CONTROL AND ACTUATING DEVICES FOR MECHANICAL SYSTEMS

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ASSIGNMENT 3: CONTROL OF TRAJECTORY OF THE WORKPIECE OF A MACHINE TOOL

The system in figure 1 represents the scheme of a machine tool. The shaft of an electric DC motor (with inductance La, resistance Ra and motor constant Kψ) is connected to a ball screw (characterised by a transmission ratio τ) that move that table of the machine. On the table is fixed a workpiece. The energy dissipation in the system is modelled considering a viscous type of dissipation proportional to the square of table speed with damping coefficient cd. The flexibility of the motor shaft is taken into account through a torsional spring of constant k_T (and damping cT). The data of the system are reported in Table 1. The machine is controlled to move the workpiece of 1 m in 1 s following a 1/3, 1/3, 1/3 motion law in speed.

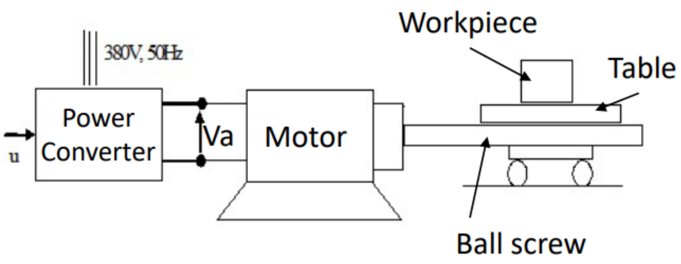


Figure 1 – Scheme of a machine-tool

System data			
Final displacement	h	[m]	1
Final time	t_f	[s]	3
Motor mass moment of inertia	Jm	[kgm^2]	0.005
Motor shaft torsional stiffness	k_T	[Nm/rad]	1000
Motor shaft torsional damping	c_T	[Nms/rad]	2
Motor resistance	Ra	[Ω]	1.2
Motor inductance	La	[H]	0.008
Motor constant	Kψ	[Vs/rad]	0.8
Ball screw ratio	τ	[m/rad]	0.1/(2*π)
Ball screw efficiency	η	[-]	1
Equivalent coefficient for energy dissipation	c_d	[Ns/m]	10
Machine table mass + workpiece mass	$m_t + m_p$	[kg]	40

Table 1 – System data

CASE A: Neglect the dynamics of the motor (La=0) and consider the motor shaft as rigid (kt→∞).

$$\left(\frac{J_m}{\tau^2} + M\right)\ddot{x} + (c_d)\dot{x} = \frac{C_m}{\tau}$$

$$J_x\ddot{x} + \left(c_d + \frac{k_\psi}{\tau^2 R_a}\right)\dot{x} = \frac{k_\psi}{\tau R_a}V_a \rightarrow J_x\,\dot{v} + \left(c_d + \frac{k_\psi}{\tau^2 R_a}\right)v = \frac{k_\psi}{\tau R_a}V_a \textit{ eq. differenziale del 1° ordine in v}$$

1) **FF control**

$$\begin{cases}\dot{x} = \dot{x}_{ref} \\ \ddot{x} = \ddot{x}_{ref}\end{cases} \rightarrow J_x\ddot{x}_{ref} + \left(c_d + \frac{k_\psi}{\tau^2 R_a}\right)\dot{x}_{ref} = f_{FF}(t) \rightarrow V_a = \frac{f_{FF}(t)}{k_\psi}\tau R_a = \text{Tension input model defined}$$

We decide at priori the control law applied to the model based on the reference input (INVERSE DYNAMICS). Particularly, the control force is obtained by using the ***same values of the mechanical system***, so the response follows exactly the input of the system, both in terms of displacement and velocity (no uncertainties in the definition of the system parameters). This is also possible since there aren't ***any unknown disturbances*** that modify the dynamic characteristics of the system and cannot handled by the FF control strategy.

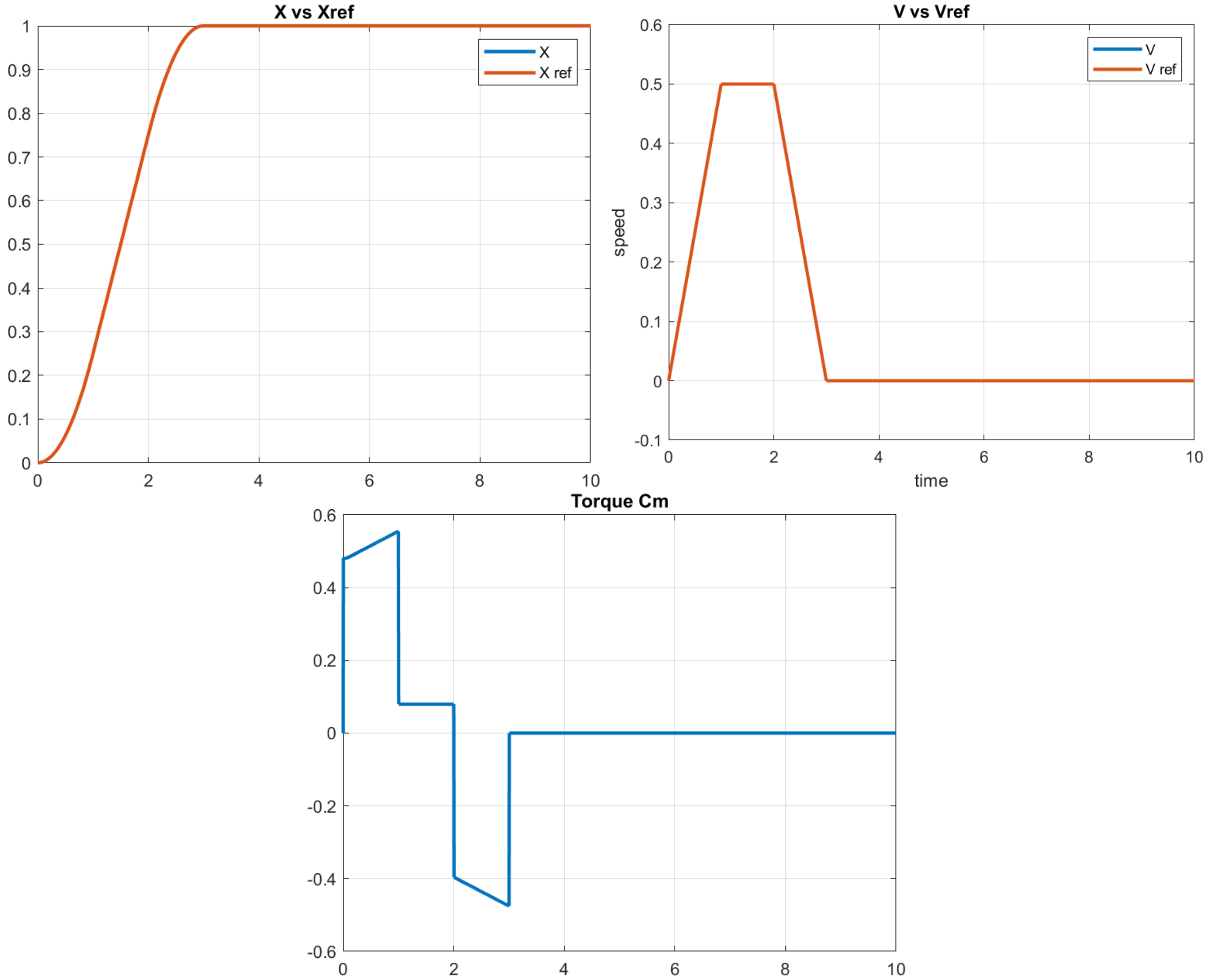


Fig. 1 – CASE A – (1) FF control

Note that the FF ***can't affect the stability*** of the system. In this case it is good having a stable system, but it is not possible to modify the dynamics of the system (the response performances cannot be improved).

2) **FB control – PI controller on the speed $\dot{x}$ of the mass M**

The goal of the control law is to make the steady state error equal to 0 and to improve the dynamic of the system without undesirable effect on the stability (adding a pole in the origin, *PI must be tuned carefully*).
In this case the outputs don't follow exactly the reference because the **FB slows the response** (negative exponential). Higher the error, higher will be the correction computed by the control. The torque Cm in this case is very similar to the FF case but smoother. The steady state error as anticipated is nil.

