The background of the slide features a dark, star-filled space scene. A large, semi-transparent sphere, resembling a planet or a black hole, is positioned on the right side. From its center, several bright, glowing, and wavy lines radiate outwards towards the left, representing gravitational waves. The colors range from deep blues and purples to bright yellows and oranges at the wave crests.

# Lecture 1: ULDM searches

## *Gravitational signatures*

Elisa G. M. Ferreira

Kavli IPMU & University of Sao Paulo

ISAPP School - *Quantum Fluids in the Universe*

June 14, 2023

# Outline

## Lecture 1: Cosm. Signatures 1

### Part I:

- Evidences
- ULDM models
- Observational signatures
- Gravitational Bounds

### Part II:

*Practice!*

#### Notebook 1: linear

- Linear observational signature of ULDM:  
suppression of structures

## Lecture 2: Cosm. Signatures 2

### Part I:

- Cont. ULDM gravitational bounds
- Future of grav. observations

### Part II:

#### Notebook 2: non-linear

- Consequences on small scales

## Lecture 3: interac. with SM

- Cont. ULDM gravitational bounds
- DM Superfluid
- Interaction of ULDM with SM
  - Axion/ALPs interaction in astrophysical systems
  - Direct detection

# *Disclaimer*

- Impossible to cover all the possible searches there are now! But I will do my best to give a general view.
- Biased review of the ULDM field
- Field that is changing rapidly, so my apologies for not mentioning your model or reference
- Lectures are going to have a practical component. Hope it is useful in your research!

Units of mass, energy and momentum = eV  
Length =  $\text{eV}^{-1}$

BUT sometimes (astro/cosmology)  
1 parsec (pc)  $\sim 3 \times 10^{16}$  m

Natural units( $c = \bar{h} = 1$ )  
 $1 \text{ kg} \rightarrow 5 \times 10^{35} \text{ eV}$   
 $1 M_{\odot} \rightarrow \sim 10^{66} \text{ eV}$

# *Further reading*

*Reviews!!!*

Main reference for gravitational searches:

- **Elisa Ferreira**, *Ultra-light dark matter*, The Astronomy and Astrophysics Review. 29 (2021) 1, 7, arXiv:[2005.03254](#)

Other very good reviews

- Lam Hui, *Wave dark matter*, Ann.Rev.Astron.Astrophys. 59 (2021) 247-289, arXiv: [2101.11735](#)
- Jens C., Niemeyer *Small-scale structure of fuzzy and axion-like dark matter*, Prog.Part.Nucl.Phys. 103787, arXiv: [1912.07064](#)
- David Marsh, *Axion cosmology*, Phys.Rept. 643 (2016) 1-79, arXiv: [1510.07633](#)

Reference for non-gravitational searches:

- Francesca Chadha-Day et al., *Axion dark matter: What is it and why now?*, Sci.Adv. 8 (2022) 8, abj3618, arXiv: [2105.01406](#)

+ many references (*in the slides*)

# Outline

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  - Direct detection

# *Evidences for dark matter*

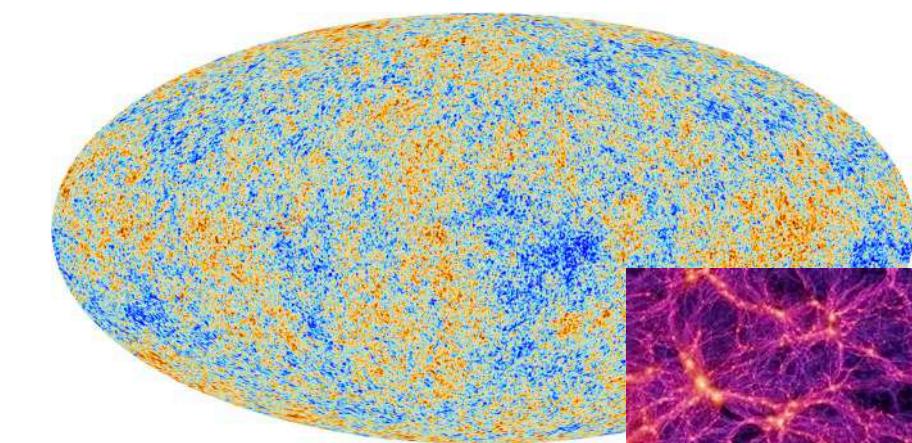
We can observe its effects in

Galaxies

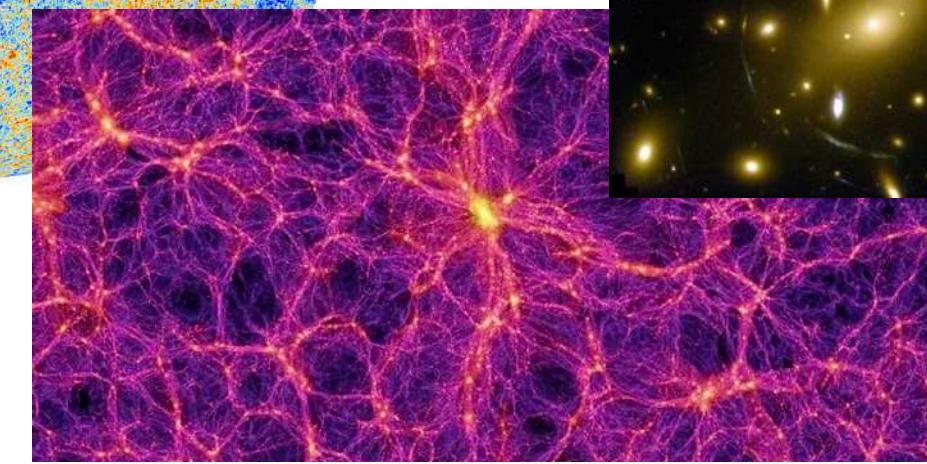


NASA and ESA

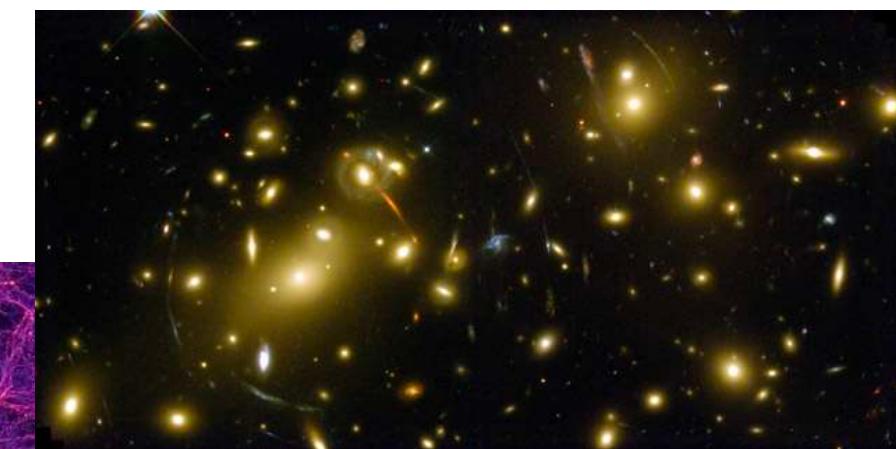
CMB+LSS



ESA and the Planck Collaboration

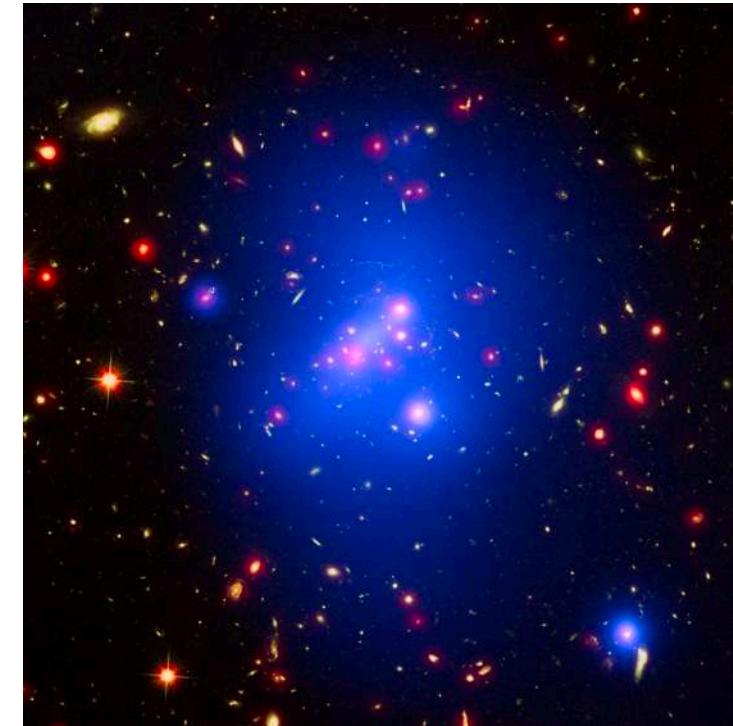


Springel & others / Virgo Consortium



NASA and ESA

Clusters

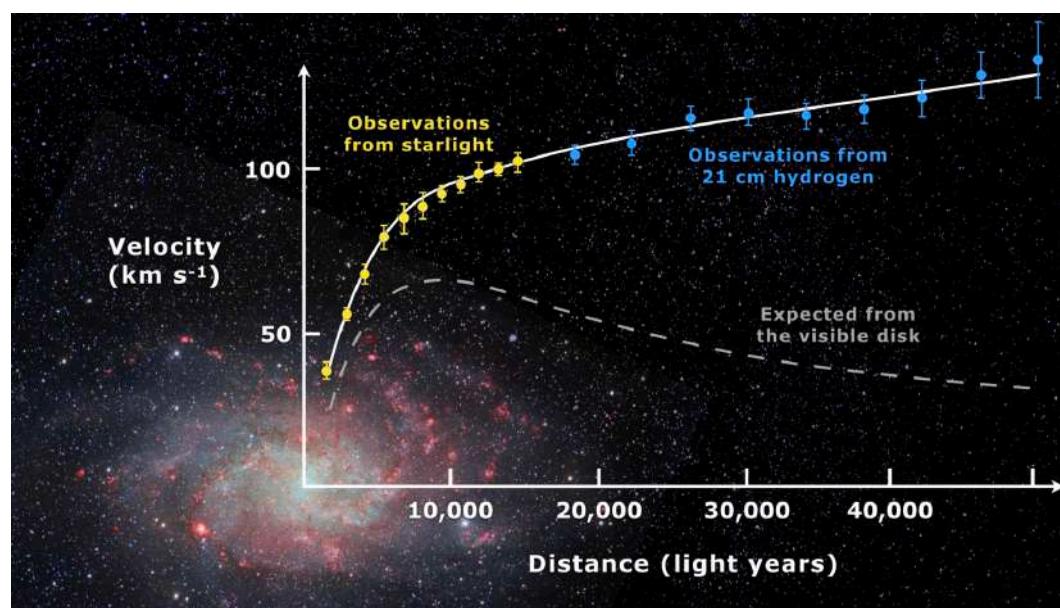


CC BY 4.0

Huge amount of evidence  
From all scales

# Evidences for dark matter - properties

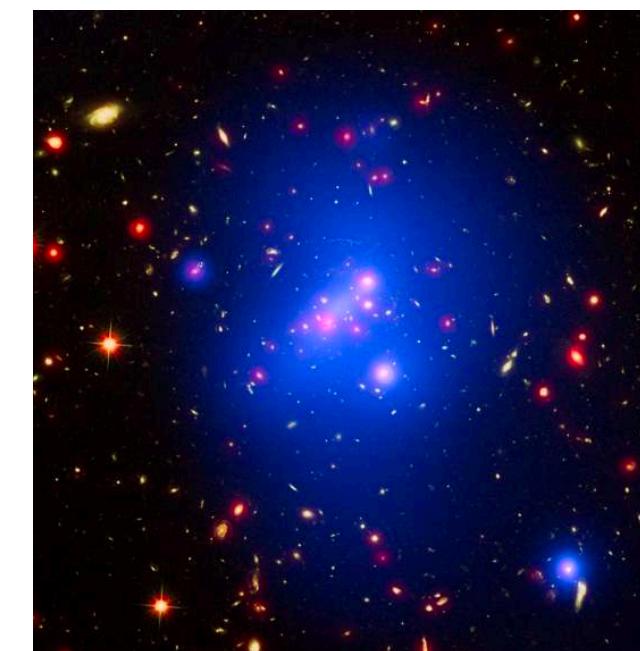
## Galaxy rotation curves



Credit: Mario De Leo

- Mass fraction
- Distribution

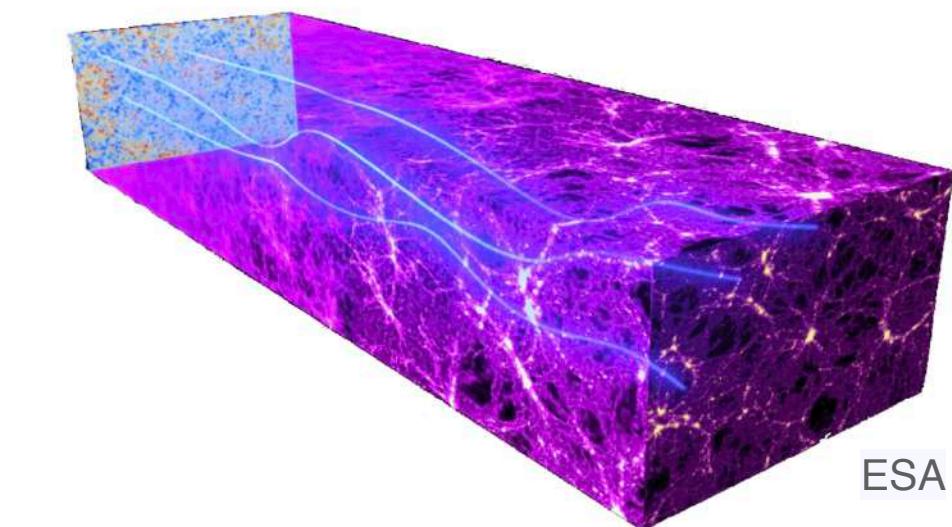
## Clusters



CC BY 4.0

- Mass fraction
- Distribution

## Lensing



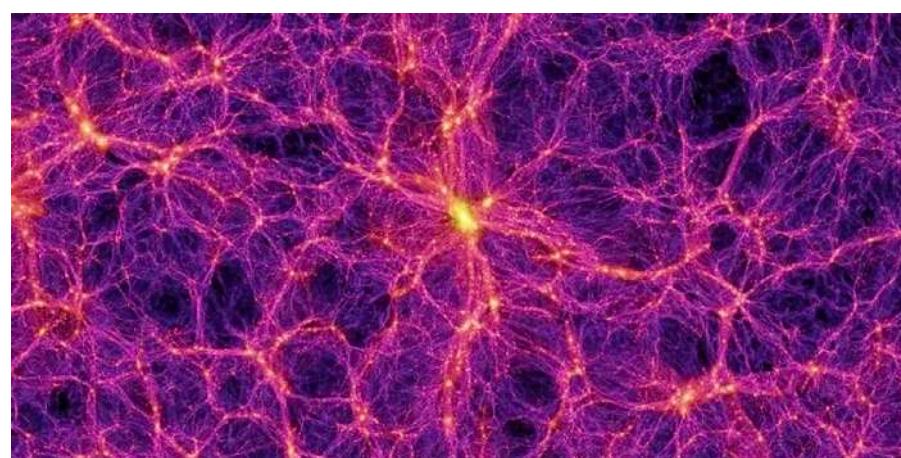
ESA

- Strong lensing
- Mass fraction
- Distribution

- Weak lensing
- Distribution
- Shape
- Structure

- Micro lensing
- Mass fraction
- Smoothness
- Structure

## Large Scale Structure

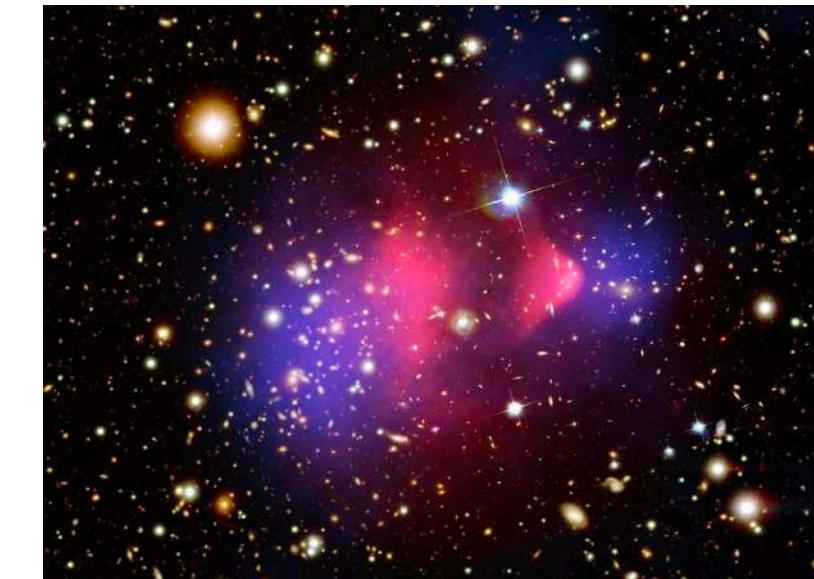


Springel & others / Virgo Consortium

### CMB/LSS

- Ratio of DM/collisional matter
- Thermal history

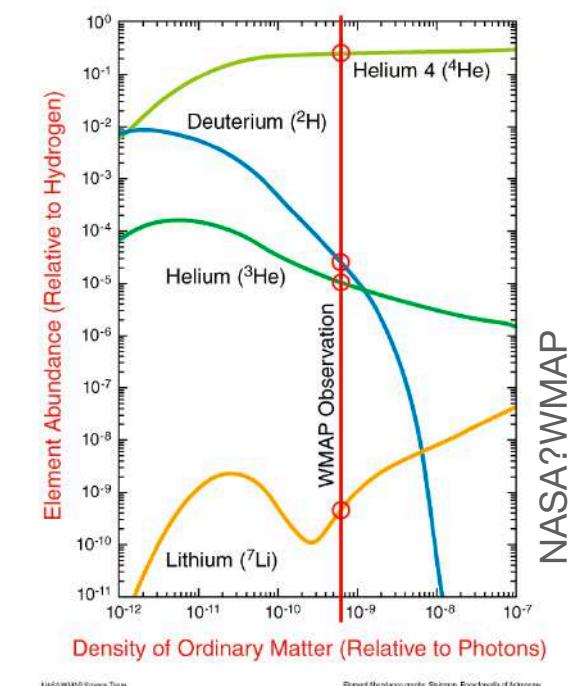
## Cluster collision



NASA/CXC/CfA and NASA/STScI

- Distribution
- Separation from collisional matter
- Self-interaction

## Big Bang Nucleosynthesis

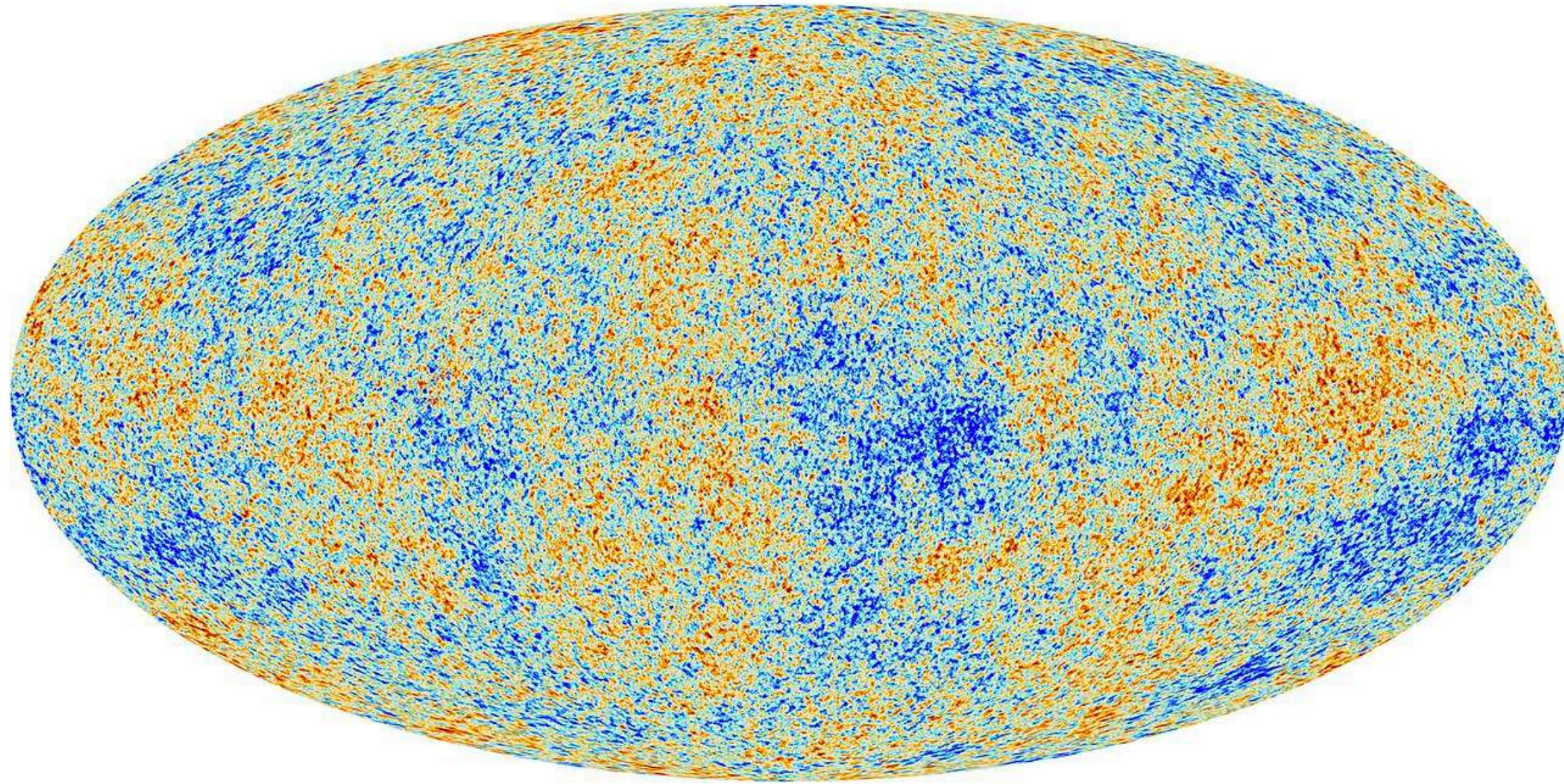


NASA/WMAP  
Based on Fukugita and Shimasaku, MNRAS, 2000

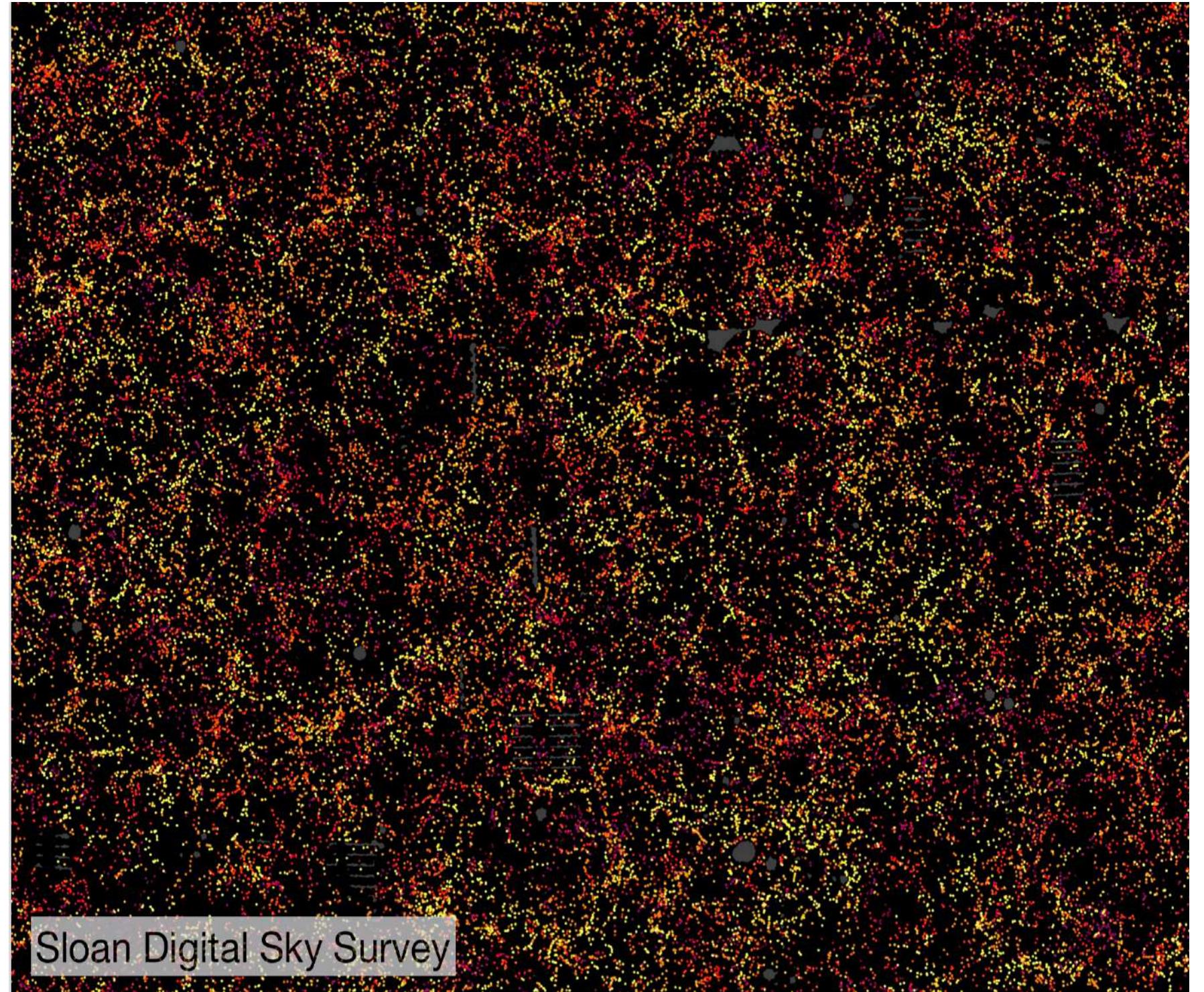
- Amount of baryons

# *Large scale structure*

Cosmic Microwave Background

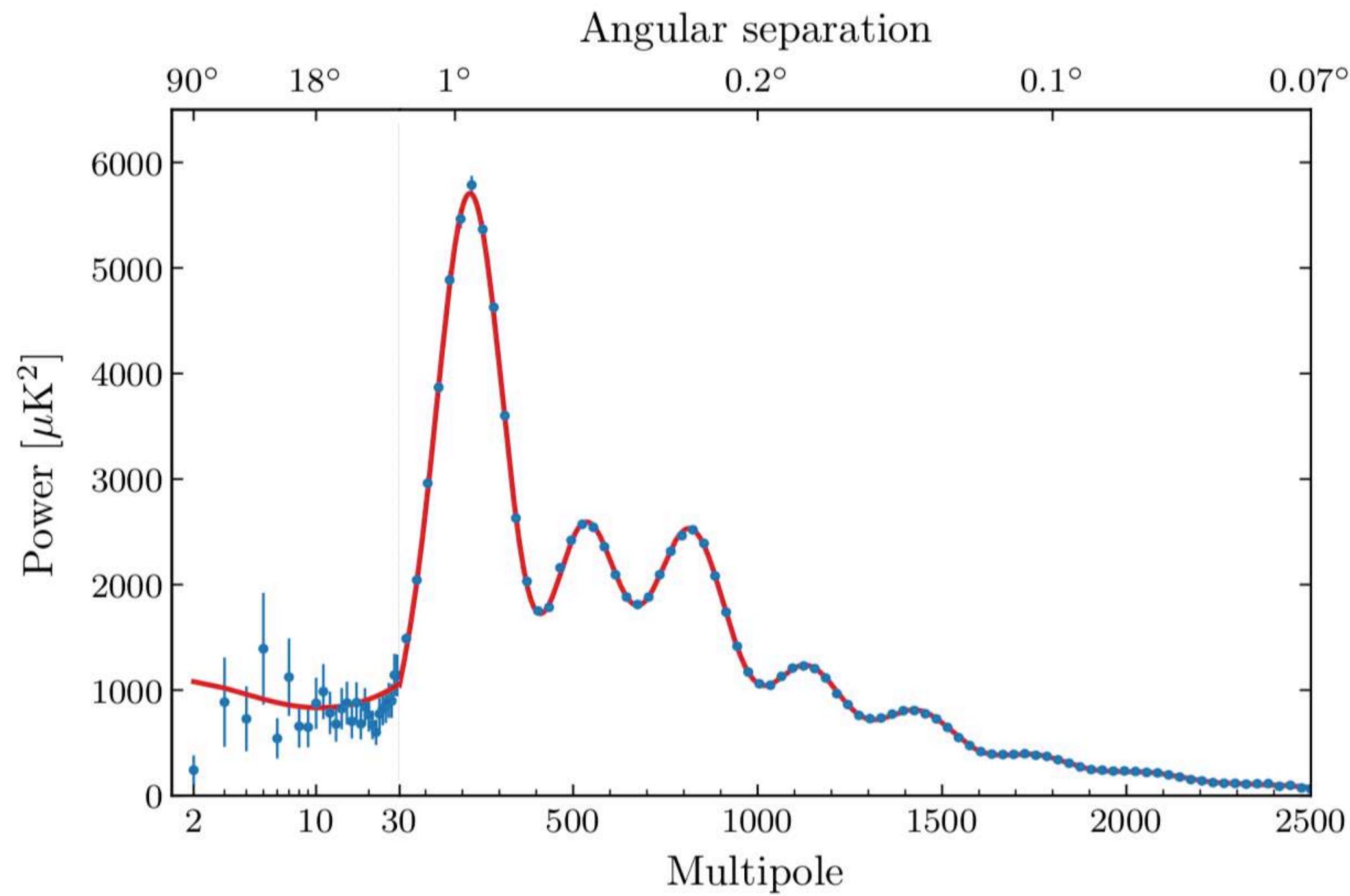


Large Scale Structure

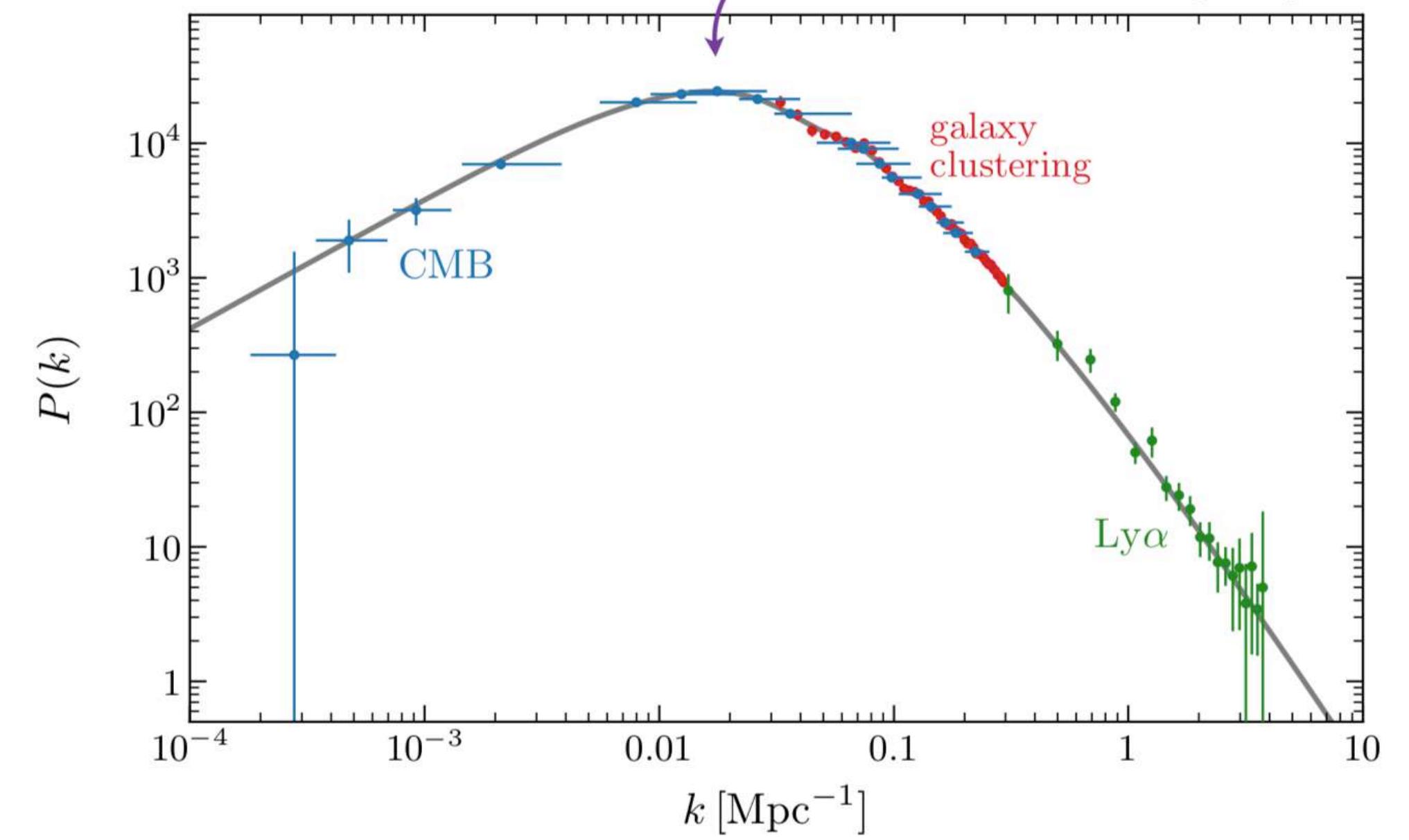


# Large scale structure

Cosmic Microwave Background



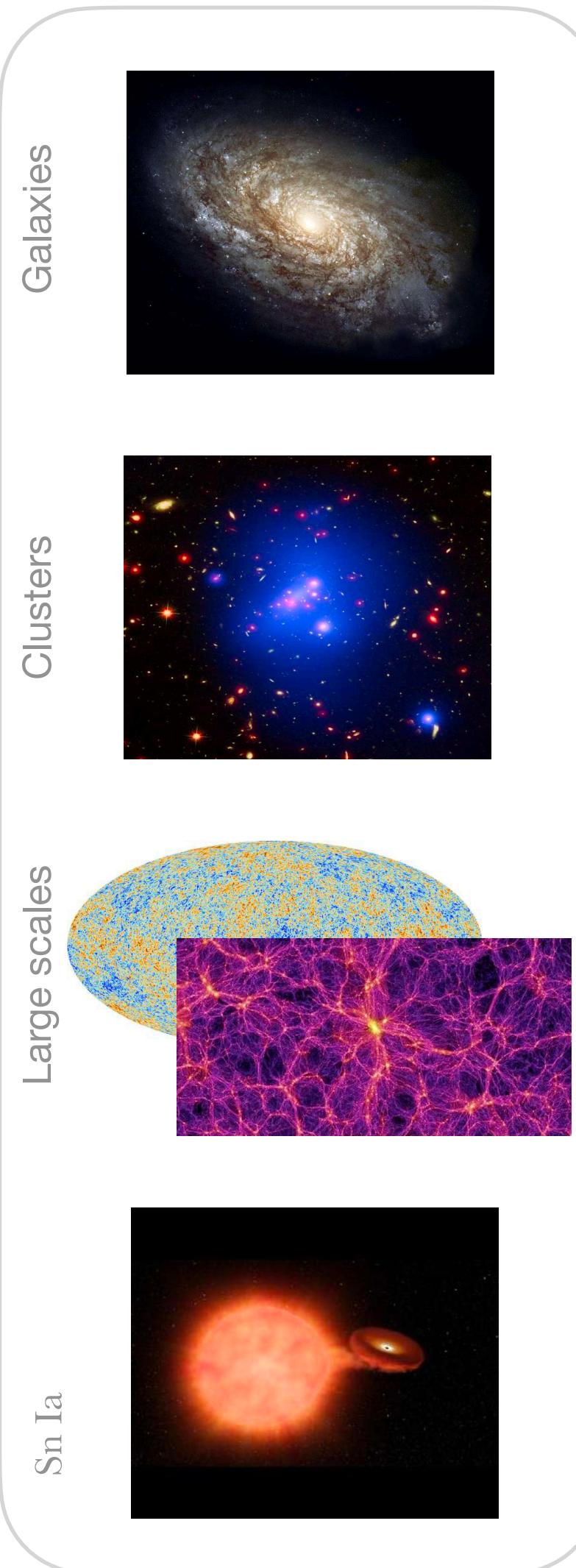
Large Scale Structure



$$\Omega_m = 0.308 \pm 0.012$$

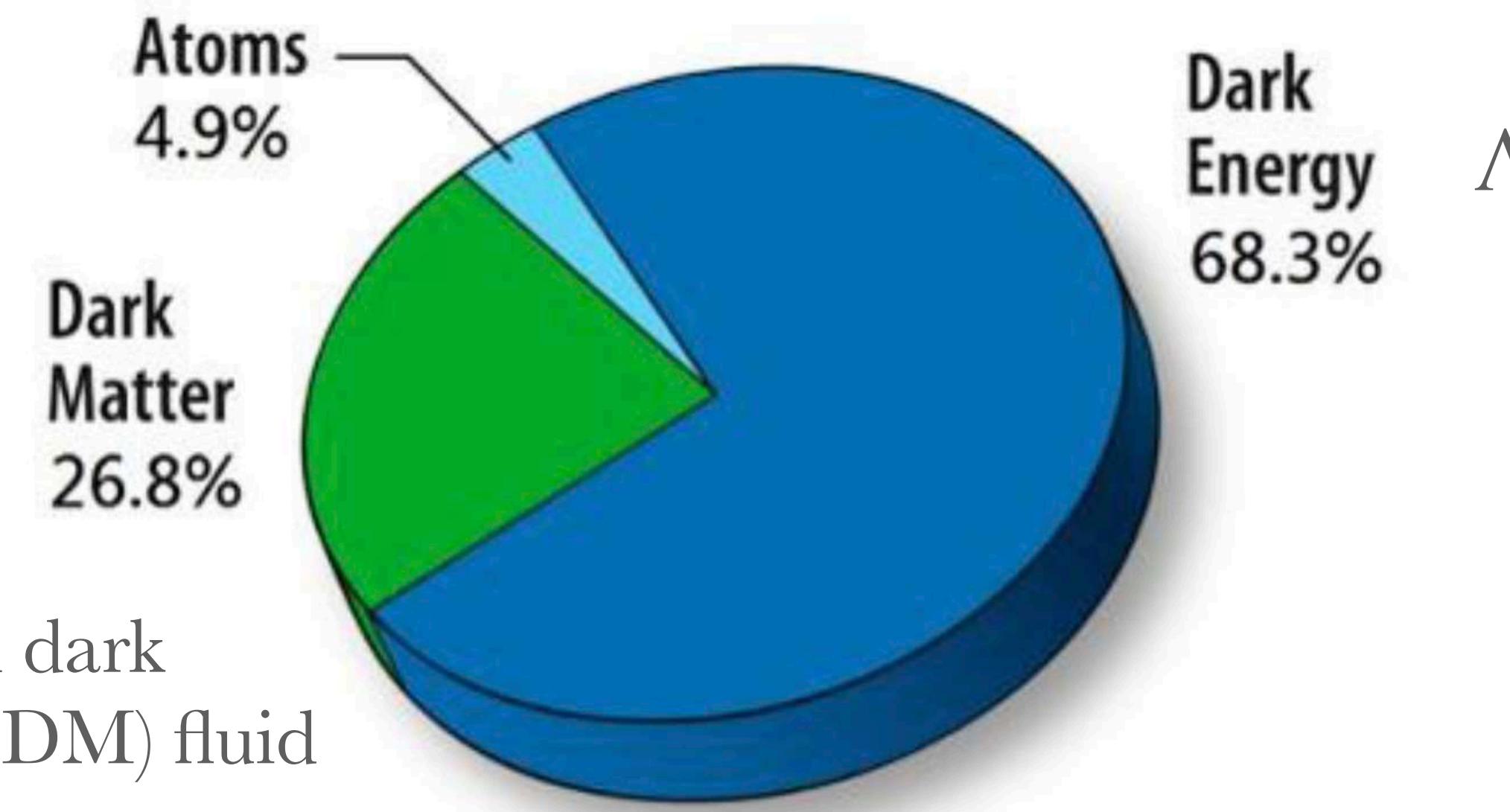
(*Planck 2018*)

# *What we **know** about dark matter*



$\Lambda$ CDM – the **standard cosmological model**

Successful description of our universe with 6 free parameters, tested to sub-percent precision.



$\Lambda$ CDM  
simple but exotic model!

# *Cold dark matter*

- **Cold:** moves much slower than  $c$
- **Pressureless:** gravitational attractive, clusters
- **Dark** (transparent): no/weakly electromagnetic interaction
- **Collisionless:** no/weakly self-interaction or interaction with baryons
- **Abundance:** amount of dark matter today known

CDM on large scales described by a ***perfect fluid***:

$$\begin{aligned} \text{Backg.: } & \rho, P \\ & w = P/\rho \end{aligned}$$

$$\text{with } P = 0 \Rightarrow w = 0$$

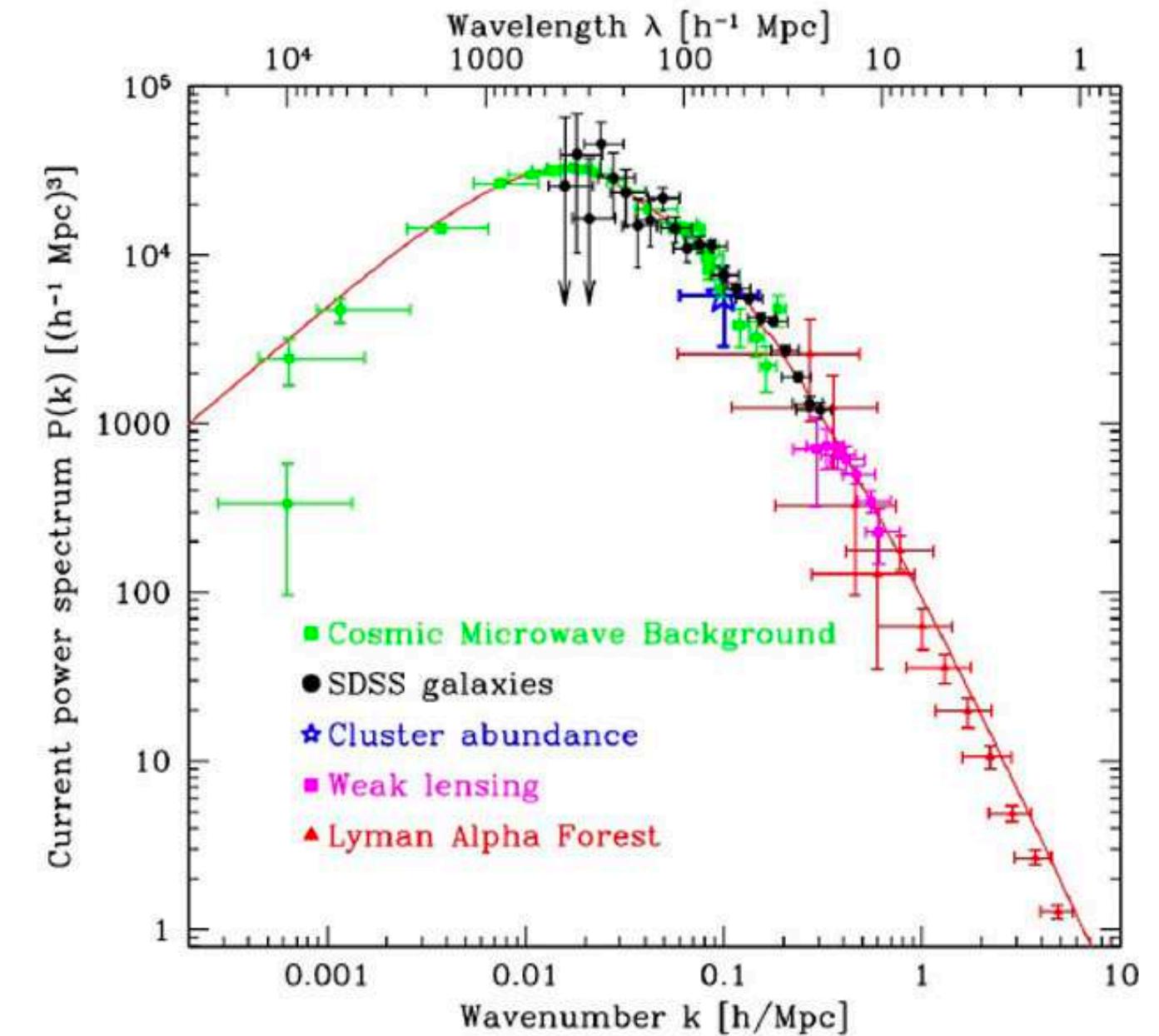
$$\Rightarrow \rho \propto a^{-3}$$

$$\text{Pert.: } \delta, \theta$$

$$\text{with } c_s \sim 0$$

# *Cold dark matter*

- **Cold**: moves much slower than  $c$
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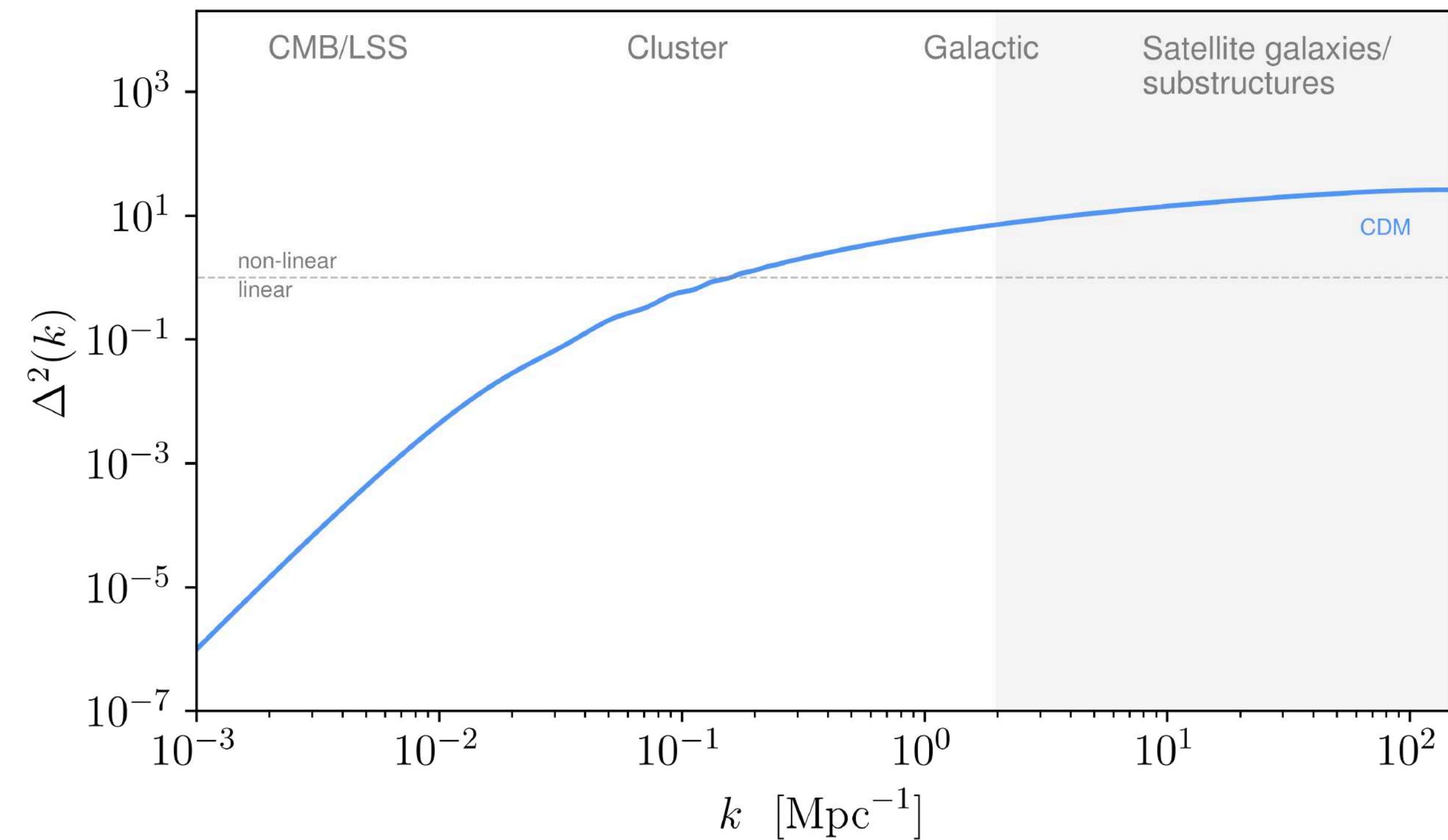
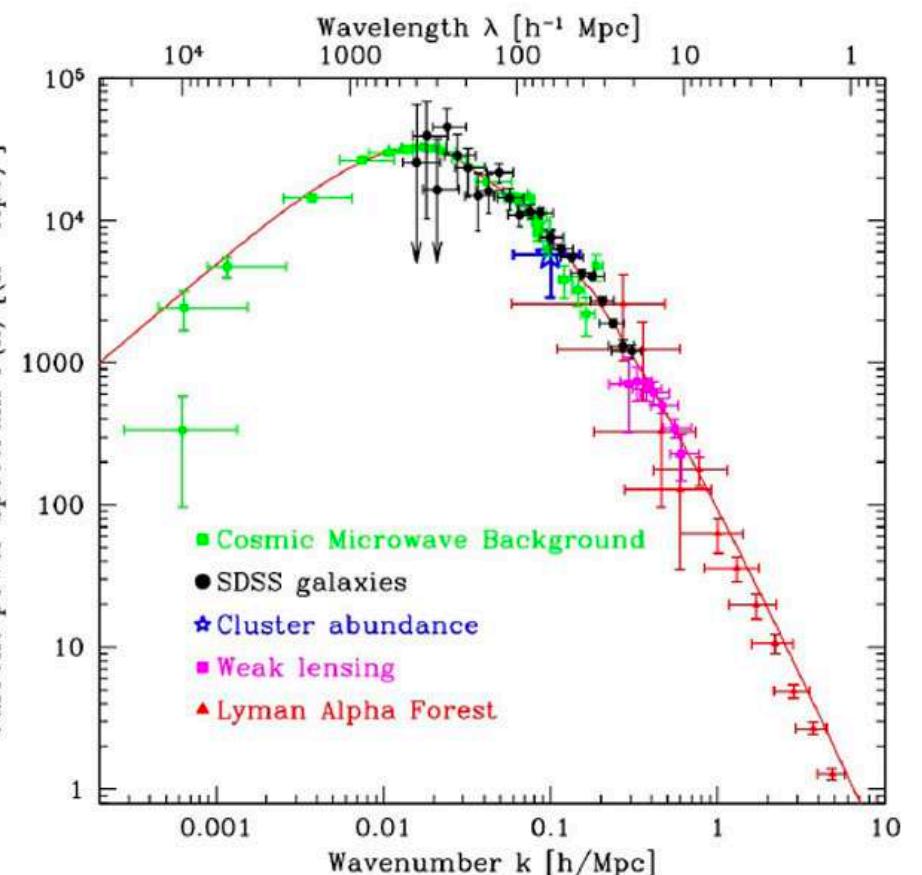
Many observational probes for  $k \sim 10^{-3} - 10 \text{ Mpc}^{-1}$   
range of redshift  $z < 3 - 4$

Incredible agreement to CDM!

# *What we know about dark matter*

Properties:

From LSS:



Measure PS well until scales  
 $k \sim 10 - 20 \text{ Mpc}^{-1}$

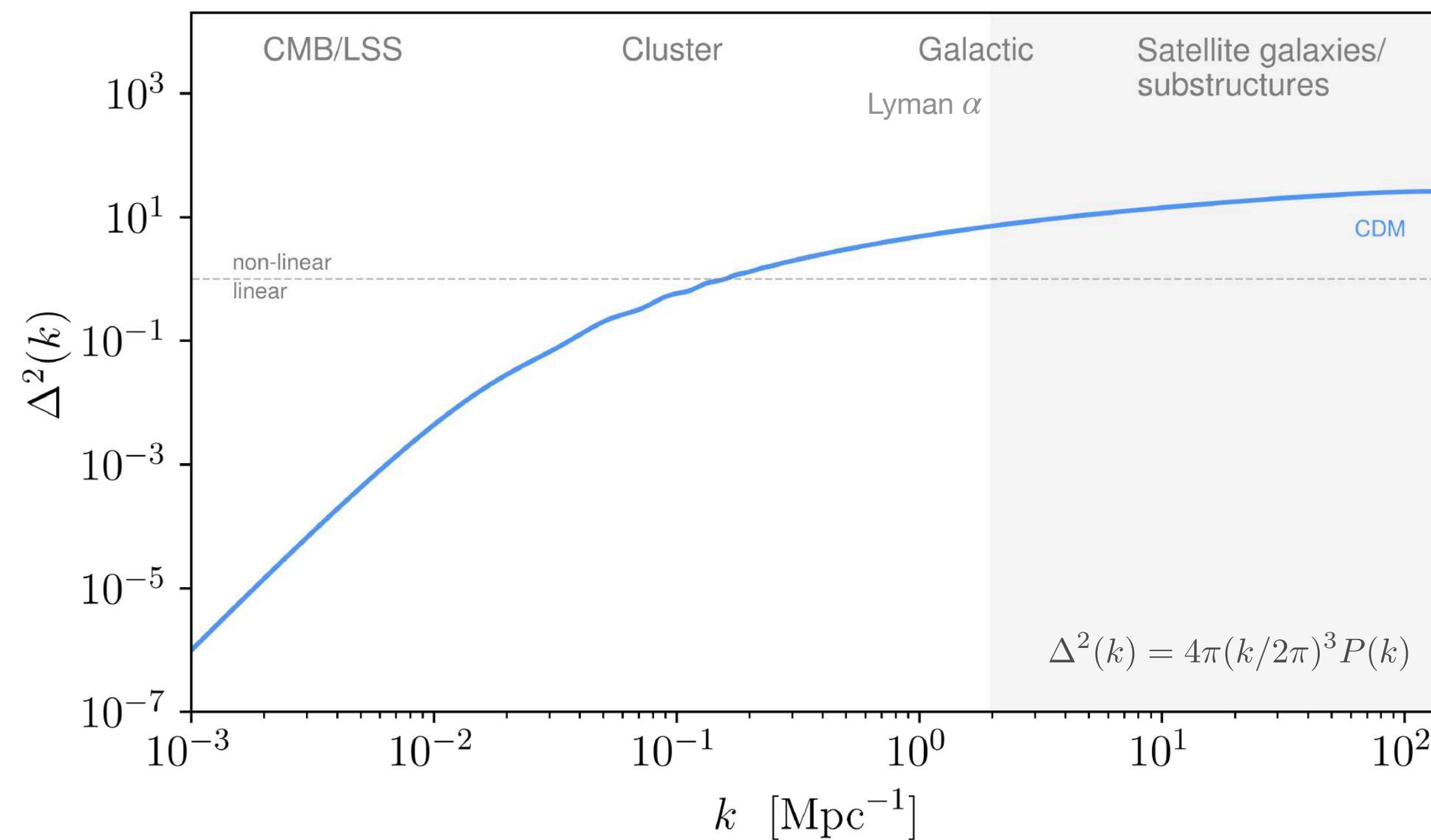
Dimensionless power spectrum

$$\Delta^2(k) = 4\pi(k/2\pi)^3 P(k)$$

# What we *know* about dark matter

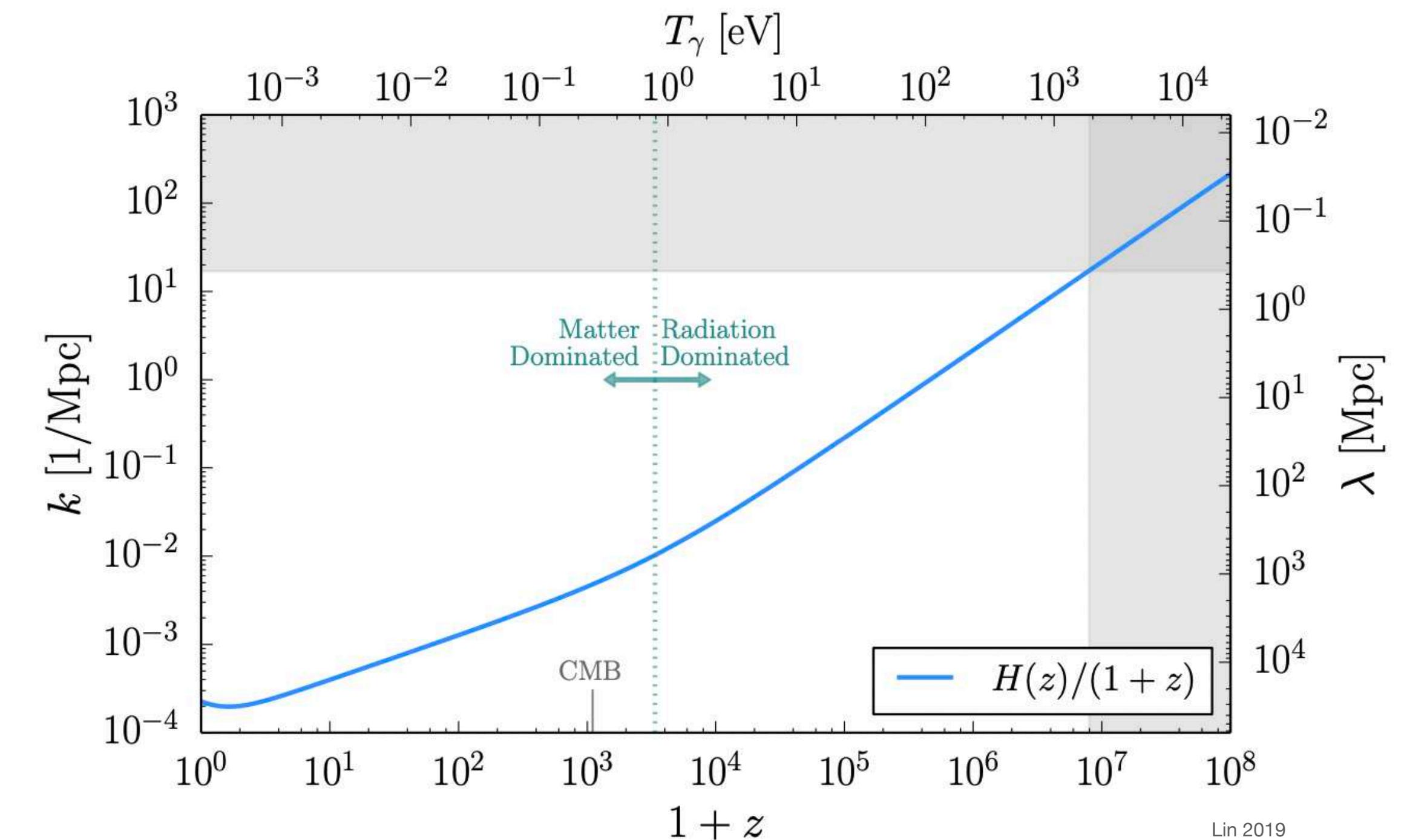
Properties:

$$T_\gamma = T_{\gamma,0}(1+z)$$



CDM pert. ( $c_s = 0$ ) inside **Hubble radius**:

$$\delta \propto \begin{cases} \log a & \text{rad. domination} \\ a & \text{matter domination} \end{cases}$$



Perturbation modes enter the **Hubble radius**

$$\lambda_{phys} = a/k = H^{-1}$$

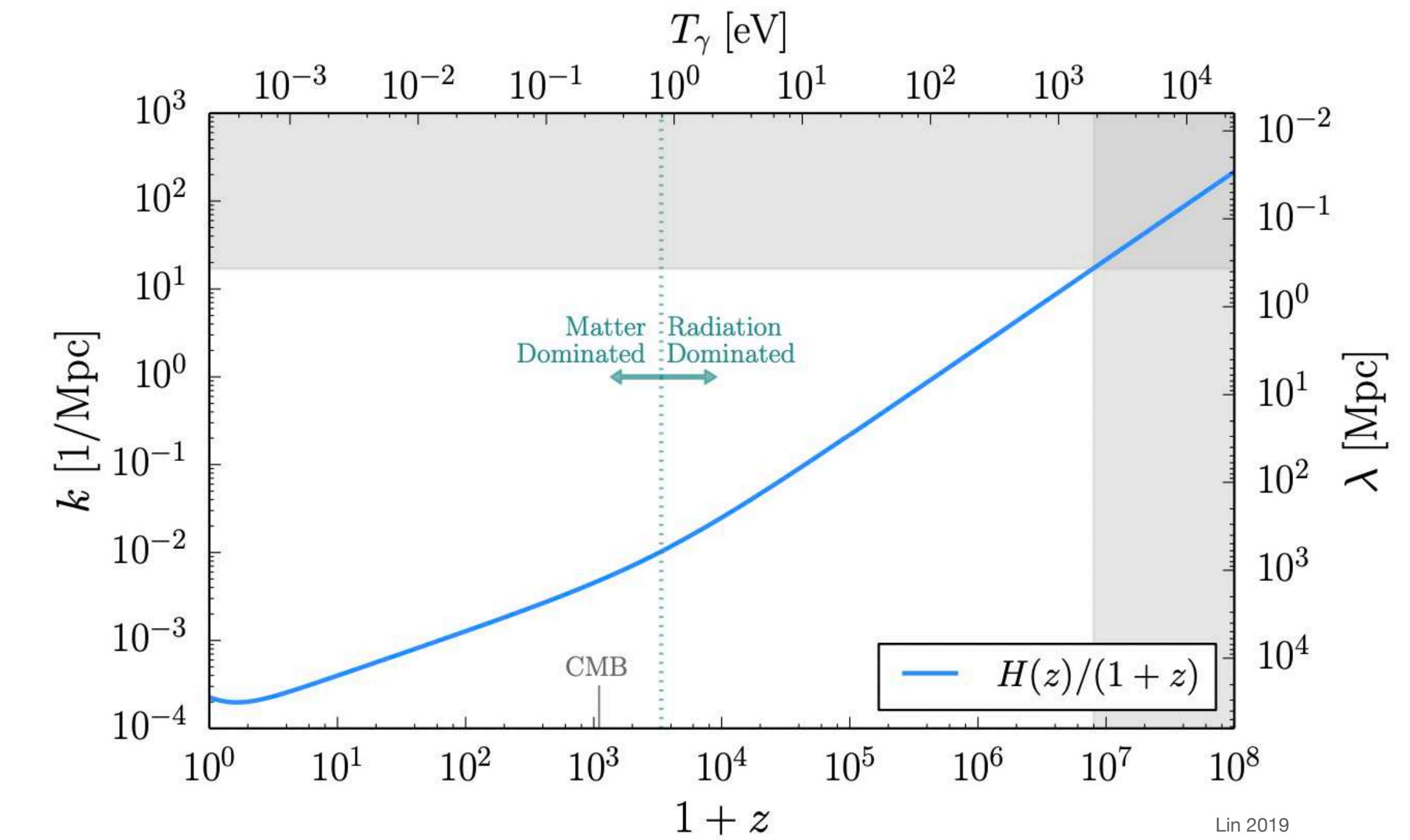
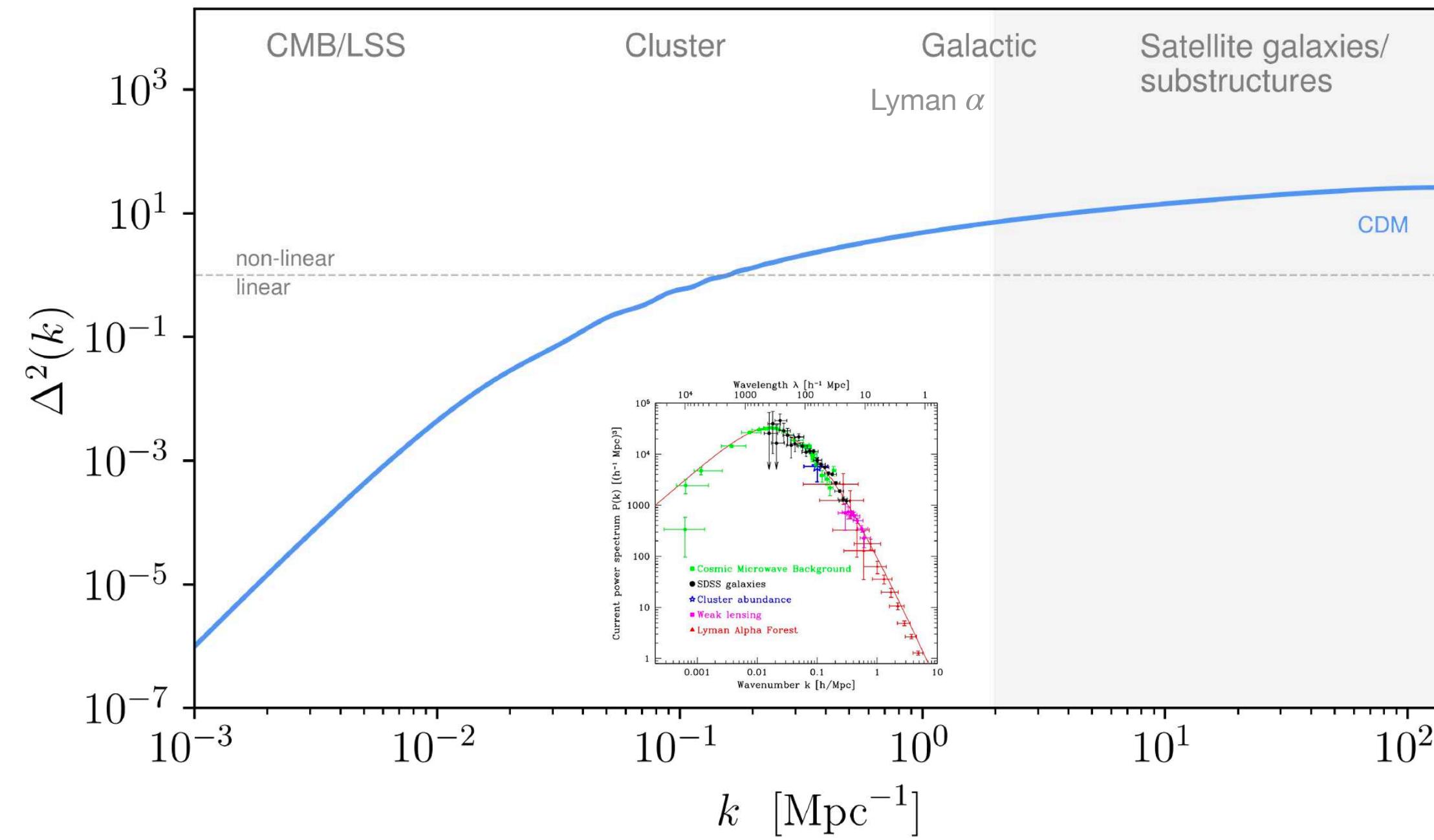
$$k = aH = H/(1+z)$$

After this, the density pert. of **CDM** start to evolve, **grow** - contribute to the PS

# What we *know* about dark matter

Properties:

$$T_\gamma = T_{\gamma,0}(1+z)$$

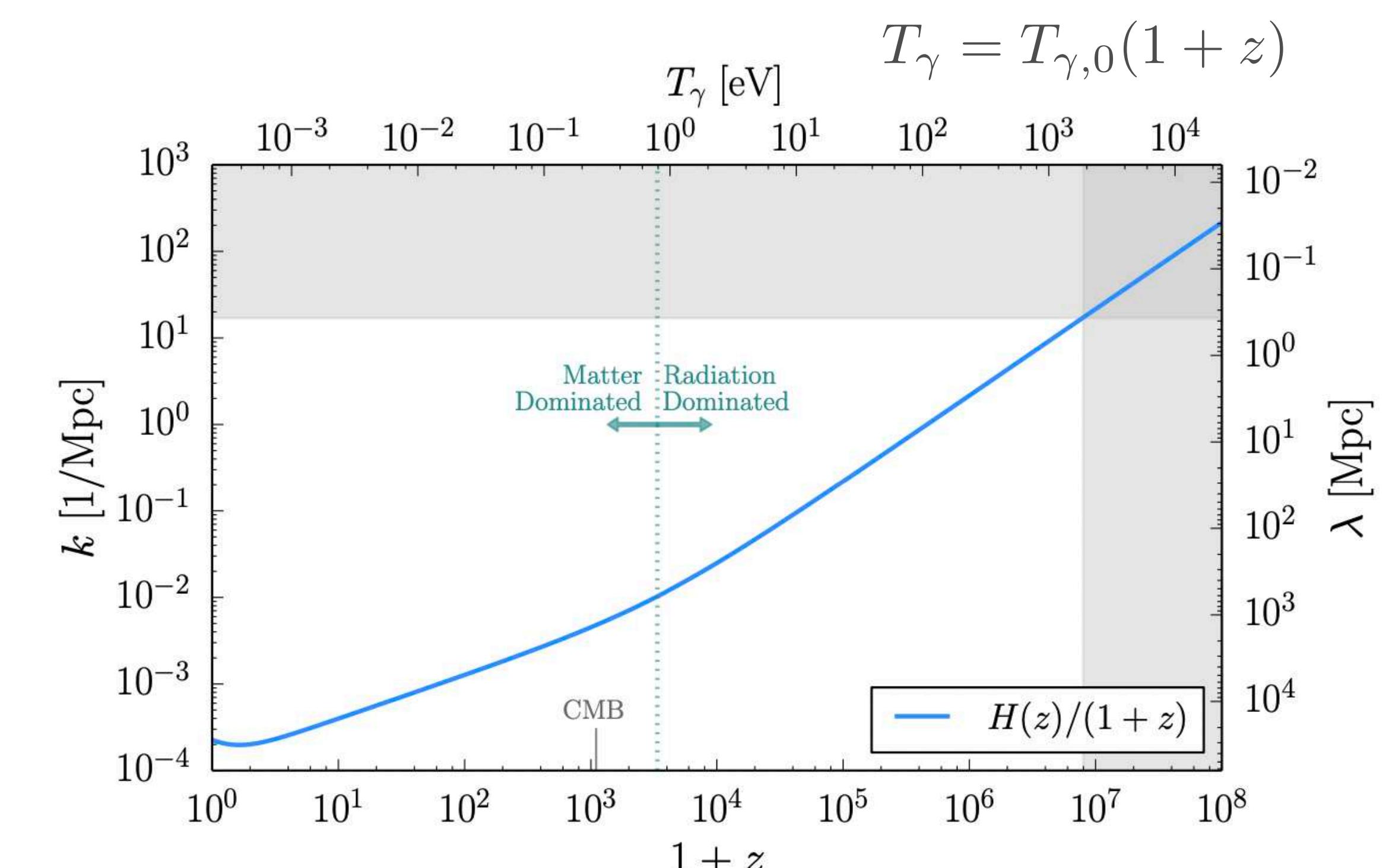
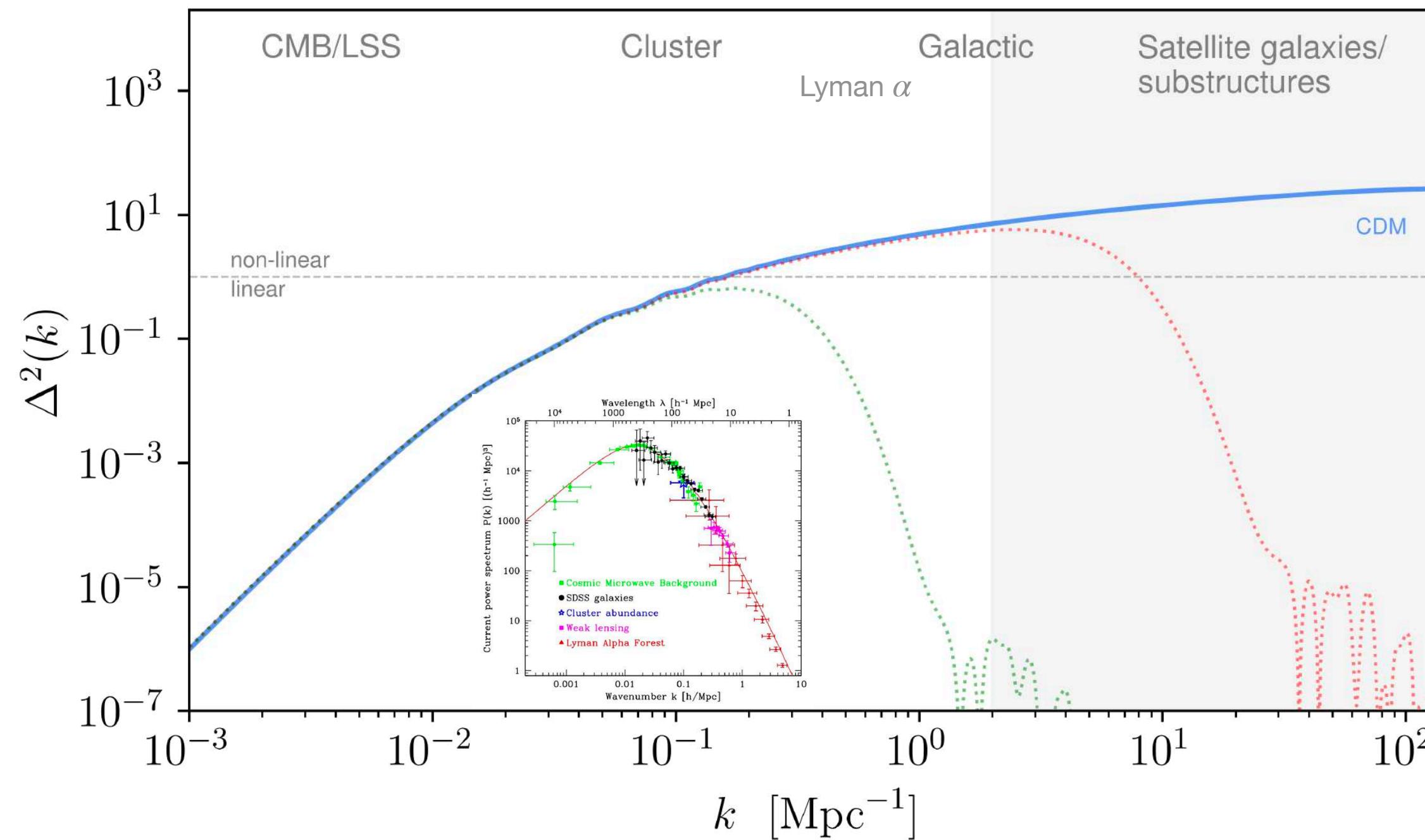


Perturbation modes enter the Hubble radius       $\lambda_{phys} = a/k = H^{-1}$   
 $k = aH = H/(1+z)$

So we can describe the observations, all the modes in the white region ( $< 10 \text{ Mpc}^{-1}$ ) are inside the **Hubble radius** and contribute to the PS, and are very precisely described by CDM  $\Rightarrow$  **cold and pressureless**

# What we *know* about dark matter

Properties:



Lin 2019

If **DM relativistic (or hot)** when  $z < 10^7$ , this mode is inside  $R_H$ , so it will contribute to the PS - since relativ. pert. DO NOT cluster, we would have a **suppression in the power spectrum** for  $k < 10 - 20 \text{ Mpc}^{-1}$  - *not in agreement with observations!*

⇒ DM has to be non-relativistic before  $z = 10^7$

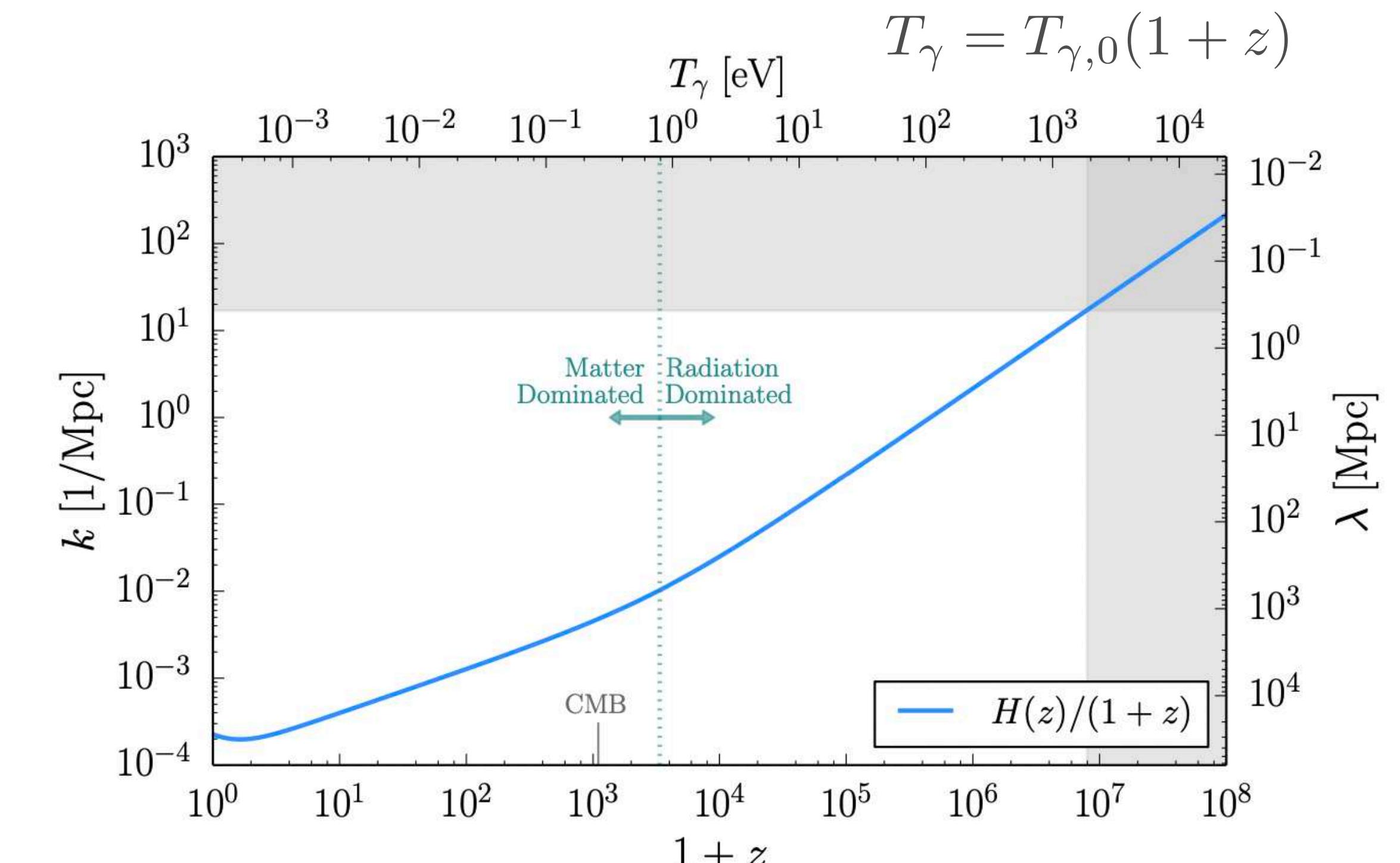
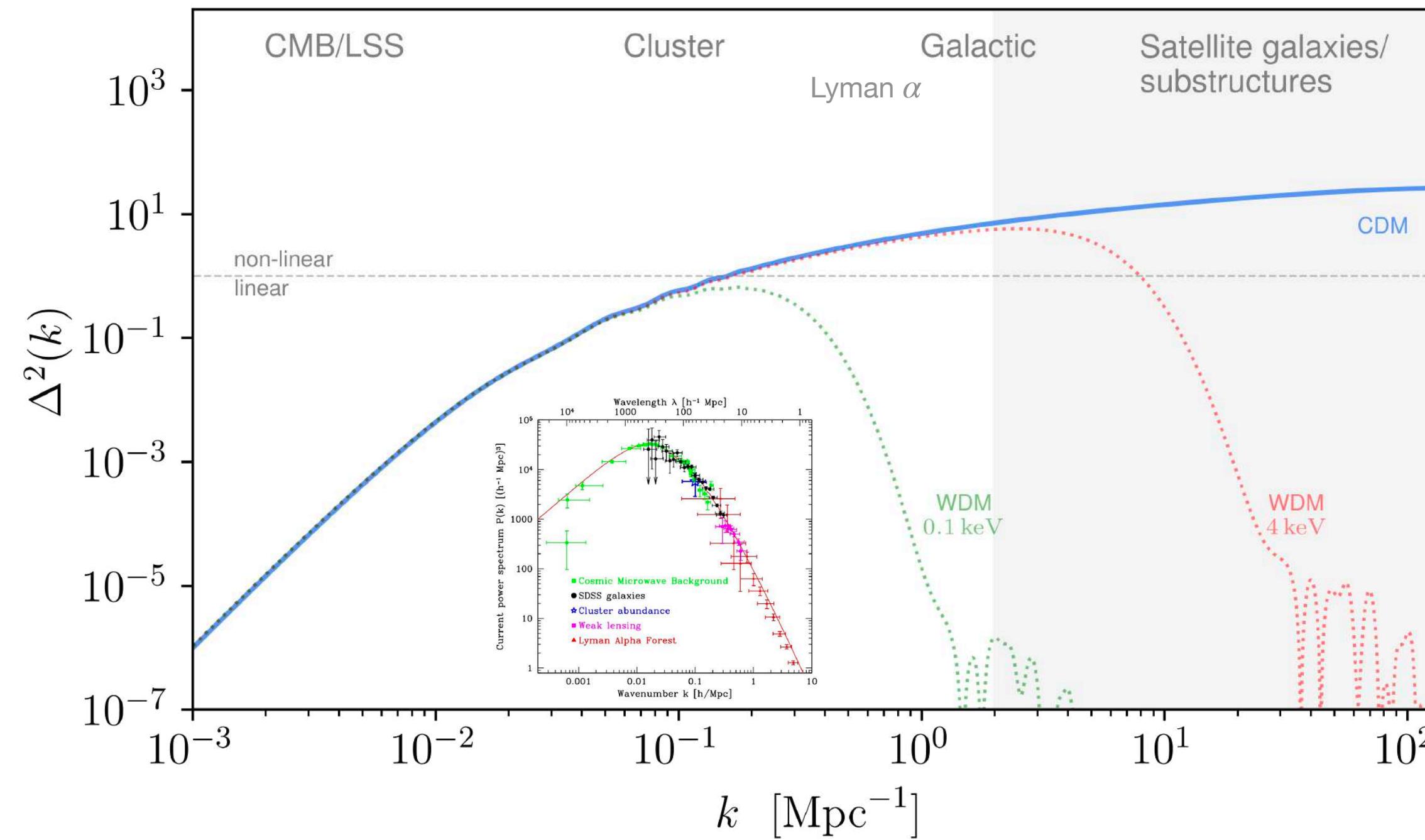
If **DM in thermal equilibrium** with the baryon-photon plasma ( $T_{dm} = T_\gamma$ )

⇒  $m_{dm} > \text{keV}$

WDM bound

# What we *know* about dark matter

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Lin 2019

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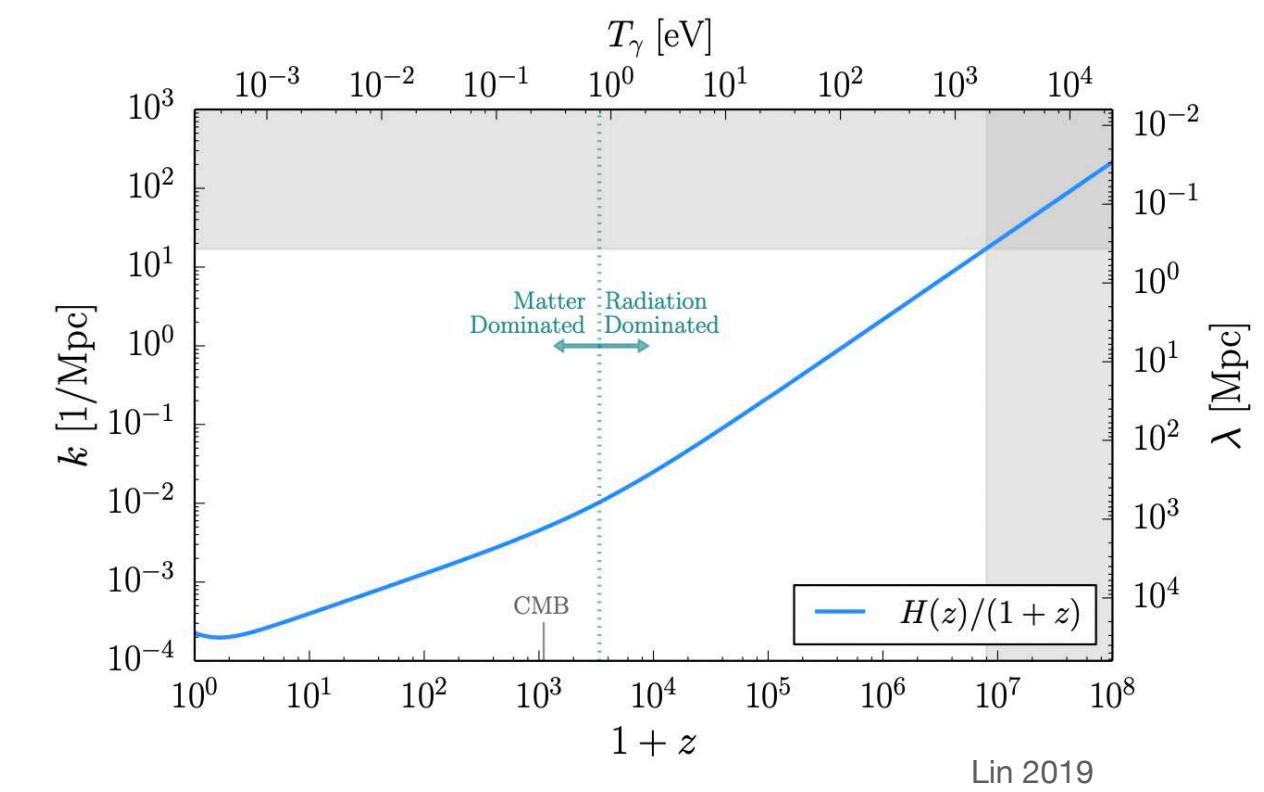
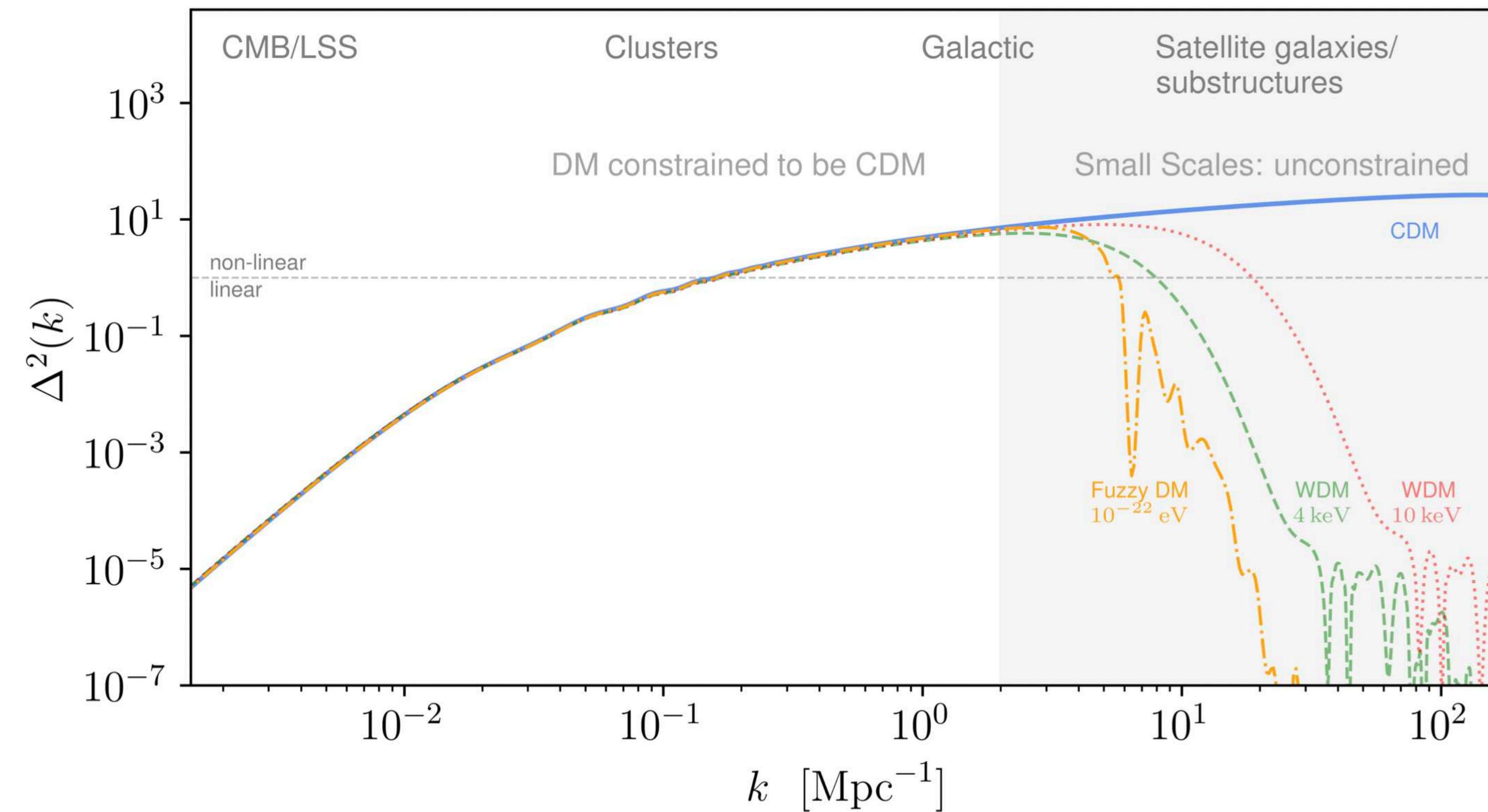
If **DM in thermal equilibrium** with the baryon-photon plasma ( $T_{dm} = T_\gamma$ )

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WDM bound

# What we *know* about dark matter

Properties:



Deviations from CDM in the highlighted region are allowed, since highly unconstrained!

