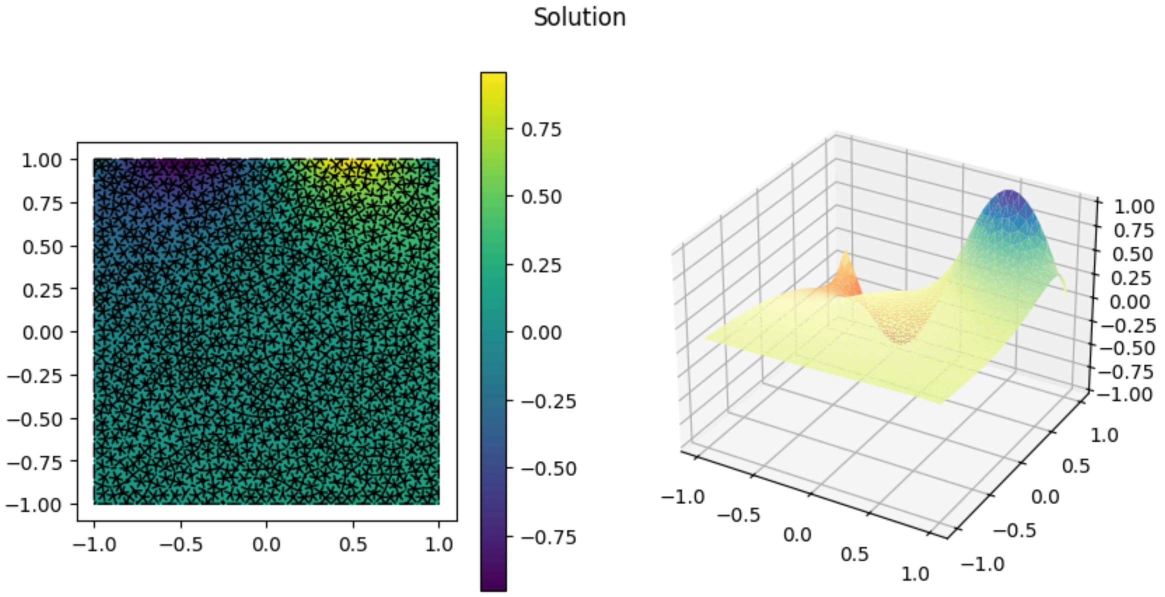
Let us compare it with the full solution.

What do I have to do?

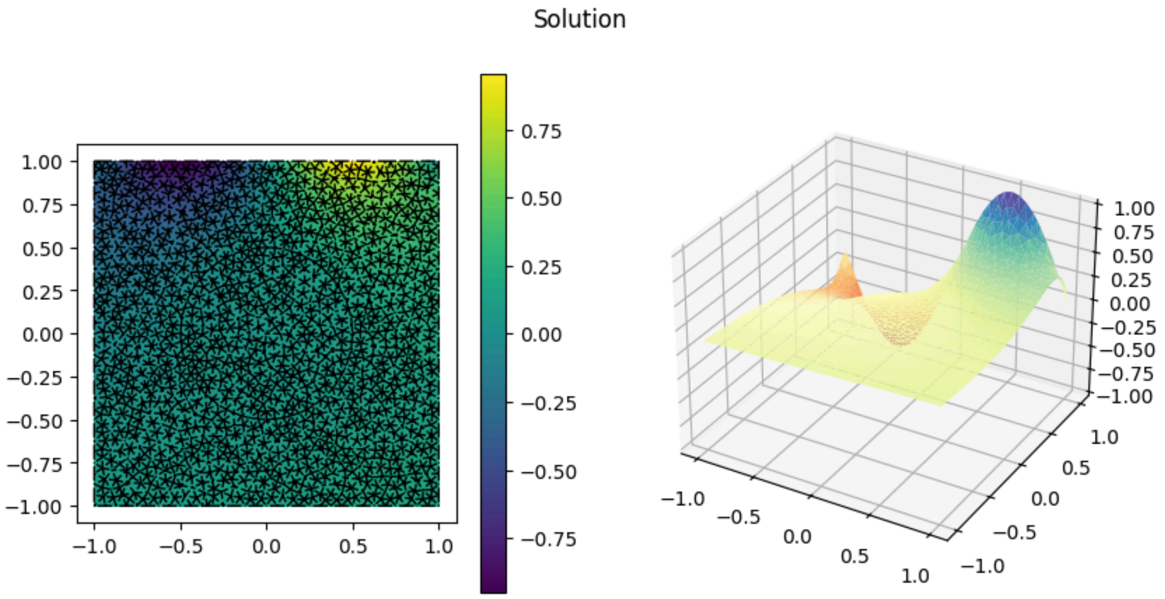
In [29]:
```python
x_test = [[6., .1, 1.]] # Test a new parameter
x_test = np.float32(x_test)
x_test = torch.tensor(x_test)

reduced_solution = np.asarray(net(x_test).detach().numpy())[0] # To make comparison wrt the other

print(reduced_solution) # In dimension 9
```

```
[ 0.7663319 -3.6407268  7.041379  -4.36864   17.088823   7.298813
 -0.7655187 -2.172597  -1.567853 ]
```

