CS3311 Homework 3 Due date: **Friday**, September 27, 2019, 8:59am
Submission: Typed, on Canvas (scanned submissions are not allowed)

The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

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1. *(65 points)* Consider the following Python program that computes the sum of first n odd numbers and verifies that the sum is equal to n^2.

```
# Implements: Sum of first n odd numbers is n^2

import operator

n = input('Pick a number greater than or equal to 0: ') # upper limit
print ('You entered n = %s. \n' % n)

pair = [1, 1] # the basis pair
print ('pair is %s \n' % pair)

for i in range (1,n+1): # executes for i = 1, 2, ..., n
pair[0] = pair[0] + 1
pair[1] = pair[1] + pair[0]*2 - 1
print ('pair is %s \n' % pair)

if operator.eq (pair[1],pair[0]**2):
print "True: the second member is the square of the first."
else:
print "False: the second member is not the square of the first."
```

Prove using induction that the program always prints
"`True: the second member is the square of the first.`"

You must present the proof based on the code and on the number of iterations of the `for` loop. Clearly label the **basis**, **inductive hypothesis**, and **inductive step**.

Please flip the page for additional questions.

Basis: The basis step is made by running the loop 0 times. The value of pair after running the loop 0 times is $[1, 1]$. $1 = 1^2$ there for the basis step is true

Inductive hypothesis assume $[x, y]$ are the value of pair after n iterations of the loop, and holds the propriety that $y = x^2$

Inductive step let $[z, a]$ be the value of pair after $n+1$ iterations of the loop. Because $[z, a]$ is made after n+1 iterations of the loop then $[z, a] = [x+1, y+(x+1)*2-1]$ Evaluate $y+(x+1)*2-1$

$$\begin{aligned} y + (x+1) * 2 - 1 &= x^2 + (x+1)2 - 1 && \text{Inductive hypothesis} \\ &= x^2 + 2x + 2 - 1 \\ &= x^2 + 2x + 1 \\ &= (x+1)^2 \end{aligned}$$

This shows that $a = z^2$ for $n + 1$ iterations of the loop. This completes the inductive step.
The Basis step holds and the inductive step holds when assuming the inductive hypothesis therefor by mathematical induction it holds true for all n iterations of the loop.

2. *(25 points)* Write the results of the following set concatenations.

(a) $\{b\}\{a, aa, aaa, \ldots\}$
$\{ba, baa, baaa, \ldots\}$

(b) $\{b\}\{\lambda, a, aa, aaa, \ldots\}$
$\{b, ba, baa, baaa, \ldots\}$

(c) $\{b\}\{a\}^*$
$\{b, ba, baa, baaa, \ldots\}$

(d) $\{b\}\{a\}^*\{b\}$
$\{bb, bab, baab, baaab, \ldots\}$
(e) $\{a\}\{a\}^*$
$\{a, aa, aaa, aaaa \ldots\}$

3. *(10 points, 1+4 points each)* State whether the following equalities are true or false *(1 points)*. Explain your answer (4 points).

(a) $\{a\}^*\{b\}^* = \{a\}^* \cup \{b\}^*$
False, because $\{a\}^* \cup \{b\}^*$ dose not concatanate $\{a\}^*$ with $\{b\}^*$ it only makes:
$\{\lambda, a, aa, aaa \ldots, b, bb, bbb, \ldots\}$
(b) $\{a\}^* \cup \{b\}^* = \{b\}^* \cup \{a\}^*$
True, because $\cup$ is comunitive