# What we *know* about dark matter

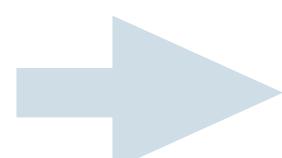
- Dark (transparent/neutral): DM does not interact electromagnetically
- Collisionless: no/weakly self-interaction; non-interacting

If DM had a *charge*  $e$ :

- Suppression of the power spectrum

Charged DM particles interact with the Standard Model via a small coupling through the photon

If the DM is coupled with the baryon-photon plasma during *recombination*, the DM density fluctuations can be washed out due to the radiation pressure and the photon diffusion (Silk damping). The BAO structure will also be directly altered through the coupling.



Interactions of DM with SM particles and itself at early times would **suppress** the power spectrum, since the radiation pressure of the baryons and photons would prevent DM density perturbations from growing and lead to the presence of “dark acoustic oscillations”

Current bounds:

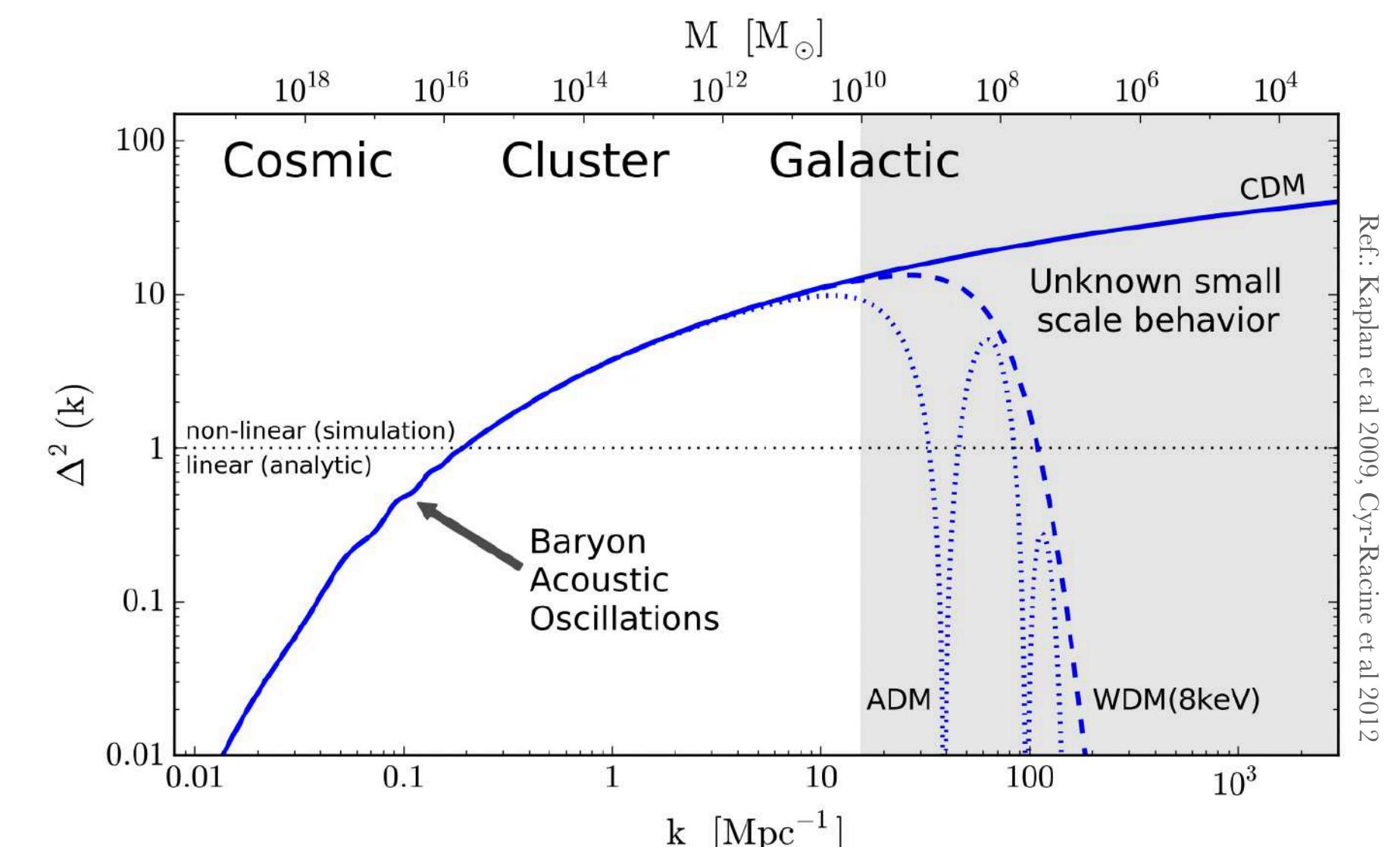
Charge:

Self-interaction

DM has neutral or charge < mili-charge!

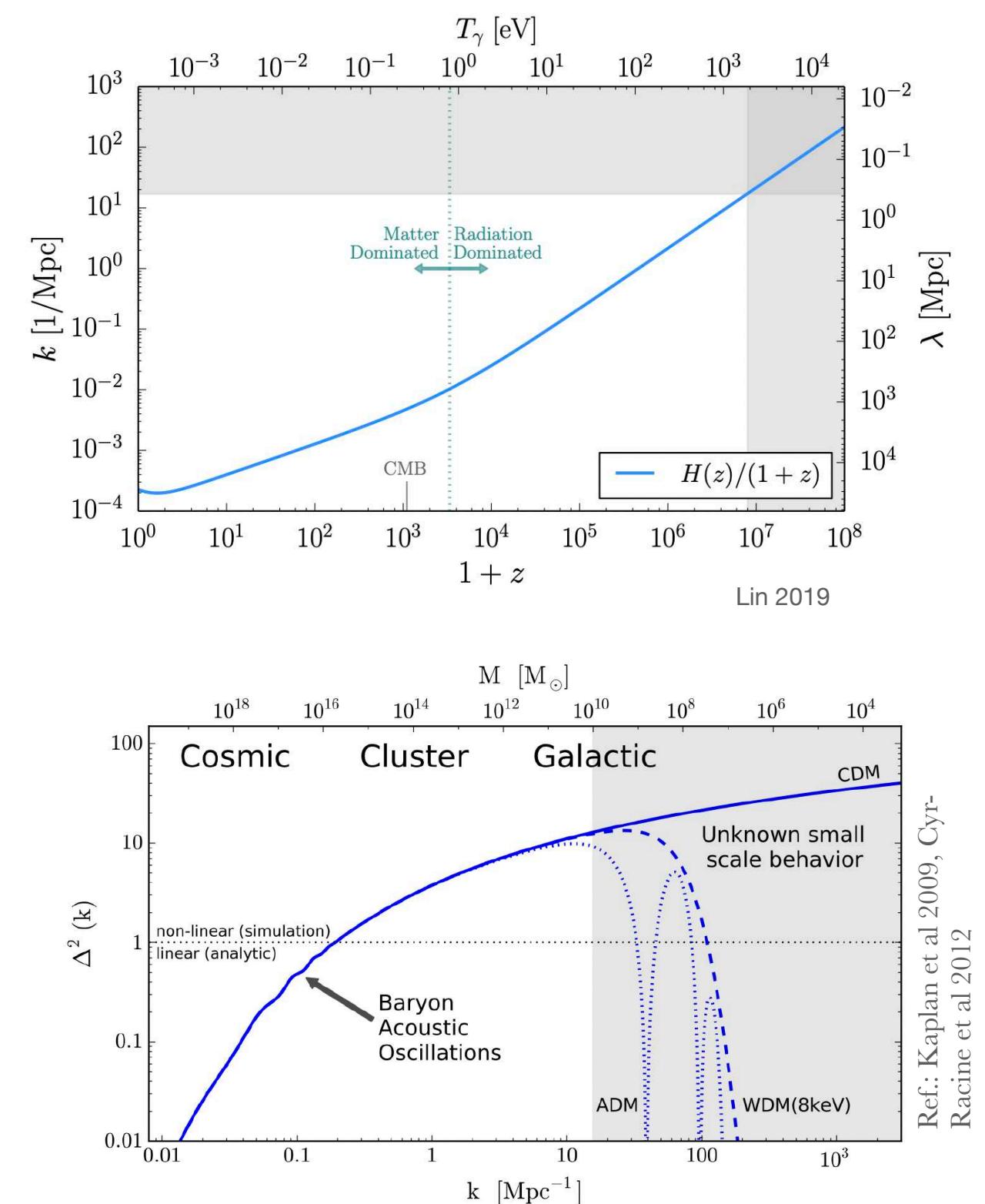
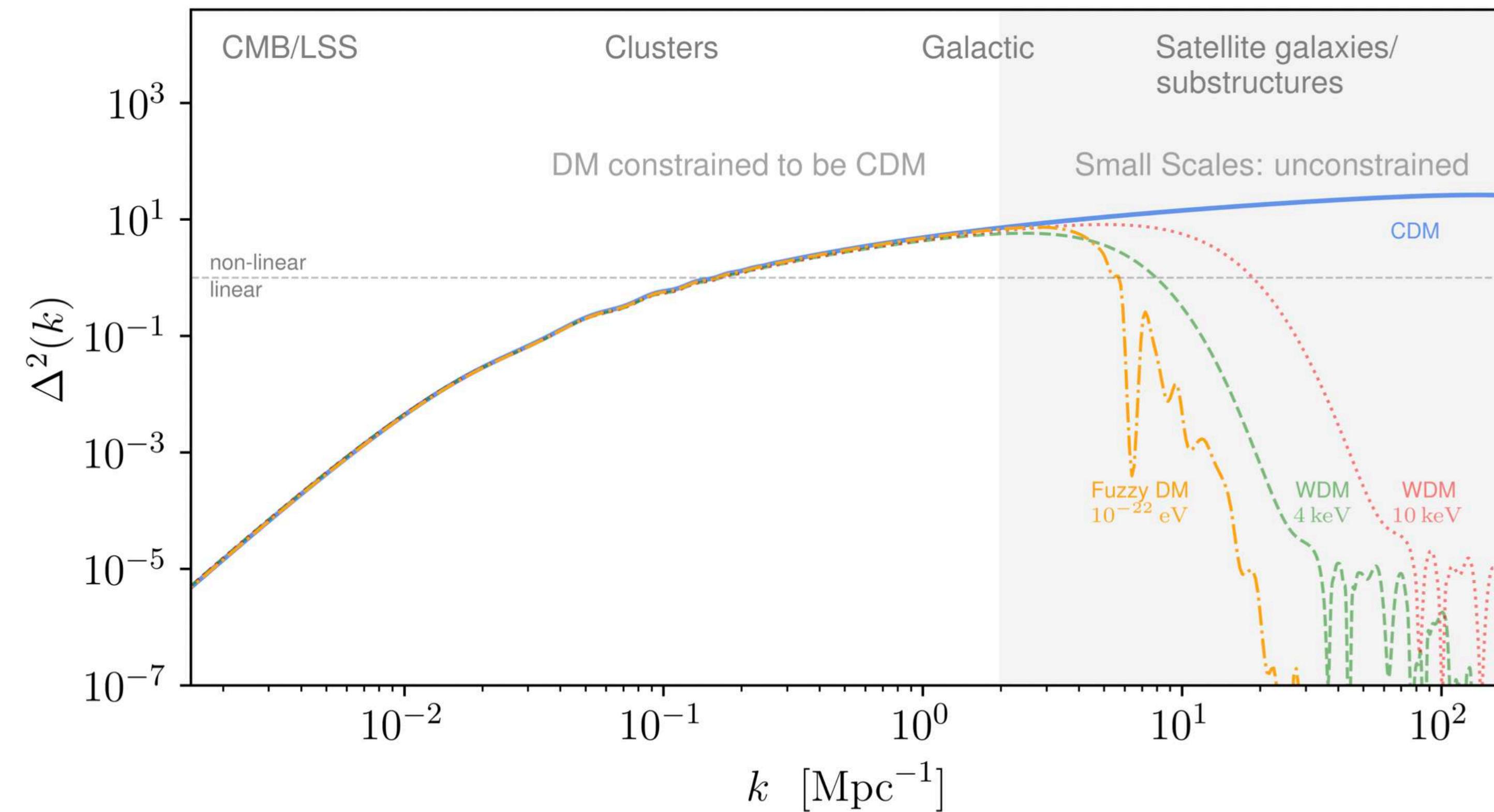
$$\sigma/m_{dm} < 0.13 \text{ cm}^2/\text{g}, \quad \sigma/m_{dm} < 0.35 \text{ cm}^2/\text{g}$$

*From: measured core densities  
from strong lensing*



# What we *know* about dark matter

Properties:



Deviations from CDM in the highlighted region are allowed, since highly unconstrained!

# *What we don't know about dark matter*

- Gold
- Pressureless
- Dark
- Collisionless

CDM on large scales



How cold it is?

WDM  
 $m \sim \text{keV}$



Cluster on all scales?

Milicharged  
DM

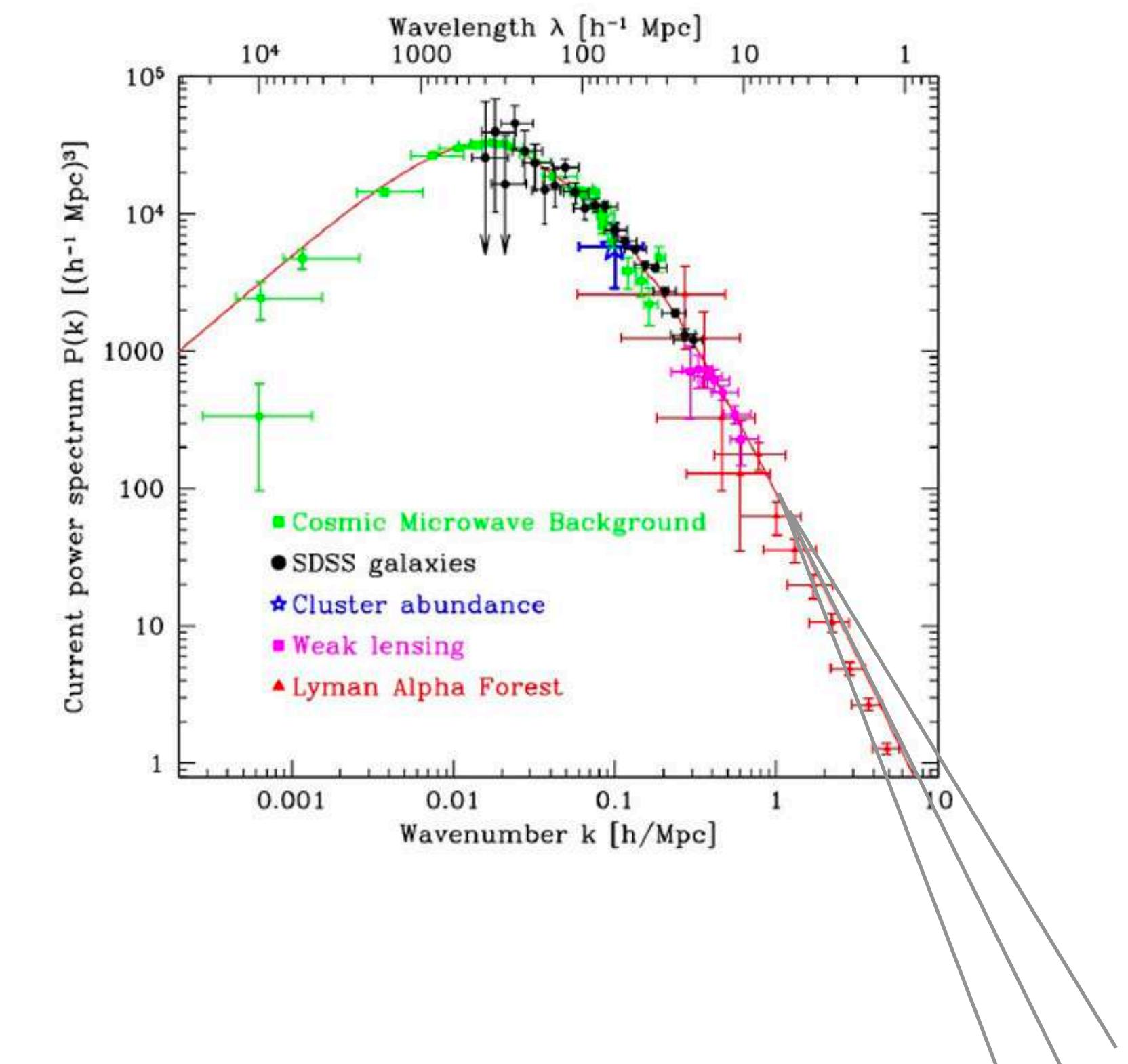


Non-gravitational  
interaction?

SIDM



How small self-interaction?

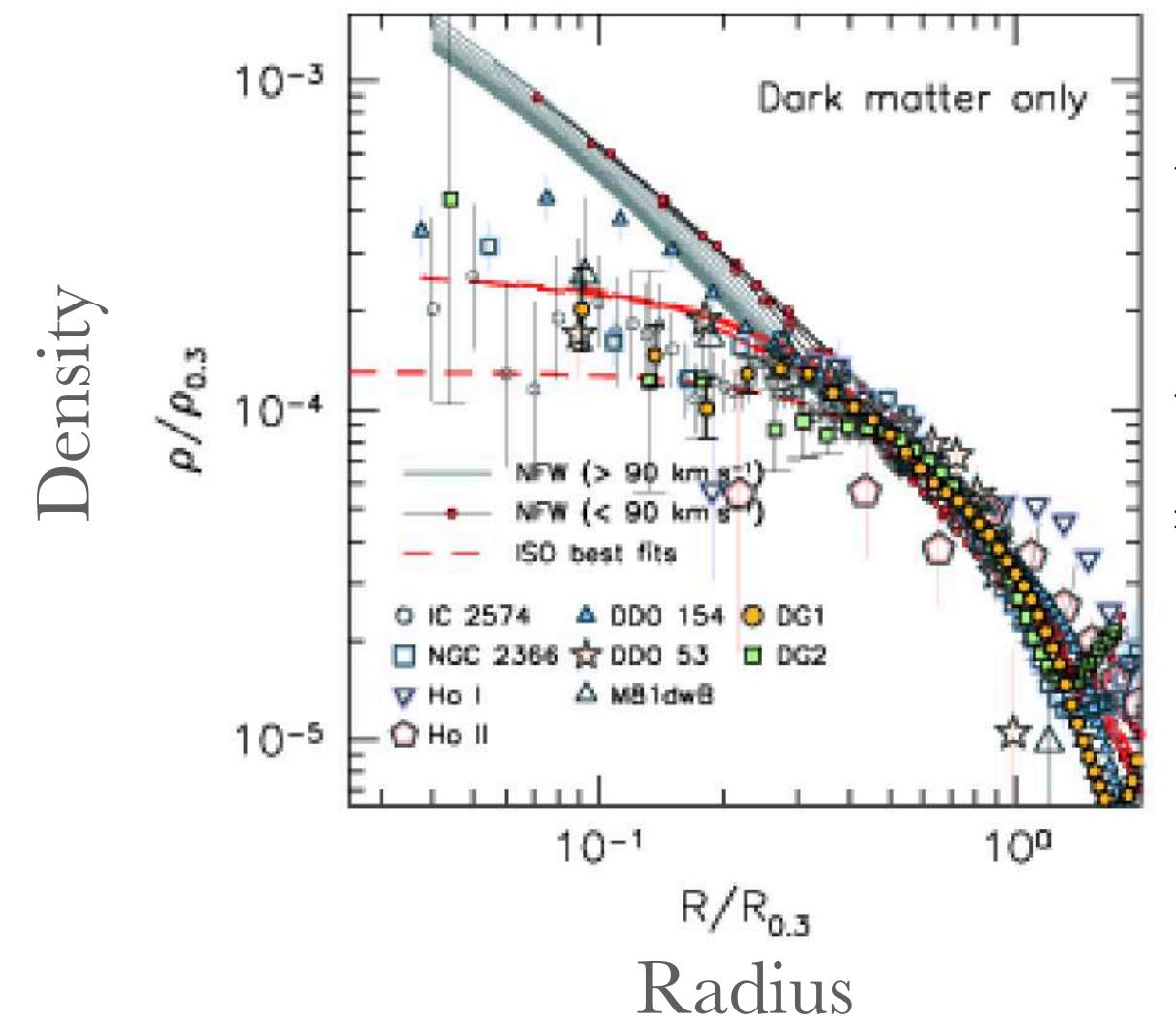


Small scale behavior: still weakly constrained and small scale challenges

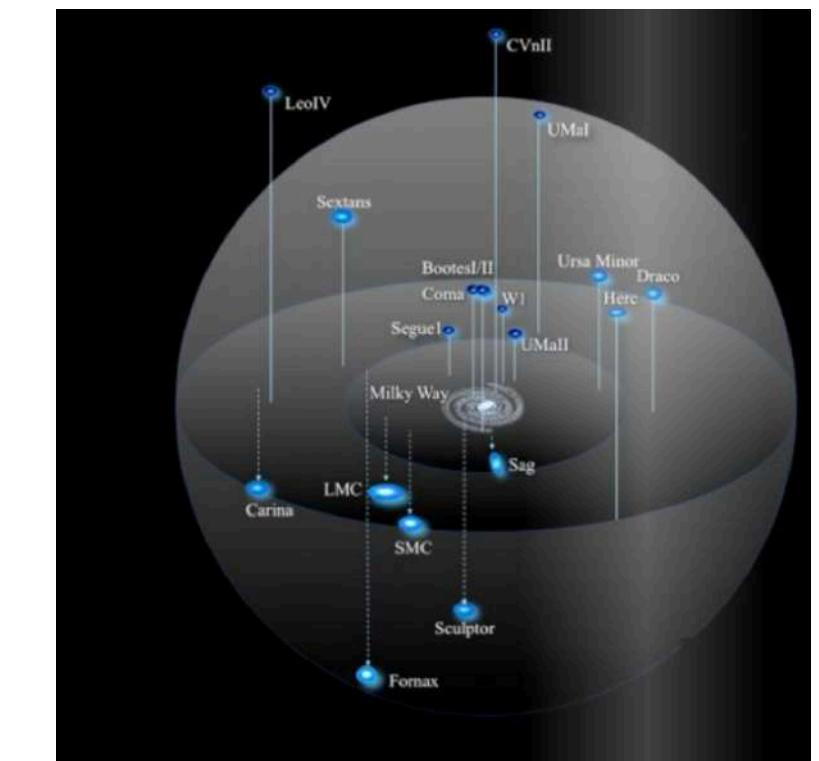
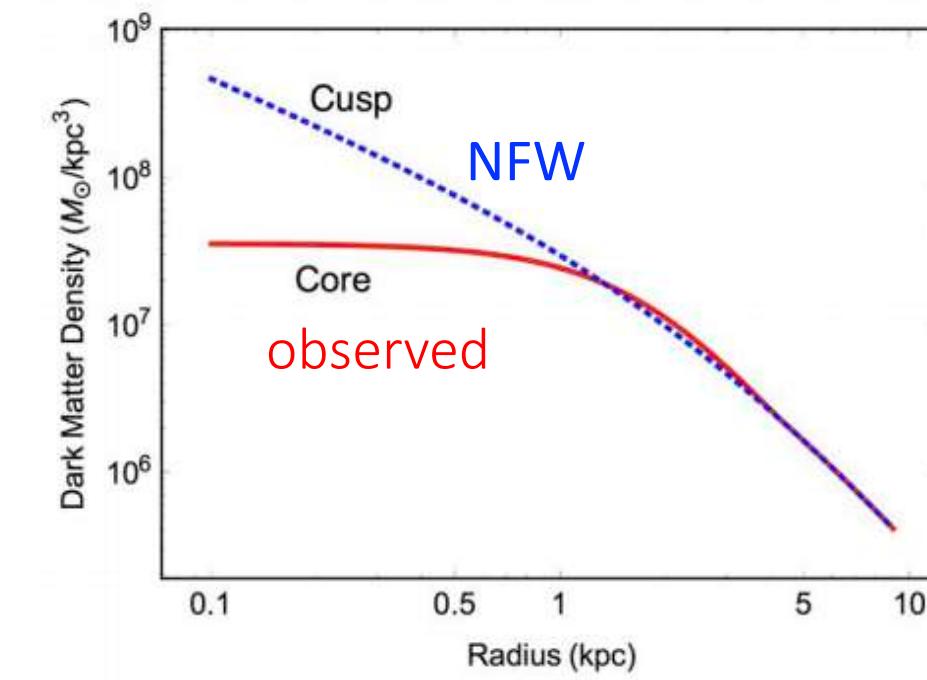
Small scale curiosities: **cusp-core**, missing satellites, BTFR, ...

# *Small scale challenges*

## Cusp-core



## CDM - NFW profile



## Missing satellites

Incompatibility between the # of satellites predicted by simulations using **LCDM** and the # of **observed** satellites

## Regularity/diversity of rotation curves

# *Regularity and diversity of rotation curves*

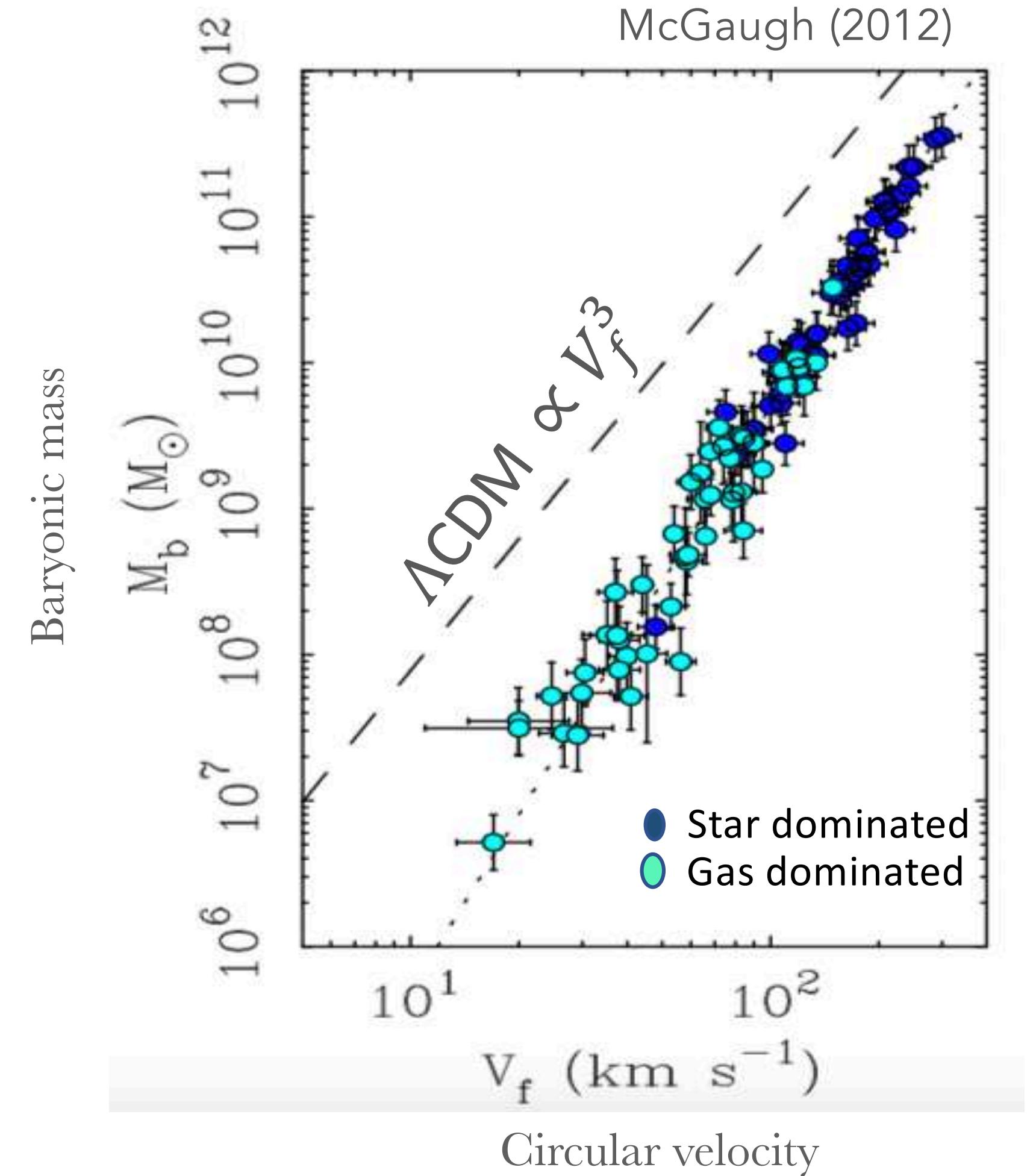
- Baryonic Tully-Fisher relation (BTFR)

Remarkably **tight** scaling relations between dynamical and baryonic properties.

Other scaling relations:

- ✓TRF
- ✓RAR - Radial acceleration relation
- ✓...

$$a_0 \simeq \frac{1}{6} H_0 \simeq 1.2 \times 10^{-8} \text{ cm/s}^2 = 2.7 \times 10^{-34} \text{ eV.}$$



Dark matter-

Large scales: CDM

Small scales:

- Feedback: Within  $\Lambda$ CDM

- Star formation
- Stellar evolution
- Sn rates
- BH and AGN feedback
- Stellar feedback
- ...

Questions:

- Can it solve all these?
- $\neq$  simulations,  $\neq$  parametrizations
- Enough feedback?
- Explains tight scaling relation?

- MOND:

Modified Newtonian Dynamics

Empirical relation

$$a = \begin{cases} a_N^b, & a_N^b \gg a_0. \\ \sqrt{a_N^b a_0}, & a_N^b \ll a_0. \end{cases}$$

*Curiosity: Baryons drive the dynamics!*

Works extremely well for: (1) rotation curves; (2) scaling relations

BUT:

~~MOND without DM~~

Problems explaining large scales

- Modify dark matter:

DM with different properties on small scales

- SIDM (Self-interacting DM)

*Solve cusp-core and missing satellites*

- WDM (Warm DM)

*Solve missing satellites*

# *What we know about dark matter*

- Gold



How cold it is?

- Pressureless



Cluster on all scales?

- Dark



Non-gravitational interaction?

- Collisionless



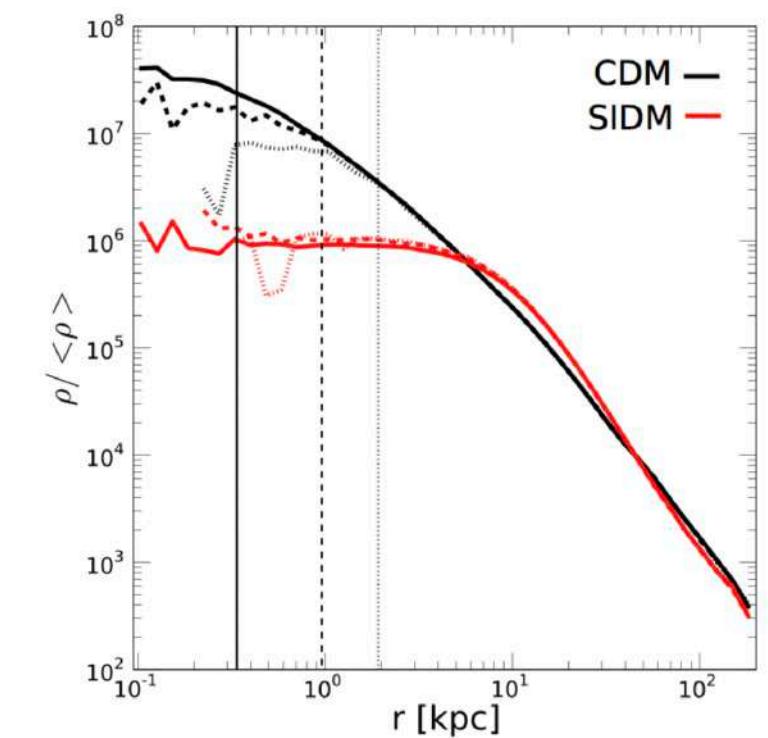
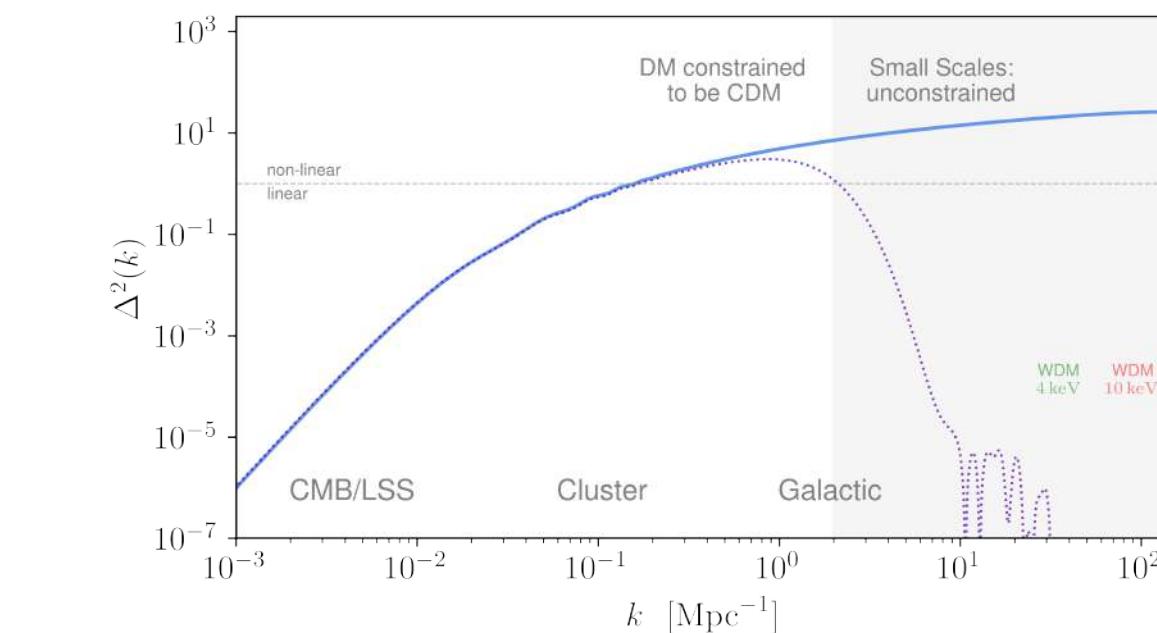
How small self-interaction?

$$\text{Solve SSP: } \sigma/m_{dm} \sim 1 \text{ cm}^2\text{g}$$

CDM on large scales!

Small scale behavior: still weakly constrained and small scale challenges

Small scale curiosities: **cusp-core**, missing satellites, BTFR, ...

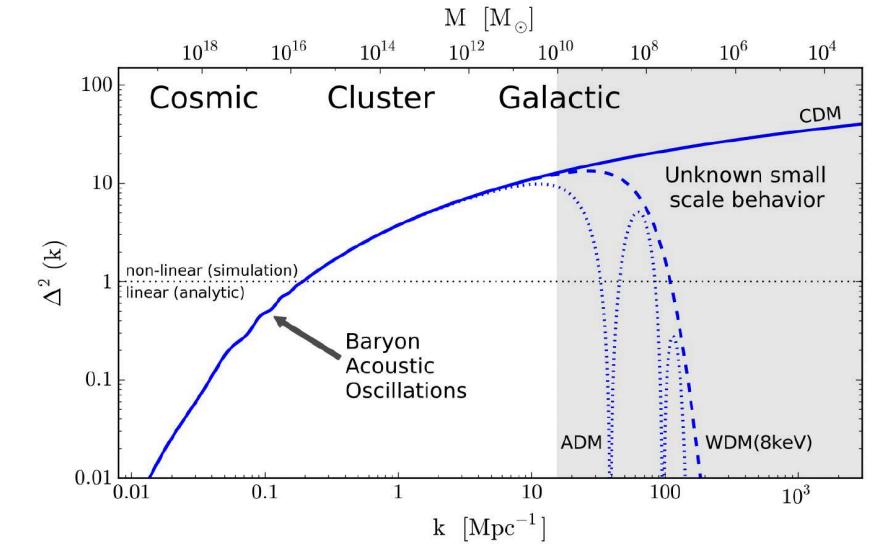


Adhikari et al. 2022

# *DM builder's guide*

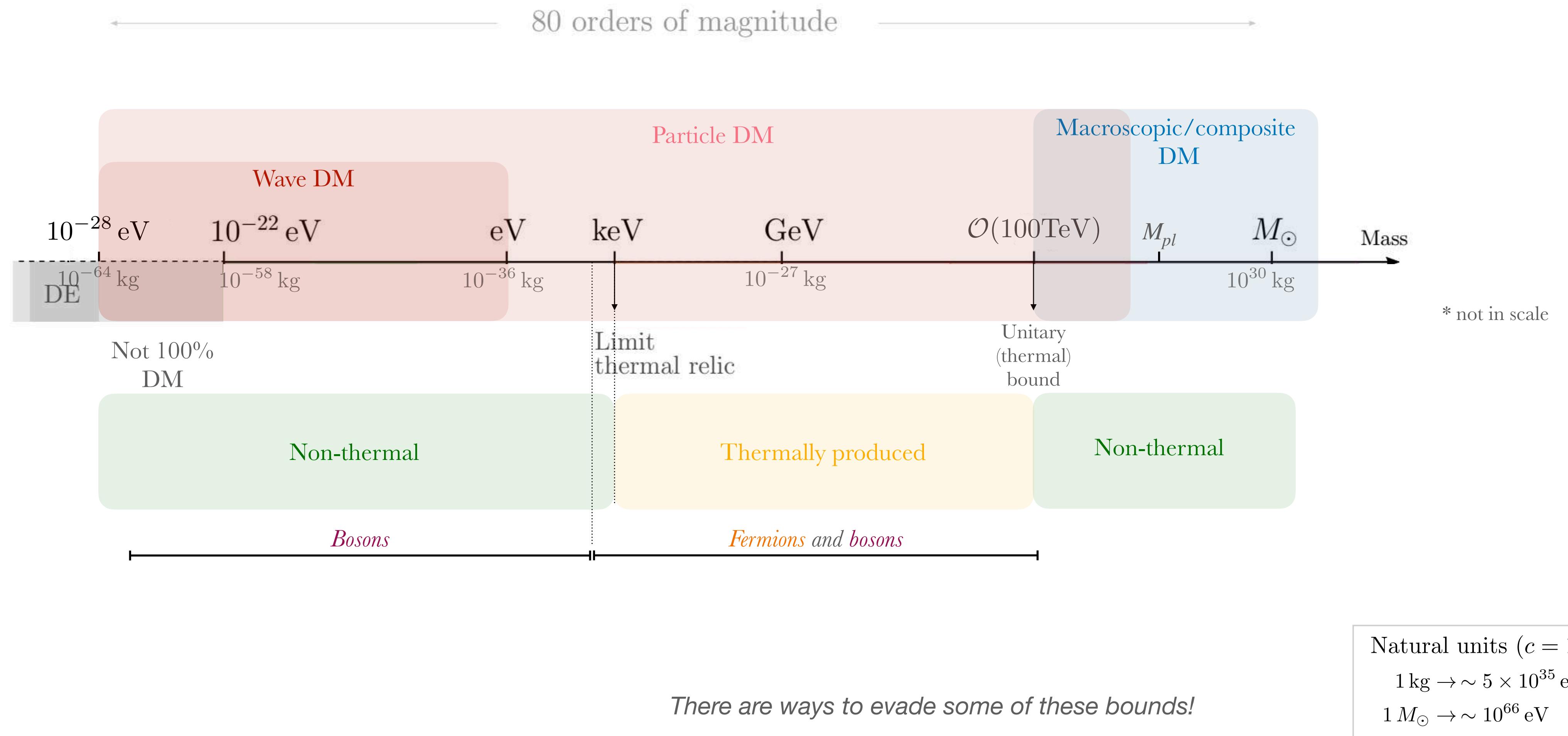
## *Pre-requisites for a dark matter candidate*

- Cold or warm
  - Thermal candidate:  $m_{dm} \geq \text{keV}$  Or produced cold by a non-thermal mechanism
  - Has to be non-relativistic at BBN
- Reproduce large and small scale distribution
  - Clusters like pressure-less fluid on large scales  $k \lesssim 10 \text{ Mpc}^{-1}$
  - Clustering on scales smaller than  $k \gtrsim 10 \text{ Mpc}^{-1}$  highly unconstrained
- Non-interacting or weakly interacting
  - (Dark, collisionless)
  - Can have a small electromagnetic interaction. Bound < **milicharge**
  - Can have a **self interaction**. Bounds:  $\sigma/m_{dm} < 0.13 \text{ cm}^2/\text{g}$ ,  $\sigma/m_{dm} < 0.35 \text{ cm}^2/\text{g}$
  - Can interact via the **weak force**
- Abundance  $\Omega_m = 0.308 \pm 0.012$  (*Planck 2018*)
- Stable If it is a particle, it has to be stable with lifetime of DM should be much greater than the age of the universe



# Mass scale of dark matter

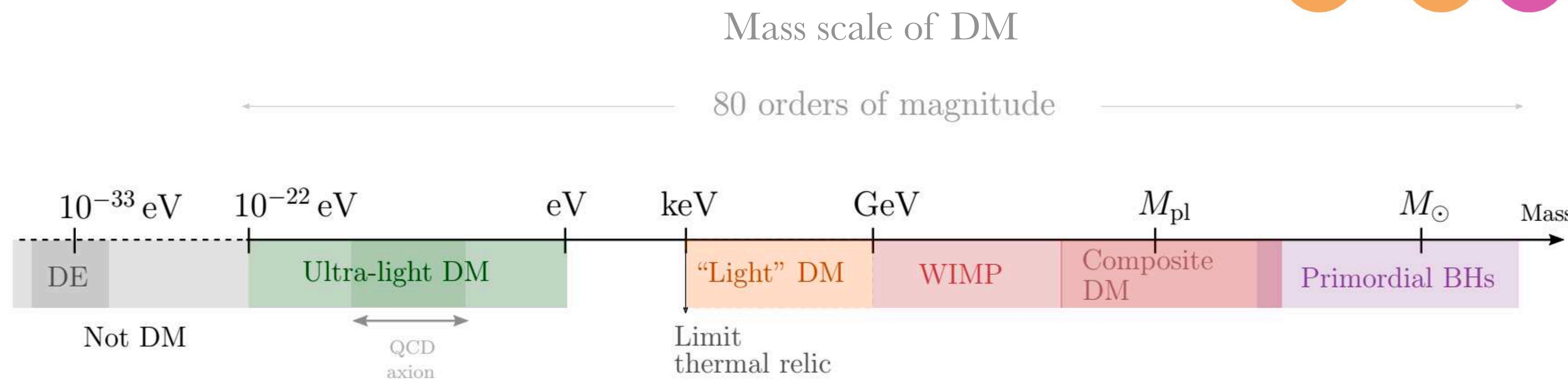
We can use observations of LSS and galaxies to put bounds in the “particle” physics properties, like mass and spin, of the DM candidate



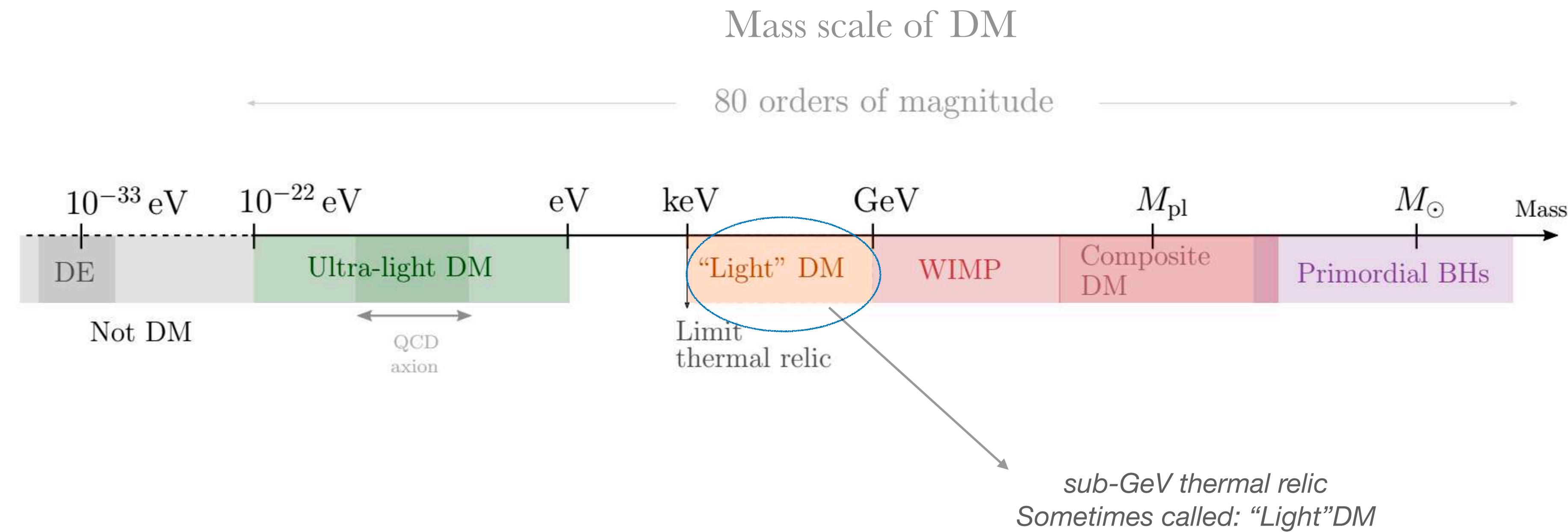
# Landscape of dark matter models

- What is DM? What is the nature of DM?

State of the “art”

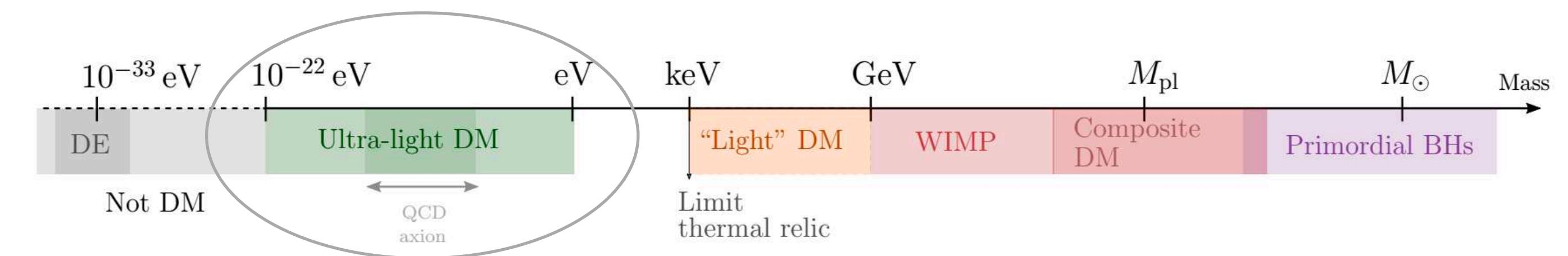


# Landscape of dark matter models

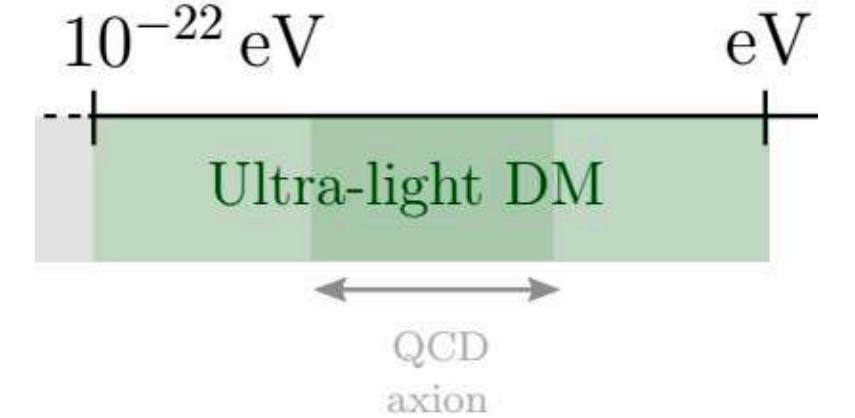


BUT, for us...

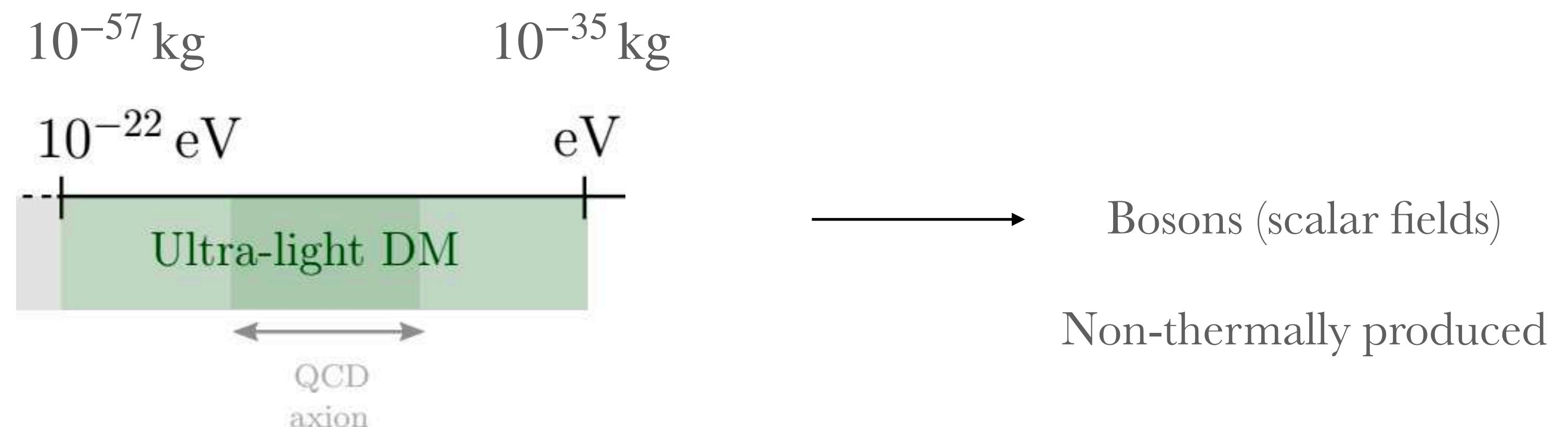
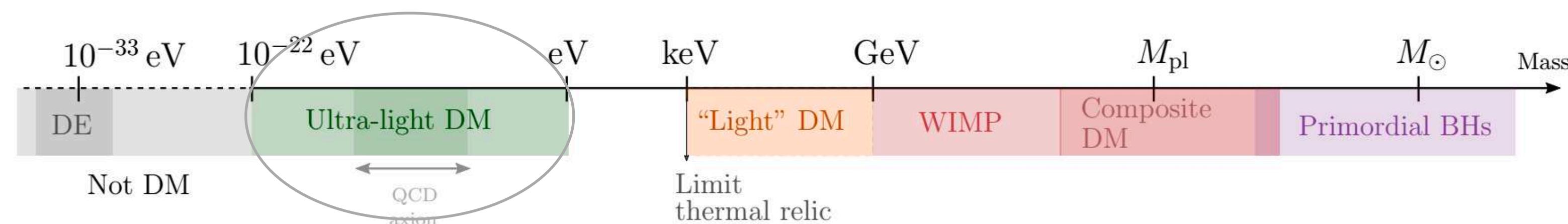
# *Ultra-light dark matter*



# *Ultra-light dark matter*



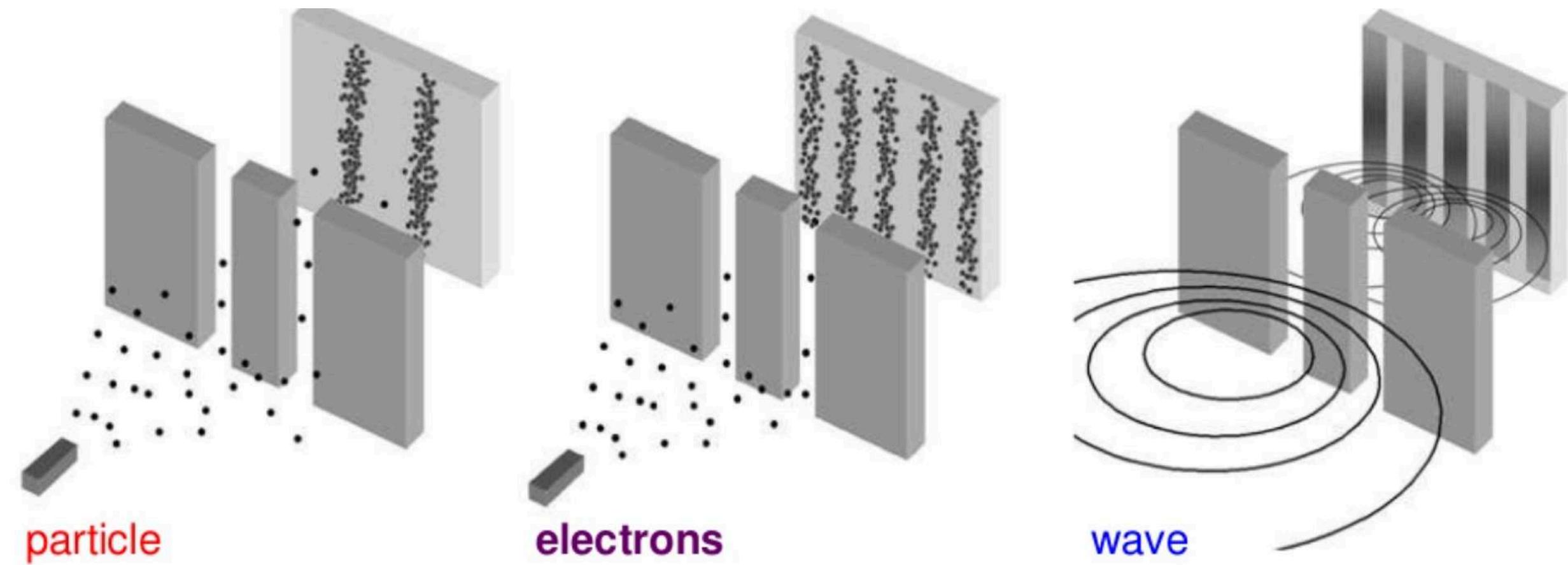
Ultra-light candidate, cold  $\longrightarrow$  Large  $\lambda_{\text{dB}} \sim 1/mv$   
 Lightest possible candidate for DM



# Wave-Particle duality

All matter exhibits a wave behaviour

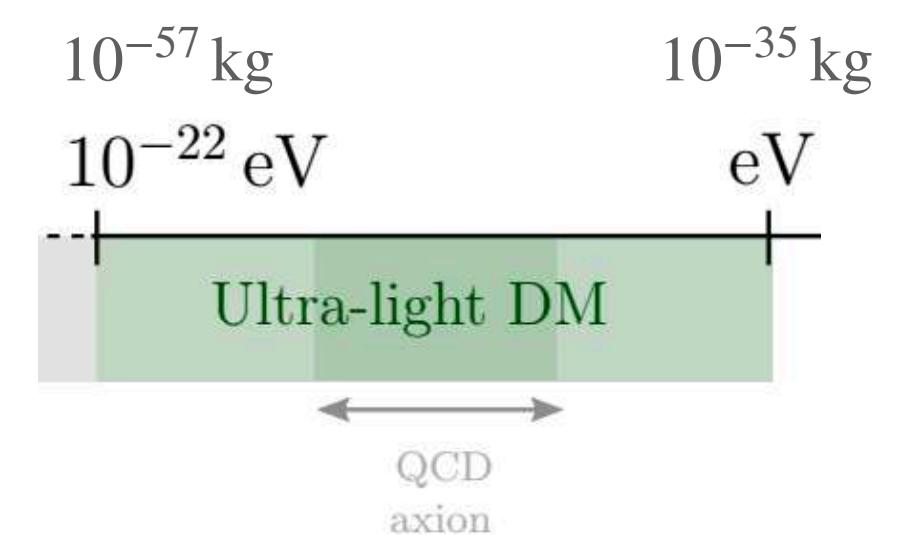
De Broglie 1924



$$\lambda_{dB} \sim \frac{1}{mv}$$

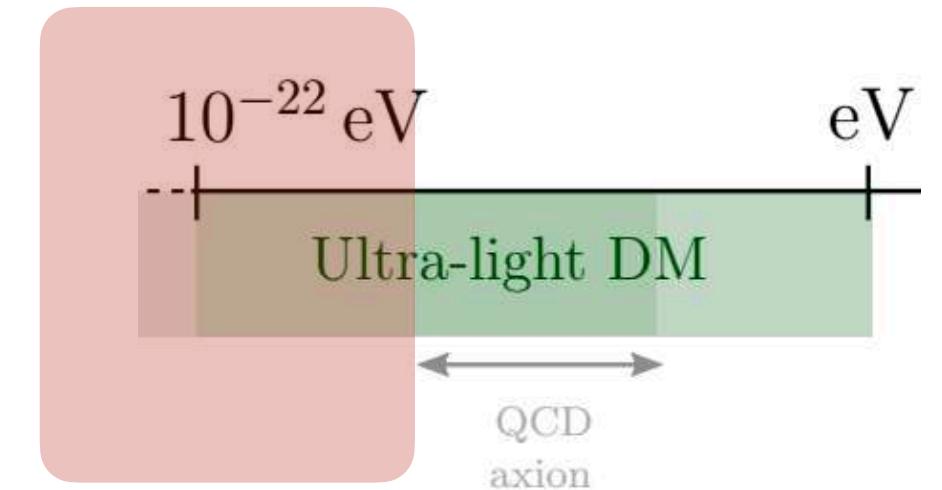
$$\lambda_{dB} \sim 1/\sqrt{2\pi mk_B T}$$

	Mass (kg)	Speed (m/s)	$\lambda_{dB}$ (m)
Accelerated e-	$9.1 \times 10^{-31}$	$5.9 \times 10^6$	$1.2 \times 10^{-10}$
Golf ball	0.045	220	$4.8 \times 10^{-30}$



$$\lambda_{dB}^{ULDM} \sim pc - kpc$$

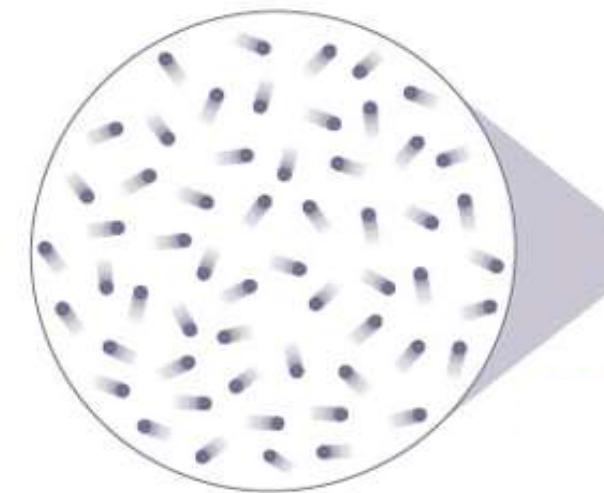
# *Ultra-light dark matter*



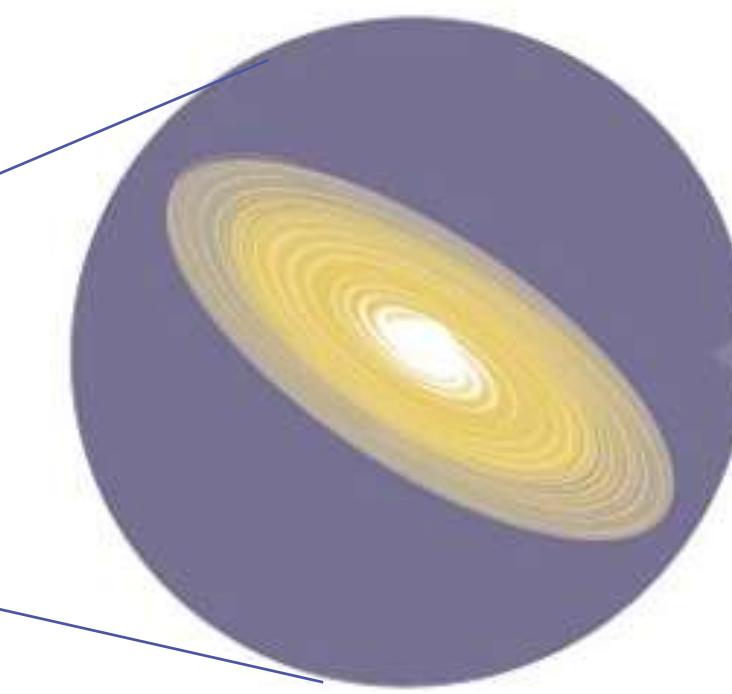
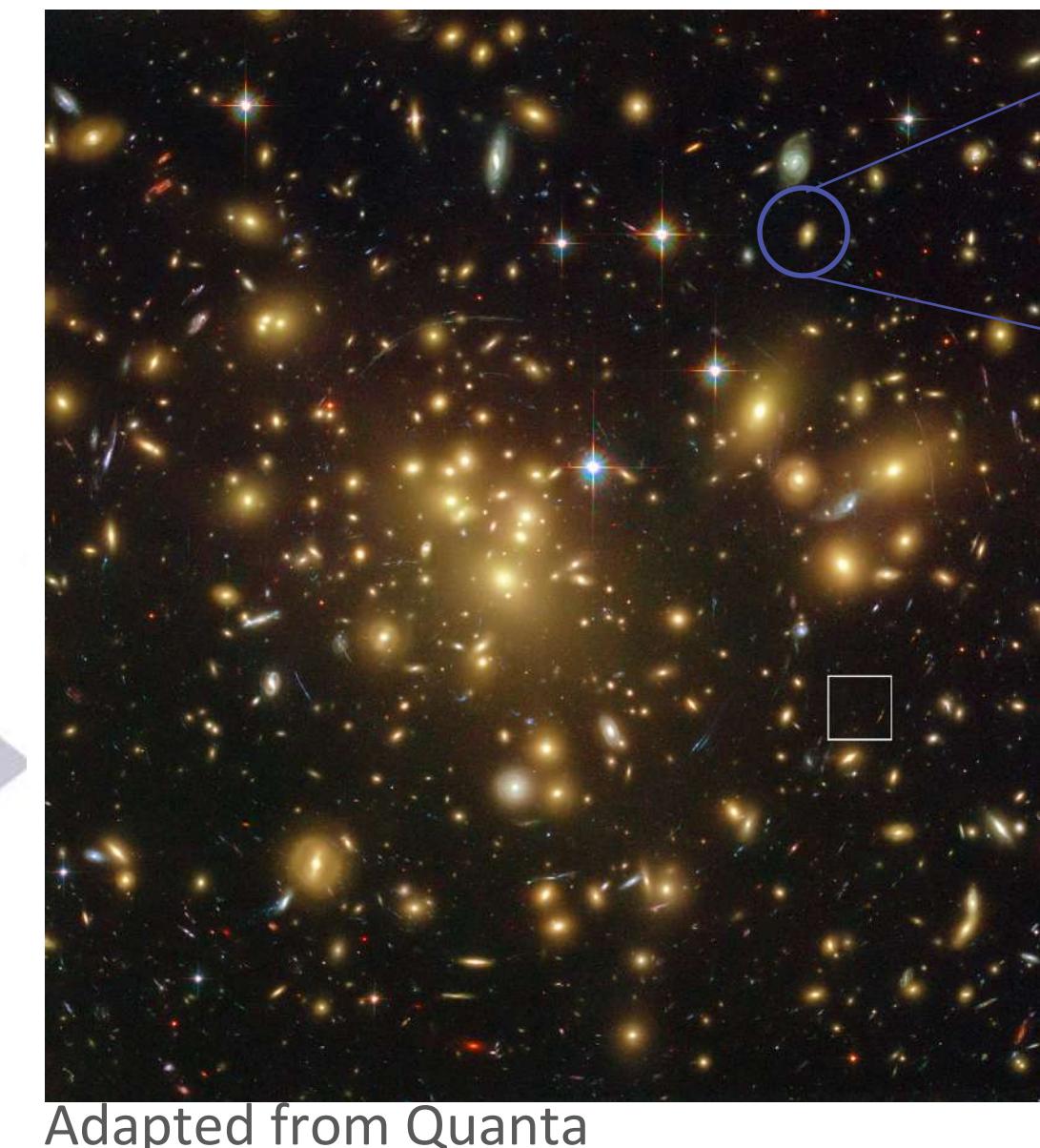
Ultra-light candidate

Large  $\lambda_{dB} \sim 1/mv$

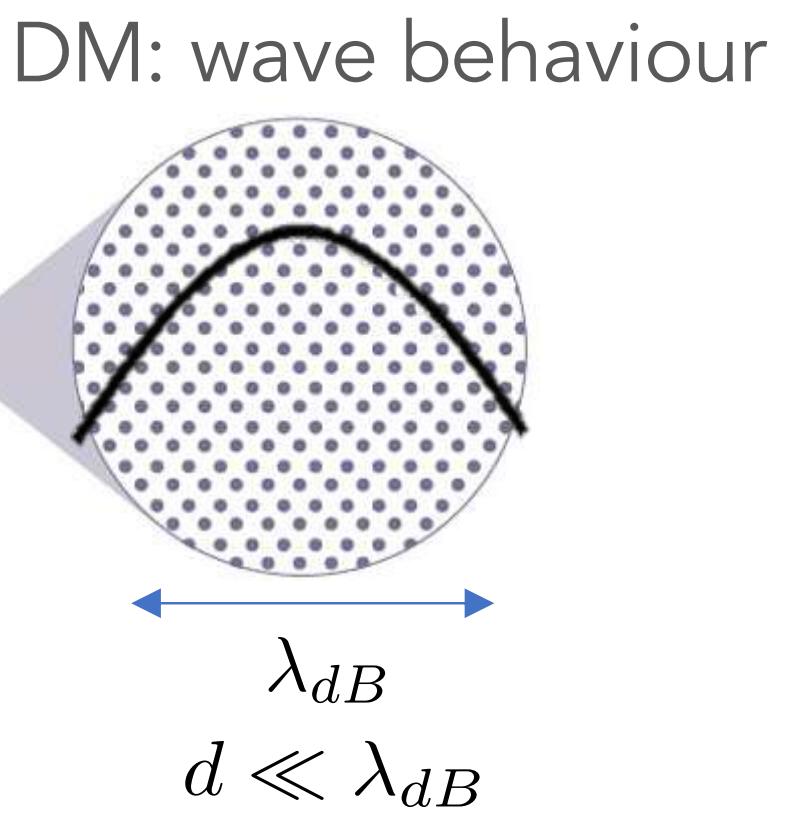
**Large** scales:  
DM behaves like standard  
particle DM (**CDM**).



DM: particles  
 $d \gg \lambda_{dB}$



Galaxy halo



**Small** scales:  
DM behaves like a **wave**

$10^{-60} \text{ kg}$

$10^{-25} \text{ eV} \lesssim m \lesssim \text{eV}$

$10^{-35} \text{ kg}$

$\lambda_{dB}^{ULDM} \sim \text{pc} - \text{kpc}$

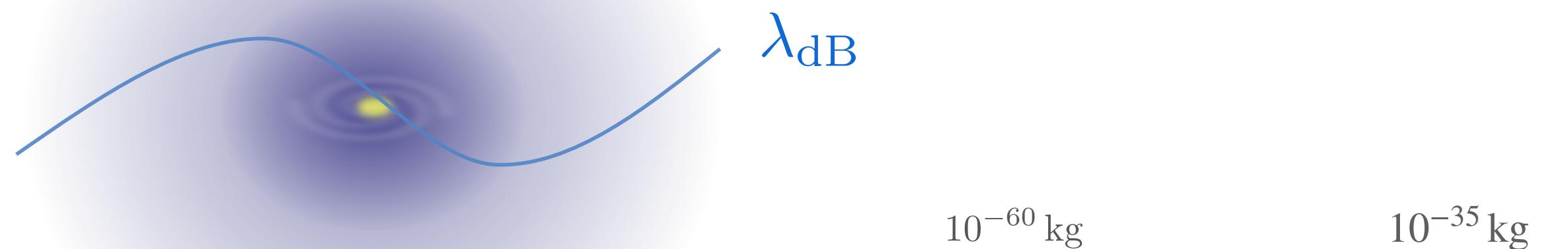
# How light is ultra-light?

“Ultra-light dark matter”, EF, 2020.

