CONTROL AND ACTUATING DEVICES FOR MECHANICAL SYSTEMS

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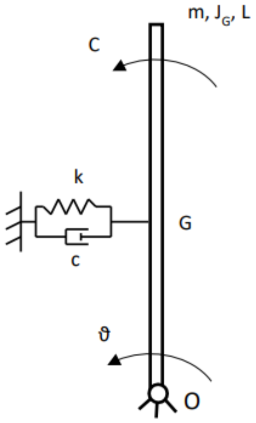
ASSIGNMENT 1: CONTROL OF 1-DOF MECHANICAL SYSTEMS

The mechanical system represented in the figure lies in the vertical plane. It is composed of a uniform beam with mass m, mass moment of inertia J_G about its centre of mass G and length L. The beam is pinned to the ground in O and it is connected to the ground in its centre of mass G by a spring of stiffness constant k and a viscous damper of damping constant c. A torque C is applied to the beam.



Parameter	Value
m [kg]	3
L [m]	4
c [Ns/m]	10
k [N/m]	See Table 2

Table 1 – System data

Stiffness k	k [N/m]
k_1	150
k_2	Such that the equivalent stiffness K*=0
k_3	13

Table 2 – Values of stiffness k

After having derived the linearised equation of motion of the system representing the small oscillations about the static equilibrium position ϑ_o= 0 (upwards vertical beam):

$$\left(\frac{mL^2}{4}+J_G\right)\ddot{\vartheta}+\left(\frac{cL^2}{4}\right)\dot{\vartheta}+\left(\frac{kL^2}{4}-\frac{mgL}{2}\right)\vartheta=C(t)$$

$$m^*\ddot{\vartheta}+c^*\dot{\vartheta}+k^*\vartheta=C$$

analyse the system considering the following types of control:

1) **Proportional control** (stability and performances of the system in the time domain and Laplace domain for increasing values of Kp, considering the 3 values of k reported in Table 2)
2) **Proportional derivative control**
3) **Proportional integral control**
    a. effect of different positions of the zero with respect to the poles of the uncontrolled system

CASE 1: Proportional control (P) of the angular position of the beam

$$m^*\ddot{\vartheta} + c^*\dot{\vartheta} + k^*\vartheta = C \rightarrow m^*\ddot{\vartheta} + c^*\dot{\vartheta} + (k^* + k_p)\,\vartheta = k_p\,\vartheta_{ref}$$

→ **LAPLACE**: $(m^*s^2 + r^*s + k^*)\Theta(s) = kp(\Theta_{ref}(s) - \Theta(s))$

$\boldsymbol{k^* > 0}$	**Stable system $\forall kp$**
$\boldsymbol{k^* = 0}$	**Stable system $\forall kp$**
$\boldsymbol{k^* < 0}$	**Stable for ↑ kp**

$\mathbf{k^* > 0} \rightarrow \mathrm{k_1} = 150\,{}^{\mathrm{N}}/_{\mathrm{m}}$

$\frac{kL^2}{4} > \frac{mgL}{2} \rightarrow k^* = 541.14\,Nm > 0$ → *uncontrolled system is asymptotically stable*

$\omega_n = 5.8156\,\frac{rad}{s}$

$\lambda_{1,2} = -1.2500 \pm 5.6797i$ → **Underdamped system**

The use of the proportional control will increase the natural frequency of the controlled system (the imaginary part of the eigenvalues will change, not the real part since there is no effect on α, P controller acts only on the stiffness parameter)

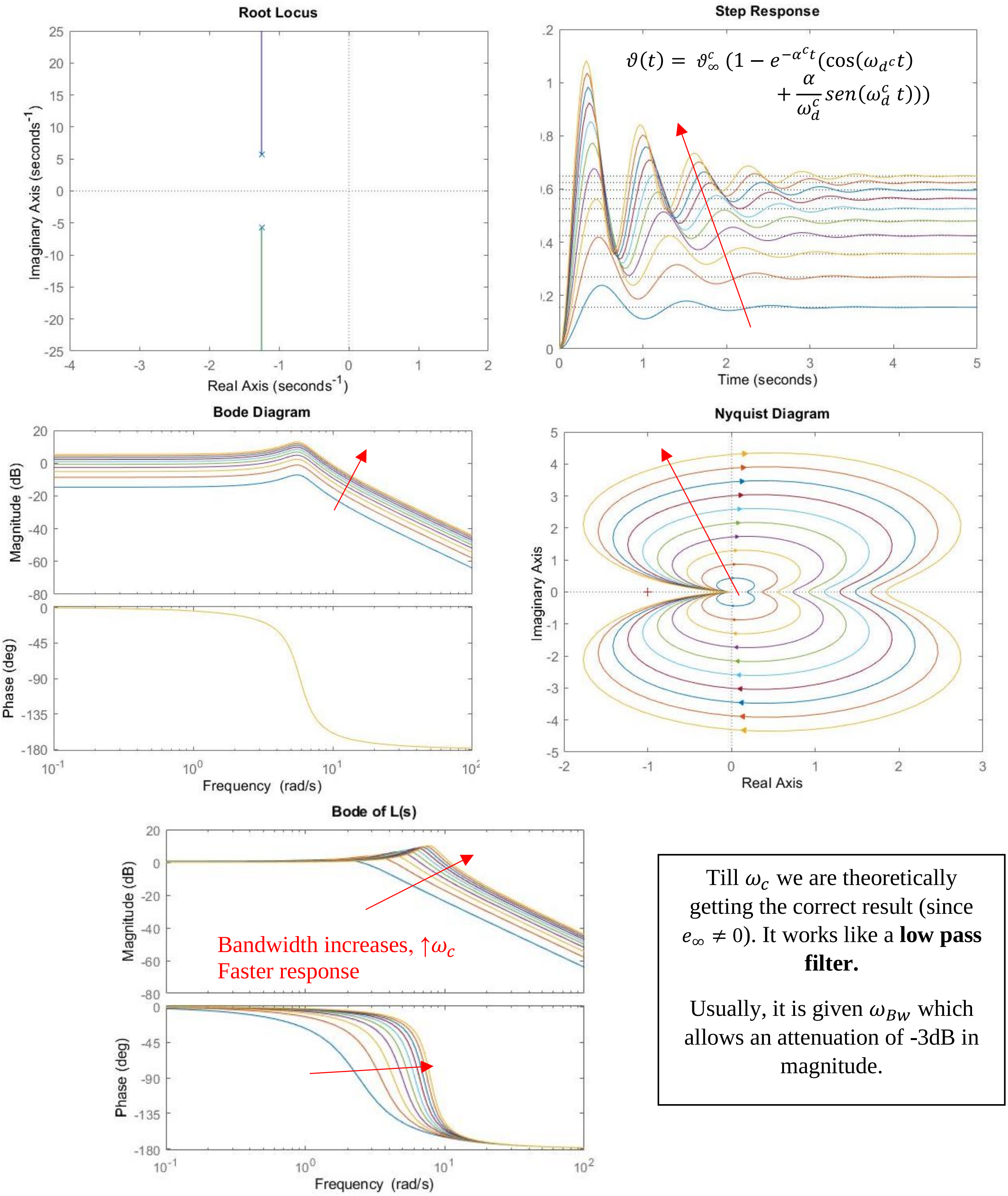
	Kp = 100
In time domain:	$\omega_n^c = \sqrt{\frac{k^*+k_p}{m^*}} = 6.3302\,{}^{rad}/_{s} > \omega_n$ $\omega_d^c > \omega_d \rightarrow t_r \downarrow$ $\lambda_{1,2}^c = -h^c\,\omega_n^c \pm i\omega_n^c\sqrt{1-h^{c2}} = -1.2500 \pm 6.2055\,\mathrm{i}$ $h^c < h \rightarrow PO \uparrow$ $\alpha_c = \alpha \rightarrow t_s =$
Passing in Laplace domain:	$GH^c = \frac{100}{16\,s^2 + 40\,s + 541.1}$ $\mu^c = -14.662 = 20\log\left(\frac{k_p}{k^*}\right)$ $L^c = \frac{100}{16\,s^2 + 40\,s + 641.1}$ $p_{1,2}^c = -1.2500 \pm 6.2055i$ $\vartheta_\infty^c = 0.1560 = \frac{k_p}{k^* + k_p} \rightarrow e_\infty \downarrow$

*Tab. 3 – Characteristics parameters of **P controller** applied to a 1 DOF mechanical system having **k* > 0** both in time and Laplace domain.*

From the figures we can notice that increasing k_p:

- From the **ROOT LOCUS** we see that for k_p that goes to plus infinity, the system will be always stable. (Note that: *the first ones identified with the "x" are the poles of the uncontrolled system*)
  - $\theta_a = \pm\frac{\pi}{2}\,[n = 2, m = 0, k = 1, q = 2]\ and\ \sigma_a = -\frac{c^*}{2m^*} = -\alpha$
- Analysing the **STEP RESPONSE,** increasing k_p increases the natural frequency of the system and the overshoot, while the adimensional dumping h decreases therefore we have increased vibrations, and a lower the steady state error. Since the poles of the controlled system are always CC, we have always an oscillating system.
- The **BODE** amplitude moves upwards as k_p becomes larger, the bandwidth widens and the cross point with 0dB is moved on the right. The phase doesn't change. The stability isn't affected. We can use bode criterion having a minimum phase GH function crossing the 0dB only one time.
  - $\begin{cases} p_m > 0 \\ |G_m|_{dB} \rightarrow \infty \end{cases} \forall kp$ → stable
  - Note that: *MATLAB plot the **actual Bode diagram**, not the asymptotic one, therefore we can see the resonance due to the 2 complex conjugate poles.*
- In **NYQUIST** diagram we notice that increasing k_p we enlarge the amplitude of the diagram, but we never get any encirclement around (-1,0) → the system remains stable.

- $\begin{cases} \tilde{N} = 0 \\ P = 0 \end{cases}$ $\forall kp \rightarrow$ stable



Till ω_c we are theoretically getting the correct result (since $e_\infty \neq 0$). It works like a **low pass filter.**

Usually, it is given ω_{Bw} which allows an attenuation of -3dB in magnitude.

*Fig. 1 – Diagrams resulting from a **P controller** applied to a 1 DOF mechanical system having **k* > 0** and **vector of kp coefficients [100:100:1000]***

$$\mathbf{k^* = 0} \rightarrow \mathrm{k_2} = \frac{\mathrm{mgL} * 4}{2 * \mathrm{L}^2} = \frac{2\mathrm{mg}}{\mathrm{L}}$$

$\frac{\boldsymbol{kL^2}}{\mathbf{4}} = \frac{\boldsymbol{mgL}}{\mathbf{2}} \rightarrow \lambda\left(\lambda + \frac{c^*}{m^*}\right) = 0 \begin{cases} \lambda_1 = 0 \\ \lambda_2 = -\frac{c^*}{m^*} = -2.5 \end{cases}$

Therefore, the **uncontrolled** system is just stable (***neutral equilibrium*** = we can stay at the equilibrium with 0 control force, having 0 steady state error → *we have a pole in the origin*). On the other hand, the **controlled** system is asymptotically stable since $k_p > 0$ *always*, and if it is sufficiently large, we have an oscillating system.

