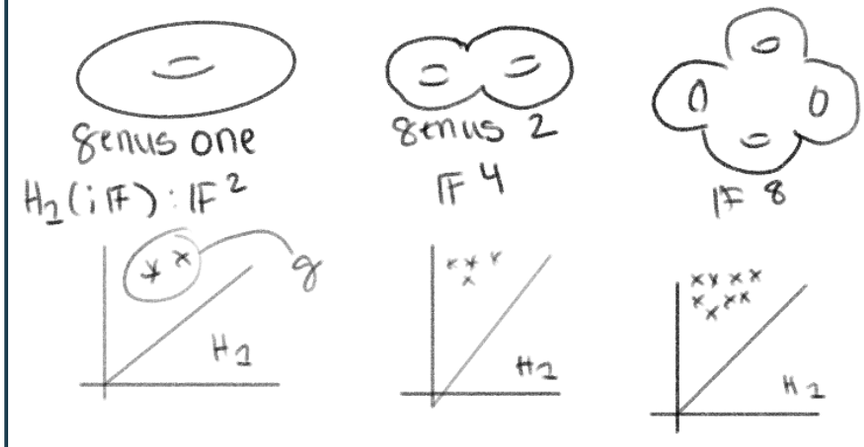
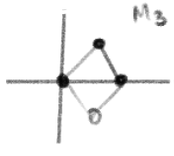
Persistance Dgm: Genus

The way we find the genus from our data is to plot the persistance dgm and find the # of points in data sample $M_0(1,1,1,1,1)$

$$\text{genus } g = H_1(;\mathbb{F}) \cong \mathbb{F}^{2g}$$





M_3 $(0, 1, \frac{1+i\sqrt{2}}{2})$ we want $(\frac{1+i\sqrt{2}}{2}, 0, 1)$

in this we realize we cannot complete this task since if we were to label vectors as (a,b,c) we want $(b,a,-c)$ but we cannot change just a,b we have to do a,b,c

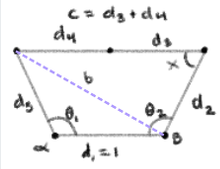
Orientation blocks the 2 triangles from being equivalent.

Theorem 2.1

only way to prove equivalence have to have the same standard proof

finding θ in the code

Error in the code. too many errors from the np.random for the θ in the pentagon (equilateral pentagon specifically in M^8). So instead we find the limits of the θ's by finding the max of θ_1 through α and subtracting it from θ_2



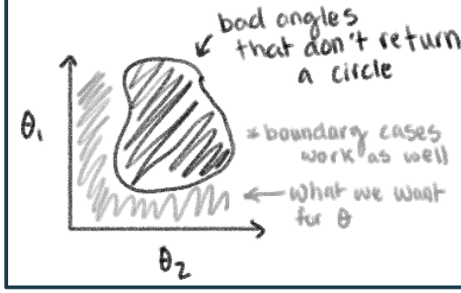
for $d(1,\dots,5)$ we find α s.t.

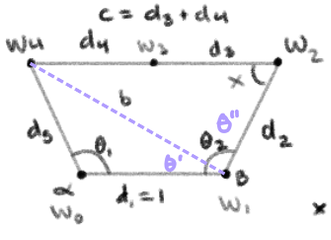
$$c(\alpha_d, \beta_d) = d_4 + d_3$$

$$\theta_1 = \text{np.random.uniform}(0, c(\alpha_d, \beta_d))$$

$$\theta_2 = \text{np.random.uniform}(0, c(\alpha_d, \beta_d) - \theta_1)$$

Max Sum angle





* $\theta_2 = \theta'' + \theta'$

$$\cos(\theta) = \frac{d_1^2 + b^2 - d_5^2}{2 d_1 b}$$

× $b^2 = d_5^2 + d_1^2 - 2 d_5 d_1 \cos(\theta_1)$

$b^2 = d_5^2 + 1 - 2 d_5 \cos\theta_1$

× $\cos(\theta'') = (1 + b^2 - d_5^2) / 2b$

$$\cos(\theta'') = \frac{(1 + d_5^2 + 1 - 2 d_5 \cos(\theta_1)) - d_5^2)}{2(d_5^2 + 1 - 2 d_5 \cos\theta_1)^{1/2}}$$

× $c^2 = b^2 + d_2^2 - 2 b d_2 \cos(\theta_2 - \theta')$

$c^2 = d_5^2 + 1 - 2 d_5 \cos\theta_1 + d_2^2 - 2(d_5^2 + 1 - 2 d_5 \cos\theta_1)^{1/2} d_2 \cos(\theta_2 - \theta_1)$ ... => $\theta' = \dfrac{\cos^{-1}(1 - d_5 \cos\theta_1)}{(d_5^2 + 1 - 2 d_5 \cos(\theta_1))^{1/2}}$

$c^2 \le (d_4 + d_3)^2$

$d_2^2 + 1 - 2 d_5 \cos\theta_1 + d_2^2 - 2(d_5^2 + 1 - 2 d_5 \cos\theta_1)^{1/2} d_2 \cos(\theta_2 - \theta') \le (d_4 + d_3)^2$

$c \le d_4 + d_3$

finding d - lengths of p

Using the law of cosines we can find the lengths needed for the pentagon

- However distance for the equilateral p. is a variable we input into the code.

