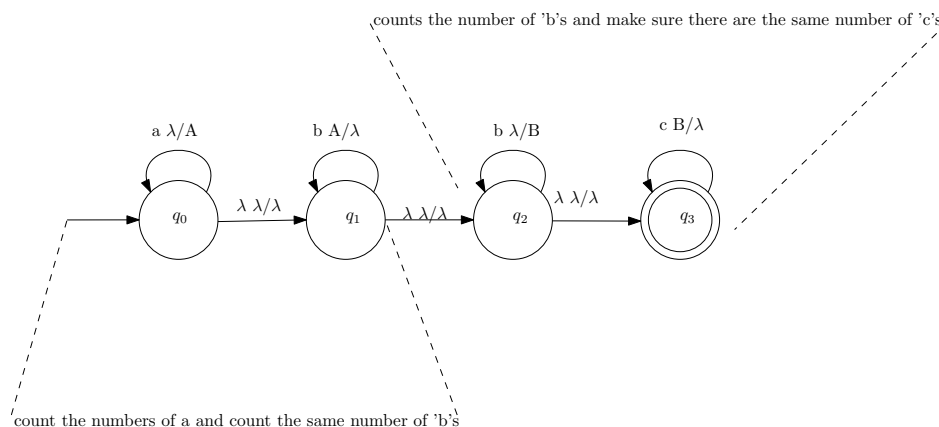


The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

1. (20+5 points) Construct a PDA that recognizes the following language.

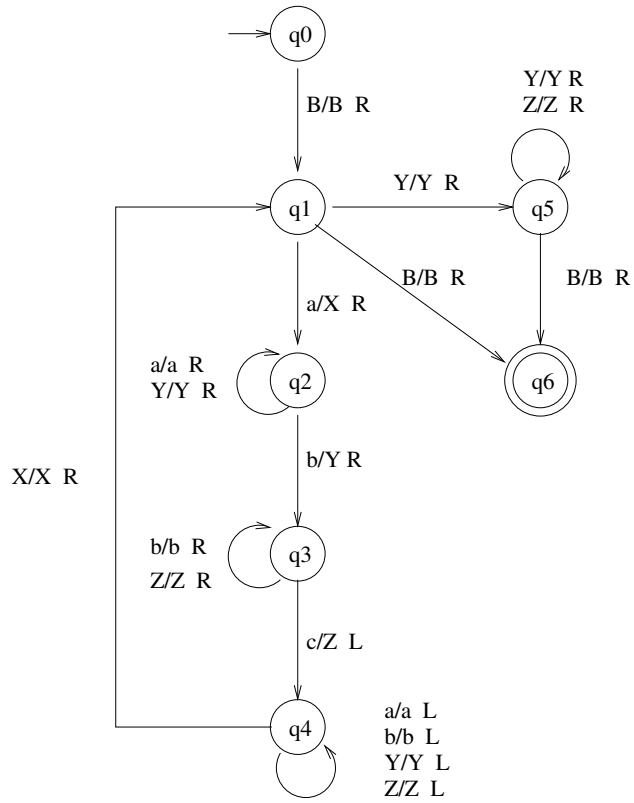
Explain how the PDA works: write the algorithm it follows, label the specific portions of the machine with the task performed (5 points).

$$\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i + k = j\}$$



The algorithm this PDA uses the fact that $j = i + k$ and turns b^j to b^{i+k} and then to $b^i b^k$ making $\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i + k = j\}$ in to $\{a^i b^i b^k c^k \mid i, k \geq 0\}$ so one now must only match the b to the number of 'a's and the number of 'c's.

2. (25 points) Let M be the TM in Example 8.2.2 on page 261 (the machine for $a^i b^i c^i$). Show the computation sequence for the strings abc and $aabc$.



abc

- | | | | | |
|---|---|---|---|---|
| B | a | b | c | B |
| ↑ | | | | |
- | | | | | |
|---|---|---|---|---|
| B | a | b | c | B |
| | ↑ | | | |
- | | | | | |
|---|---|---|---|---|
| B | X | b | c | B |
| | | ↑ | | |
- | | | | | |
|---|---|---|---|---|
| B | X | Y | c | B |
| | | | ↑ | |
- | | | | | |
|---|---|---|---|---|
| B | X | Y | Z | B |
| | | | | ↑ |
- | | | | | |
|---|---|---|---|---|
| B | X | Y | Z | B |
| | ↑ | | | |
- | | | | | |
|---|---|---|---|---|
| B | X | Y | Z | B |
| | | ↑ | | |