	Kp = 10	Kp = 100
In time domain:	$\omega_n^c = 0.7906\ rad/s$ $\lambda_1 = -2.2182$ $\lambda_2 = -0.2818$ (*Overdamped* system)	$\omega_n^c = \sqrt{\frac{k_p}{m^*}} = 2.5\ rad/s$ $\lambda_{1,2}^c = -h^c\ \omega_n^c \pm i\ \omega_n \sqrt{1-h^{c^2}}$ $= -1.2500 \pm 2.1651$ i (*Underdamped* system)
Passing in Laplace domain:	$GH^c = \frac{10}{16\,s^2 + 40\,s}$ $L^c = \frac{10}{16\,s^2 + 40\,s + 10}$ $p_1^c = -2.2182$ $p_2^c = -0.2818$ $\vartheta_\infty^c = \frac{k_p}{k_p} = 1 \rightarrow e_\infty = 0$	$GH^c = \frac{100}{16\,s^2 + 40\,s}$ $L^c = \frac{100}{16\,s^2 + 40\,s + 100}$ $p_{1,2}^c\ -1.2500 \pm\ 2.1651i$ $\vartheta_\infty^c = 1 = \frac{k_p}{k_p} \rightarrow e_\infty = 0$ (This comes from the characteristic of the system, not of the control)

Tab. 4 – Characteristics parameters of ***P controller*** *applied to a 1 DOF mechanical system having* ***k* = 0*** *both in time and Laplace domain*

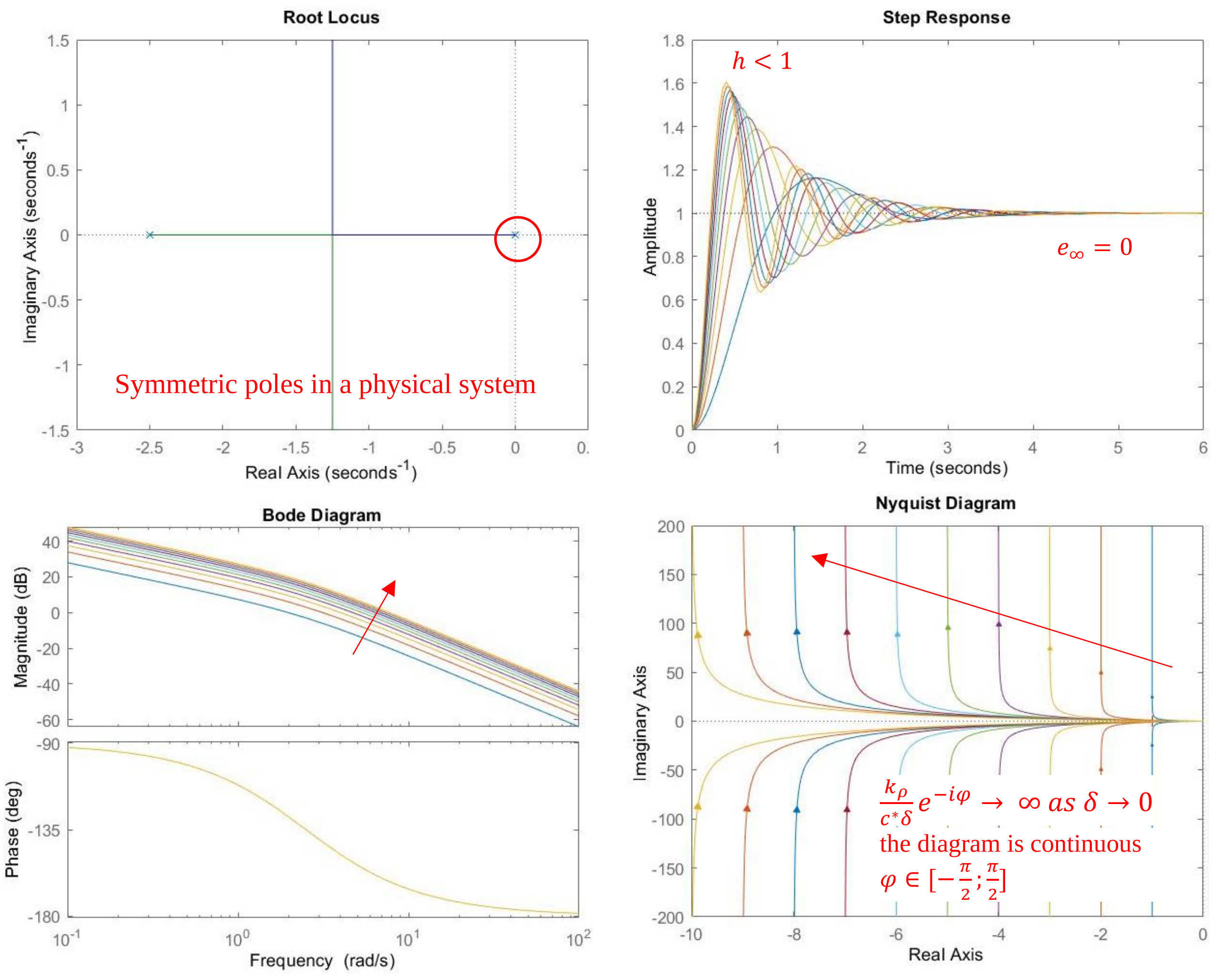


Fig. 2 – Diagrams resulting from a ***P controller*** *applied to a 1 DOF mechanical system having* ***k* = 0*** *and* ***vector of kp coefficients [100:100:1000]***

According to what we have seen in the theory, if $k^* = 0$ it means that we get a pole in the origin.

- In the **ROOT LOCUS** we can see that for increasing value of k_p , the system remains always stable. For low value of k_p system results overdamped, otherwise it's underdamped = the 2 real poles reach the asymptotes and then they depart from the real axis as 2 CC poles. Because of the pole in the origin, the steady state response is equal to 1. Increasing k_p we get an increase in the natural frequency and overshoot while the adimensional damping decreases, therefore vibrations start to rise. We can also find the position of the breakaway point on the Re axis.
  - $\theta_a = \pm\frac{\pi}{2}$ $[n = 2, m = 0, k = 1, q = 2]$
  - $\sigma_a = -\alpha$
  - $\downarrow k_p$ real poles (overdamped system)
  - $\uparrow k_p$ complex conjugate poles (underdamped system)
- **BODE** starts with a slope of $-20dB/dec$ and with a -π in phase. The system remains stable. Increasing k_p: system remains stable, bode amplitude is moved upwards, so the bandwidth enlarges. The phase doesn't change, it is the actual diagram of the phase.
  - $\begin{cases} p_m > 0 \\ |G_m|_{dB} \to \infty \end{cases}$ $\forall kp$ → stable
- In **NYQUIST** for higher values of k_p the amplitude increases without any encirclements around (-1, 0). Having a pole in the origin the *cs closed path* must change (**Cauchy principle of arguments**).
  - $\begin{cases} \tilde{N} = 0 \\ P = 0 \end{cases}$ $\forall kp$ → stable

<u>*Note that*</u> changing the range in which k_p varies, for example $k_p = 3 \div 30$, the response does not show oscillations, resulting in the case of overdamped system.

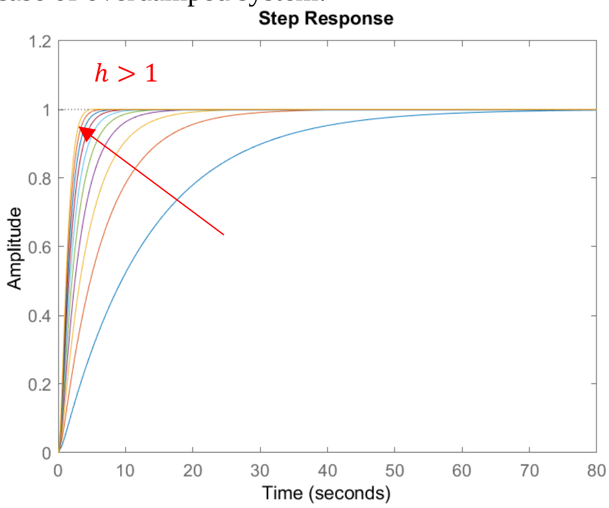
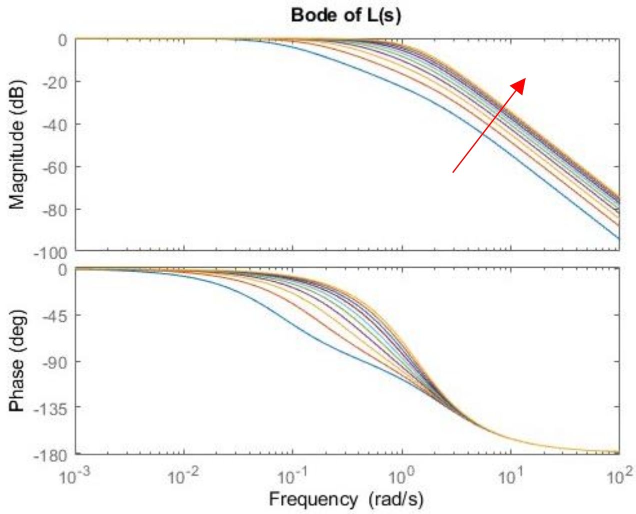


*Fig 3. – Diagram of the step response of **a P controller** applied to a 1 DOF mechanical system having **k* = 0** and **vector of kp coefficients [3:3:30]***



Till ω_c we are really getting the correct result since $e_\infty = 0$. This is because the system is stable, not asymptotically. The approximation of L is true.

- No distortions of amplitude in the quasi-static zone

*Fig. 4 – Bode diagrams of L resulting from a **P controller** applied to a 1 DOF system having **k* = 0** and **vector of kp [100:100:1000]***

$\mathbf{k^* < 0} \rightarrow \mathrm{k_3} = 13\ \mathrm{N/m}$

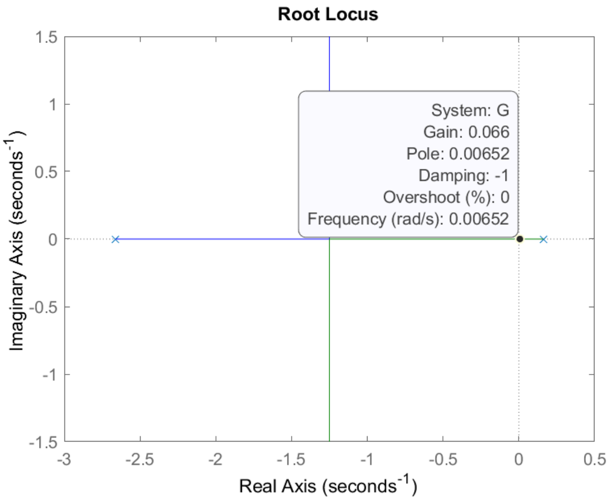
$\frac{kL^2}{4} > \frac{mgL}{2} \rightarrow k^* = -6.86\ Nm < 0$ → *uncontrolled system is statically unstable.*

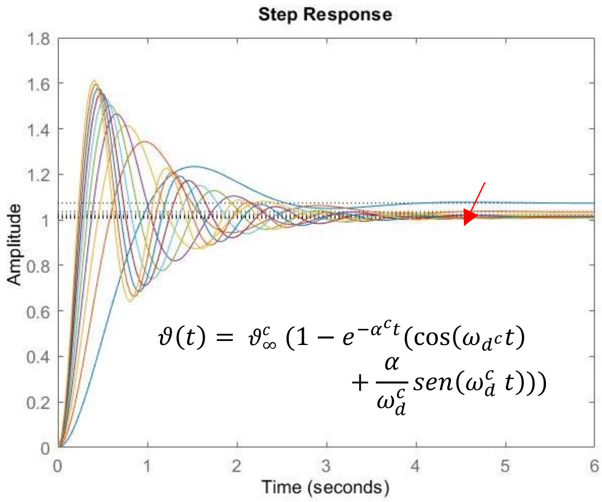
$\lambda_1 = 0.1611\ and\ \lambda_2 = -2.6611$ → in fact, since $\sqrt{\Delta} > |-\alpha|$, we have 2 real eigenvalues, one of them is positive. The use of the proportional control will switch the nature of the eigenvalues, as a matter of fact if k_p is increased enough the system from overdamped becomes underdamped ($\omega_n^c \uparrow$).

	Kp = 100
In time domain:	$\lambda_{1,2}^c = -h^c\omega_n^c \pm i\ \omega_n\sqrt{1-h^2} = -1.2500 \pm 2.0637i$ (Underdamped system) $\boldsymbol{k_p} \uparrow$ **:** $\uparrow \omega_n^c = \sqrt{\frac{k^*+k_p}{m^*}} = 2.4127\ rad/s$ $\omega_d^c \uparrow$ so $t_r \downarrow$ $h^c \downarrow and\ PO \uparrow$ $\alpha_c = so\ t_s =$
Passing in Laplace domain:	$GH^c = \frac{100}{16\ s^2 + 40\ s - 6.86}$ $\mu^c = \frac{k_p}{k^*} < 0 \rightarrow \lvert\mu_c\rvert > 1$ $L^c = \frac{100}{16\ s^2 + 40\ s + 94.14}$ $p_{1,2}^c = -1.2500 \pm 2.0637i$ $\vartheta_\infty^c = 1.0737 = \frac{k_p}{k^* + k_p} > 1 \rightarrow k_p \uparrow, \vartheta_\infty^c \downarrow, e_\infty \downarrow$

*Tab. 5 – Characteristics parameters of **P controller** applied to a 1 DOF system having **$k^* < 0$** both in time and Laplace domain.*

In this case, because of $k^* < 0$ (**static instability**), the uncontrolled system has one pole with positive real part ($Re\{p_1\} = \alpha_1$) therefore the *Bode criterion can't be used* because GH isn't anymore a minimum phase system. For the evaluation of stability, we can use only Nyquist and Root locus.





With a k_p *that provide a gain* $\mu^c = 20\log(k_p/k^*) \geq 0.066$ we can stabilise the system.