Pentagon Generator Code,

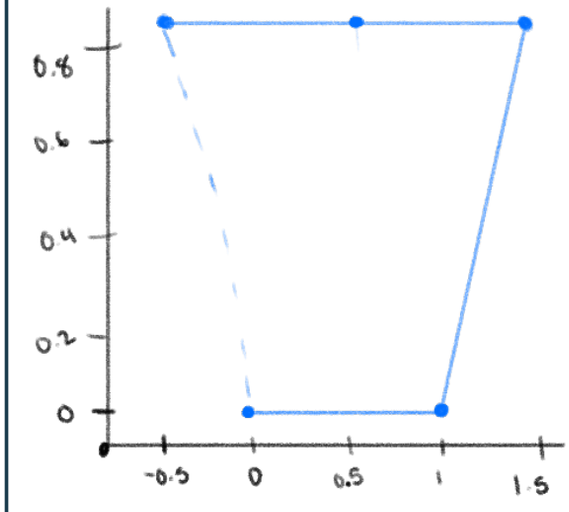
Create a function that takes in d_1, d_2, d_3, d_4, d_5 and determines the maximum θ_1 s.t a pentagon is formed.

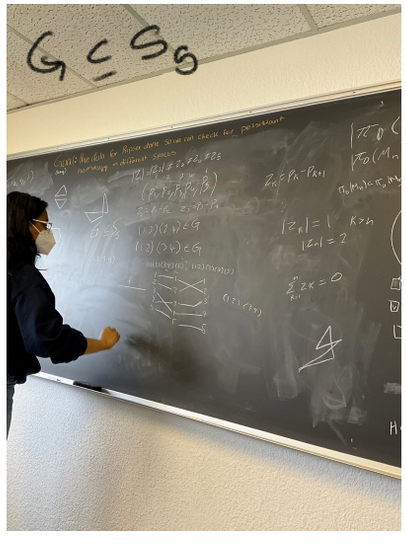
Max sum angle AD (adjacent)

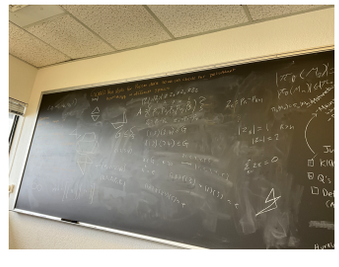
in $\mathbb{C}$ plane and still in M_5

$M(_(____))$

↑ how many generated graphs







Whats Next Slide: Stage 2

changing order of vertices to make multiple ⏢ so we can make multiple of those.

However those are not the same pentagons.

This is a Pentagon: Dear god

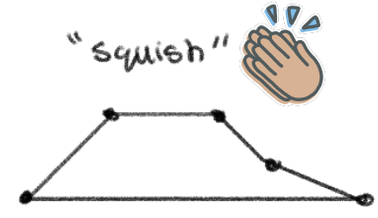
Theres more:

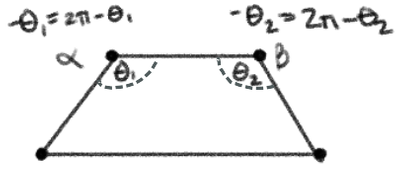
No!

Not in M^5

Degenerate pentagons

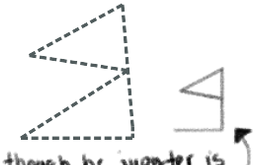
"squish"





+ still α and β

spider fangs



though bc jupyter is annoying it plots to

flipping our known pentagon

taking previous pentagon from code.

Orient it so it is flipped upside down and generate more pentagons

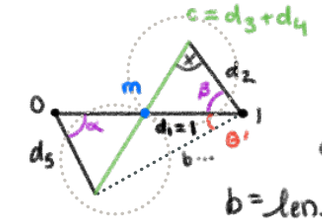
In $\mathbb{C}$ plane and still in M_5

Max sum angle OP

through solving we find we cannot preserve edge lengths so they are equivalent in following pentagon orientation.

In order to find max sum angle of opposite sides in this case

We apply the several side angle theorems for triangles we can find the angles needed for pentagon parameters



$x, d_2, c \Rightarrow b$

$b, c, d_2 \Rightarrow \theta' + \beta$

$d_5, d_1, b \Rightarrow \theta'$

$d_5, d_1, b \Rightarrow \alpha$

Input: distances start Δ

$b = \text{len_side}(d_2, c, x)$

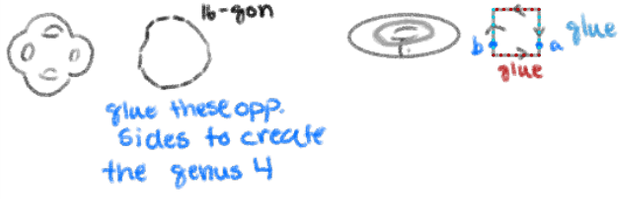
$\theta' + \beta = \text{angle_from_side}(b, d_2, c)$