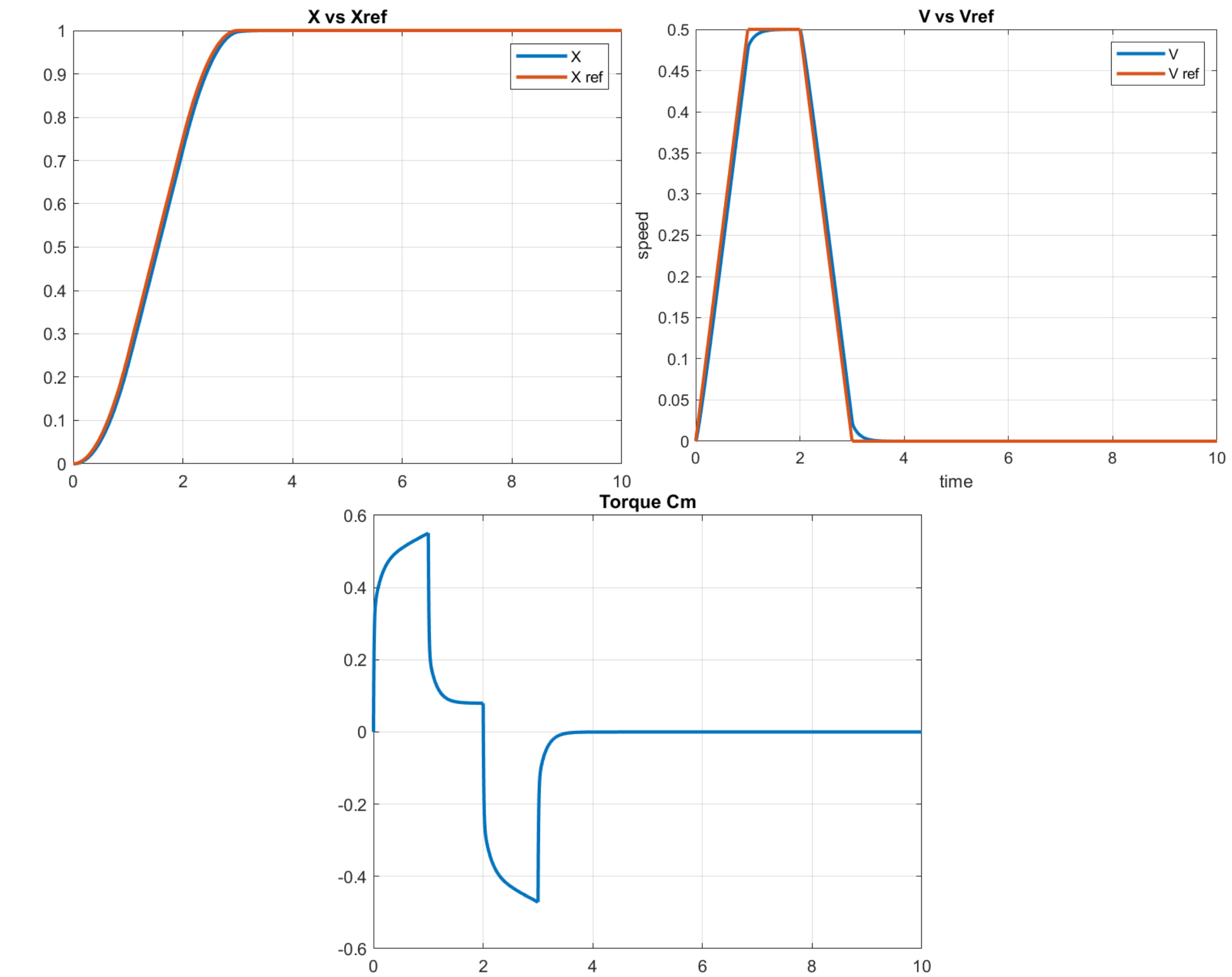


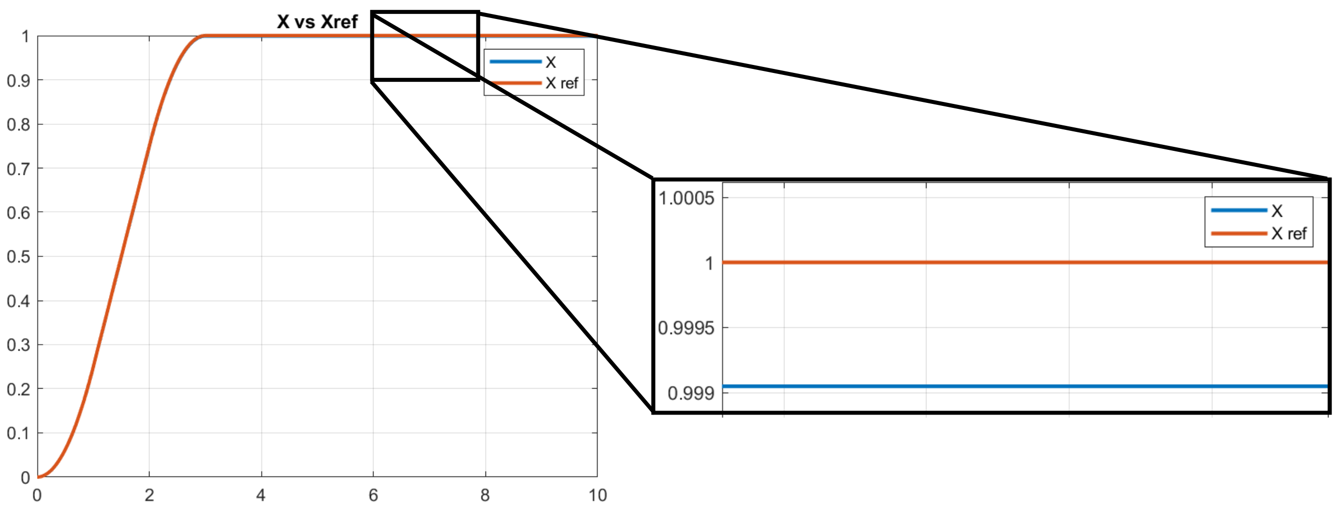
*Fig. 2 – CASE A – (2) FB control (**kp = 100**)*

3) **FF control considering uncertainties on system parameters**: $c'_d = 80\%\ c_d$

Uncertainties on the dumping coefficient since it is generally obtained experimentally without using models.

If the numerical model that we are using to obtain the reference law of the FF force is not perfectly representing the system, we cannot have the perfect representation of the control law as before.

Moreover, in this case, is possible to notice that the output is a little bit different and lower with respect to the reference. ***The control force is lower*** than the one necessary, so displacement, speed and torque assume lower values than the reference. ***The reference is not exactly reproduced in output.***



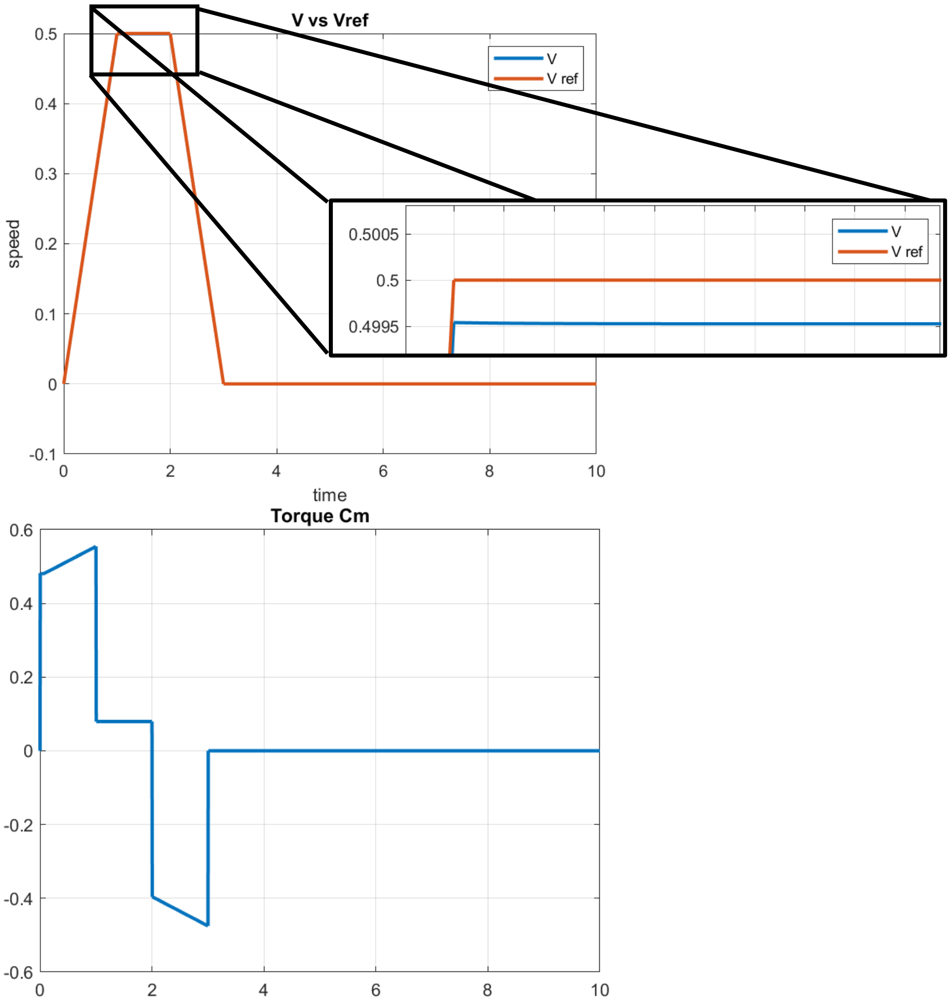
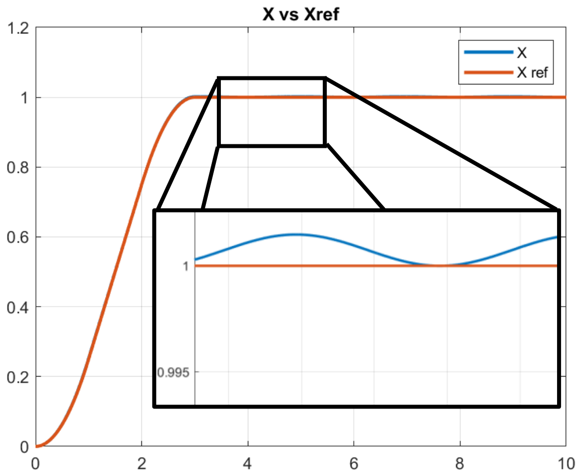
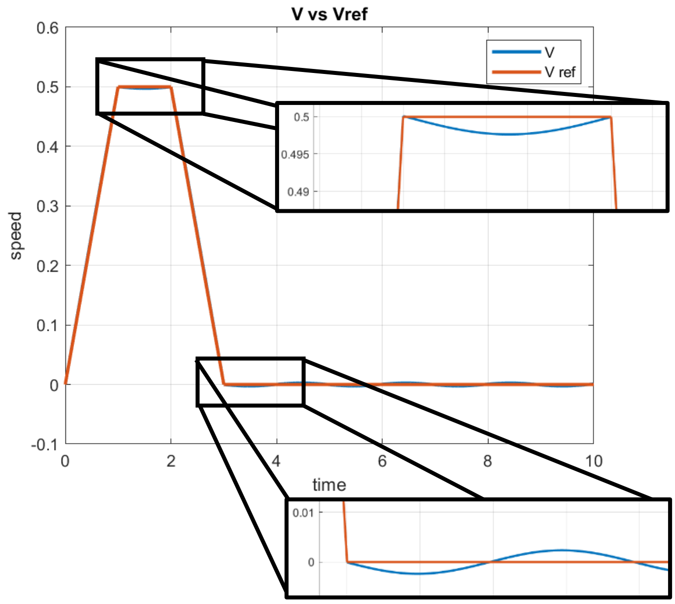


Fig. 3 – CASE A – (3) FF control with uncertainties in the model (cd' = 80%cd)

4) **Presence of a disturbance force applied on the machine table: sinusoidal disturbance (unknown input on the system).**

4.1 - FF control: The output signal mainly affected by the disturbance is the torque Cm: there is a little sinusoidal behaviour. Also, for what concern speed, V and Vref have different behaviours. As a matter of fact, FF doesn't handle in a proper way the unknown disturbance because it is ***not able to modify the dynamic characteristics of the system***.





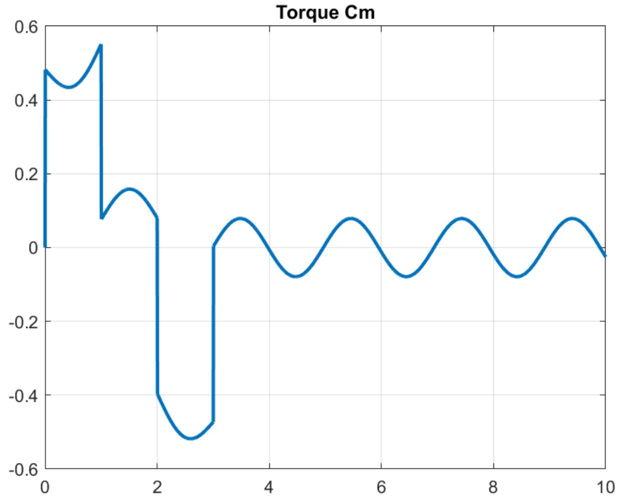


*Fig. 4 – CASE A – (4.1) FF control with sinusoidal disturbance applied having **f = 20/(2π) rad/s***

<u>Increasing the frequency</u> of the disturbance, neither speed nor torque are affected by it. Indeed, with a FF control, disturbances at high frequency are completely attenuated.

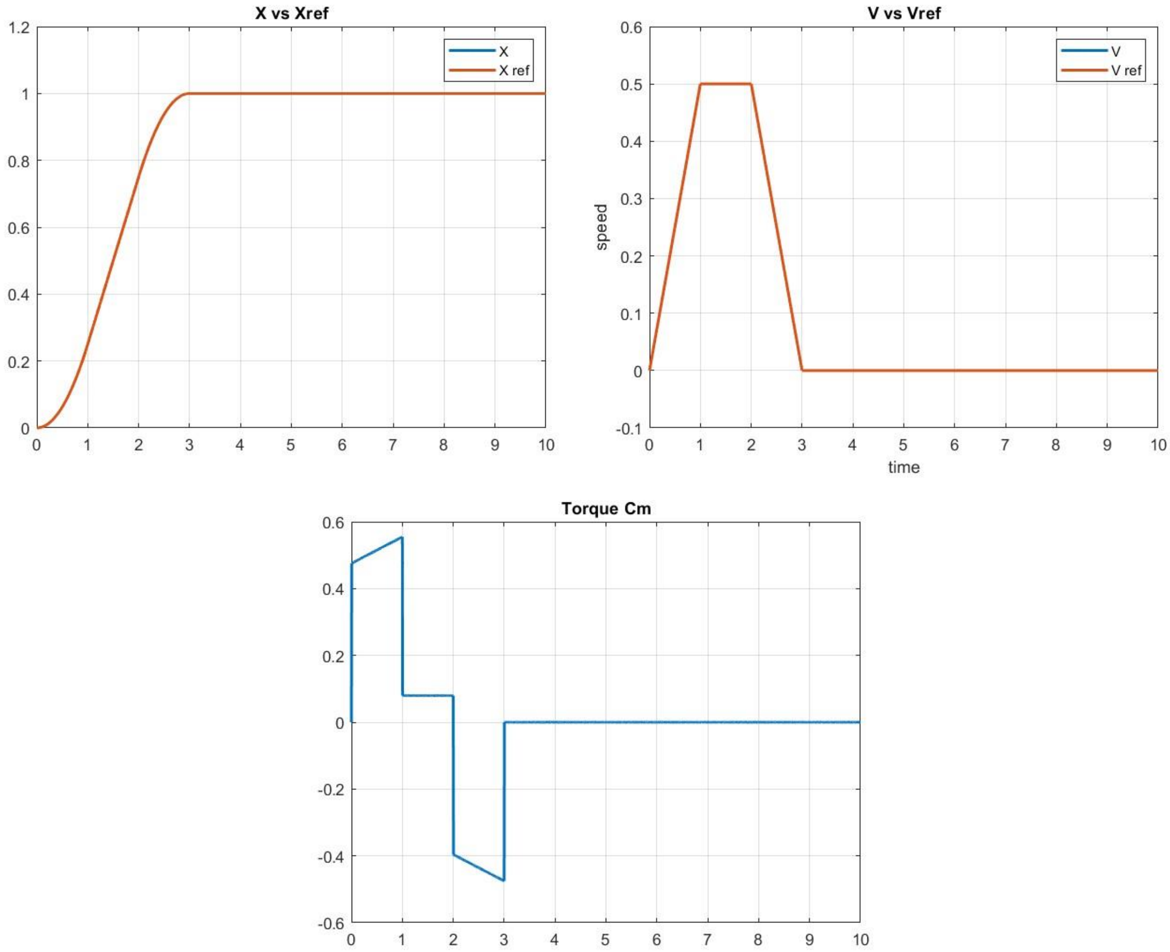


*Fig. 5 – CASE A – (4.1) FF control with sinusoidal disturbance applied having **f = 350000/(2π) rad/s***

4.2 - FB control: The FB control strategy can modify the dynamics of the system reducing the effect of the disturbance, therefore increasing the kp value, ***we can reject the disturbance effect***.
This different behaviour can be seen analysing the output plot setting kp = 10 and then kp = 100. In the first case position, speed and torque diagrams differ a lot from the desired output.