Behave as wave on galactic scales:

- $\lambda_{dB}$  must be **smaller** than the halo

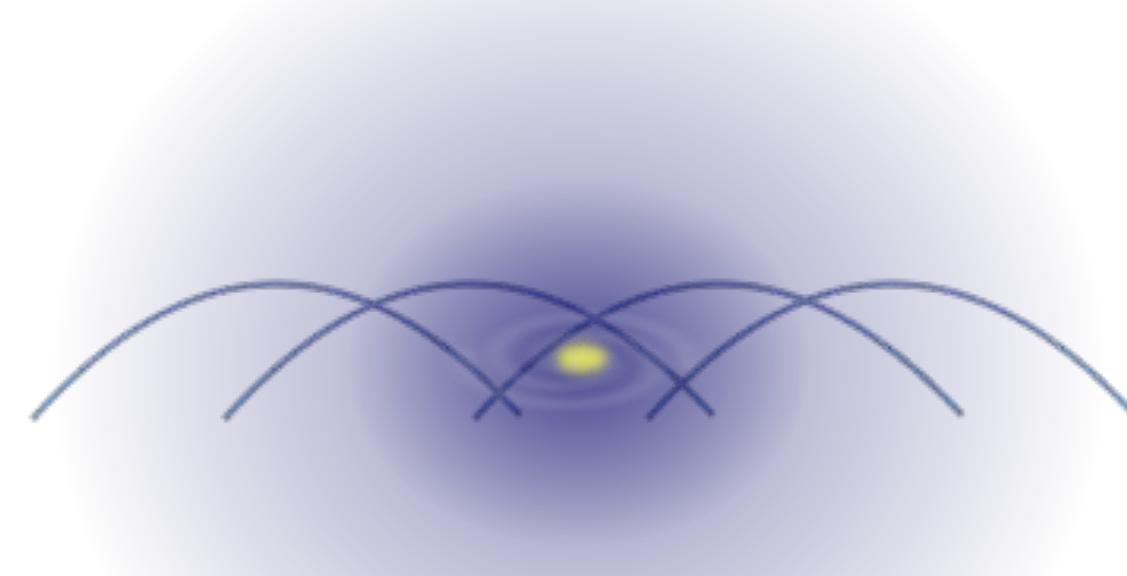
$$\lambda_{dB} < R_{\text{halo}}$$
$$\Rightarrow m \gtrsim 10^{-25} \text{ eV}$$



$$10^{-25} \text{ eV} \lesssim m \lesssim \text{eV}$$

- $\lambda_{dB}$  **overlap** to be of halo size

$$\lambda_b \sim \frac{1}{mv} \geq d \sim \left( \frac{m}{\rho_{vir}} \right)^{\frac{1}{3}}$$
$$\Rightarrow m \leq 2 \text{ eV}$$



$$\lambda_{dB}^{ULDM} \sim \text{pc} - \text{kpc}$$

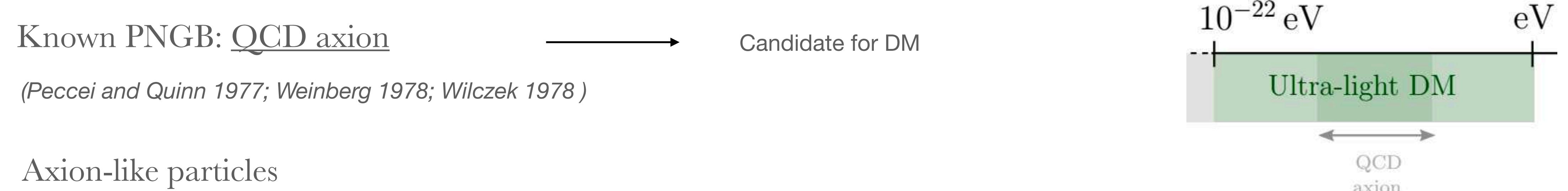
*Recap*

“Light DM”theory

# *Motivation: particle physics*

## ULDM candidates

- Natural candidate for a light scalar field is a pseudo-Nambu Goldstone boson (breaking of an approximate symmetry)



### Axions or Axion like particles (ALP)

Axions and ALPs are pseudo Nambu Goldstone bosons from the spontaneous symmetry breaking of a  $U_{\text{PQ}}(1)$  ( $U(1)$ ) symmetry, and are described by the complex field:  $\Psi = v e^{i\phi/f_a}$

$$v_{0,ssb} = f_a/\sqrt{2} \quad \longrightarrow \quad \phi \rightarrow \phi + c$$

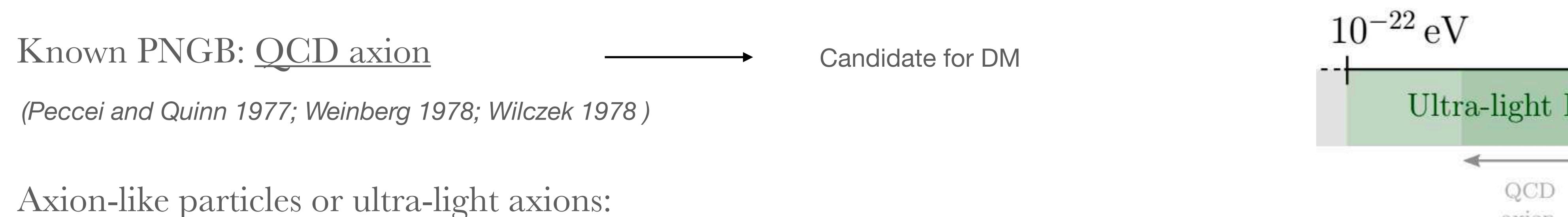
Non-perturbative effects (from string theory or instantons) induce a potential:

$$V(\phi) = \Lambda_a^4 [1 - \cos(\phi/f_a)] \xrightarrow{\phi \ll f_a} \frac{1}{2} m^2 \phi^2 + \frac{g}{4} \phi^4 + \dots$$

# *Motivation: particle physics*

## ULDM candidates

- Natural candidate for a light scalar field is a pseudo-Nambu Goldstone boson



Axion-like particles or ultra-light axions:

- ALPs expected in string theory (*Arvanitaki et al., Svrcek, Witten*)
- Can generate PNGB that are ultra-light
- Formation mechanism: needs to have a relic abundance that gives the correct DM abundance

*Non-thermal mechanism (e.g. mis-alignement)*

$$\Omega_{axion} \sim 0.15 \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{7/6} \theta_1^2$$

$$\Omega_{ALP} \sim 0.1 \left( \frac{f_a}{10^{17} \text{ GeV}} \right)^2 \left( \frac{m}{10^{-22} \text{ eV}} \right)$$

# *Motivation: particle physics*

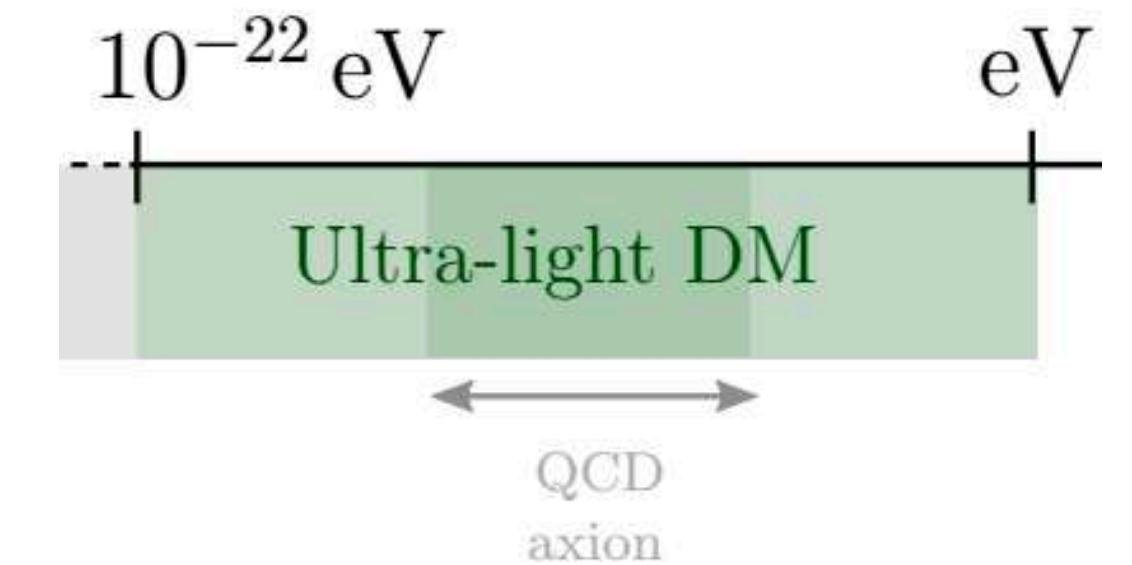
## ULDM candidates

- Natural candidate for a light scalar field is a pseudo-Nambu Goldstone boson

Known PNGB: QCD axion Candidate for DM  
(*Peccei and Quinn 1977; Weinberg 1978; Wilczek 1978*)

Axion-like particles or ultra-light axions:

- ALPs expected in string theory *(Arvanitaki et al., Svrcek, Witten)*
- Can generate PNGB that are ultra-light
- Formation mechanism: needs to have a relic abundance that gives the correct DM abundance  
*Spin-0: Non-thermal mechanism (e.g. misalignment)*



Vector FDM: challenging in the ultra-light regime

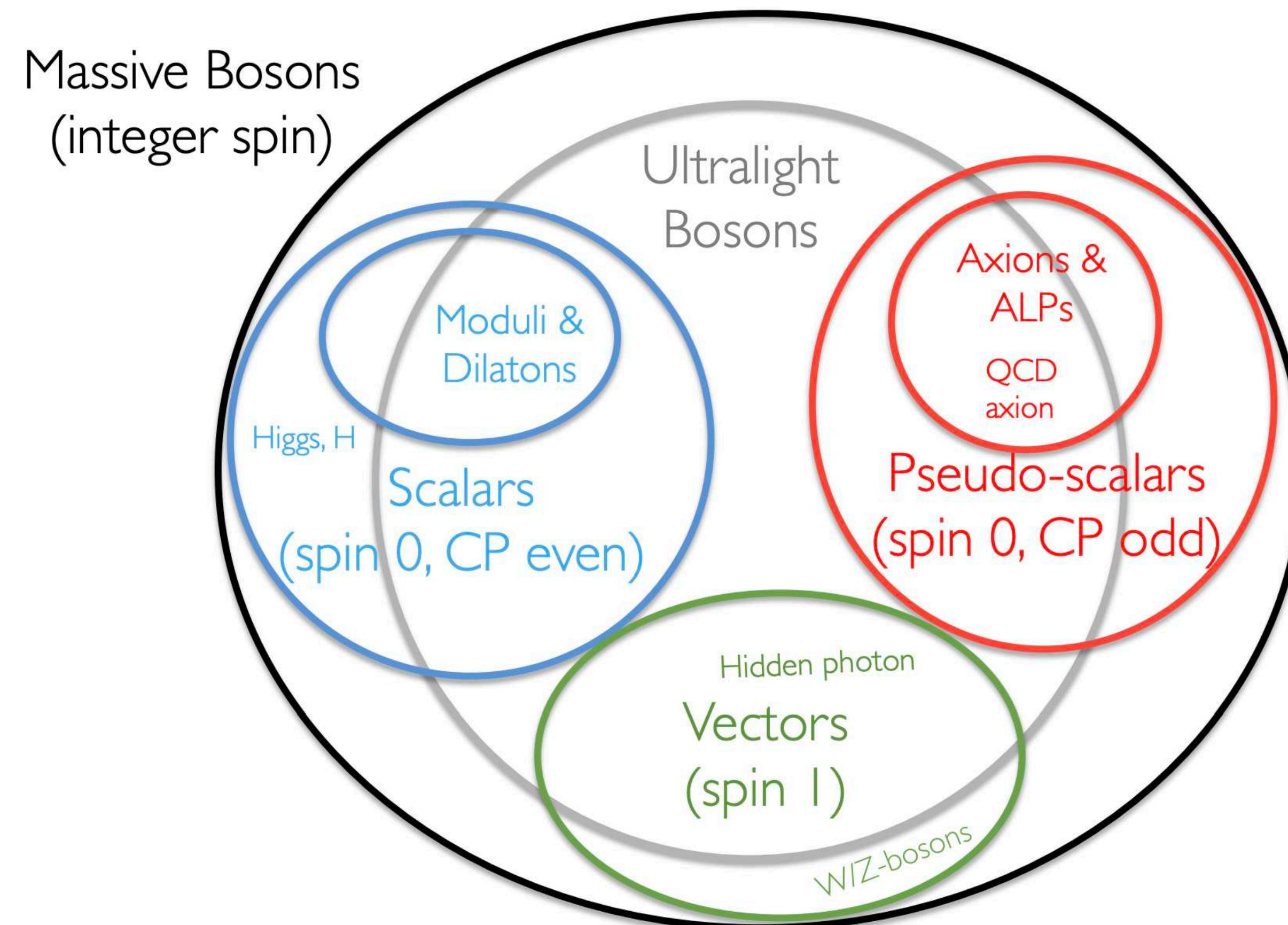
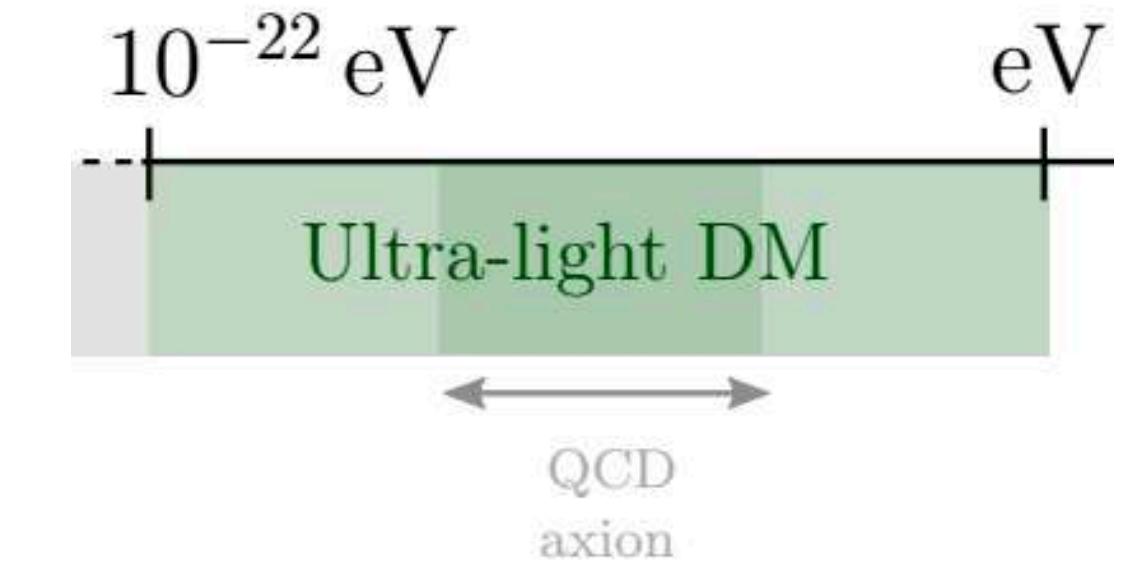
(e.g. from misalignment requires non-minimal couplings to Ricci scalar -> viol. of unitarity long. graviton-photon scattering; oscillating Higgs or oscillating misaligned axion - resonant production - choices for couplings for right abundance)

Spin 2 FDM: (e.g bigravity)

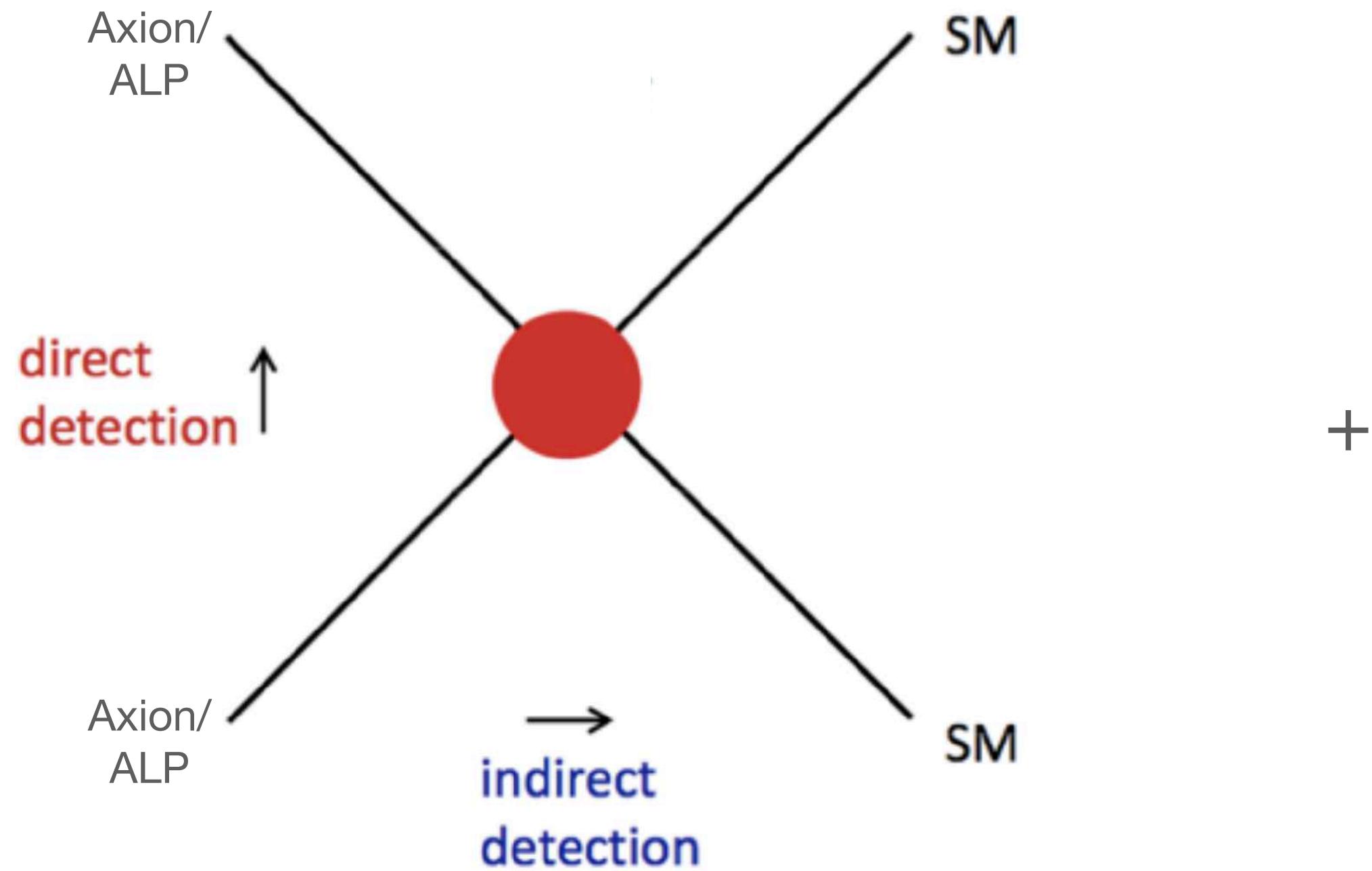
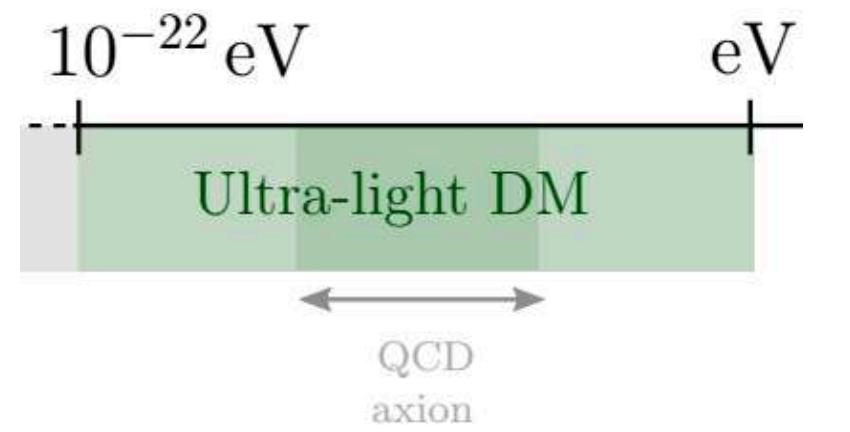
# *Motivation: particle physics*

## ULDM candidates

Many extensions of the Standard Model predict additional massive bosons

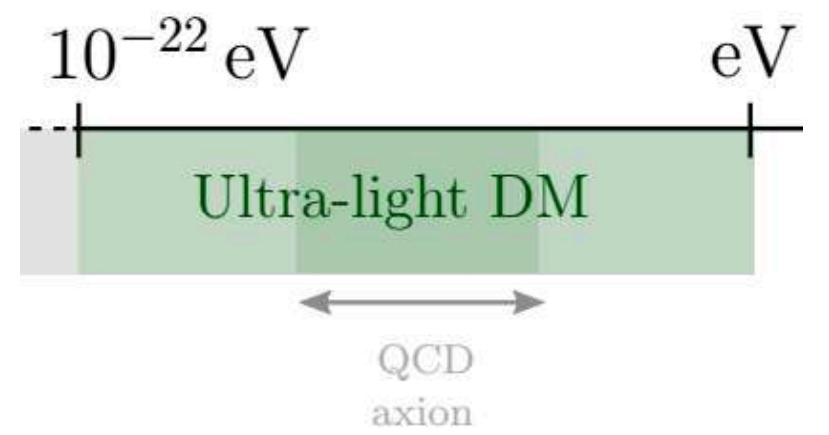


# How to search for *ULDM*?

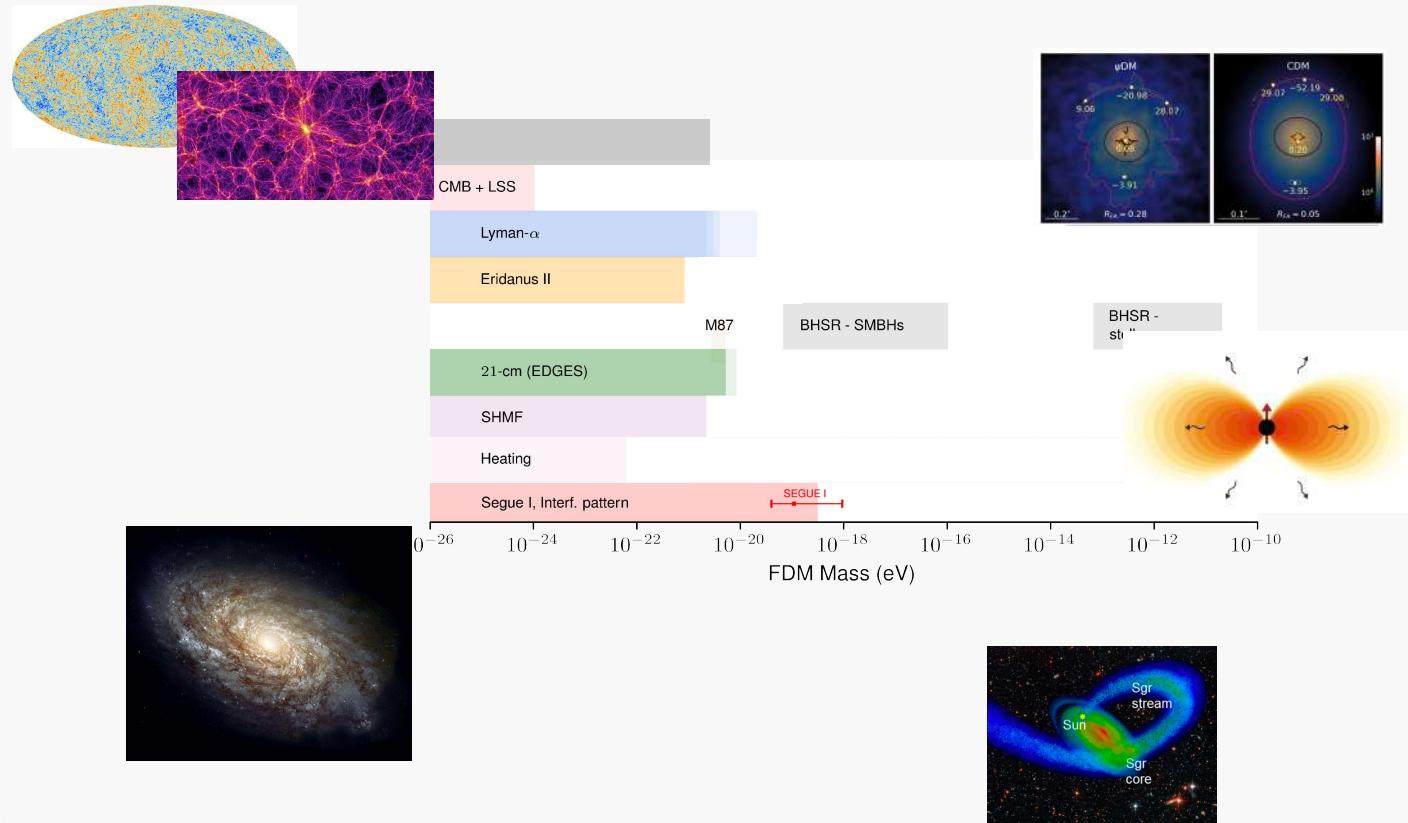


Gravitationally  
Cosmological and astrophysical searches

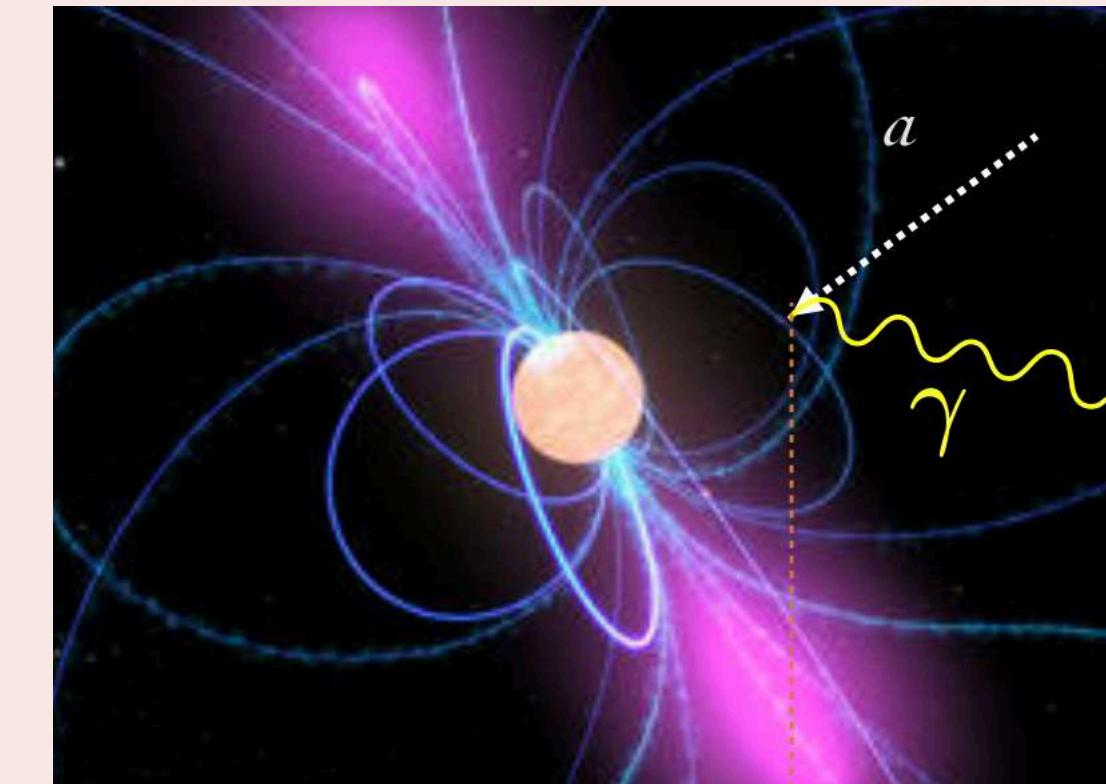
# How to search for axions/ALPs?



## Cosmological and astrophysical searches

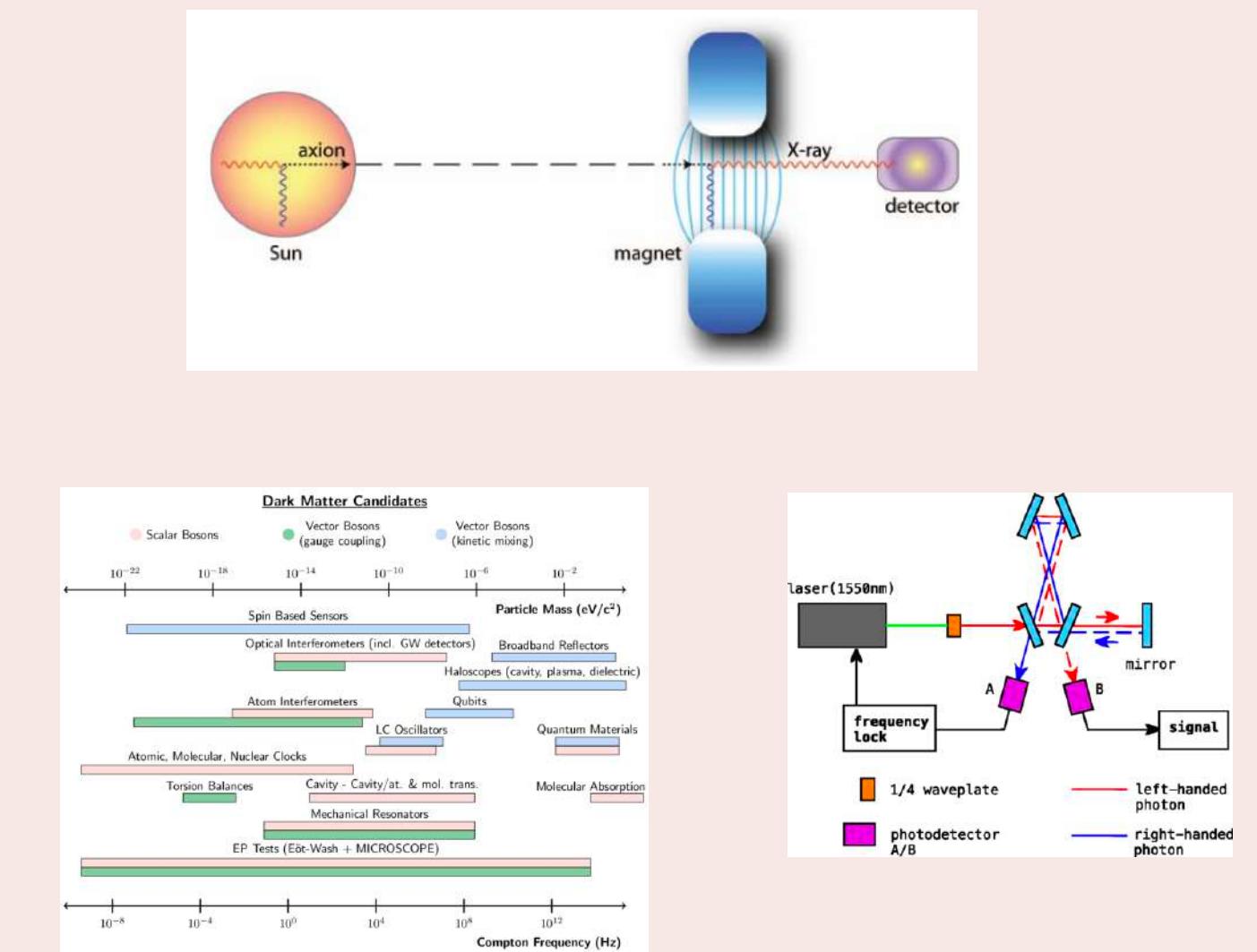


## Indirect detection



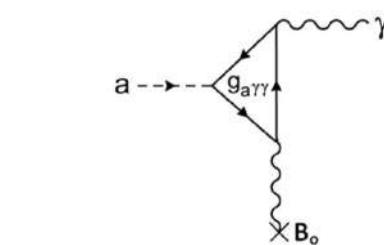
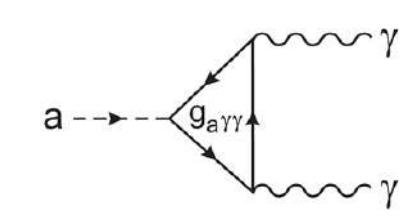
## "Direct detection"

### Axion/ALPs experiments



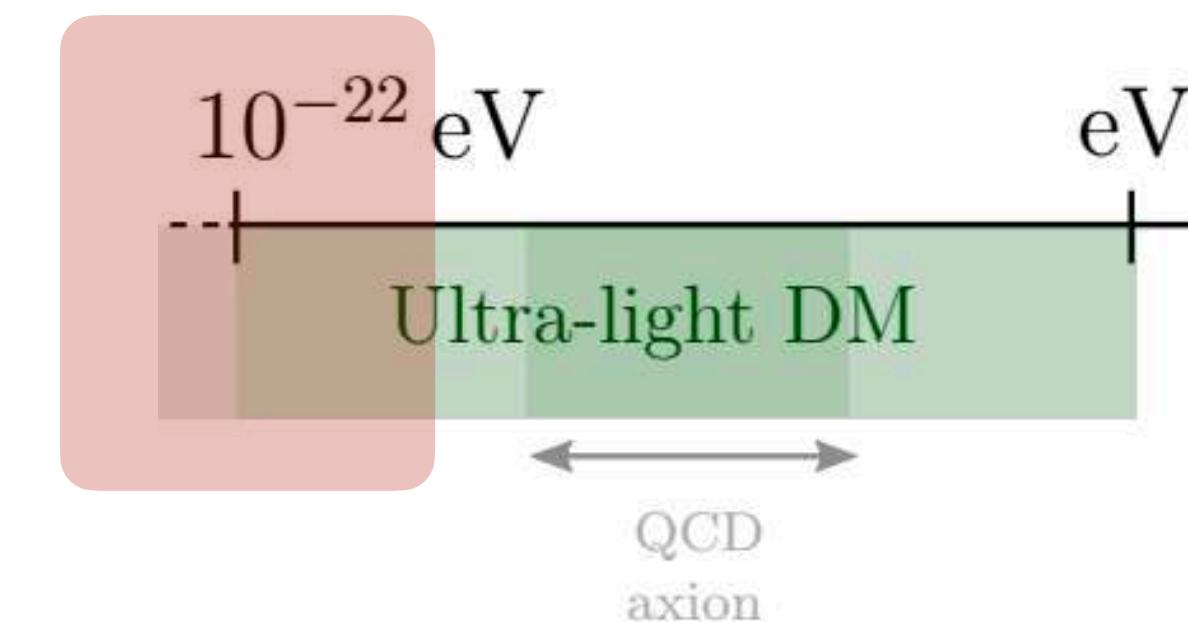
## Gravitational

## Interactions with the SM

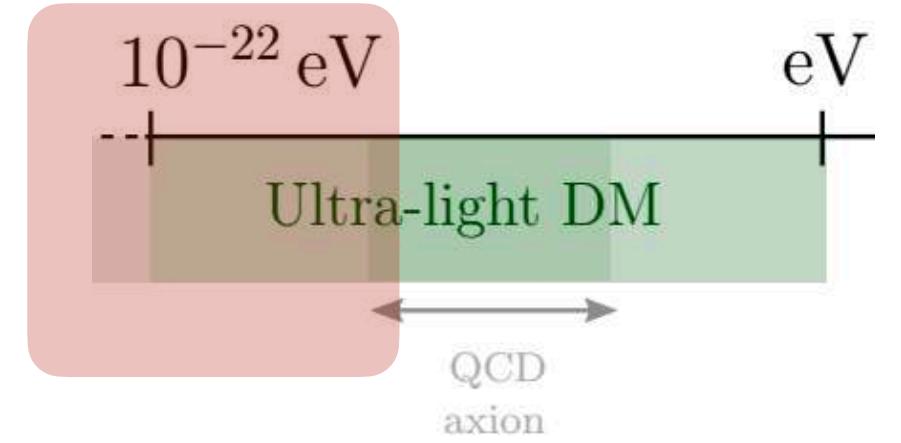


# *Gravitational signatures*

*Cosmological and astrophysical*



# *Ultra-light Dark Matter -classes*



3 classes:

## Fuzzy DM (FDM)

- Gravitationally bounded ultra-light scalar field model
- Condensation under gravity (BEC)

$m$

DOFs

## Self Interacting FDM (SIFDM)

- Presence of (weakly) self-interaction
- Condensation under gravity + SI (superfluid)

$m \quad g$

## DM Superfluid

- Forms a superfluid in galaxies
- MOND behaviour interior of galaxies

Axion and ALP (axion like particles)

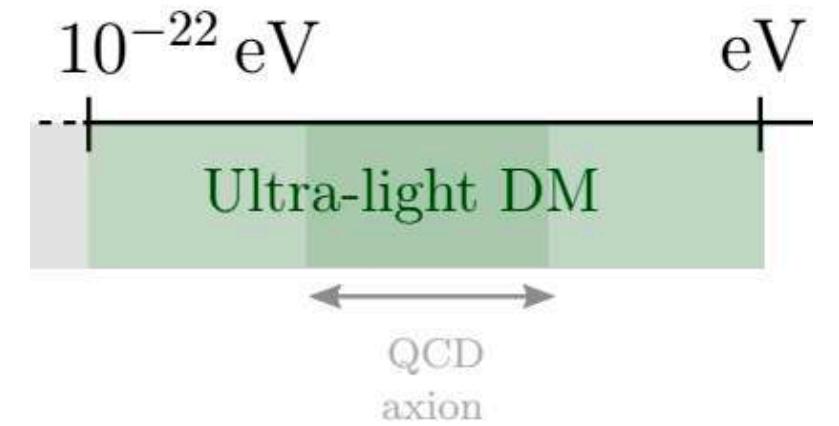
$$i\dot{\psi} = \left( -\frac{1}{2m} \nabla^2 + \frac{g}{8m^2} |\psi|^2 - m\Phi \right) \psi$$

$$\mathcal{L} = P(X)$$

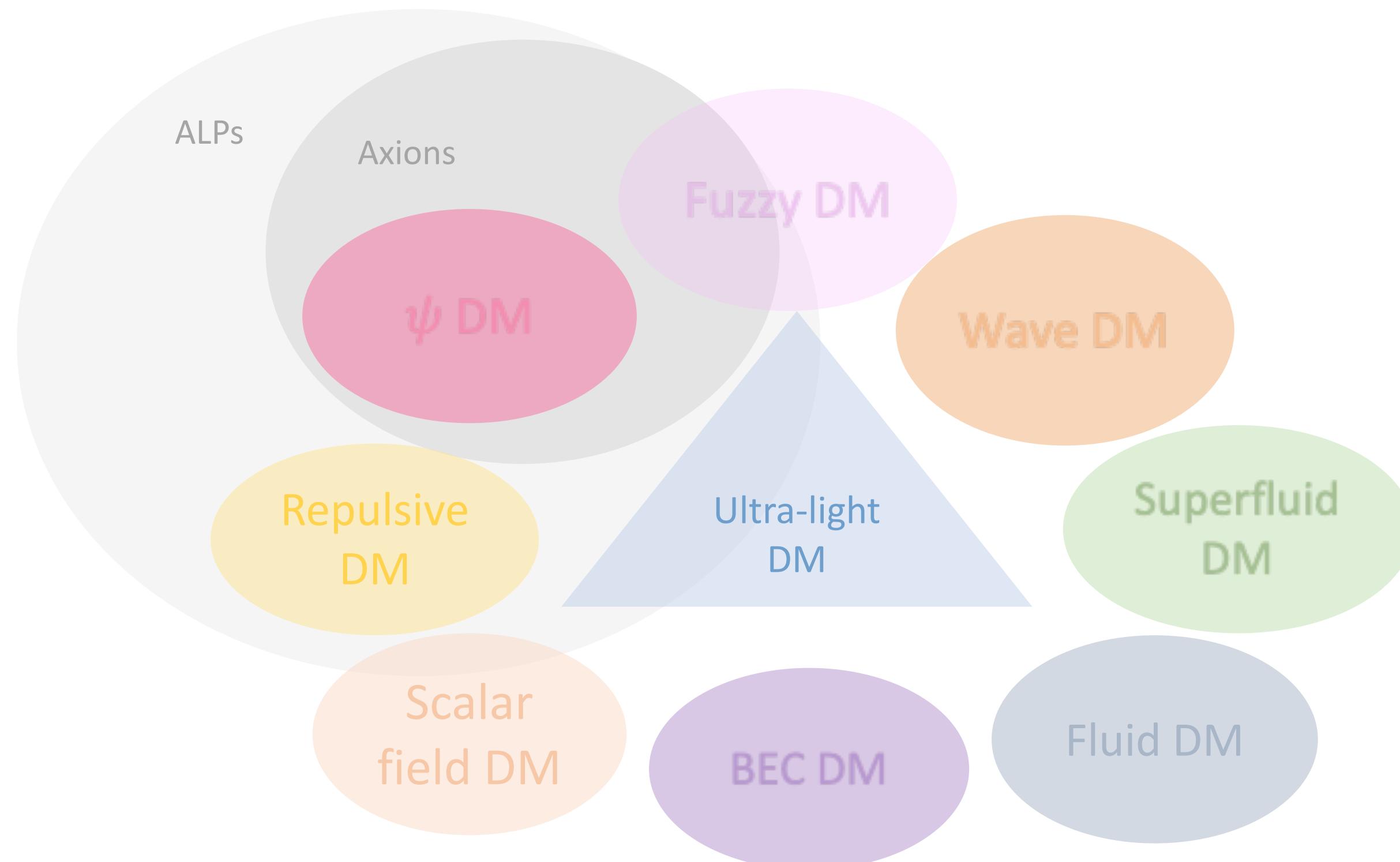
→ Connection with condensed matter and particle physics!

“*Ultra-light dark matter*”, **E.Ferreira**, 2020. The Astronomy and Astrophysics Review.

# *Ultra-light Dark Matter - models*

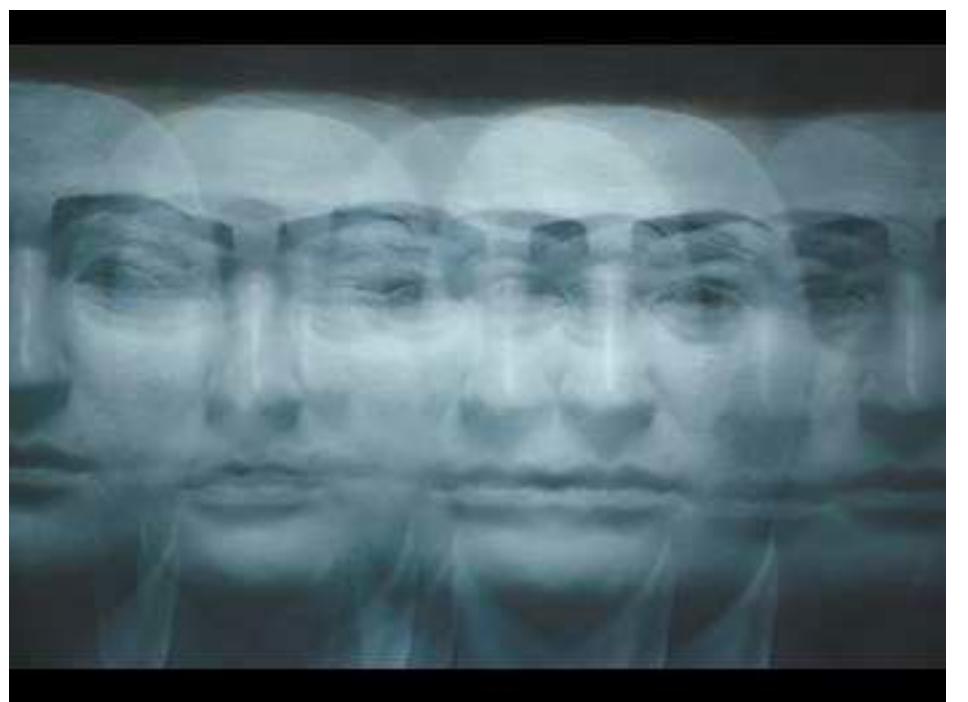


There are many ways to have a DM with this property → many ULDM models in the literature  
However, each of these models presents a different dynamics on small scales - different **phenomenology**

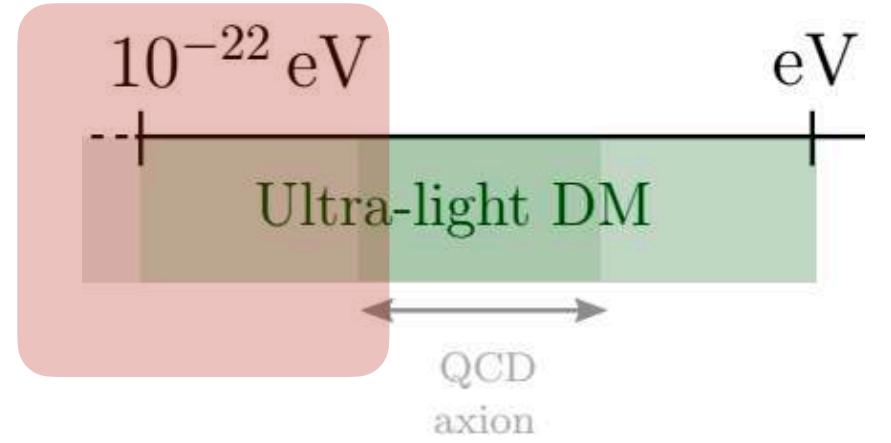


# Fuzzy dark matter

## Self interacting fuzzy dark matter



# Fuzzy dark matter



## Fuzzy DM (FDM)

- Gravitationally bounded ultra-light scalar field model
- Condensation under gravity (BEC)

$m$

## Wave DM Ultra-light axions

## Self Interacting FDM (SIFDM)

- Presence of (weakly) self-interaction
- Condensation under gravity + SI (superfluid)

$m \quad g$

Hu W, Barkana R, Gruzinov A (2000 a,b)

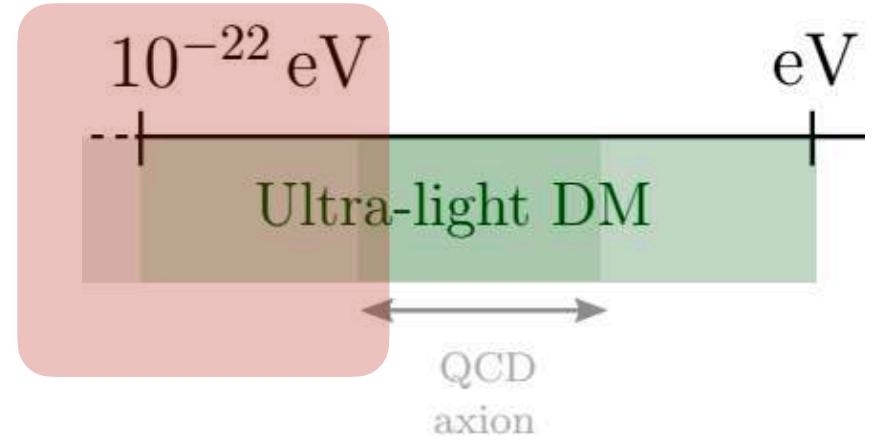
(Reviews: EF (2021), J. Niemeyer (2019), L. Hui (2021))

Idea:

$$m_{\text{fdm}} \sim 10^{-22} \text{ eV}$$

address the small scale problems+ rich phenom.

# Fuzzy dark matter



## Fuzzy DM (FDM)

- Gravitationally bounded ultra-light scalar field model
- Condensation under gravity (BEC)

$m$

## Wave DM Ultra-light axions

Focus more on spin 0 particles here!

(Some of the grav. phenom. is carried for vectors, for example)

- Spin 0 - FDM
- Spin 1 - Vector FDM
- Higher spin FDM

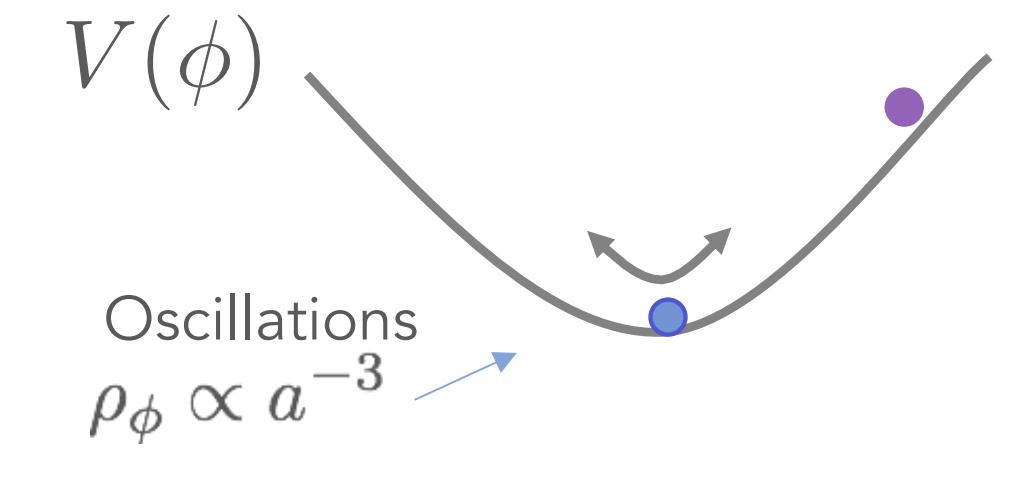
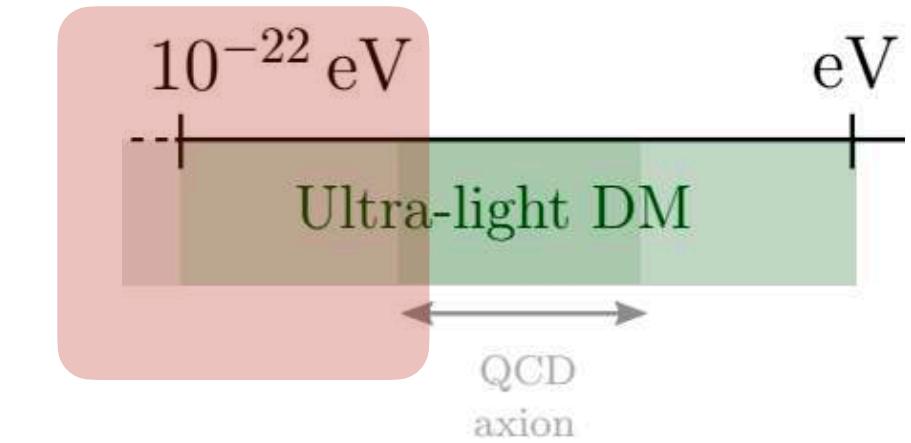
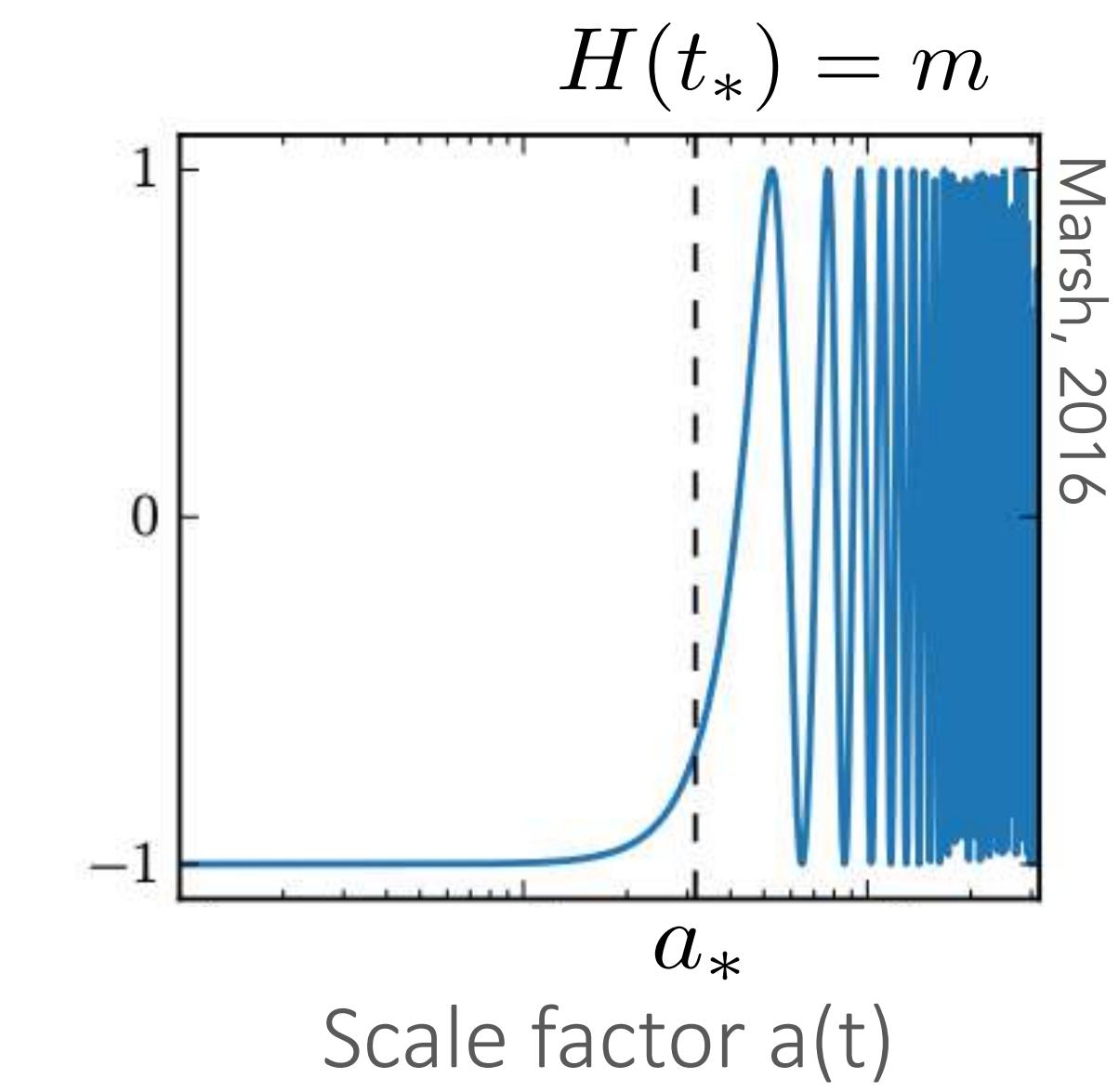
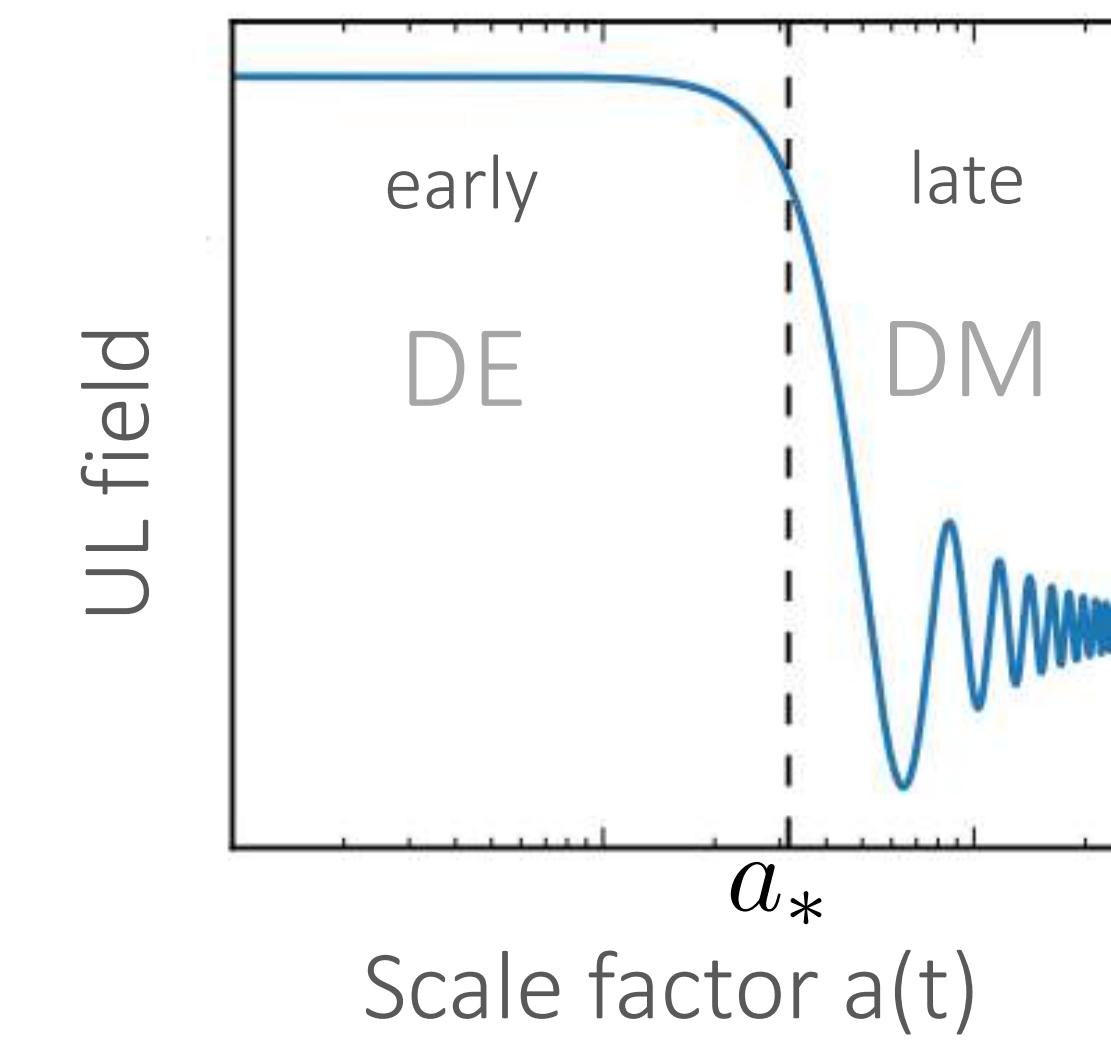
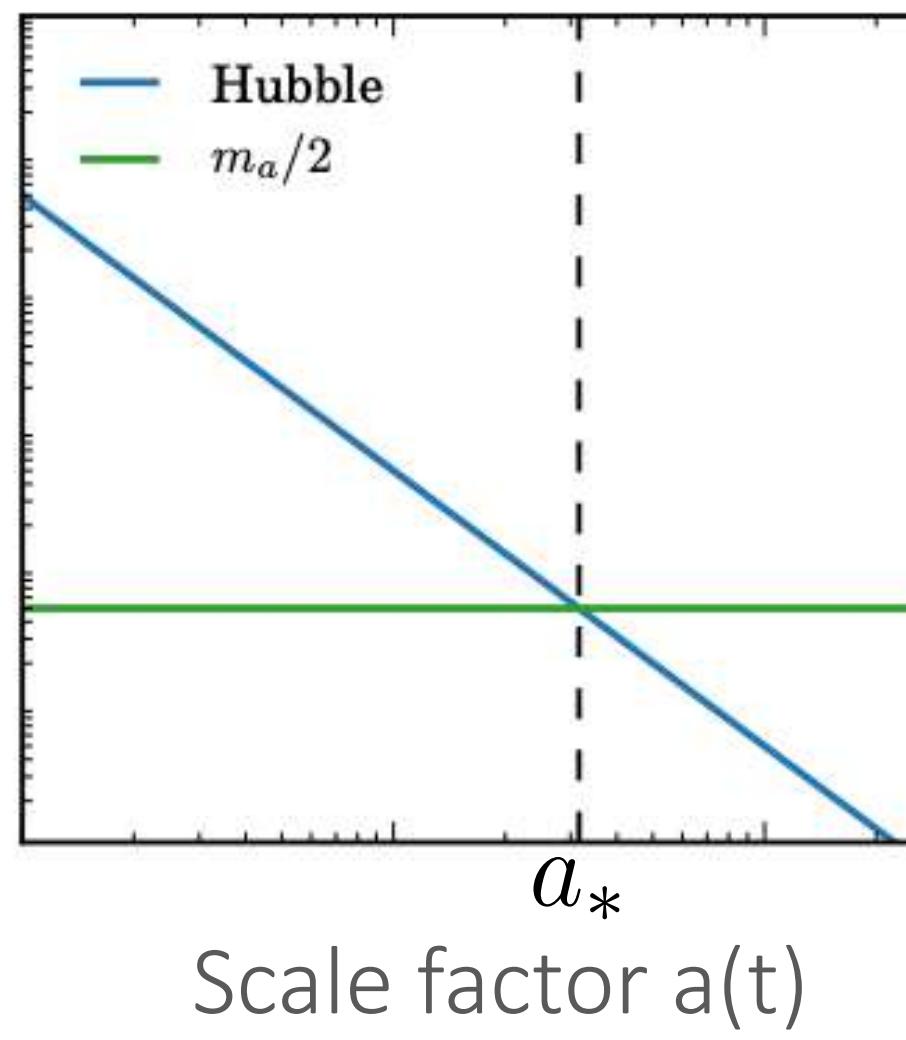
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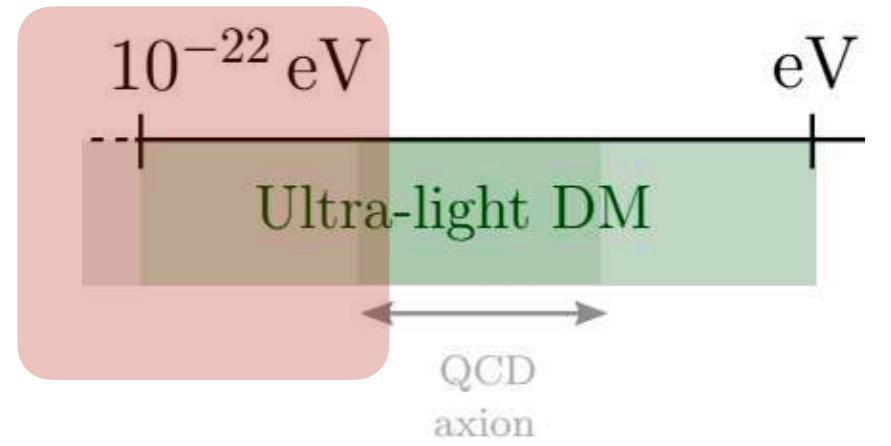
# *Cosmological evolution*

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

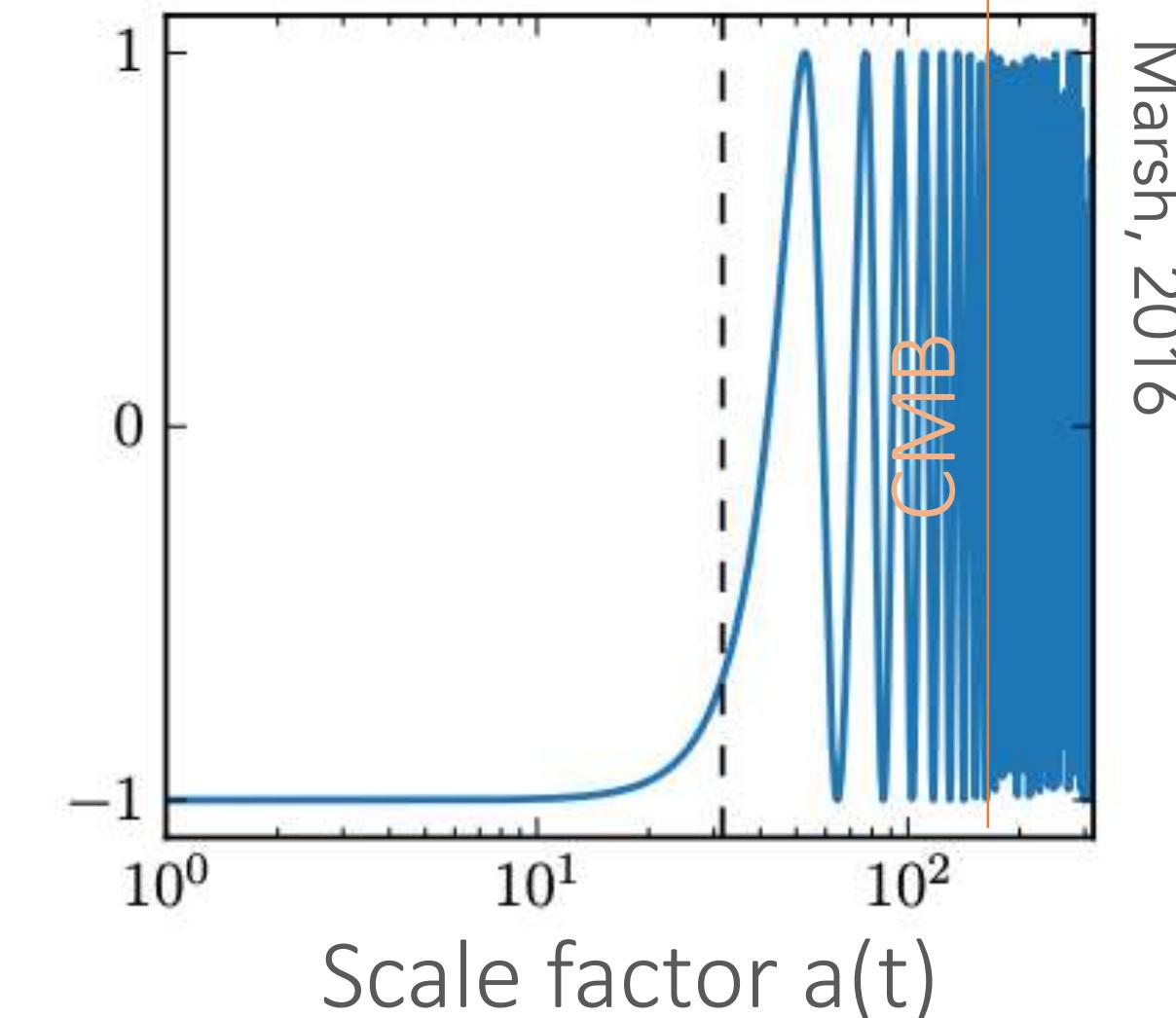
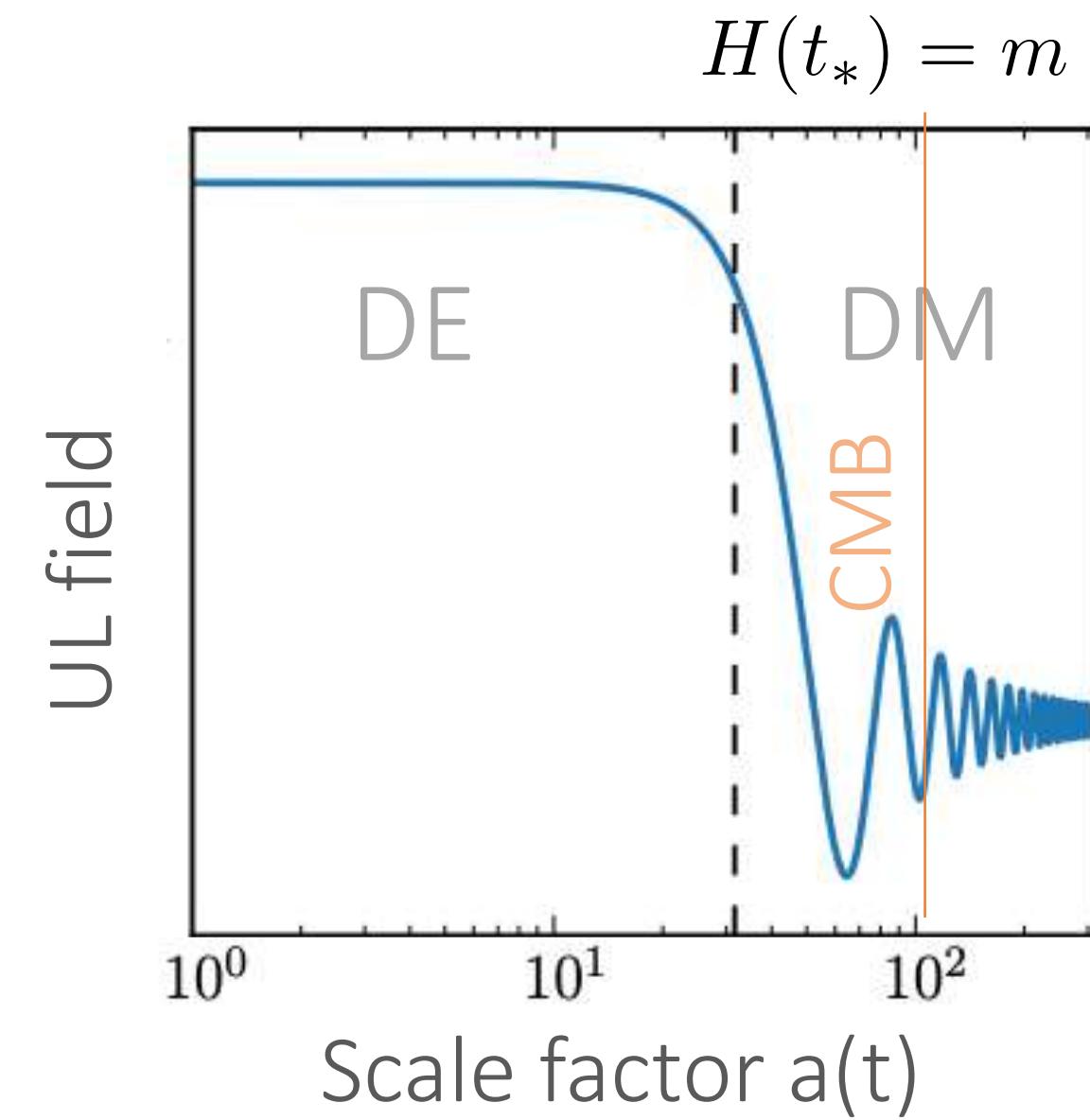
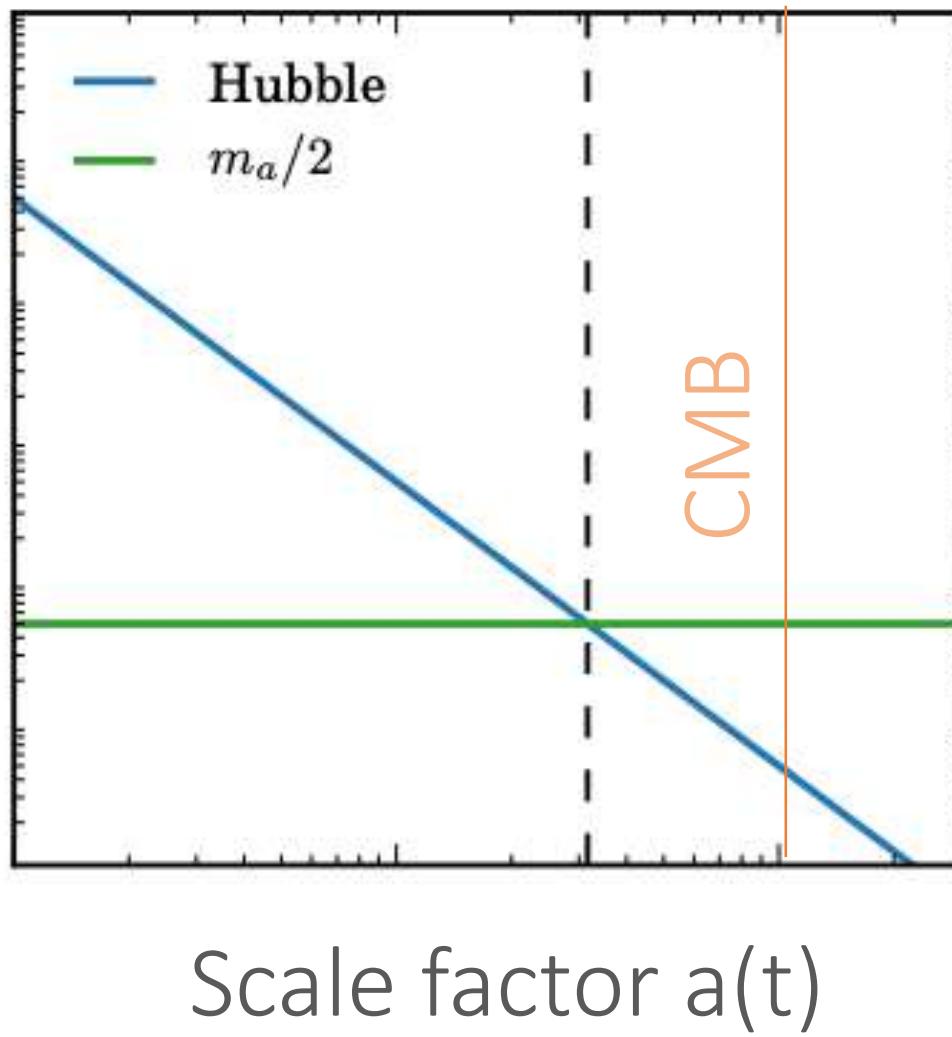
$$\left[ \begin{array}{ccc} H \gg m & \xrightarrow{\quad} & \phi_{\text{early}} = \phi(t_i) & \xrightarrow{\quad} & \omega = -1 & \text{DE} \\ H \ll m & \xrightarrow{\quad} & \phi_{\text{late}} \propto e^{imt} & \xrightarrow{\quad} & \langle \omega \rangle = 0 & \text{DM} \end{array} \right]$$



# Cosmological evolution

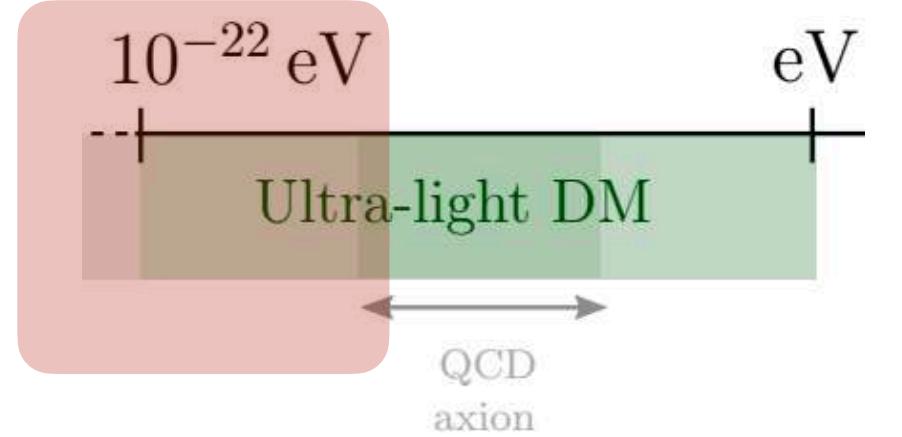


In order to **behave like DM**: start oscillating before matter-radiation equality



$$m > 10^{-28} \text{ eV} \sim H(a_{\text{eq}})$$

# Structure formation - non-relativistic regime



Evolution on small scales: take non-relativistic regime of the theory, relevant for structure formation.

Schrödinger-Poisson system : describe the FDM and the SIFDM

$$\left\{ \begin{array}{l} i\dot{\psi} = \left( -\frac{1}{2m}\nabla^2 + \frac{g}{8m^2}|\psi|^2 - m\Phi \right) \psi \\ \nabla^2\Phi = 4\pi G(m|\psi|^2 - \bar{\rho}) \end{array} \right.$$

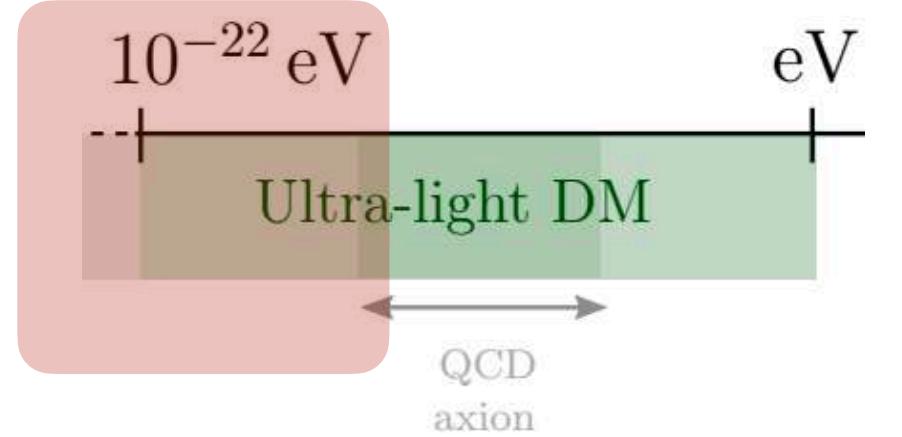
Schrödinger equation  
(Gross-Pitaevskii)

Poisson equation

$g = 0 \longrightarrow$  FDM  
 $g \neq 0 \longrightarrow$  SIFDM

Fundamentally different than  
CDM/WDM/SIDM!

# Structure formation - non-relativistic regime



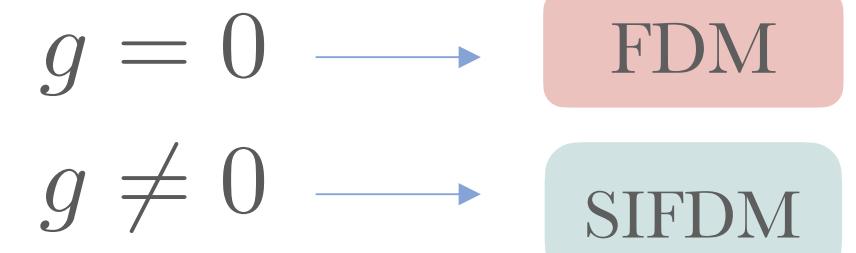
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Schrödinger equation  
(Gross-Pitaevskii)

Poisson equation



Fundamentally different than  
CDM/WDM/SIDM!

Madelung equations  $(\psi \equiv \sqrt{\rho/m} e^{i\theta} \text{ and } \mathbf{v} \equiv \nabla\theta/m)$

$$\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\dot{\mathbf{v}} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{m} \left( V_{grav} - P_{int} - \frac{1}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right)$$

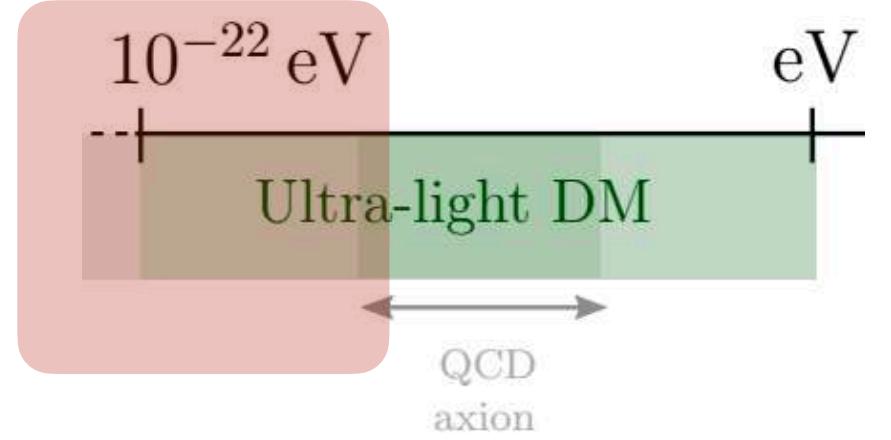
$$P_{int} = K\rho^{(j+1)/j} = \frac{g}{2m^2}\rho^2$$

$$\frac{1}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}$$

Quantum pressure

FLUID  
DESCRIPTION

# Structure formation - non-relativistic regime



Evolution on small scales: take non-relativistic regime of the theory, relevant for structure formation.

Schrödinger-Poisson system : describe the FDM and the SIFDM

$$\left\{ \begin{array}{l} i\dot{\psi} = \left( -\frac{1}{2m}\nabla^2 + \frac{g}{8m^2}|\psi|^2 - m\Phi \right) \psi \\ \nabla^2\Phi = 4\pi G(m|\psi|^2 - \bar{\rho}) \end{array} \right.$$

Schrödinger equation  
(Gross-Pitaevskii)

Poisson equation

$g = 0 \longrightarrow$  FDM  
 $g \neq 0 \longrightarrow$  SIFDM

Fundamentally different than  
CDM/WDM/SIDM!