$\theta' = \text{angle_from_side}(b, d_1, d_5)$

$\alpha = \text{angle_from_side}(d, d_5, b)$

Output: $\alpha + (\theta' + \beta) - \theta'$

$\theta' = \gamma$ in implamentation



Next steps: modeling on 4n-gon

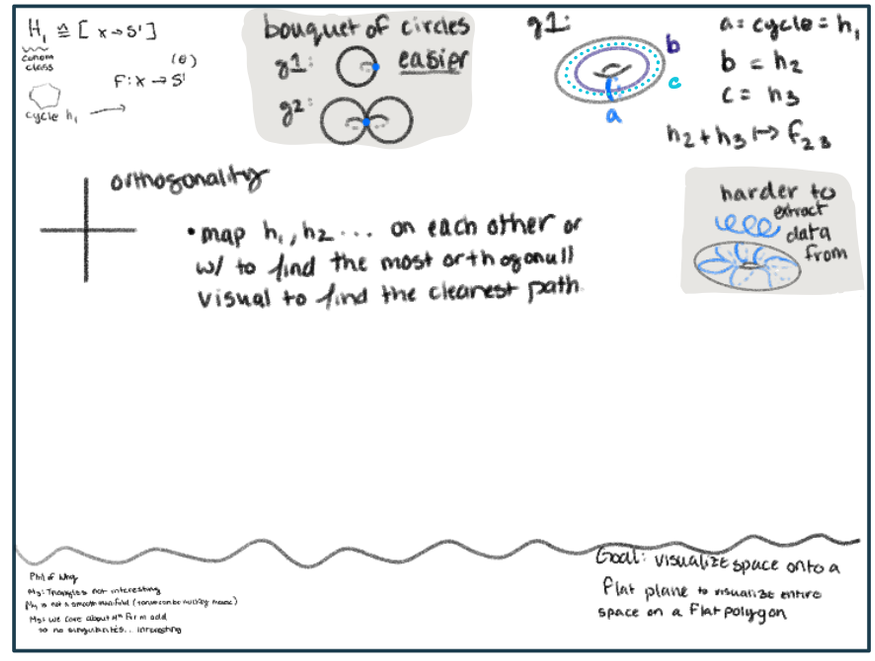
the genus 4+1 has the following n-gons that are the clean unfurled versions of the data. We want to plot the data points on opposite sides (gluing sides) and show the data points on the 3D plane for the purpose of getting a cleaner data set.

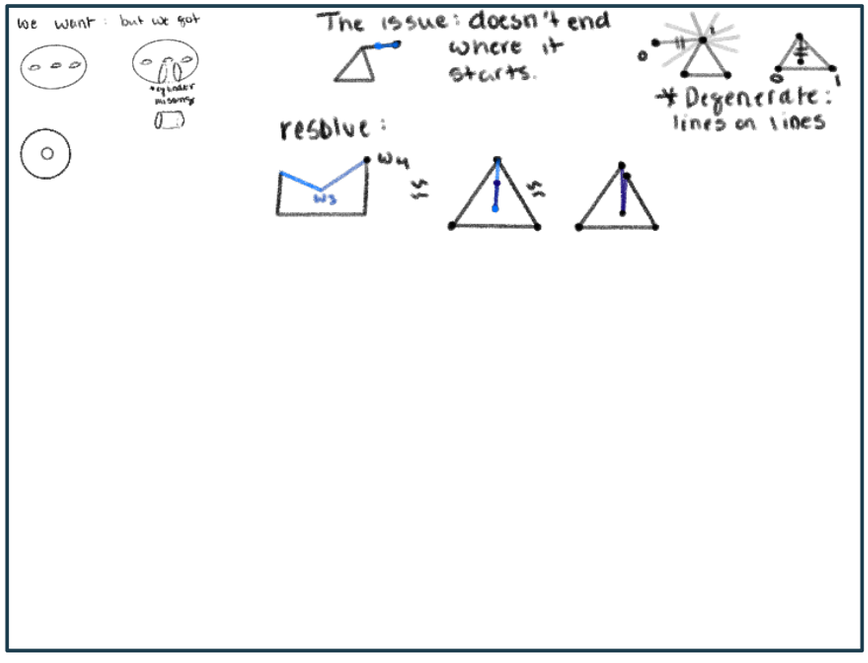
choosing Geom. Ind. cohomology classes

Want to visualize what the space of pentagons looks like (has applications in Biology). Specifically we want to find 3 cohomology classes to find $\mathbb{R}^3$ since we cannot project onto $\mathbb{R}^5$ because we want to get a more indicative pic of the space of M_5 w/ differing distances.

- Δ in $\mathbb{C}^5$ so get down to $\mathbb{R}^3$ while preserving enough to find the topological space
- use cohom classes to find the differences in geometric spaces

Klaus & Kojima shows the example of mapping onto $\mathbb{R}^3$





Error in code: Gaps

We have gaps in the code where we can't get specific pentagons. So in order to generate the degenerate pentagons we set the diffent bases of $(0,1)$. However this only works on the equilateral cases. if the degenerate edge is not = then it is not a polygon. So instead use past polygon and collapse it.