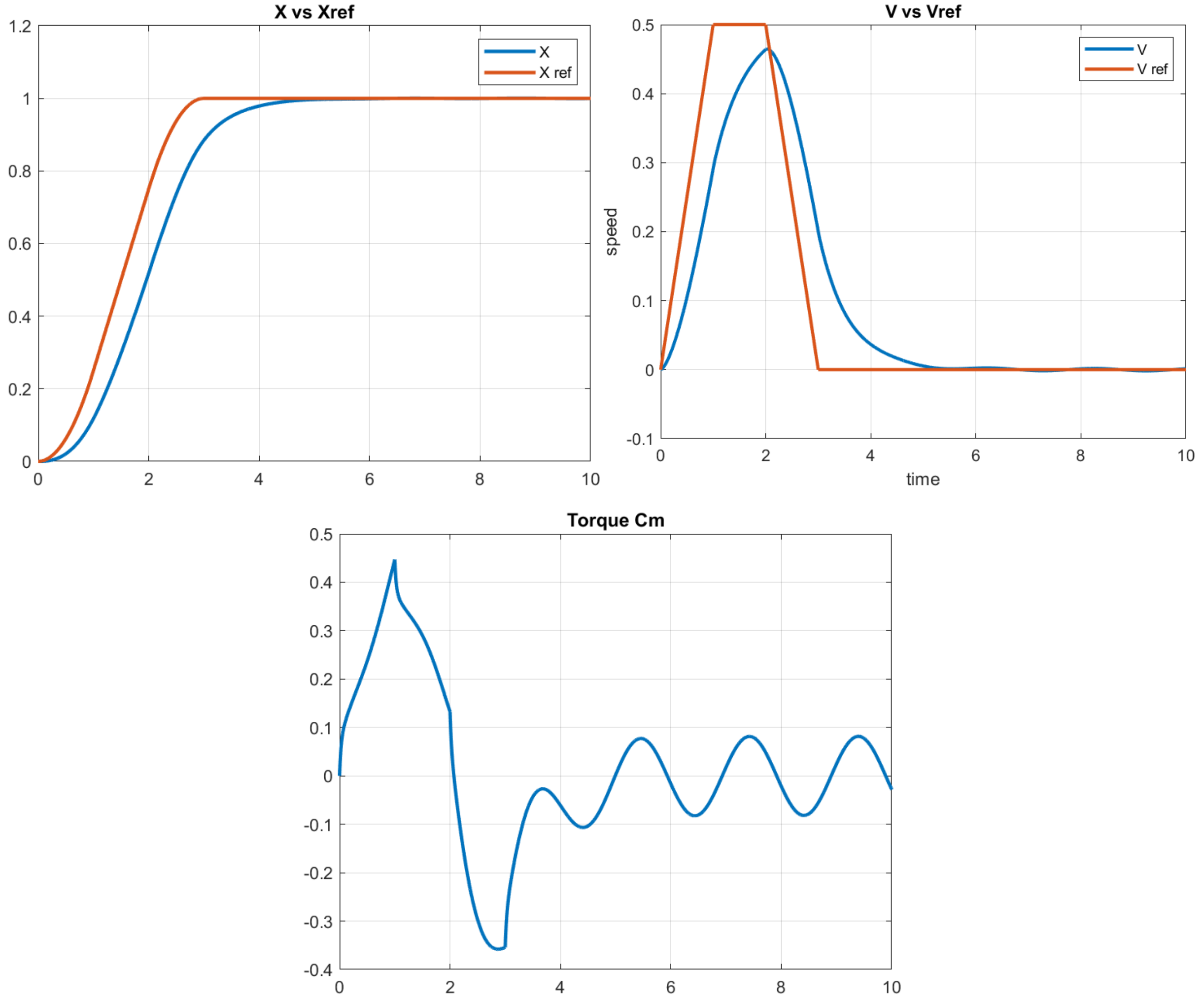
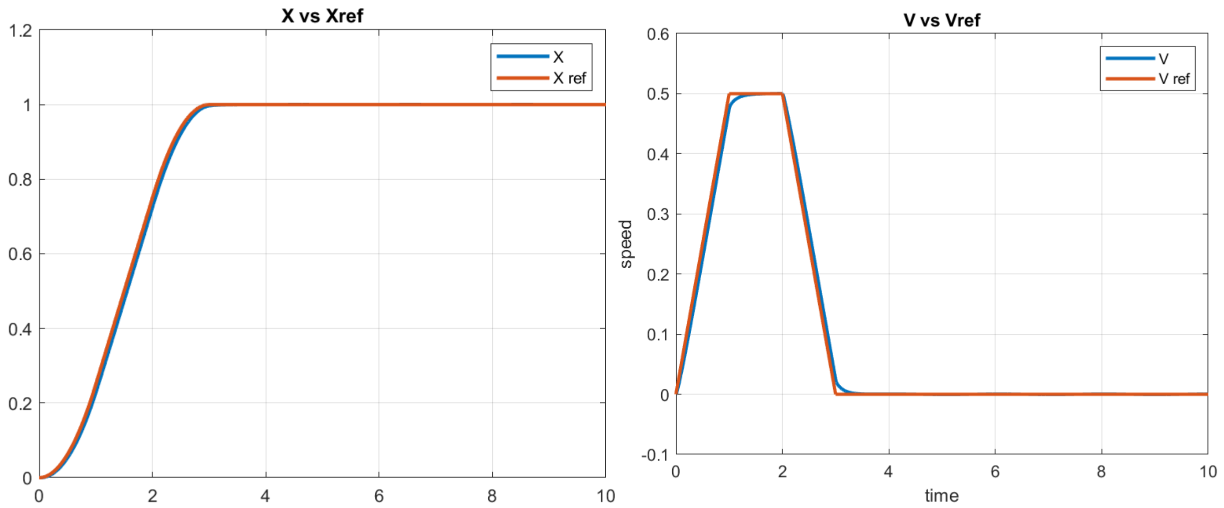


Fig. 6 – CASE A – (4.2) FB control with sinusoidal disturbance applied $f = 20/(2\pi)$ rad/s having ***kp = 10***

While having **kp = 100**, disturbances don’t affect the speed and position signals, only some effects are still present in the torque Cm.



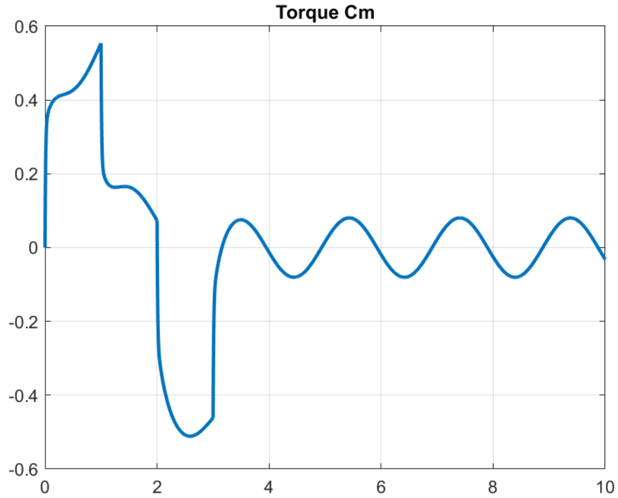


Fig. 7 – CASE A – (4.2) FB control with sinusoidal disturbance applied f = 20/(2π) rad/s having ***kp = 100*****.**

Note that: as for the FF control, if the disturbance isn’t present in the bandwidth of the system, it is almost totally attenuated.

5) **Presence of noise on the measurement in the application of the FB control law**: **white noise.**
The noise acts as an input for the controller because the transfer function that we have between the actual variable and the noise, except for the phase, is the same as the one between the signal and the reference. If we improve the way in which the system follows the reference, we also improve the way in which the system follows the noise. Choosing a noise amplitude equal to 10^{-8}, thus very low, it can be noticed that it affects mainly the torque.

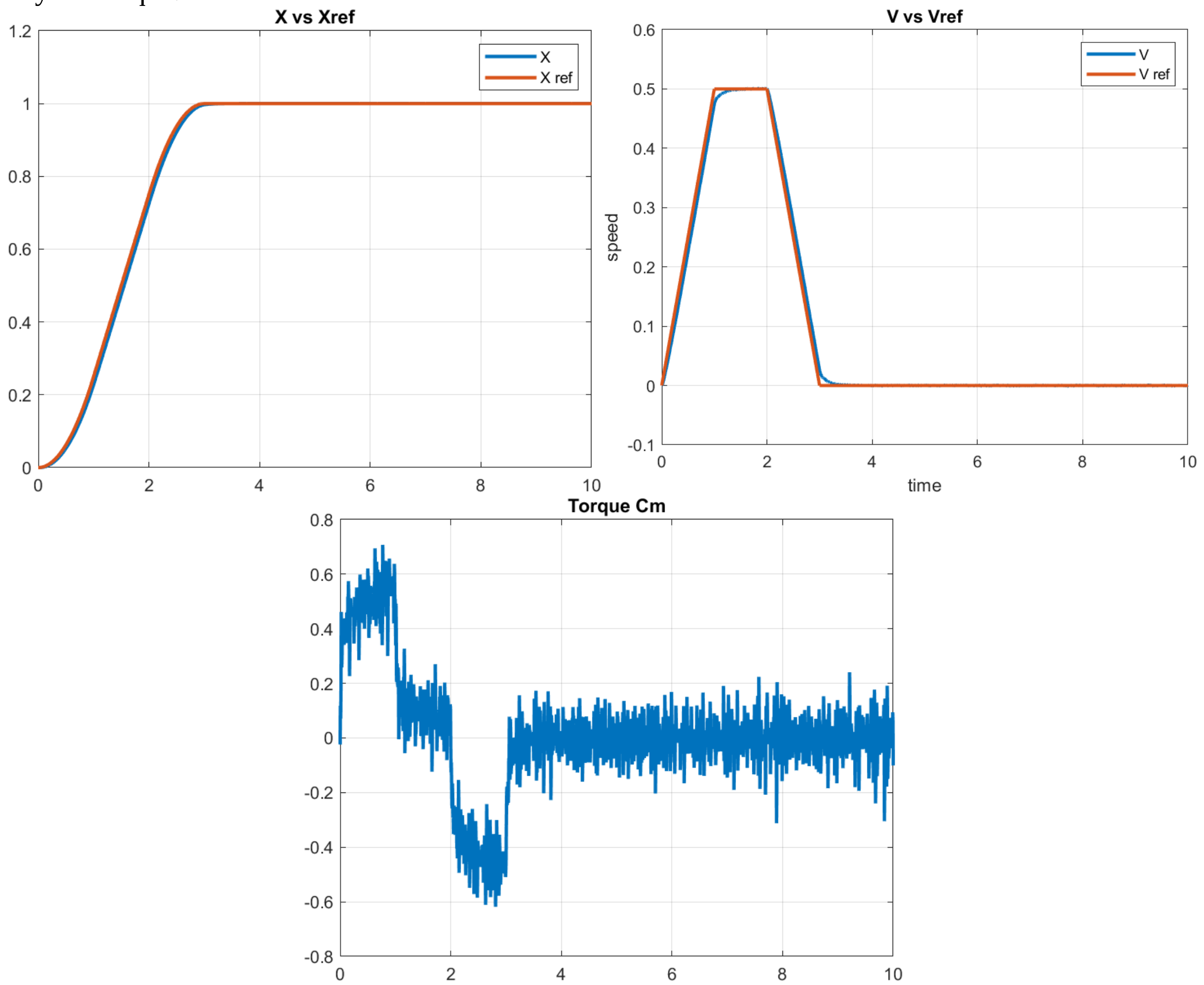


Fig. 8 – CASE A – (5) FB control with white noise affecting the measurement ***noise power = 10^-8***

Increasing the noise power till 10^{-5} is possible to observe that this amplitude value influences more the outputs with a significant effect on the torque signal, which appears completely distorted, and different from the behaviour above without any noises.