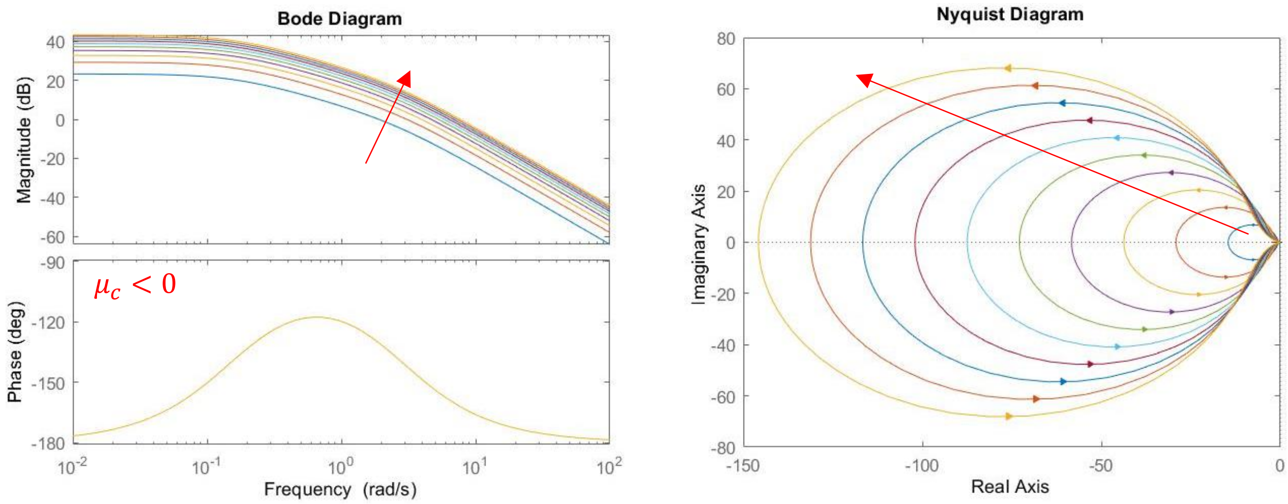
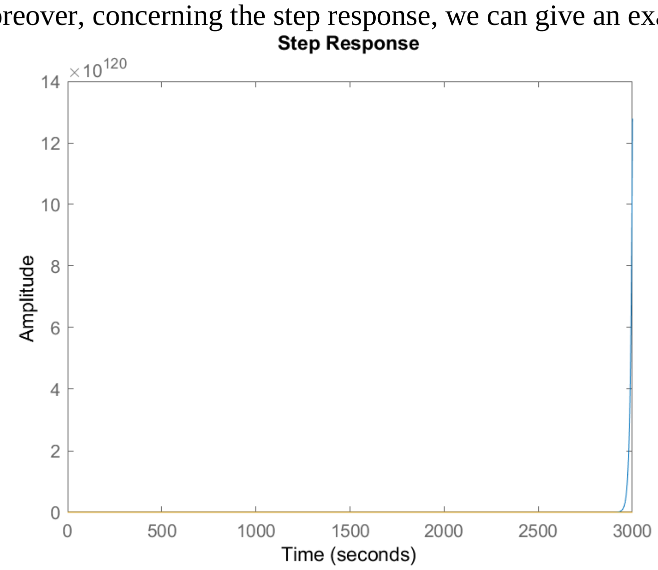


*Fig. 5 – Diagrams resulting from a **P controller** applied to a 1 DOF mechanical system having **k* < 0** and **vector of kp [100:100:1000]***

- **NYQUIST DIAGRAM**: we have 2 different conditions:
  - $\downarrow k_p : \begin{cases} \tilde{N} = 0 \\ P = 1 \end{cases}$ → unstable controlled system
  - $\uparrow k_p : \begin{cases} \tilde{N} = 1 \\ P = 1 \end{cases}$ → stable controlled system
    - Note that $\tilde{N}$ are encirclements in the counterclockwise direction.
- **ROOT LOCUS**: if k_p is low, one pole remains in the positive real plane, as soon as k_p increase the system becomes stable, firstly as an overdamped system (no oscillations, poles real and negative), then as an underdamped system (oscillations occurs since poles become complex conjugates)
  - $n = 2, m = 0, q = 2, k = 1 \rightarrow \vartheta_a = \pm\pi/2$
  - $\sigma_a = -{c^*}/{2m^*} = -\alpha$
  - Break away point can be computed: $N'_{GH} D_{GH} - D'_{GH} N_{GH} = 0$
- **STEP RESPONSE**: it can be noticed that when k_p is low, the system is overdamped; and since $k^* < 0$ → $\vartheta^c_\infty > 1$
  - $k_p \uparrow, \vartheta^c_\infty \downarrow, e_\infty \downarrow$

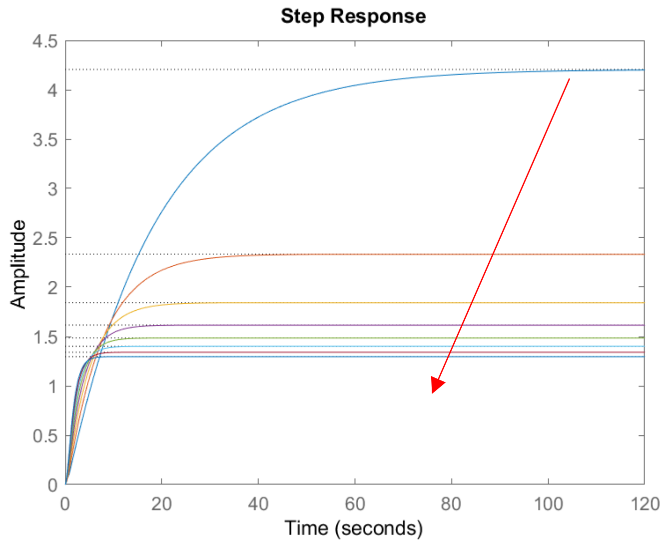
Moreover, concerning the step response, we can give an example of unstable (static) response having low kp:



Consequently, if we do not use a control gain kp high enough, the controlled system is not stable: there is an exponential with positive exponent in the final transient step response.

$$\vartheta_g(t) = Ae^{\alpha_1^c t} + Be^{-\alpha_2^c t}$$

*Fig. 6 – Step response of a **P controller** applied to a 1 DOF mechanical system having **k* < 0** and **vector of kp coefficients [3:3:30]***



On the other hand, for kp greater than almost 7, the controlled system is stable and overdamped.

Fig. 7 – Step response of a ***P controller*** *applied to a 1 DOF mechanical system having* ***k* < 0*** *and* ***vector of kp coefficients [9:3:30]***

CASE 2: Proportional derivative control (PD) of the angular position of the beam

$$m^*\ddot{\vartheta} + c^*\dot{\vartheta} + k^*\vartheta = C \rightarrow m^*\ddot{\vartheta} + (c^* + k_d)\,\dot{\vartheta} + (k^* + k_p)\,\vartheta = k_p\,\vartheta_{ref} + k_d\,\dot{\vartheta}_{ref}$$

$$\rightarrow \textbf{LAPLACE}: (m^*s^2 + r^*s + k^*)\Theta(s) = kp(1 + T_d s)\left(\Theta_{ref}(s) - \Theta(s)\right)$$

We change the damping of the controlled system, increasing it; deriving the reference the rise time is lower. Tuning Td we can introduce the zero of the controllers before or after the poles having different effects as shown below:

	↓ frequency zero: Td ↑ → |z| <|p|	↑ frequency zero: Td ↓ → |z| > |p|
$k^* > 0$	Stable system $\forall kp$	Stable system $\forall kp$
$k^* = 0$	**Stable system $\forall kp$**	**Stable system $\forall kp$**
$k^* < 0$	Stable for ↑ kp	Stable for ↑ kp

$k^* > 0$

<table>
<tr><td></td><td colspan="2">Kp – Kd (Td*Kp)</td></tr>
<tr><td rowspan="2">In time domain:</td><td colspan="2">$h < 1$: the uncontrolled response is underdamped
$\omega_n^c = \sqrt{\frac{k^*+k_p}{m^*}} > \omega_n$ as a P controller
$\omega_d^c = \omega_n^c\sqrt{1 - {h^c}^2}$
$\lambda_{1,2}^c = -\frac{c^* + k_d}{2m^*} \pm \sqrt{\left(\frac{c^* + k_d}{2m^*}\right)^2 - \frac{k^* + k_p}{m^*}}$</td></tr>
<tr><td>↑ kp, fixed kd:
ω_d^c, ω_n^c ↑ → t_r ↓
h^c ↓ → PO ↑
$\alpha_c =$ → $t_s =$
e_∞ ↓ since ϑ_∞ ↑
$\dot{\vartheta}\,(0^+) = \frac{k_d}{m^*}$ doesn't change</td><td>kd ↑, fixed kp:
h^c ↑ → PO ↓
the 2 solutions can switch from CC to real and negative
$\omega_n^c = (e_\infty =)$
but ω_d^c ↓ due to h^c ↑
$\alpha_c = \frac{c^* + k_d}{2 * m^*}$ ↑ → t_s ↓
$\dot{\vartheta}\,(0^+) = \frac{k_d}{m^*}$ ↑ → t_r ↓</td></tr>
</table>

Tab. 3 – Characteristics parameters of ***P controller*** *applied to a 1 DOF system having* ***k* < 0*** *both in time and Laplace domain*

Td=1: the zero is at very low frequency so it is before the two cc poles.

- We can apply the **BODE** criterion and we can see that the system is always stable because pm>0 always, in fact the phase arrives to $-\pi$ from the upper part.
- In **NYQUIST** we have no encirclement around (-1,0)
    - $\begin{cases} \tilde{N} = 0 \\ P = 0 \end{cases}$ $\forall kp$ → stable
- In the **ROOT LOCUS** we have that for high value of kp the two cc poles become real, and one goes to $+\infty$ and the other to 0. If we exaggerate with the value of kp the system doesn’t oscillate anymore.
- Analysing the **STEP RESPONSE**, we have that increasing kp: increase ω_n, the overshoot, and the vibrations because h decreases, and the system remains always stable.

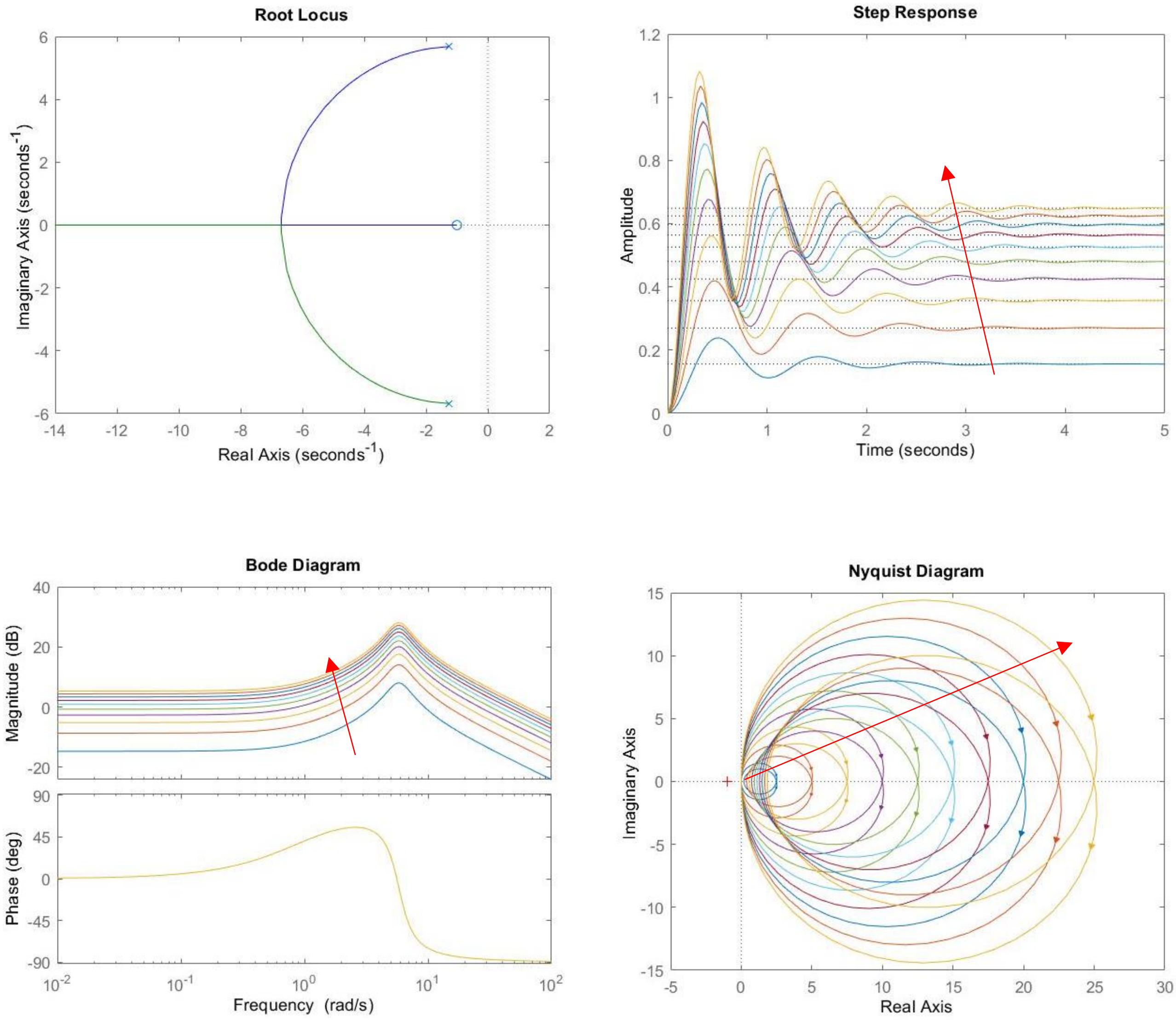


Fig.9 – Diagrams resulting from a *__PD controller__* *applied to a 1 DOF mechanical system having* ***__k* > 0__*** *and* ***vector of kp [100:100:1000] and Td = 1***

Having a focus on the time response of the system increasing k_d, it can be noticed that the time derivative of $\dot{\vartheta}(0^+) = k_d\,/m^* \neq 0$, therefore this term speed up the response of the system allowing a decrease in the rise time (higher promptness).