## HOMEWORK

From the relativistic action of a scalar field in FRW

$$S_\phi = \int d^4x \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

Write the perturbed equation in Newtonian gauge

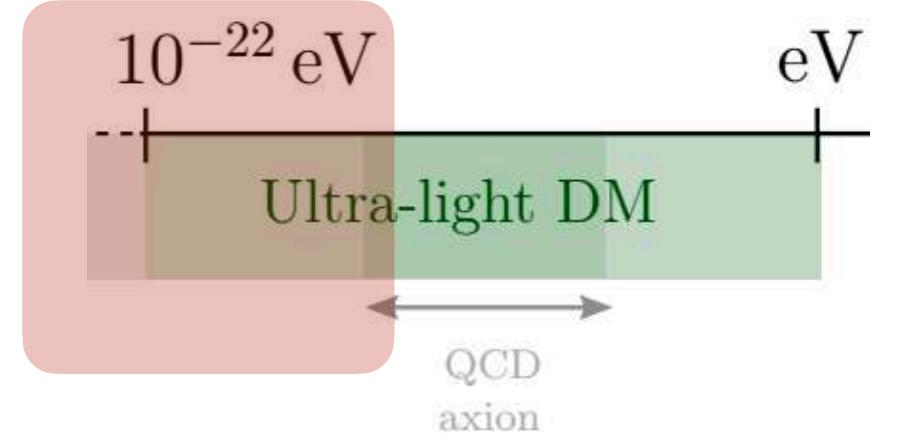
$$S_\phi = \int d^4x a^3 \left[ \frac{1}{2}(1 - 4\Phi) \dot{\phi}^2 - \frac{1}{a^2} (\partial_i \phi)^2 - (1 - 2\Phi)V(\phi) \right]$$

Compute the non-relativistic action and EoM (Schrödinger-Poisson system)

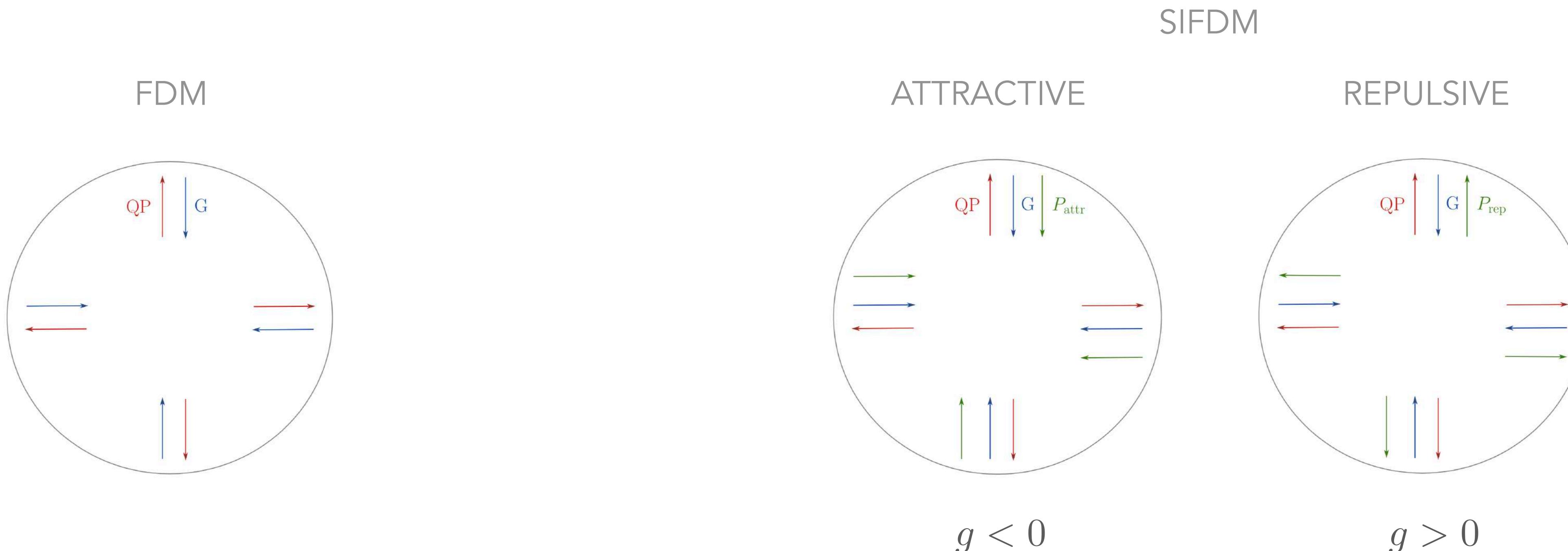
$$S = \int d^4x \left[ \frac{i}{2} (\psi \partial_t \psi^* - \psi^* \partial_t \psi) - \frac{|\nabla \psi|^2}{2m} - \frac{g}{16m^2} |\psi|^4 - m(\psi \psi^* - \langle \psi \psi^* \rangle) \Phi - \frac{a}{8\pi G} (\partial_i \Phi)^2 \right]$$

Refs.: EF, “ULDM” review  
Niemeyer 2019 review

# Structure formation - perturbation and stability



Competition between gravity and pressure (quantum pressure and interaction)



$$\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0$$

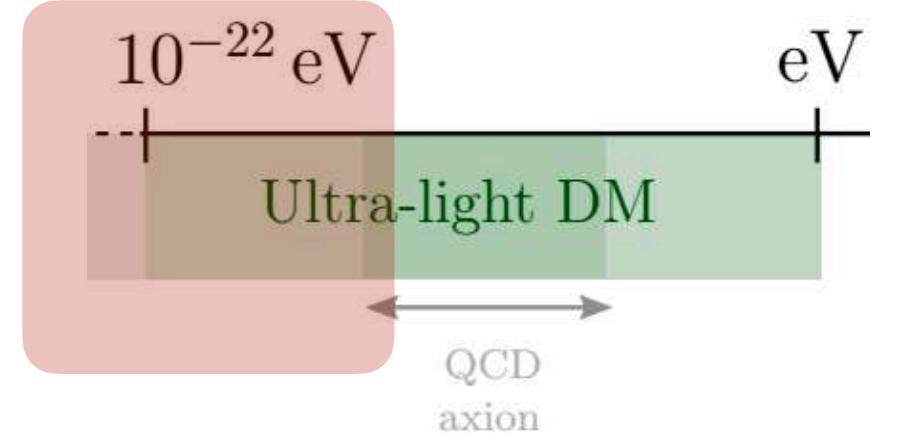
$$\dot{\mathbf{v}} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{m} \left( V_{grav} - P_{int} - \frac{1}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right)$$

$$P_{int} = \frac{g}{2m^2} \rho^2$$

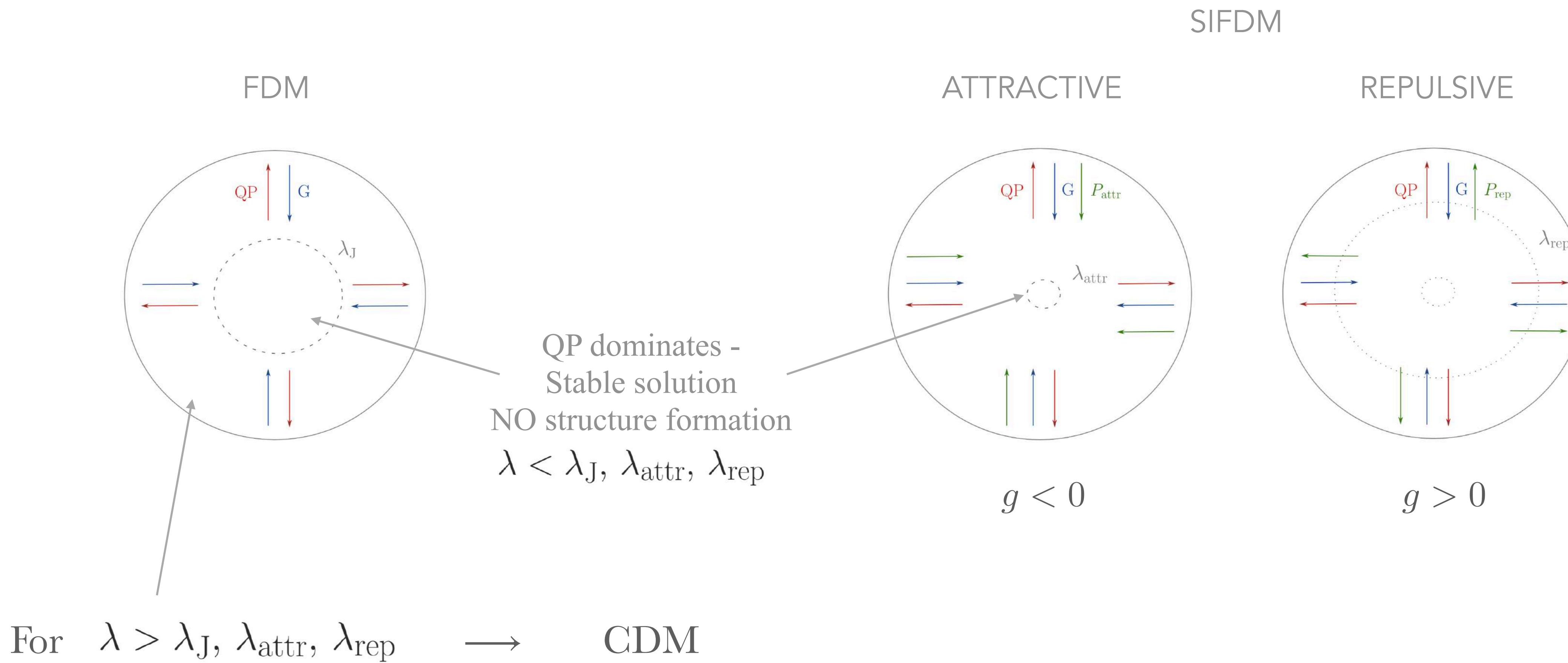
$$\frac{1}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}$$

Quantum pressure

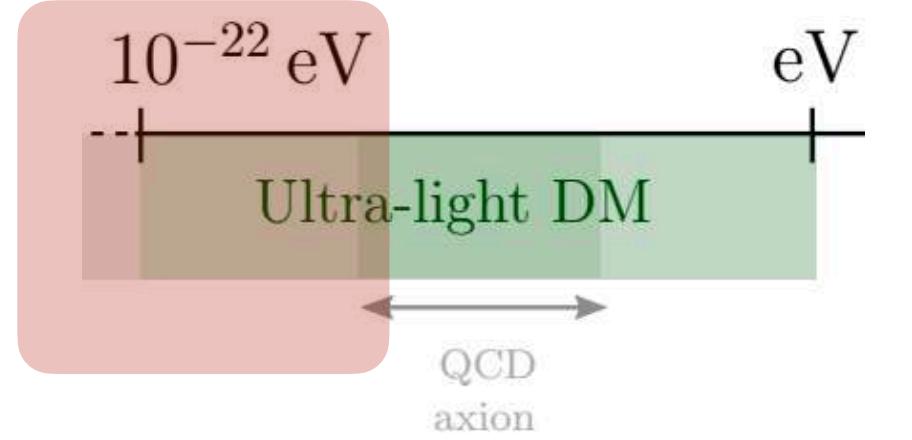
# Structure formation - perturbation and stability



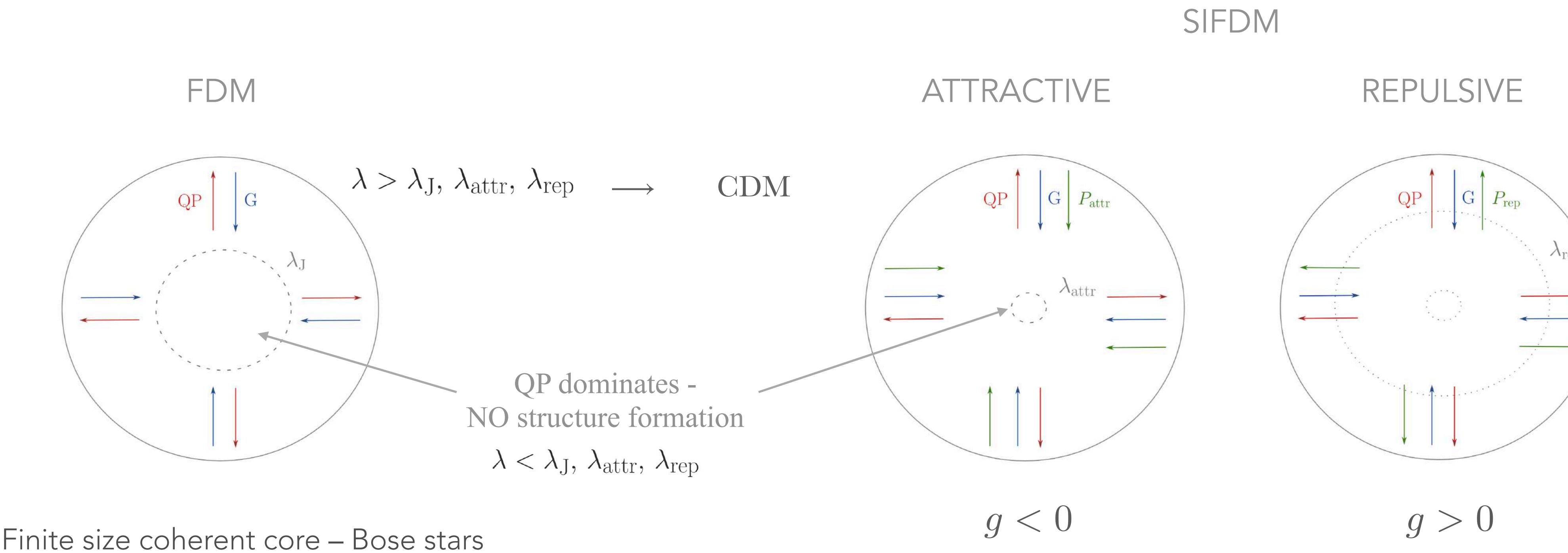
Finite clustering scale (Jeans length)- no structure formation on small scales  
( $\text{CDM}, \lambda_J$  effectively zero)



# Structure formation - perturbation and stability



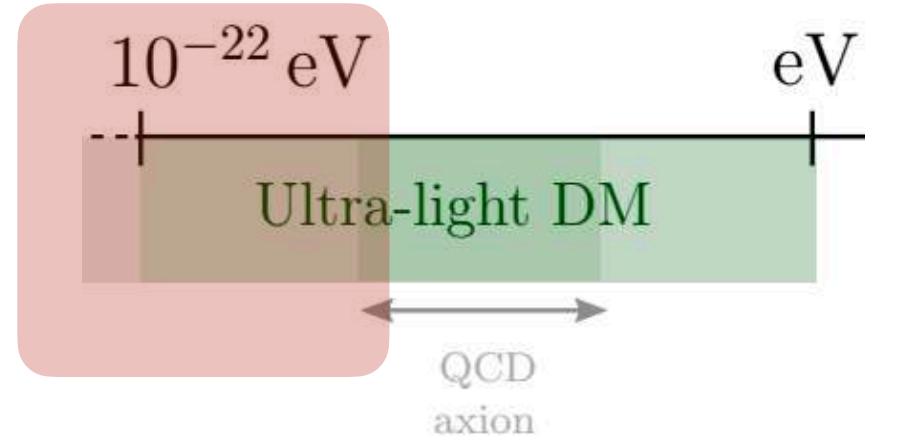
Finite clustering scale - no structure formation on small scales



$$m \leq 10^{-20} \text{ eV} \Rightarrow \lambda_{dB} > \mathcal{O}(\text{kpc})$$

Galactic scales

# Structure formation - perturbation and stability



Finite clustering scale - no structure formation on small scales

In the limit where only self-interaction is important:

$$i\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{g}{8m^2}|\psi|^2\psi.$$

We can decompose as:  $\psi(\mathbf{x}, t) = \psi_c(t) + \delta\bar{\Psi}(\mathbf{x}, t)$

Homogeneous:

$$i\dot{\psi}_c = \frac{g}{8m^2}|\psi_0|^2\psi_c$$

Periodic solution

$$\psi_c(t) = \psi_0 e^{-i\mu_c t}$$

where  $|\psi_0|^2 = n_0$  fixes the amplitude and  $\mu_c = gn_0/8m^2$

Perturbations:

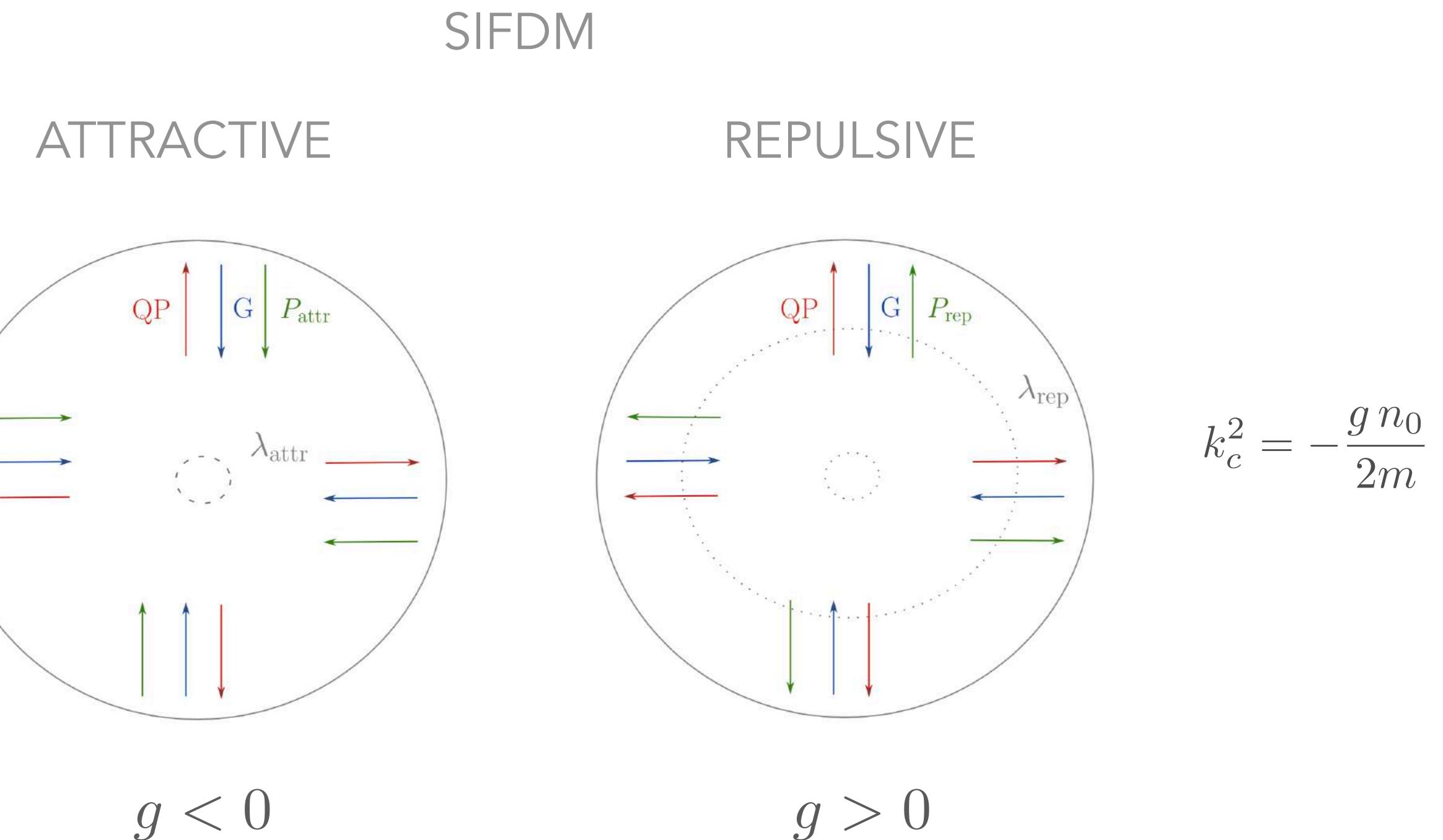
$$i\delta\Psi = -\frac{1}{2m}\nabla^2\delta\Psi + \frac{gn_0}{8m^2}(\delta\Psi + \delta\Psi^*)$$

Rewriting  $\delta\Psi$  as  $\Psi = A + iB$

$$\Rightarrow \frac{d}{dt} \begin{pmatrix} A_k \\ B_k \end{pmatrix} = \begin{pmatrix} 0 & \frac{k^2}{2m} \\ -\frac{k^2}{2m} - \frac{gn_0}{4m^2} & 0 \end{pmatrix} \begin{pmatrix} A_k \\ B_k \end{pmatrix}$$

↓ Dispersion relation

$$\omega_k^2 = \frac{gn_0}{4m^2} \frac{k^2}{2m} + \frac{k^4}{4m^2}$$



Solution:

$(\omega_k^2 > 0)$	$\delta\Psi_k = Z(\omega_k + \zeta_k)e^{i\omega_k t} + Z^*(\omega_k - \zeta_k)e^{-i\omega_k t}$
$(\omega_k^2 < 0)$	$\delta\Psi_k = c_1(\gamma_k - i\zeta_k)e^{\gamma_k t} + c_2(\gamma_k + i\zeta_k)e^{-\gamma_k t}$

# Structure formation - perturbation and stability

**HOMEWORK**

## EXERCISE:

In the limit where only self-interaction is important:

$$i\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{g}{8m^2}|\psi|^2\psi.$$

We can decompose as:  $\psi(\mathbf{x}, t) = \psi_c(t) + \delta\bar{\Psi}(\mathbf{x}, t)$

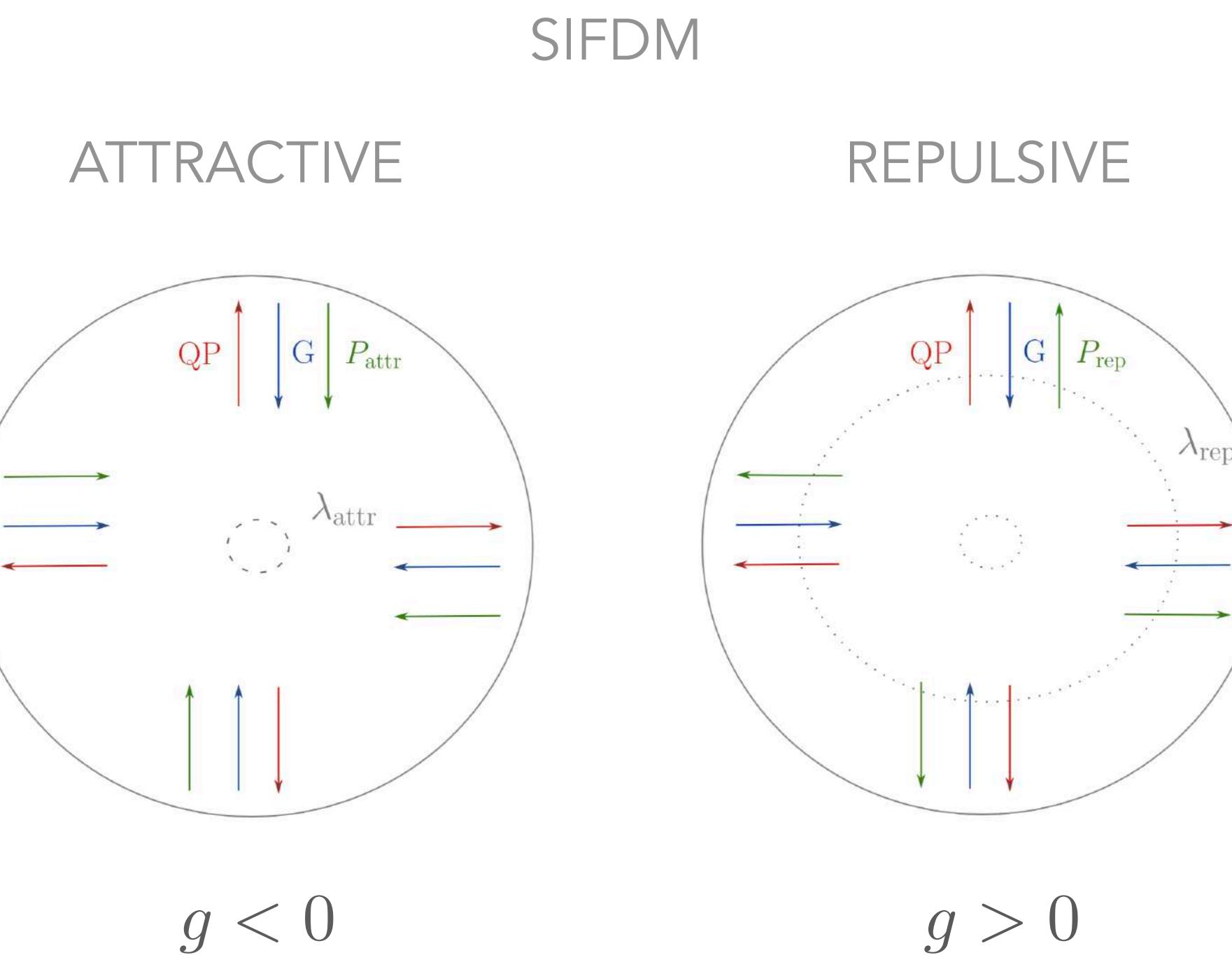
Homogeneous:

$$i\dot{\psi}_c = \frac{g}{8m^2}|\psi_0|^2\psi_c$$

$$\psi_c(t) = \psi_0 e^{-i\mu_c t}$$

Perturbations:

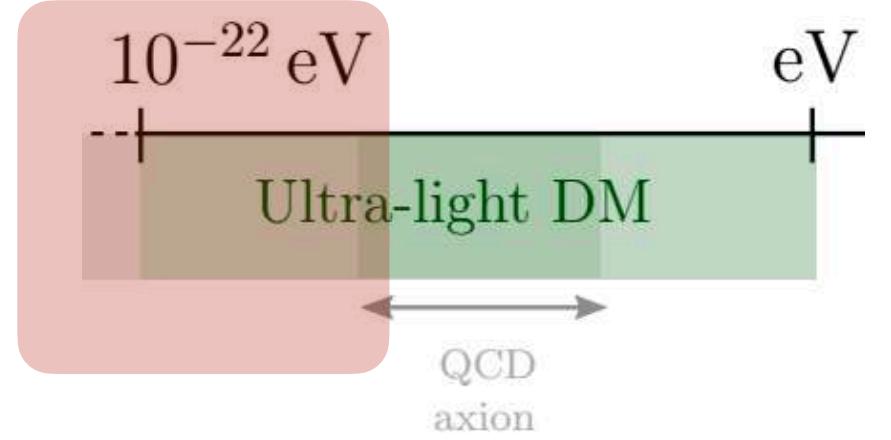
$$i\delta\Psi = -\frac{1}{2m}\nabla^2\delta\Psi + \frac{gn_0}{8m^2}(\delta\Psi + \delta\Psi^*)$$



$$k_c^2 = -\frac{g n_0}{2m}$$

Derive the solutions to the Schrodinger equation above for an attractive and repulsive potential.  
Identify the different scales of the problem and where we have clustering or a stable, oscillatory solution.

# Structure formation - perturbation and stability



Finite clustering scale - no structure formation on small scales

ATTRACTIVE

$$k_c^2 = -\frac{|g| n_0}{2m}$$

$$k > k_c (\lambda < \lambda_c) \Rightarrow$$

Solution oscillates and is stable

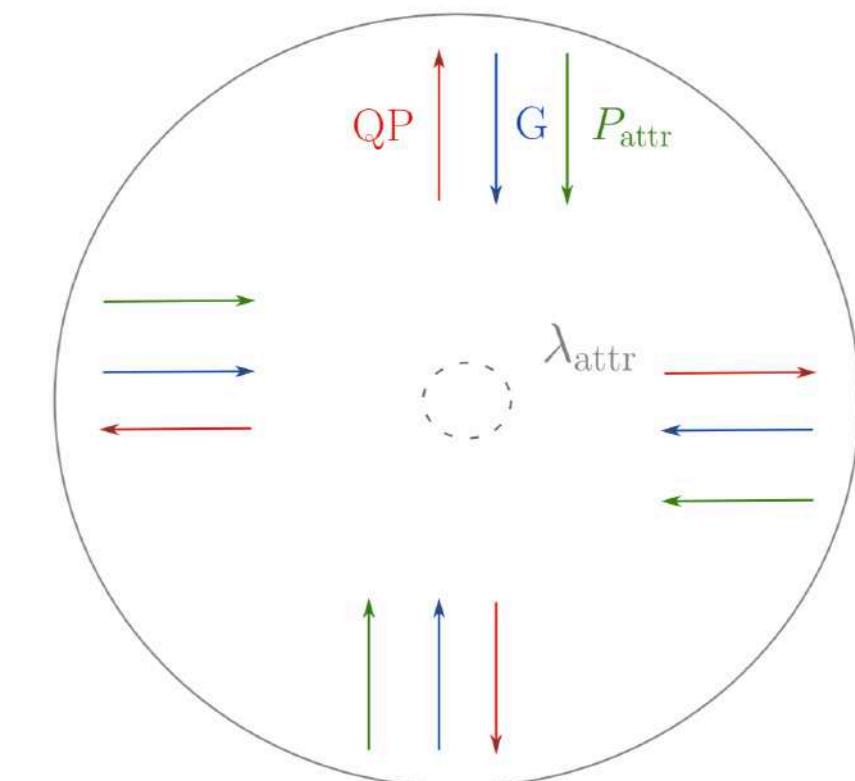
This stable configuration, however, is different than in the case for repulsive interaction, forming a localized object, with maximum size given by  $\lambda_c$

$$k < k_c (\lambda > \lambda_c) \Rightarrow$$

Exponential growth (like CDM)

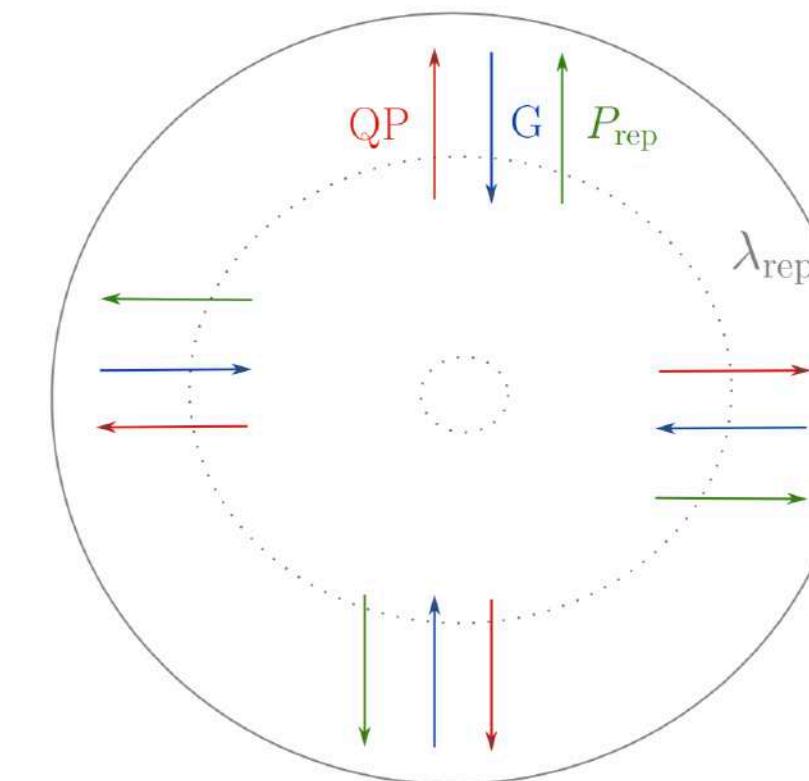
SIFDM

ATTRACTIVE



$$g < 0$$

REPULSIVE



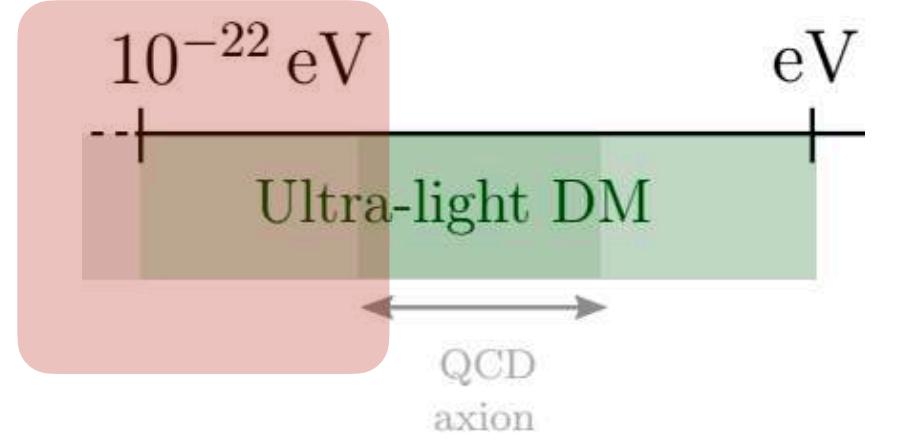
$$g > 0$$

$$k_c^2 = -\frac{g n_0}{2m}$$

For **attractive** interactions can only form **localized clumps** (solitons)

QCD axion:  $m \sim 10^{-5} \text{ eV}$   
 $\lambda_a \sim -10^{-48}$   $\rightarrow l_{soliton} \sim 10^{-5} \text{ kpc}$

# Structure formation - perturbation and stability



Finite clustering scale - no structure formation on small scales

REPULSIVE  $g > 0$

$$k_c^2 = \frac{g n_0}{2m}$$

Homogeneous configuration is always stable, and it is always going to be described by an oscillatory solution, either if  $\lambda$  is bigger or smaller than  $\lambda_c$

Dispersion relation

*Long wavelength regime*

$$\lambda \gg \lambda_c$$

*Short wavelength regime*

$$\omega_k \simeq c_s k$$

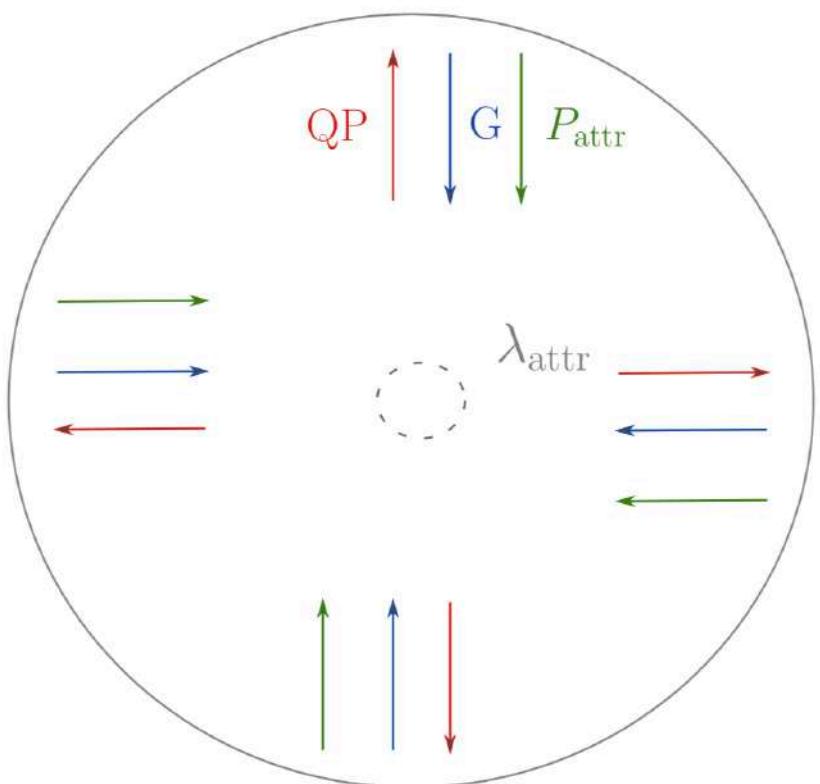
*Long range - superfluid*

$$\omega_k \simeq k^2/2m$$

*Free massive particle*

SIFDM

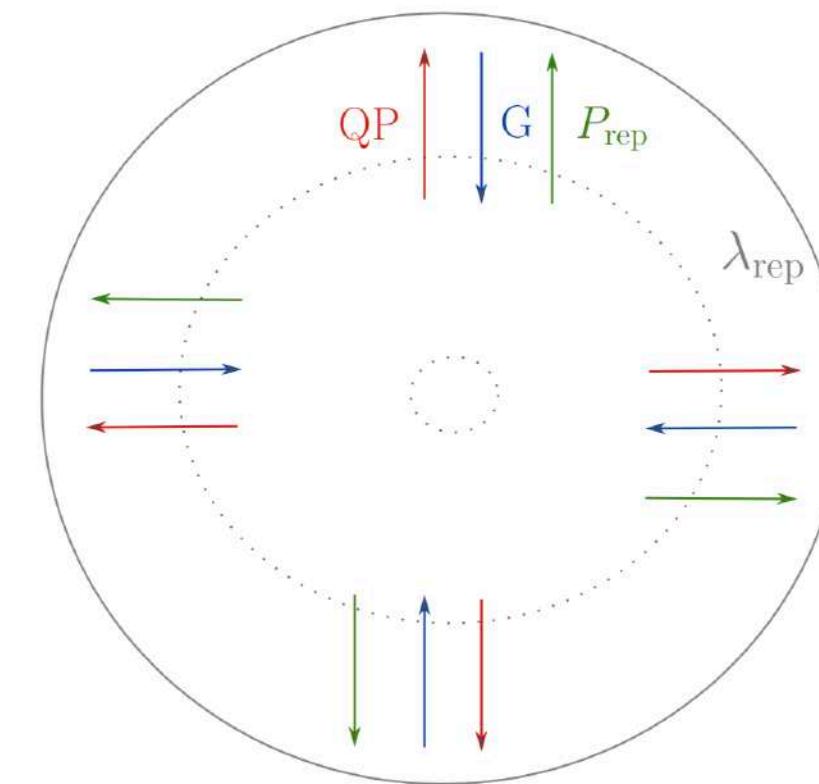
ATTRACTIVE



$$g < 0$$

Superfluid!

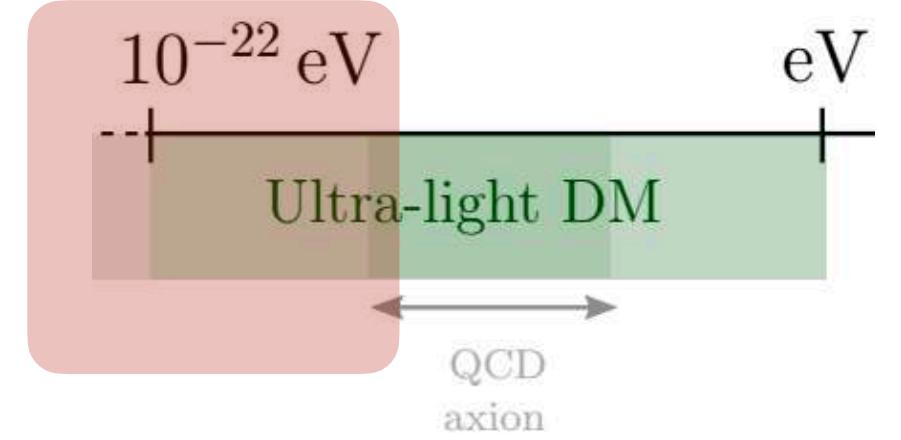
REPULSIVE



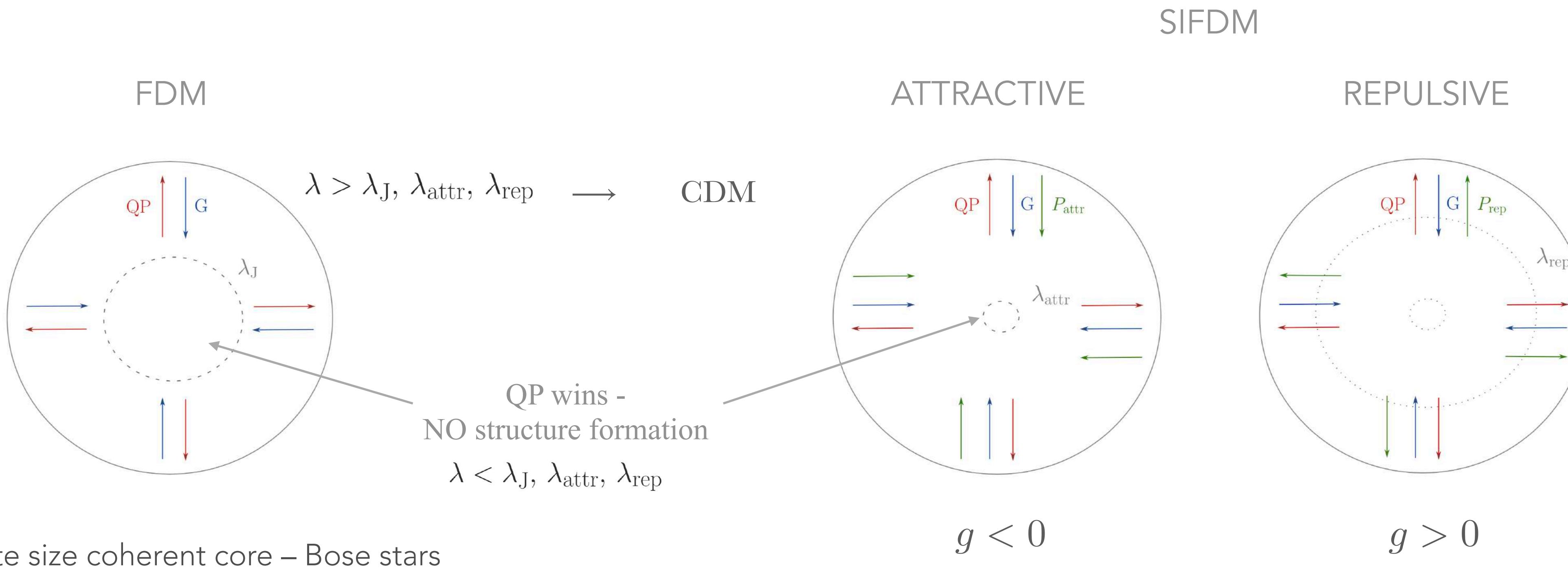
$$g > 0$$

$$k_c^2 = -\frac{g n_0}{2m}$$

# Structure formation - perturbation and stability



Finite clustering scale - no structure formation on small scales



$$\lambda_J = 55 \left( \frac{m}{10^{-22} \text{ eV}} \right)^{-1/2} \left( \frac{\rho}{\bar{\rho}} \right)^{-1/4} (\Omega_m h)^{-1/4} \text{ kpc}$$

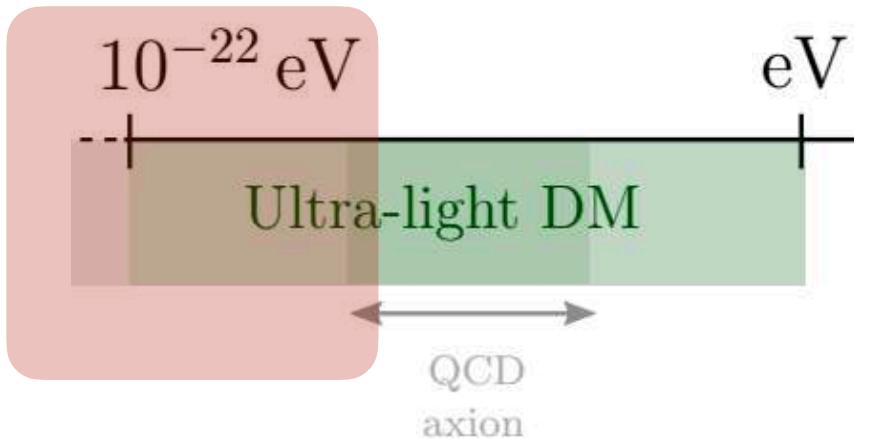
$m \leq 10^{-20} \text{ eV} \Rightarrow \lambda_{dB} > \mathcal{O}(\text{kpc})$

Galactic scales

$g > 0$	$\rightarrow \forall \lambda$	Solution oscillates. Condensate (long range)
$g < 0$	$\rightarrow \begin{cases} \lambda > \lambda_* \\ \lambda < \lambda_* \end{cases}$	Structures grow. No condensate. Solution oscillates. Condensate (finite size)

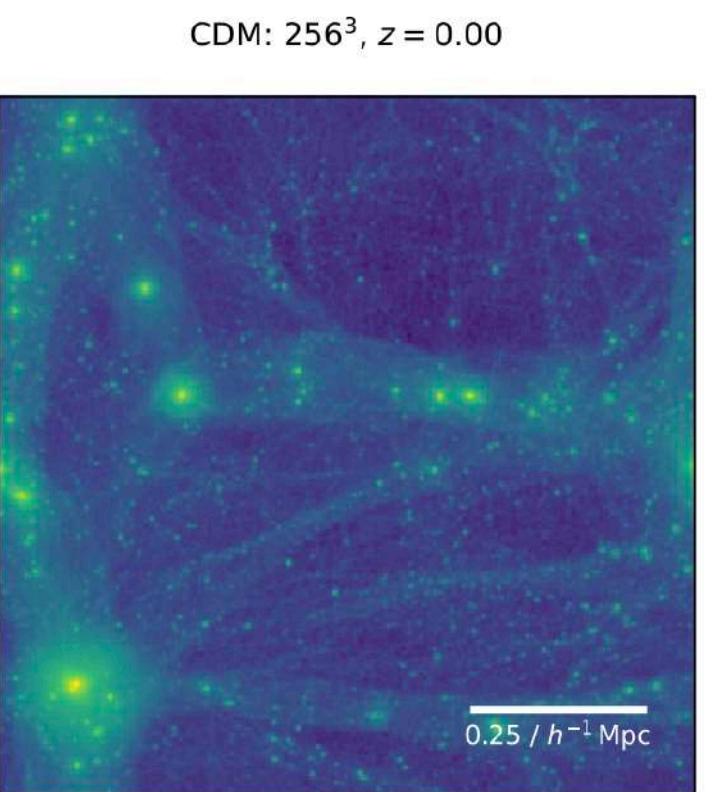
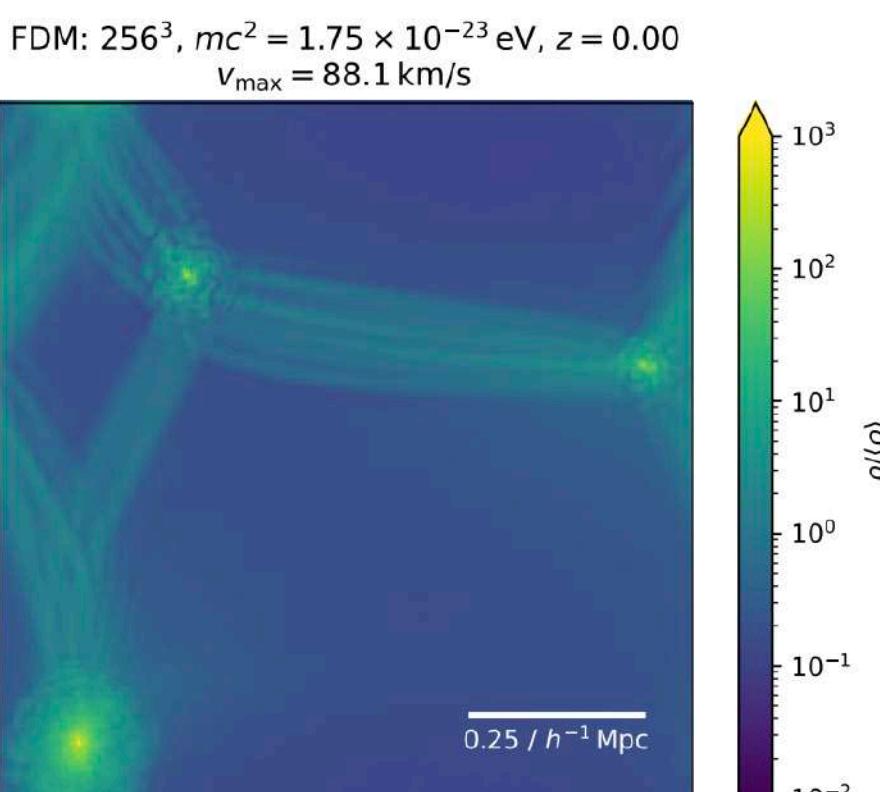
# Phenomenology

## RICH PHENOMENOLOGY ON SMALL SCALES



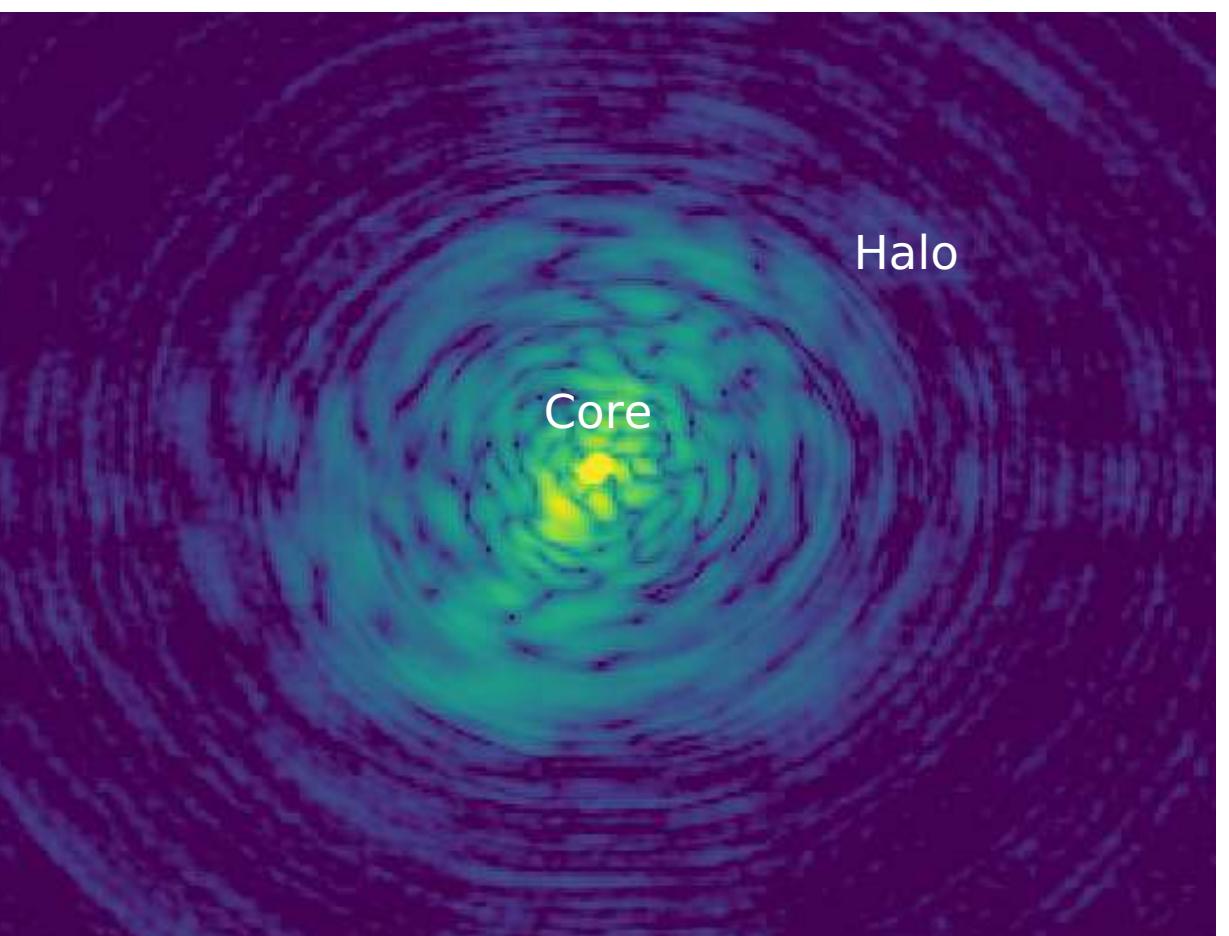
\* Focus only in gravitational signatures

### Suppression of small structures

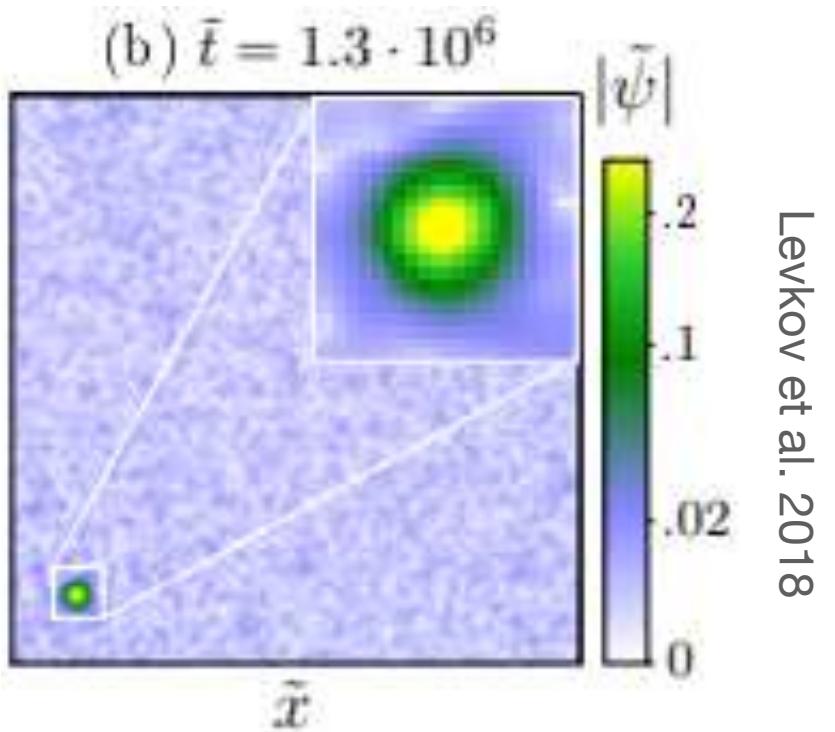


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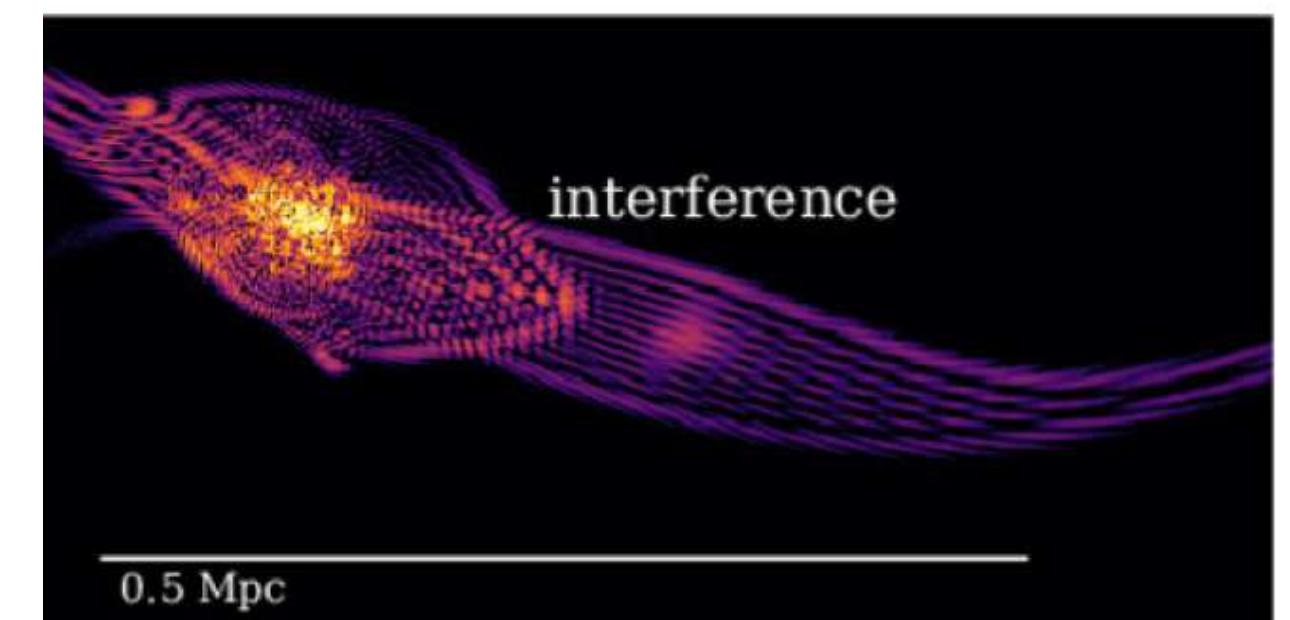
### Formation of a solitonic core



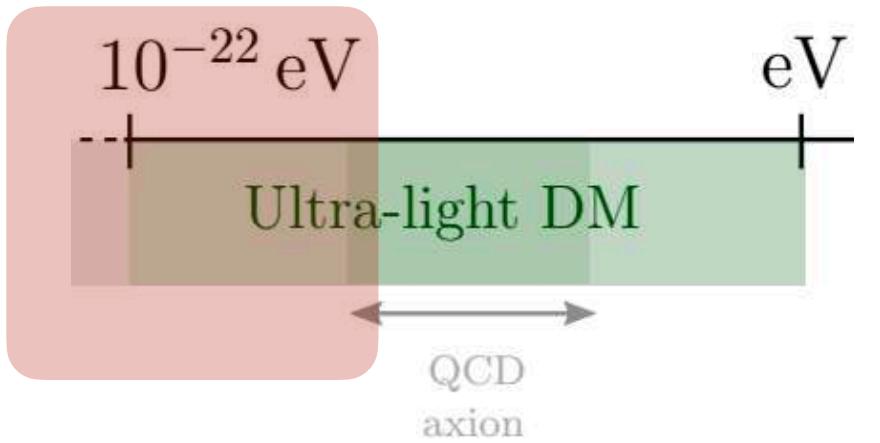
### Dynamical effects



### Wave interference

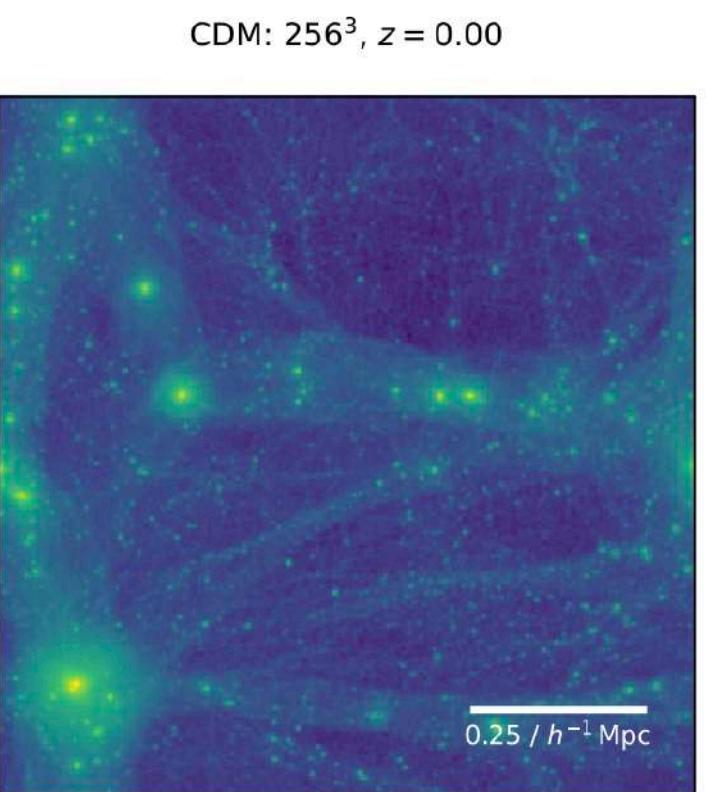
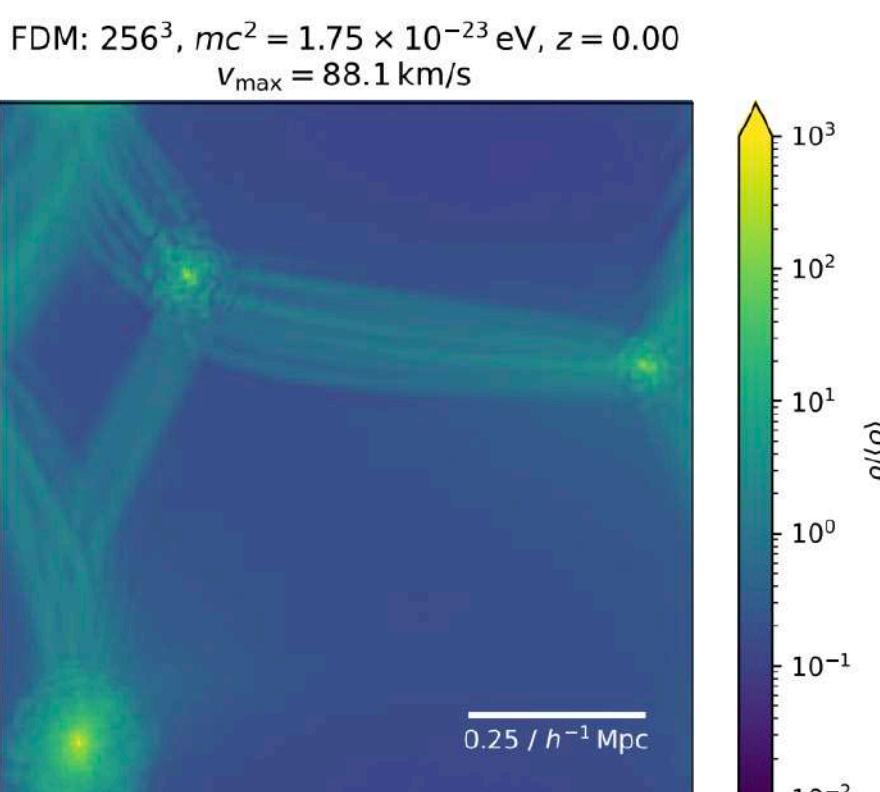


# Phenomenology



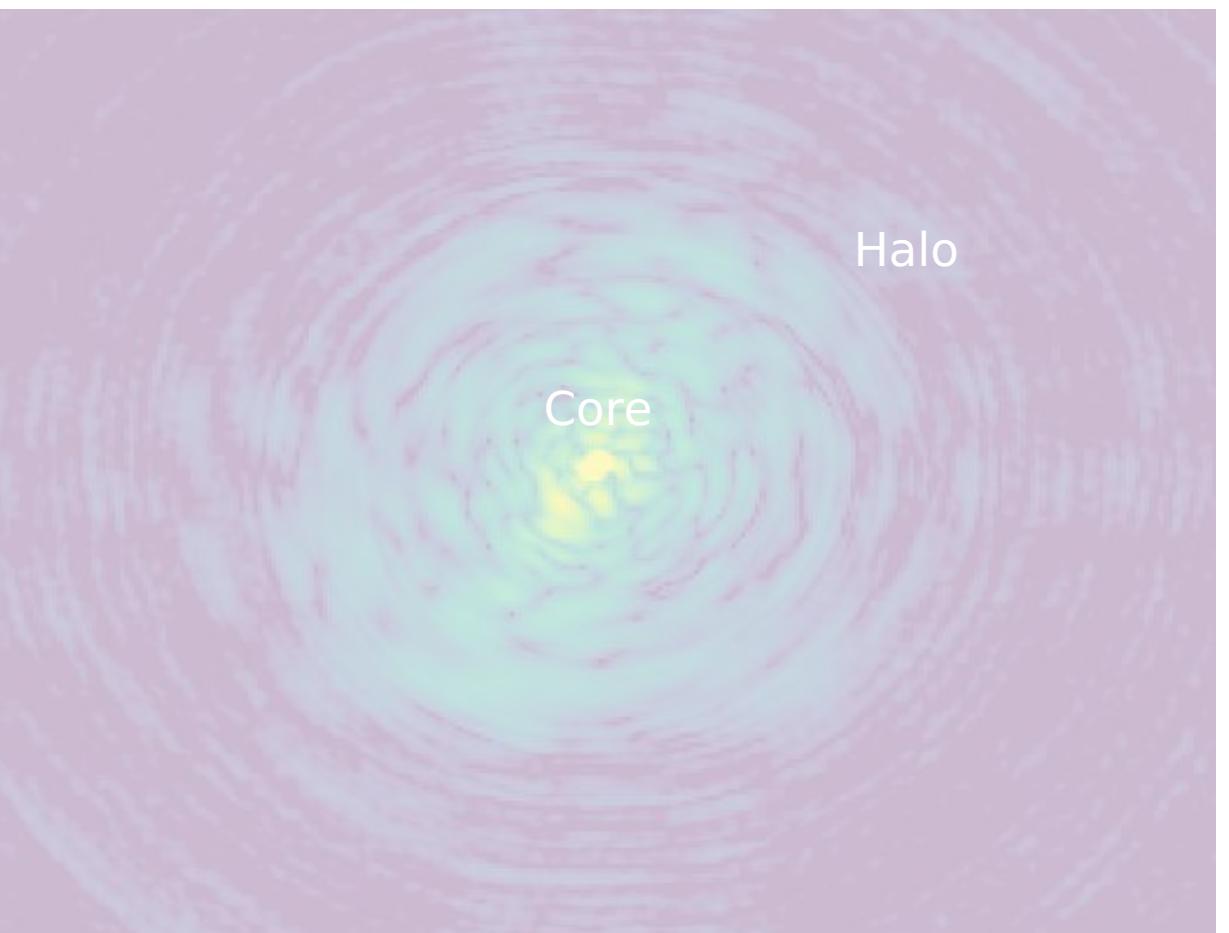
## RICH PHENOMENOLOGY ON SMALL SCALES

### Suppression of small structures

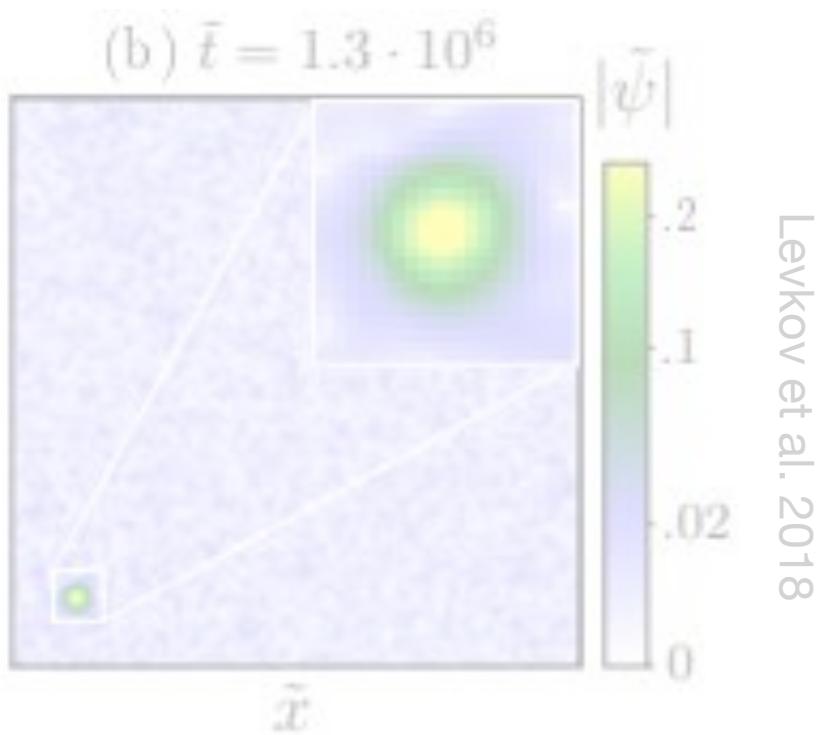


S. May et al. 2021

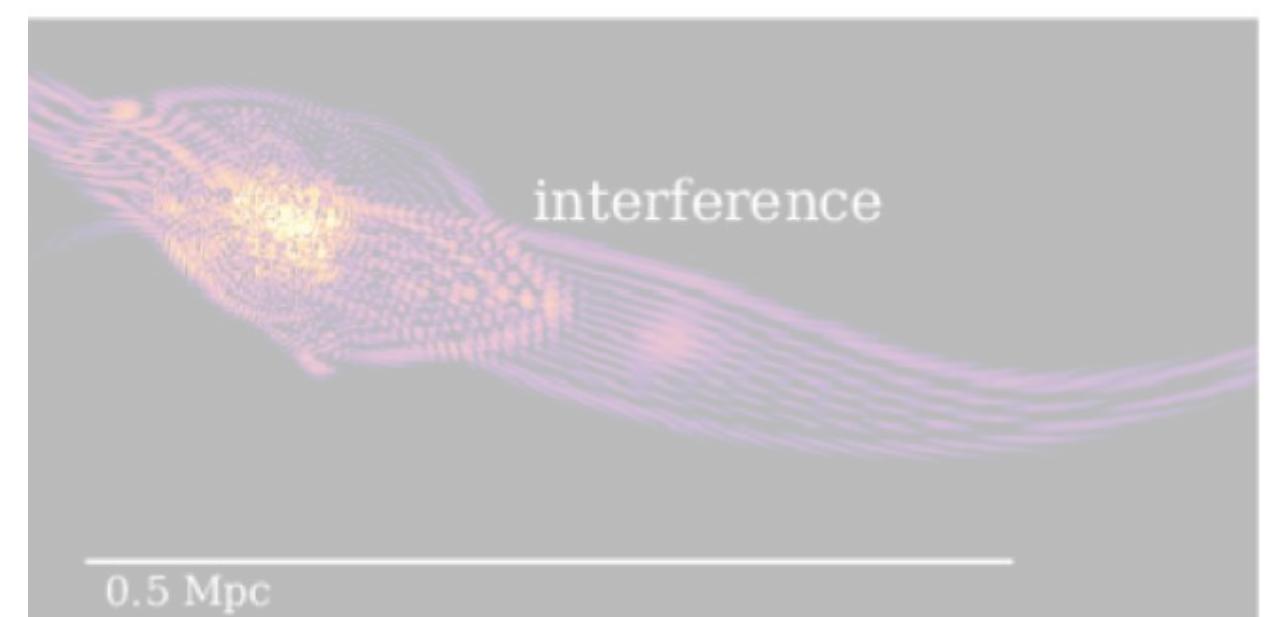
### Formation of a solitonic core



### Dynamical effects



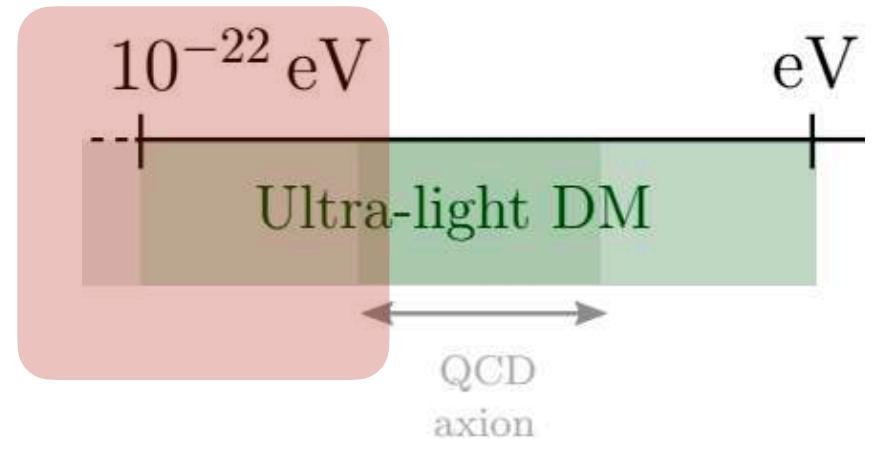
### Wave interference



Mocz et al. 2017

# Phenomenology

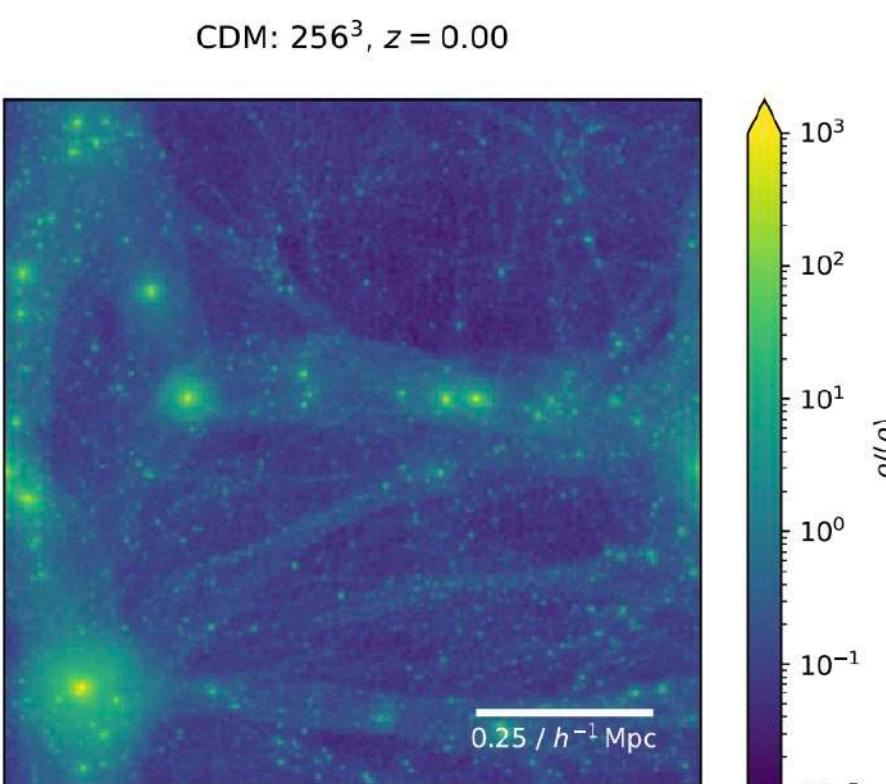
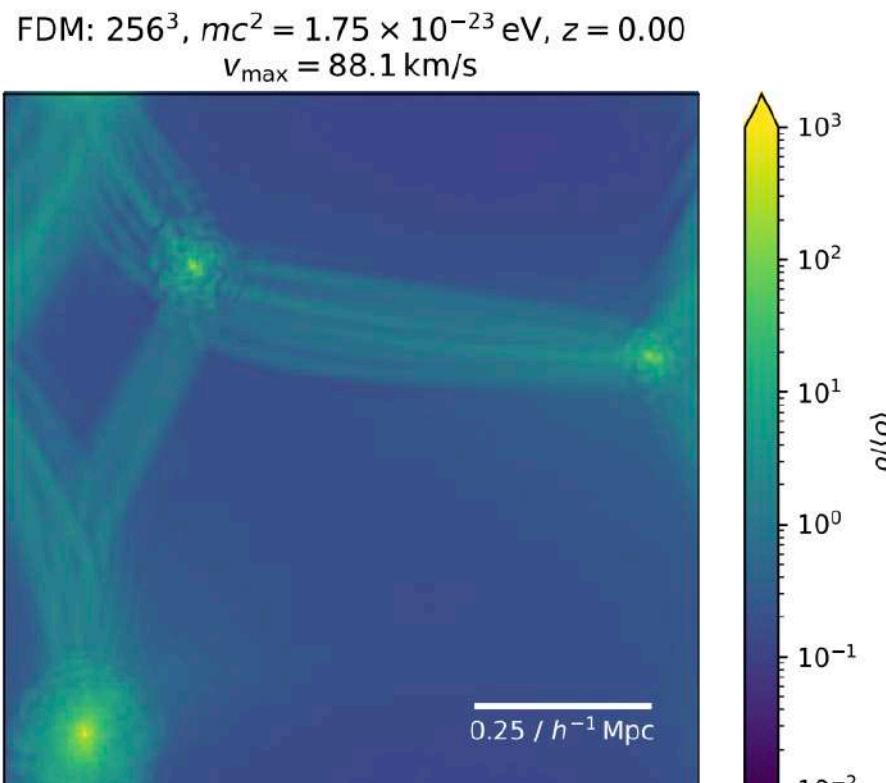
## Suppression of small structures



Finite Jeans length  $\lambda_J$  or  $\lambda_{\text{attr}}, \lambda_{\text{rep}}$

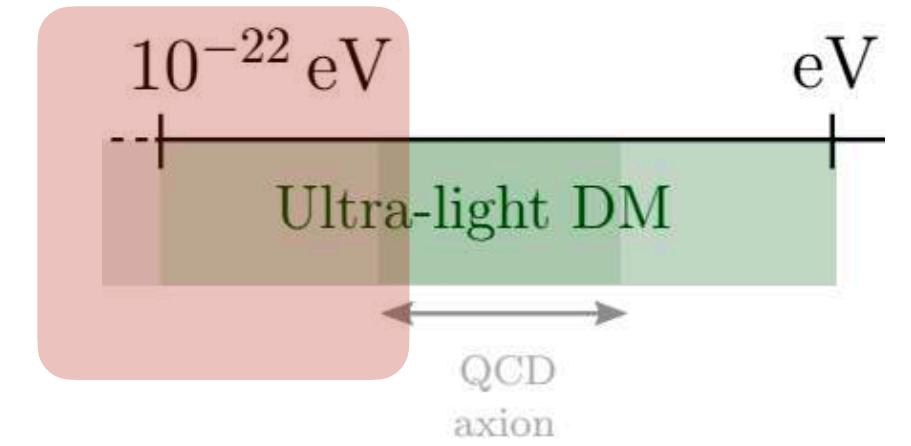


No small scale structure



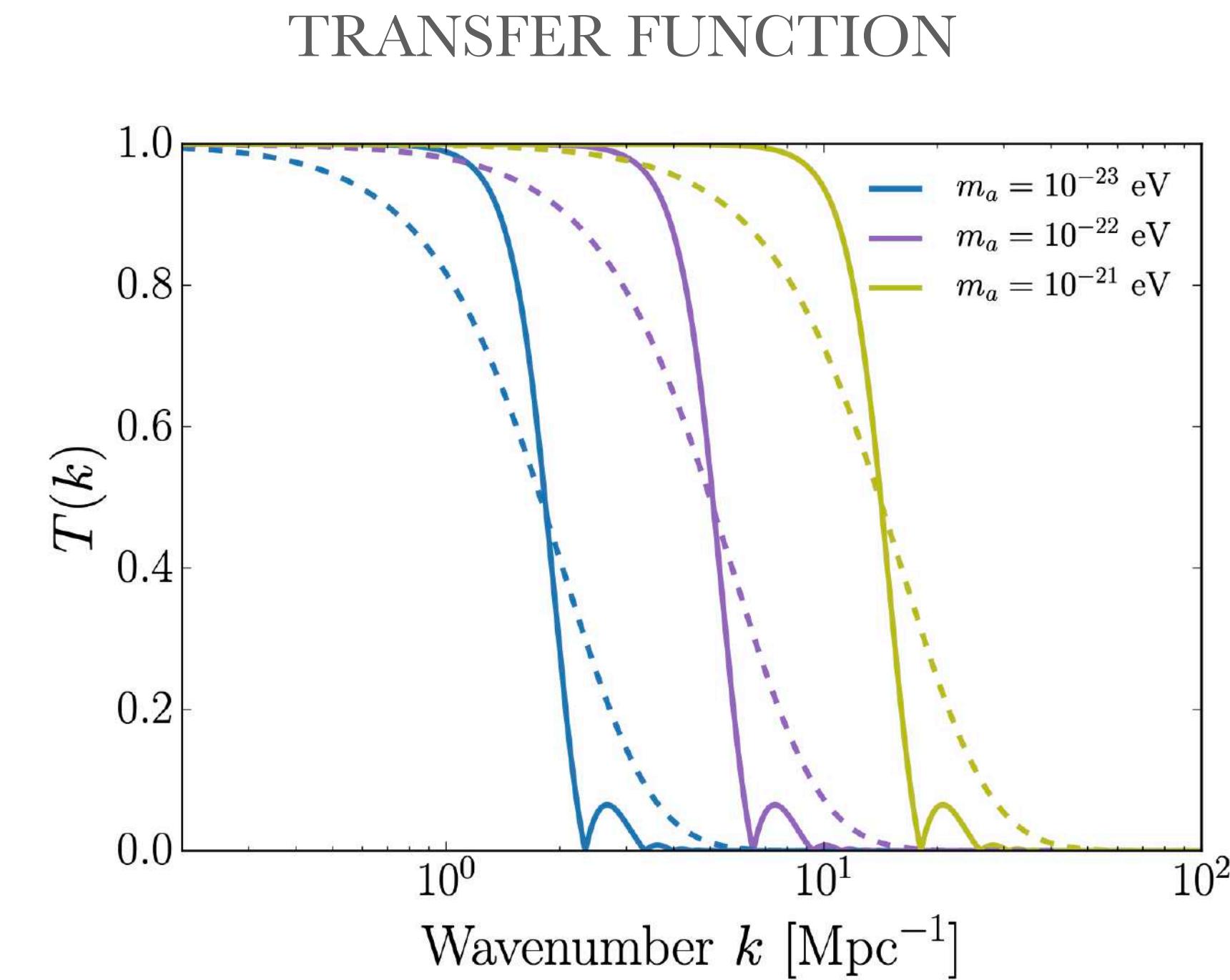
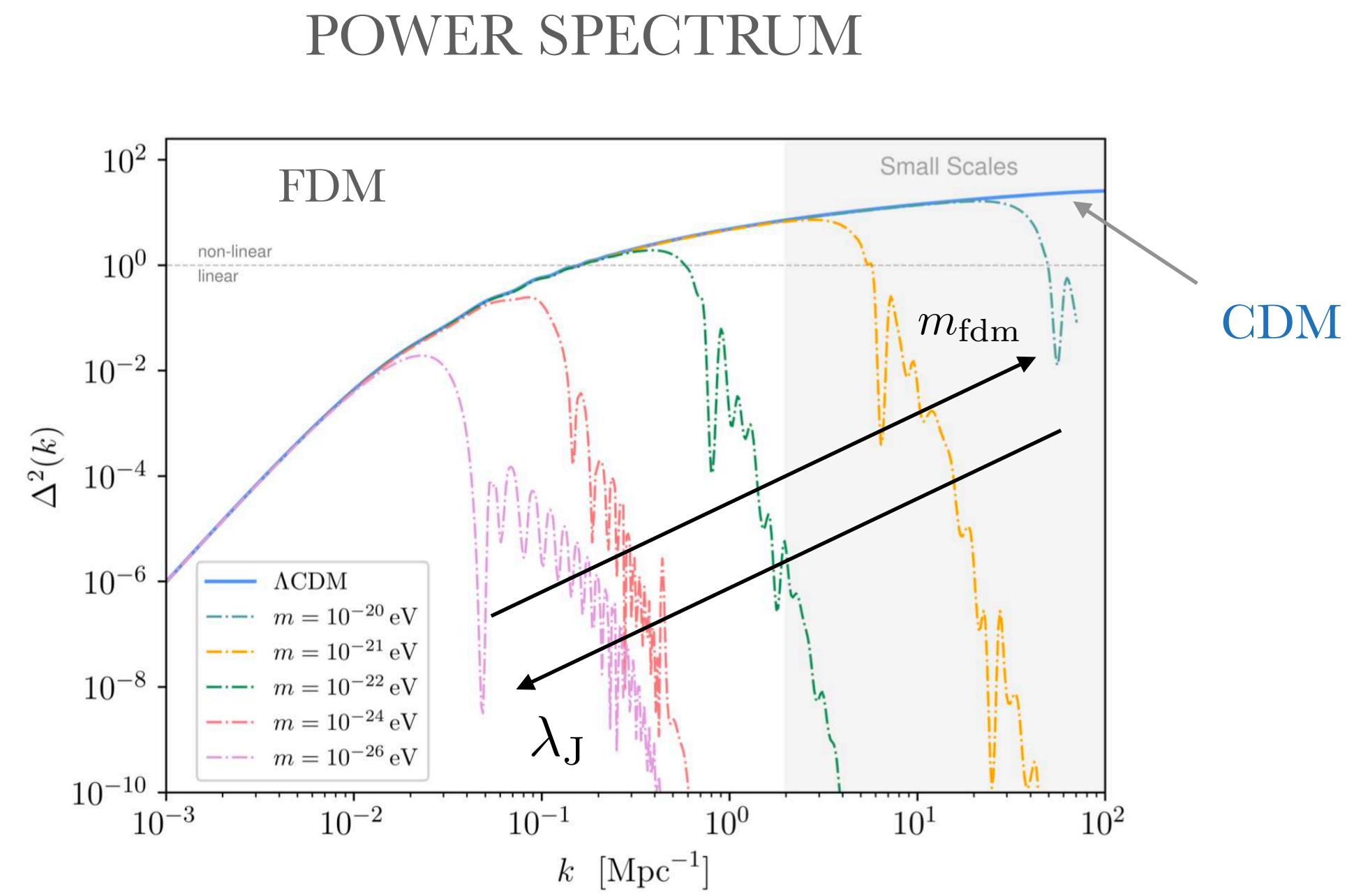
# Phenomenology