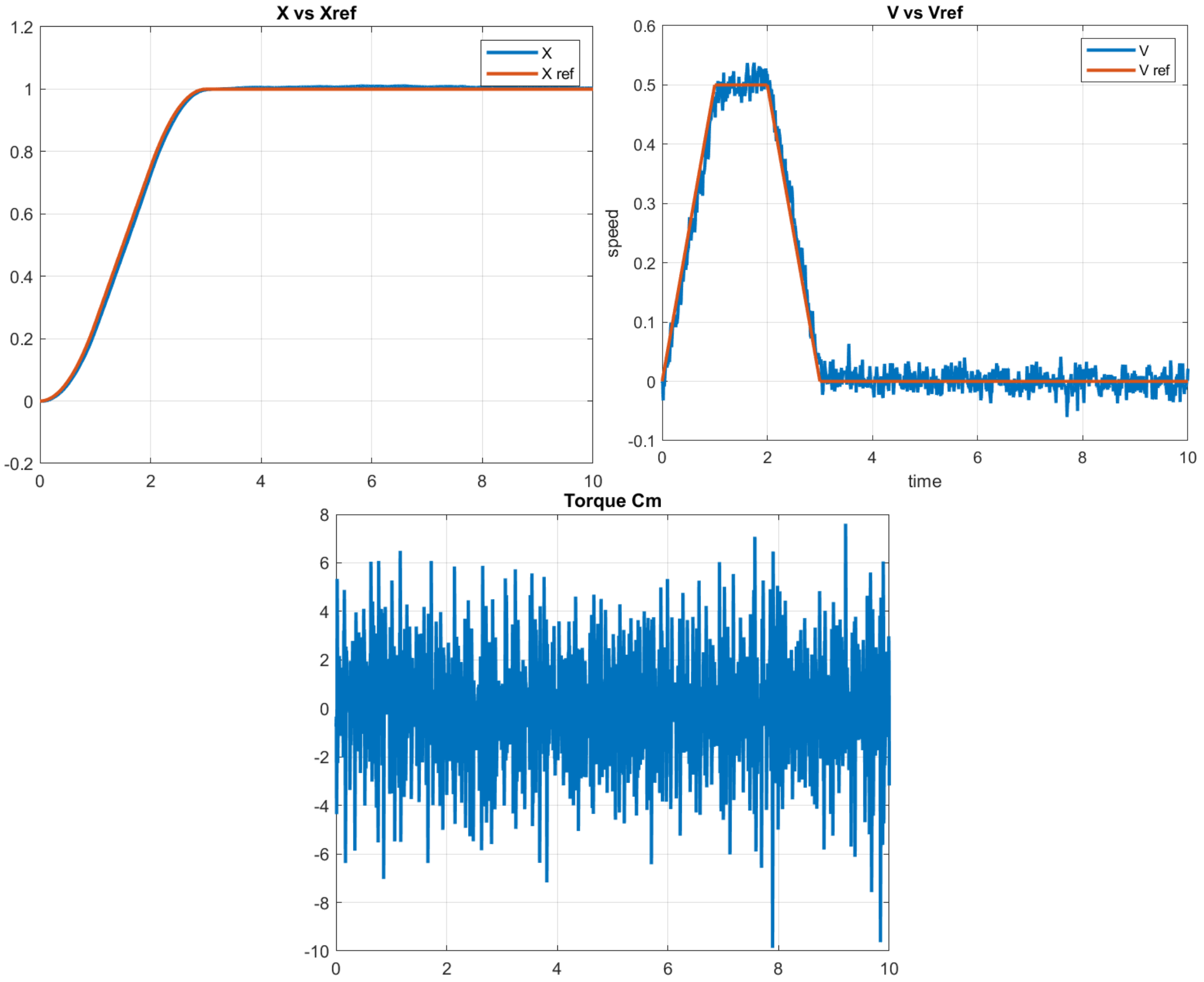


*Fig. 9 – CASE A – (5) FB control with white noise affecting the measurement **noise power = 10^-5***

CASE B: <u>Consider the dynamics of the motor</u> (La≠0) and consider the motor shaft as rigid (kt→∞).

1) **FF control, using the same tension input Va computed in A-1**

The input tension Va is the same obtained at the point A-1 imposing:

$$\begin{cases} \dot{x} = \dot{x}_{ref} \\ \ddot{x} = \ddot{x}_{ref} \end{cases}$$

$$J_x \ddot{x}_{ref} + \left(c_d + \frac{k_\psi}{\tau^2 R_a} \right) \dot{x}_{ref} = f_{FF}(t) \rightarrow V_a = \frac{f_{FF}(t)}{k_\psi} \tau R_a$$

Therefore, in this expression we are not considering the actuator’s dynamic that instead has been considered in the formulation of the ***mathematical model of the motor*** since:

$$(J_x s + c_d)\, V(s) = \frac{C_m}{\tau} = \frac{k_\psi I_a}{\tau} \quad with \quad I_a = \frac{V_a - \frac{k_\psi}{\tau}\dot{x}}{L_a s + R_a}$$

That’s why we won’t expect the system response to perfectly follow the reference (FF force not considering the dynamics, model considers the dynamics).

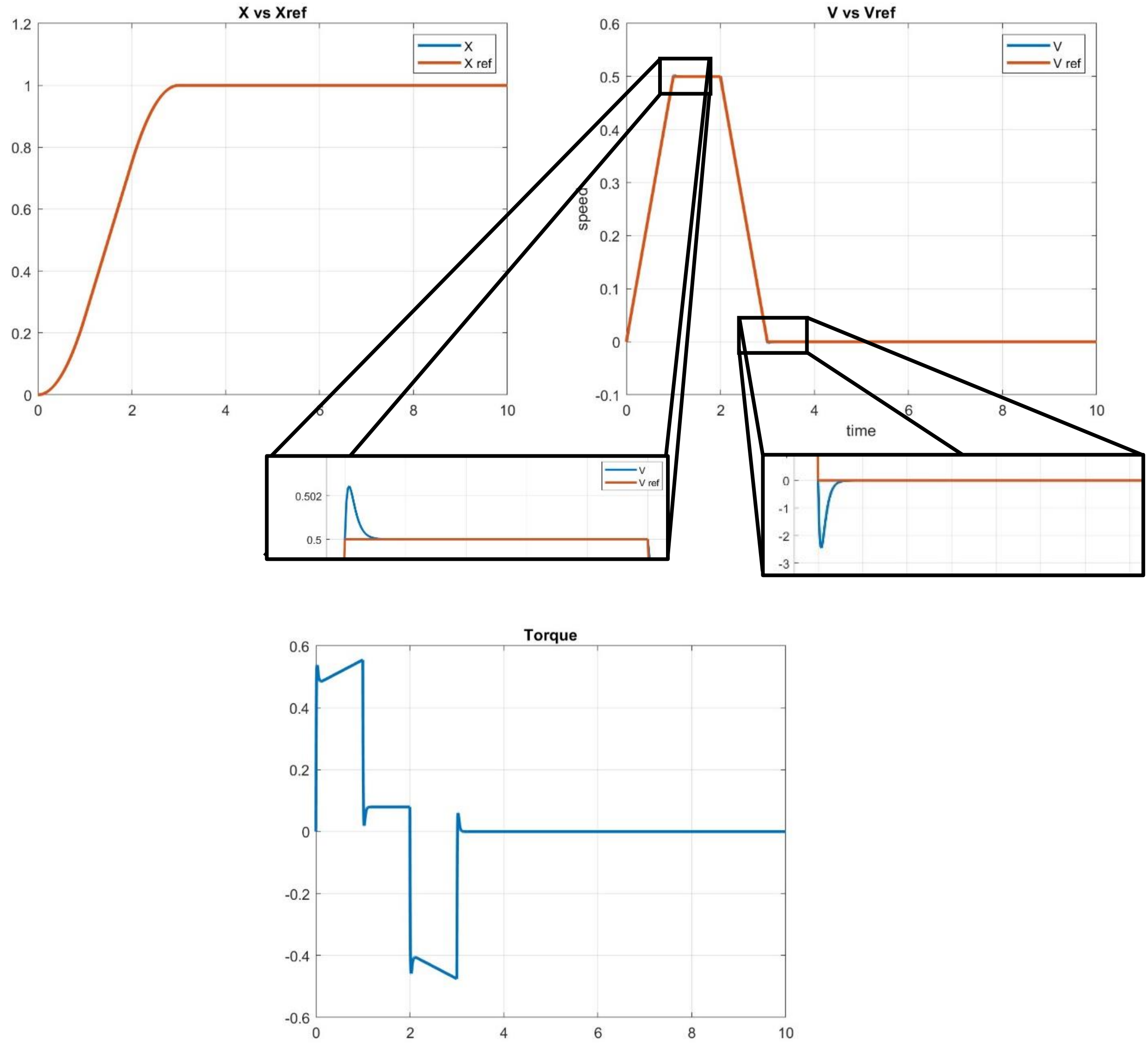


Fig.10 – CASE B – (1) FF control

The peaks in the torque’s graph may be due to the fact that the FF force is not considering the motor’s dynamic while the model it is.

2) FB control – PI controller on the speed $\dot{x}$ of the mass M

As done in CASE A, we applied a PI control on the linear speed of the mass M in order to nullify the steady state error and improve the dynamic of the system avoiding undesirable effect on the stability *choosing carefully the position of the zero*. Here as well the system’s response will be **slowed** by the FB control. In this case it’s possible to observe the effect of the actuator’s dynamic especially on the torque.

