CS3311 Homework 1

Due: Wednesday, September 11, 2019, 8:59am
Submission: Typed, pdf on Canvas
(scanned submissions are not allowed)

The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

Consider the following sets for questions **1** and **2**:

$$X = \{a, 2, \{a\}, [a], [a, a], [\emptyset, a]\} \qquad Y = \{a, 3, [a], [a, a], \emptyset, [a, \emptyset]\}$$

1. *(20 points)* Write out each of the sets listed below.

(a) $X \cup Y$
$\{a, 2, \{a\}, [a], [a, a], [\emptyset, a], 3, \emptyset, [a, \emptyset]\}$

(b) $X \cap Y$
$\{a, [a], [a, a]\}$

(c) $X - Y$
$\{2, \{a\}, [\emptyset, a]\}$

(d) $P(X - \{[a, a], [\emptyset, a]\})$
$\{\{\}, \{a\}, \{2\}, \{\{a\}\}, \{[a]\}, \{a, 2\}, \{a, \{a\}\}, \{a, [a]\}, \{2, \{a\}\}, \{2, [a]\}, \{\{a\}, [a]\}, \{a, 2, \{a\}\}, \{a, 2, [a]\}$
$\{a, \{a\}, [a]\}, \{2, \{a\}, [a]\}, \{a, 2, \{a\}, [a]\}\}$

(e) $\{a, 1\} \times \{a, [a], \{a\}, \emptyset\}$
$\{[a, a], [a, [a]], [a, \{a\}], [a, \emptyset], [1, a], [1, [a]], [1, \{a\}], [1, \emptyset]\}$

2. *(20 points)* State whether the following propositions are TRUE or FALSE.

(a) $a \in X$
True
(b) $\{a\} \in X$
True
(c) $a \in Y$
True
(d) $\{a\} \in Y$
False
(e) $\emptyset \in X$

False
(f) $\emptyset \in Y$
True
(g) $\emptyset \subseteq Y$
True
(h) $\{\emptyset\} \subseteq X$
False
(f) $\{\emptyset\} \subseteq Y$
True
(j) $\{[a, a]\} \in X \times X$
True

Please turn the page over for additional questions.

3. *(20 points)*

(a) Write the first 5 elements of the set S_1 defined recursively. Put the basis elements as the first members. Assume that the arithmetic computations defined in the recursive step will be performed to obtain the new elements of S_1.

> **(i) Basis:** $[1, 1] \in S_1$
>
> **(ii) Recursive step:** If $[n, m] \in S_1$, then $[n + 1, m + 2(n + 1) - 1] \in S_1$.
>
> **(iii) Closure:** S_1 consists of exactly the elements that can be obtained by starting with the basis elements of S_1 and applying the recursive step finitely many times to construct new elements of S_1.

$S_1 = \{[1, 1], [2, 4], [3, 9], [4, 16], [5, 25]\}$

(b) Write the first 6 elements of the set S_2 defined recursively. Put the basis elements as the first members. The first member of the basis sequence is a number, and the second member is a string. The recursive step performs an arithmetic addition on the first member and string concatenation on the second member. The symbols a, b are characters, not variables.

> **(i) Basis:** $[1, a] \in S_2$ and $[1, b] \in S_2$
>
> **(ii) Recursive step:** If $[n, w] \in S_2$, then $[n + 2, awa] \in S_2$ and $[n + 2, bwb] \in S_2$.
>
> **(iii) Closure:** S_2 consists of exactly the elements that can be obtained by starting with the basis elements of S_2 and applying the recursive step finitely many times to construct new elements of S_2.

$S_2 = \{[1, a], [1, b], [3, aaa], [3.bab], [3, aba], [3, bbb]\}$

4. *(5+15 points)*

Consider the following infinite set A.

$A = \{1, 5, 13, 29, 61, \ldots\}$

(a) Describe the pattern to obtain an element from the previous element.

(b) Give a recursive definition of the set A.
(i) Basis: $1 \in A$
(ii) Recursive step: If $n \in A$, then $(n * 2) + 3 \in A$.
(iii) Closure: A consists of exactly the elements that can be obtained by starting with the basis elements of A and applying the recursive step finitely many times to construct new elements of A.

5. *(5+15 points)*

Consider the following infinite set B.

$B = \{a, babb, bbabbbb, bbbabbbbbb, \ldots\}$

(a) Describe the pattern to obtain an element from the previous element.
if $w \in B$ then $bwbb \in B$

(b) Give a recursive definition of the set B.

(i) Basis: $a \in B$
(ii) Recursive step: If $w \in B$, then $bwbb \in B$.

(iii) Closure: B consists of exactly the elements that can be obtained by starting with the basis elements of B and applying the recursive step finitely many times to construct new elements of B.