## Suppression of small structures



Finite Jeans length  $\lambda_J$  or  $\lambda_{\text{attr}}, \lambda_{\text{rep}}$

Suppresses small scale structure



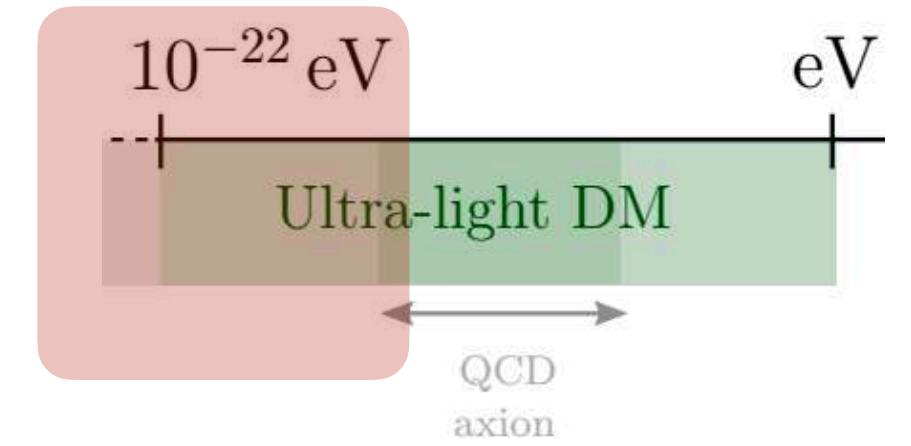
$$P_X(k, z) = T_X^2(k, z) P_{\Lambda CDM}(k)$$

$$\begin{cases} T_{WDM} = [1 + (\alpha k)^{2\mu}]^{-5/\mu} \\ T_{FDM} = \frac{\cos x_J^3(k)}{1+x_J^8(k)} \end{cases}$$

- Degenerate with WDM

# Phenomenology

## Suppression of small structures

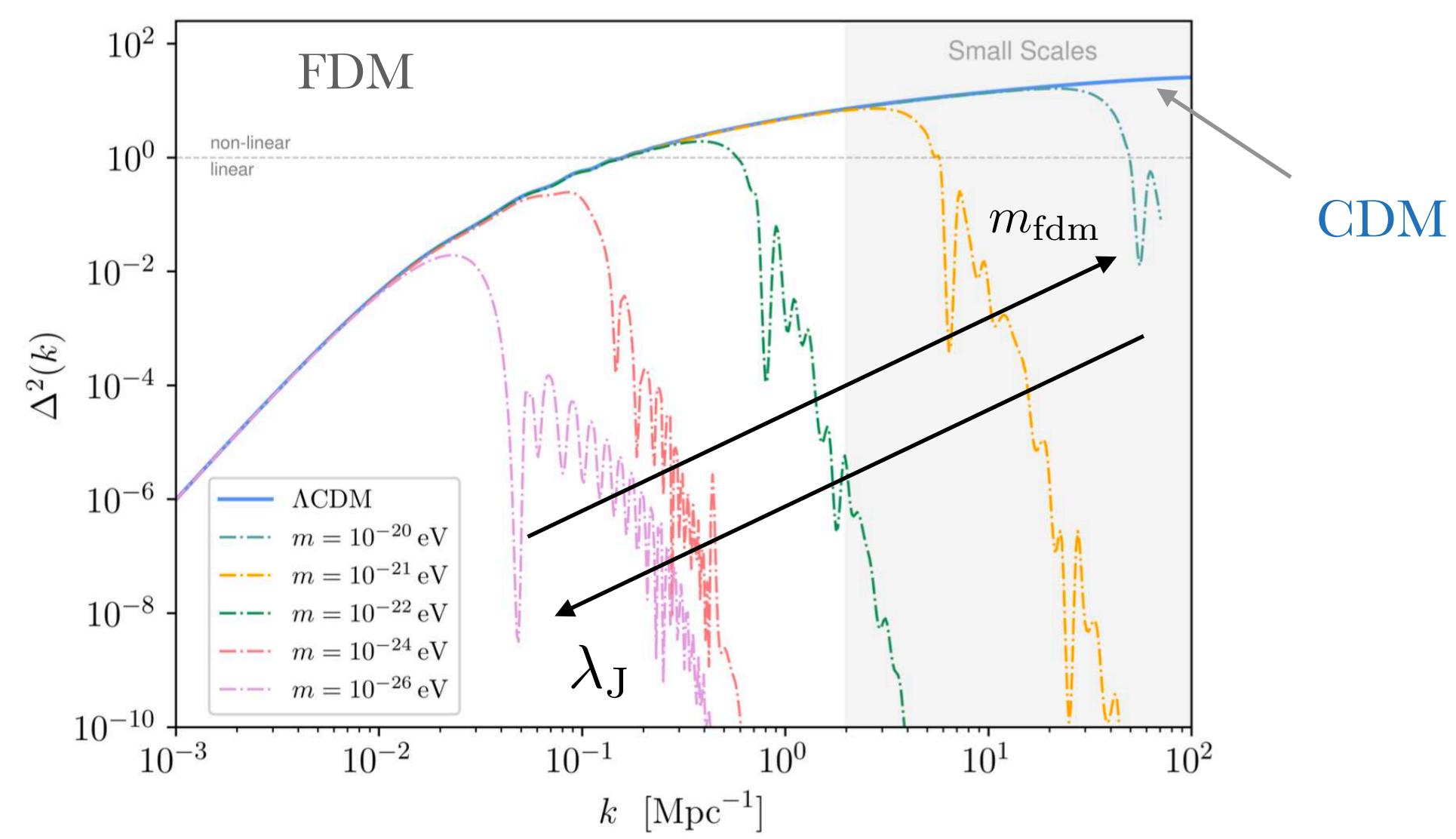


Finite Jeans length  $\lambda_J$  or  $\lambda_{\text{attr}}, \lambda_{\text{rep}}$

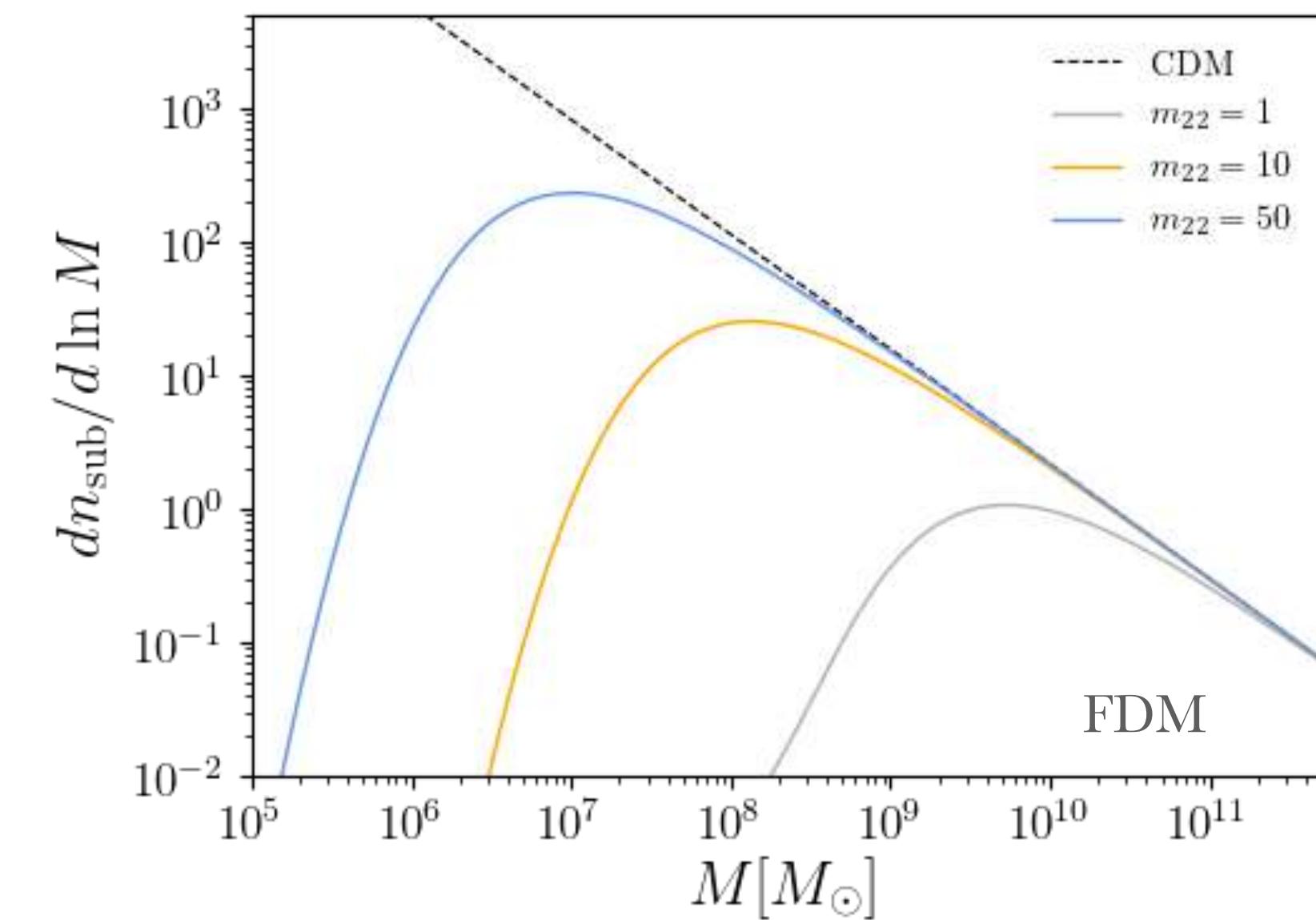


Suppresses small scale structure

POWER SPECTRUM



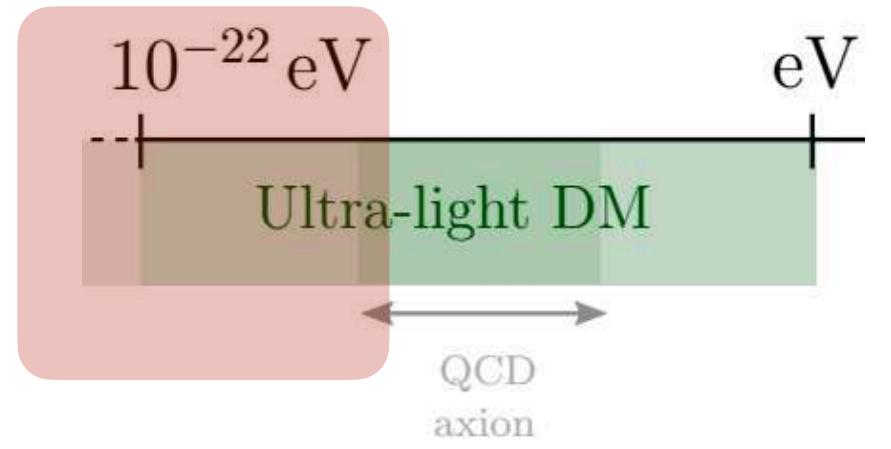
(sub) HALO MASS FUNCTION



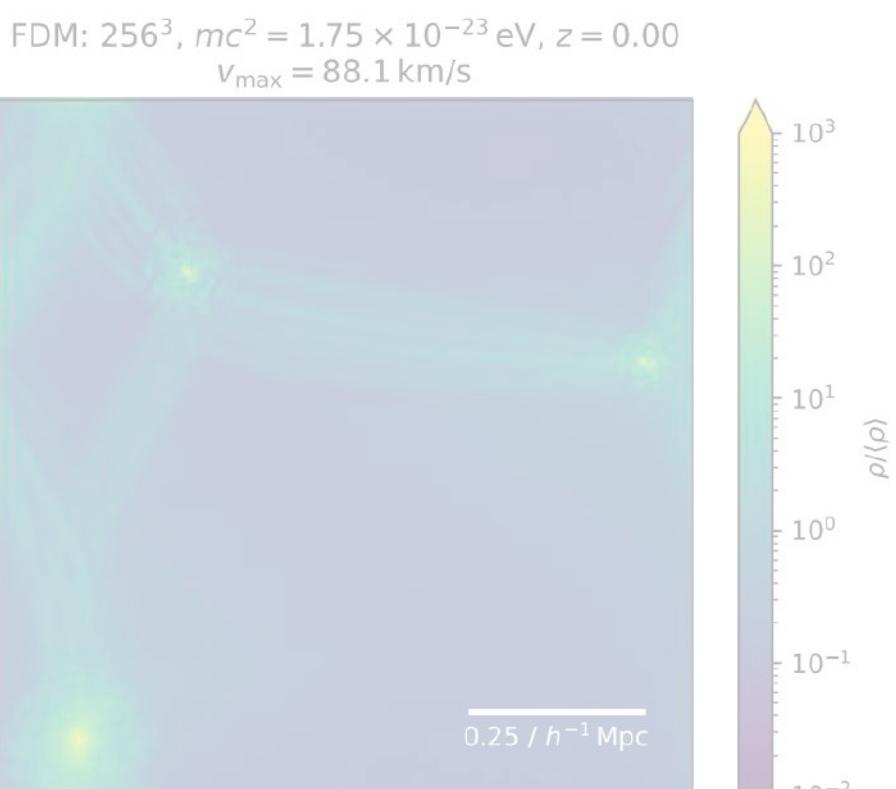
What about the PS from inflation? Practice session!

# Phenomenology

## RICH PHENOMENOLOGY ON SMALL SCALES

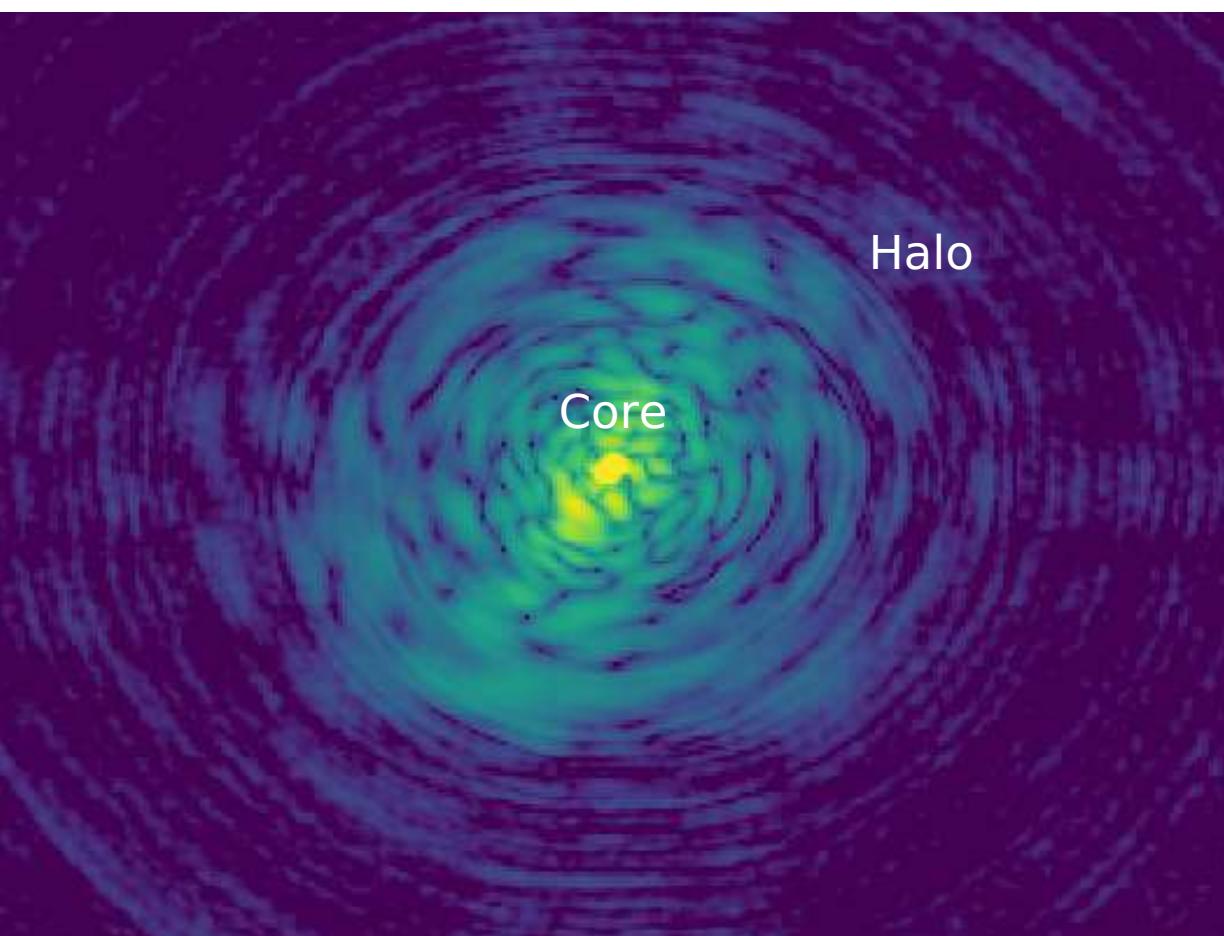


Suppression of small structures

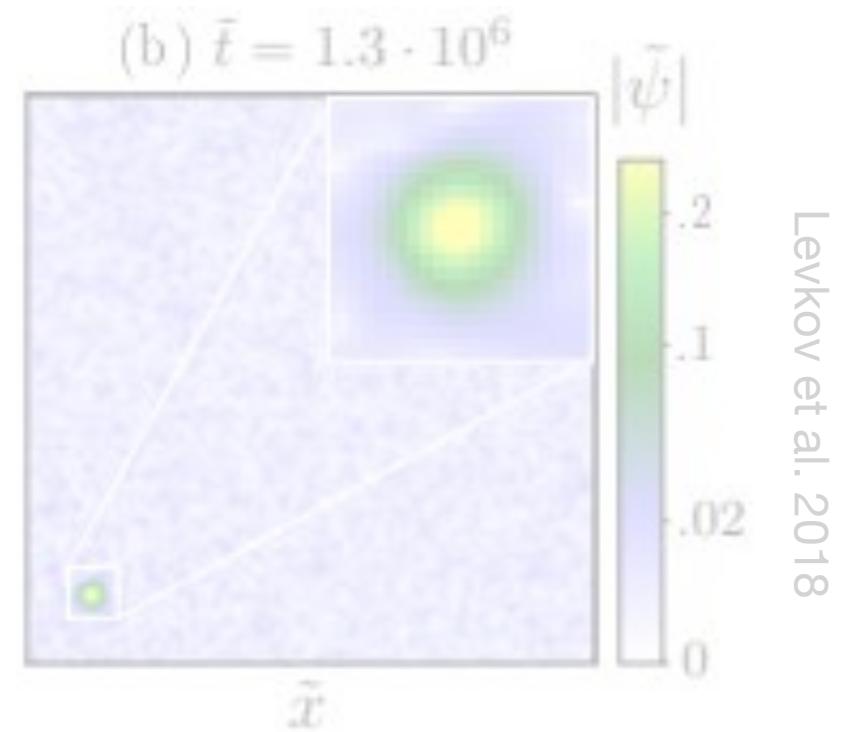


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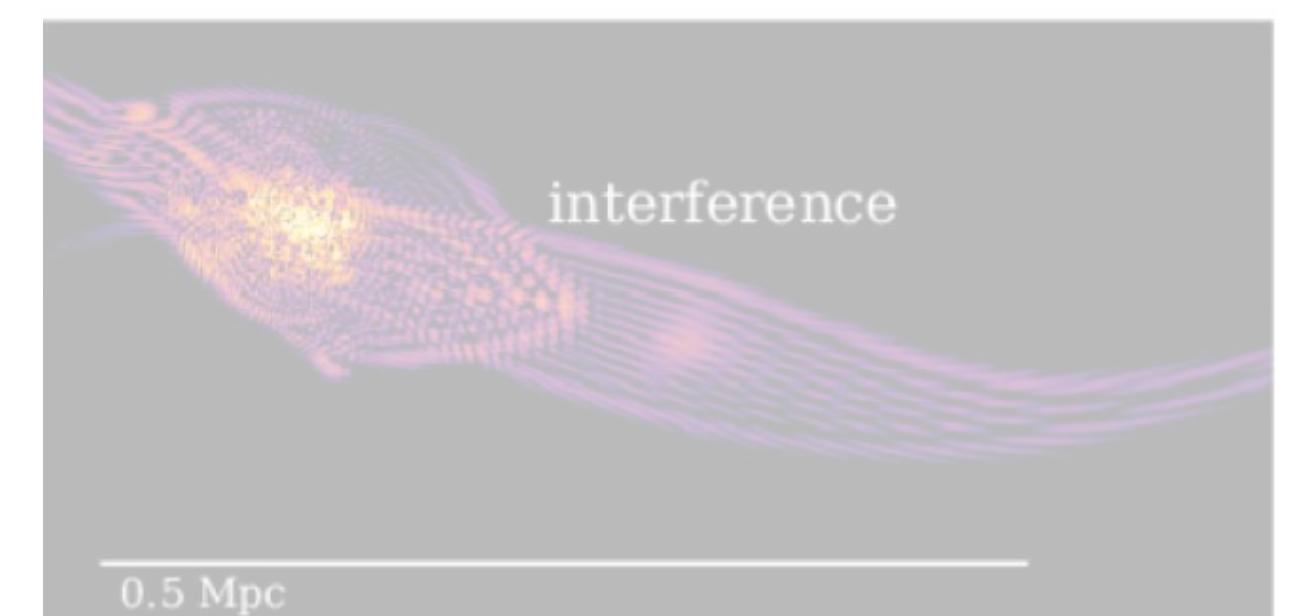
Formation of a solitonic core



Dynamical effects



Wave interference

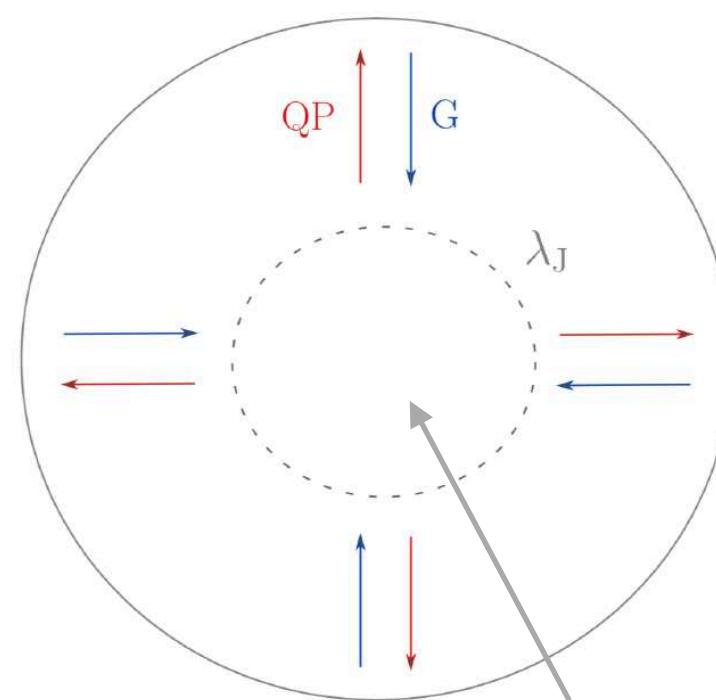
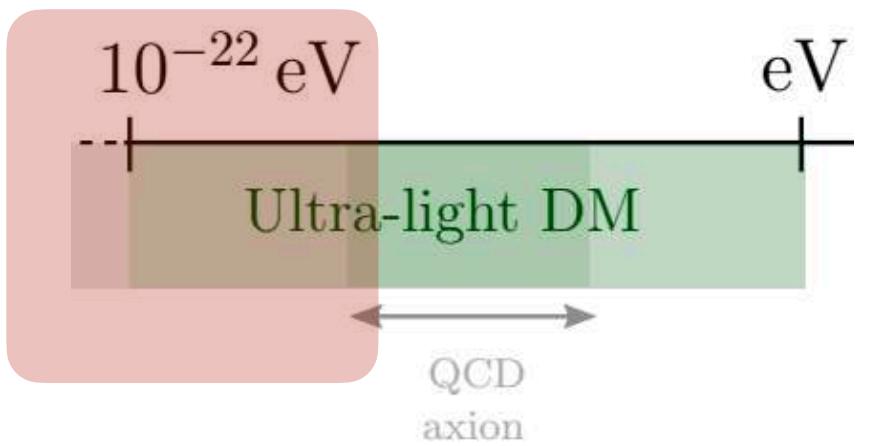
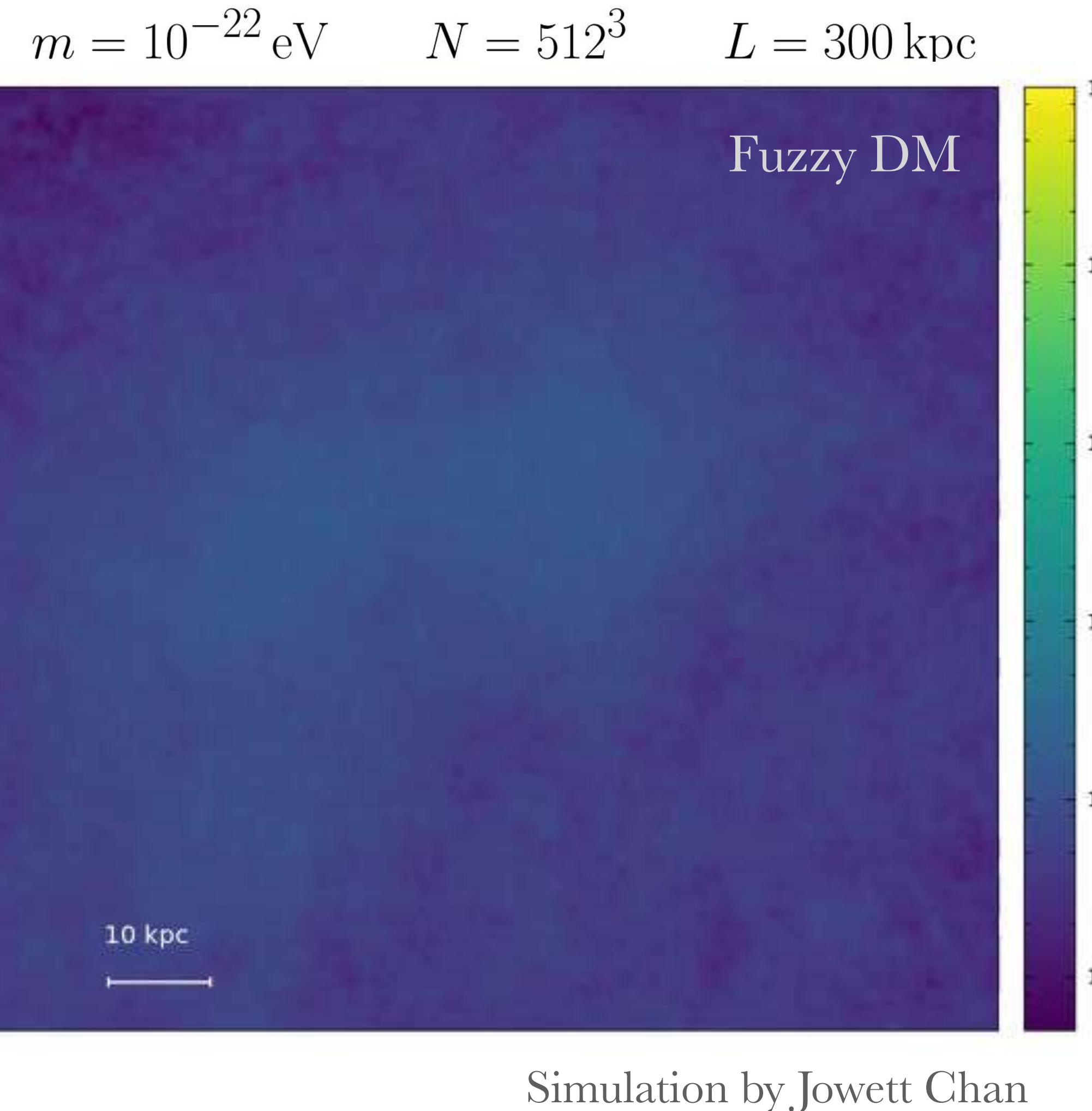


Mocz et al. 2017

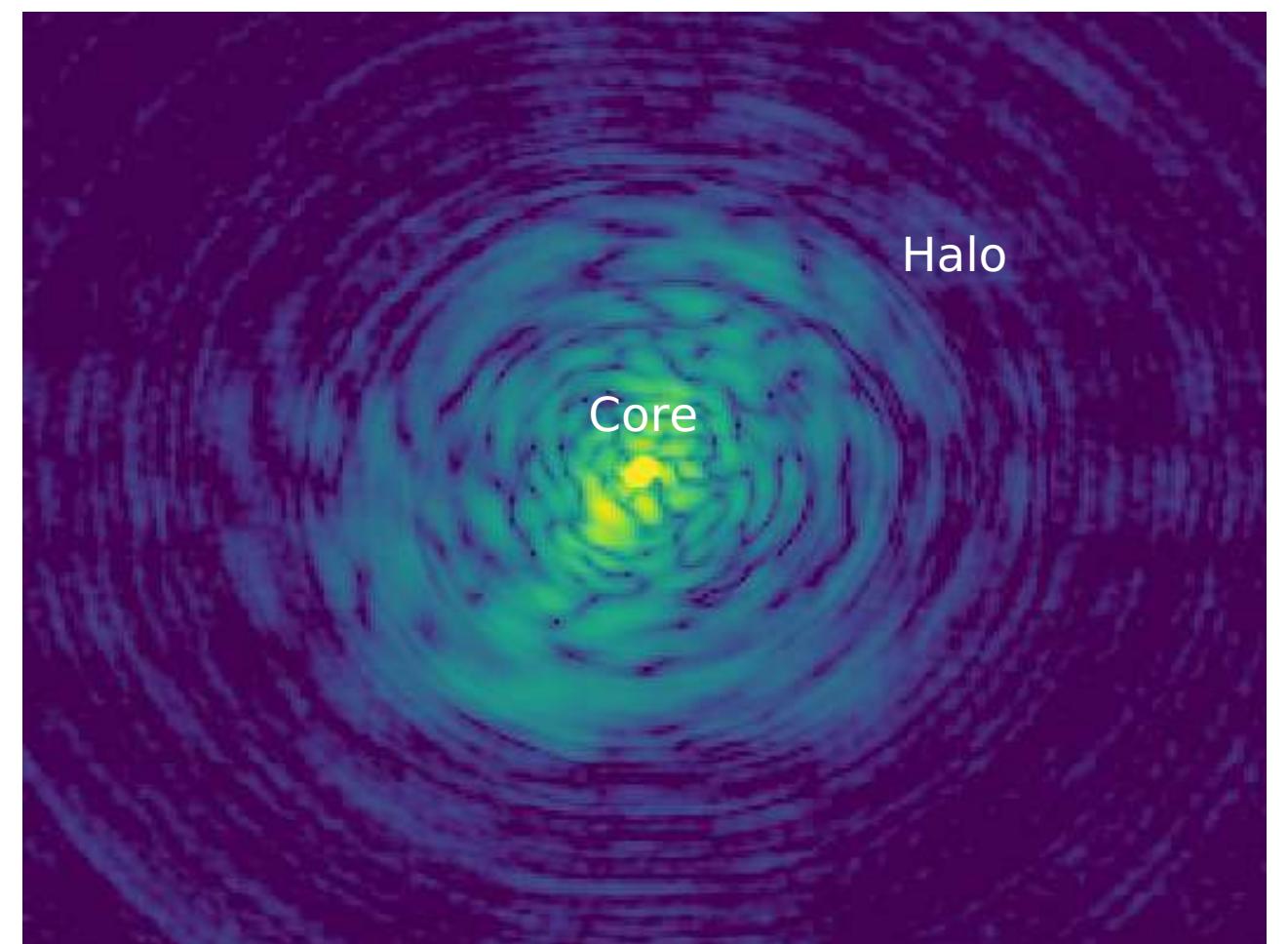
# Phenomenology

## Formation of cores

NON-LINEAR  
evolution: need  
simulations

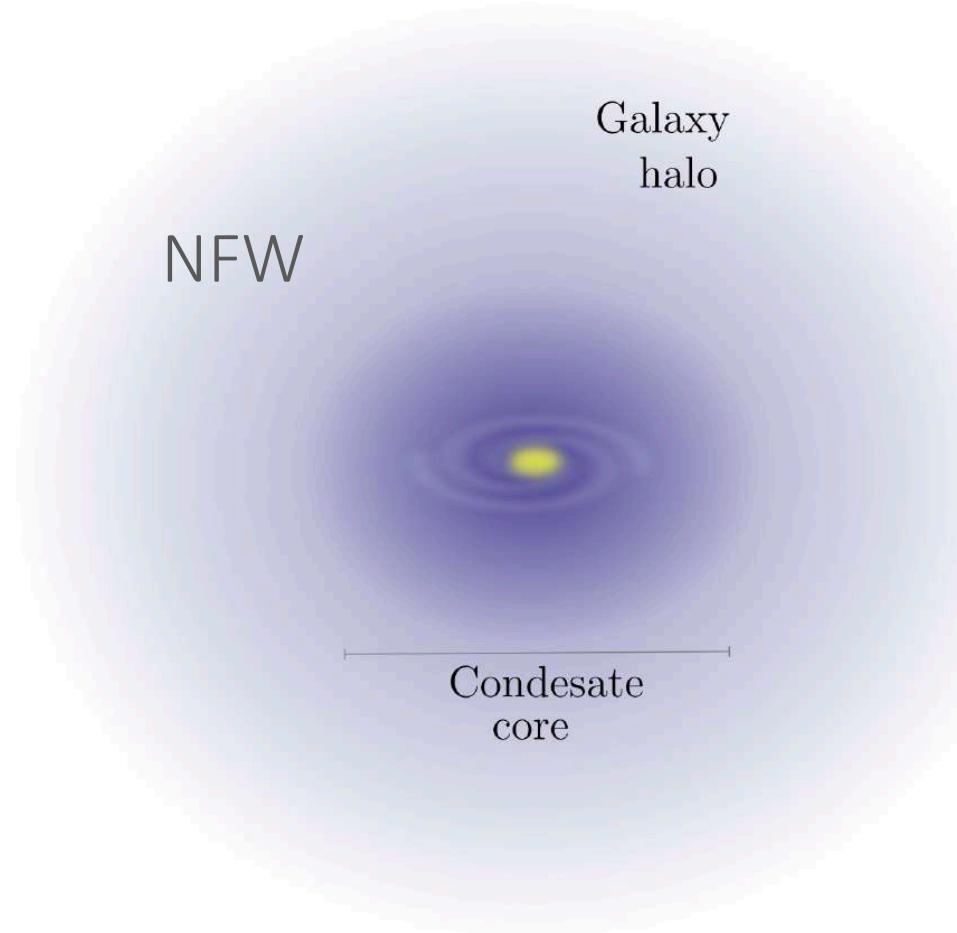


NO structure formation  
Stable, oscillating solution

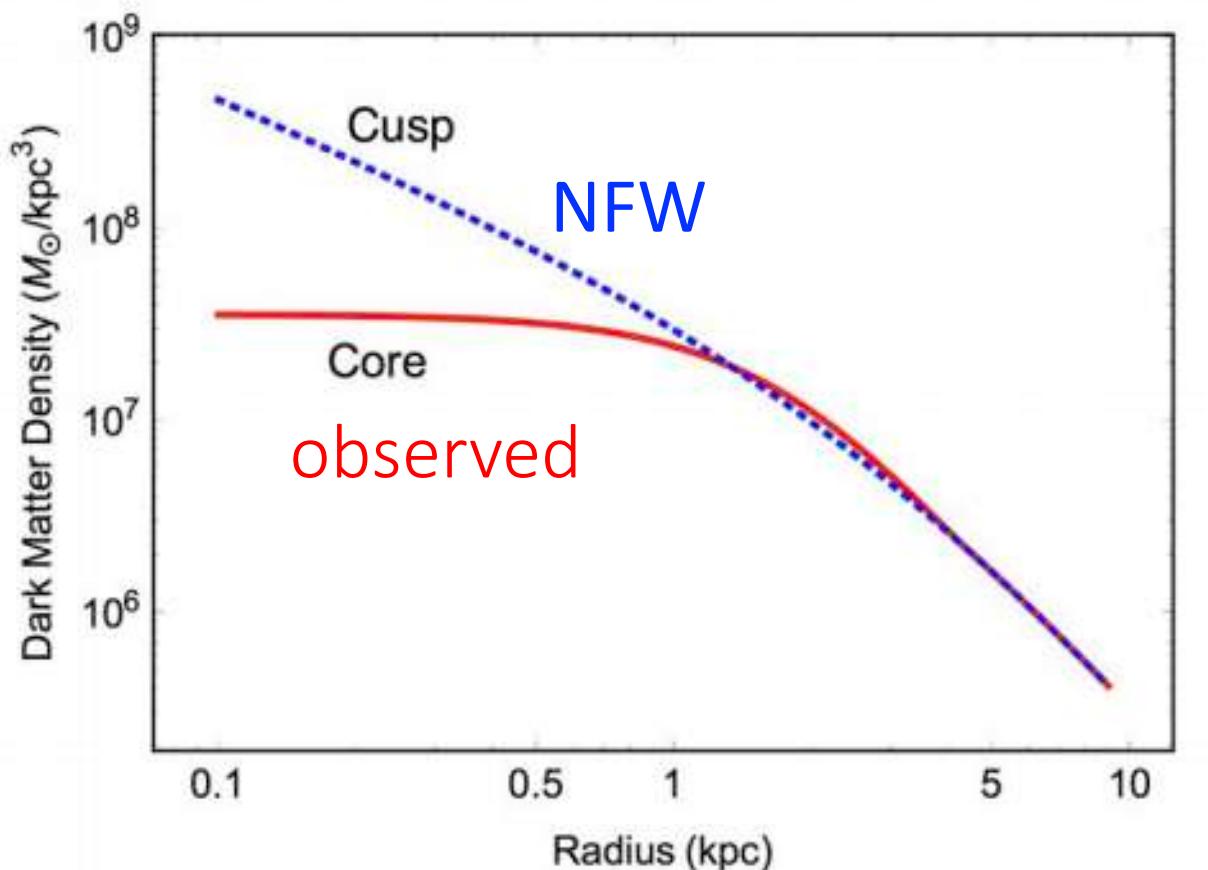
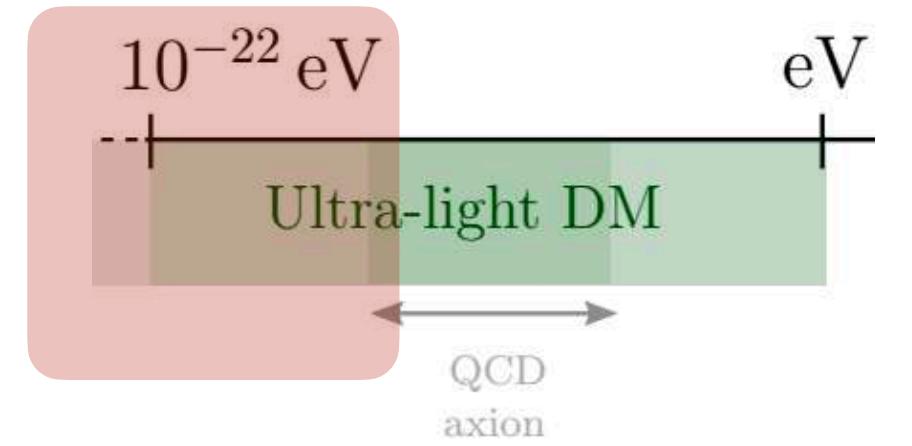


# Phenomenology

## Formation of cores



$$\rho(r) \simeq \begin{cases} \rho_c & \text{for } r \leq r_c \\ \rho_{\text{NFW}} & \text{for } r \geq r_c \end{cases}$$



FDM

From simulations Schive et al. 2014, fitting function:

Stable core solution

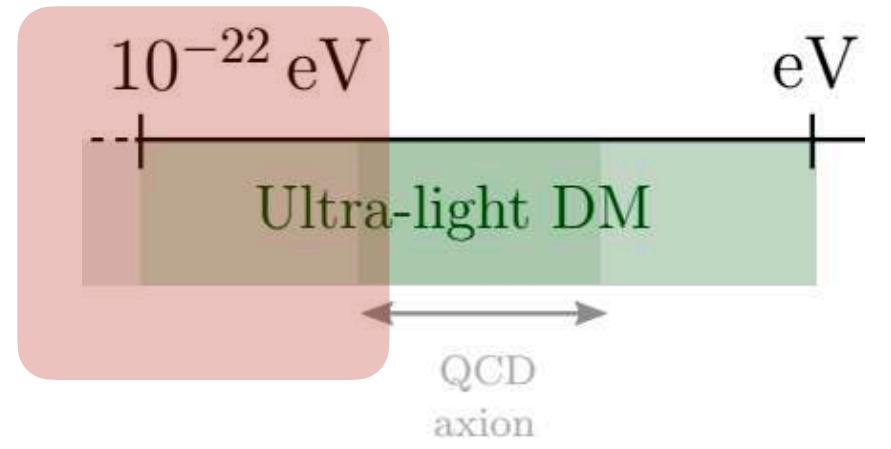
$$\rho_c \simeq \frac{1.9 \times 10^{-2}}{[1 + 0.091(r/R_{1/2,c})^2]^8} \left(\frac{m}{10^{-22} \text{ eV}}\right)^{-2} \left(\frac{r_c}{\text{kpc}}\right)^{-4} M_\odot \text{ pc}^{-3},$$

$$r_c \simeq 0.16 \left(\frac{m}{10^{-22} \text{ eV}}\right)^{-1} \left(\frac{M}{10^{12} M_\odot}\right)^{-1/3} \text{ kpc}.$$

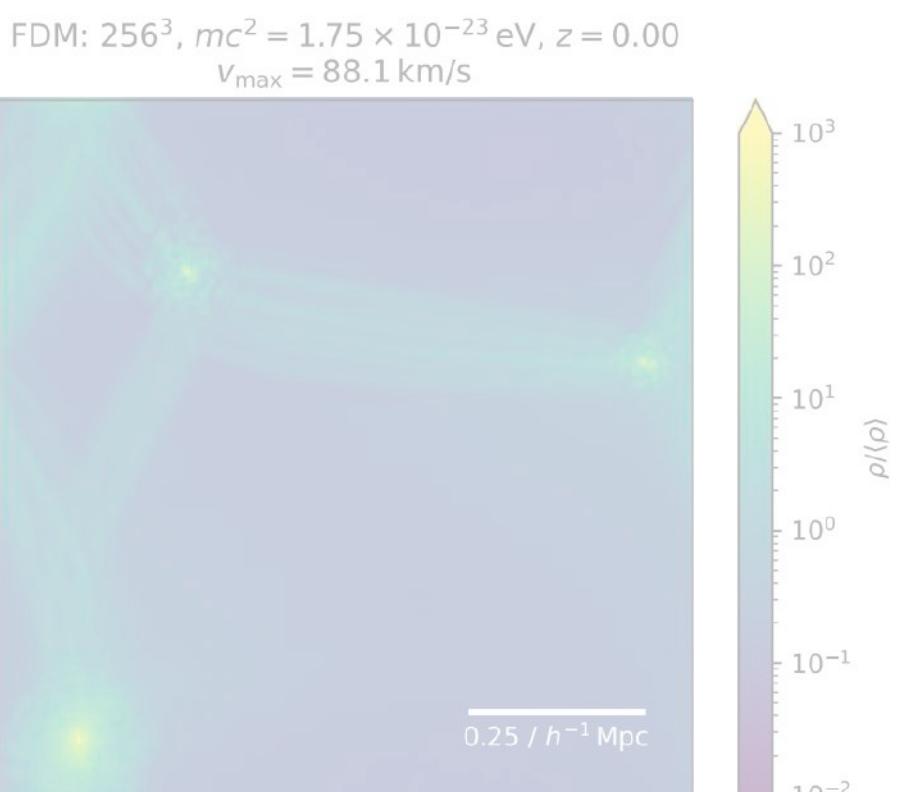
Relations used to compare  
with observations

# Phenomenology

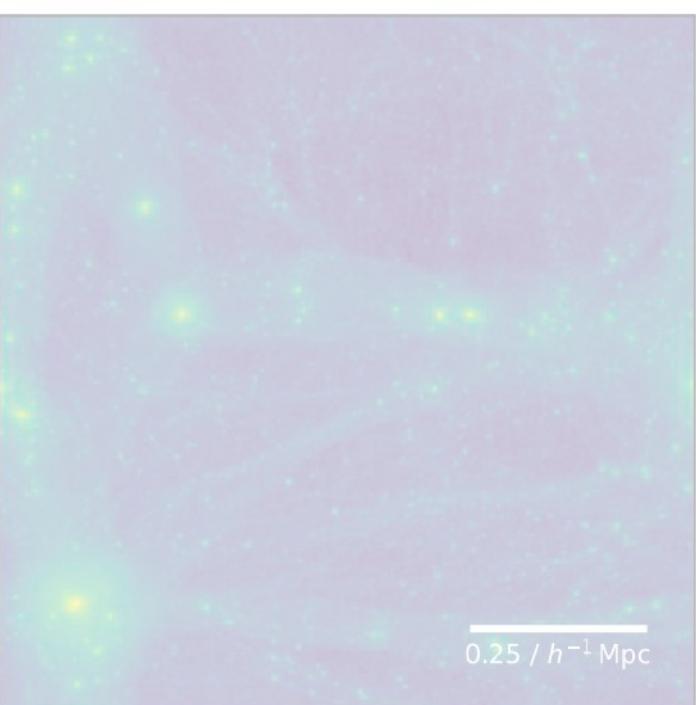
## RICH PHENOMENOLOGY ON SMALL SCALES



Suppression of small structures

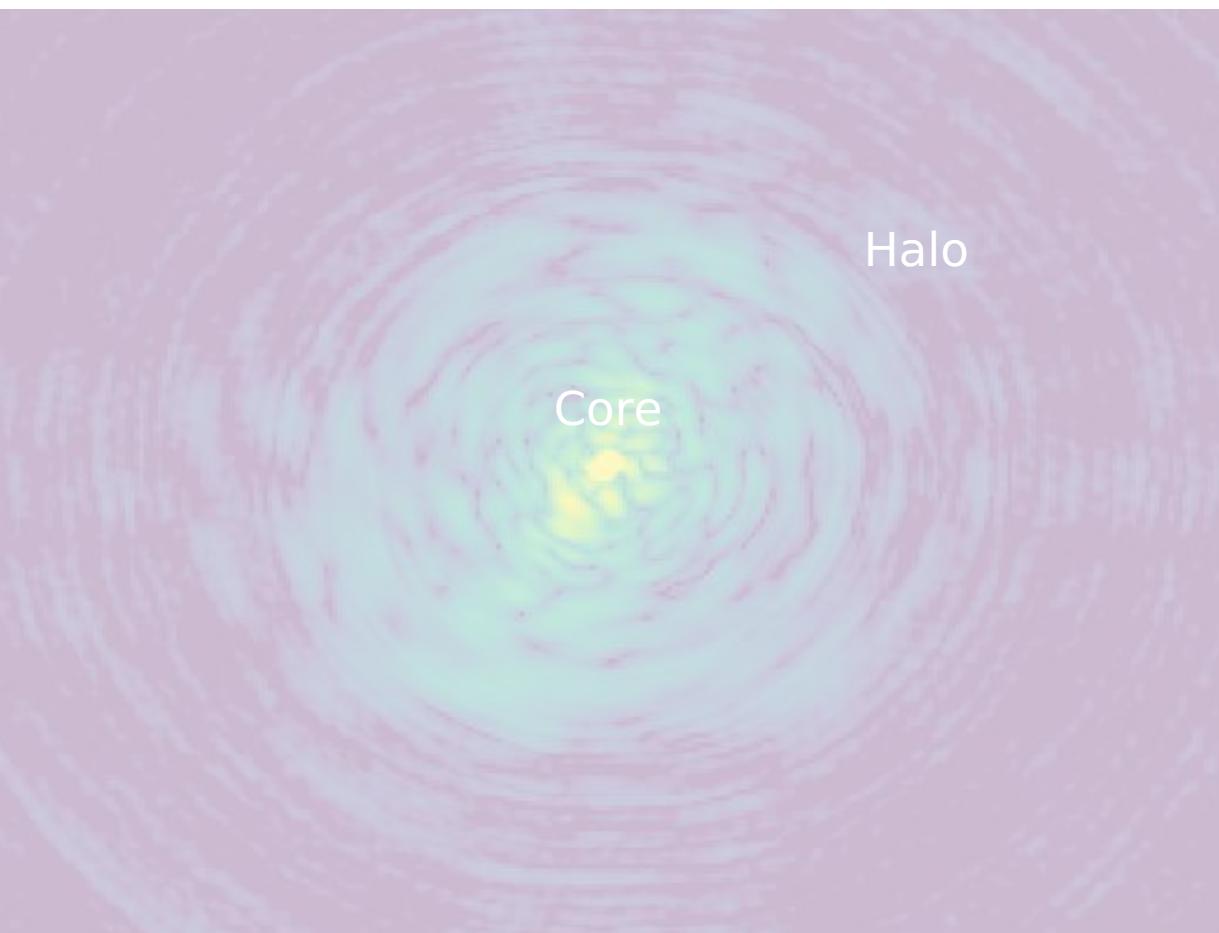


CDM:  $256^3$ ,  $z = 0.00$

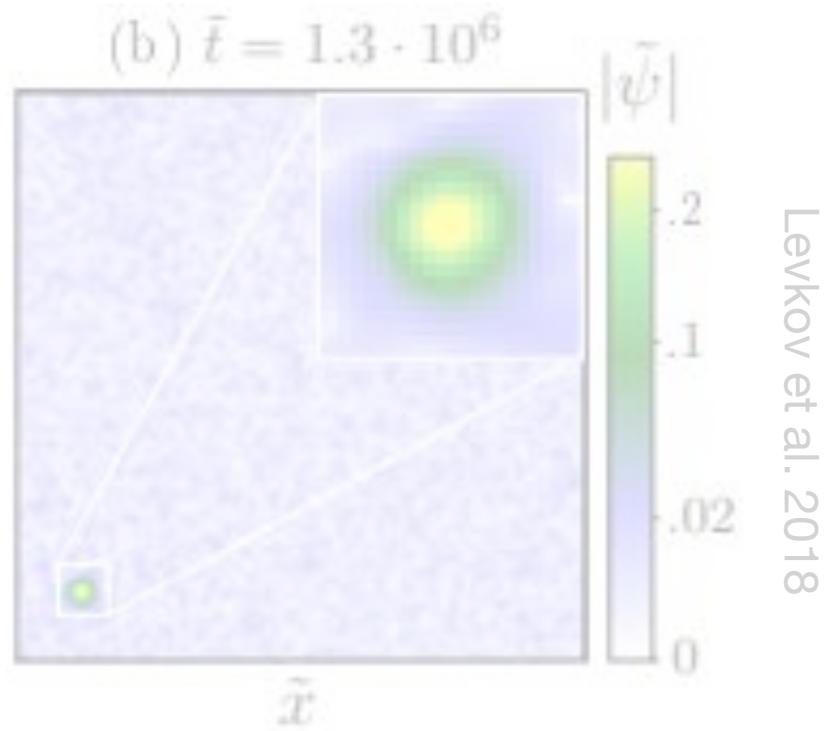


S. May et al. 2021

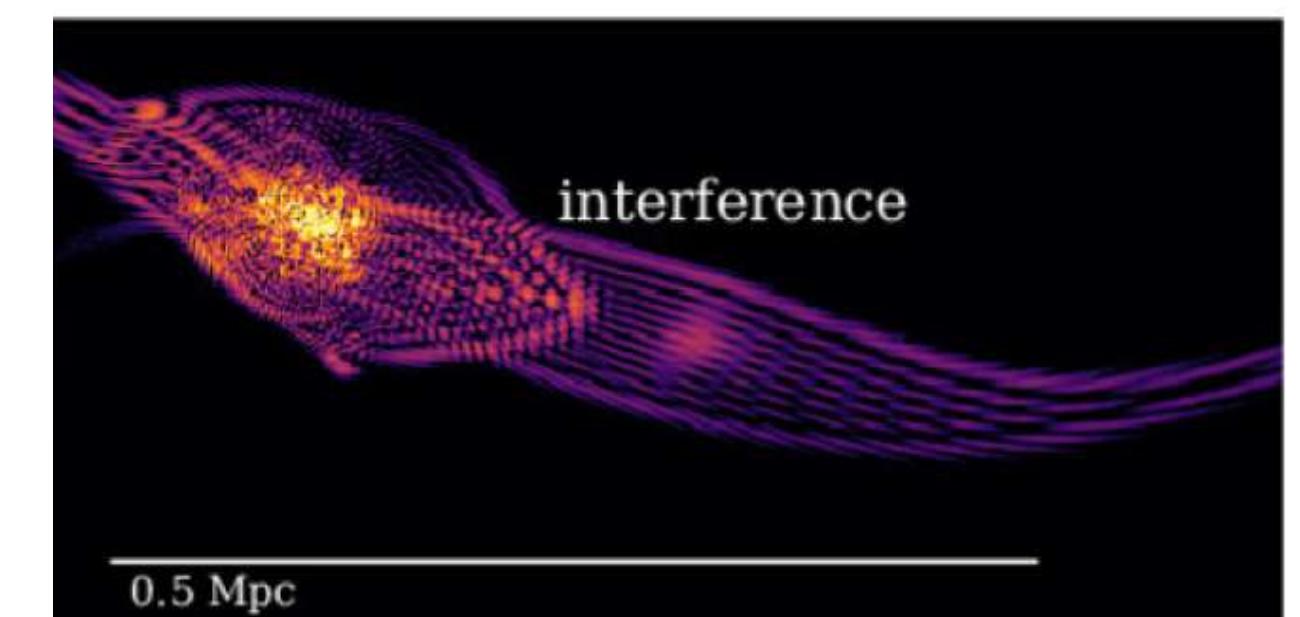
Formation of a solitonic core



Dynamical effects



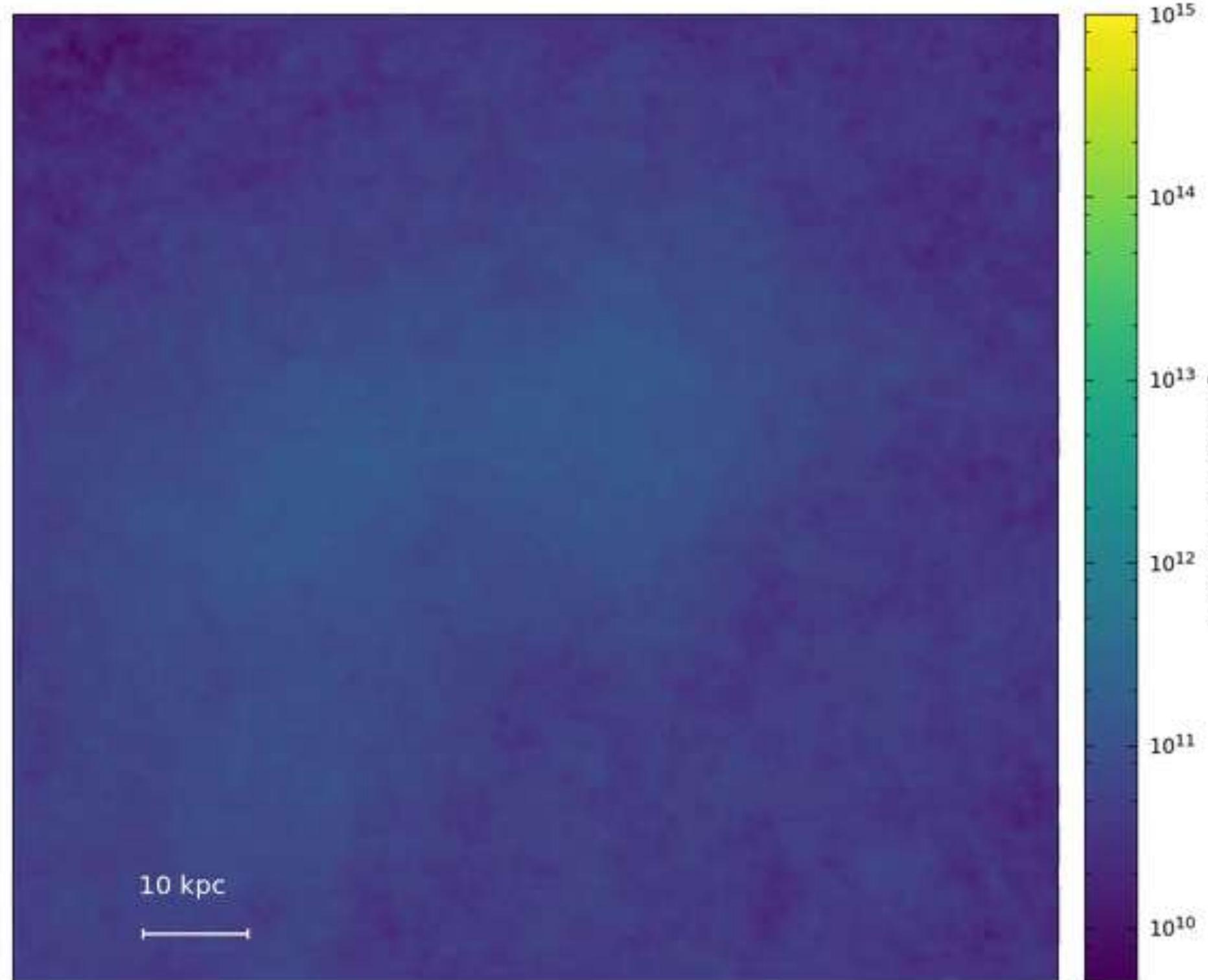
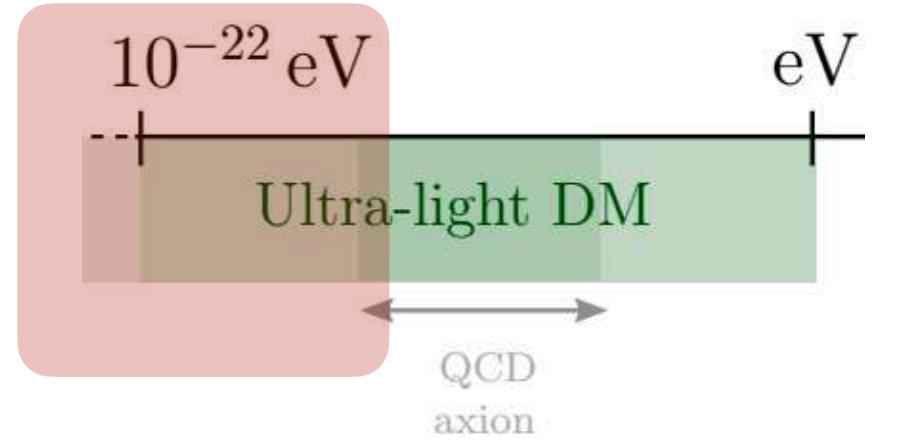
Wave interference



Mocz et al. 2017

# Phenomenology

Wave interference: granules and vortices

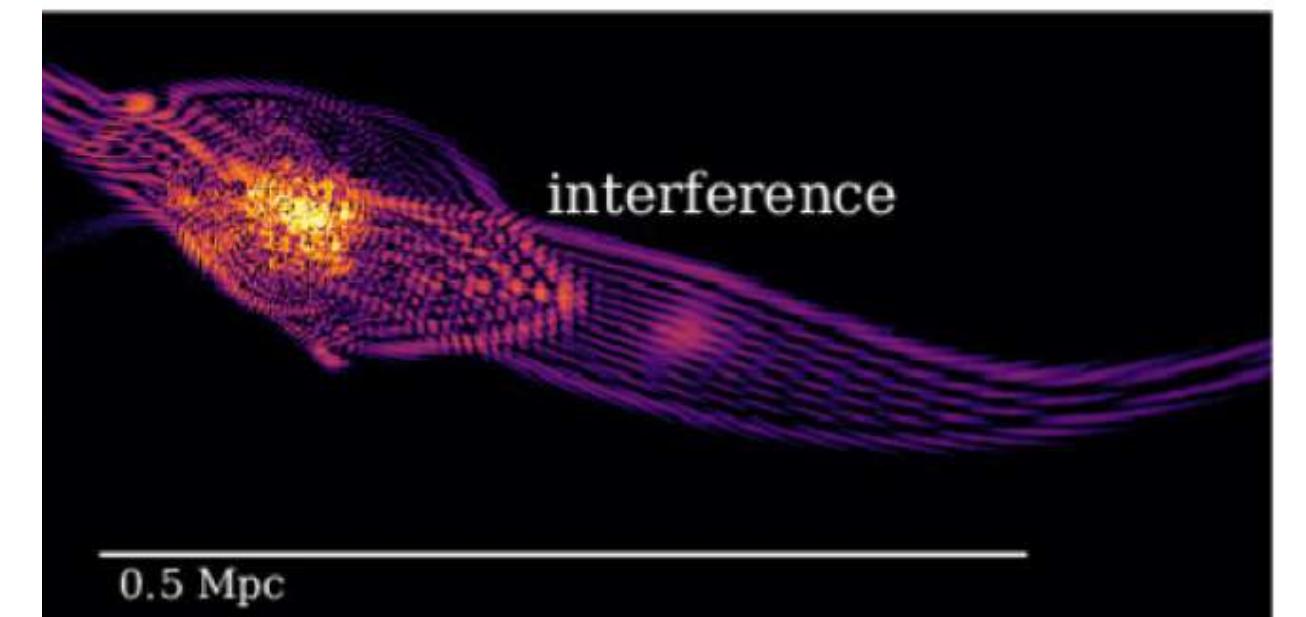
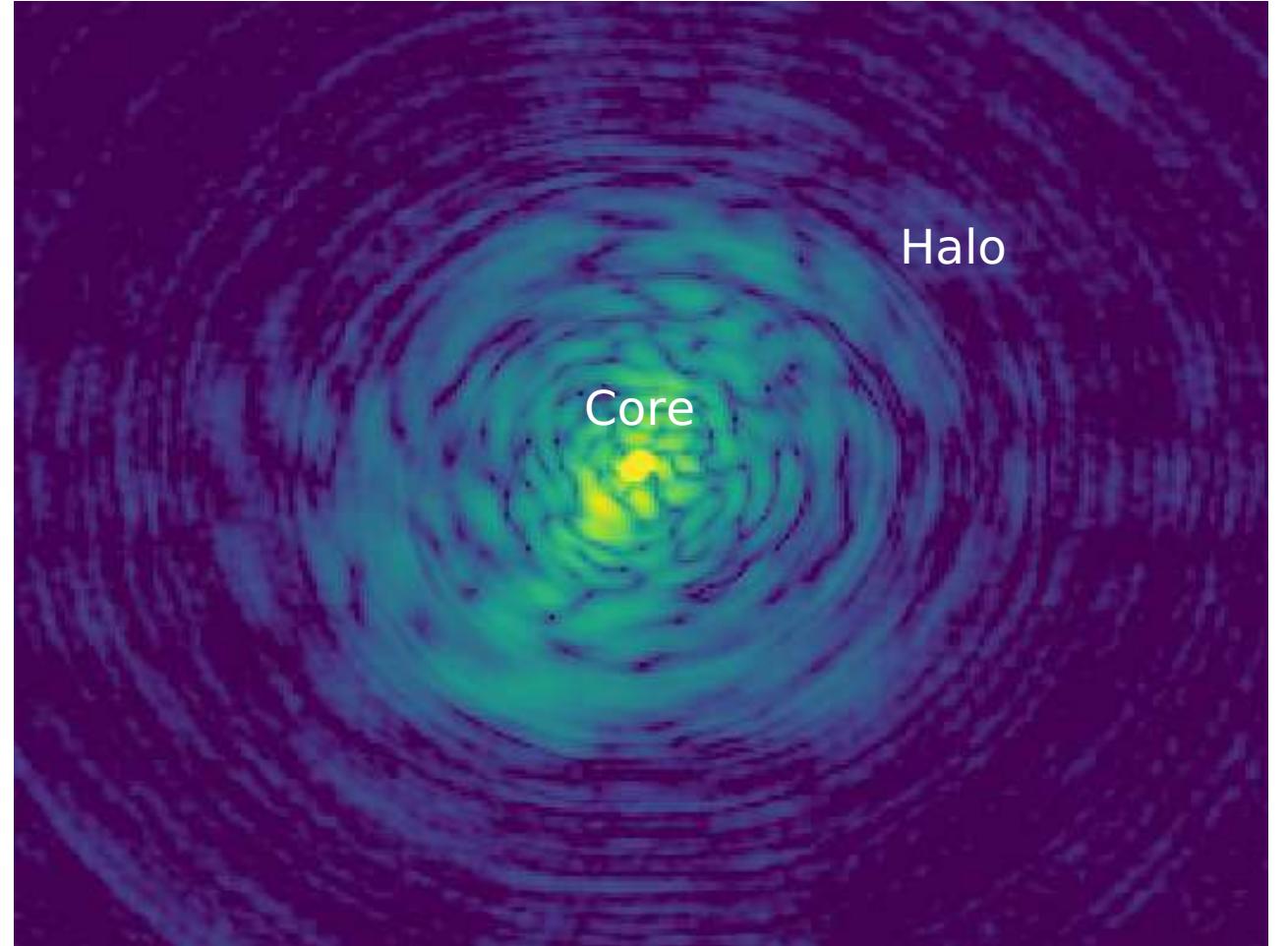


Simulation by Jowett Chan

Order one fluctuations in density  $\longrightarrow$

Constructive interference: **granules**  
Destructive interference

$$\sim \lambda_{\text{dB}}$$



Mocz et al. 2017

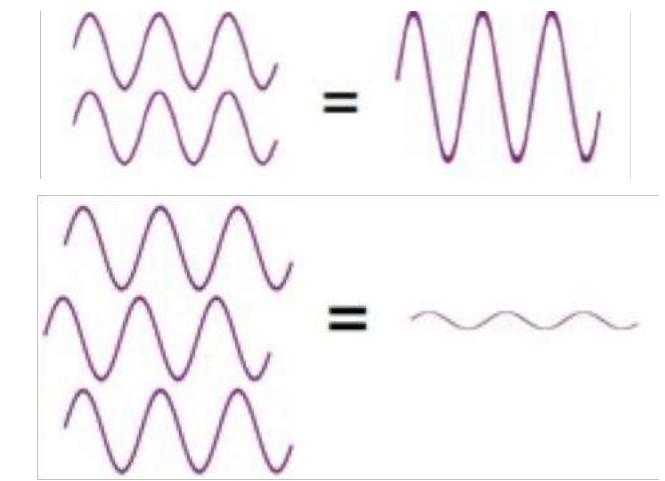
Hard to observe!

# *Vector, higher spin or multicomponent FDM*

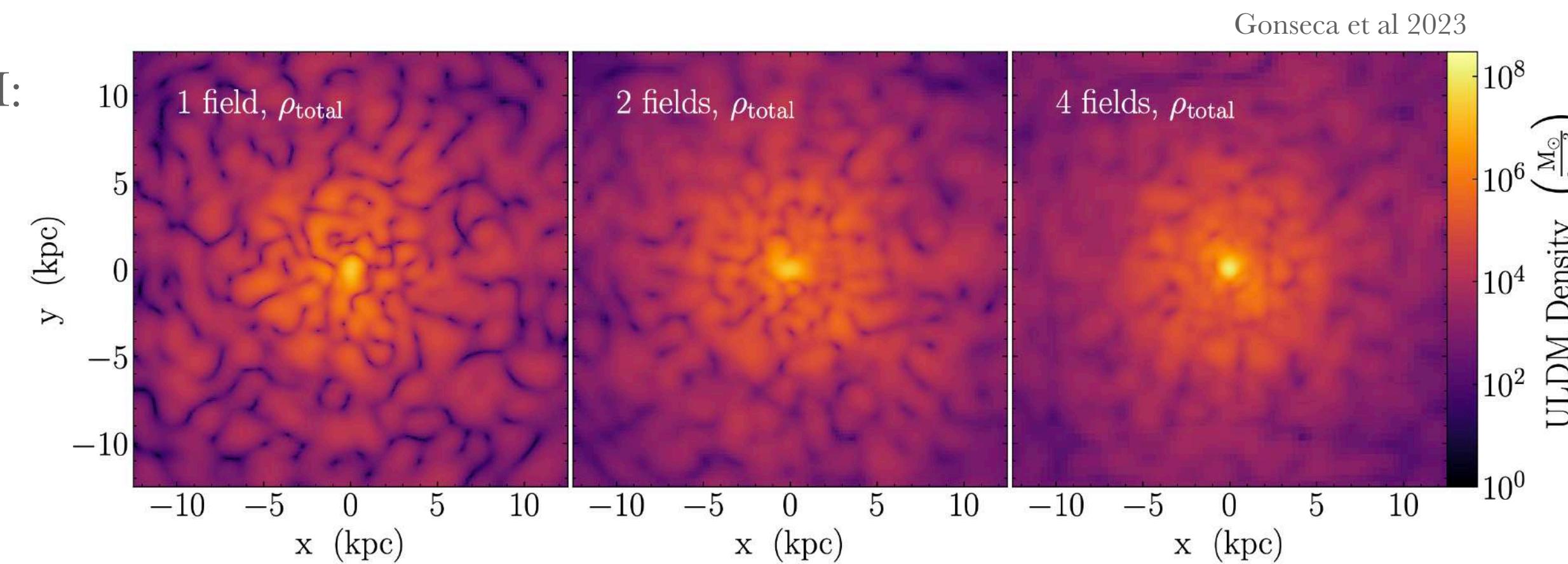
ULDM or ULA are a coherent wave - same frequency and constant phase difference

Multiple coherent waves

Interference patterns



For ULDM:



Gonseca et al 2023

Multiple FDM or VFDM (or higher spin s FDM)  
*attenuates* the granule amplitude by

$$\frac{[\delta\rho/\rho]_{\text{nfdm},s}}{[\delta\rho/\rho]_{\text{fdm}}} \propto \frac{1}{\sqrt{(2s+1)}} = \frac{1}{\sqrt{N}}$$

(Amin et al 2022)

Vector (and higher-spin) FDM Amin et al 2022

(Vector FDM = 3 x same mass FDM (spin 0))

Multicomponent FDM Gonseca et al 2023

# Phenomenology

## Vortices

Vortices are sites where the fluid velocity has a non-vanishing curl

Two ways:

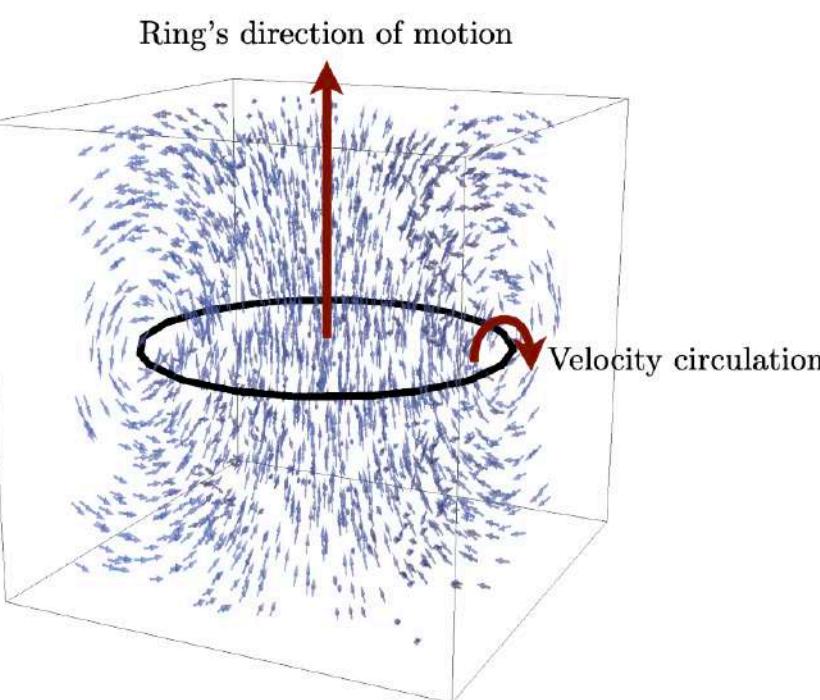
- regions where the density vanishes
- transfer of angular momentum (superfluids only)

## Fuzzy DM

Interference of waves leads to **vortices** - where there is **destructive interference**

General defet in 3D

$$\mathcal{C} = \frac{1}{m} \oint_{\partial A} d\theta = \frac{2\pi n}{m}$$



$$(\psi \equiv \sqrt{\rho/m} e^{i\theta} \text{ and } \mathbf{v} \equiv \nabla\theta/m)$$

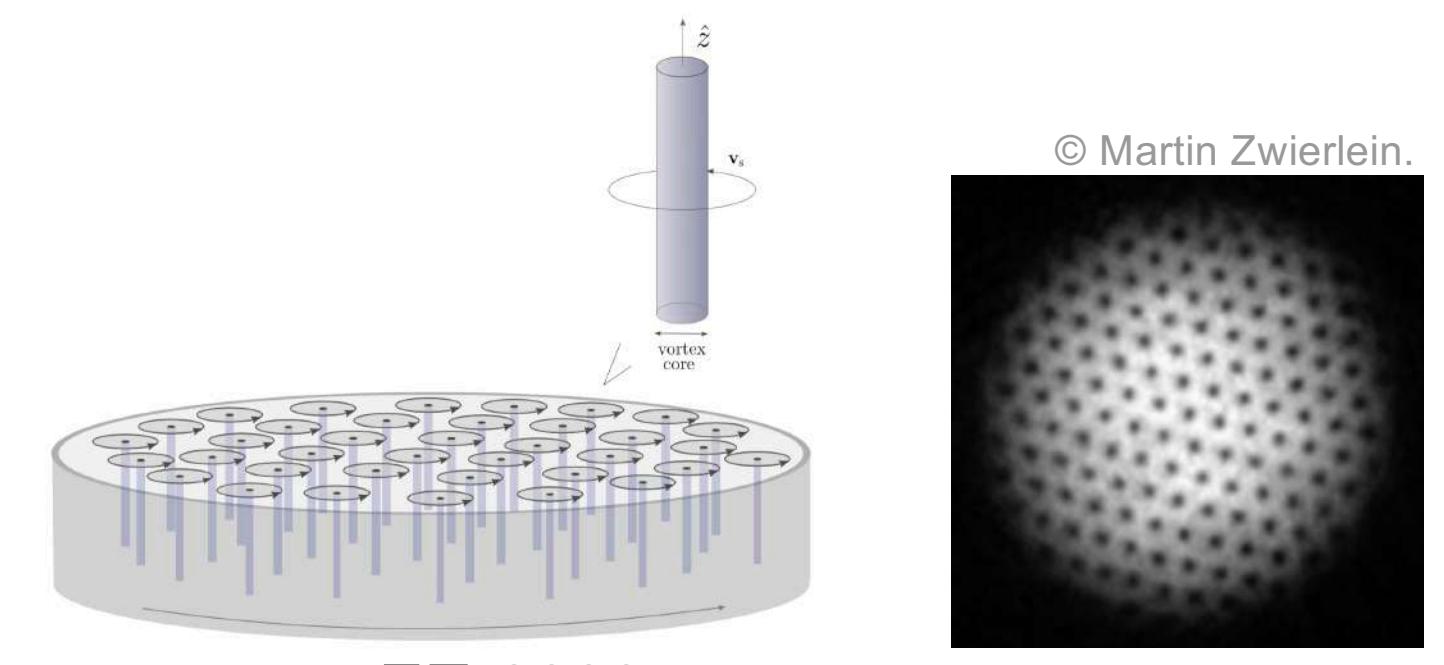
$$\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\dot{\mathbf{v}} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{m} \left( V_{grav} - P_{int} - \frac{1}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right)$$

Vel. field is a gradient flow  $\longrightarrow$  irrotational fluid, no vorticity

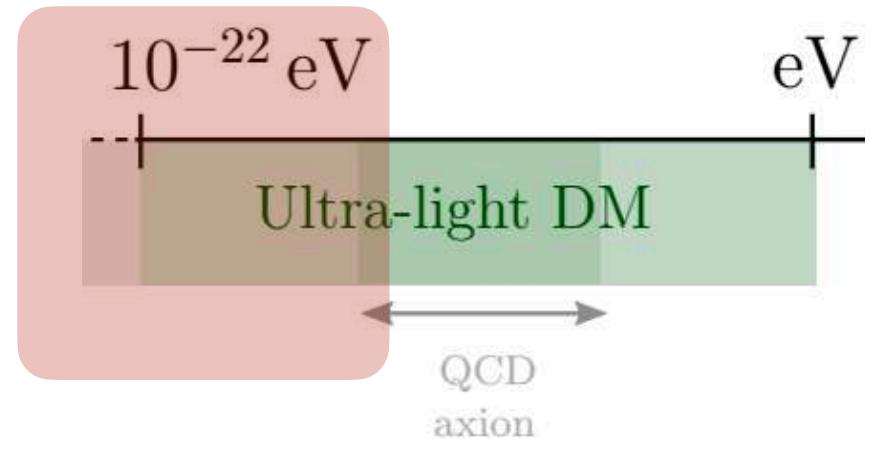
## Self-interacting Fuzzy DM

Superfluid cannot rotate uniformly. If the superfluid rotates faster than the critical vel., network of vortices are formed.