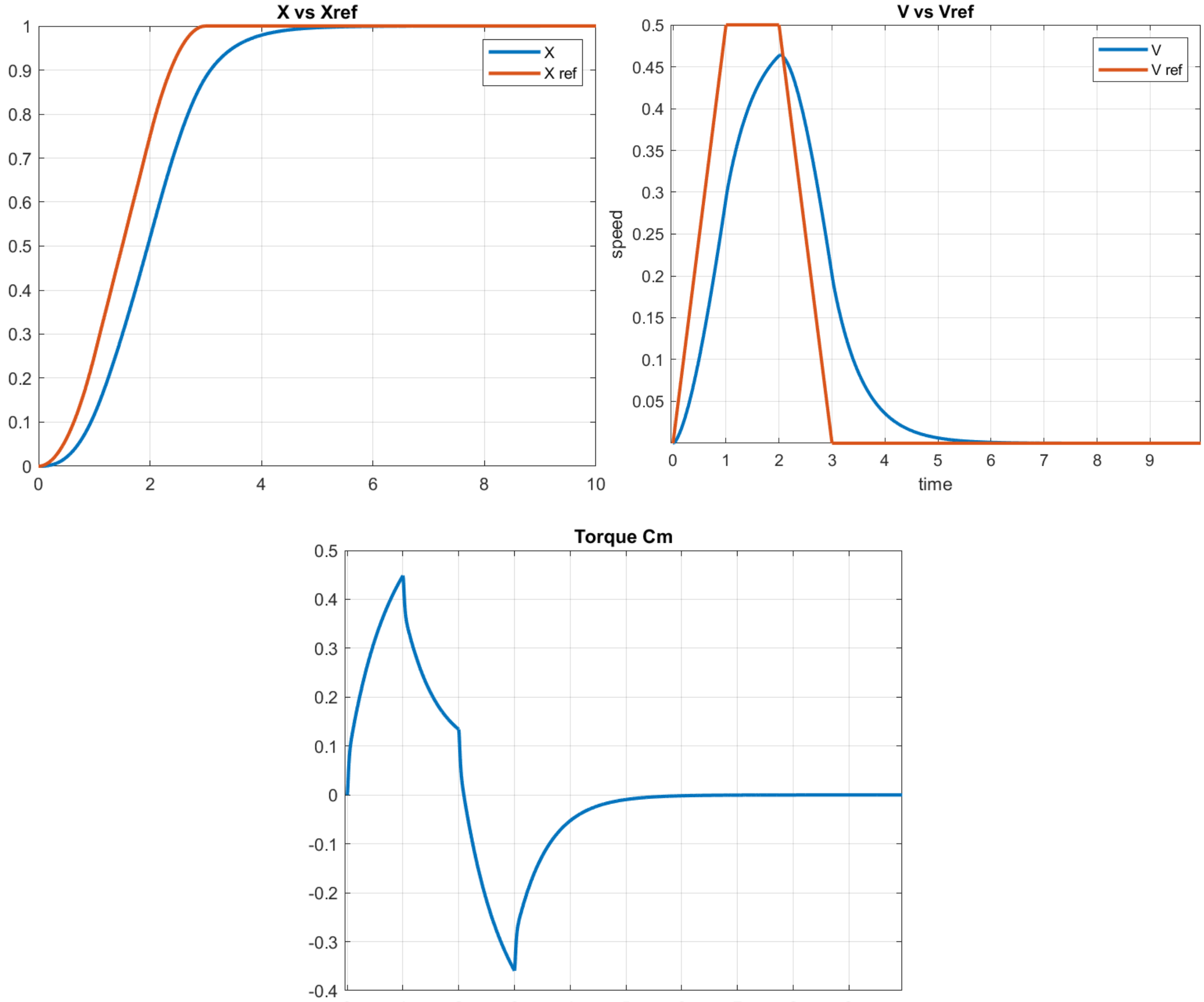
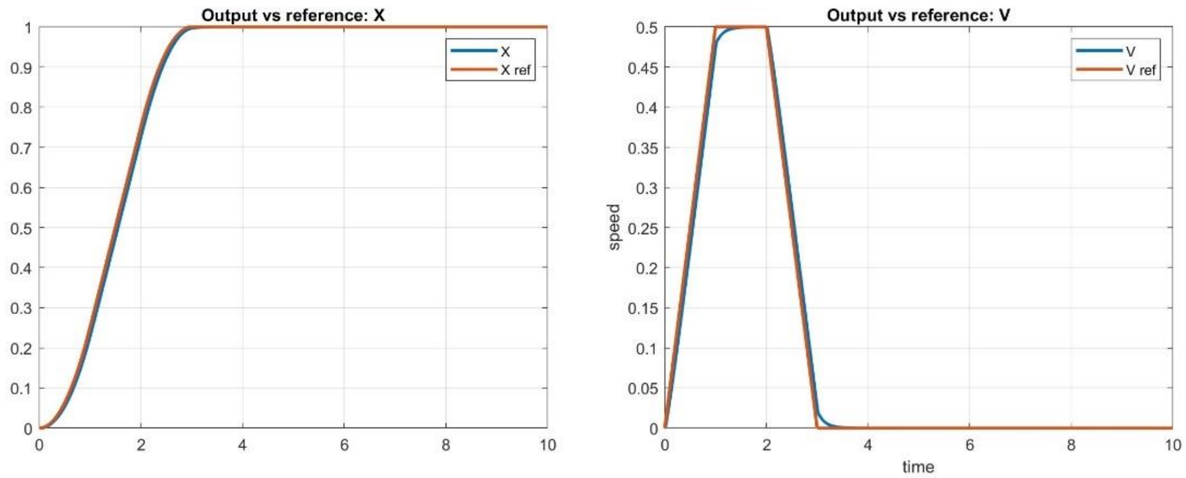


Fig.11 – CASE B – (2) FB control ***kp = 10***

Increasing kp the differences between output and reference signals are minimised.



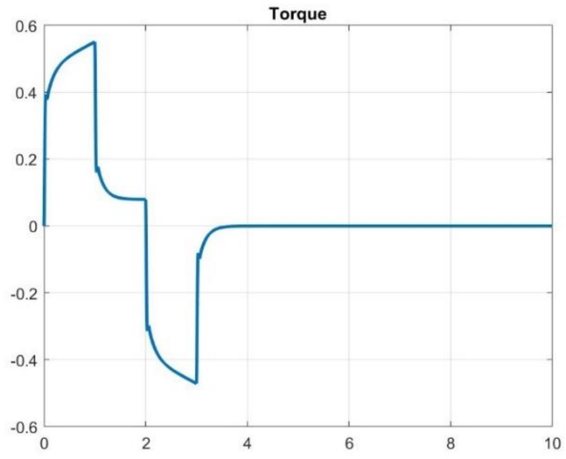
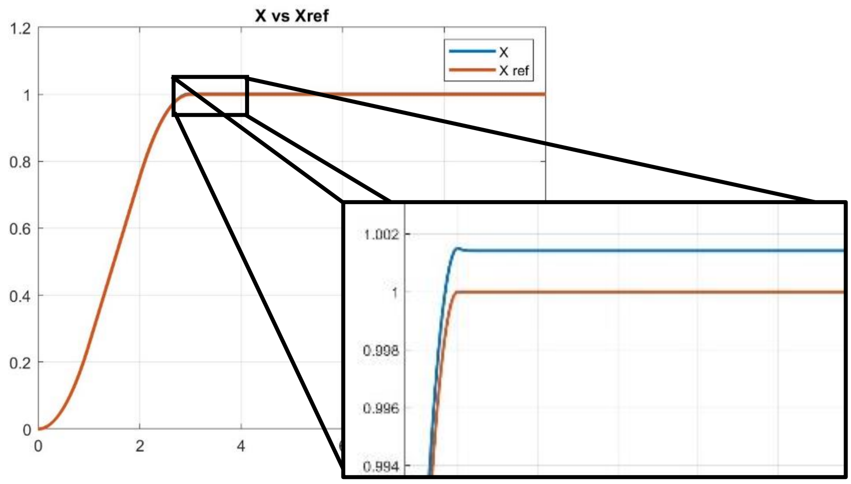
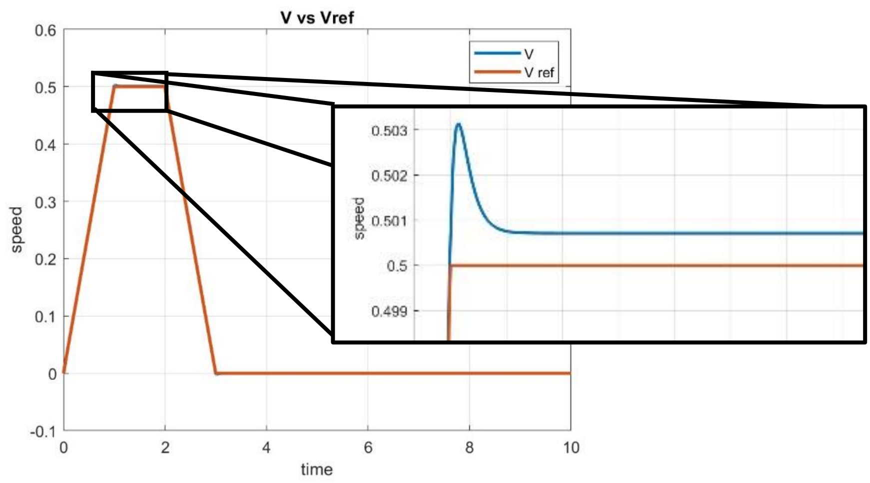


Fig.12 – CASE B – (2) FB control ***kp = 100***

3) **FF control considering uncertainties on system parameters**: $\boldsymbol{c_d' = (c_d + 30\%\ c_d)}$
We can see the effect on both speed and displacement, which achieve higher values than the reference (since the ***FF force will be higher than the needed one***). Peaks are associated with what stated before.





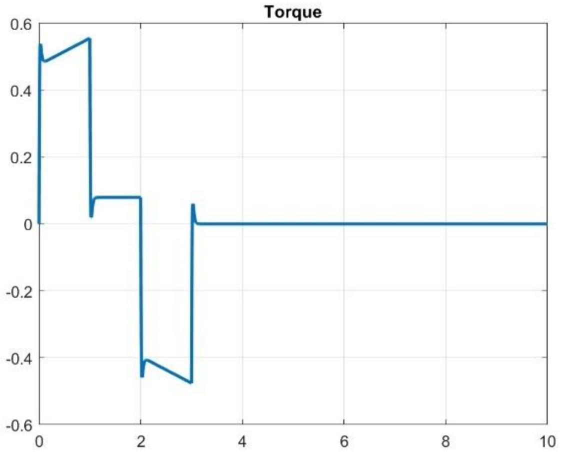


Fig.13 – CASE B – (3) FF control with uncertainties in the model’s parameters (cd’= cd + 30% cd)

4) Presence of a disturbance force applied on the machine table: sinusoidal disturbance (non-controlled input on the system).

4.1 - FF control: As previously observed, the FF doesn’t handle in a proper way the unknown disturbance because it is not able to modify the dynamic characteristics of the system.

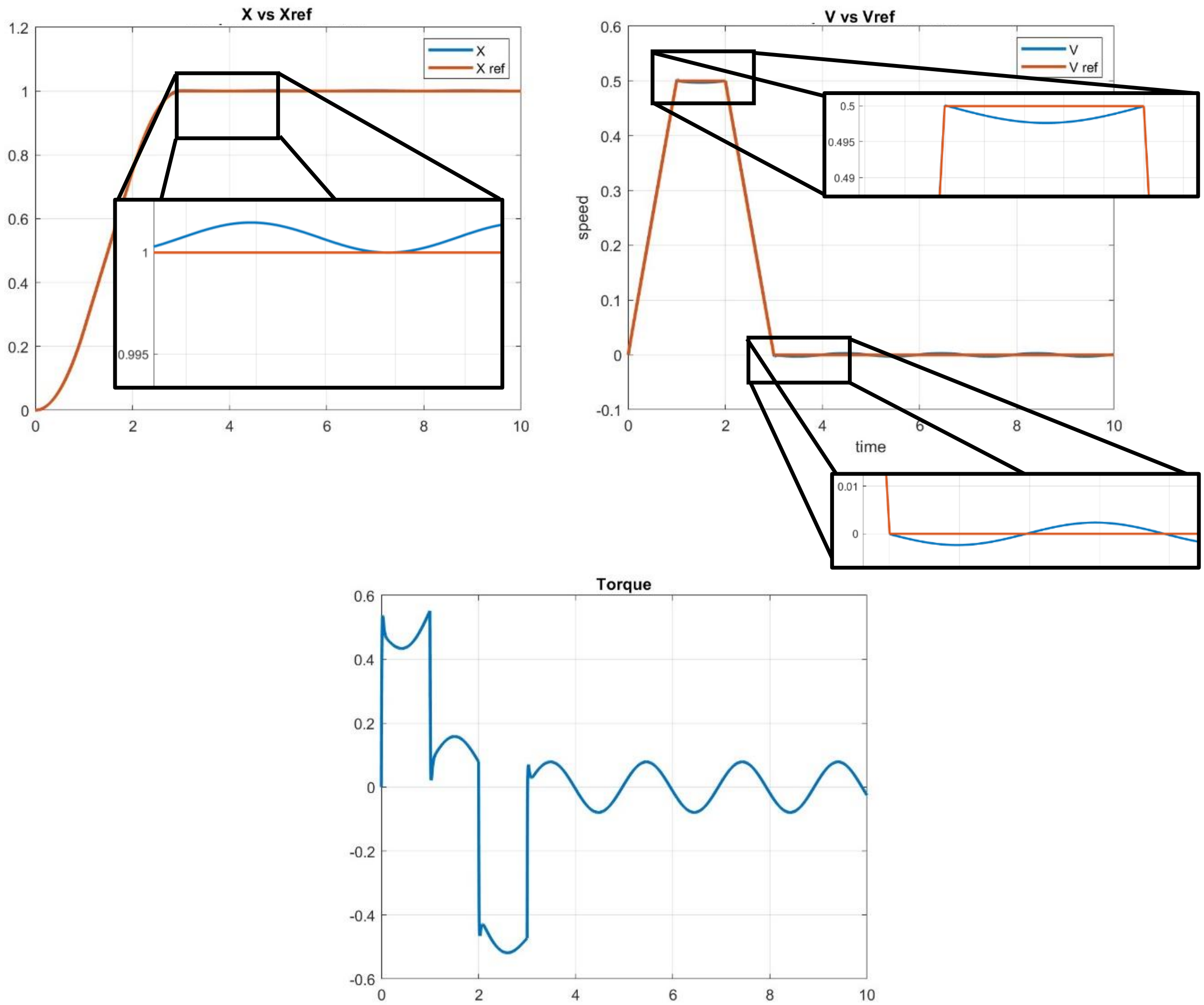


Fig.14 – CASE B – (4.1) FF control with a sinusoidal disturbance affecting the response of the system, ***f = 20/(2π) rad/s.***

But <u>increasing the disturbance frequency,</u> neither speed nor torque are affected by it. Indeed, with FF control, disturbances at high frequency are completely attenuated.

