CS3311 Homework 2 Due date: **Monday**, September 16, 2019, 8:59am
Submission: Typed, on Canvas (scanned submissions are not allowed)

The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

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1. *(100 points)*

Consider the set S_1 constructed recursively. The first member of the basis sequence is a number, and the second member is a string. The recursive step performs an arithmetic addition on the first member and a string concatenation on the second member. The symbols a, b are characters, not variables.

> **(i) Basis:** $[1, a] \in S_1$ and $[1, b] \in S_1$
>
> **(ii) Recursive step:** If $[n, w] \in S_1$, then $[n + 2, awa] \in S_1$ and $[n + 2, bwb] \in S_1$.
>
> **(iii) Closure:** S_1 consists of exactly the elements that can be obtained by starting with the basis elements of S_1 and applying the recursive step finitely many times to construct new elements of S_1.

Use induction to prove that the first member of each pair represents the length of the string which is the second member. In other words, for all $[n, m] \in S_1$ $n =$length(m).

The induction proof should present the following three components clearly:

> Basis
> Inductive hypothesis
> Inductive step

(1)

> **Basis:** The first members in the set is $[1, a]$ and $[1, b]$. 1=length(a) and 1=length(b) therefor the basis of induction holds
> **Inductive hypothesis:** Assume that each member of S_1 which has been formed by applying the recursive step n times, are in the form $[n, m]$ and has the property that$n =$length(m) property
> **Inducive step:** let ω be an element in S_1 that was made by applying the recursive step n time, ω is of the form $[n, m]$ and has the property that $n =$length(m). The next element,x that will be added to set S_1 after $n + 1$ steps. To obtain y apply the recurisve step once on ω. $y = [n + 2, ama]$ or $y = [n + 2, bmb]$. Both definitions add 2 to n and grows the length of m by two. That means that if $n =$ length(m) then $n + 2 =$length(cmc) where c is either a or b. we assumed $n =$ length(m) in

the inductive hypothesis and therefor, conclude by mathematical induction that for all $[n, m] \in S_1$ $n =$length(m).