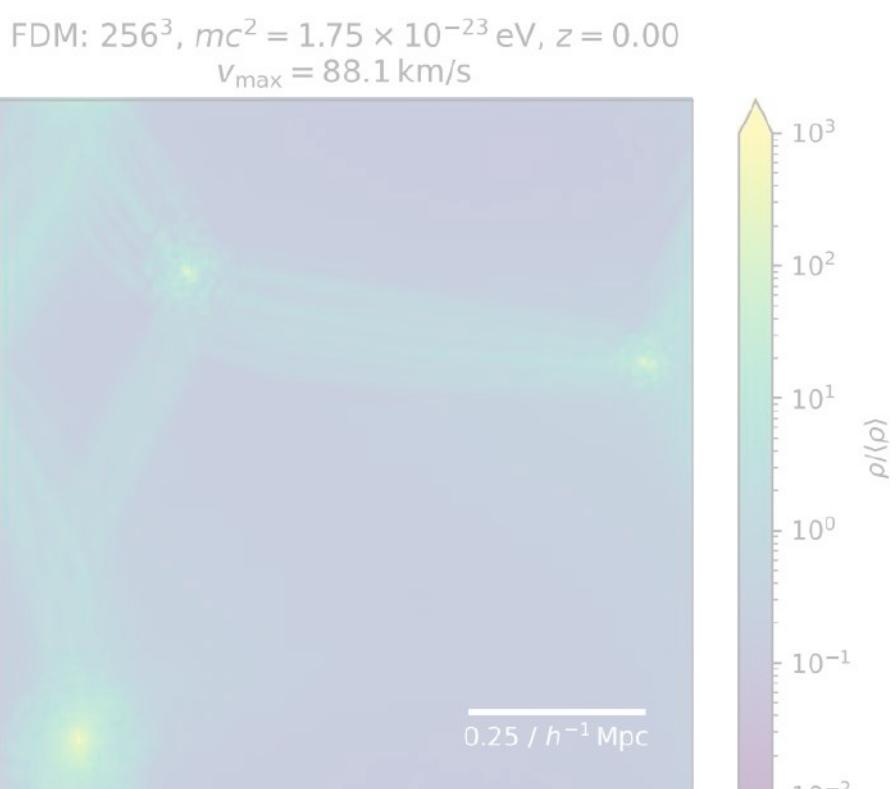


# Phenomenology

## RICH PHENOMENOLOGY ON SMALL SCALES

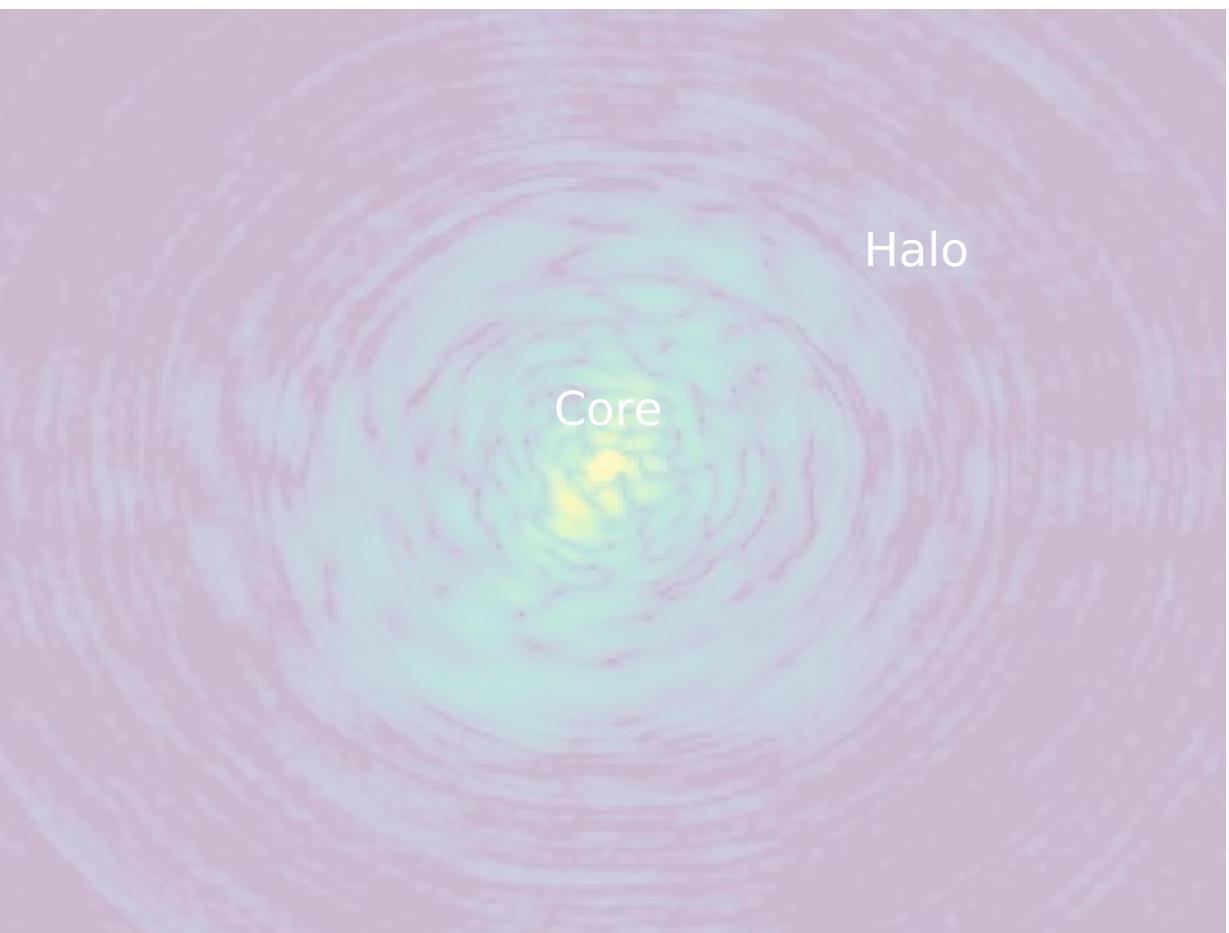


Suppression of small structures

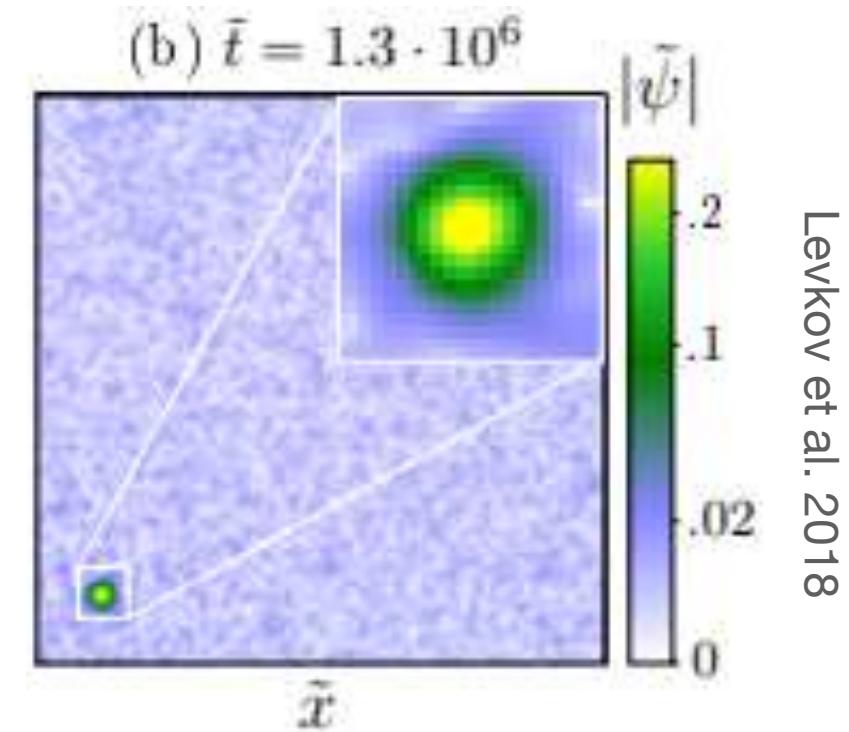


S. May et al. 2021

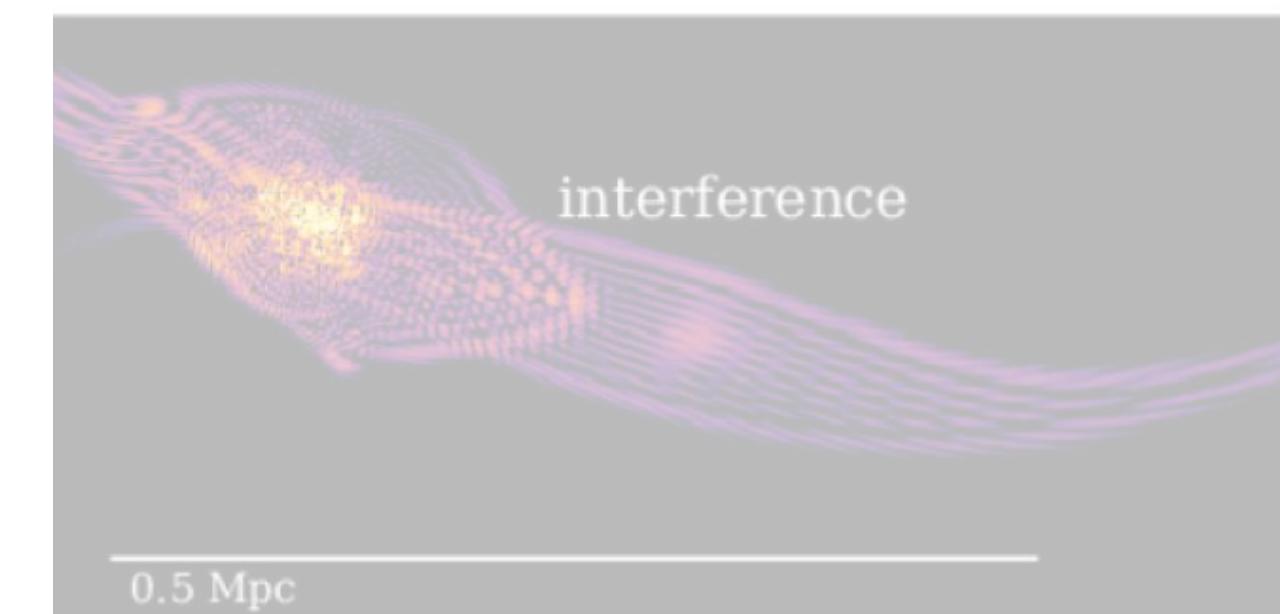
Formation of a solitonic core



Dynamical effects



Wave interference

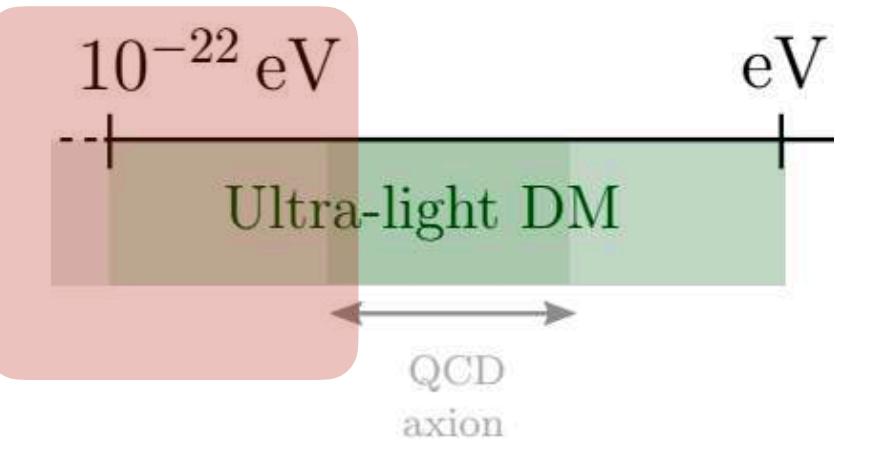


Mocz et al. 2017

# *Phenomenology*

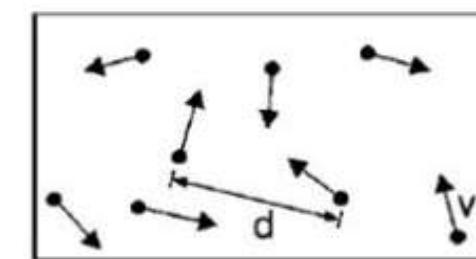
## Dynamical effects

Relaxation, oscillation, friction, and heating

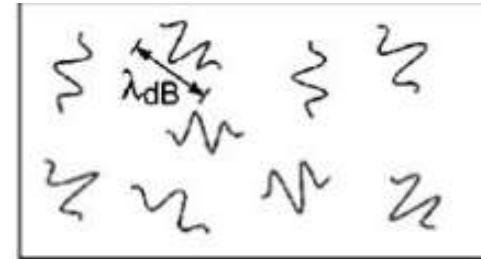


# Bose Einstein Condensate

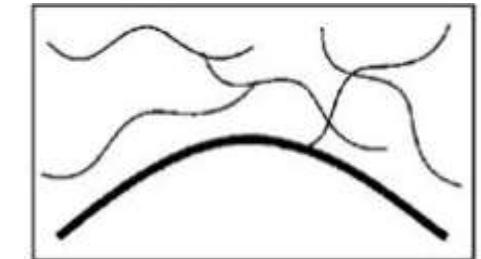
- Bose Einstein condensate (BEC): macroscopic occupation of the ground state
- At low temperatures, each particle wave function overlap - single wave function describes the entire fluid.



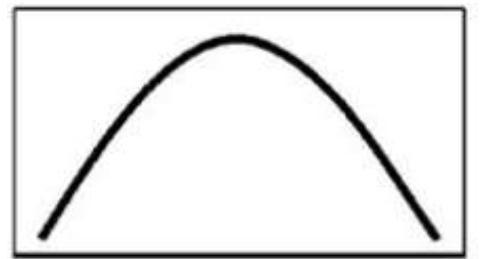
High temperature  
Thermal velocities



Low temperature  
 $\lambda_B \sim T^{-1/2}$   
"wave packets"



$T = T_c$   
BEC  
"matter wave overlap"  
 $d \sim \lambda_{dB}$



$T = 0$   
Pure BEC  
"giant matter wave"

## Superfluid

- Appears at low T after the superfluid condenses into a BEC.
- Effective dynamics: fluid flows without friction



## Description

Mean field approximation:

Large N, dilute

"wavefunction of the condensate"

$$\hat{\Psi}(\mathbf{r}, t) = \psi(\mathbf{r}, t) + \delta\hat{\Psi}(\mathbf{r}, t)$$

classical field

with  $\psi(\mathbf{r}, t) = \langle \hat{\Psi}(\mathbf{r}, t) \rangle$   
Fixed  $n_0 = |\psi(\mathbf{r}, t)|^2$

small perturbation: describes depletion of the condensate



Credit: Peking University

$$i\partial_t \psi(\mathbf{r}, t) = \left( -\frac{\nabla^2}{2m} + V_{trap}(\mathbf{r}) + U_0 |\psi(\mathbf{r}, t)|^2 \right) \psi(\mathbf{r}, t)$$

Non-linear Schrödinger equation - Gross-Pitaevskii equation

# Phenomenology

## Dynamical effects

Relaxation, oscillation, friction, and heating

### Formation of a BEC / superfluid

- Thermalization (and condensation) *seem* to happen inside the galaxy!
- Formation of a **soliton** (ground state) or **Bose star** in the interior of galaxies

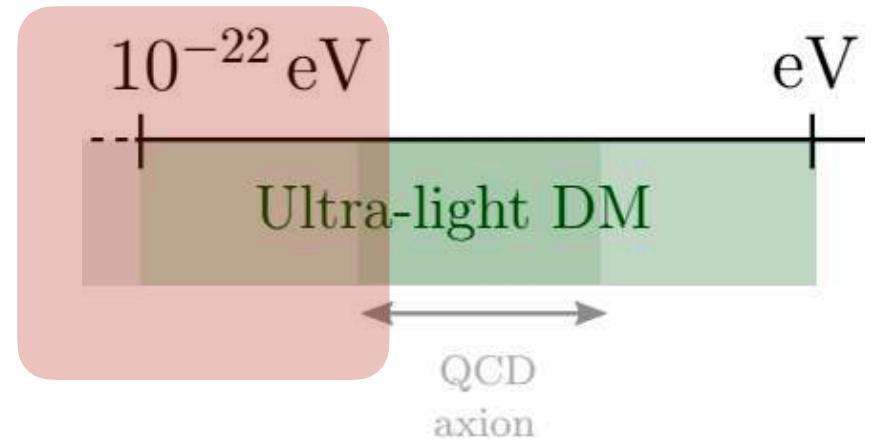
- Formation of a condensate and a core occur from gravitational interaction.

Condensation/relaxation time:  $\tau_{\text{gr}} \gg \tau_{\text{int}}$

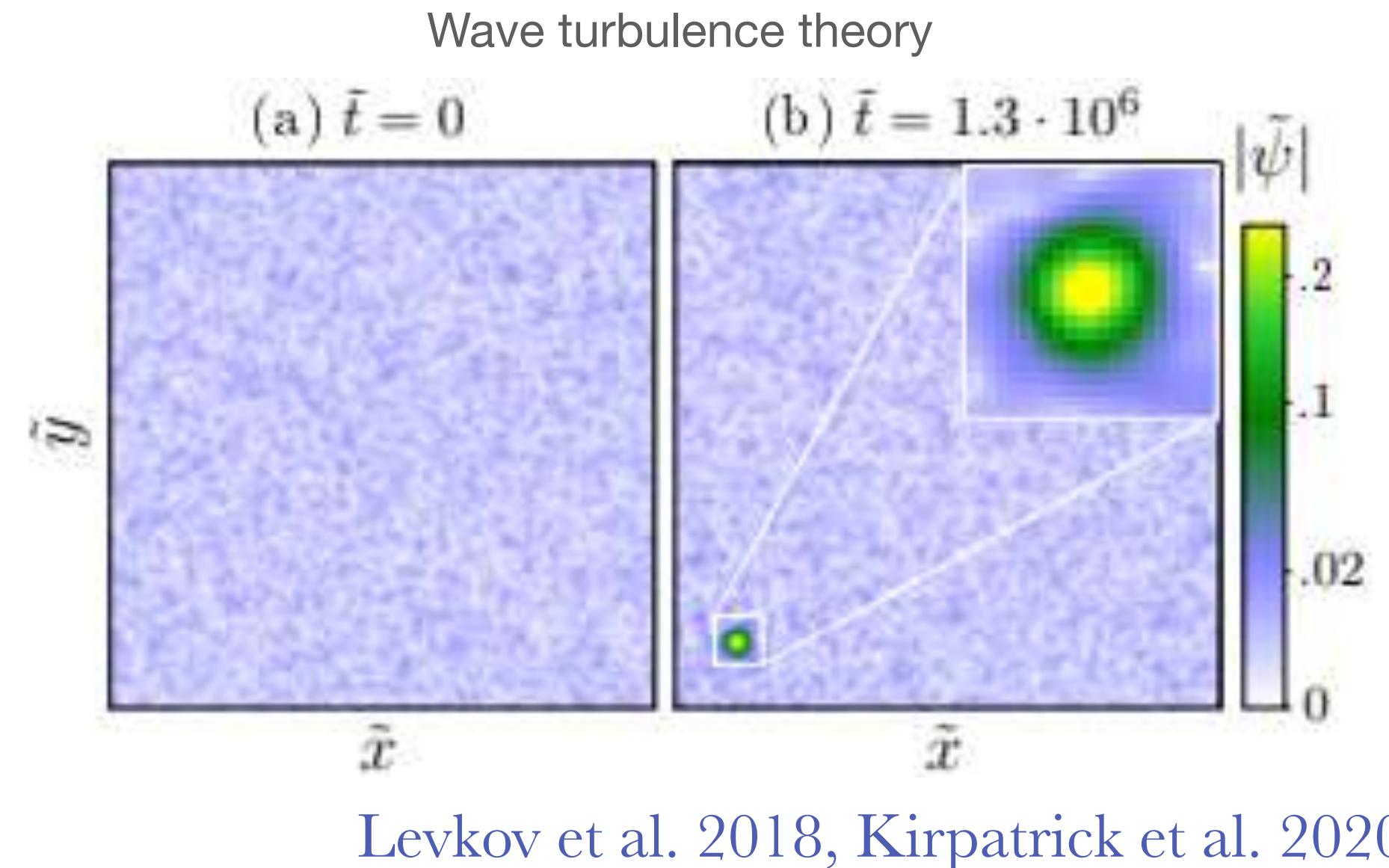
$$\tau_{\text{gr}} \sim 10^6 \text{ yr} \left( \frac{m}{10^{-22} \text{ eV}} \right)^3 \left( \frac{v}{30 \text{ km/s}} \right)^6 \left( \frac{\rho}{0.1 M_\odot/\text{pc}^3} \right)^{-2}$$

$$\tau_{\text{int}} = \frac{1}{\sqrt{8}|g|n}$$

Smaller than the age of the universe!

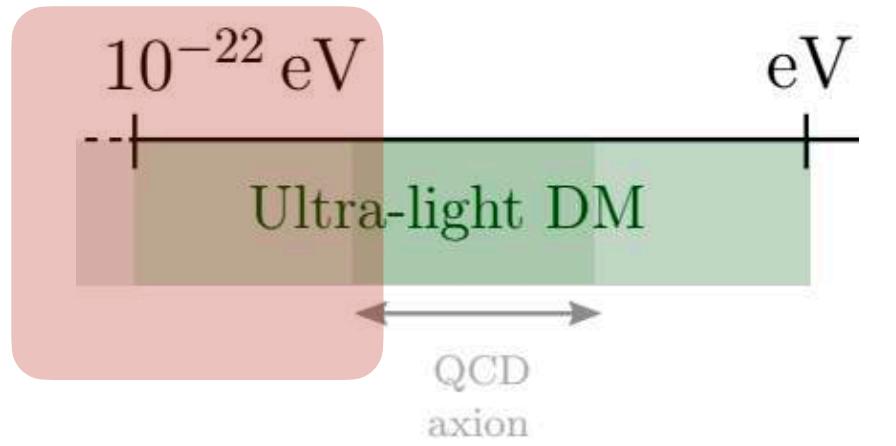


A. Guth M. Hertzberg, C. Prescod-Weinstein (2014)



# Phenomenology

## Dynamical effects

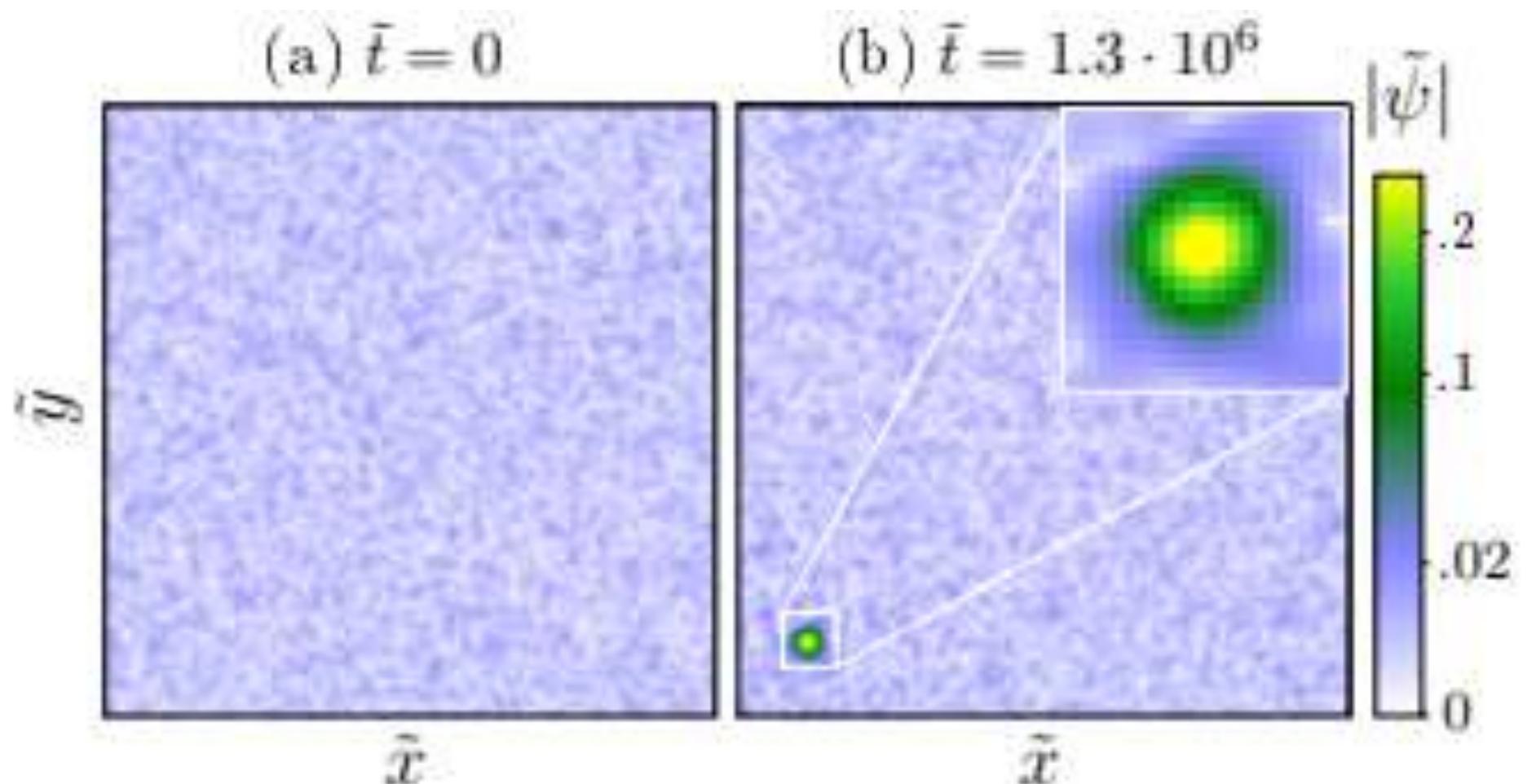


Relaxation, oscillation, friction, and heating

Formation of a BEC / superfluid

*Open question!*

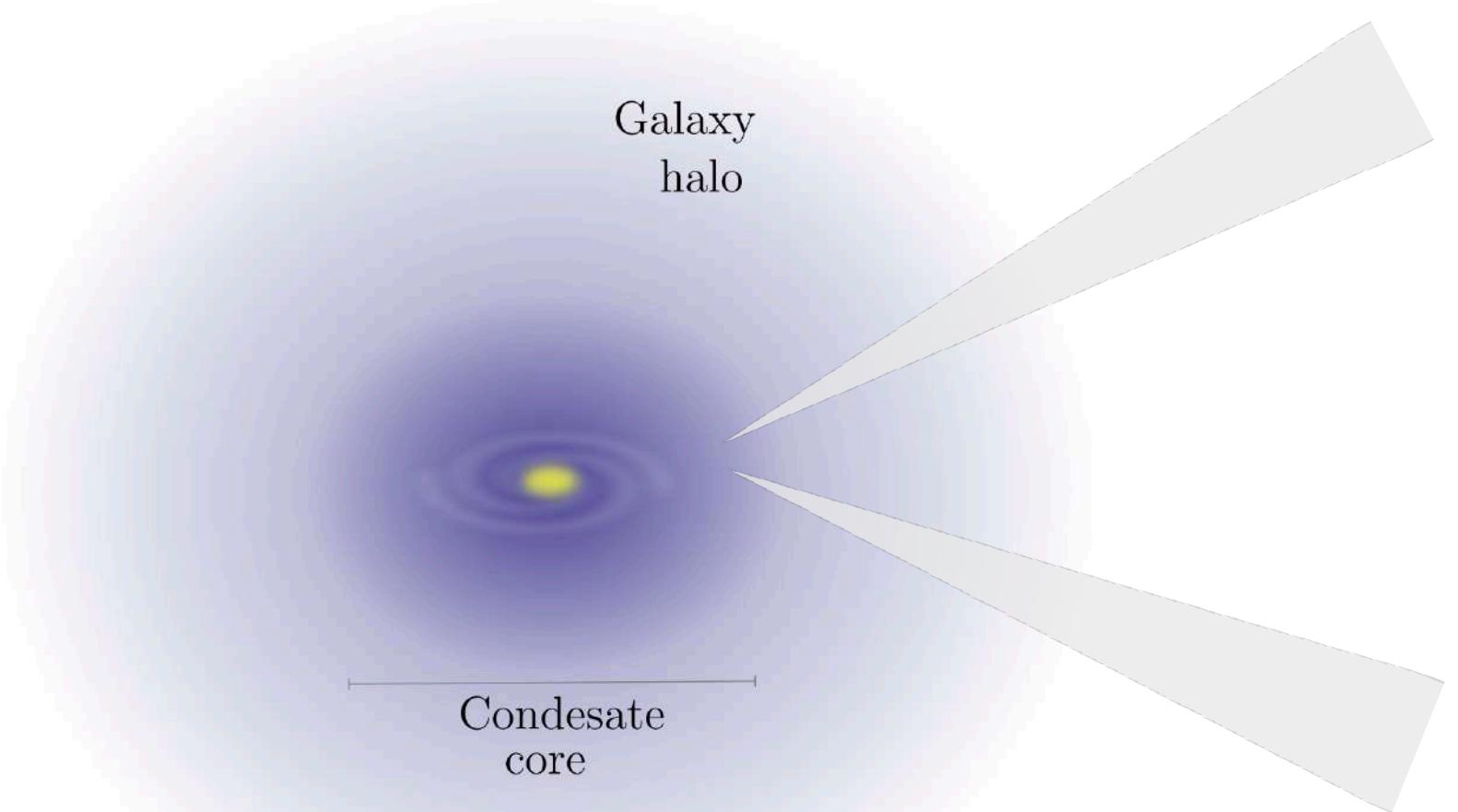
- Need theoretical work to describe *analytically* the formation of these solitons
- Cosmologically, classical or quantum field?



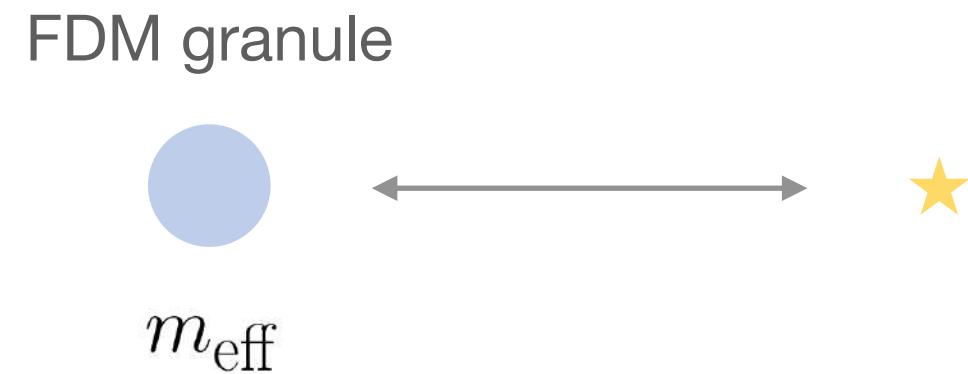
# Phenomenology

## Dynamical effects

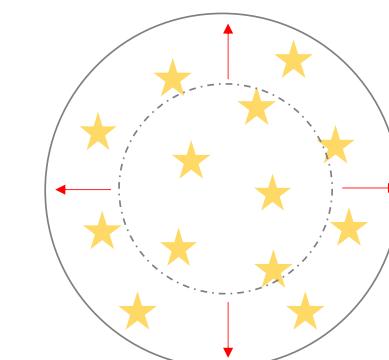
Relaxation, oscillation, friction, and heating



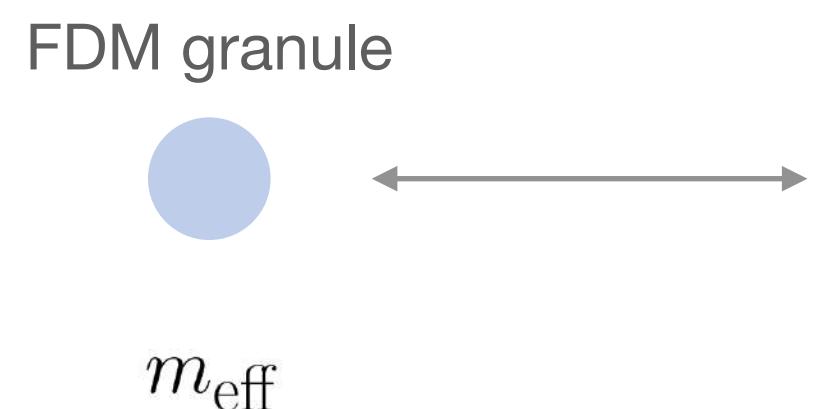
### Heating



System (star)  
gains energy



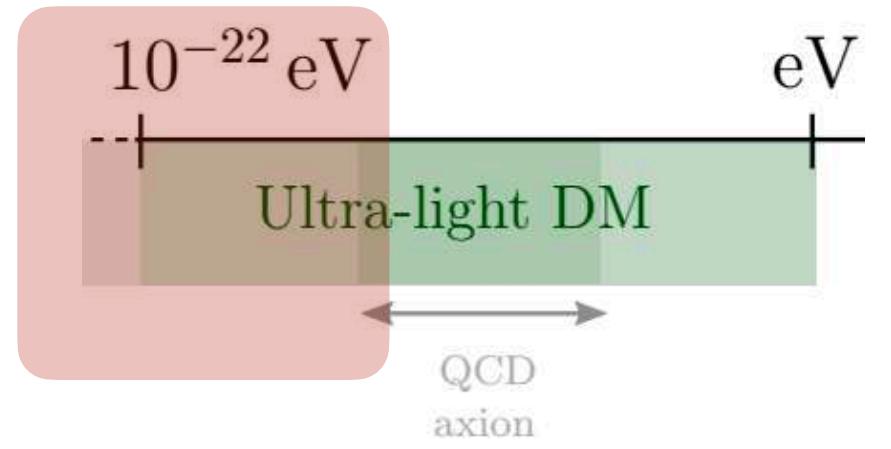
### Friction



System (GC or BH)  
loses energy

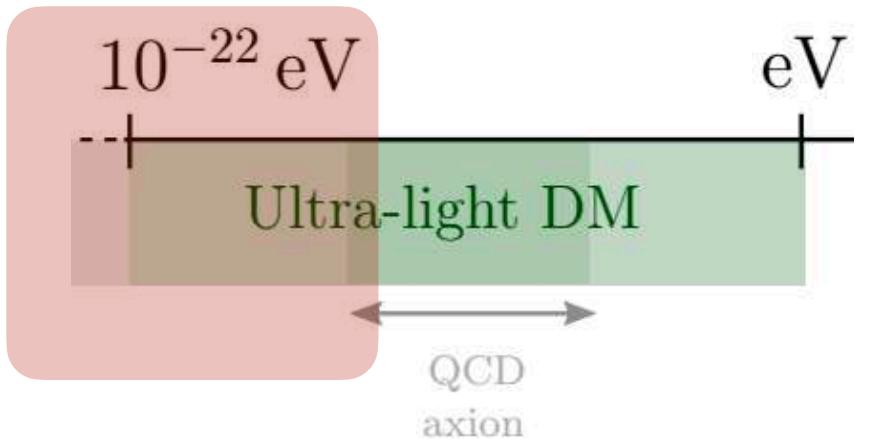


Globular cluster



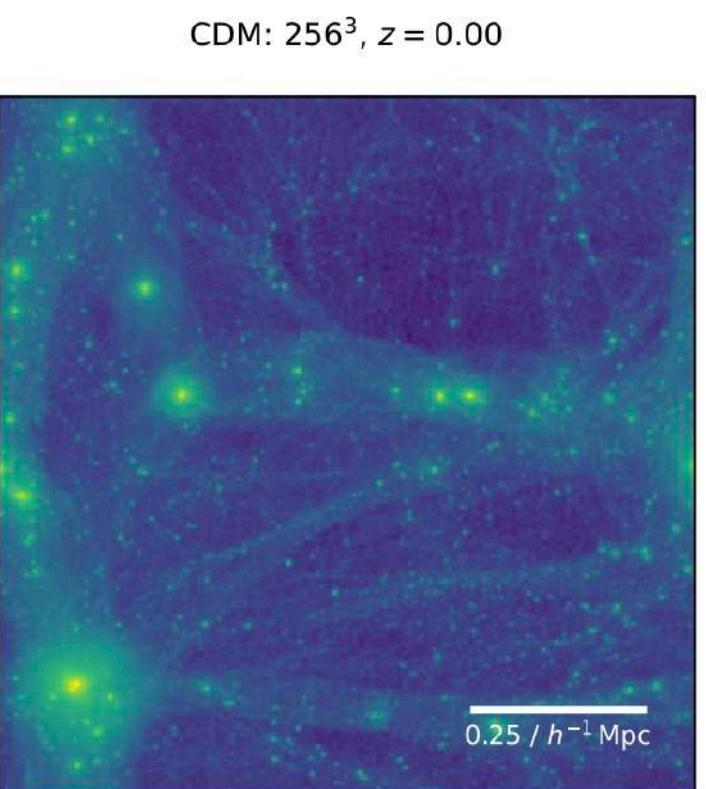
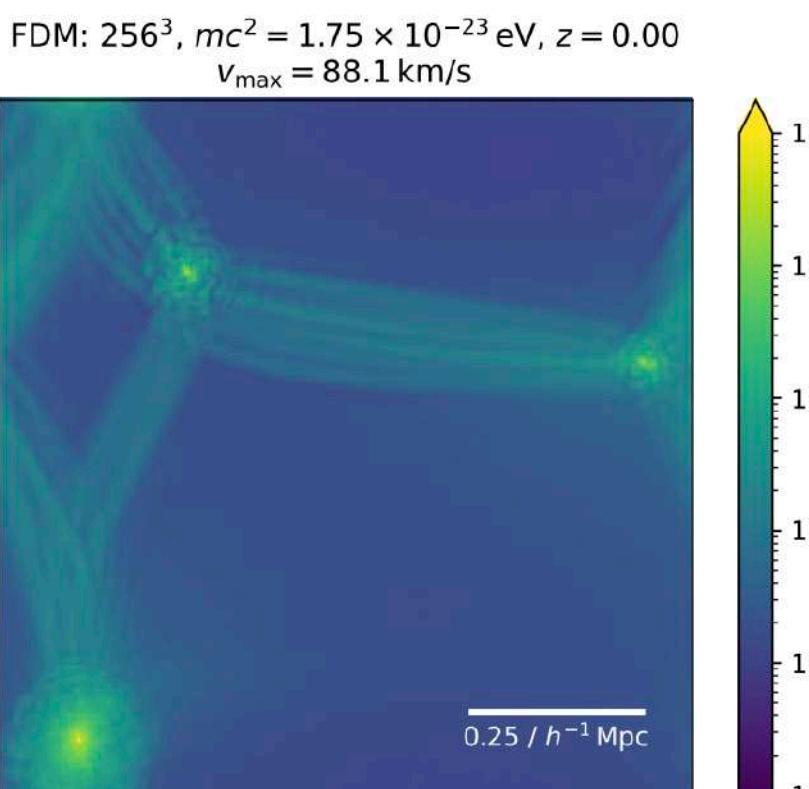
# Phenomenology

## RICH PHENOMENOLOGY ON SMALL SCALES



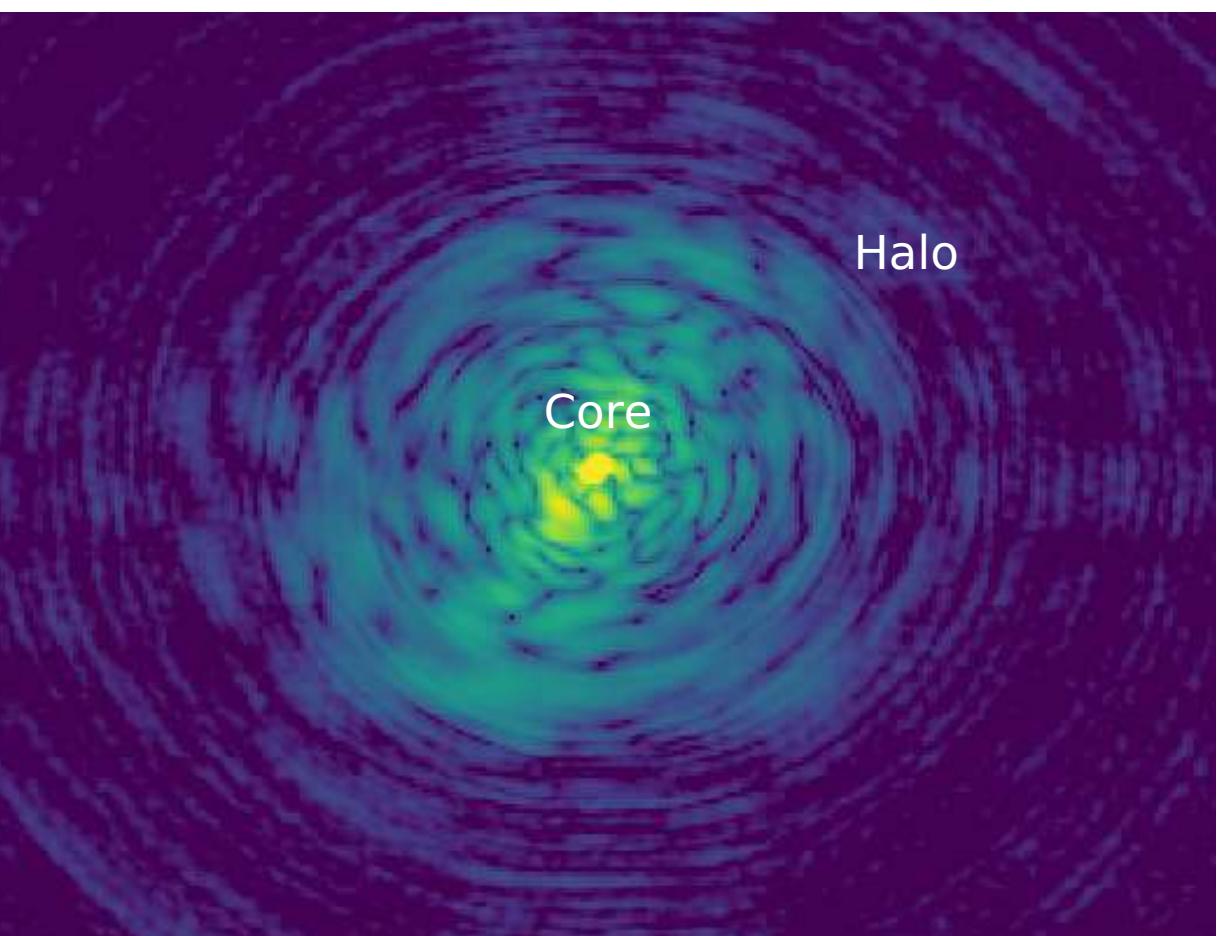
\* Focus only in gravitational signatures

### Suppression of small structures

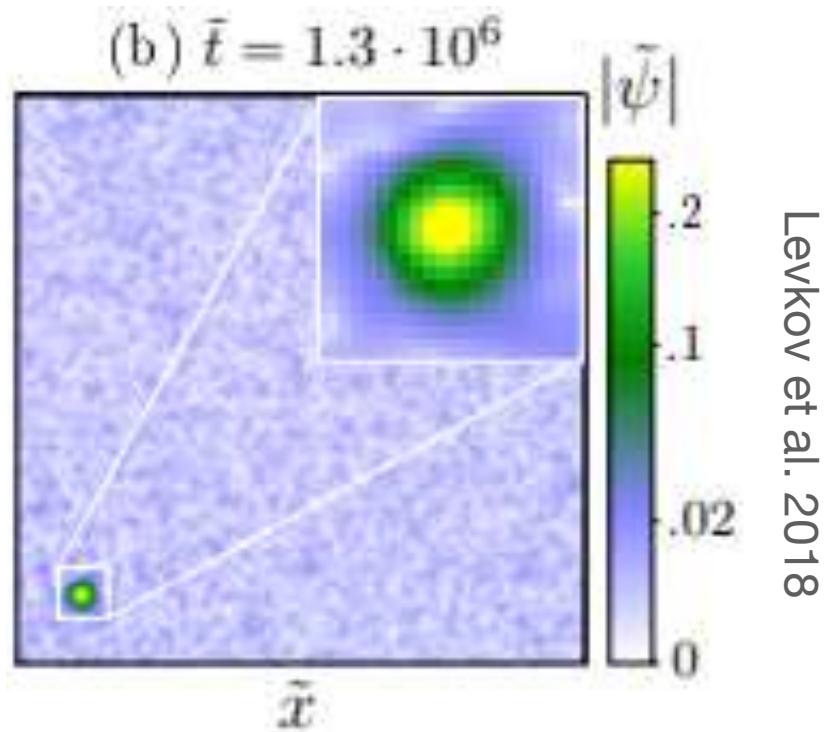


S. May et al. 2021

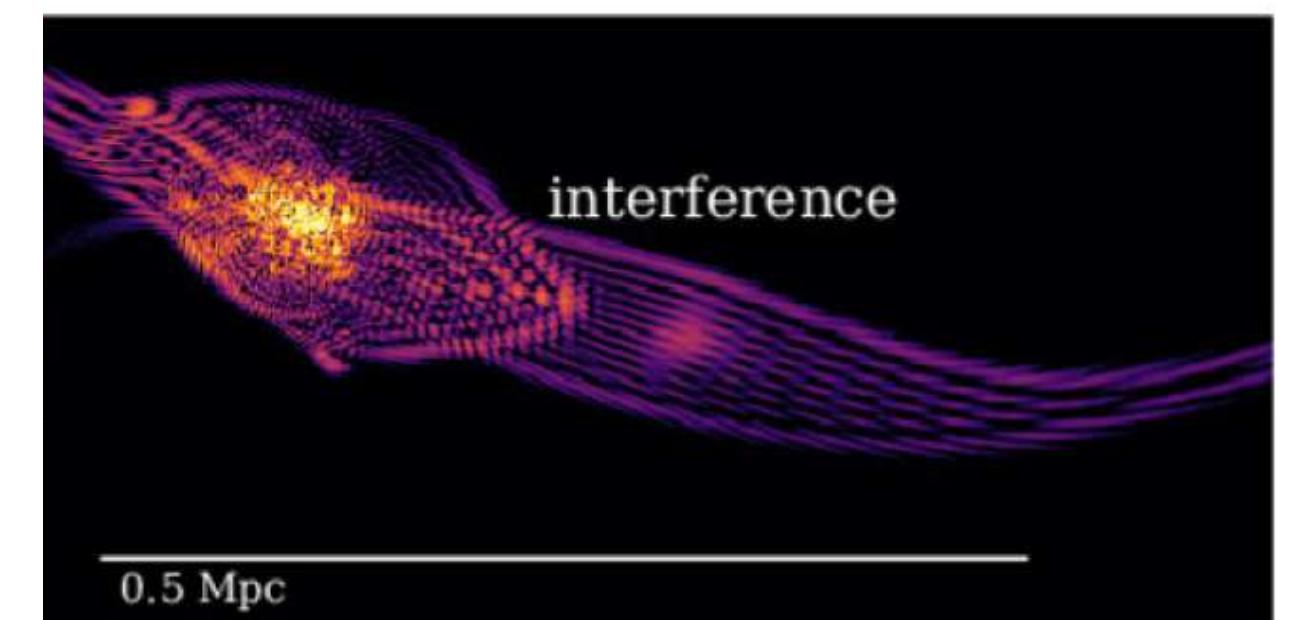
### Formation of a solitonic core



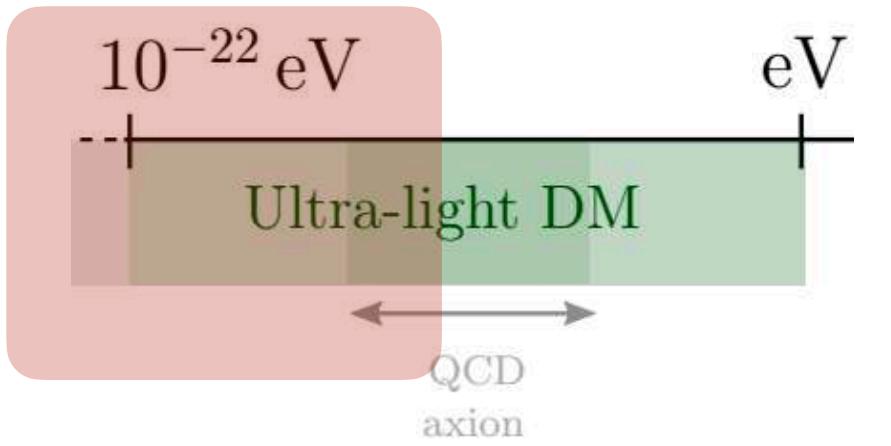
### Dynamical effects



### Wave interference



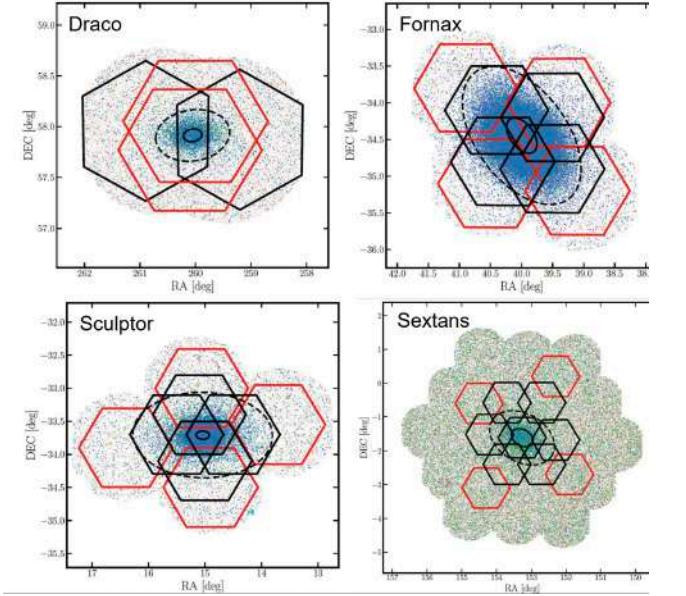
# Observational implications and constraints



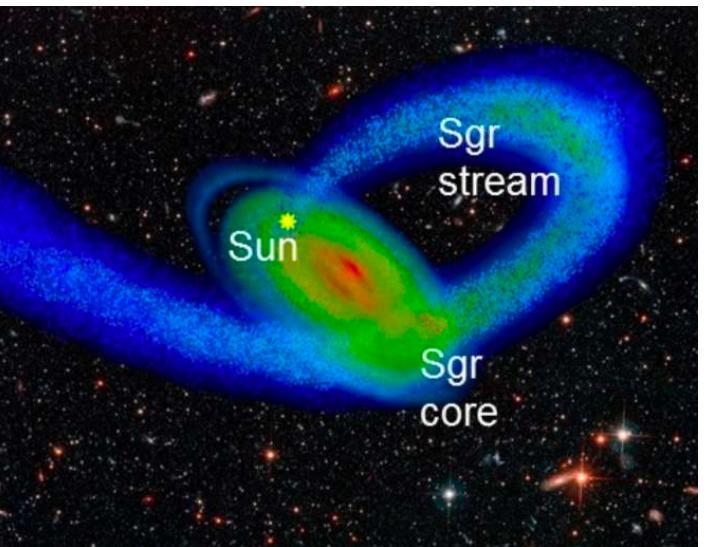
## Galaxies



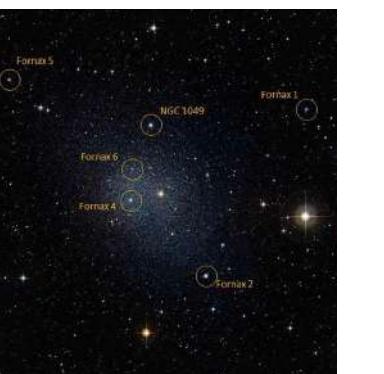
Dwarfs



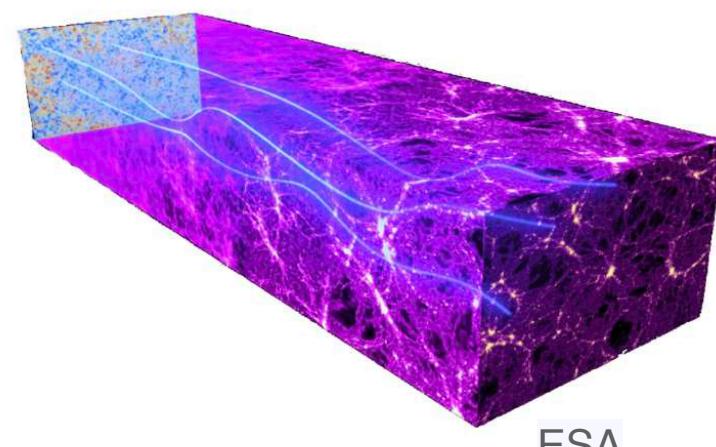
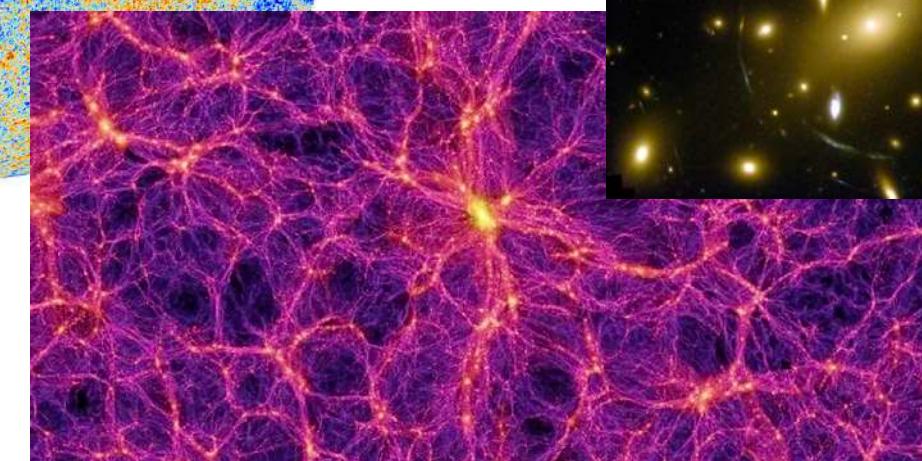
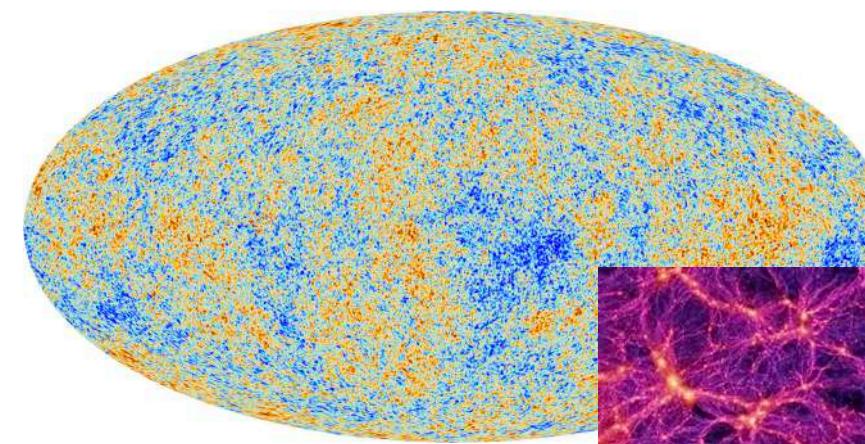
Stellar stream



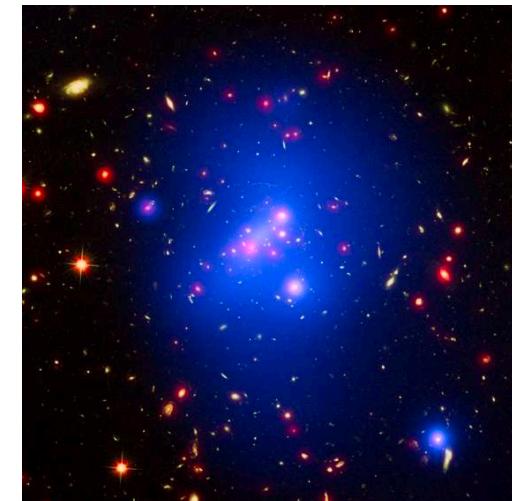
Globular clusters



CMB+LSS



Clusters



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NASA and ESA

*interlude*

*Cosmological and astrophysical **probes** of dark matter*

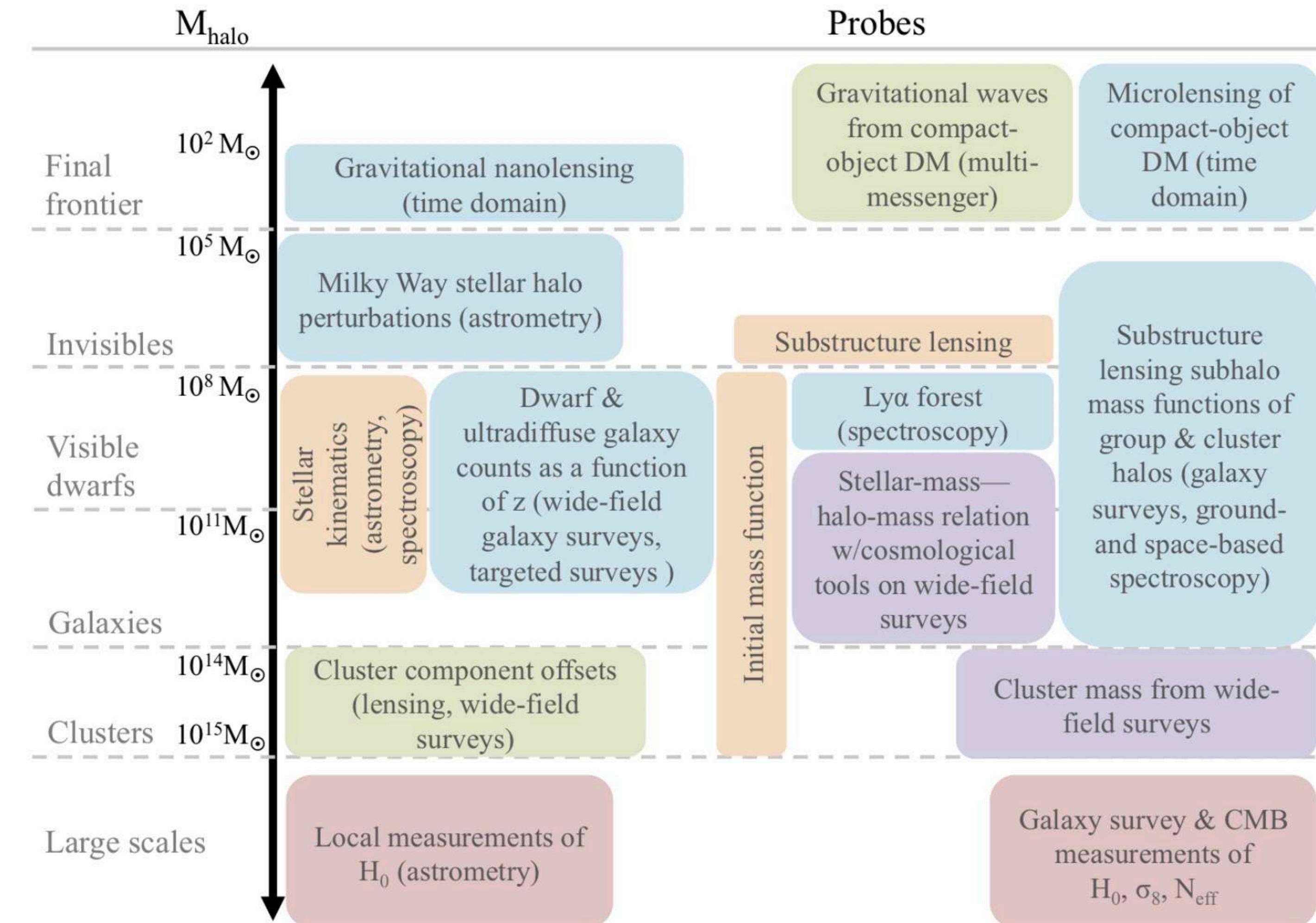
*interlude*

*Cosmological and astrophysical **probes** of dark matter*

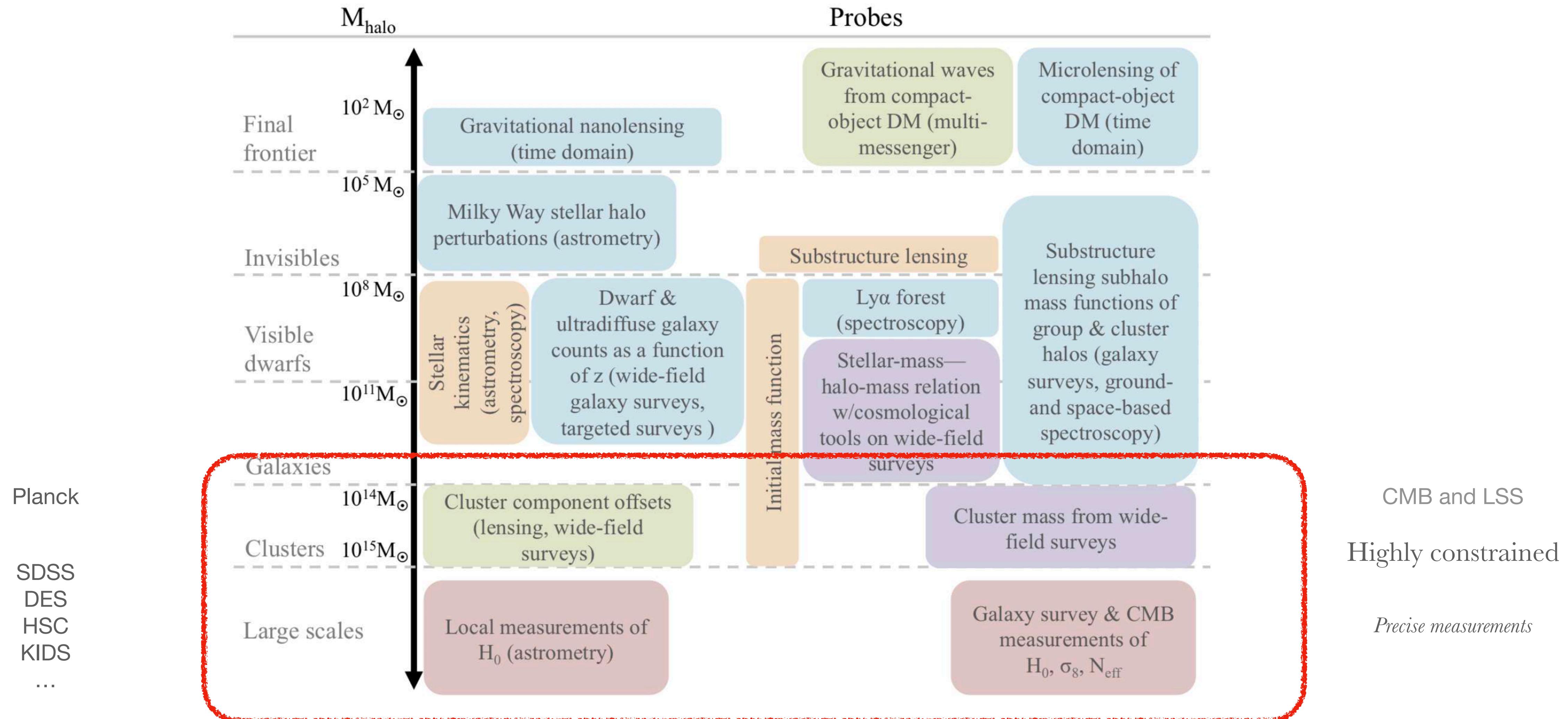
Good reference:

"*Gravitational probes of dark matter physics*", Matthew R. Buckley , Annika H. G. Peter, 2017

# Cosmological and astrophysical probes of dark matter



# Cosmological and astrophysical probes of dark matter

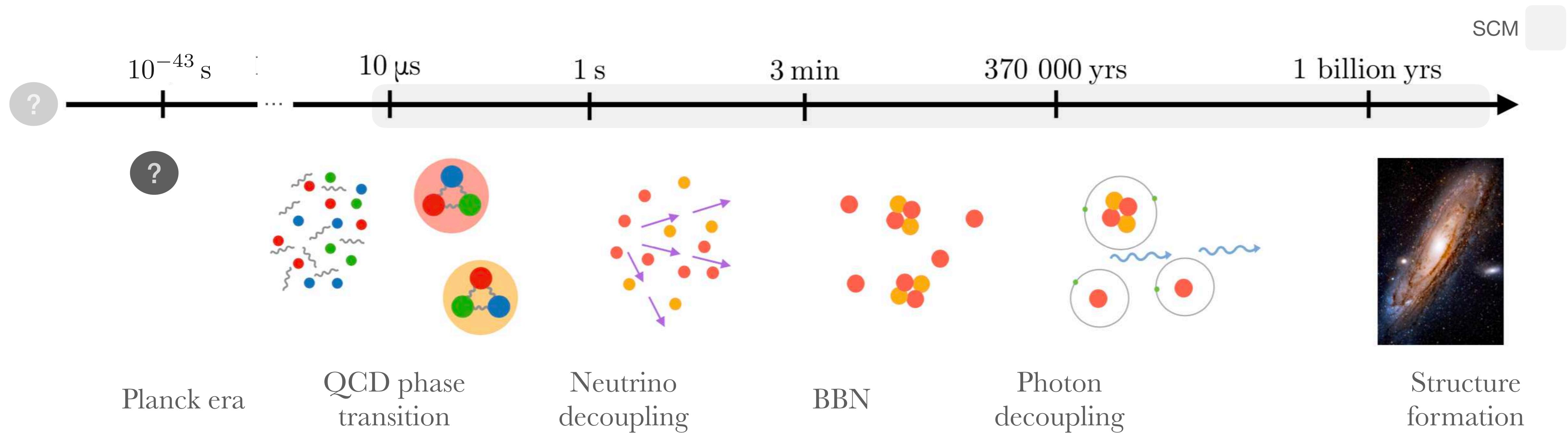


# *Cosmological and astrophysical probes of dark matter*

CMB and LSS

# *Thermal history of the universe*

The universe “started” **hot** e **dense** → As it **cools**, the structures we know start to form



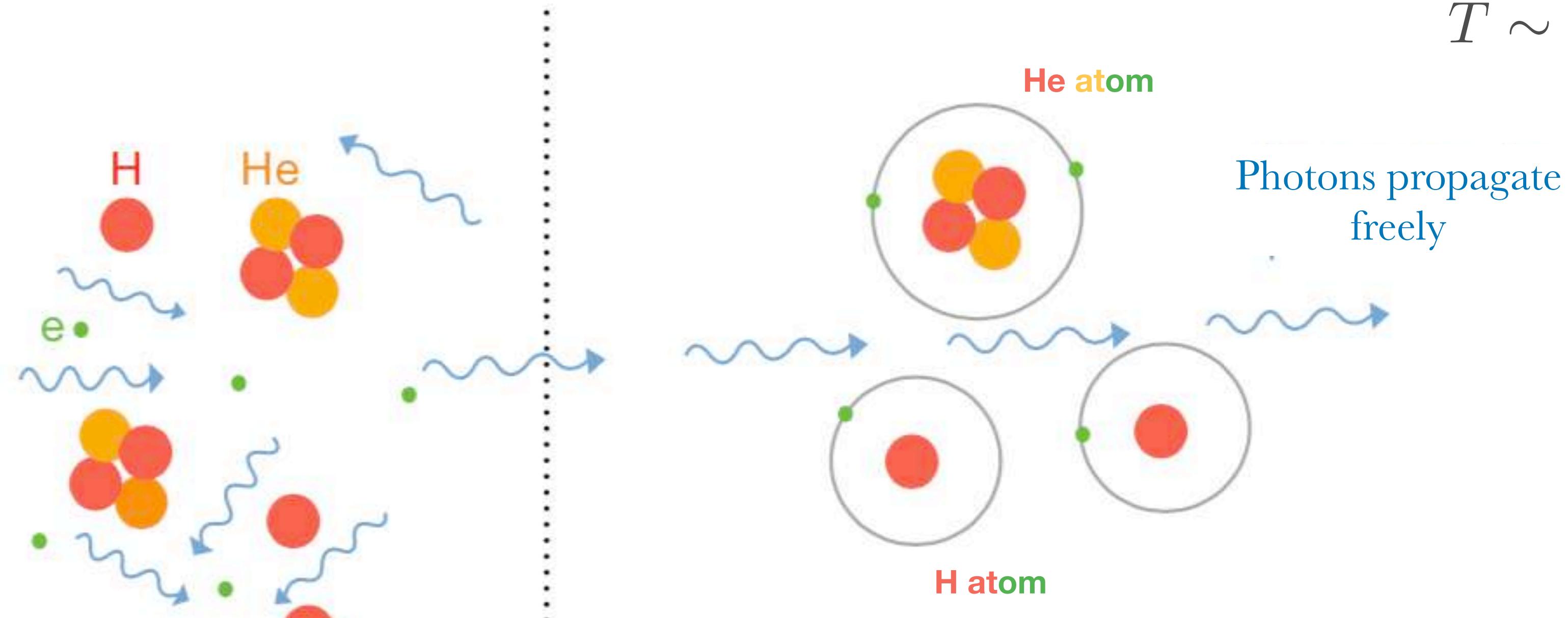
Crédito: D. Baumann

- (Most of them) observationally confirmed by independent measurements

# Recombination and photon decoupling

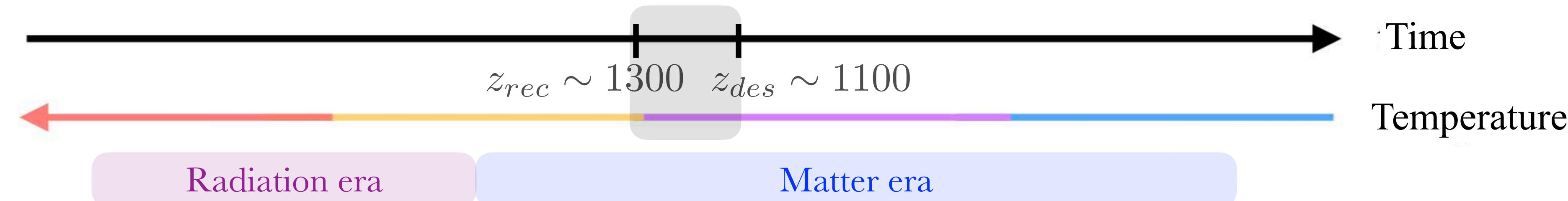
$t \sim 370000$  yrs

$T \sim 3000$  K

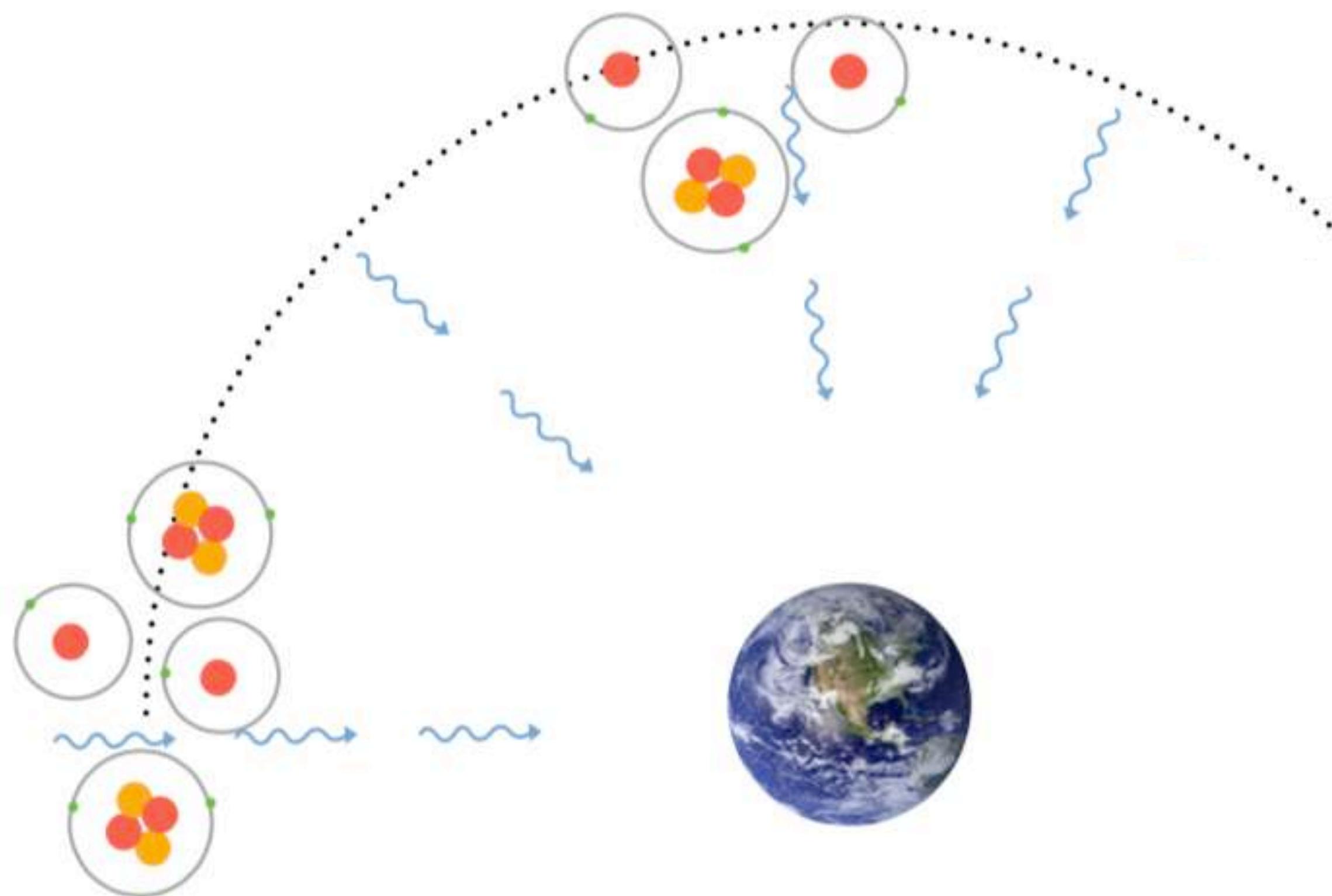


Plasma (“soup”) of coupled H, He, électrons and radiation - thermal equilibrium  
- universe is opaque: radiation cannot escape!

Atoms are formed!  
Charged electrons bound with n H and He nucleus



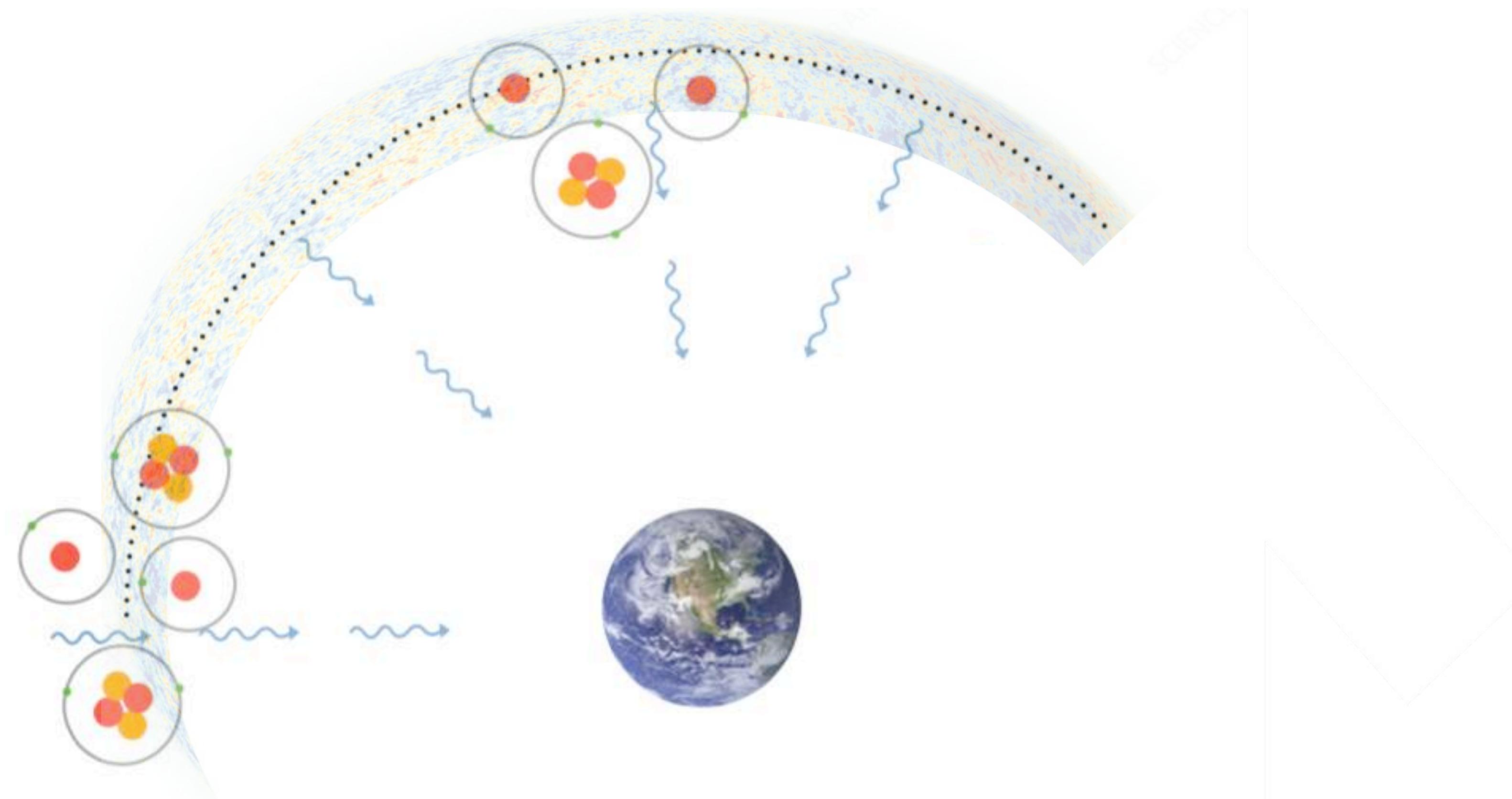
These photons are the first light of our universe...



Crédito: D. Baumann

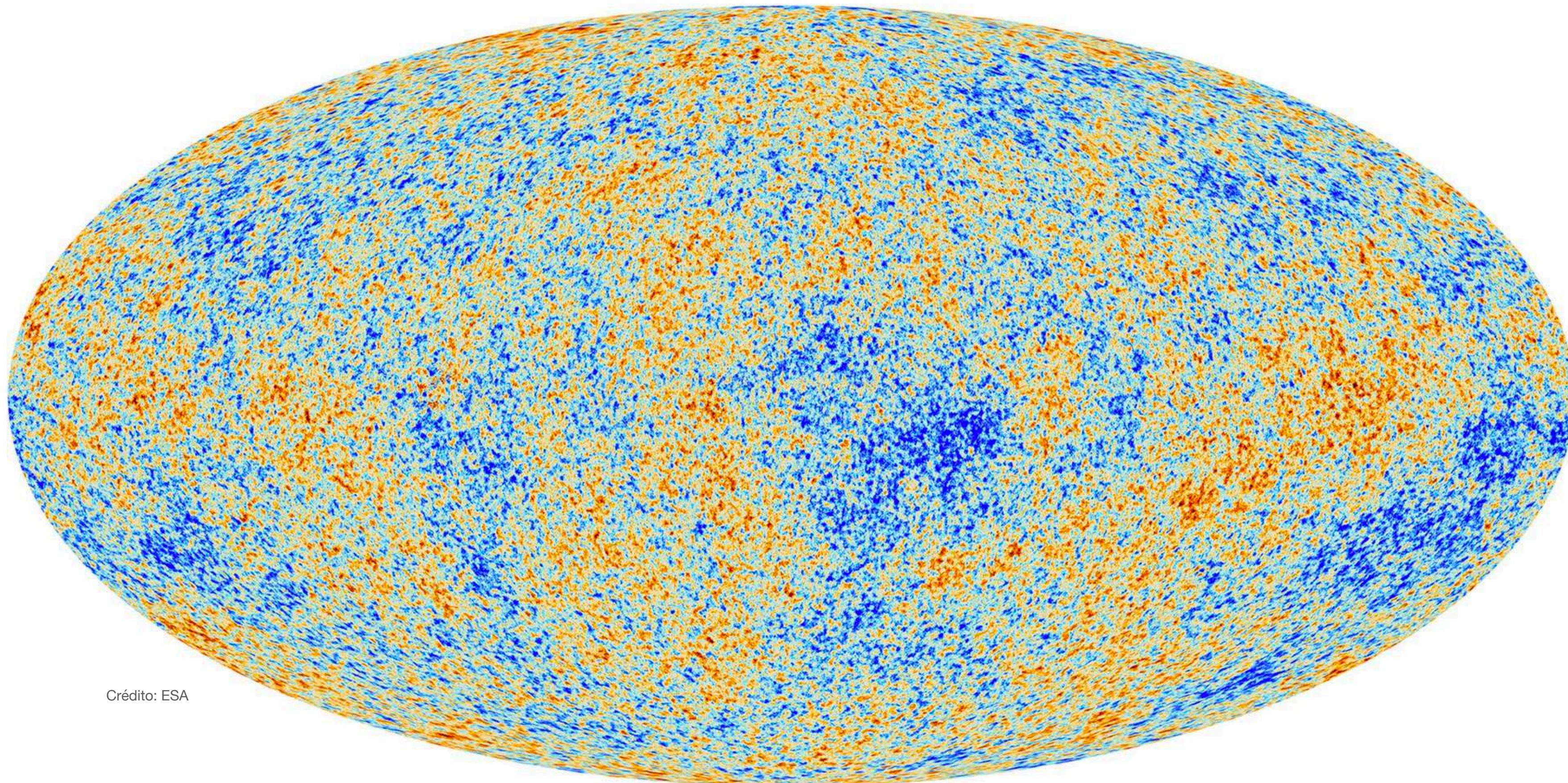
... e tell us how the universe was at early times.

