2

a

$$\begin{aligned} P(X \leq 2) &= P(X=0) + P(X=1) + P(X=2) \\ &= .4 + .3 + .12 \\ &= .85 \end{aligned}$$

b

$$\begin{aligned} P(X > 1) &= P(X=2) + P(X=3) + P(X=4) + P(X=5) \\ &= .15 + .1 + .05 \\ &= .3 \end{aligned}$$

c

$$\begin{aligned} \mu_X &= \sum_x x P(X=x) \\ &= 0(P(X=0)) + 1(P(X=1)) + 2(P(X=2)) + 3(P(X=3)) + 4(P(X=4)) \\ &= 0(.4) + 1(.3) + 2(.15) + 3(.1) + 4(.05) \\ &= 1.1 \end{aligned}$$

d

$$\begin{aligned} \delta_X^2 &= \sum_X (X^2) P(X=x) - \mu_X^2 \\ &= (0^2(P(X=0)) + 1^2(P(X=1)) + 2^2(P(X=2)) + 3^2(P(X=3)) + 4^2(P(X=4)) - 1.1^2 \\ &= (0^2(.4) + 1^2(.3) + 2^2(.15) + 3^2(.1) + 4^2(.05) - 1.1^2 \\ &= 1.39 \end{aligned}$$

4 a

probability mass function must sum up to 1 that is $\sum_X P(X=x) = 1$

$$\begin{aligned} \sum_X P_1(X=x) &= .2 + .2 + .2 + .3 + .1 + .1 = .9 \\ \sum_X P_2(X=x) &= .1 + .3 + .3 + .2 + .2 = 1.1 \\ \sum_X P_3(X=x) &= .1 + .2 + .4 + .2 + .1 = 1 \end{aligned}$$

There for P_3 could be a possible probability mass function of X.

8

a

$$\begin{aligned} F(x) &= P(X \le x) \\ P(X \le 2) &= F(2) \\ &= .83 \end{aligned}$$

b

$$\begin{aligned} P(X > 3) &= 1 - P(X \le 3) \\ &= 1 - F(3) \\ &= .05 \end{aligned}$$

c

$$\begin{aligned} P(X = 1) &= F(1) - F(0) \\ &= .72 - .41 \\ &= .31 \end{aligned}$$

d

$$\begin{aligned} P(X = 0) &= P(X \le 0) \\ &= F(0) \\ &= .41 \end{aligned}$$

e

$$\begin{aligned} P(X = 0) &= F(0) = .41 \\ P(X = 1) &= F(1) - F(0) = .31 \\ P(X = 2) &= F(2) - F(1) = .11 \\ P(X = 3) &= F(3) - F(2) = .12 \\ P(X = 4) &= F(4) - F(3) = .05 \end{aligned}$$

$P(X = 0)$ has the highest probability.

24

a

$$\begin{aligned} \int_{-\infty}^{\infty} f(t)\mathrm{dt} = 1 \\ 1 &= 0 + \int_1^{\infty} \frac{C}{x^3}\mathrm{dx} \\ &= \frac{C}{2x^2}\Big|_1^{\infty} \\ &= \frac{C}{2(\infty)^2} - \frac{C}{-2(1)^2} \\ C &= 2 \end{aligned}$$

b

$$\mu_X = \int_{-\infty}^{\infty} x f(x) \mathrm{dx}$$

$$\begin{aligned}\mu_X &= 0 + \int_1^{\infty} x \frac{2}{x^3} \mathrm{dx} \\ &= 2\end{aligned}$$

c

$$\begin{aligned}F(x) &= \int_{-\infty}^{x} f(t) \mathrm{dt} \\ &= \int_1^{x} \frac{2}{t^3} \mathrm{dt} \\ &= 1 - \frac{1}{x^2} \text{ for} x \geq 1\end{aligned}$$

for $x < 1, F(x) = 0$

$$F(x) = \begin{cases} 0 & \text{x<0} \\ 1 - \frac{1}{x^2} & \text{x} \geq 1 \end{cases}$$

d

$$\begin{aligned}.5 &= \int_{-\infty}^{x_m} f(X) \mathrm{dx} \\ &= \int_{-\infty}^{x_m} \frac{2}{x^3} \mathrm{dx} \\ &= 1 - \frac{1}{x_m^2} \\ 1.412 &= x_m\end{aligned}$$

e

$$\begin{aligned}\mathrm{PM}_{10} = P(X \leq 10) &= F(10) \\ &= 1 - \frac{1}{(10)^2} \\ \mathrm{PM}_{10} &= .99\end{aligned}$$