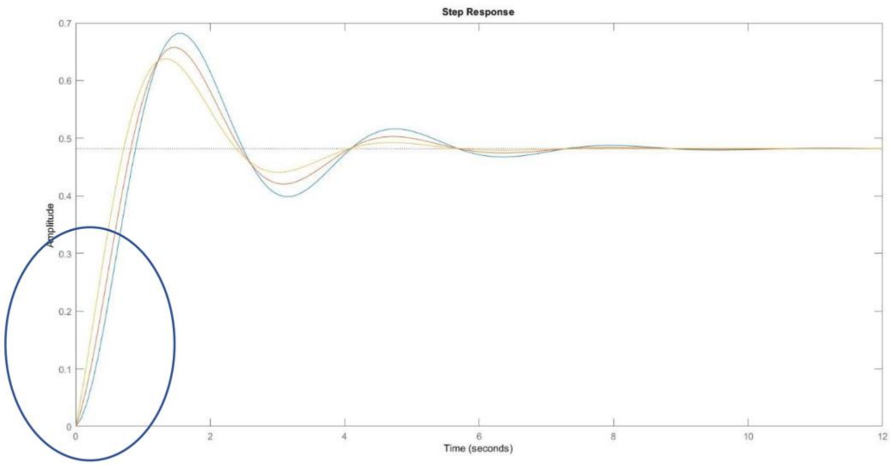


*Fig. 10 – Step response resulting from a PD **controller** applied to a 1 DOF mechanical system having **k* > 0** increasing Td*

This case having |zi| < |all the poles| will be much more stable ↑ k_p than the following case where |zi| > |all the poles|, moreover it has also a **greater bandwidth** and a faster response of the control system.

Td=0.02: the zero is placed at high frequency with respect the cc poles, but *stability isn't affected*. From **BODE** criterion: $p_m > 0, G_m \rightarrow \infty$. In both cases we have that e_∞ is reduced increasing k_p . We must underline that the derivative effect is very low having the zero at ↑ frequency.

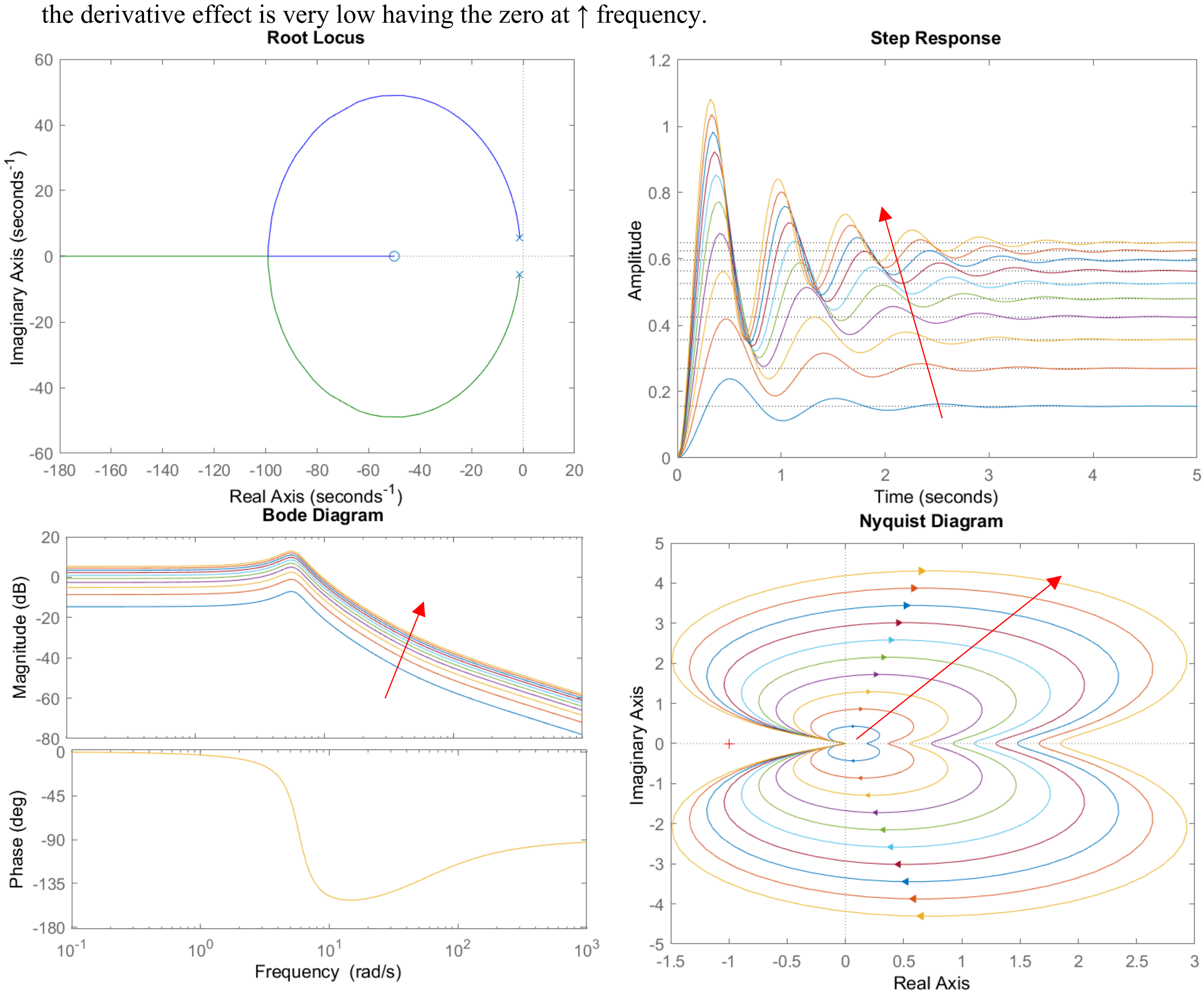


*Fig. 11– Diagrams resulting from a PD **controller** applied to a 1 DOF ì system having **k* > 0** and **vector of kp [100:100:1000] and Td = 0.02***

If the zero is before the poles, we have the ***phase lead*** problem. It is an anticipation of the phase due to the zero introduced by the control. This bump depends on the distance between the poles and the zero. Analysing the step response, we obtain a very fast response at the beginning, reaching slowly θ_∞. We can reduce this effect moving the zero closer to the 2 poles.

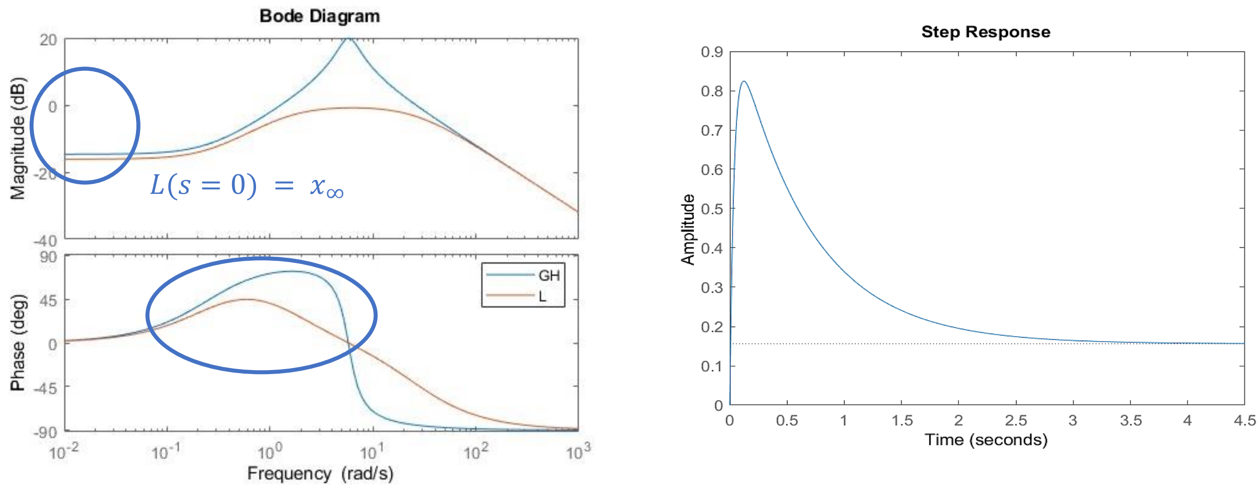


*Fig. 12 – Bode diagrams of GH and L and the step response resulting from a PD **controller** having **k* > 0**, **kp =100, Td = 4***

$\mathbf{k^* = 0}$

Td = 4: we have a pole in the origin given from the system itself that present a neutral static equilibrium.

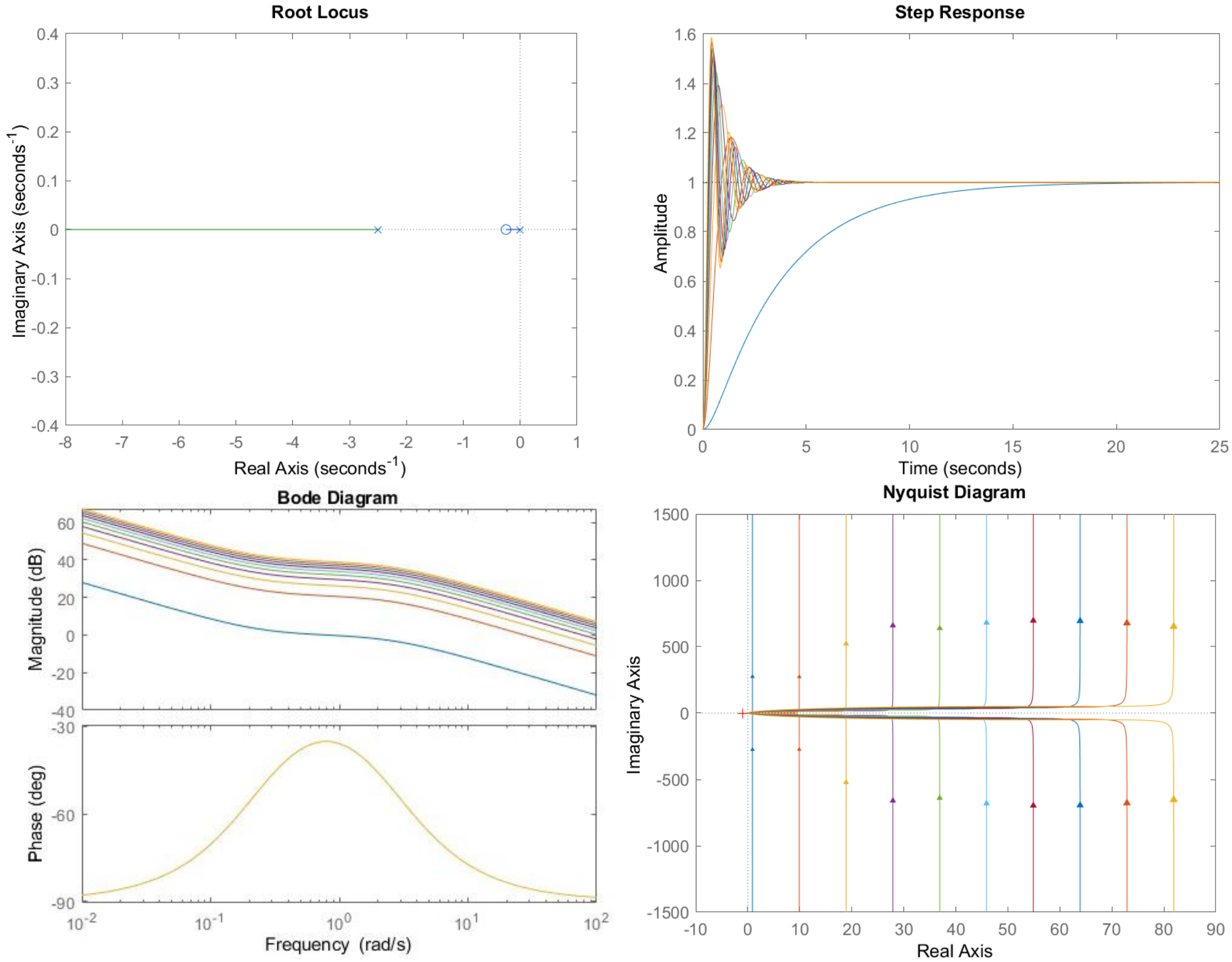


*Fig. 12 – Bode diagrams of GH and L and the step response resulting from a PD **controller** having **k* = 0**, **kp =[10:100:1000], Td = 4***

Td = 0.4: in order to reduce the phase lead phenomenon that appears increasing kp (the pole goes towards infinite, far from the zero).