# Cosmic Microwave Background (*CMB*)



Given the expansion of the universe, we observe these photons in microwave.

# Cosmic Microwave Background (*CMB*)



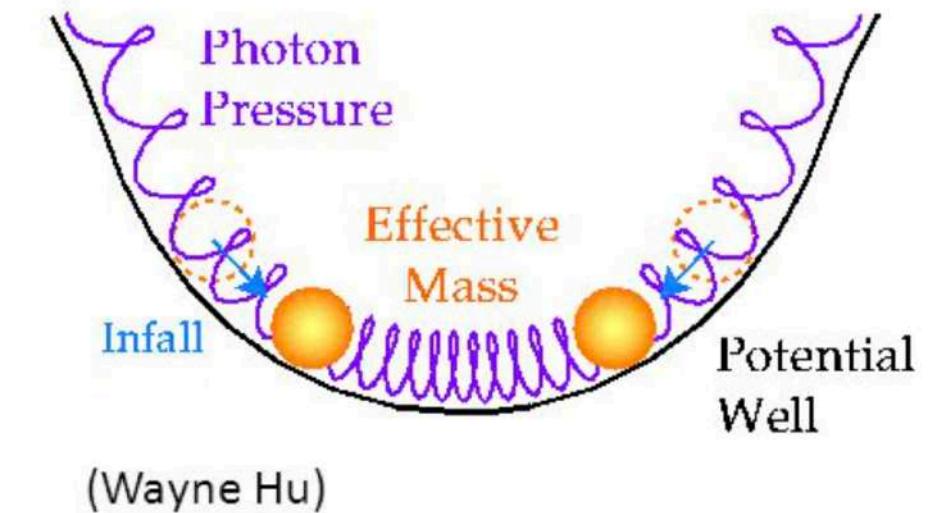
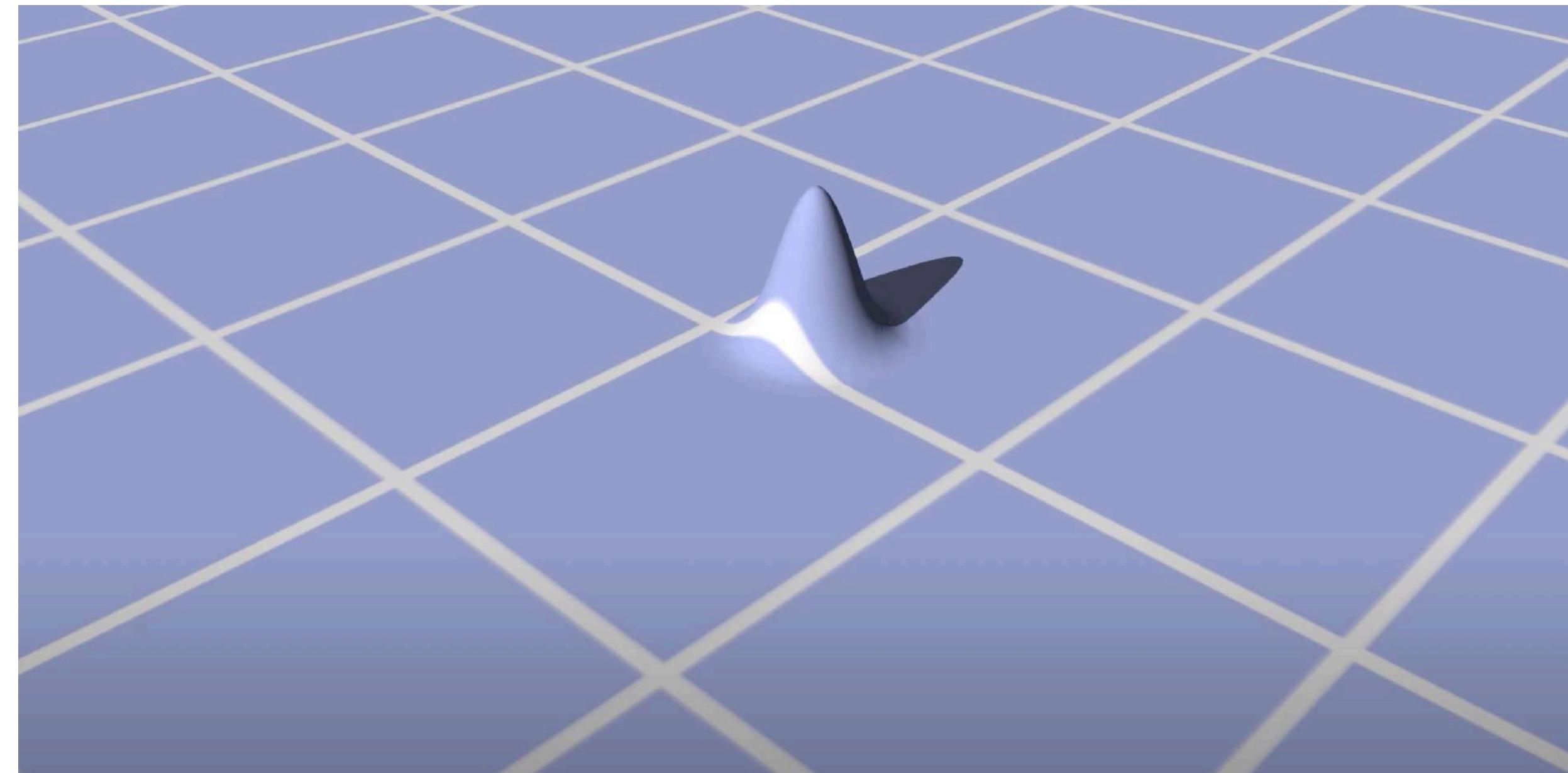
Crédito: ESA

Temperature 2.7 K. Small fluctuations - initial condition for the structures of our universe

# Baryon Acoustic Oscillation (*BAO*)

- Oscillation in the baryon-photon fluid: pressure vs gravity
- This wave propagates until matter/radiation decoupling
- Its signature is imprinted in the CMB and the distribution of galaxies

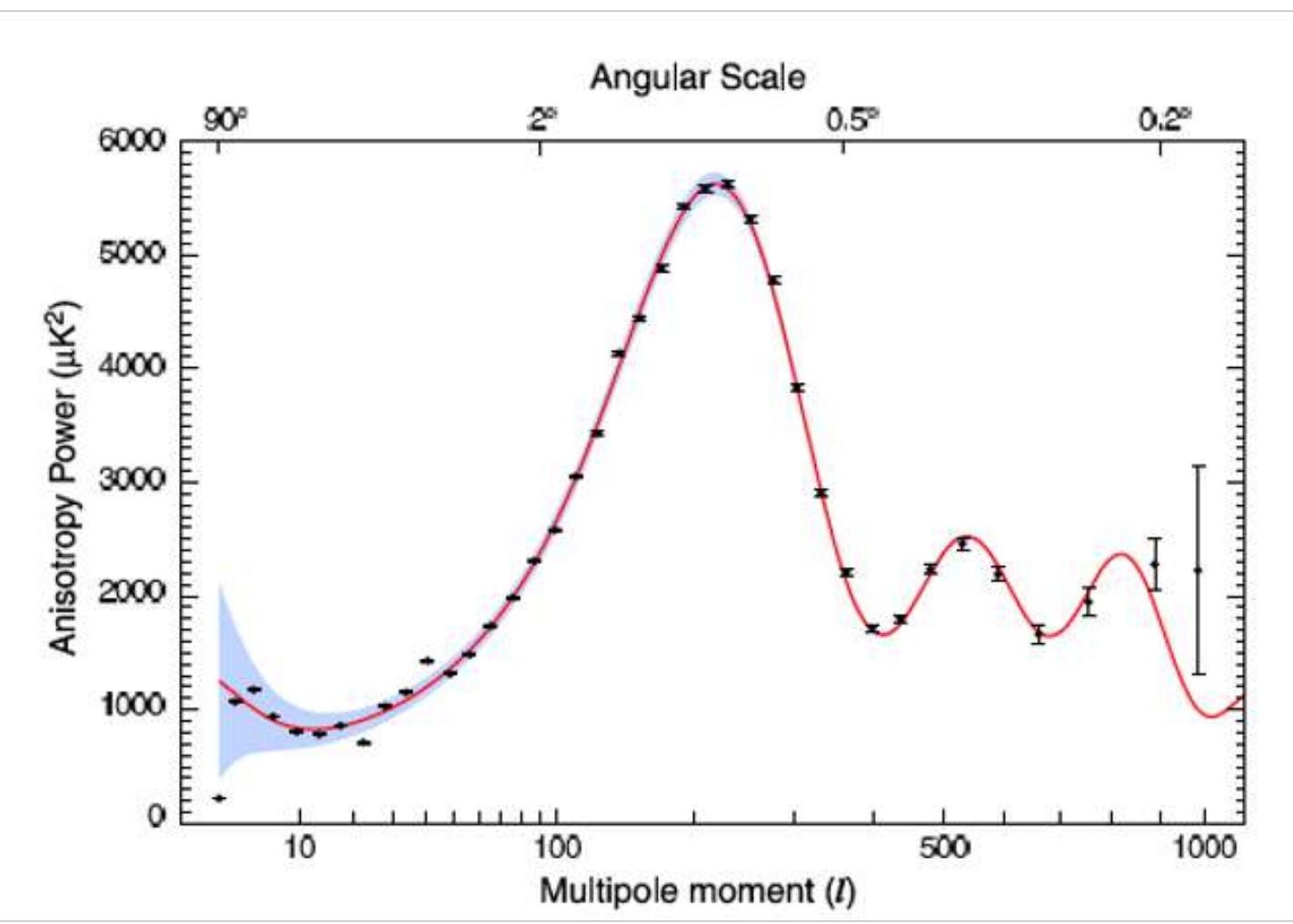
Scale known to 0.2% precision from  
CMB power spectrum ( $147.4 \pm 0.3$  Mpc)



Crédito: CASTRO

# Baryon Acoustic Oscillation (BAO)

## Cosmic Microwave Background



Scale known to 0.2% precision from  
CMB power spectrum ( $147.4 \pm 0.3$  Mpc)

Using CMB and other LSS probes, can constraint the parameters with incredible precision.

$$\Omega_b = 0.0484 \pm 0.0003$$

$$\Omega_m = 0.308 \pm 0.012$$

$$\Omega_\Lambda = 0.692 \pm 0.012$$

$$n_s = 0.9626 \pm 0.0057$$

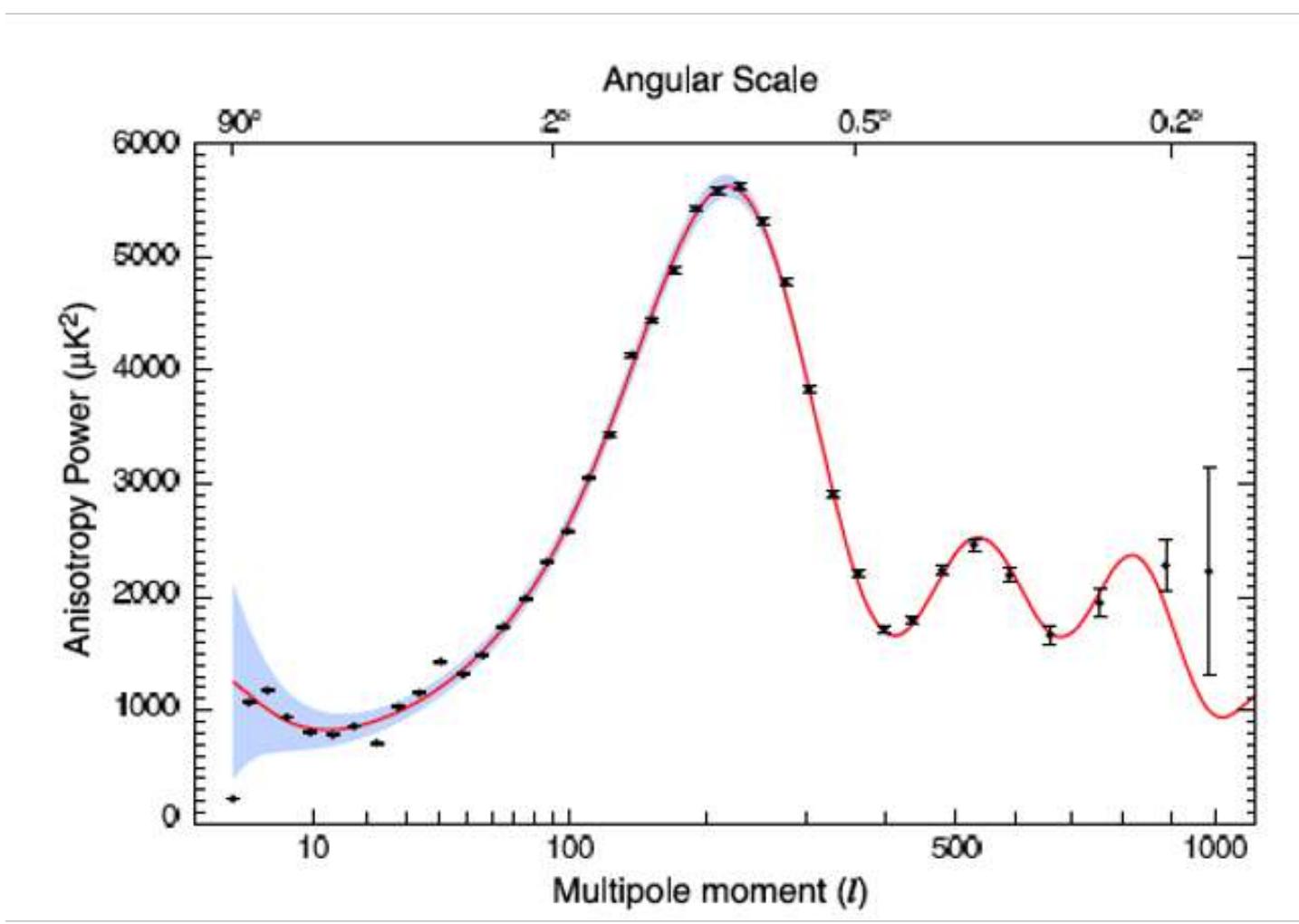
$$10^9 A_s = 2.092 \pm 0.034$$

$$\tau = 0.0522 \pm 0.0080$$

Standard cosmological model - **LCDM model**

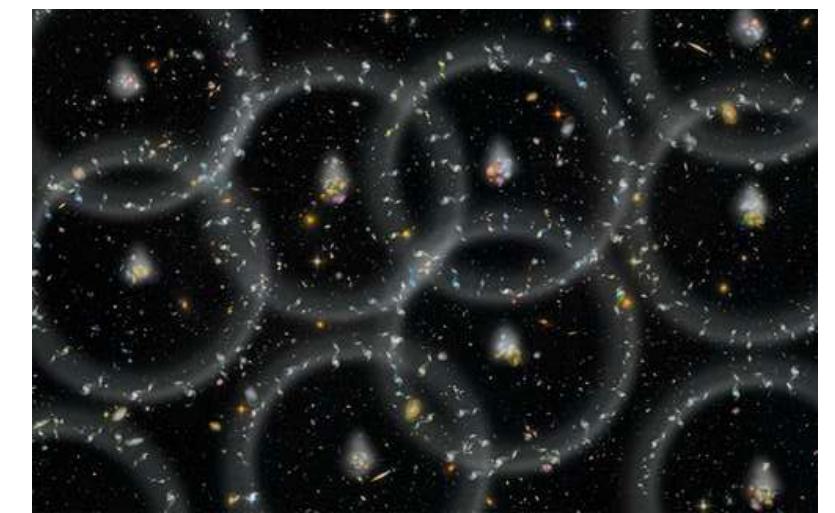
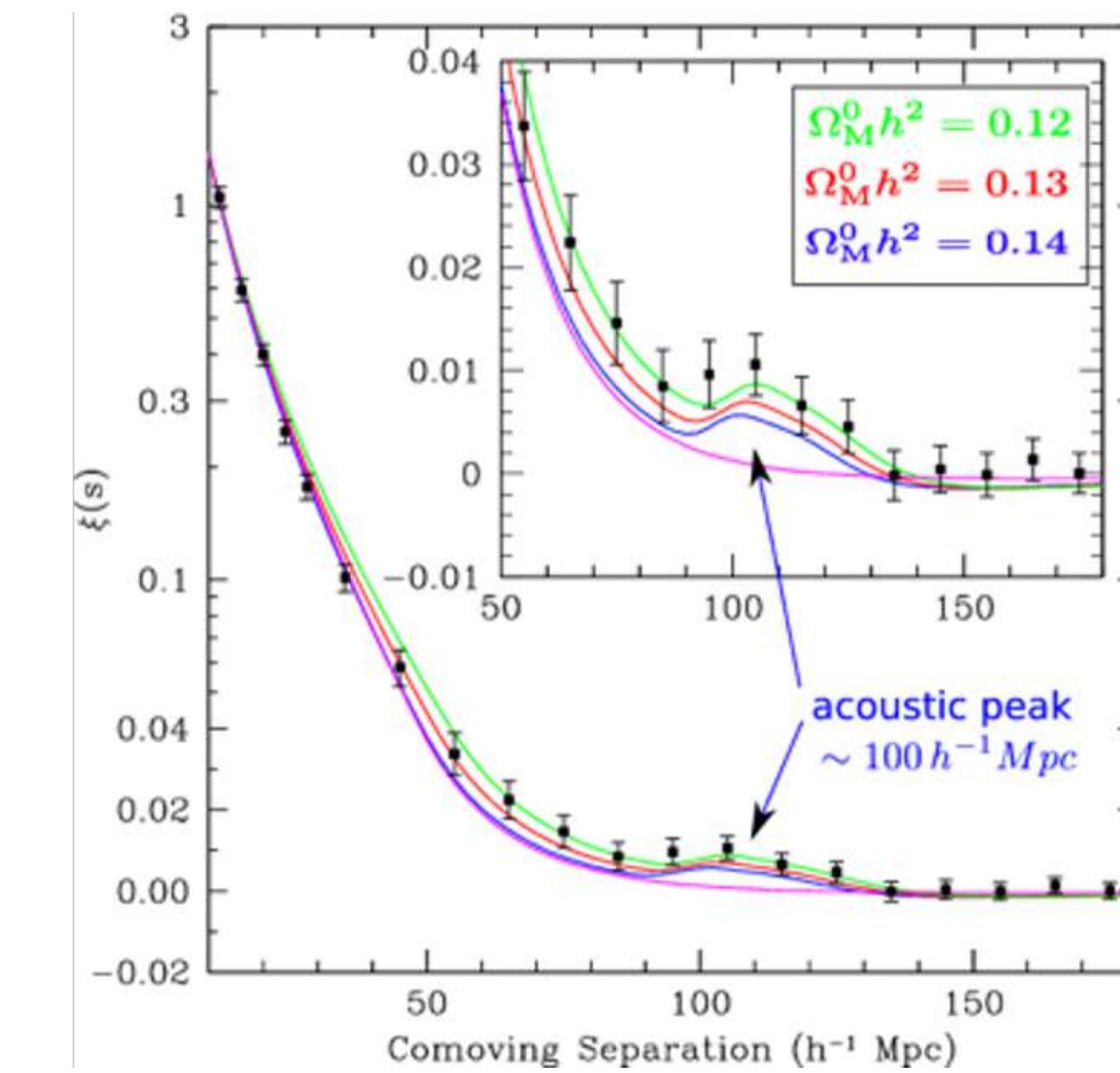
# Baryon Acoustic Oscillation (BAO)

Cosmic Microwave Background



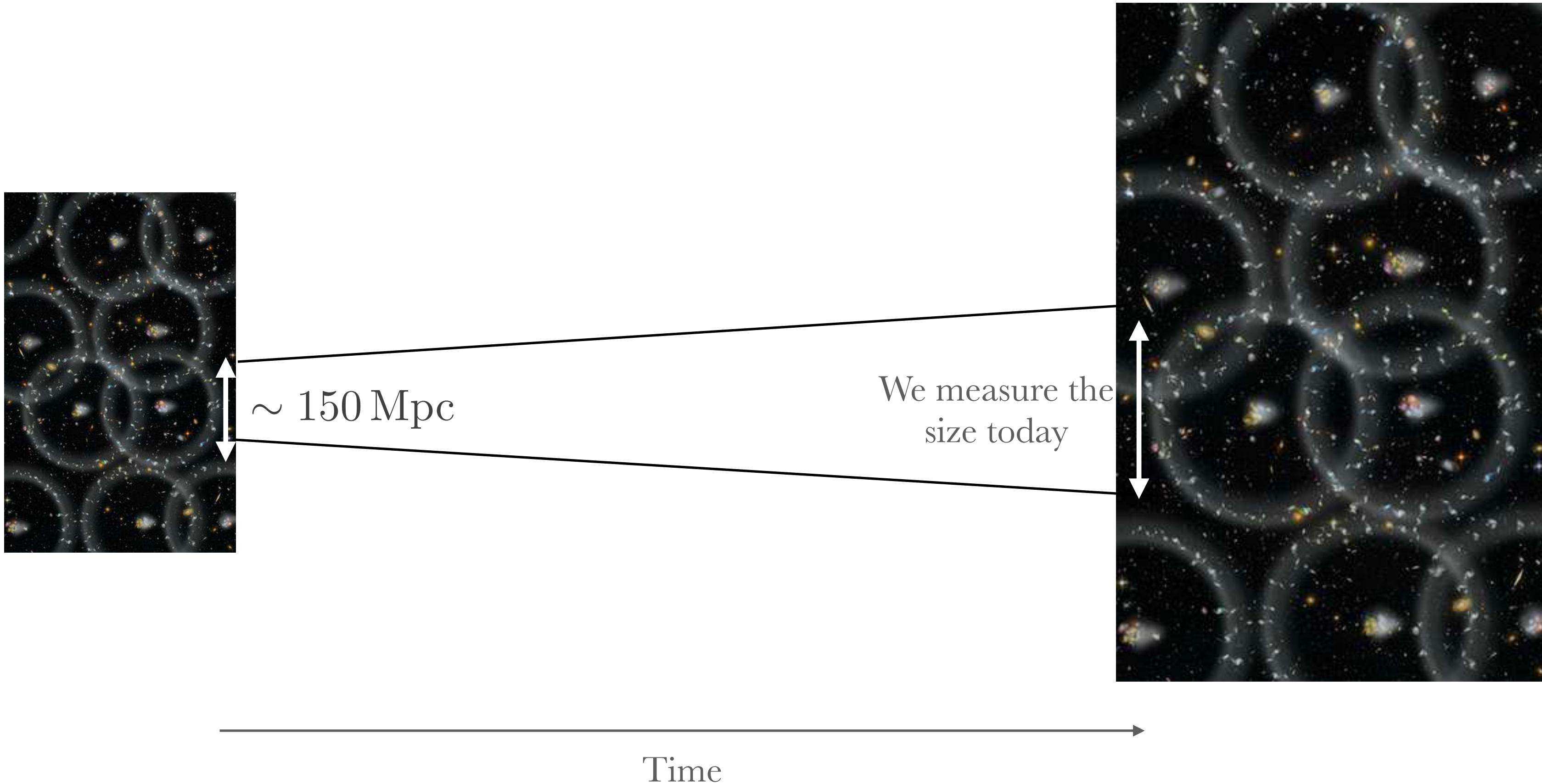
Scale known to 0.2% precision from  
CMB power spectrum ( $147.4 \pm 0.3$  Mpc)

Large Scale Structure



Credit: LBNL

# *Baryon Acoustic Oscillation (BAO)*



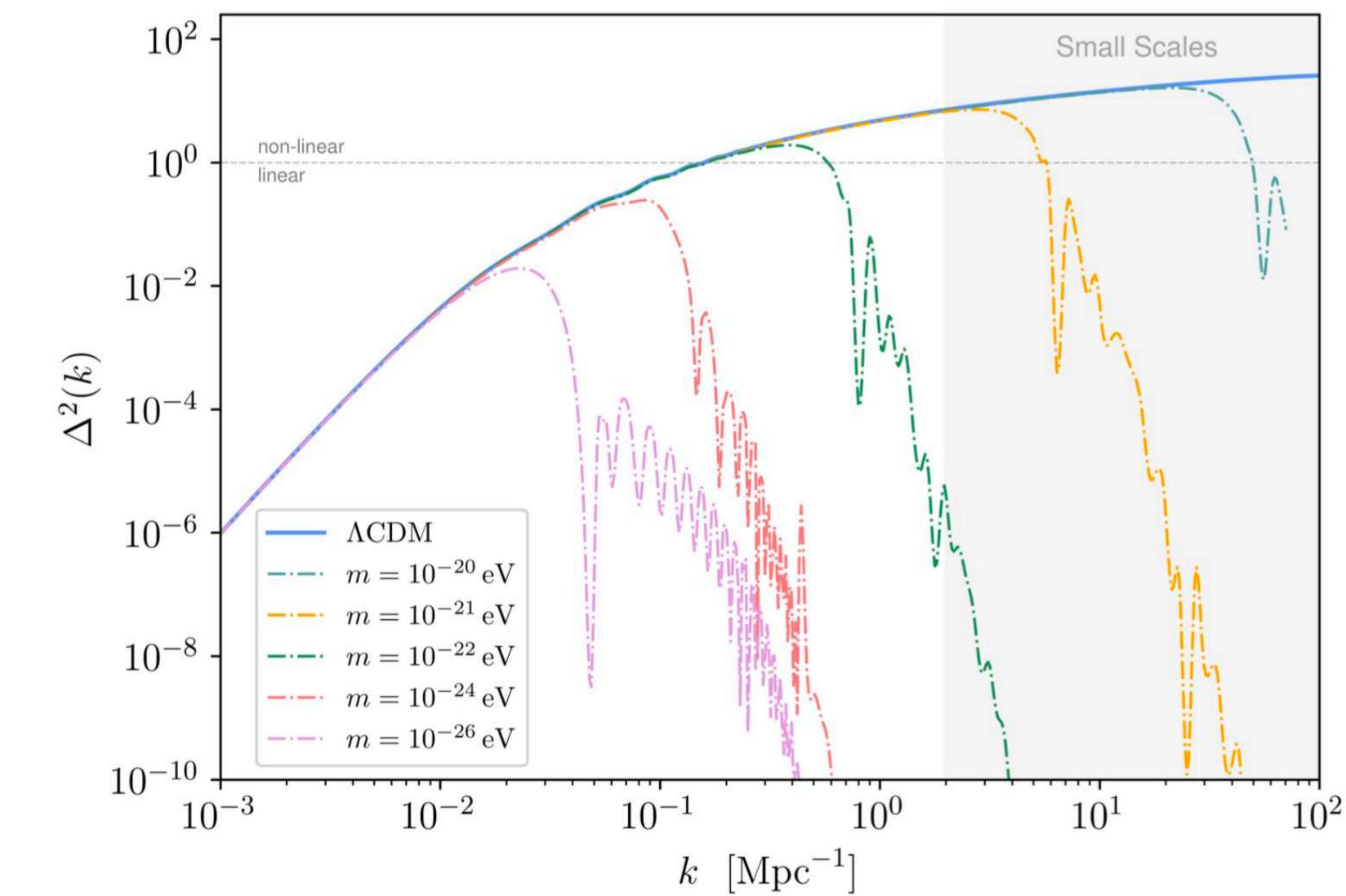
- BAOs are “standard rulers” to measure expansion

# *Cosmological and astrophysical probes of dark matter*

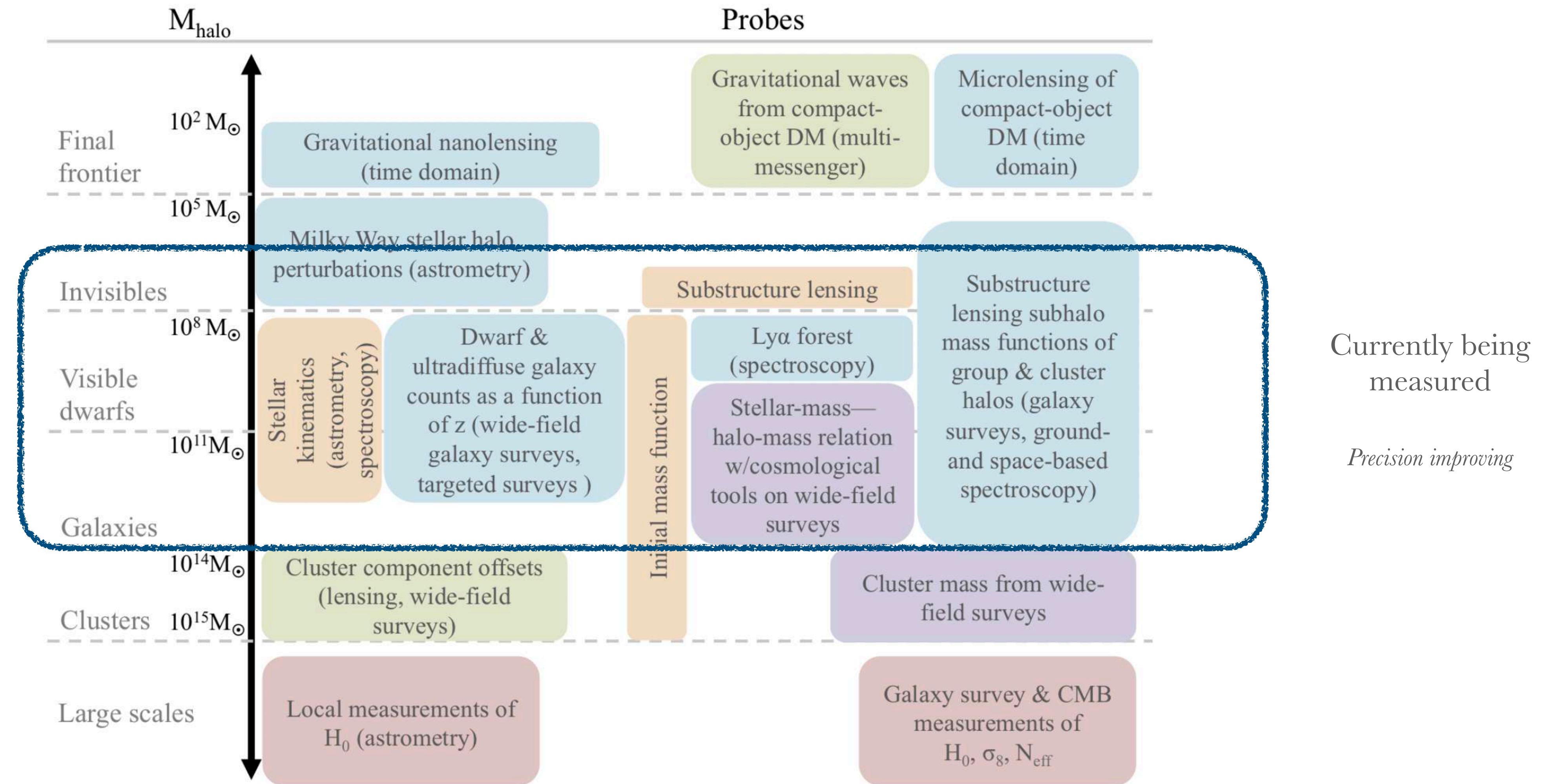
## CMB and LSS

Depends on the amount of **CDM** - a cold fluid  
Damping/suppression

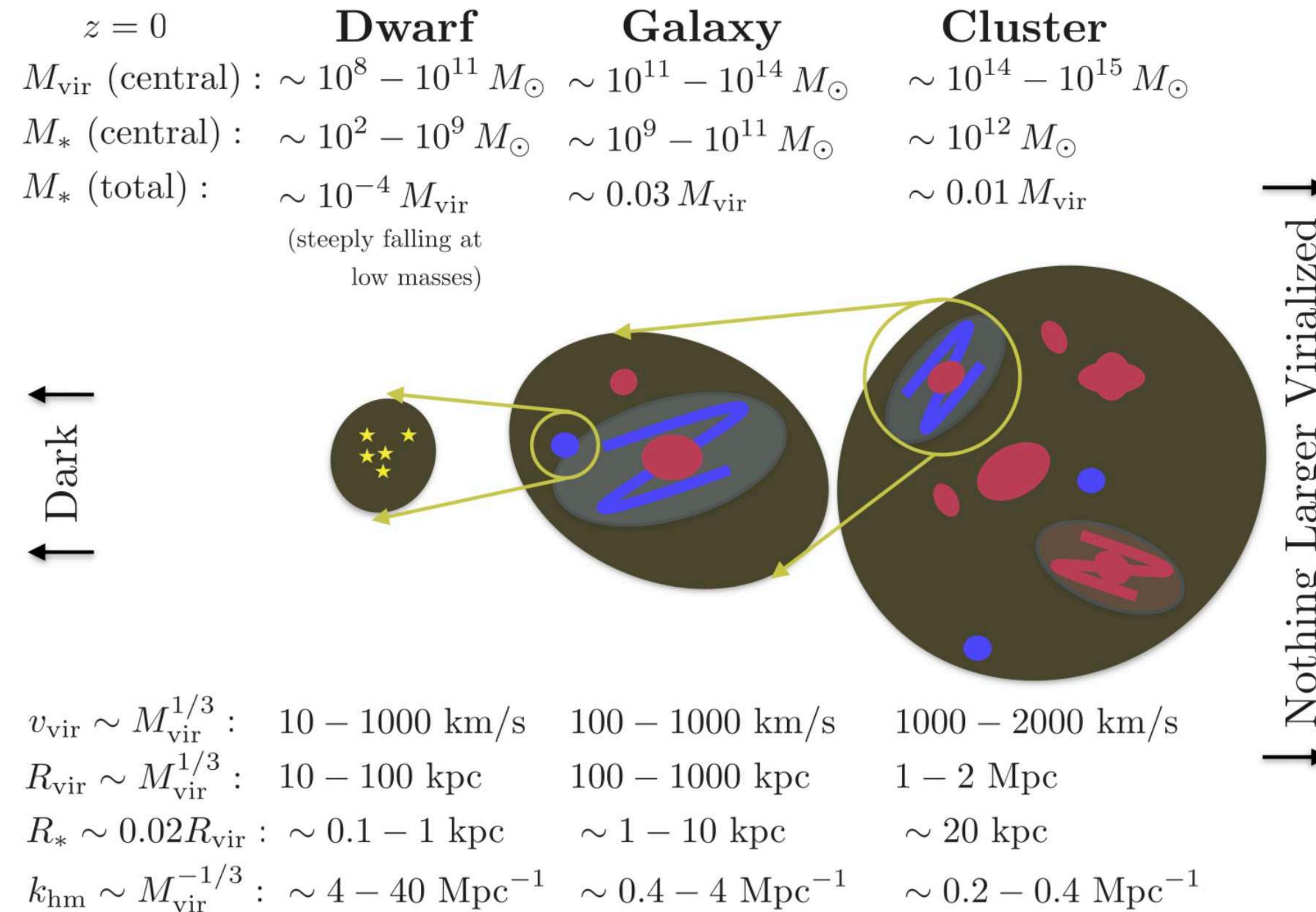
...



# Cosmological and astrophysical probes of dark matter

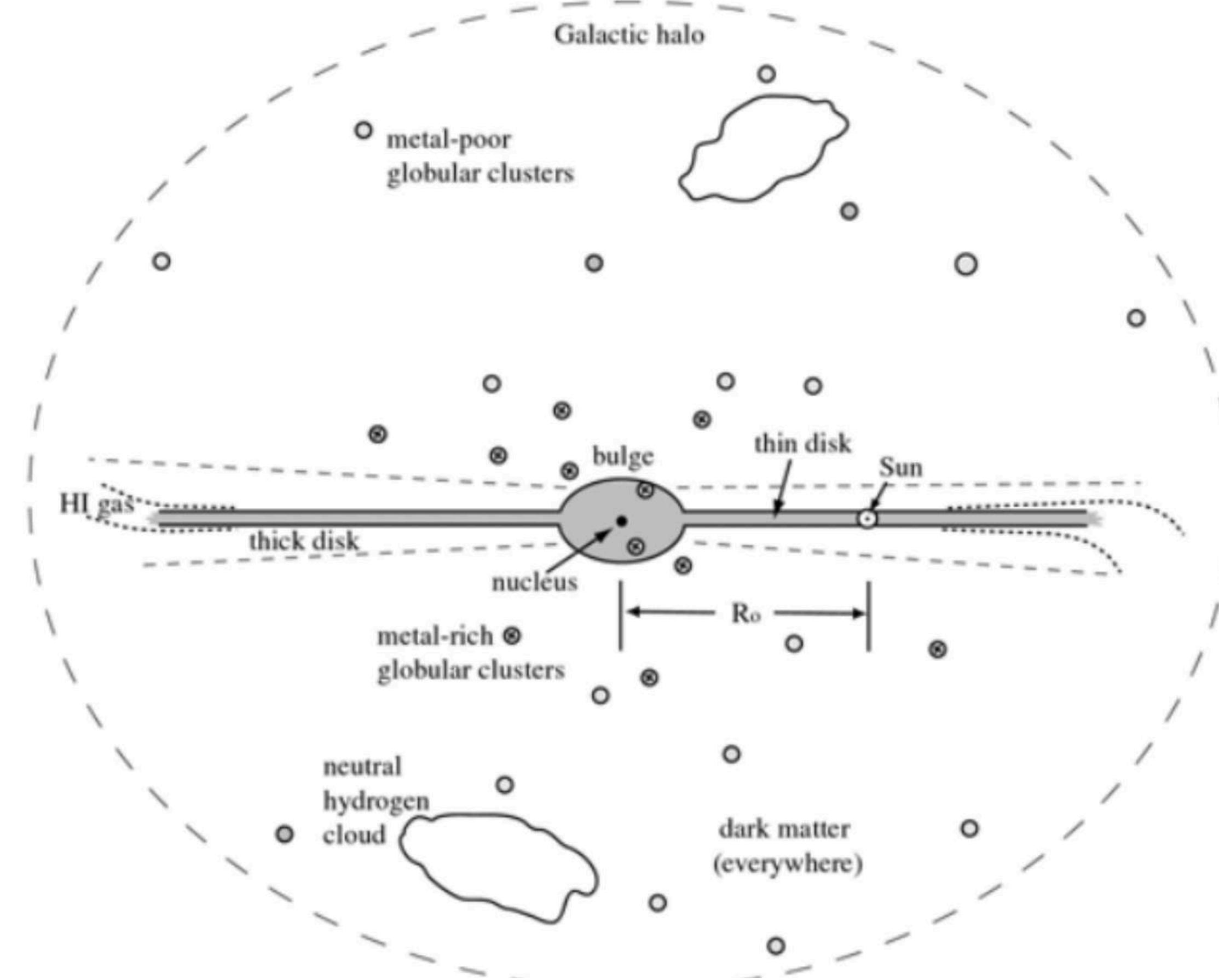


# *Cosmological and astrophysical probes of dark matter*



# *Cosmological and astrophysical probes of dark matter*

Galaxies



A schematic side view of the Milky Way

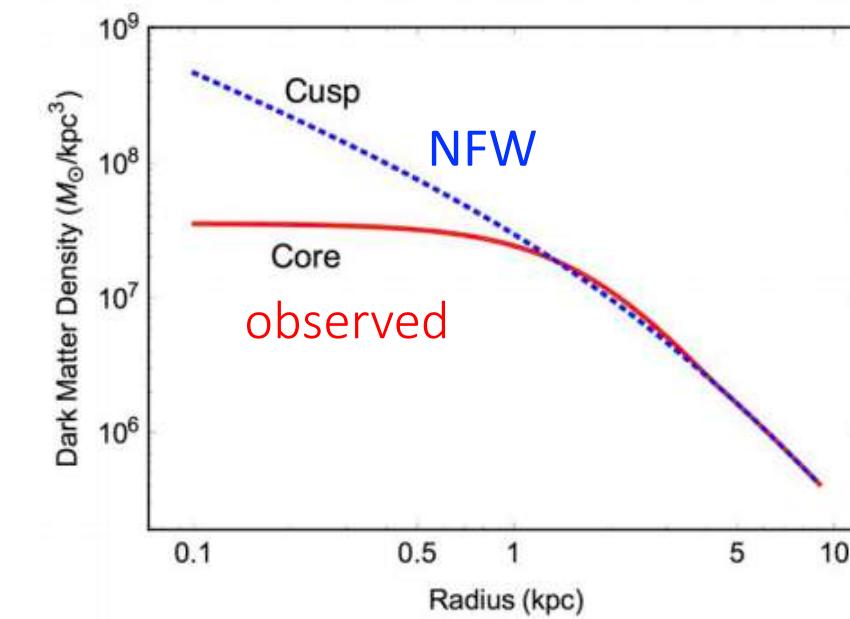
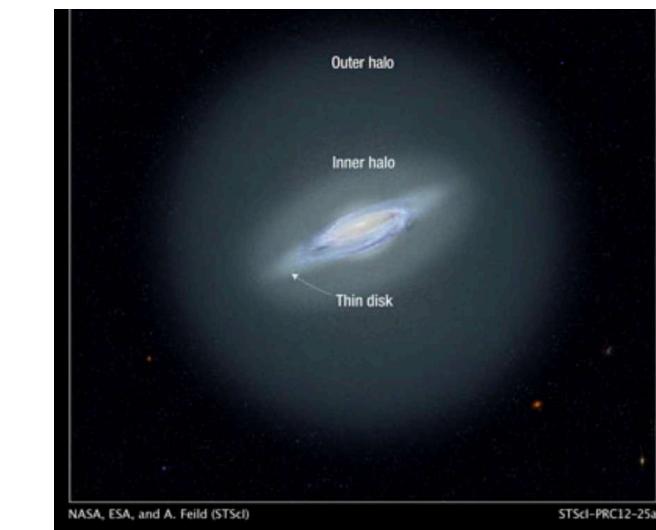
Sparke & Gallagher

# *Cosmological and astrophysical probes of dark matter*

## Galaxies/dwarf galaxies

*Local Milky Way observables and stellar streams*

- Density profile
  - MW gravitational profile
  - Dwarfs density profile
- Vertical dynamics
- Velocity dispersion
- Dynamical effect
  - Heating
  - Dynamical friction
- Counting small galaxies
- Stellar stream      (*More later*)
- ...



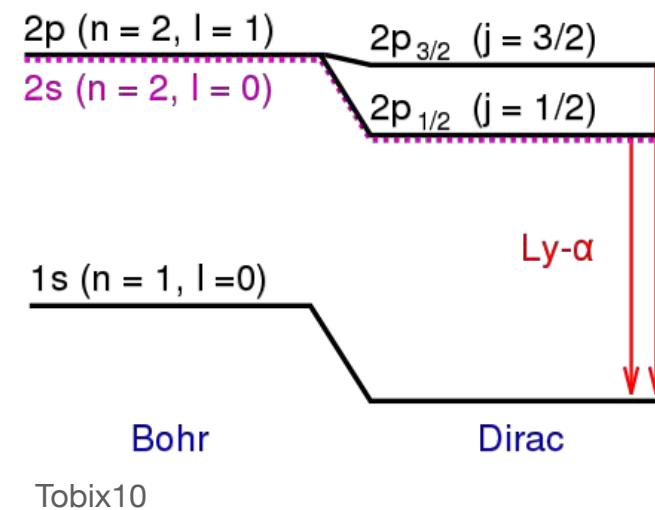
## Gravitational lensing

*(More later)*

# Cosmological and astrophysical probes of dark matter

**Lyman  $\alpha$**

Spectral line of hydrogen



**Lyman-alpha forest** is a series of absorption lines in the spectra of distant galaxies and quasars arising from the Lyman-alpha electron transition of the neutral hydrogen atom

Armengaud

$$\delta(\lambda) = \frac{f(\lambda)}{C_q(\lambda)f(z)} - 1$$

$$P_{1D}(k) = \langle (\text{FFT}(\delta))^2 \rangle$$

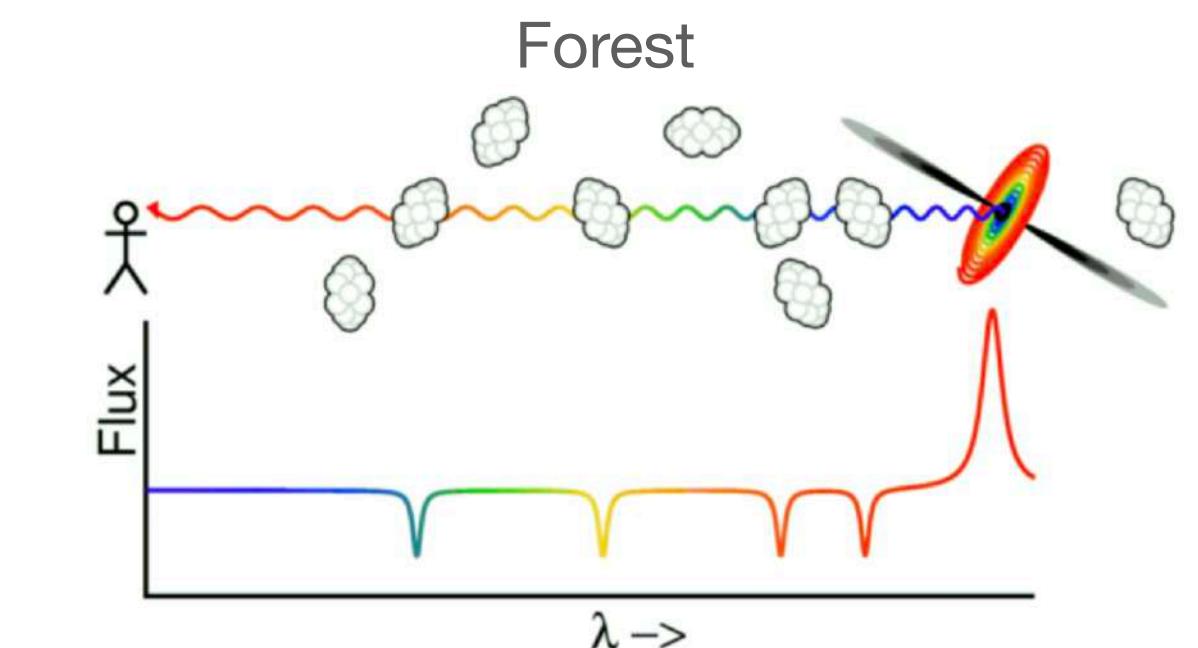
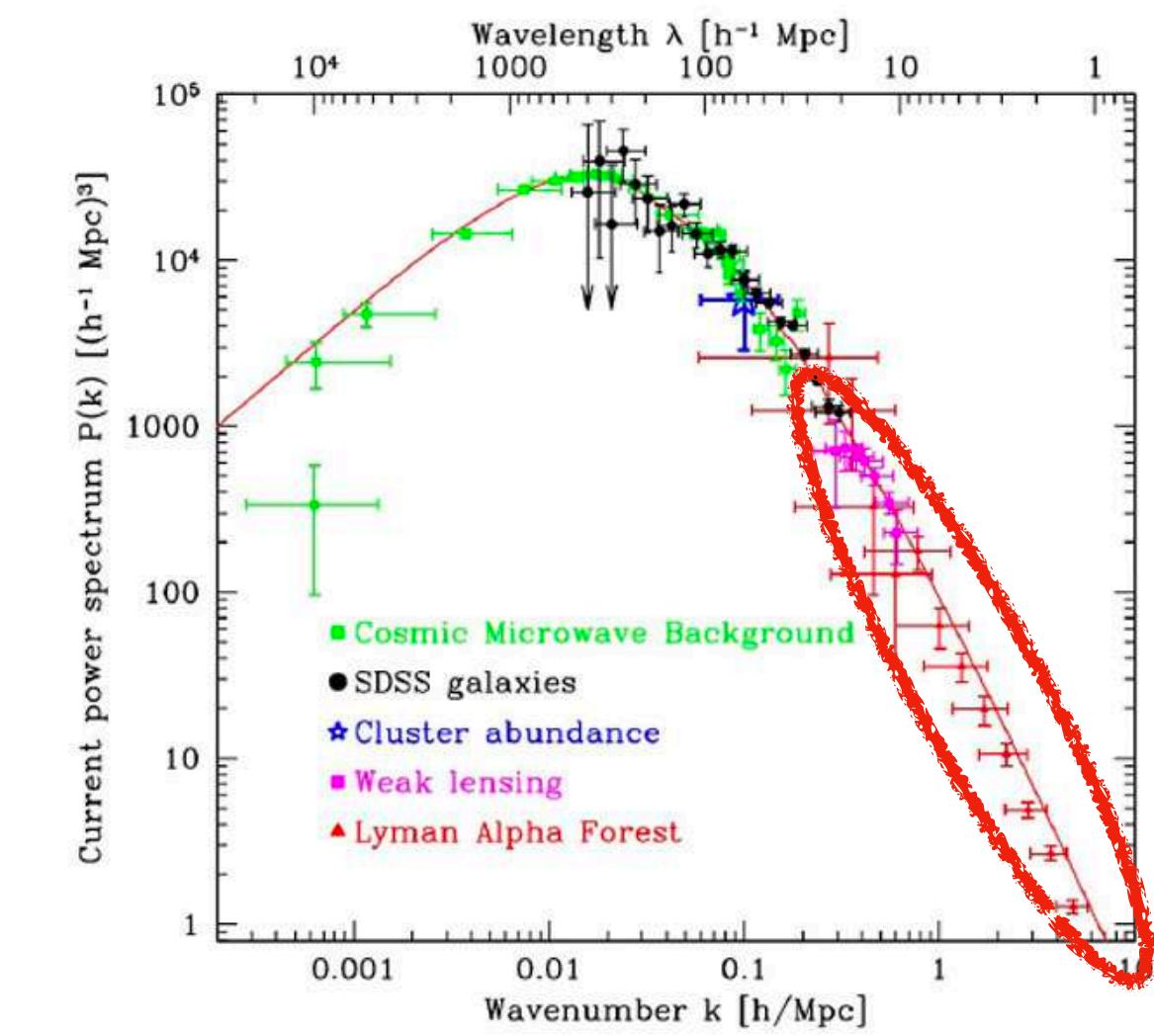
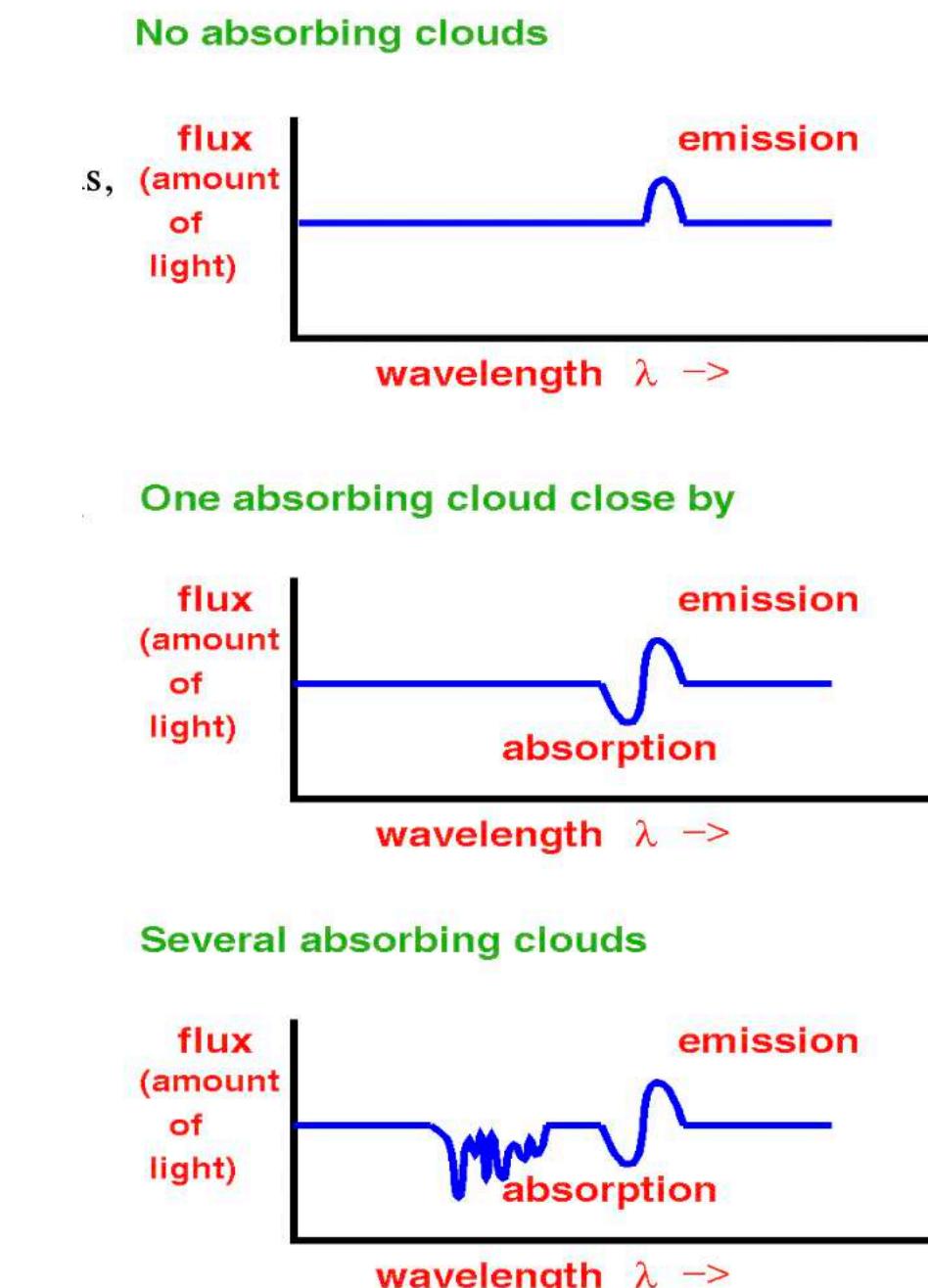
related to 3D matter  $P(k)$

Advantage: provide PS on intermediary scales

It probes the matter power spectrum in the mildly nonlinear regime over a large range of redshifts ( $z = 2 - 6$ ) down to small scales ( $1 - 80 h^{-1} \text{ Mpc}$ )

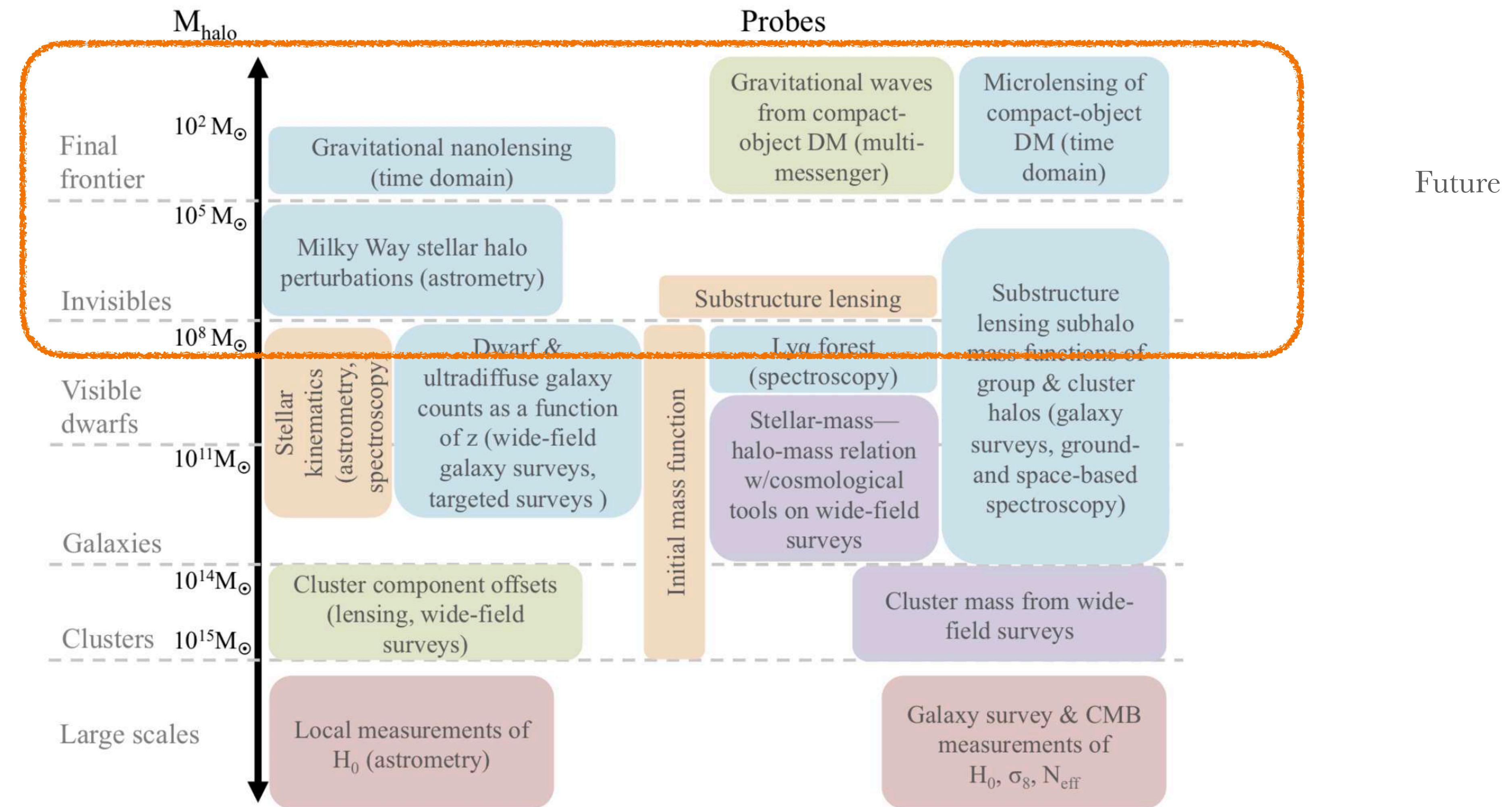
Challenges:

- Modelling/simulating of DM on NL scales
- Modelling/simulating Ly $\alpha$ 
  - Reionization

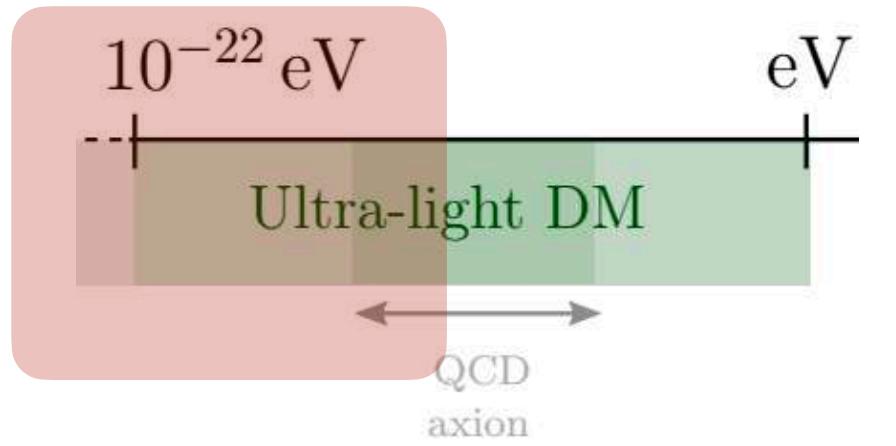


Ly $\alpha$  forest: series of absorption lines

# Cosmological and astrophysical probes of dark matter



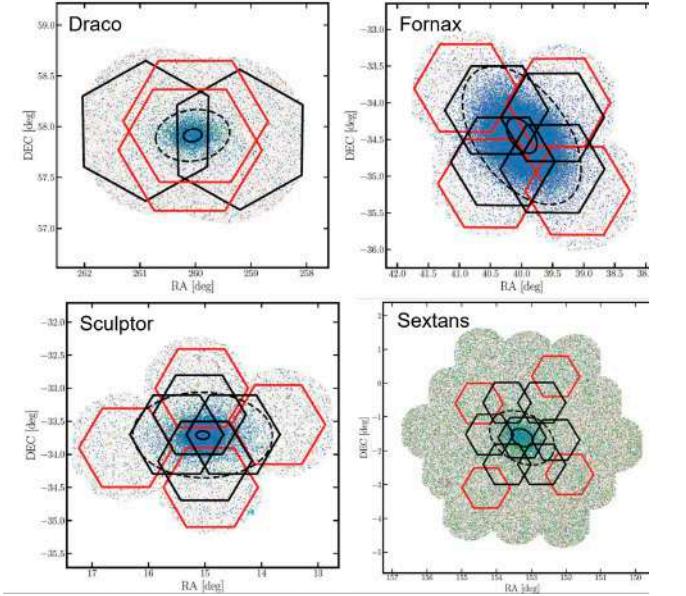
# Observational implications and constraints



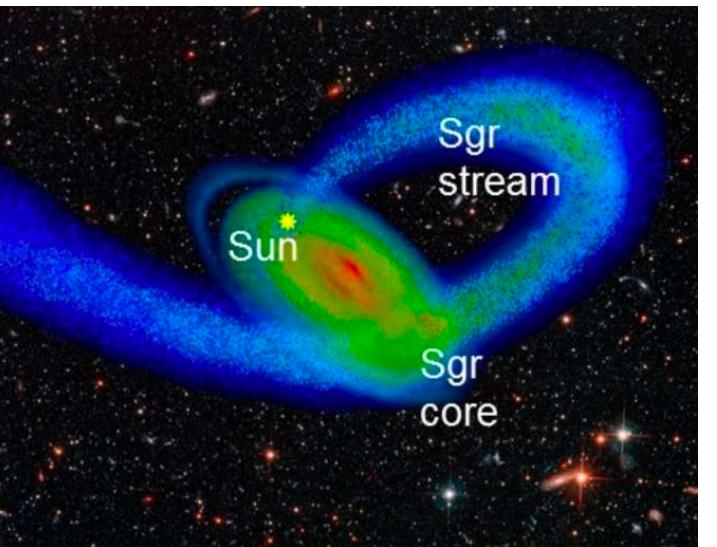
## Galaxies



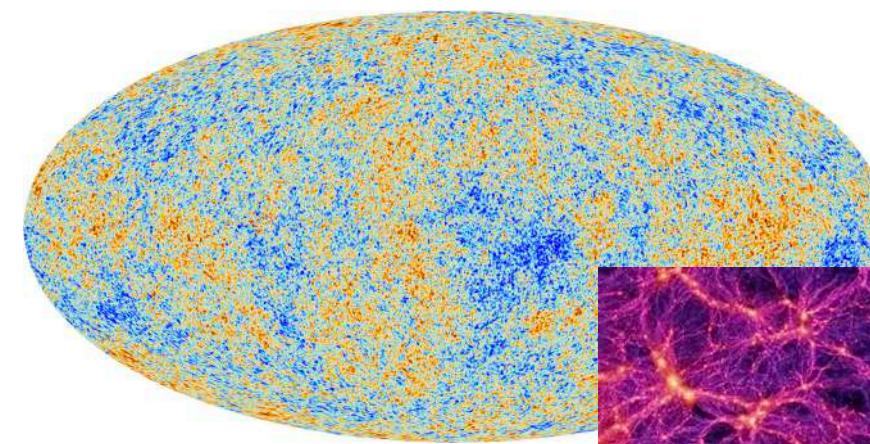
Dwarfs



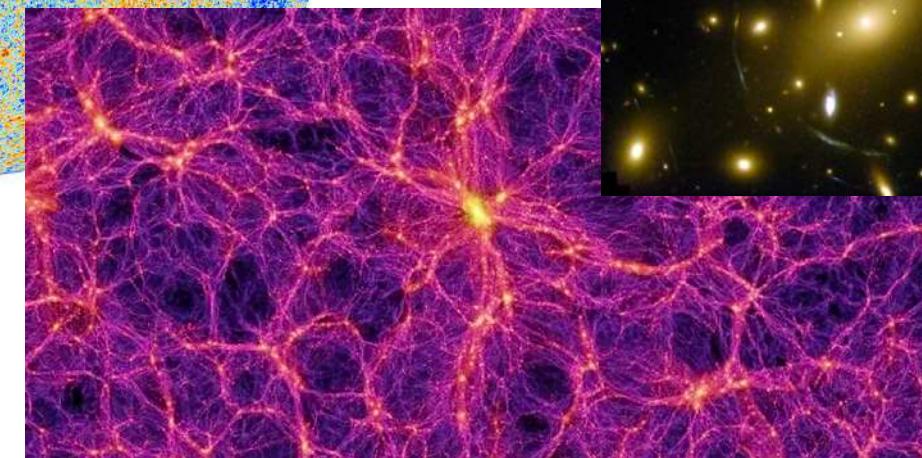
Stellar stream



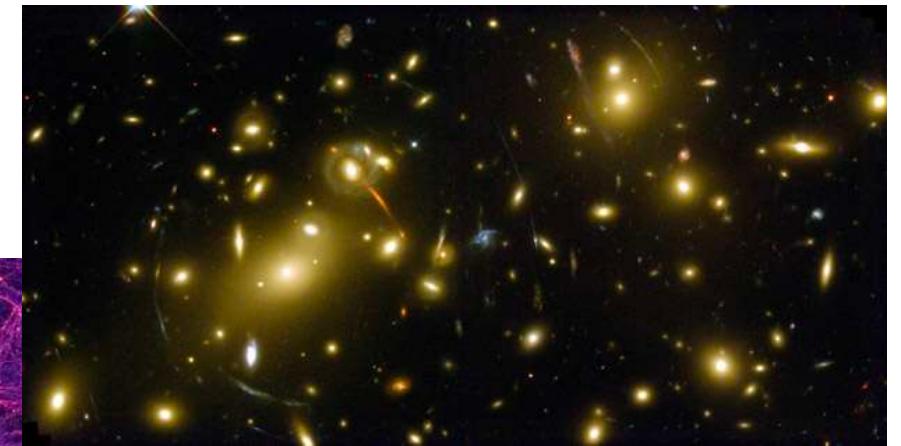
## CMB+LSS



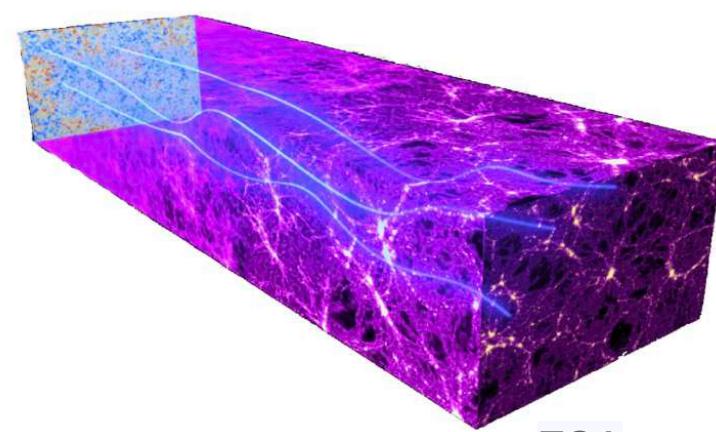
ESA and the Planck Collaboration



Springel & others / Virgo Consortium

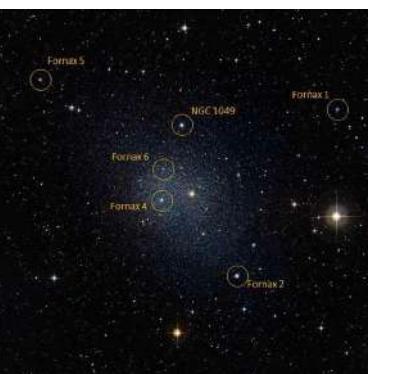


NASA and ESA

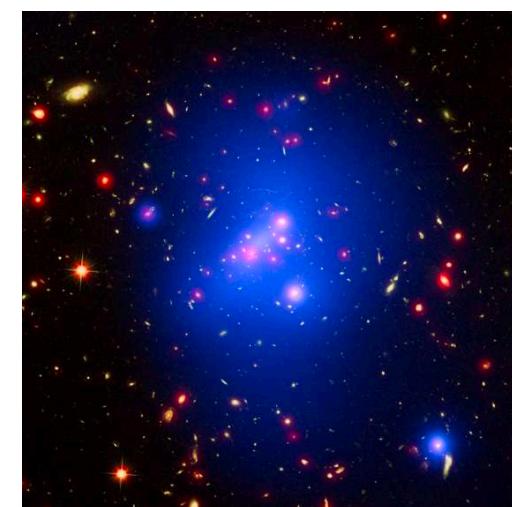


ESA

## Globular clusters



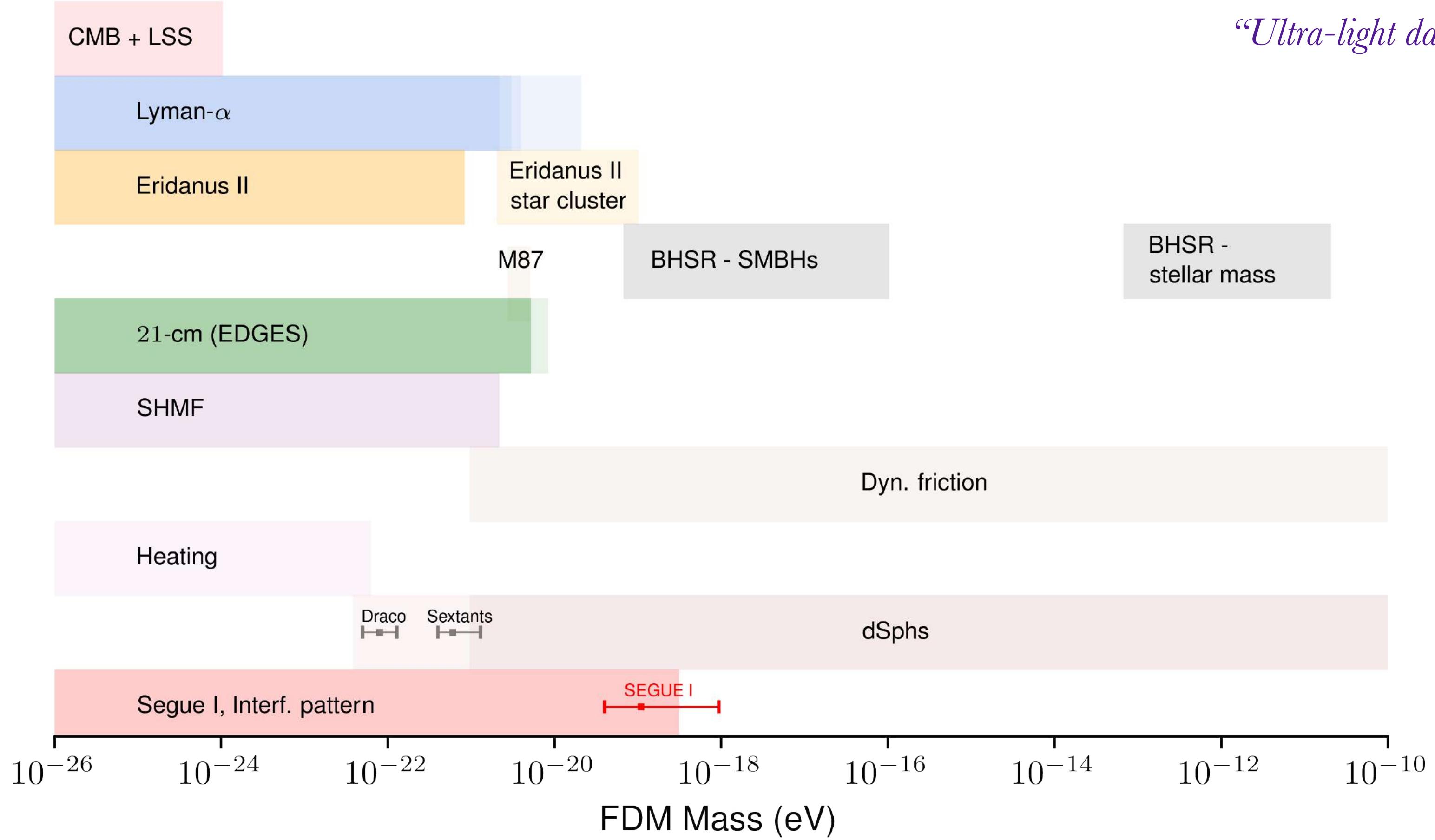
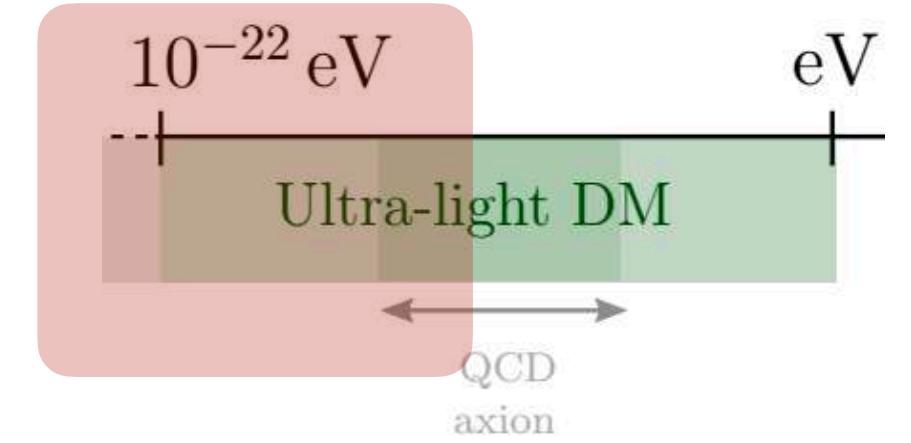
## Clusters

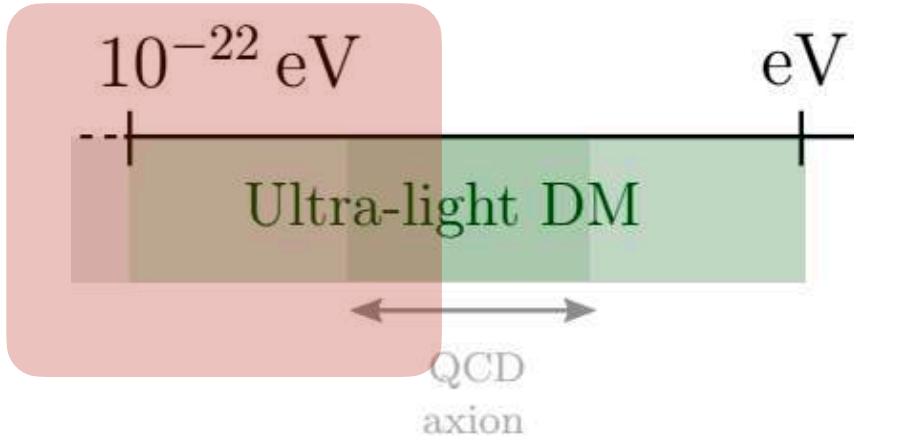


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# *Observational implications and constraints*

## *Fuzzy Dark Matter - bounds on the mass*



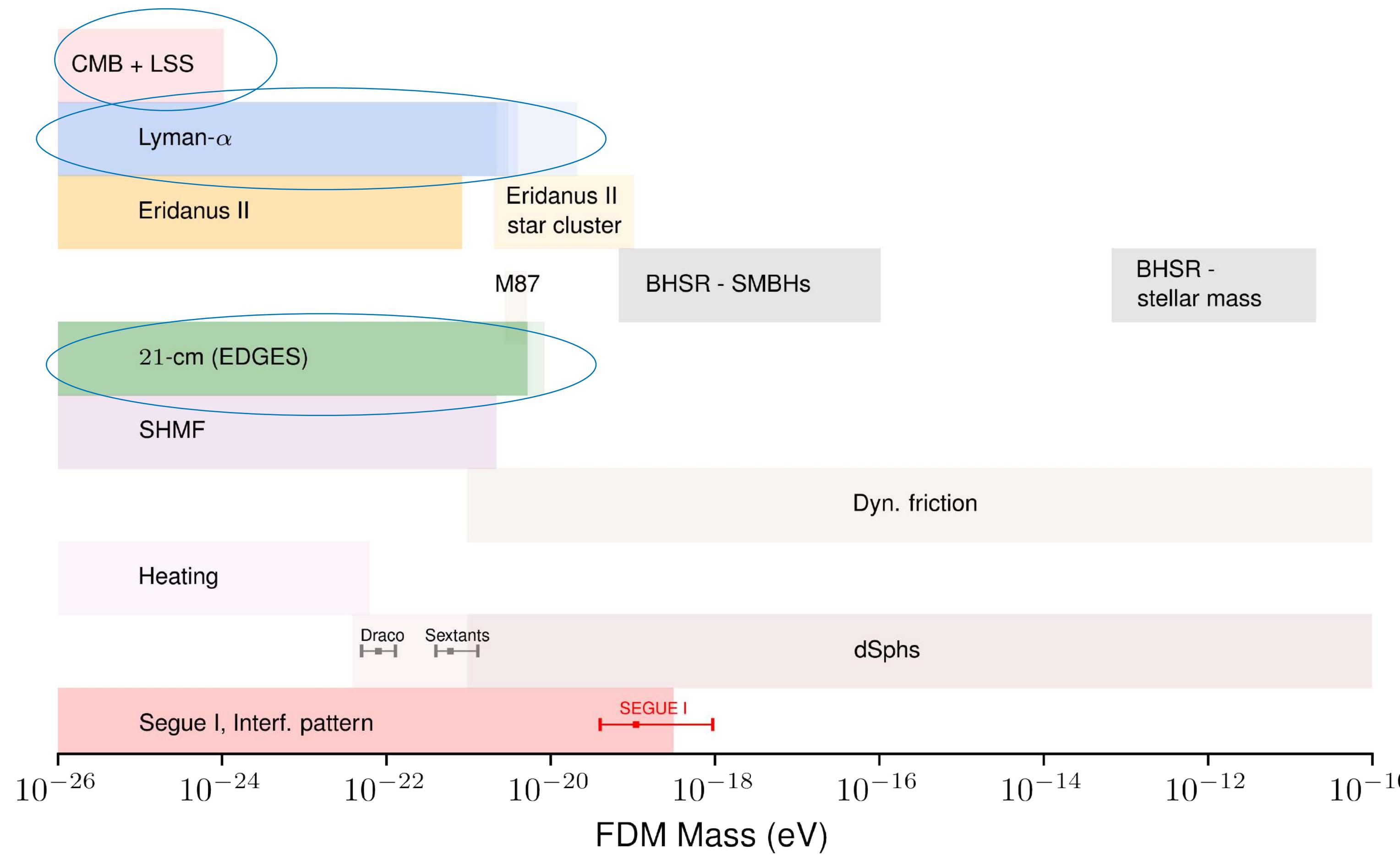
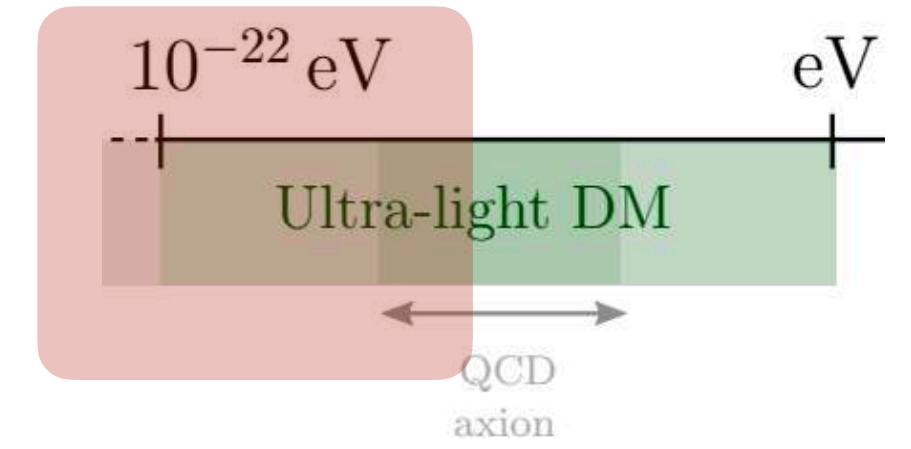