In [30]:
```python
# Project the solution to see it

nn_proj_reduced_solution = basis_functions @ reduced_solution
mu = x_test[0]
thetaA2 = mu[0].item()
thetaf1 = mu[1].item()
mu_3 = mu[2].item()
thetaA1 = 1
Dirichlet_top = gedim.AssembleStrongSolution(Dirichlet_Term, 3, problemData, lib)


gedim.PlotSolution(mesh, dofs, strongs, nn_proj_reduced_solution, Dirichlet_top)
# The problem is quite constant to zero and is also tricky for the NN
# Variability can help better to understand in which direction goes
```



In [31]:
```python
##### full #####

stiffness = thetaA1*stiffness1 + thetaA2*stiffness2
weakTerm_down = thetaf1*weakTerm_down1
f = weakTerm_down - (thetaA1*stiffnessStrong1 + thetaA2*stiffnessStrong2) @ Dirichlet_top
full_solution = gedim.LUSolver(stiffness, f, lib)

full_solution = gedim.LUSolver(stiffness, f, lib)
gedim.PlotSolution(mesh, dofs, strongs, full_solution, Dirichlet_top) # Plotting the FOM solution (the ground trouth)
```



Let us perform an error analysis and comment on the speed up!

In [32]:
```python
### compute error
import time
```

```
abs_err = []
rel_err = []
testing_set = np.random.uniform(low=P[:, 0], high=P[:, 1], size=(100, P.shape[0]))
speed_up = []

print("Computing error and speedup analysis") # Compute the erorr

for mu in testing_set:

  thetaA2 = mu[0]
  thetaf1 = mu[1]
  mu_3 = mu[2]

  #### I DO NOT NEED THE SOLVER #####
    # No need to assemble --> no solve any kind of system, just call the net
  Dirichlet_top = gedim.AssembleStrongSolution(Dirichlet_Term, 3, problemData, lib) ## label

  ##### full #####
  stiffness = thetaA1*stiffness1 + thetaA2*stiffness2
  weakTerm_down = thetaf1*weakTerm_down1
  f = weakTerm_down - (thetaA1*stiffnessStrong1 + thetaA2*stiffnessStrong2) @ Dirichlet_top


  start_fom = time.time()
  full_solution = gedim.LUSolver(stiffness, f, lib)
  time_fom = time.time() - start_fom
  # gedim.PlotSolution(mesh, dofs, strongs, full_solution, Dirichlet_top)

  #### reduced #####
  x_test = [[mu[0], mu[1], mu[2]]]
  x_test = np.float32(x_test)
  x_test = torch.tensor(x_test)

  start_rom = time.time()
  reduced_solution = np.asarray(net(x_test).detach().numpy())[0]
  time_rom = time.time() - start_rom

  speed_up.append(time_fom/(time_rom))

  proj_reduced_solution = basis_functions@reduced_solution
  # gedim.PlotSolution(mesh, dofs, strongs, proj_reduced_solution, Dirichlet_top)

  ### computing error
  error_function = full_solution - proj_reduced_solution
  error_norm_squared_component = np.transpose(error_function) @ inner_product @ error_function
  absolute_error = np.sqrt(abs(error_norm_squared_component))
  print(absolute_error)
  abs_err.append(absolute_error)

  full_solution_norm_squared_component = np.transpose(full_solution) @  inner_product @ full_solution
  relative_error = absolute_error/np.sqrt(abs(full_solution_norm_squared_component))
  rel_err.append(relative_error)
  print(relative_error)

# There are a lot of variablity in the errors
```

```
Computing error and speedup analysis
0.02821184718236291
0.014908633525250271
0.07616442397872149
0.015541296347706929
0.03766335037575535
0.009276231385921324
0.05719365989801668
0.014810570624140118
0.032620691231664285
0.013750165623966614
0.0405729203923825
0.008127863478883427
0.05900610769209333
0.012249218116076082
0.037401050686081634
0.00839834230956689
0.09512669743667924
0.019592785374797927
0.07050750194533502
0.01470807803004408
0.06127180256855271
0.012129656663648057
0.02045637606995963
0.0043147546530735055
0.03503938564104698
0.009261329038292005
0.027333812545155343
0.017505357024757827
0.03951699594194714
0.016443723627136062
0.08257323132842302
0.01638081395057979
0.06555599461016136
0.013988320972191347
0.07212403030351155
0.02072005363901202
0.03876076559983442
0.008767042409436874
1.617604829332954
0.32895825887977853
0.6837192040850134
0.13847357856053077
0.04227339745004451
0.008397214339237256
0.07107951639756235
0.014676881620522131
0.054006771452411784
0.01275873094622701
0.013584195980631736
0.002795474810406504
0.026187963547102074
0.0055709659663471895
0.014725626437153218
0.003224641106580426
0.04400968793924805
0.009126878694495221
0.11382070016326835
0.06607058559205269
0.025826928527340687
0.027011695659077774
0.12416705473559358
0.02476798895299906
0.12071354604773803
0.026734027571896756
0.848377814631573
0.23700054189685804
0.0301390785421146
0.005938991748945168
0.03994837708127216
0.01029693468220226
0.05864930779207655
0.09757275142895484
0.05222779560472152
0.01376082765536707
0.04962777831498411
0.01207310970875782
0.028332547102752438
0.006070245816001522
0.09597048445667407
0.018731500462683046
0.0806101128565935
0.01631531949678877
0.0508171262637631
0.010785368019487871
0.07579283997743448
0.014916081319311943
0.03032970539556055
0.008434558880418979
0.013676386742586626
0.015857943963793236
0.057536609127093984
0.01224344250687304
0.024063544788619797
```