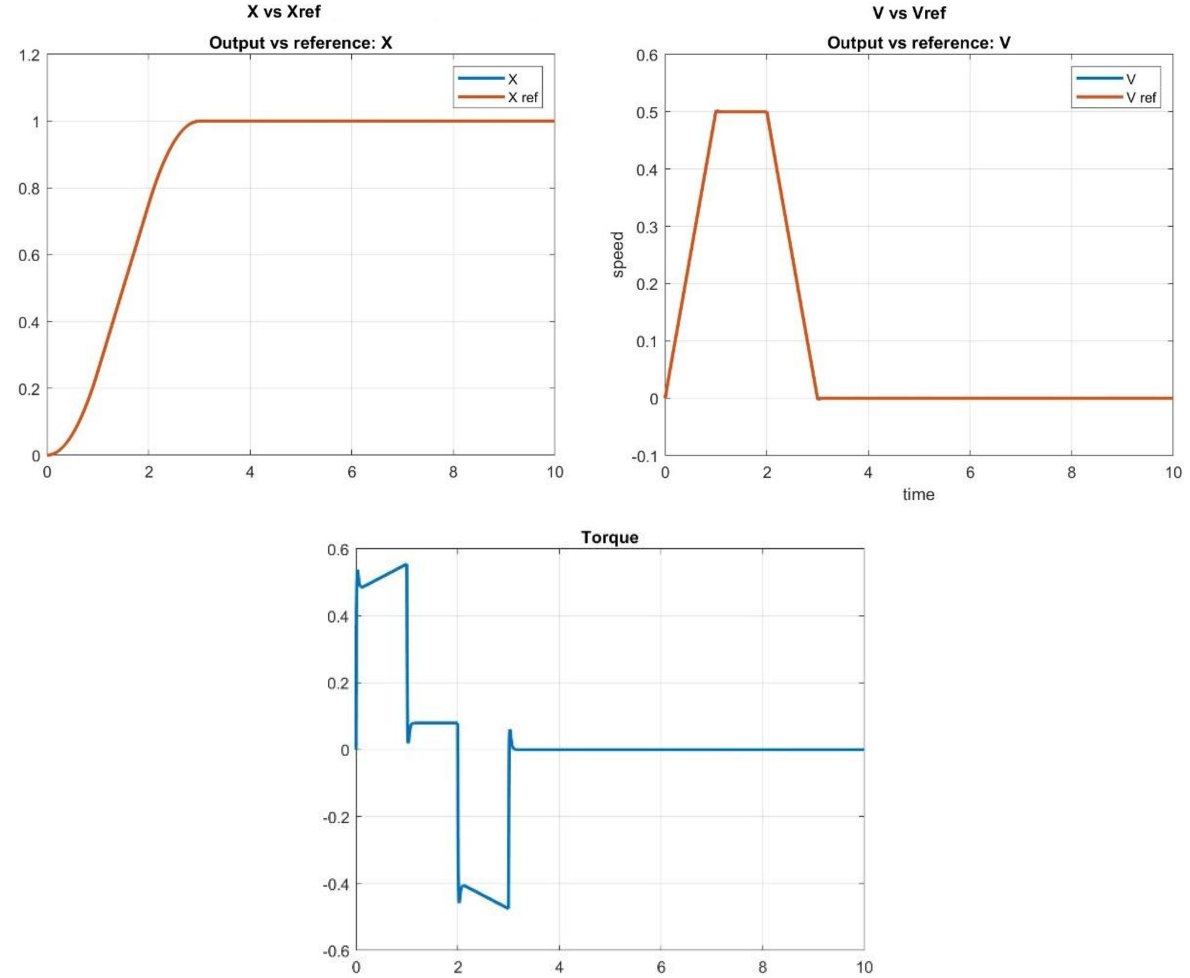
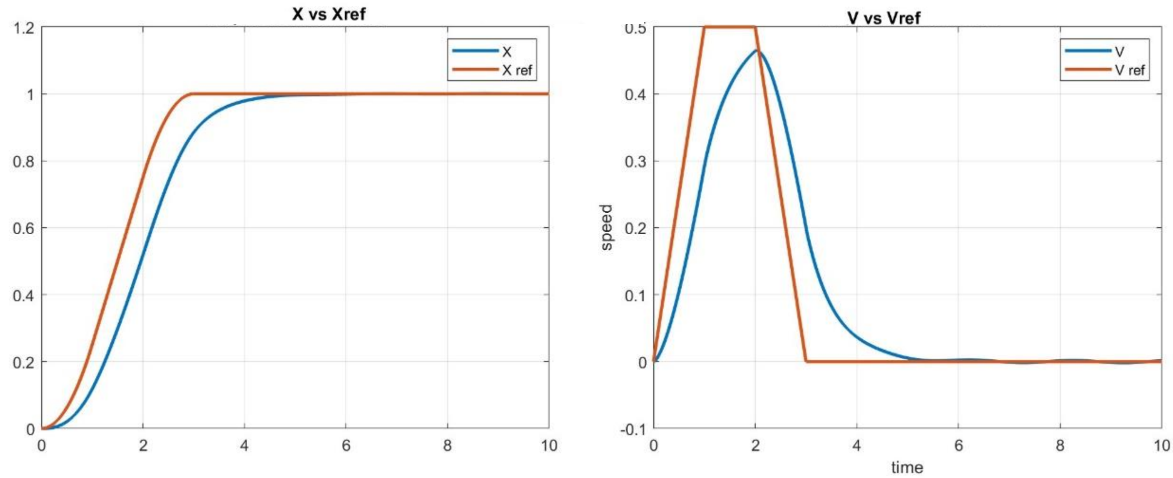


*Fig.15 – CASE B – (4.1) FF control with a sinusoidal disturbance at high frequency affecting the response of the system, **f = 350000/(2π) rad/s.***

4.2 - FB control: The FB control strategy ***can modify the dynamics*** of the system reducing the effect of the disturbance, therefore increasing the kp value, we can reject the disturbance effect. As previously described with a control gain Kp=10 the system is not able to reject the disturbance



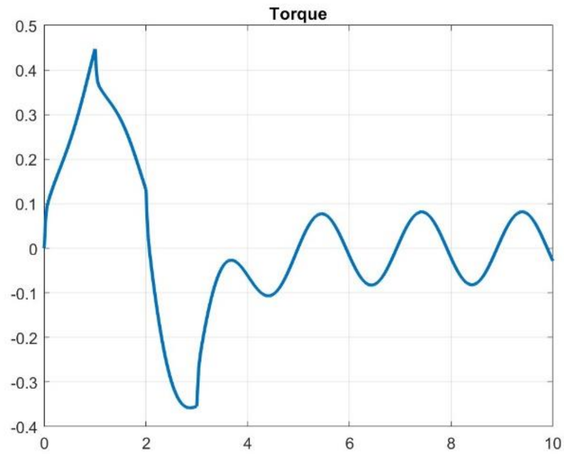


*Fig.16 – CASE B – (4.2) FB control with a sinusoidal disturbance f = 20/(2π) rad/s, **kp = 10***

While having kp = 100, ***disturbances don’t affect the speed and position signal***, only some effects are still present in the torque Cm.

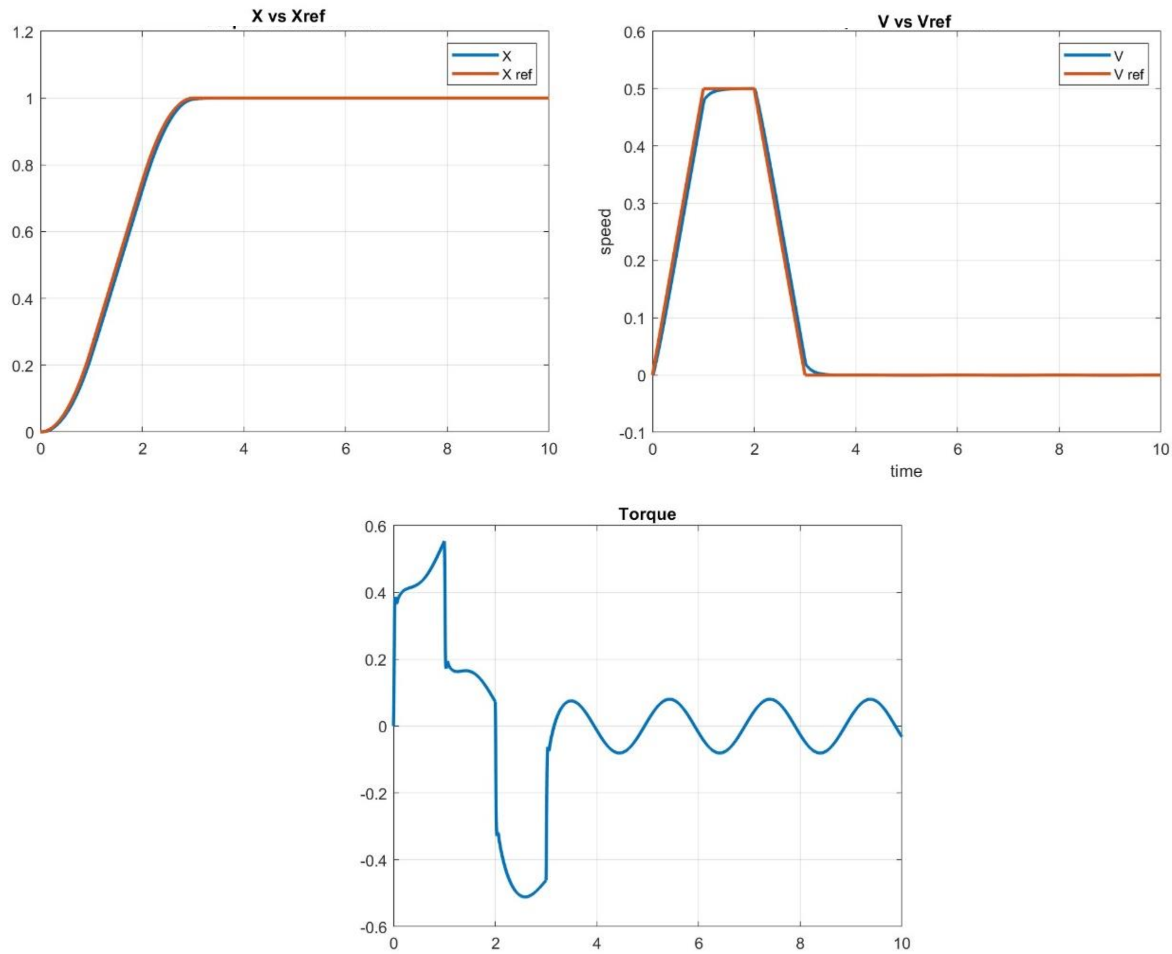


*Fig.17 – CASE B – (4.2) FB control with a sinusoidal disturbance f = 20/(2π) rad/s, **kp = 100***

5) **Presence of noise on the measurement in the application of the FB control law**: **white noise.**
The response is affected by the noise at low power (noise amplitude equal to 10^{-8}). The noise acts as an input for the controller because the transfer function that we have between the actual variable and the noise, except for the phase, is the same as the one between the signal and the reference.