- | | | | | |
|---|---|---|---|---|
| B | X | Y | Z | B |
| | | | ↑ | |

- | | | | | |
|---|---|---|---|---|
| B | X | Y | Z | B |
| | | | | ↑ |

aabc

- | | | | | | |
|---|---|---|---|---|---|
| B | a | a | b | c | B |
| ↑ | | | | | |

- | | | | | | |
|---|---|---|---|---|---|
| B | a | a | b | c | B |
| | ↑ | | | | |

- | | | | | | |
|---|---|---|---|---|---|
| B | X | a | b | c | B |
| | | ↑ | | | |

- | | | | | | |
|---|---|---|---|---|---|
| B | X | a | Y | c | B |
| | | | ↑ | | |

- | | | | | | |
|---|---|---|---|---|---|
| B | X | a | Y | Z | B |
| | | | | ↑ | |

- | | | | | | |
|---|---|---|---|---|---|
| B | X | a | Y | Z | B |
| | ↑ | | | | |

- | | | | | | |
|---|---|---|---|---|---|
| B | X | a | Y | Z | B |
| | | ↑ | | | |

There are additional questions on the nextpage.

3. (20+5 points) Construct a TM that takes an input consisting of a sequence of a 's followed by fewer or equal number of b 's; and outputs a string where the number of b 's is the same as the original number of a 's.

The input format is: $\{a^i b^j \mid i, j \geq 0 \text{ and } i \geq j\}$

The output format is: $\{a^i b^i \mid i \geq 0\}$

For example:

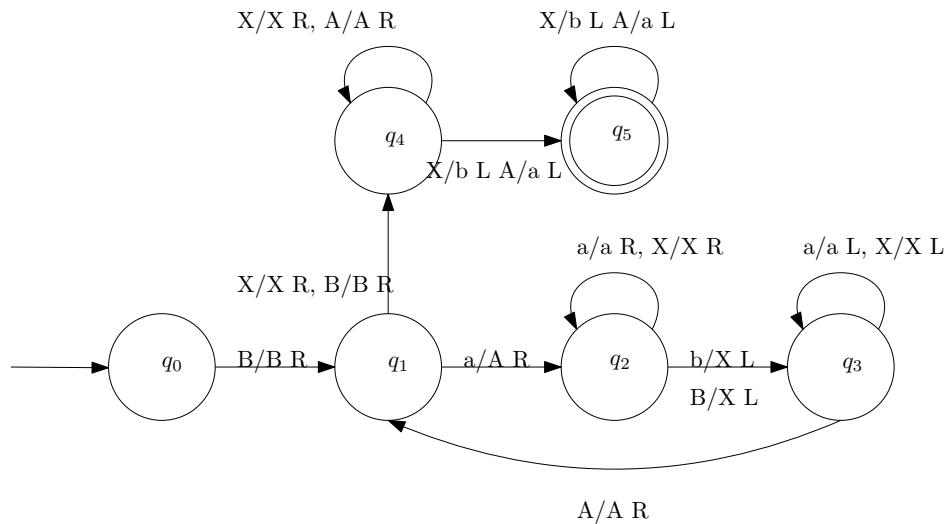
If the input is ' $BaaaaabbB$ ', the output should be ' $BaaaaabbbbB$ '.

If the input is ' $BaaabbbbB$ ', the output should stay the same: ' $BaaabbbbB$ '.

You may assume that the input will be in the desired format. There is no need to check for errors.

Write the high-level algorithm executed by the machine and label the sections (5 points).

1. move right to the first a and replace it with A if it is not replace all A with a and all X with b
2. keep moving the right to the first b and or B and replace it with a X
3. move to the left till A and repeat



4. (20+5 points) Construct a TM that accepts the following language.

Write the high-level algorithm executed by the machine and label the sections (5 points).

$\{a^i b^j c^k \mid i + j = k\}$

1. move right to a mark A move to the first c and mark it C

2. move left to the first A or X move right if a mark A if b mark X if C got to accept
3. check if there are any unmarked cells