0.0059847118000395565
0.06714673296915058
0.014161498823338203
0.0158390565678505
0.00351061268557254
0.10958906682472694
0.02512548460668675
0.07364061497463294
0.016042582923767874
0.03712234313207918
0.026629184662925128
0.18778004412376584
0.04096727218872616
0.073777323859217
0.015066172058200638
0.16487615440226092
0.035400075104476424
0.07141392493678779
0.015539143216420522
0.0806946941639414
0.18077789236094427
0.05335103303966353
0.017673243401288254
0.11511707916205155
0.024982797651049587
0.06083735599622492
0.013544730967062614
0.04618391024426444
0.01391309194879253
0.08708278588102569
0.03686957888036641
0.016344155356257144
0.0033485565402340254
0.016097016850897573
0.003814105035193591
0.021730814466563324
0.021399425189198167
0.0432727196424626
0.0200445618367447
0.049014847497908236
0.02242070571935231
0.05319665180134205
0.01103159474043813
0.03933850711836065
0.024427266351966796
0.059283806168710575
0.017123443343034655
0.04029981816534516
0.008459305005729228
0.111916504028514
0.06739874384843333
0.06802217628258674
0.01403614745650105
0.017945667588458933
0.006001709797772595
0.14830013814688423
0.029585892273652332
0.07350810023087762
0.015122213887077888
0.13902998554973475
0.03011458408323742
0.02869912253462392
0.007682396753743027
0.022177875693441942
0.0045974341376816856
0.030578279996783905
0.024810113788121604
0.07977351679861787
0.016355267540372146
0.01772061690014818
0.01748602118857569
0.010792307023187654
0.011486059075650102
0.026888157636357567
0.005537986544494978
0.028685813633302318
0.01325211116497352
0.05411497881739731
0.017036741554642836
0.032044132049254874
0.012080230070039399
0.04542204537910534
0.013394853151140886
0.051836454350734675
0.01089871263494314
0.04122167953060133
0.009400513101232138
0.02468839452260874
0.008687949834085553
0.18721314540977405
0.041956038855593944
0.03853943298655953
0.008858270355406662
0.055640885491766004

```
0.012030640481916658
0.03670355895898002
0.01020055555347715
0.03499766784063283
0.045967811210061285
0.0160053355085197
0.003198011996363442
0.01980685656861594
0.004062878535178802
0.18200220166313089
0.036942629016226886
0.07384122795203243
0.014784985514387804
```

In [33]:

```python
# The error increase, because we are using ML and so loose accuracy --> the better part is that we go faster
print("avarege relative error = ", np.mean(rel_err) )
print("avarege absolute error = ", np.mean(abs_err) )
print("avarege speed_up = ", np.mean(speed_up) ) # BUT we are faster!

# See the graph in notes 3 for more details on the error
```

```
avarege relative error =  0.02530663317527609
avarege absolute error =  0.08746066292620291
avarege speed_up =  36.87536589607264
```