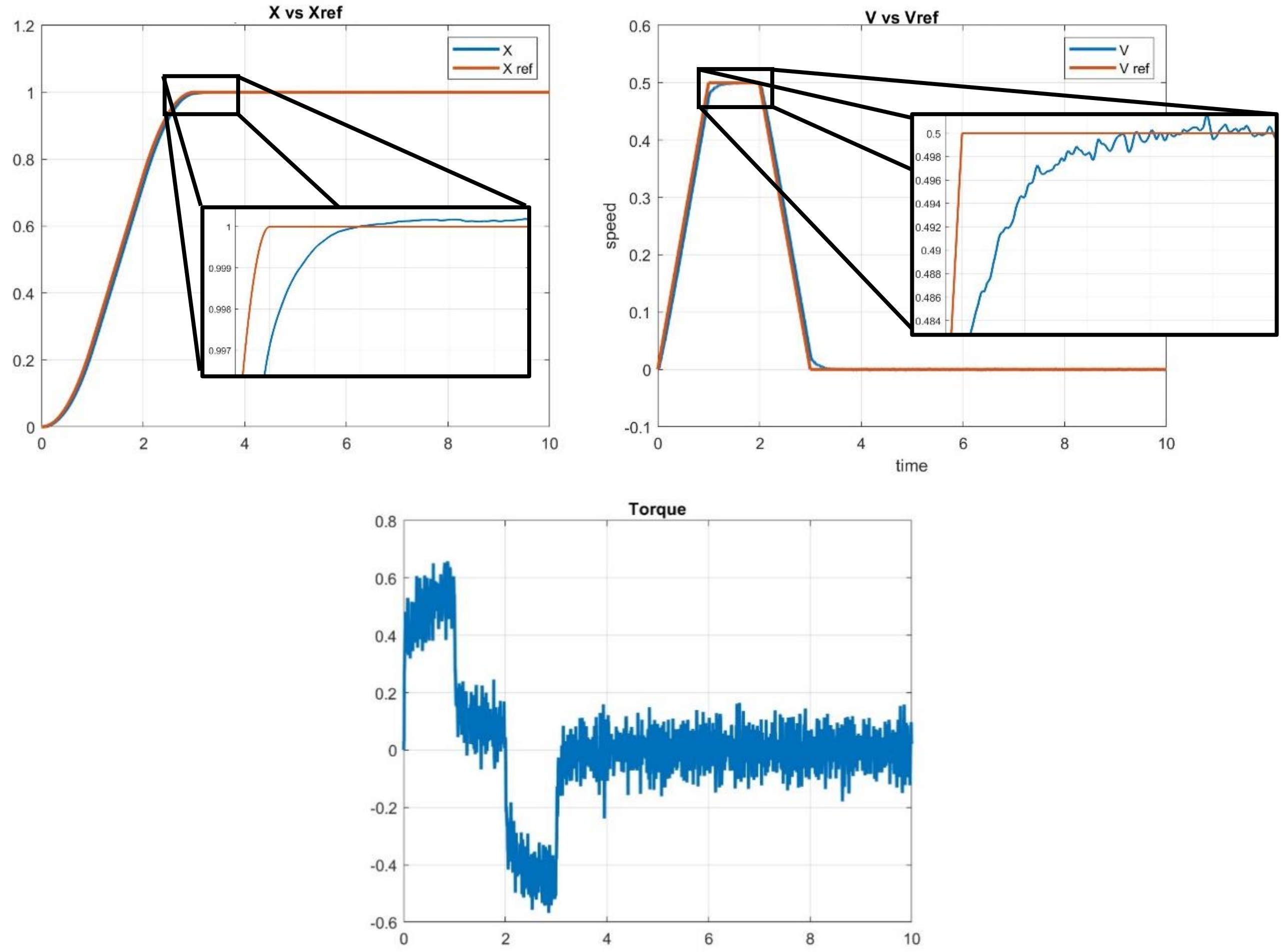


Fig.18 – CASE B – (5)FB control with a white noise with low amplitude (10 ^ -8).

CASE C: Neglect the dynamics of the motor (La=0) and consider a finite torsional stiffness for the motor Shaft (2 DOF system).

Damping and stiffness matrices are coupled.

$$\begin{bmatrix} J_m & 0 \\ 0 & M \end{bmatrix}\begin{bmatrix} \delta\ddot{\vartheta} \\ \delta\ddot{x} \end{bmatrix} + \begin{bmatrix} c_T + \frac{k_\psi^2}{R_a} & -\frac{c_T}{\tau} \\ -\frac{c_T}{\tau} & c_d + \frac{c_T}{\tau^2} \end{bmatrix}\begin{bmatrix} \delta\dot{\vartheta} \\ \delta\dot{x} \end{bmatrix} + \begin{bmatrix} k_T & -\frac{k_T}{\tau} \\ -\frac{k_T}{\tau} & \frac{k_T}{\tau^2} \end{bmatrix}\begin{bmatrix} \delta\vartheta \\ \delta x \end{bmatrix} = \begin{bmatrix} \frac{k_\psi}{R_a} \\ 0 \end{bmatrix}\delta V_a$$

1) **COLOCATED FB CONTROL (PI)**: the actuator is connected close to the sensor that measures the variable that we want to control.

$$\delta V_a = k_p\left(\delta\dot{\vartheta}_{ref} - \delta\dot{\vartheta}\ \right) + k_i\left(\int_0^t (\delta\dot{\vartheta}_{ref} - \delta\dot{\vartheta}\ )dt\right)$$

$$\begin{bmatrix} J_m & 0 \\ 0 & M \end{bmatrix}\begin{bmatrix} \delta\ddot{\vartheta} \\ \delta\ddot{x} \end{bmatrix} + \begin{bmatrix} c_T + \frac{k_\psi^2}{R_a} + \frac{k_\psi}{R_a}k_p & -\frac{c_T}{\tau} \\ -\frac{c_T}{\tau} & c_d + \frac{c_T}{\tau^2} \end{bmatrix}\begin{bmatrix} \delta\dot{\vartheta} \\ \delta\dot{x} \end{bmatrix} + \begin{bmatrix} k_T + \frac{k_\psi}{R_a}k_i & -\frac{k_T}{\tau} \\ -\frac{k_T}{\tau} & \frac{k_T}{\tau^2} \end{bmatrix}\begin{bmatrix} \delta\vartheta \\ \delta x \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{k_\psi}{R_a}k_p & \frac{k_\psi}{R_a}k_i \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \delta\dot{\vartheta}_{ref} \\ \delta\vartheta_{ref} \end{bmatrix}$$

The terms introduced by the PI controller are added on the main diagonal of the damping and stiffness matrix in the (1,1) position. This increases the stability property of the system as ↑ kp. Note that [C] and [K] remains symmetric.

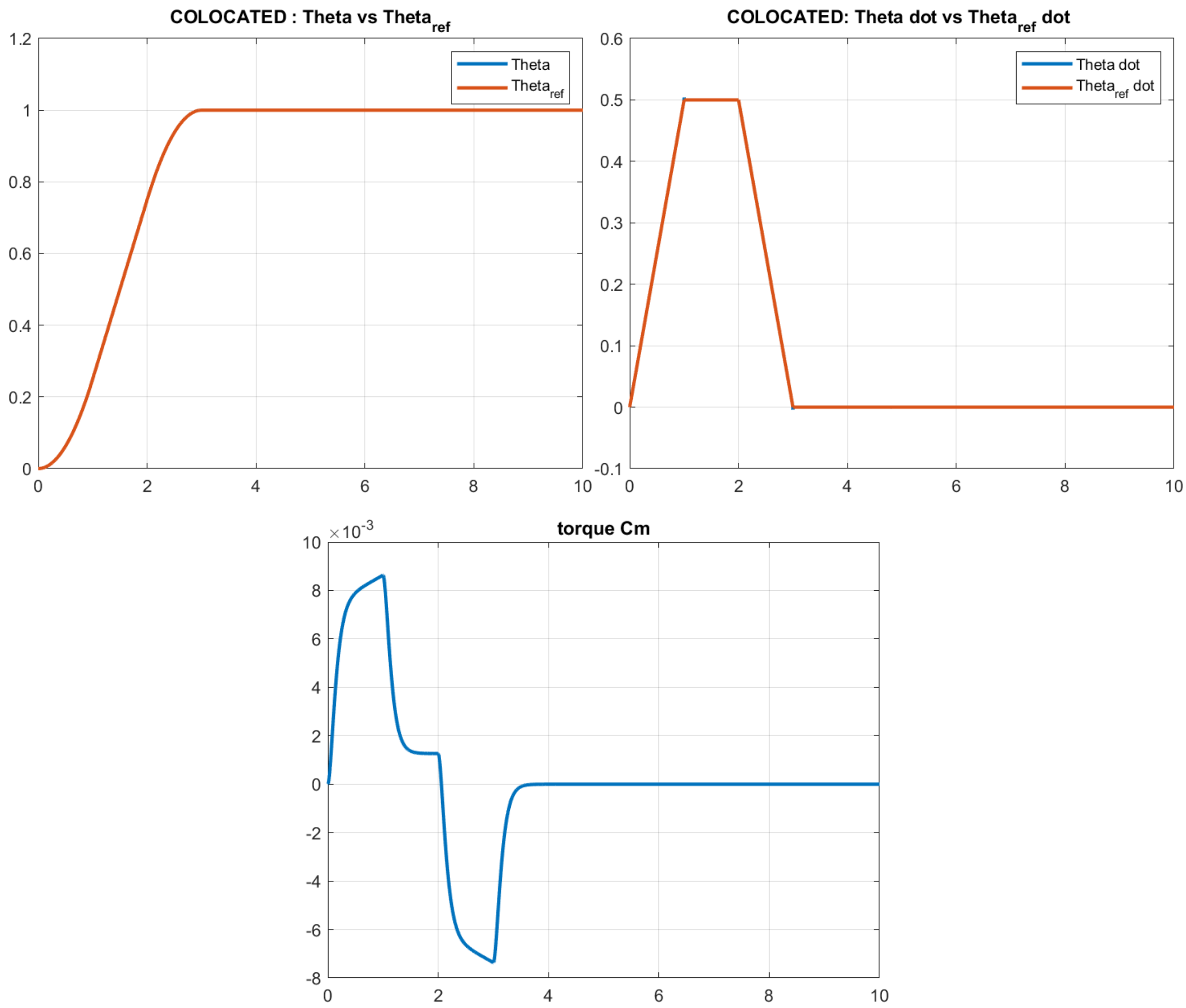


Fig.19– CASE C – (1) 2 DOF system with a colocated control and kp = 10000 and kp = 10, same results

2) **NON COLOCATED FB CONTROL**: actuator and sensor do not have the same location.

$$\delta V_a = k_p\left(\delta \dot{x}_{ref} - \delta \dot{x}\right) + k_i\left(\int_0^t \left(\delta \dot{x}_{ref} - \delta \dot{x}\right)dt\right)$$

$$\begin{bmatrix} J_m & 0 \\ 0 & M \end{bmatrix}\begin{bmatrix} \delta\ddot{\vartheta} \\ \delta\ddot{x} \end{bmatrix} + \begin{bmatrix} c_T + \frac{k_\psi^2}{R_a} & -\frac{c_T}{\tau} + \frac{k_\psi}{R_a}k_p \\ -\frac{c_T}{\tau} & c_d + \frac{c_T}{\tau^2} \end{bmatrix}\begin{bmatrix} \delta\dot{\vartheta} \\ \delta\dot{x} \end{bmatrix} + \begin{bmatrix} k_T & -\frac{k_T}{\tau} + \frac{k_\psi}{R_a}k_i \\ -\frac{k_T}{\tau} & \frac{k_T}{\tau^2} \end{bmatrix}\begin{bmatrix} \delta\vartheta \\ \delta x \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{k_\psi}{R_a}k_p & \frac{k_\psi}{R_a}k_i \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \delta\dot{x}_{ref} \\ \delta x_{ref} \end{bmatrix}$$

Introducing a not collocated control, the terms of the PI controller are added on the ***extra diagonal terms*** of the damping and stiffness matrices, therefore resulting in a no more symmetric matrices (positional force field not conservative, speed forcefield is no more completely dissipative).