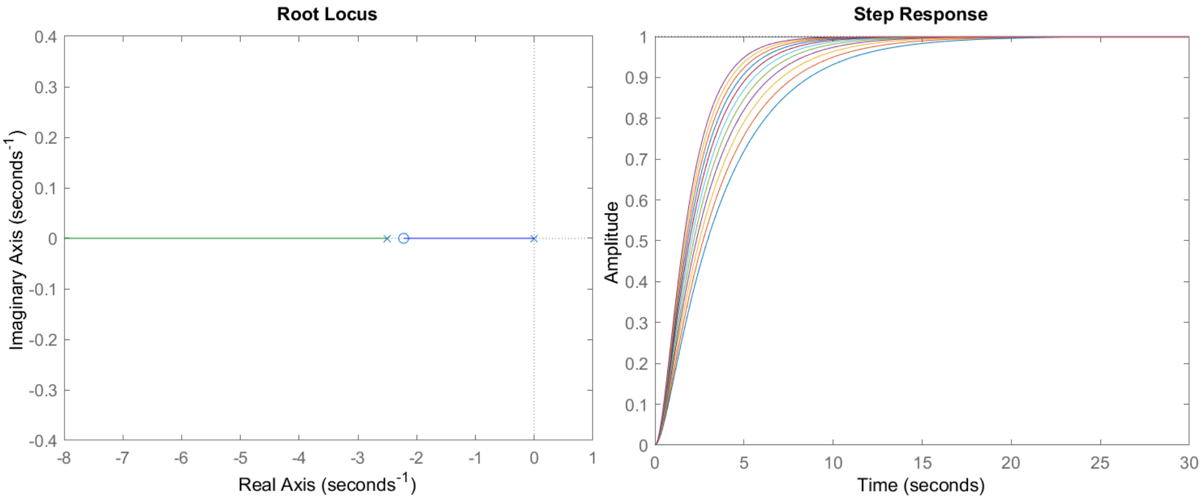
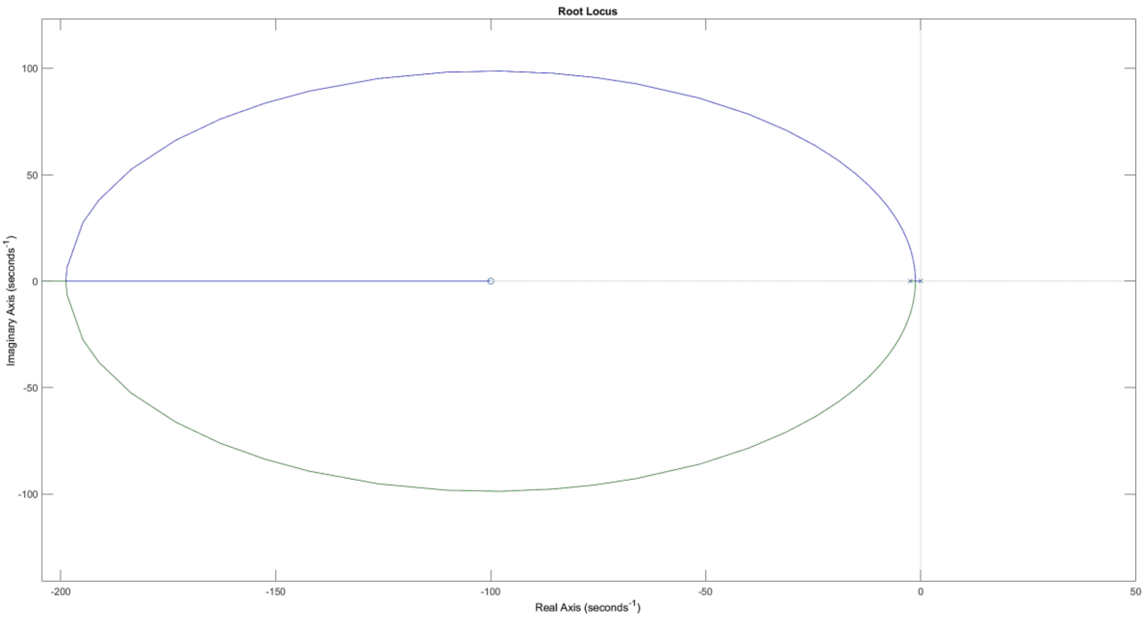
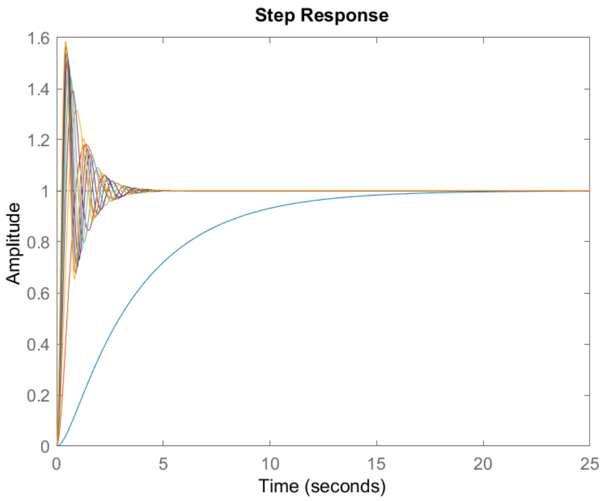
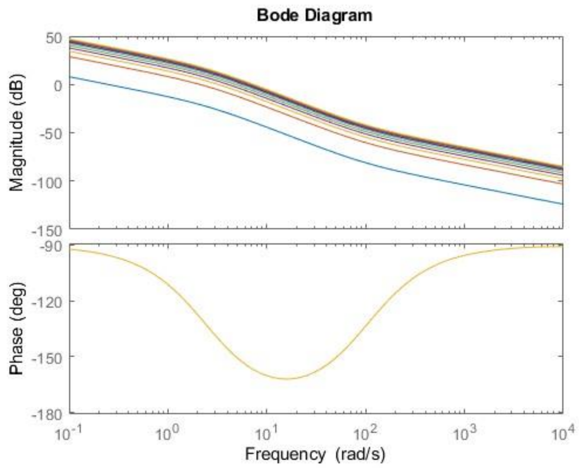


Fig. 12 – Bode diagrams of GH and L and the step response resulting from a PD ***controller*** *having* ***k* = 0, kp =[10:1:20], Td = 0.45***

Td = 0.01:







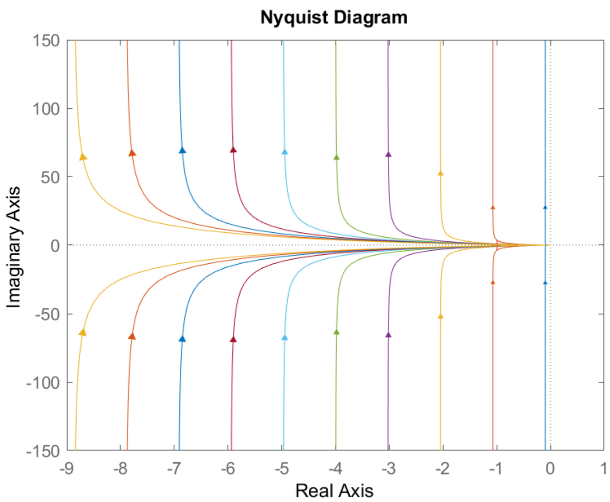


*Fig. 12 – Bode diagrams of GH and L and the step response resulting from a PD **controller** having **k* = 0, kp = [10:100:1000], Td = 0.01***

$\mathbf{k^* < 0}$

Td = 4: unstable uncontrolled system

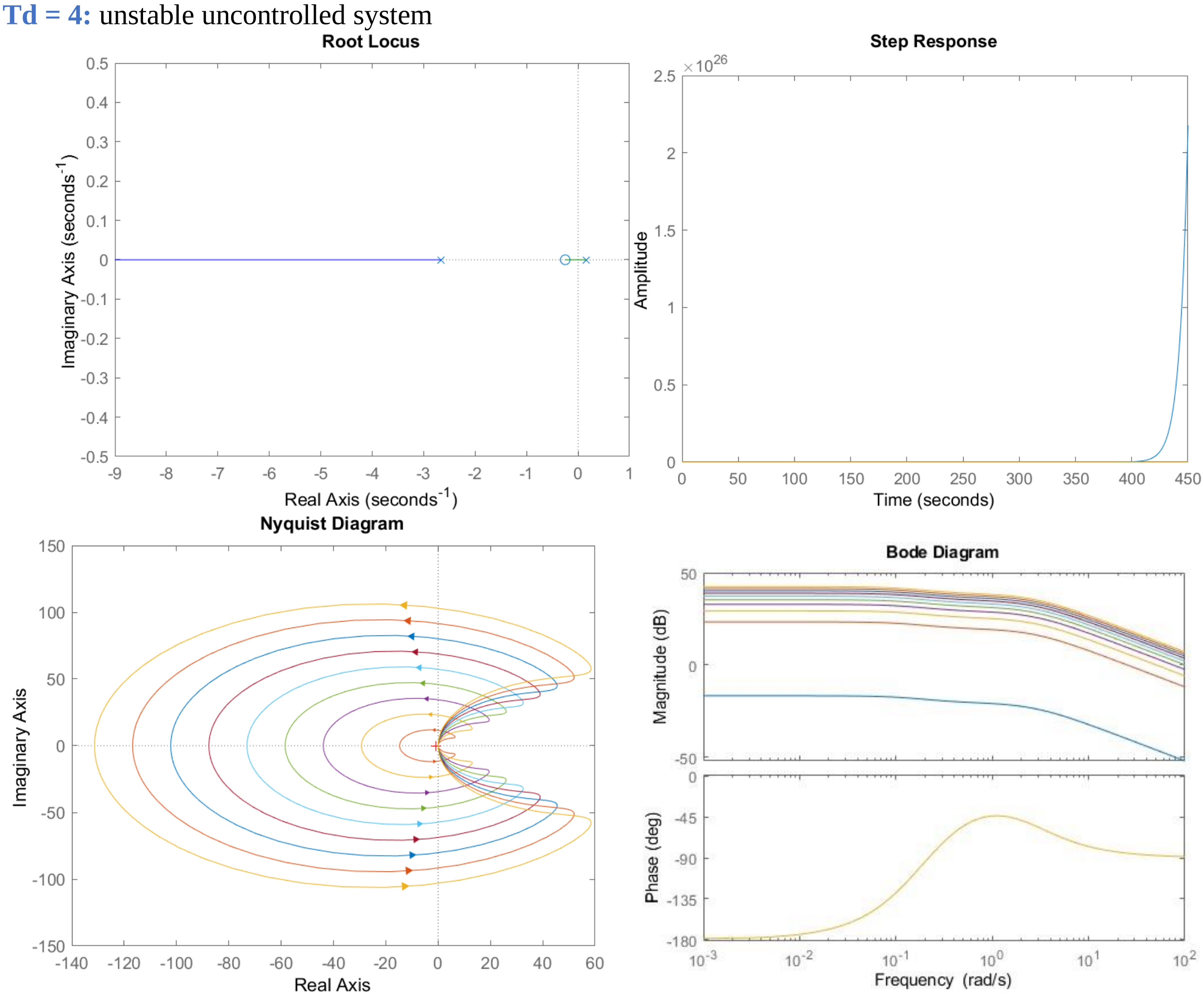


*Fig. 12 – Bode diagrams of GH and L and the step response resulting from a PD **controller** having **k* < 0, kp =[1:100:1000], Td =4***

Td = 0.01:

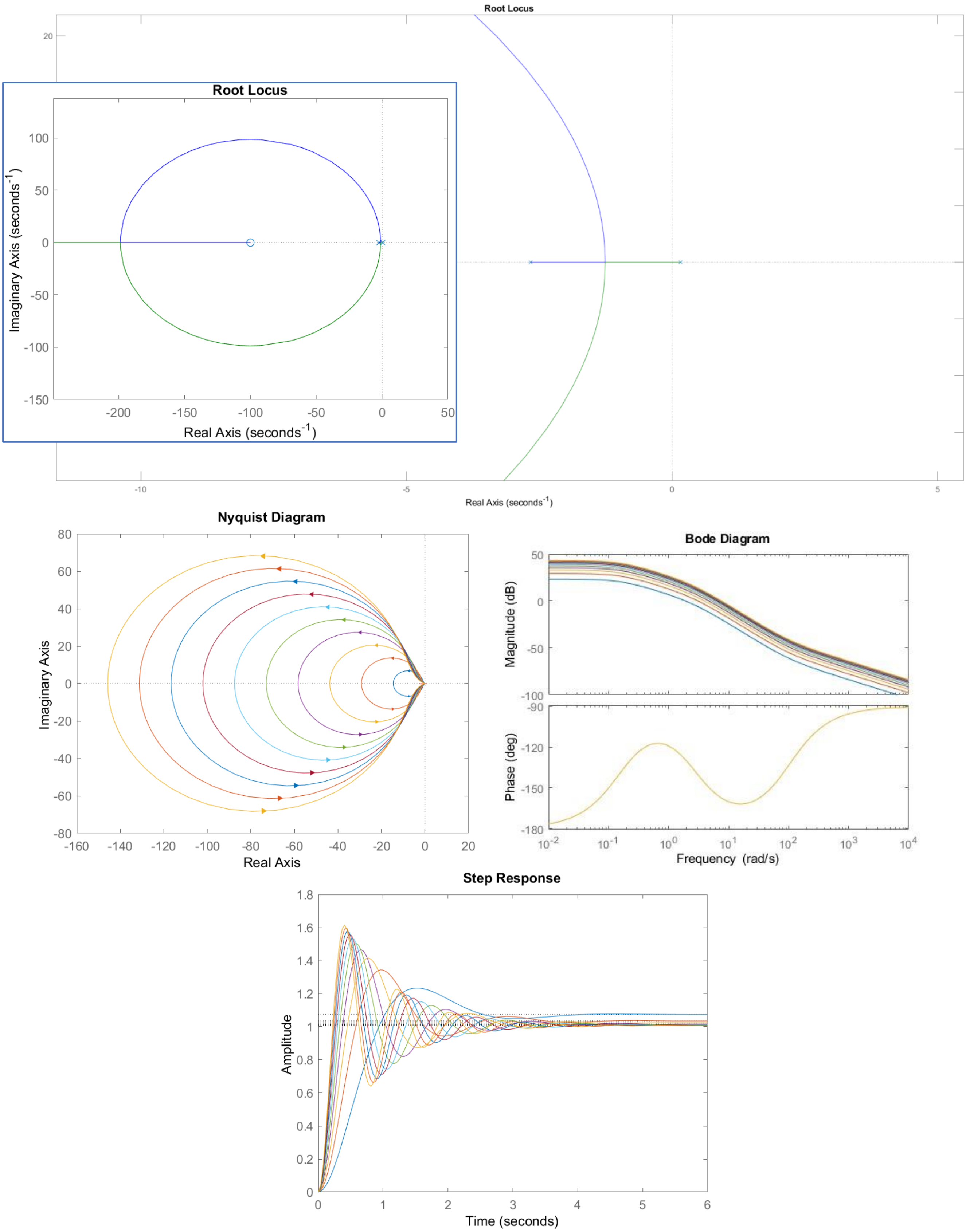


Fig. 12 – Bode diagrams of GH and L and the step response resulting from a PD ***controller*** *having* ***k* <0, kp =[100:100:1000], Td = 0.01***

CASE 3: Proportional integral control (PI) of the angular position of the beam

$$\rightarrow \textbf{LAPLACE}: (m^*s^2 + r^*s + k^*)\Theta(s) = \frac{kp(1+T_is)}{T_is}\left(\Theta_{ref}(s) - \Theta(s)\right)$$

Because of the presence of an integral operator, we have a pole in the origin, therefore a **steady state error equal to zero** and an infinite gain at zero frequency. The proportional control introduces a new state in the system.

	↓ frequency zero: Ti ↑ → |z| <|p|	↑ frequency zero: Ti ↓ → |z| > |p|
$k^* > 0$	Stable system $\forall kp$	**Stable for ↓ kp**
$k^* = 0$	Stable system $\forall kp$	Unstable $\forall kp$
$k^* < 0$	Stable for ↑ kp	Unstable $\forall kp$

$\mathbf{k^* > 0}$

$$\boldsymbol{k_i = \frac{k_p}{T_i}\begin{cases} T_i = 0.02 \rightarrow worst\ case\ |z_i| > |all\ the\ poles| \\ T_i = 20 \rightarrow best\ case\ |z_i| < |all\ the\ poles| \end{cases}}$$

Ti=20: $\omega_n = 5.8156\ rad/s$

$\lambda_{1,2} = -1.2500 \pm i\ 5.6797$

	Kp = 150
In time domain:	$\omega_n^c = 6.5724\ {}^{rad}/_{s} > \omega_n$
Passing in Laplace domain (easier analysis):	$GH^c = \dfrac{3000\ s + 150}{320\ s^3 + 800\ s^2 + 1.082\ 10^4\ s}$ $z_1 = -0.05 = -\dfrac{1}{T_i}$ $\mu^c = -14.662 = 20\log\left(\dfrac{k_p}{k^* * T_i}\right)$ $L^c = \dfrac{3000\ s + 150}{320\ s^3 + 800\ s^2 + 1.382\ 10^4\ s + 150}$ $p_1^c = -0.0109$ $p_{2,3}^c = -1.2446 \pm\ 6.4514i$ $\vartheta_\infty^c = 1 \rightarrow\ e_\infty = 0$

*Tab. 3 – Characteristics parameters of **PI controller** applied to a 1 DOF mechanical system having **k* < 0** both in time and Laplace domain having Ti = 20*

- The **BODE** criterion is allowed since GH is a minimum phase function and we see that the system is stable because:
  - $\begin{cases} p_m > 0 \\ |G_m|_{dB} \rightarrow \infty \end{cases} \forall kp$ → stable
- Increasing $k_p$the system stays stable because we enlarge the amplitude of **NYQUIST** diagram, but we don't obtain any encirclement around the (-1,0) point.
- The **STEP RESPONSE** arrives at the reference without oscillations and the error is nil because of the pole in the origin.
  - We must look at the **dominant pole (not oscillating response increasing kp)**

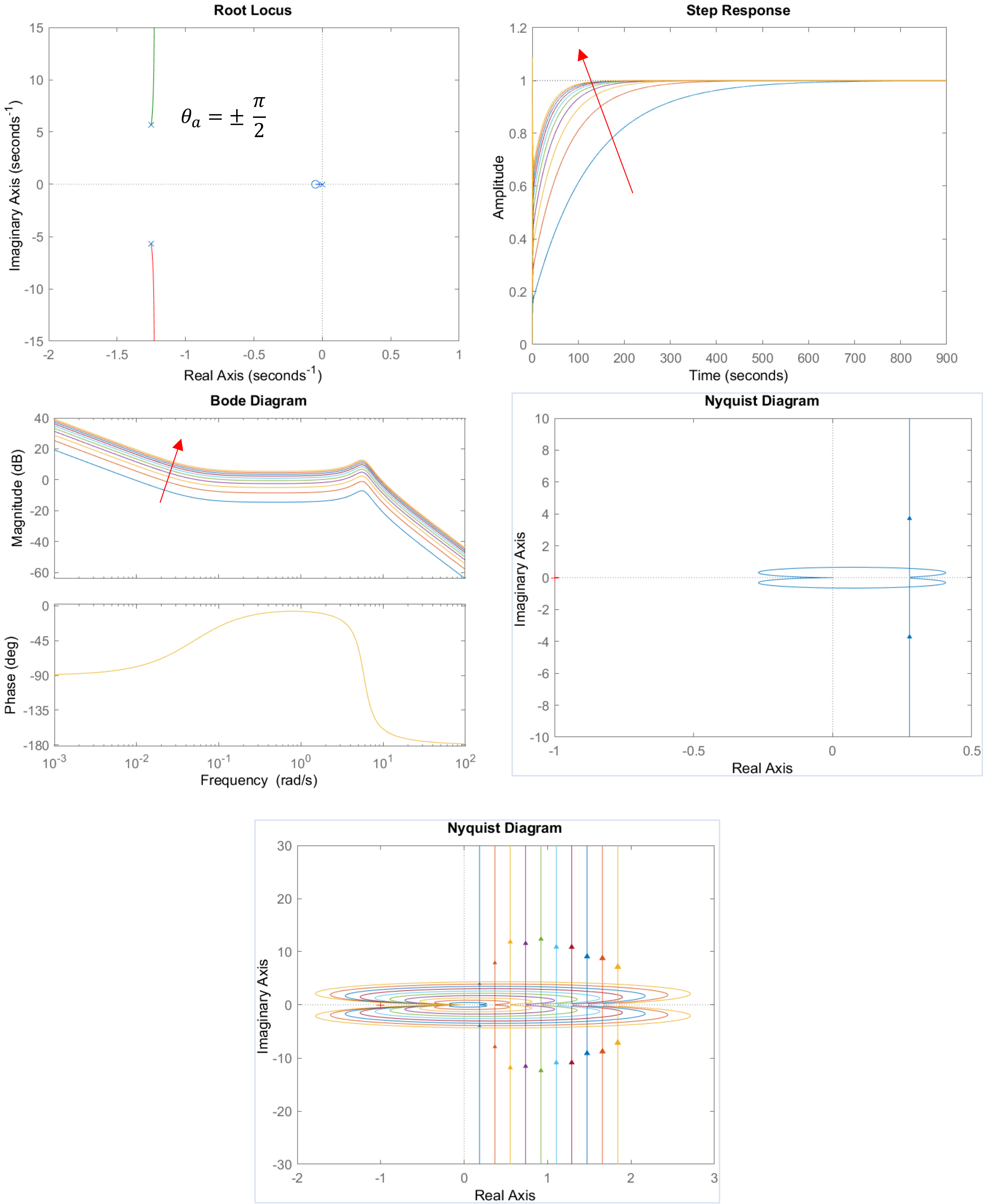


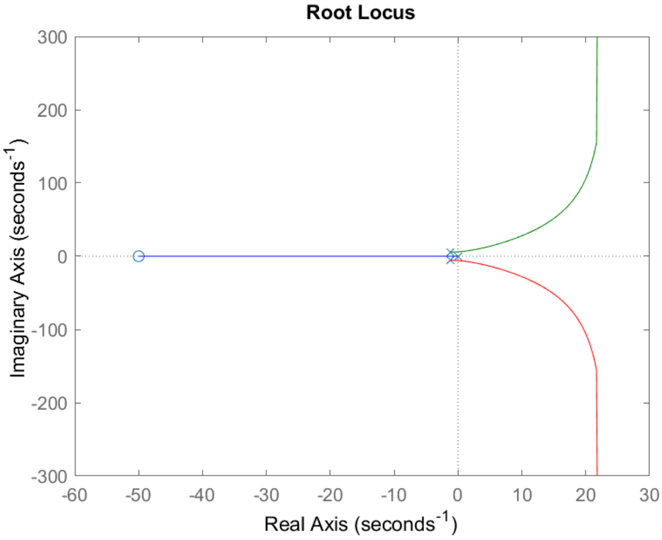
*Fig.13 – Diagrams resulting from a PI **controller** applied to a 1 DOF mechanical system having **k* > 0** and **vector of kp [100:100:1000] having Ti = 20***

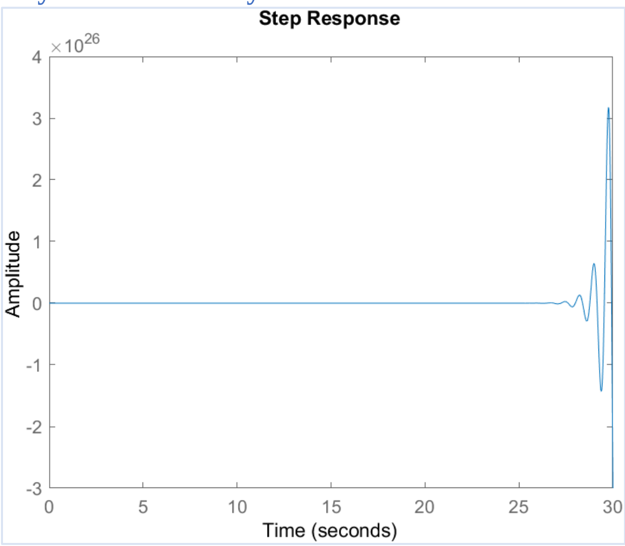
Ti=0.02: the system become unstable ↑kp because of the zero at high frequency.