*Observational implications and constraints*

Suppression of small scale structures

# *Observational implications and constraints*

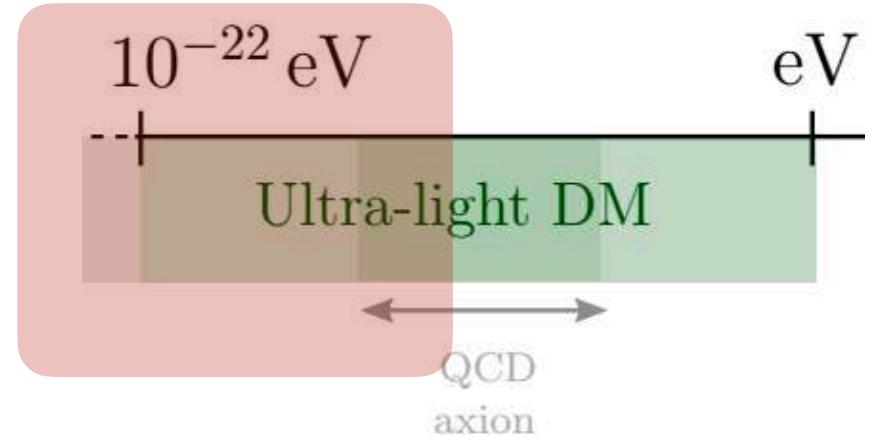
## *Fuzzy Dark Matter - bounds on the mass*



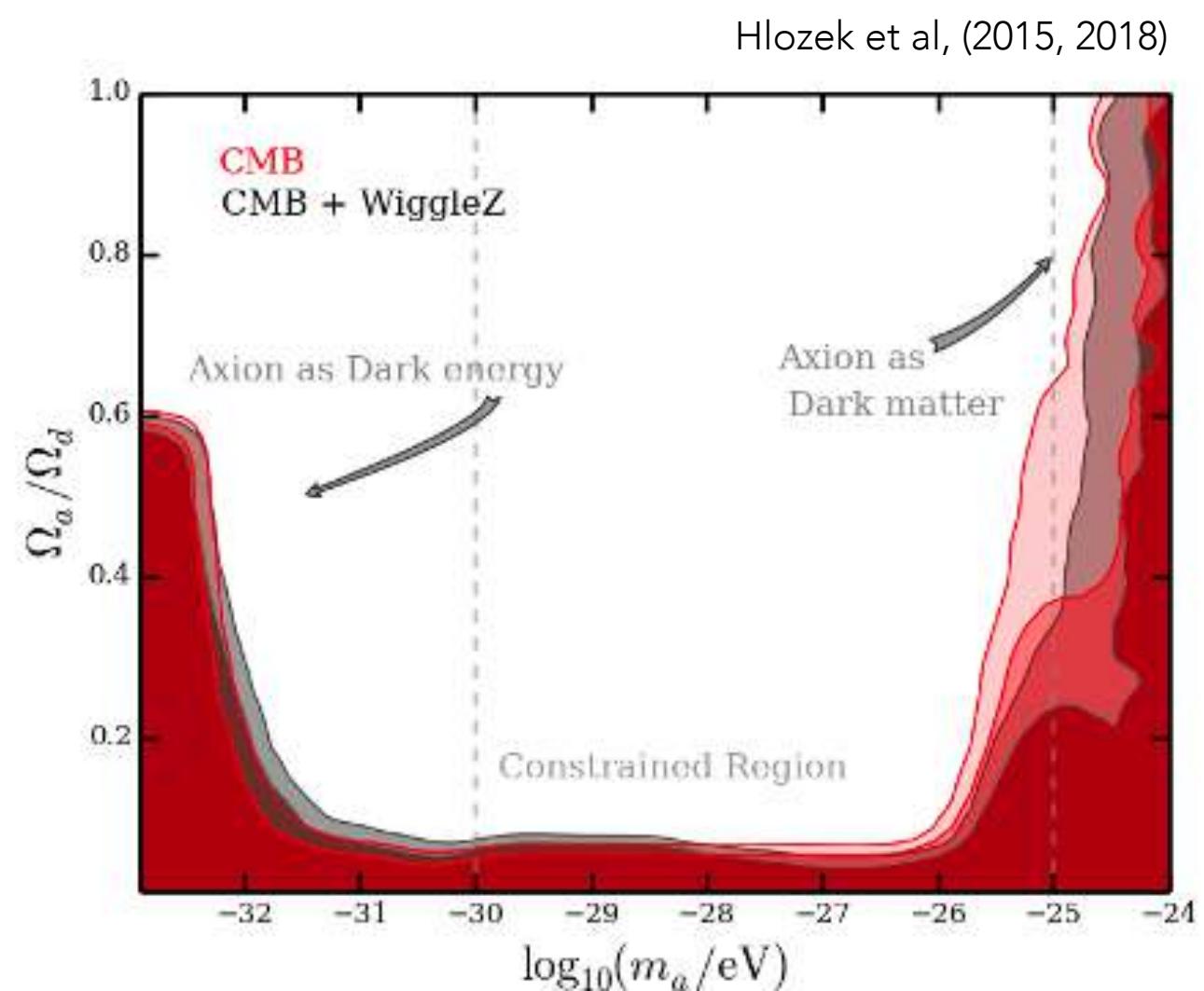
# Observational implications and constraints

## Fuzzy Dark Matter - bounds on the mass

Suppression of small structures

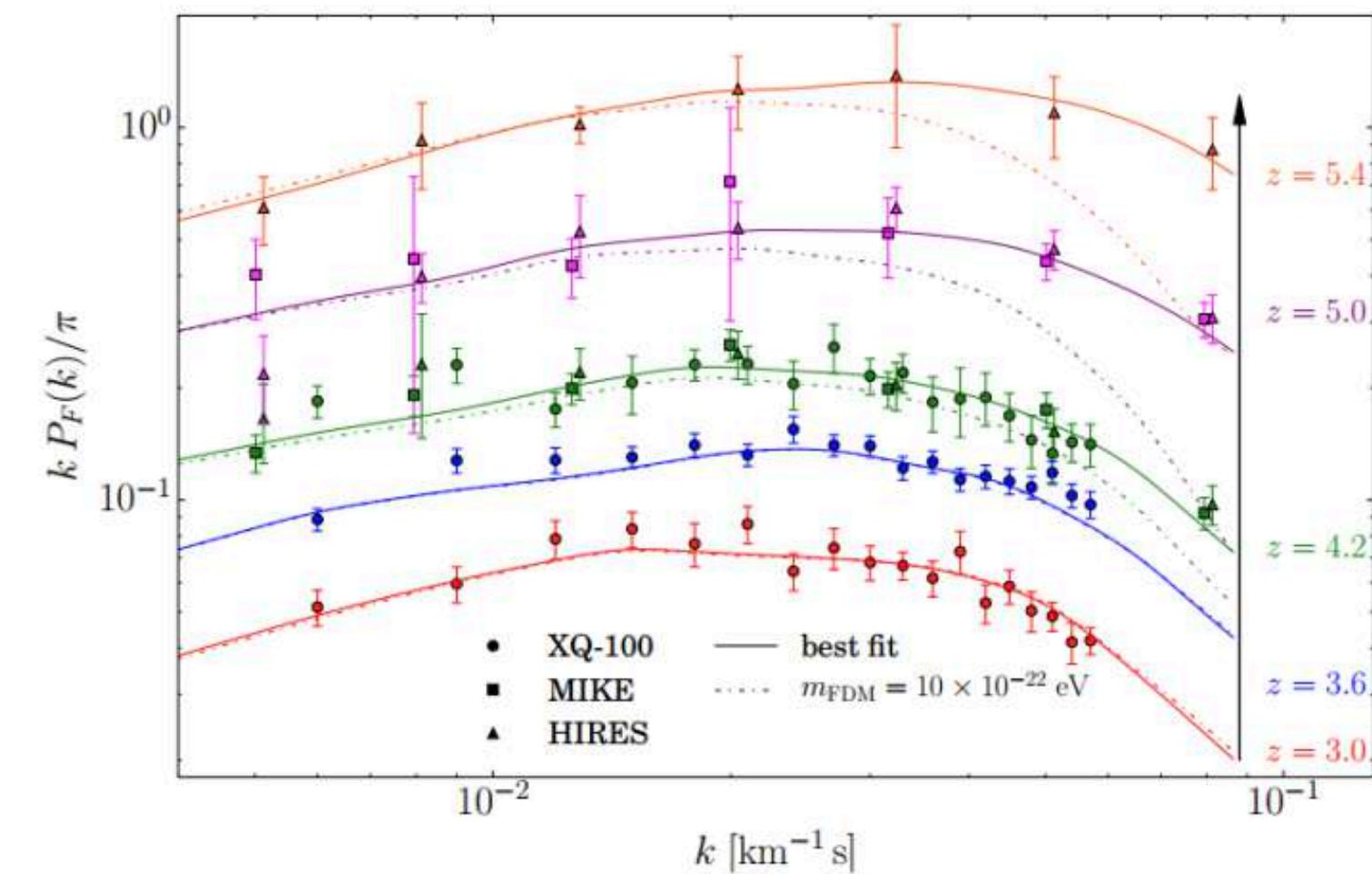


CMB/LSS



$$m \gtrsim 10^{-24} \text{ eV}$$

Lyman alpha

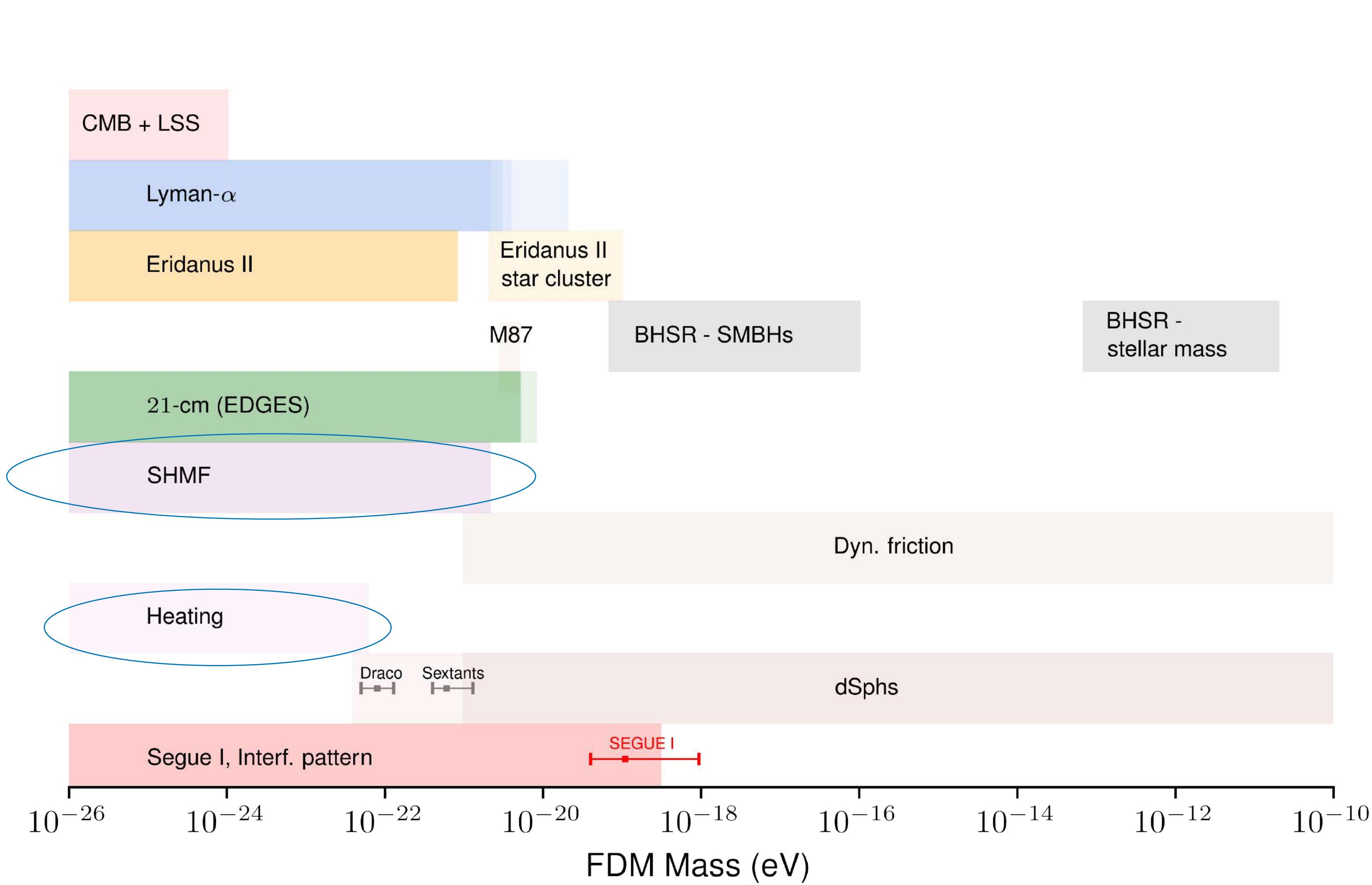


$$m \gtrsim 2 \times 10^{-20} \text{ eV}$$

so enough Mpc-scale power in Ly-a forest at  $z = 5$ .

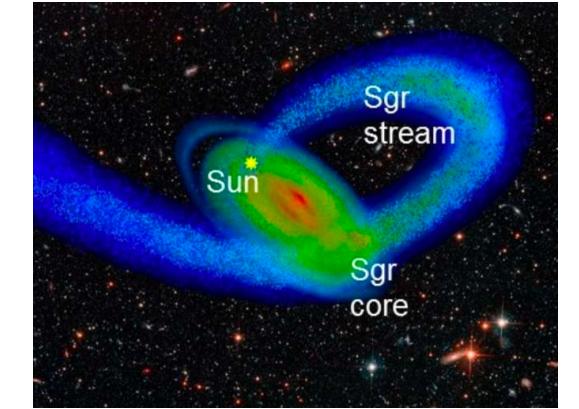
# *Observational implications and constraints*

## *Fuzzy Dark Matter - bounds on the mass*



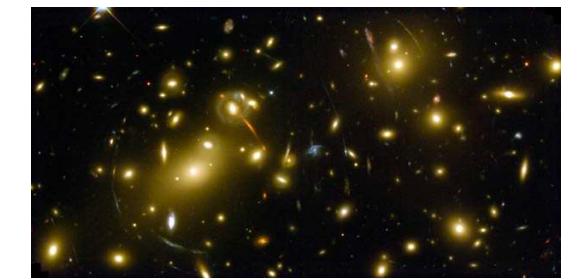
Suppression of small structures

Stellar streams



Schutz 2020: bound in the FDM SHMF using stellar streams and grav. lensing

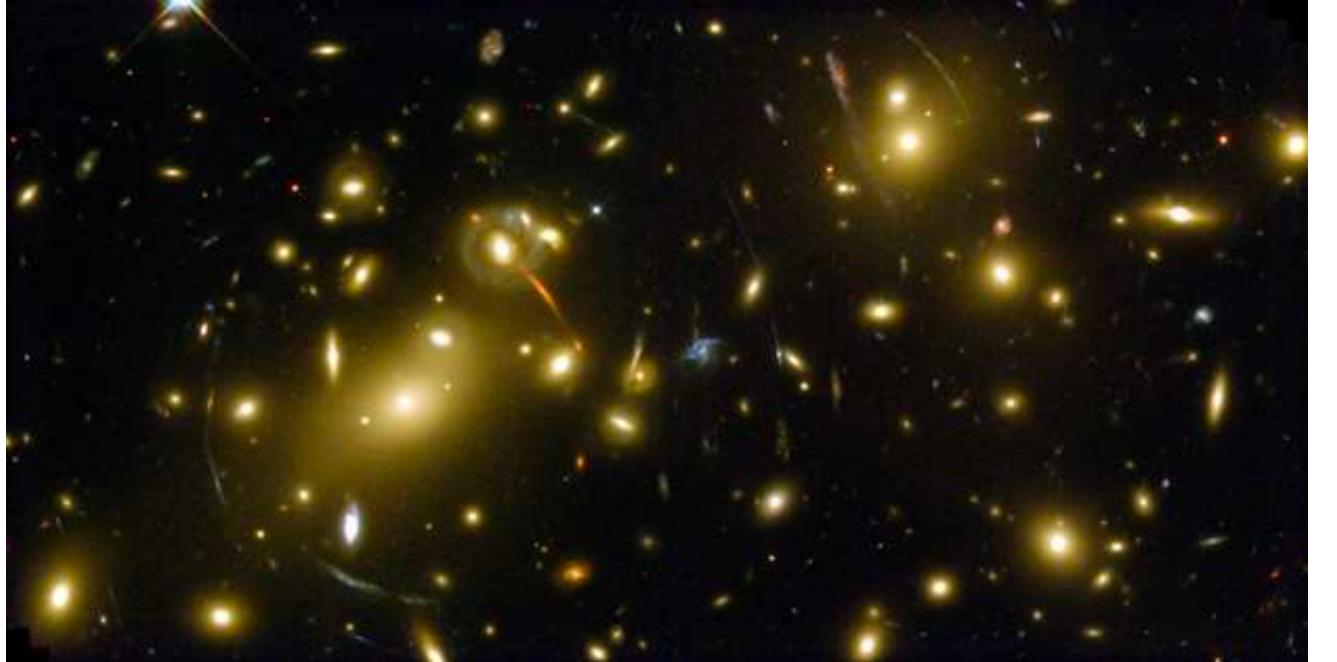
Grav. lensing



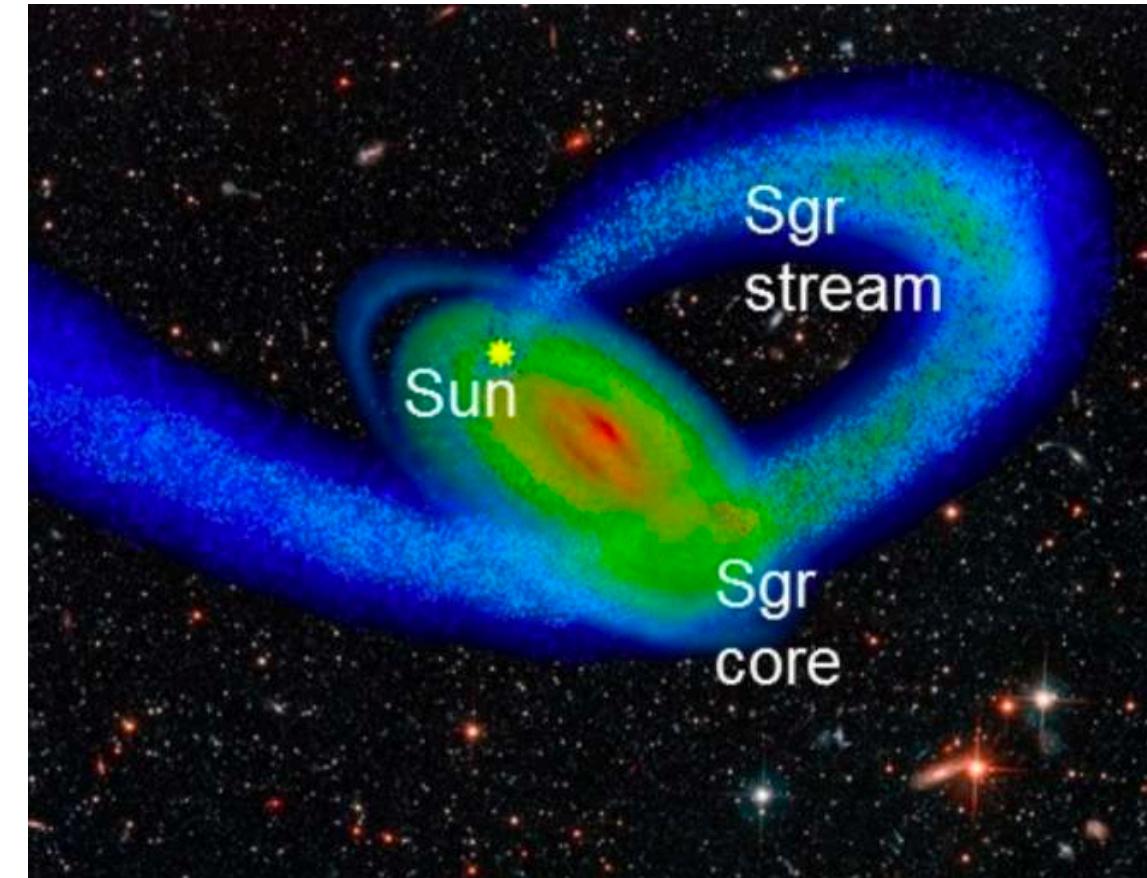
# *Gravitational probes*

Probes that are sensitive to changes in the gravitational potential - total mass

Gravitational lensing

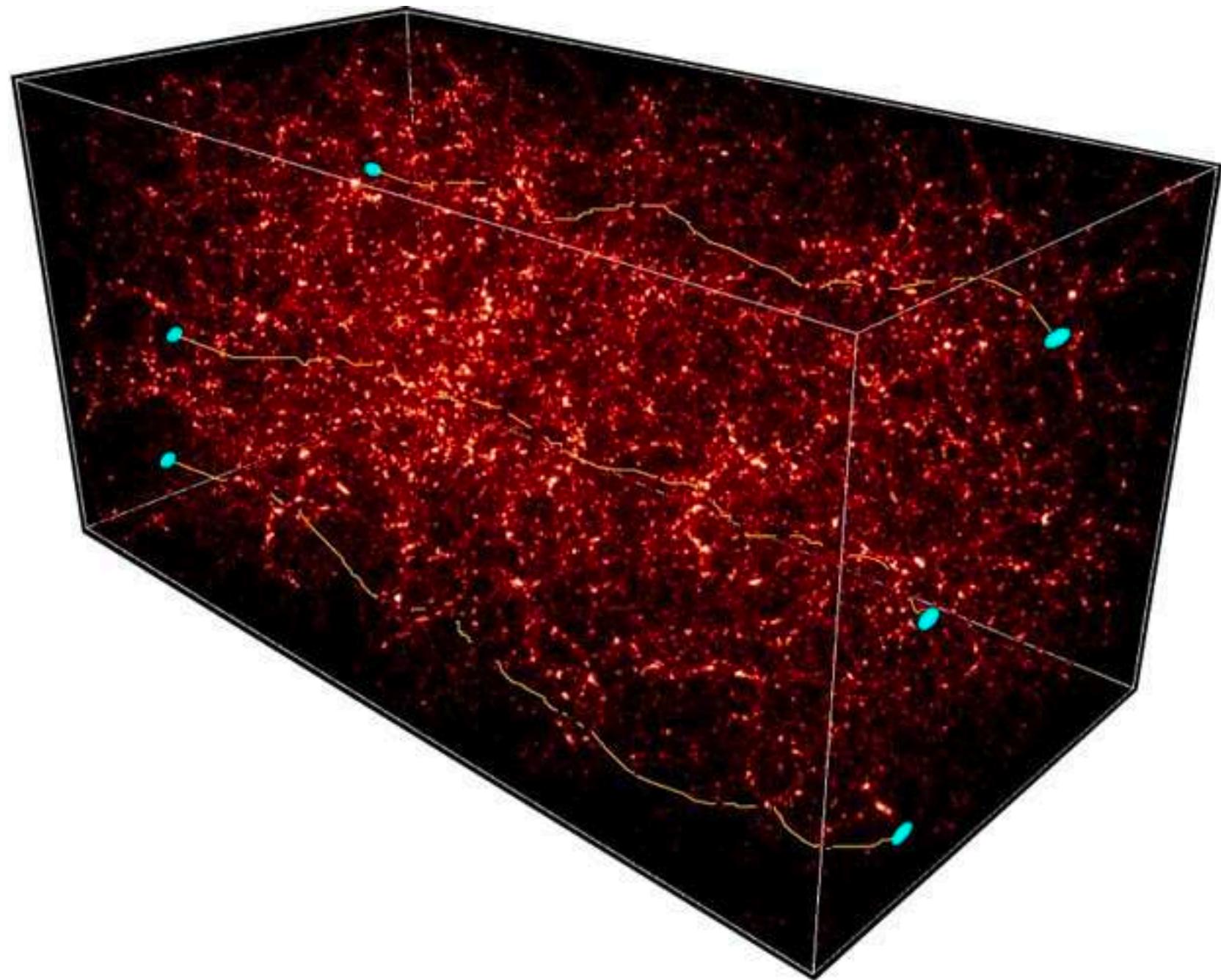


Stellar stream



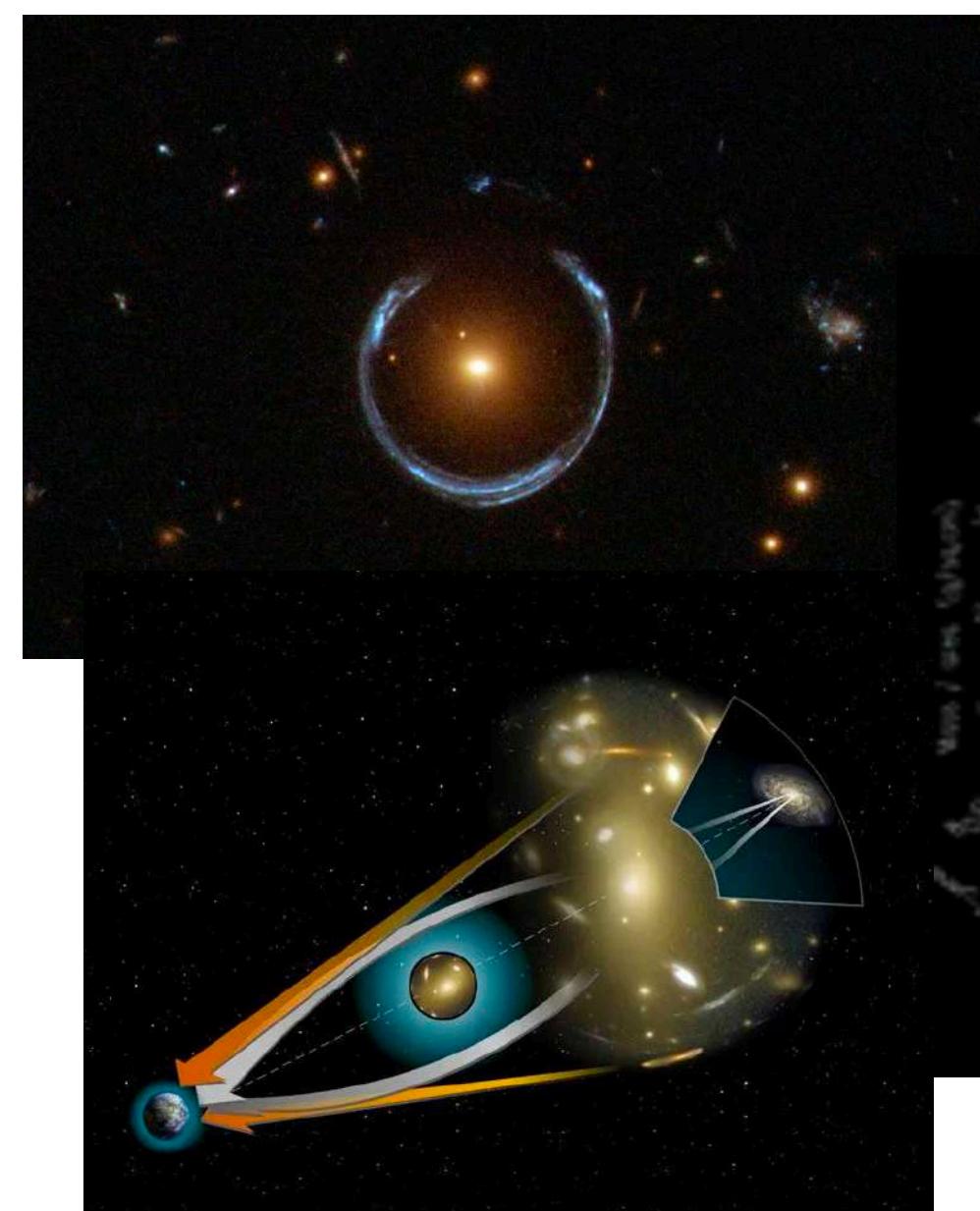
# *Gravitational lensing*

## Weak lensing



- Background image is perturbed by matter in its path
- Statistical signal
- Total mass on large scales

Image: © ESA/Hubble/NASA



Tyson, Kochanski & Dell'Antonio

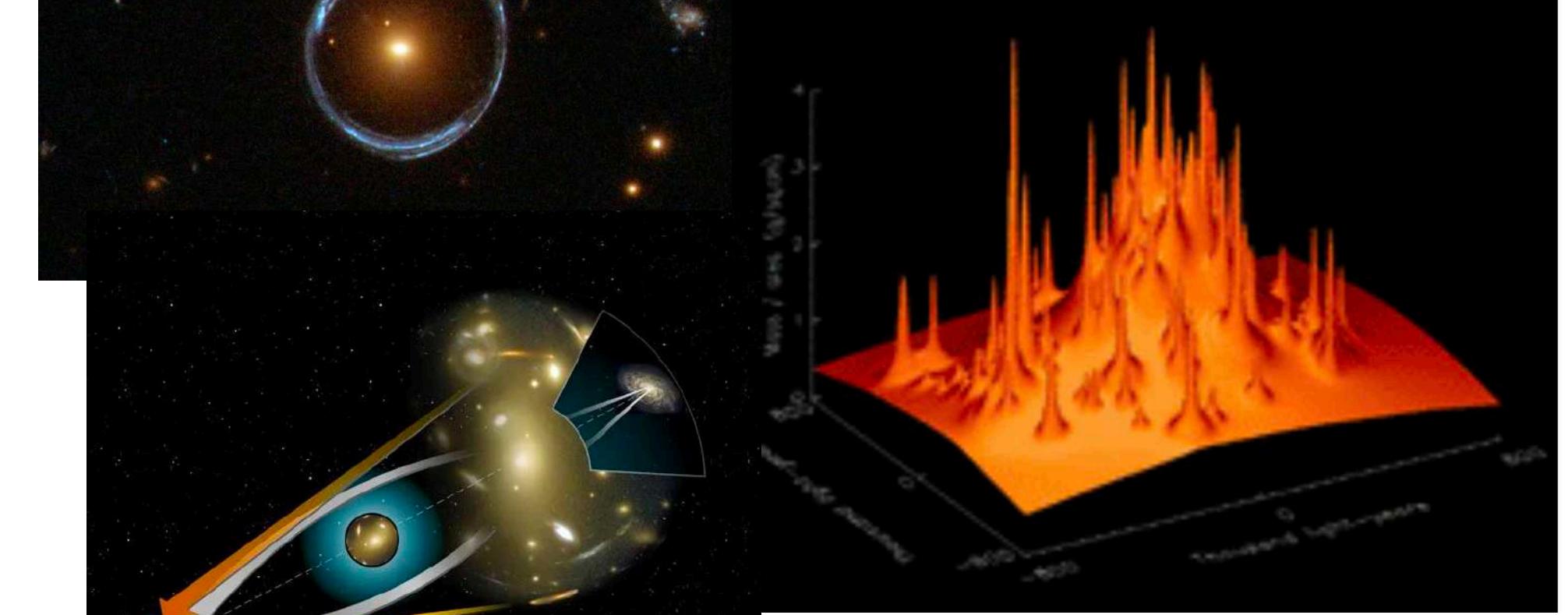


Image: © ESA/Hubble/NASA

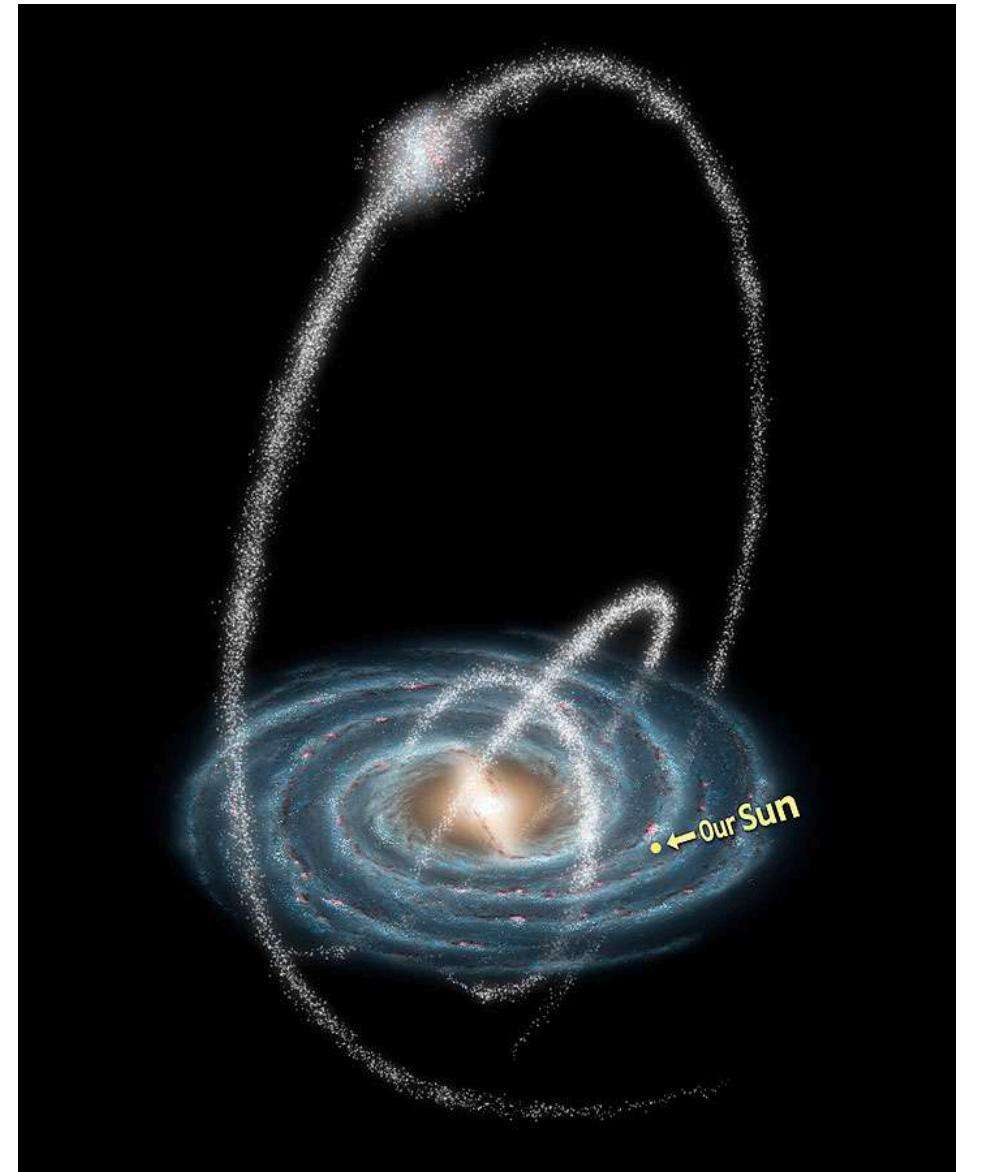
- Reconstruct the gravitational potential (total enclosed mass) of the lens
- Total mass on small scales

# *Stellar streams*

Stellar streams are a stream of stars orbiting a galaxy which are remnants of a tidally disrupted globular clusters or dwarf galaxies, that was torn apart by a more massive system.

These streams are usually thin and very long, extending to dozens of kpc across the 3-dimensions of the halo, and wrap around the disrupting galaxy.

Streams are good dynamical probes since they are initially cold and very sensitive to the **gravitational potential**

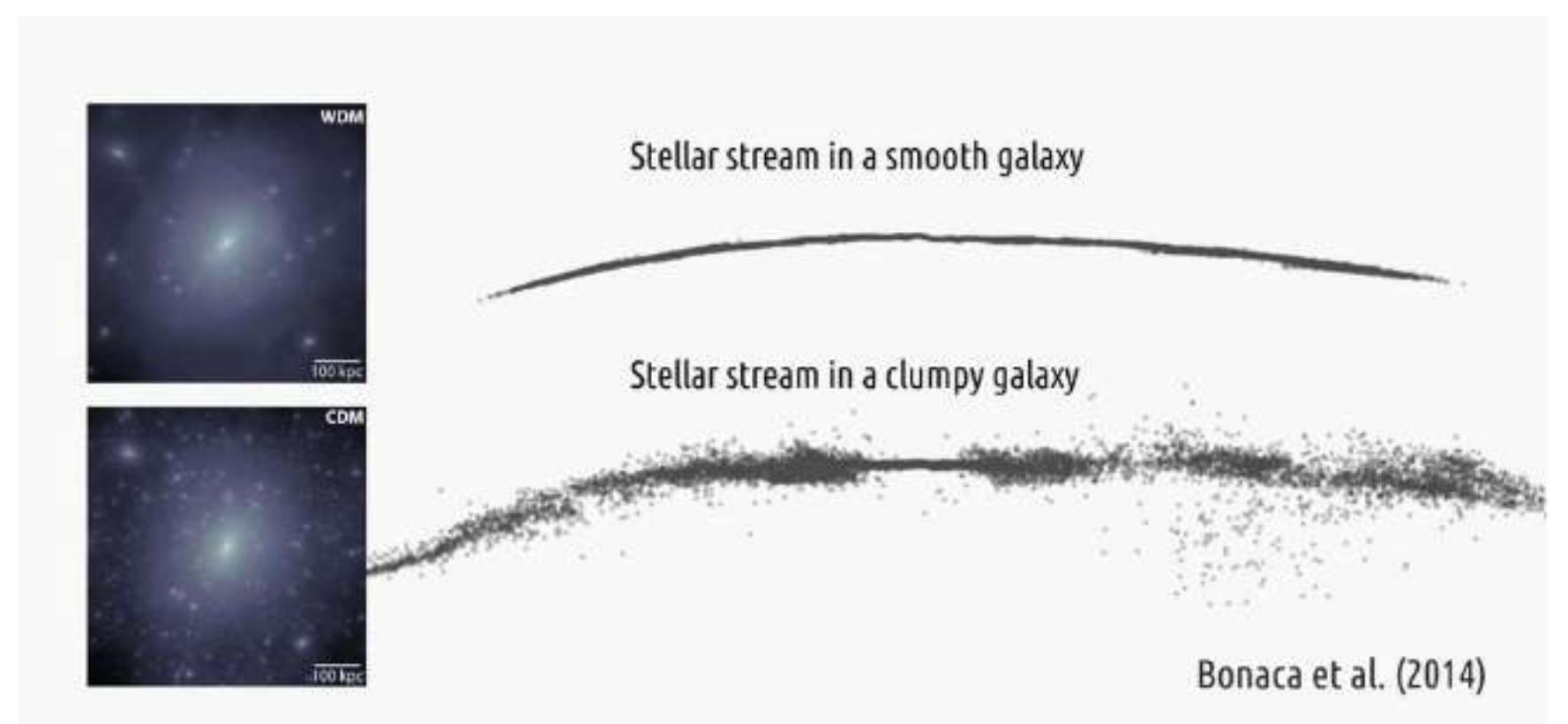


NASA

→ Substructures present in the halo when encounter the streams disturb it causing:

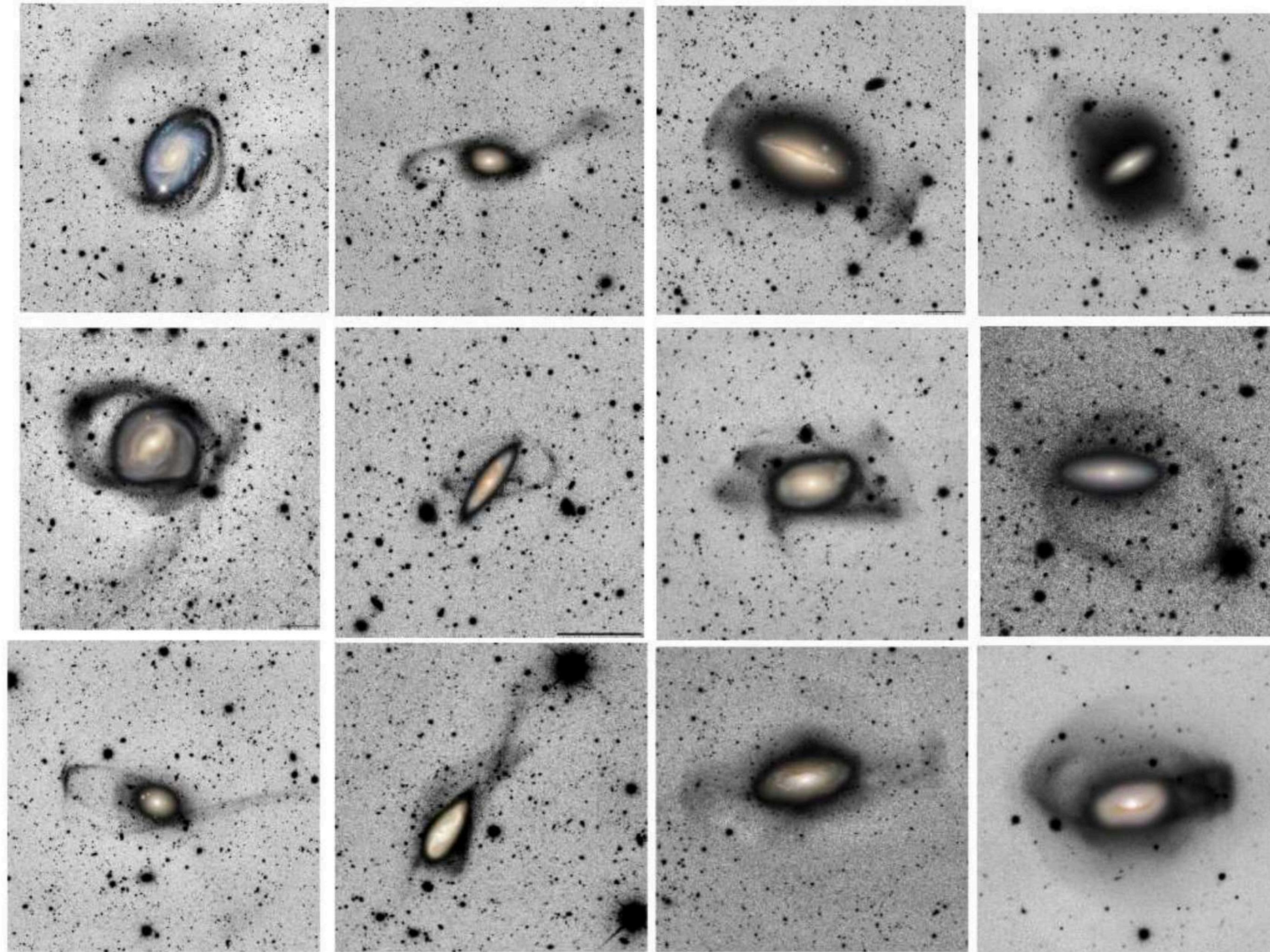
- Dynamical heating: which are changes in the velocities in the stream,
- Disturbances in the morphology of the stream: formation of gaps which are underdensities caused by the sub-halos encountered

It is estimated that we can observe gaps in streams cause by substructures with mass as low as  $M \sim 10^5 - 10^6 M_{\odot}$  (Erkal and Belokurov 2015; Erkal et al. 2016; Bovy et al. 2017)



Bonaca et al. (2014)

# *Stellar streams*



*Practice session:*

Suppression of small scale structures

# S8 tension

Ref.: K. Rogers et al 2023

Changes in the small scale paradigm can change the behaviour of DM in many scales, including cosmology

Ex.: Fuzzy DM

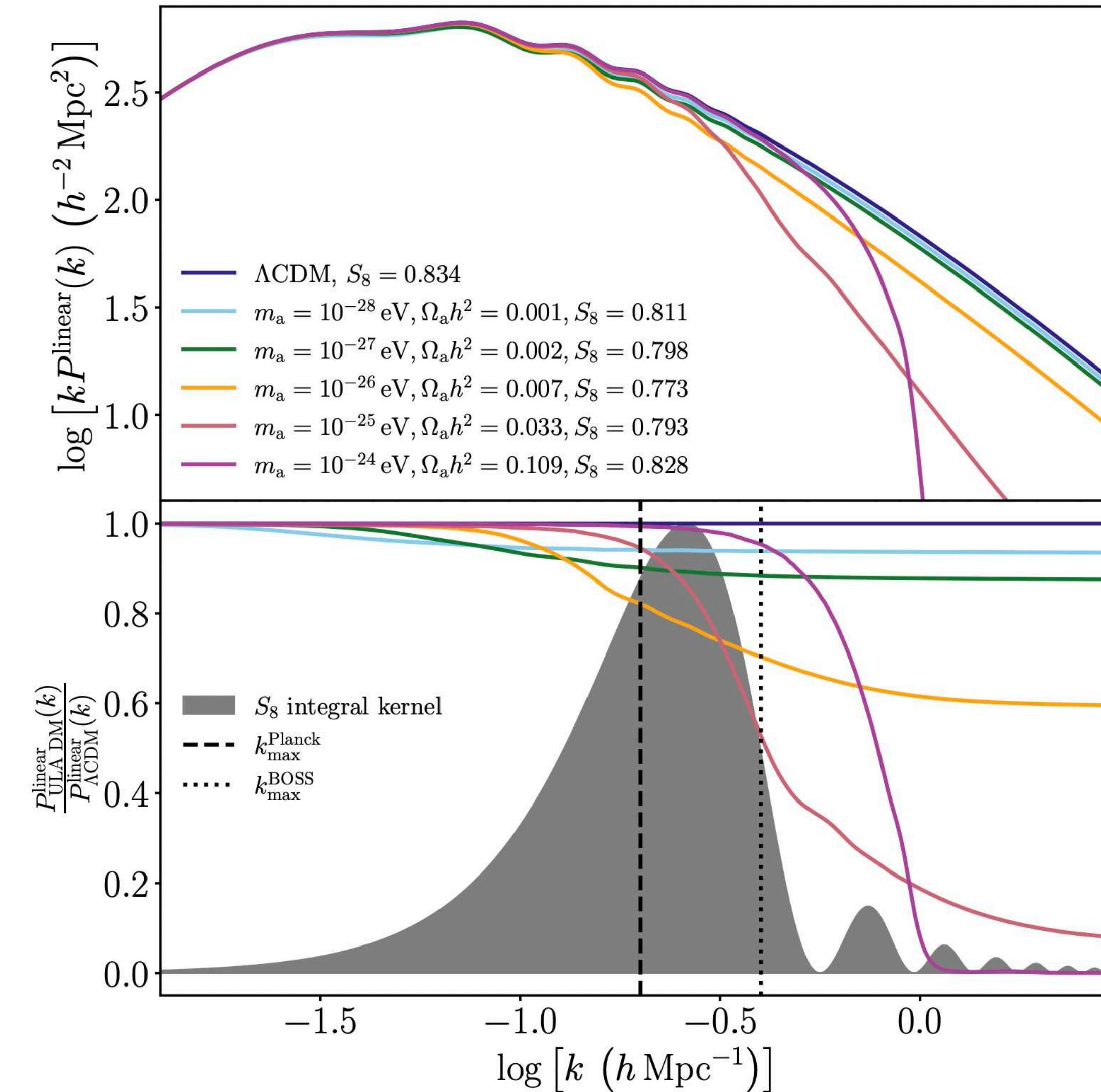
$$\sigma_8 = \int d\ln k \frac{k^3}{2\pi} W^2(k) P^{\text{linear}}(k)$$

$$S_8 = \sqrt{\frac{\Omega_m}{0.3}} \sigma_8$$

The presence of ULAs can significantly lowers S8 for:

$$m_a \in [10^{-27}, 10^{-25}] \text{ eV}$$

S8 is lowered because the Jeans scale today for  $m_a = 10^{-25} - 10^{-26}$  eV is about  $\lambda_J = 4 - 12 h^{-1} \text{ Mpc}$



# *S<sub>8</sub> tension*

Ref.: K. Rogers et al 2023

Ex.: Fuzzy DM

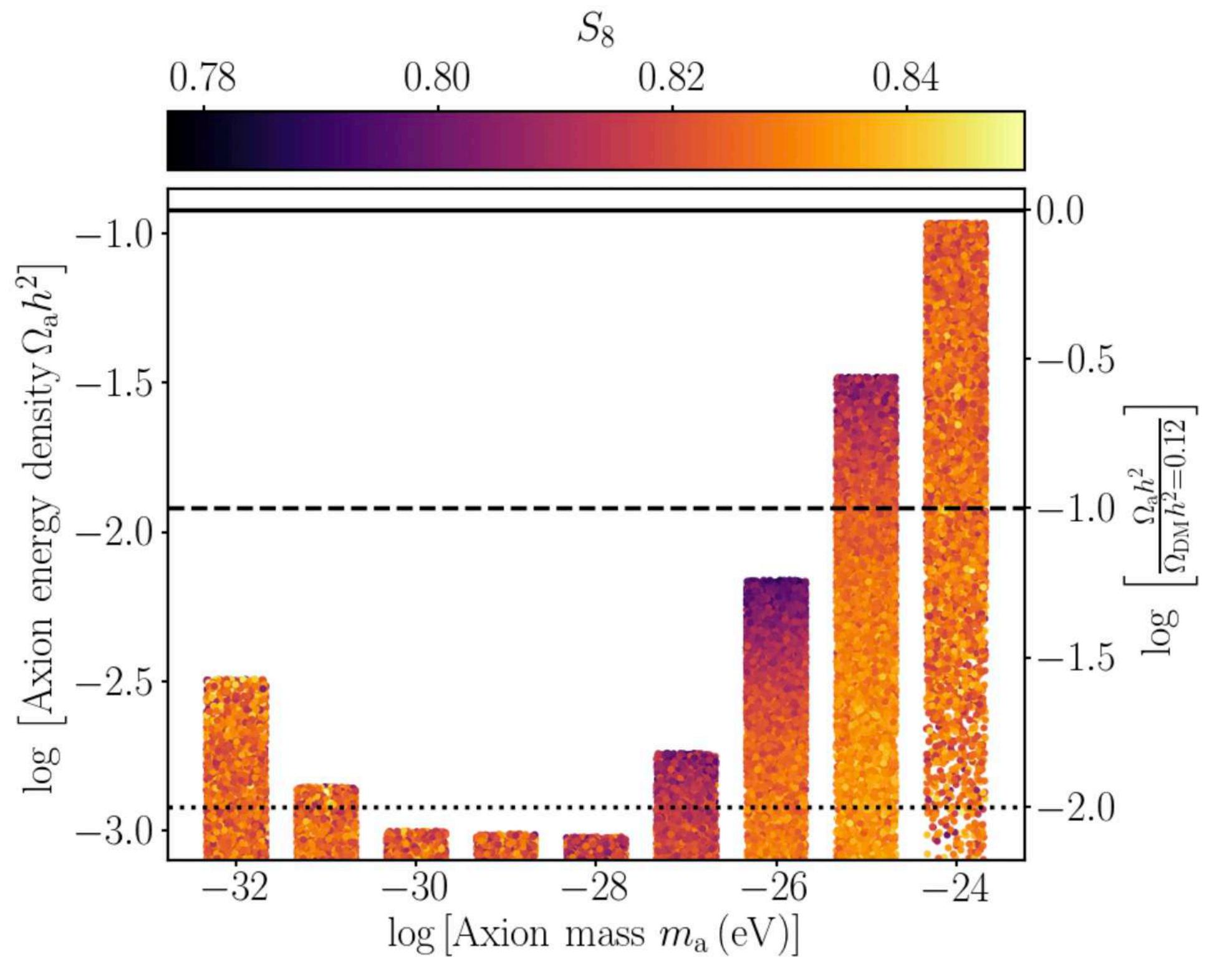
Planck + BOSS

The presence of ULAs with mass

$$10^{-28} \text{ eV} \leq m_a \leq 10^{-25} \text{ eV}$$

can improve consistency between CMB and galaxy clustering  
(reduce the S<sub>8</sub> discrepancy)

from  $2.6\sigma$  to  $1.7\sigma$



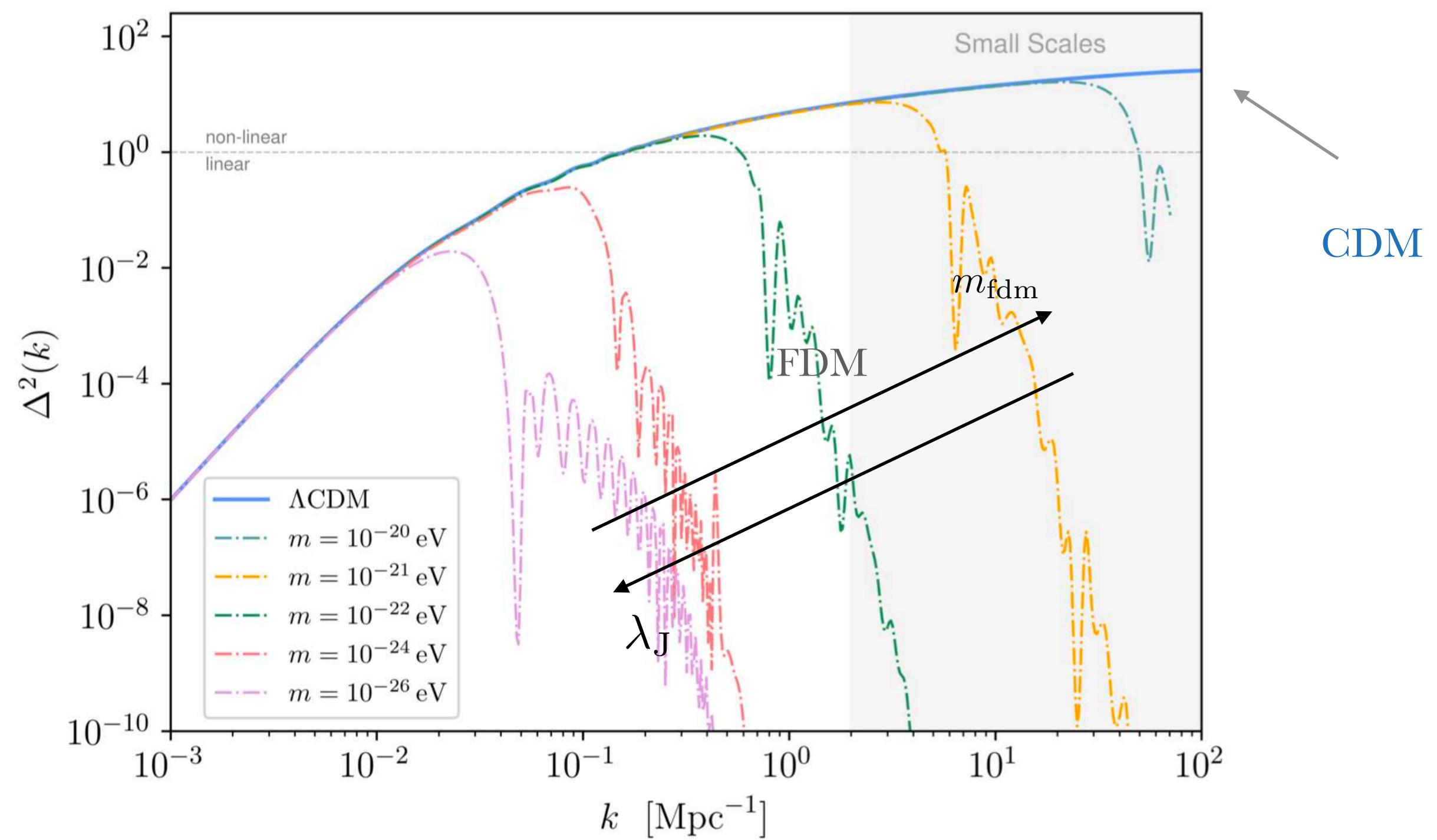
Ex.:

- H<sub>0</sub> tension: Early dark energy - axion-like particle
- Model address H<sub>0</sub> and S<sub>8</sub> tensions: “Chameleon EDE”, Karwal et al 2021

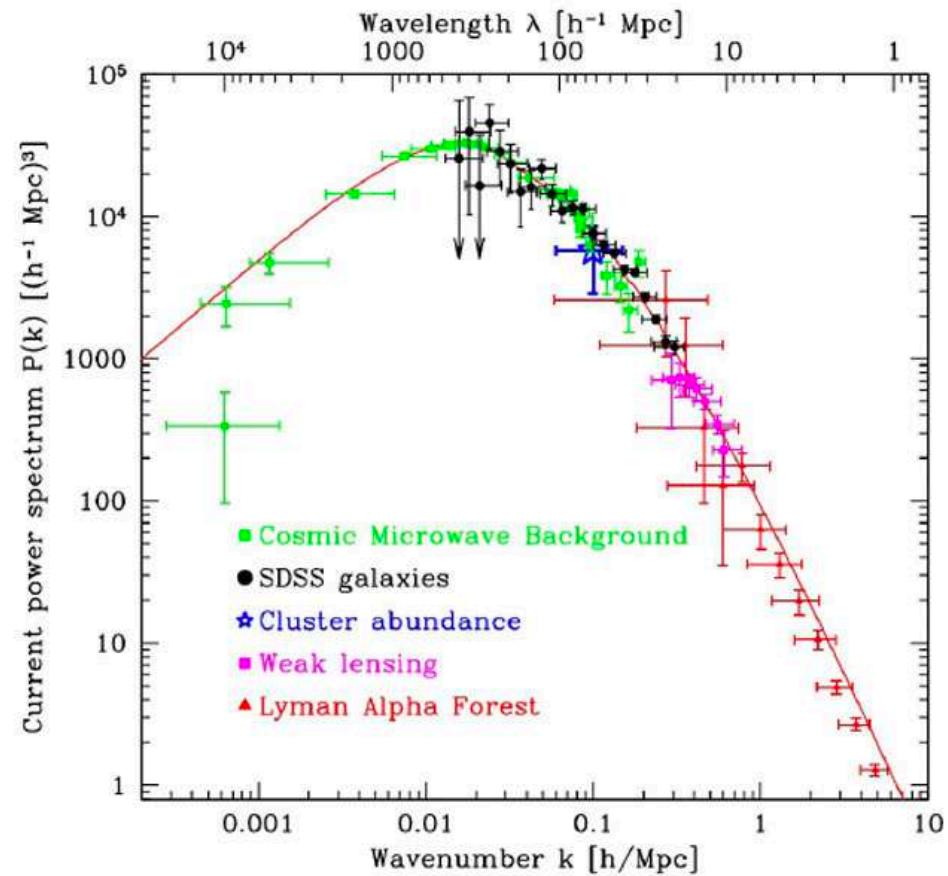
# Suppression of small scale structure

Can we probe the small scales?

## POWER SPECTRUM



Power spectrum: highly constrained for  $k > 10 \text{ Mpc}^{-1}$   
highly unconstrained for  $k < 10 \text{ Mpc}^{-1}$



Measure PS well until scales  
 $k \sim 10 - 20 \text{ Mpc}^{-1}$

Can we measure the power spectrum on scales  
 $k < 10 \text{ Mpc}^{-1}$ ?

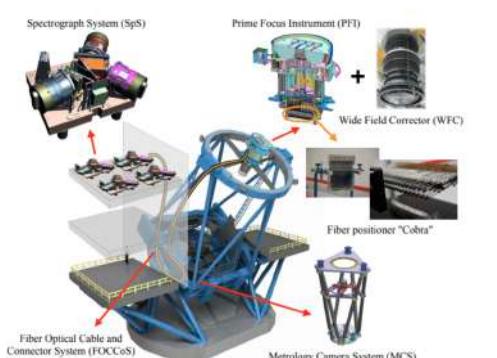
# Future - signals in cosmology

## Observations

### Photometric and spectroscopic surveys

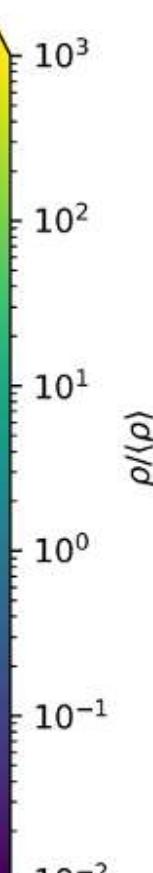
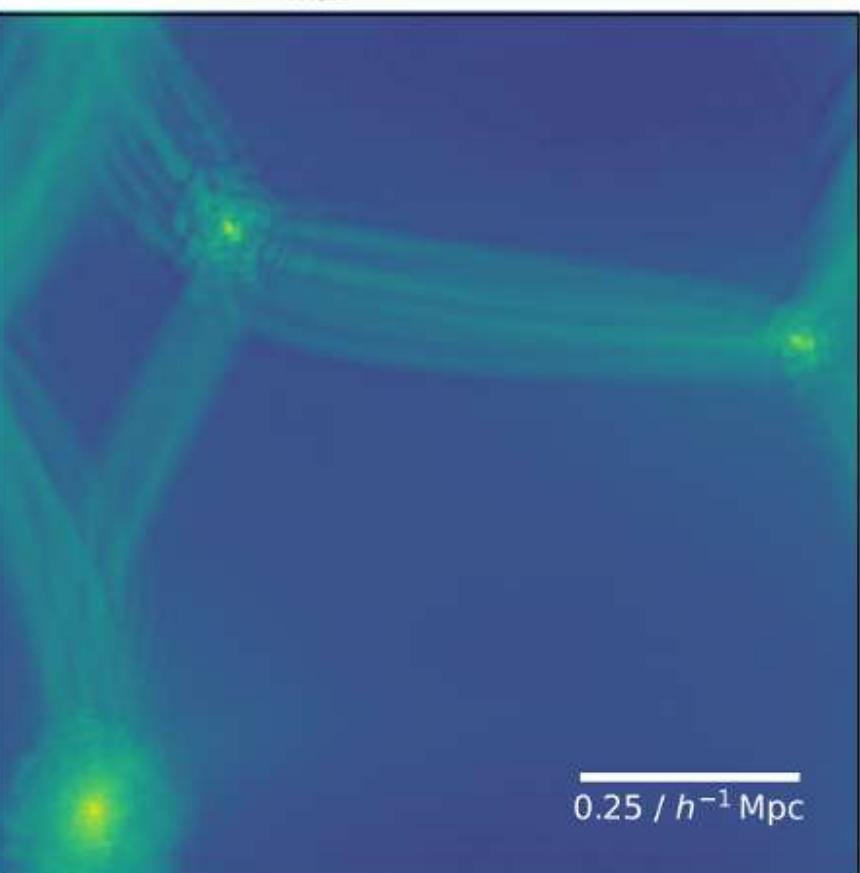


Prime Focus Spectrograph (PFS)



## Simulations

FDM:  $256^3$ ,  $mc^2 = 1.75 \times 10^{-23}$  eV,  $z = 0.00$   
 $v_{\max} = 88.1$  km/s



## CMB



## 21cm



## New probes

### Substructures

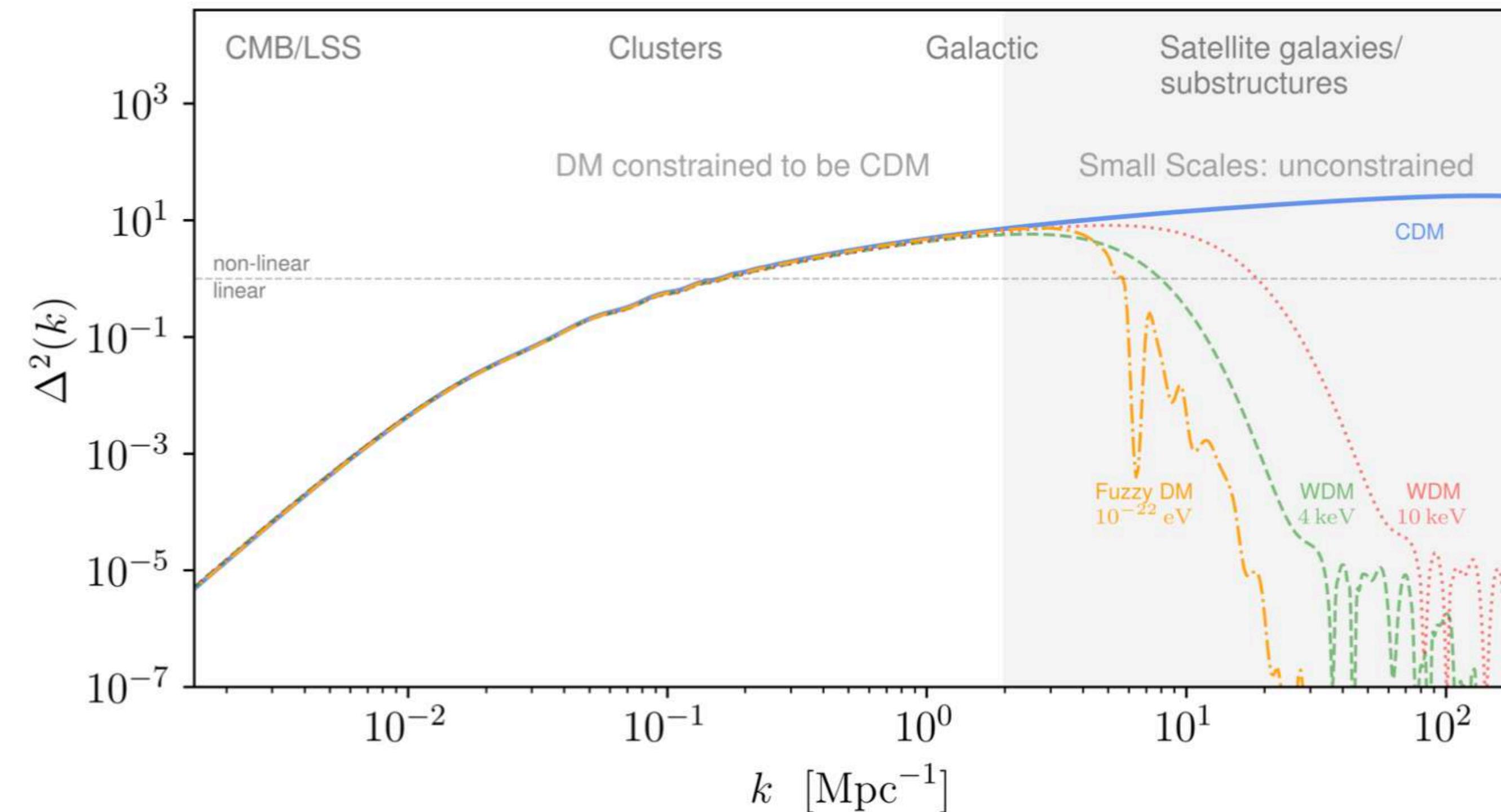
- strong lensing
- stellar streams

Small scale information from PS  
- substructure convergence PS

**CMB-S4**  
Next Generation CMB Experiment

# *Sub-galactic power spectrum*

Using gravitational probes, **strong lensing** and **stellar streams**, to describe substructures



# *Sub-galactic power spectrum*

Using gravitational probes, **strong lensing** and **stellar streams**, to describe substructures

A. Diaz Rivero, et al. (2017); Diaz Rivero, et al. , (2018)

## Substructure convergence power spectrum

Develop a formalism to compute the substructure convergence power spectrum for different populations of dark matter subhalos.

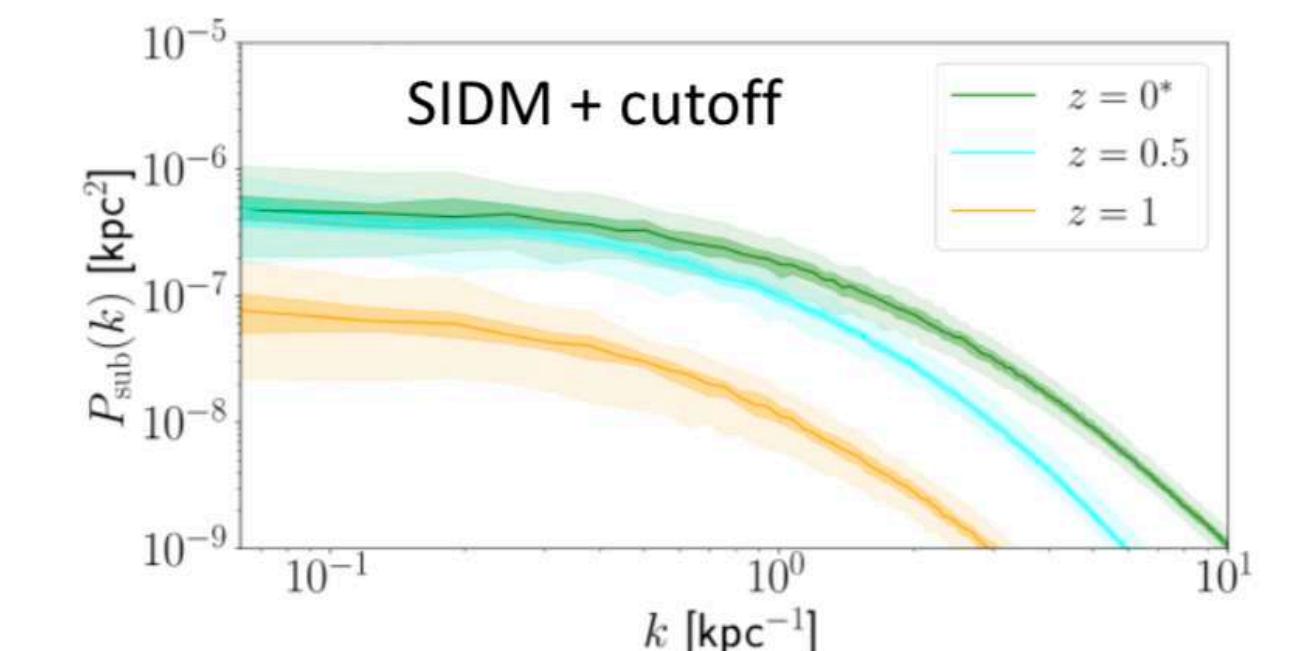
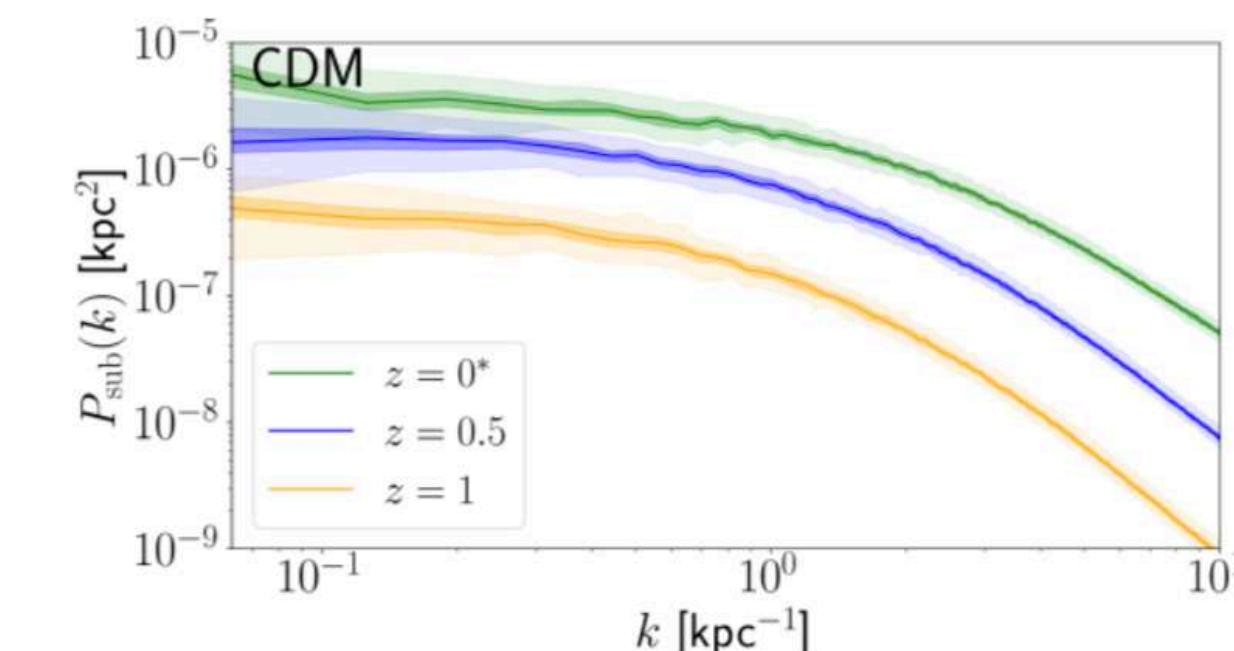
Bayer et al. (2018) ; Auger et al. 2009  
FDM: Kawai et al. (2021)

Hezaveh et al. (2016) (projected mPS by using strong lensing)

**Change of language:** instead of talking about lensing perturbations in terms of individual subhalos, look at the correlation function of the projected density field.

(based on Dvorkin's slide)

$$P_{\text{sub}}(k) = P_{1\text{sh}}(k) + P_{2\text{sh}}(k)$$



# *Sub-galactic power spectrum*

Using gravitational probes, **strong lensing** and **stellar streams**, to describe substructures

## Substructure convergence power spectrum

*Sten Delos and Fabian Schmidt (2021)*

**Stellar streams:** perturbed by passing substructure. Good gravitational probe, since given their low dynamical temperature and negligible self-interaction, it retains the memory of those encounters.

THIS WORK: Fully analytical understanding of the stream perturbations!

Power spectrum of a stream's stellar density is analytically related to that of the substructure background:

$$P_*(k, t) = \chi_* \left( k\sigma_0 t, \frac{D}{k\sigma_0^3} \right) \frac{k^2 t^2}{3} P_{\Delta v}(k, t)$$

**Stream power**      **Substructure power**

$$P_{\Delta v}(k, t) = 16\pi^4 G^2 \bar{\rho}^2 k^2 t \int_k^\infty \frac{dq}{q} \frac{\mathcal{P}(q)}{q^6} \int d^3 u \frac{f(u)}{u} \theta_H(qu - kv)$$

- Previous:
- Mostly numerical
  - Perturbations → sub-halo mass function

Relates the stellar stream perturbation to the surrounding matter distribution, from dark and luminous substructure

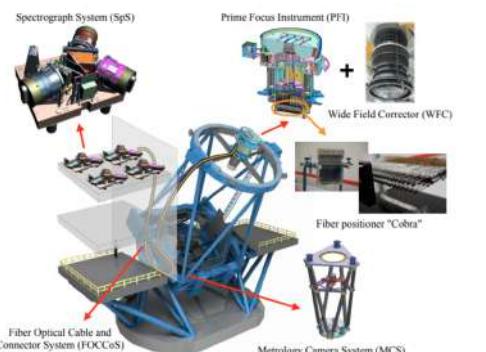
# Future - signals in cosmology

## Observations

### Photometric and spectroscopic surveys



### Prime Focus Spectrograph (PFS)



## CMB



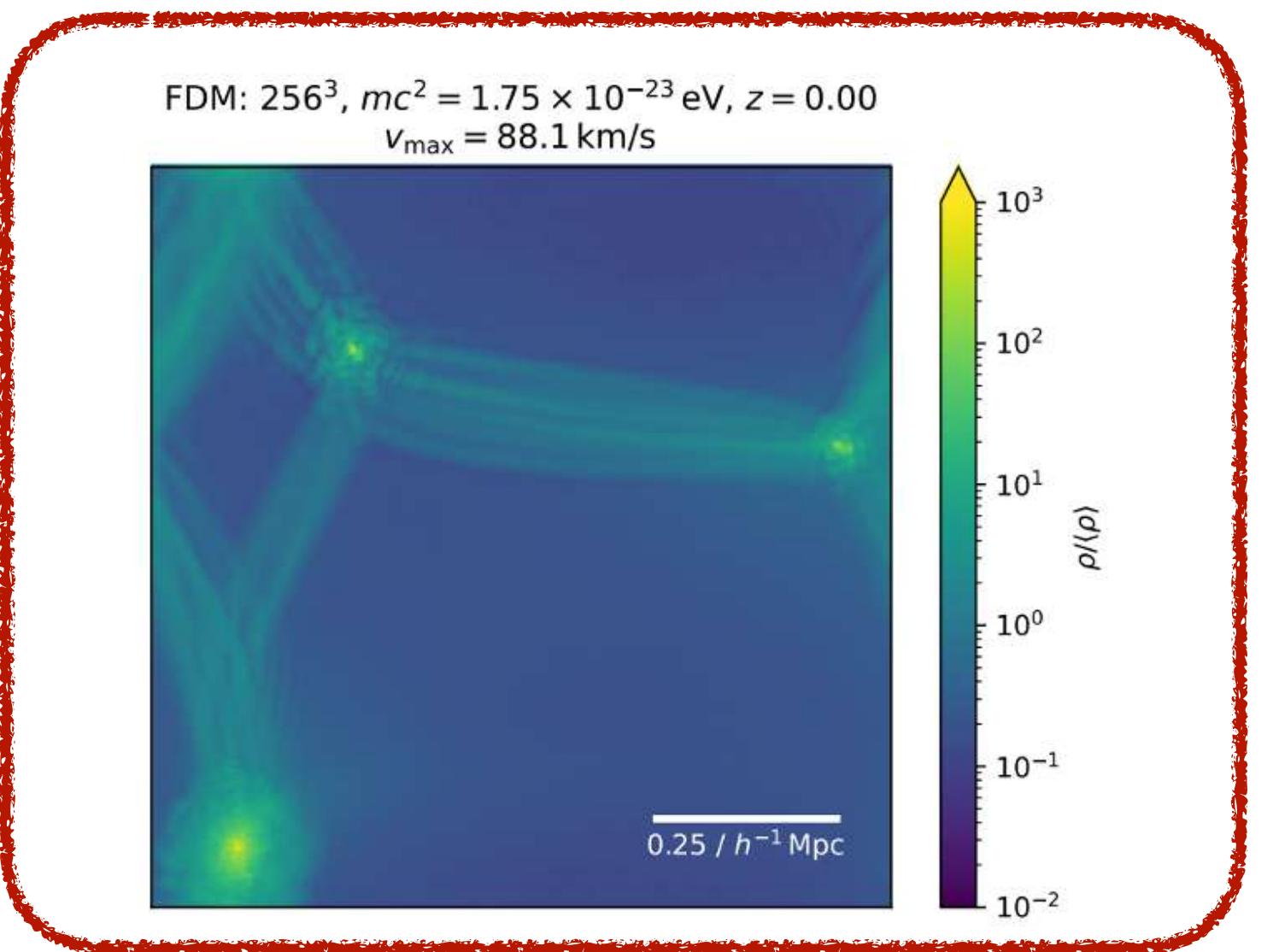
## 21cm



How to model the PS on these small scales?  
Highly non-liner scales!  
Challenge!

## Simulations

(more later)



## New probes

### Substructures

- strong lensing
- stellar streams

Small scale information from PS  
- substructure convergence PS

# *Future - signals in cosmology*

All we computed in lecture 1 was the **LINEAR** power spectrum

How to model the PS on these small scales?  
Highly non-linear scales!  
*Hard!*

We can use EFT of LSS to compute mildly non-linear scales, but smaller scales are non-linear - Need simulations!

Need to improve simulations!  
*Challenge!*  
*(more later)*

Simulations