Since $k_{12} \neq k_{21}$, we can further notice that ↑ki ($= \frac{k_p}{T_i} \rightarrow \uparrow k_p$), $k_{12}\, k_{21} < 0$, so flutter stability may arise.

The last condition that must be satisfied in order to have dynamic instability is $|k_{12}\, k_{21}| > \left(\frac{k_{11}-k_{22}}{2}\right)^2$.

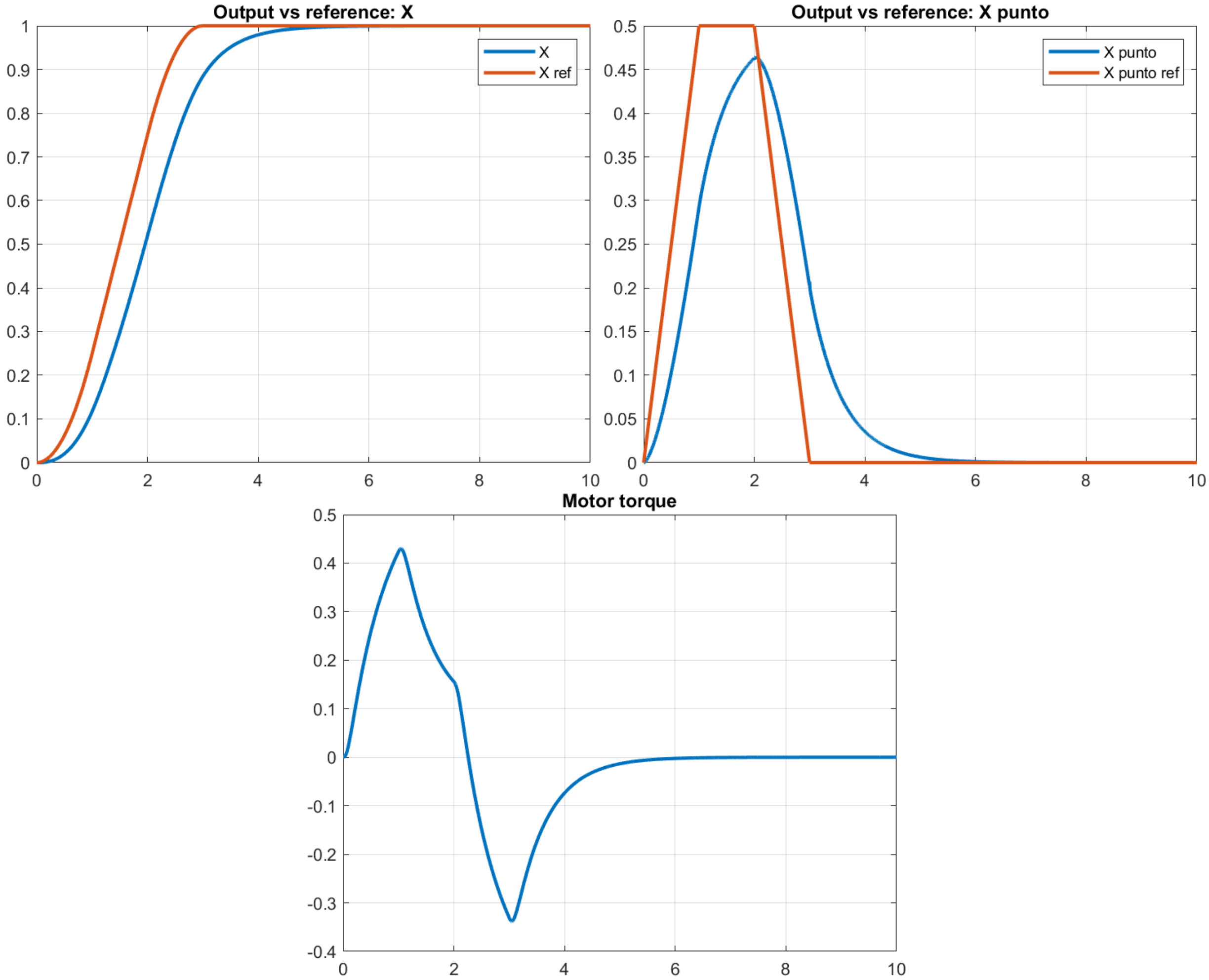
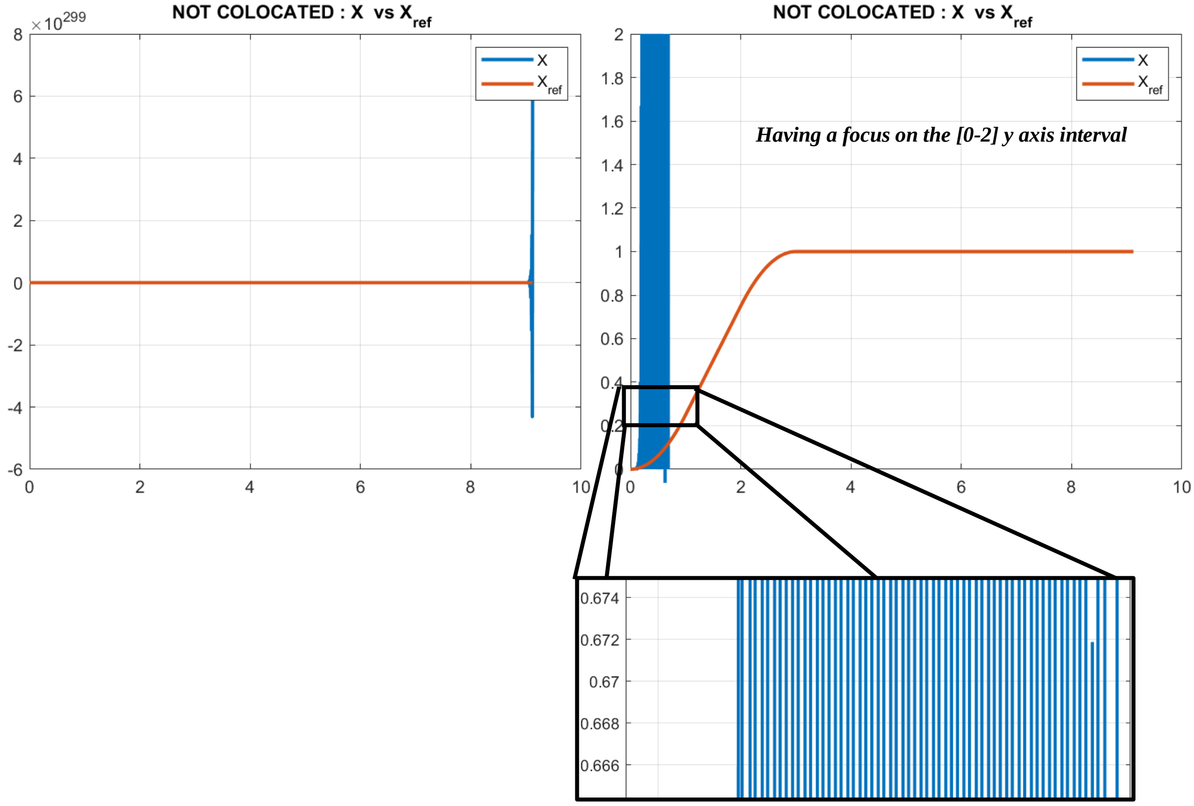


Fig.20 – CASE C – (2) 2 DOF system with a not colocated control and kp = 10



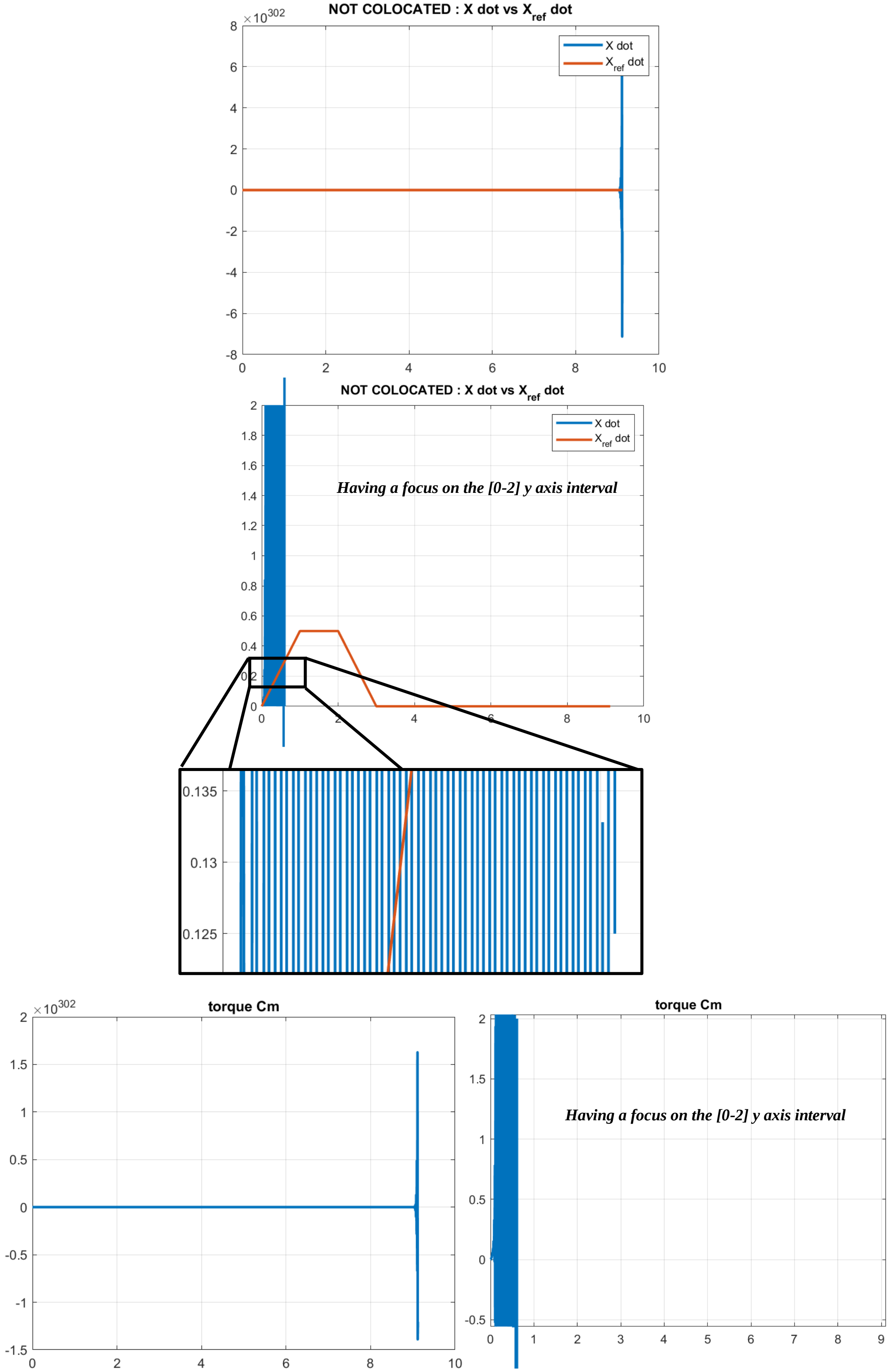


*Fig.21 – CASE C – (2) 2 DOF system with a not colocated control and **kp = 10000***