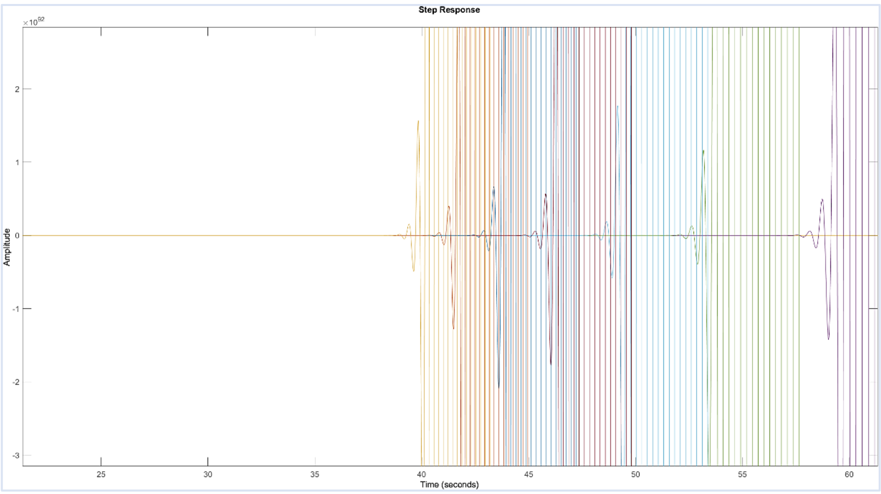
	Kp = 150
In time domain:	$\omega_n^c = 6.5724\ {}^{rad}/_{s} > \omega_n$
Passing in Laplace domain:	$GH^c = \dfrac{3\,s + 150}{0.32\,s^3 + 0.8\,s^2 + 110.82\,s}$ $z_1 = -50 = -\dfrac{1}{T_i}$ $\mu^c = 22.8350 = 20\log\left(\dfrac{k_i}{k^*}\right)$ $L^c = \dfrac{3\,s + 150}{0.32\,s^3 + 0.8\,s^2 + 13.82\,s + 150}$ $p_1^c = -6.6358$ $p_{2,3}^c = +2.0679 \pm 8.1464i$ $\vartheta_\infty^c = 1 \rightarrow e_\infty = 0$

*Tab. 3 – Characteristics parameters of **PI controller** applied to a 1 DOF mechanical system having **k* >0** both in time and Laplace having Ti = 0.02*

- Using **BODE** criterion we notice that pm < 0, but for low value of kp the system become stable because we move to the left-hand side the position of Ω_{pc}.
- Analysing the **ROOT LOCUS,** we notice that for low k_p system is stable, but having small Ti, and for increasing kp, 2 branches of the root locus move to the right half plane resulting in dynamic instability (2 cc poles unstable).
- Also the **STEP RESPONSE** tells us that there is **dynamic instability**.







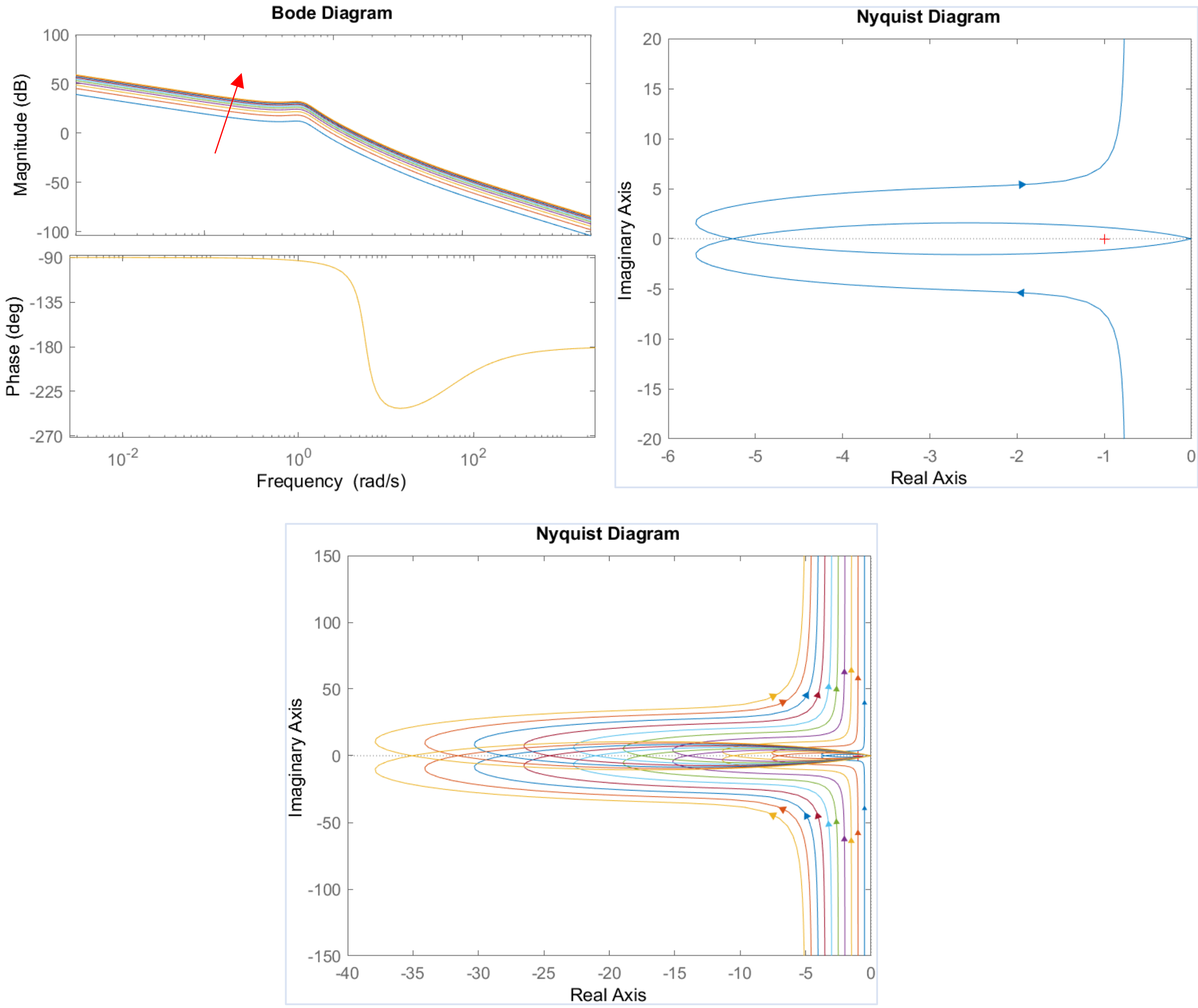


*Fig. 14 – Diagrams resulting from a **PI controller** applied to a 1 DOF mechanical system having **k* > 0** and **vector of kp [100:100:1000] having Ti = 0.02***

Note that having a pole in the origin we have to consider a modified Nyquist path to obtain a closed Nyquist diagram with semi-circular shape.

- **NYQUIST DIAGRAM**: increasing kp we have 2 encirclements in clockwise direction around (-1, 0), therefore we can stabilise the system but with ↓ kp, that means ↓ ω_{Bw} , ↓ speed of response.

It is better to choose Ti in order to be in the first case analysed for the PI controller. We need to design the controller in a precise way, otherwise we obtain an unstable controlled system.

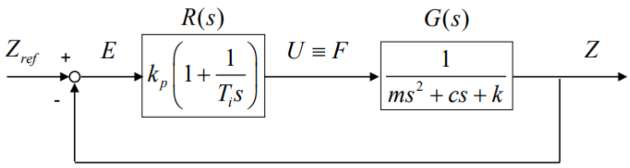


*Fig. 15 – Block diagram of the **PI controller** applied to a 1 DOF mechanical system*

The problem of the integral controller is the ***phase lag*** effect that modifies the transfer function L. It represents, in opposition to the phase lead, a distortion of the phase due to the presence of the zero placed before the poles. Therefore, the response of the system is low at lower frequencies. We ned to reduce the distance between the zero and the poles. Note that the zero must not be exactly equal to the 2 conjugate poles, since we can have instability in that case.

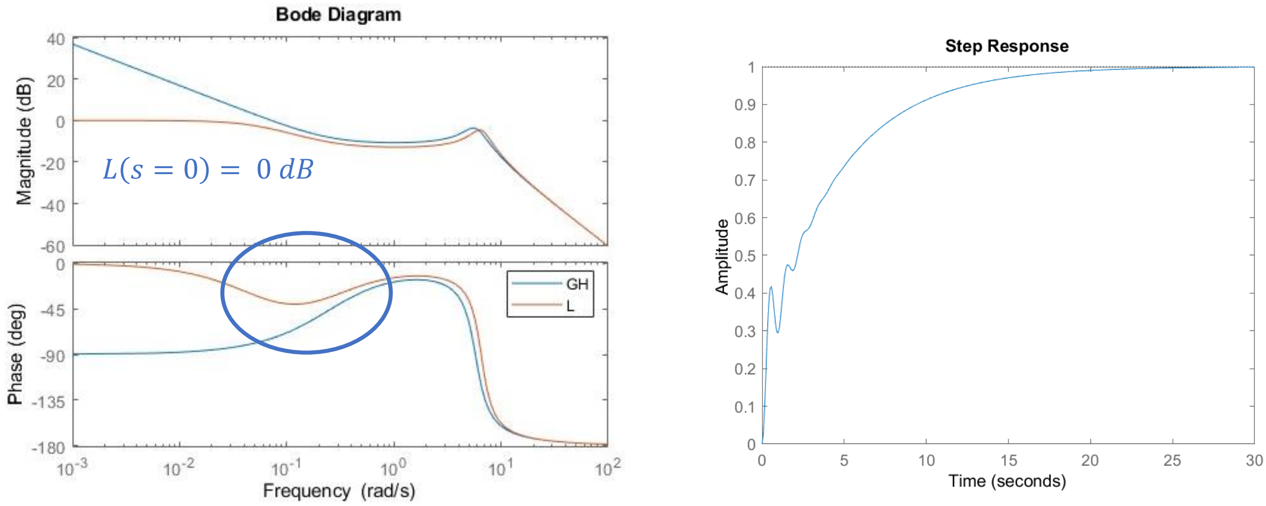


Fig. 12 – Bode diagrams of GH and L and the step response resulting from a ***PI controller*** *having* ***k* > 0, kp =150, Ti = 1.***

$\mathbf{k^* = 0}$

Ti=20: we can use Bode Criterion. In that case we have **2 poles in the origin** (one from the neutral equilibrium system and the other from the PI control)

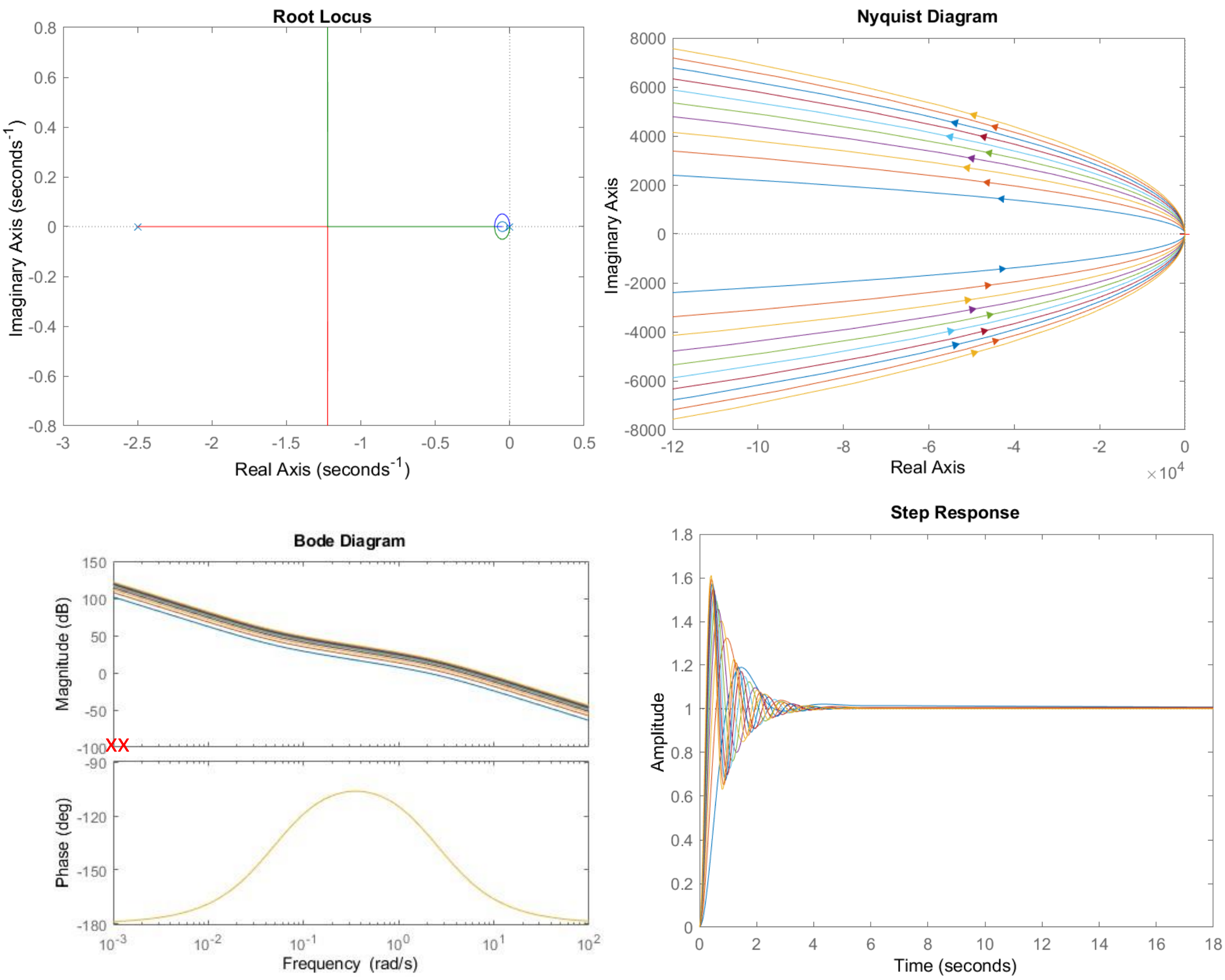


Fig. 14 – Diagrams resulting from a ***PI controller*** *applied to a 1 DOF mechanical system having* ***k* = 0*** *and* ***vector of kp =150 having Ti=20***

Ti=0.02: dynamic instability

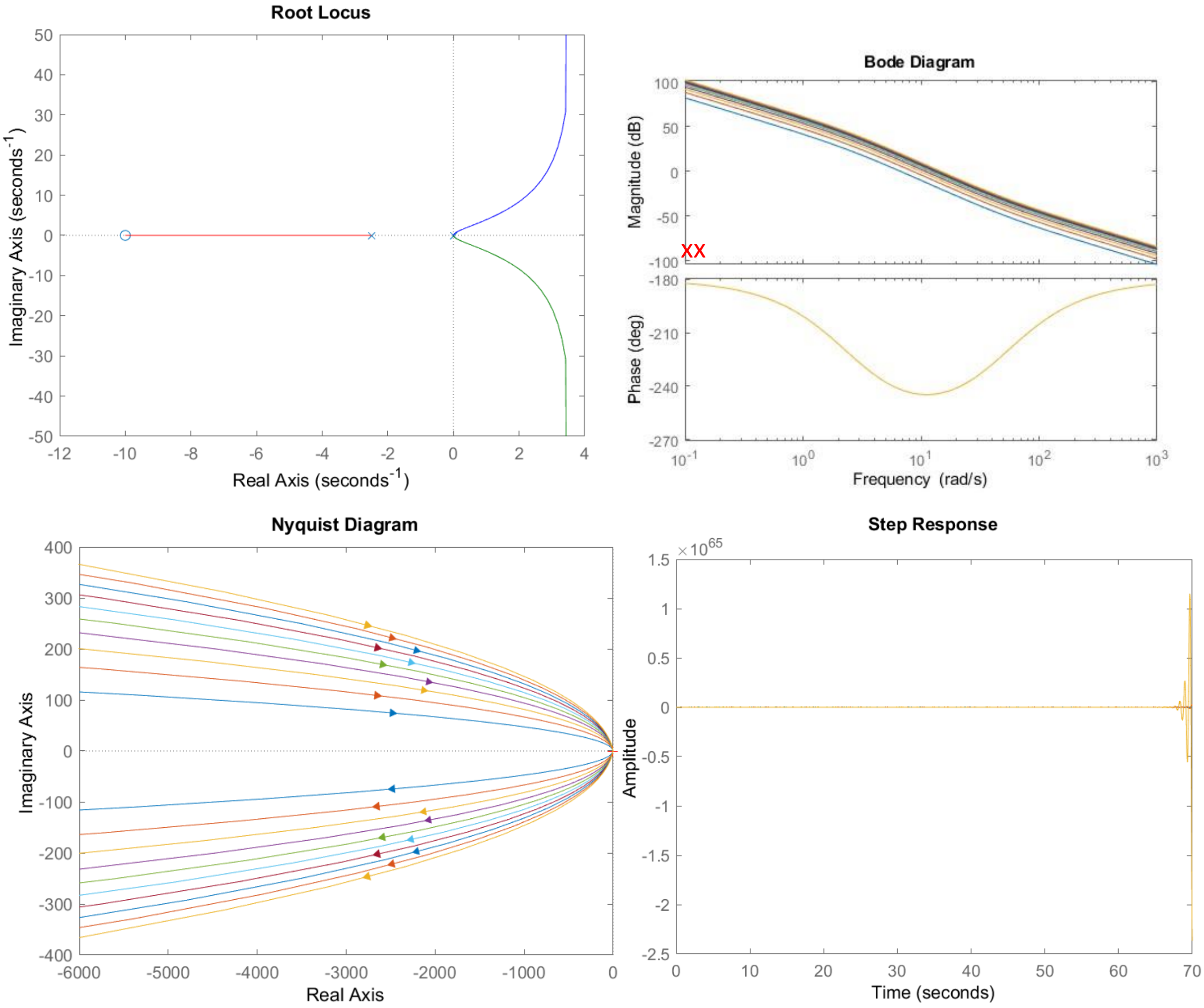


Fig. 14 – Diagrams resulting from a ***PI controller*** *applied to a 1 DOF mechanical system having* ***k* = 0*** *and* ***vector of kp [100:100:1000] having Ti = 0.02***

$k^* < 0$

Ti=20: we cannot use Bode Criterion

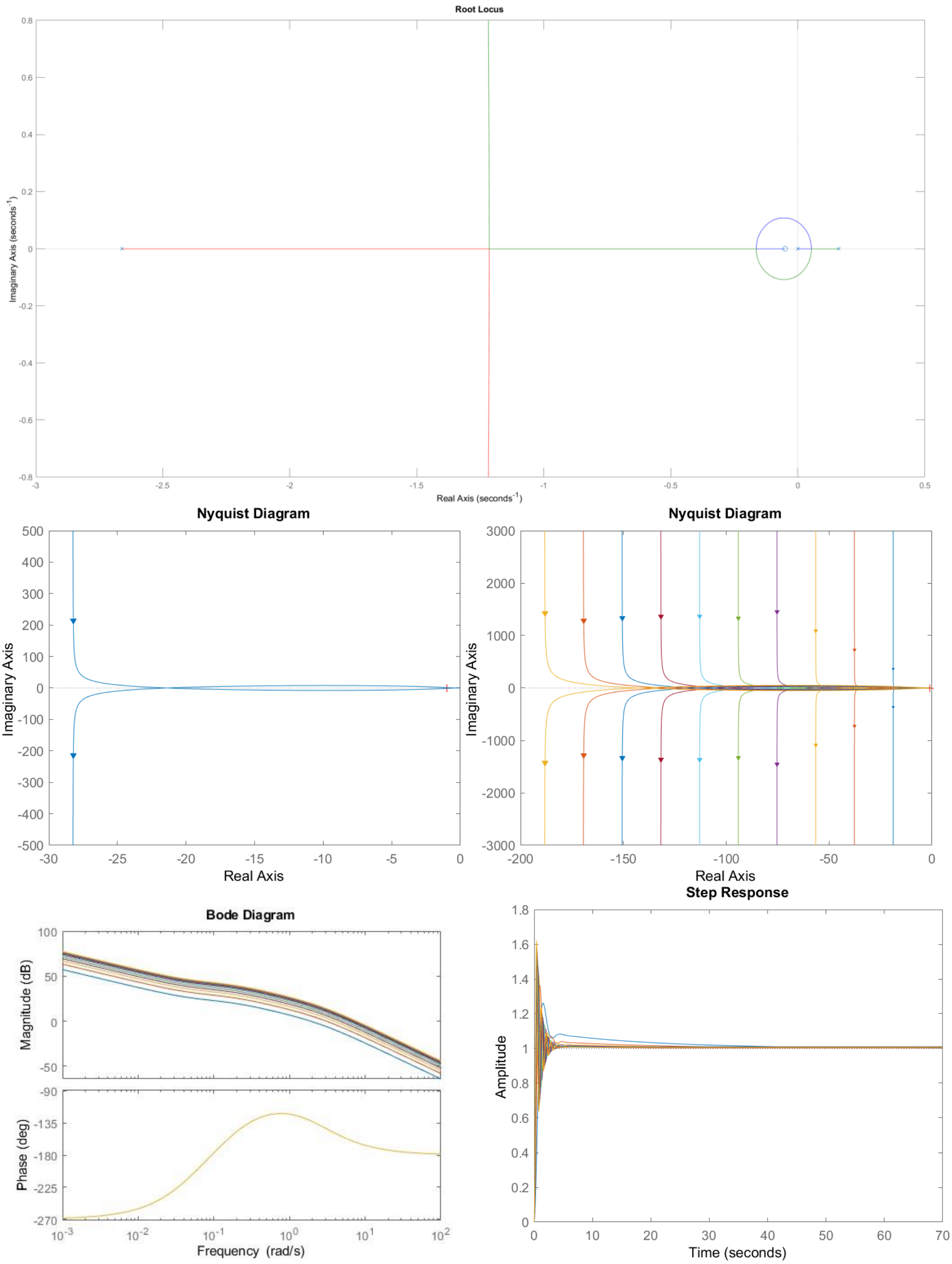


*Fig. 14 – Diagrams resulting from a **PI controller** applied to a 1 DOF mechanical system having **k* < 0** and **vector of kp [100:100:1000] having Ti=20***

Ti=0.02: dynamic instability

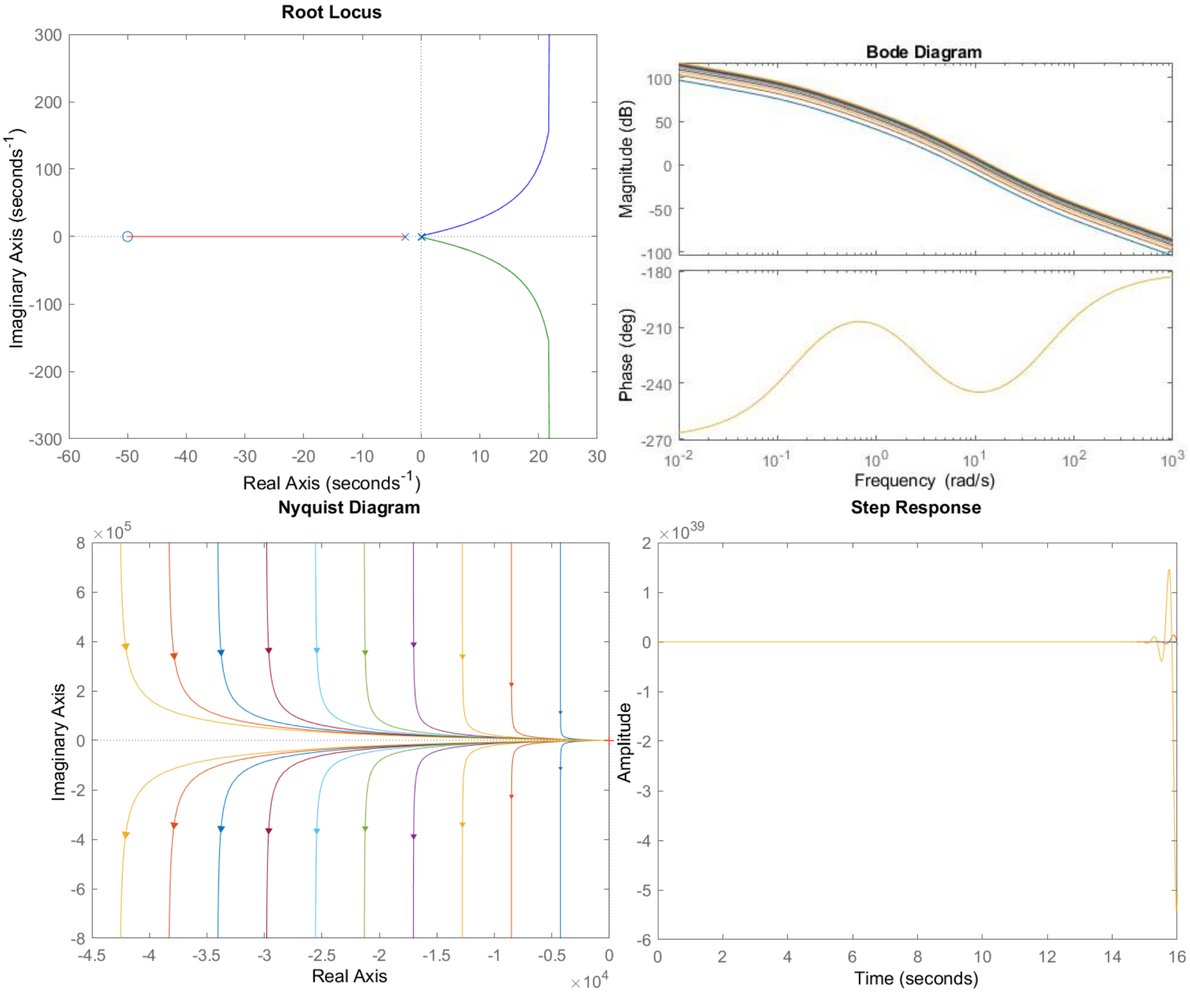


*Fig. 14 – Diagrams resulting from a **PI controller** applied to a 1 DOF mechanical system having **k* < 0** and **vector of kp [100:100:1000] having Ti=0.02***