Classical Dynamics
A Computational Simulation of the Stratos Space Jump

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(Dated: April 6, 2015)

On October 14, 2012, Felix Baumgartner performed a space dive from the upper stratosphere, jumping from 38,970m above the earths surface. He descended in free fall, breaking the sound barrier. This simulation examines the Stratos jump, taking into account the linear and quadratic drag forces Felix felt descending in the atmosphere. The simulation analyzed his fall numerically, using Euler's method to solve the dynamic drag forces.

1. BACKGROUND

The Stratos space jump proved that with careful scientific planning and engineering expertise humans can successfully break the sound barrier. The Stratos space jump teams mission were to evaluate a next generation full pressure suit from high altitudes and to obtain physiologic information of the jumper.[1] Felix became the first to ever break the sound barrier without any form of engine power [1] and also set the record for highest sky dive. He reached speeds of 843 mph before finally being slowed down by the drag force of the atmosphere. The freefall lasted a total of 4 minutes, 20 seconds before deployment of parachutes for a safe landing. After the jump, Felix described his hopes for the feat that people will use the data [for] aerospace safety [and] spark childrens interest in science. [5]

Already, commercial companies compete in the aeronautic market, the most notorious being the Elon Musk backed SpaceX. As humans become habituated in space, the biological effects on astronauts must be studied. The Stratus jump provided important physiological data, such as heart rate, oxygen intake, and reaction to altitude related sicknesses including hypoxia and ebullism. The data collected will contribute critical insight to the study and design of equipment so a human could survive emergency escape situations [1].

To safely execute the Stratos space jump, the team equipped Felix with special pressurized suit designed by the David Cark co. The suit was designed considering the many factors that Felix would face descending from 24 miles above the earth. The pressurized suit contained ventilation systems for comfort and visibility (to avoid fogging up visor). Thermal variations presented a unique challenge as Felix would face both extreme cold from the altitude as well as heat friction from the air as he fell at high speeds. The final design materials accounted for all this and also minimized bulk so that Felix had the flexibility he needed to maneuver and position himself throughout free fall. As a final precaution, the suit was equipped with an emergency drogue parachute in case Felix began to spin out of control. [1]

2. INTRODUCTION

As a physics student, this simulation provided a real life situation for modeling the complex interactions of fluid dynamics and drag as experienced by sky (or in this case space) divers. Turbulence is a Millennium Prize problem [6]. Modeling fluid mechanics and how particles in the fluid field are affected remains extremely complicated. Computational methods have made modeling and solving these nonlinear differential equations for particular sets of parameters possible (within software limits). This simple simulation utilized the Euler method of solving second order nonlinear differential equations. Programming in Matlab allowed for a numerical solution to the equations of motion Felix experienced when jumping from the stratosphere. Both Felixs path of motion and his velocity were analyzed as a function of time. The simulations were then compared to the actual scientific data collected by the instruments Felix carried during his jump.

3. MATERIALS AND METHODS

3.1. Software

This simulation primarily considered the velocity dependent force of drag. Both frictional drag in the linear regime and the inertial drag in the quadratic regime were examined. We assume that Felix is a sphere of diameter (D) 1.5 meters and of mass 200kg.

In general, Newtons laws dictate that:

$$\vec{F}(\vec{v}) = m\ddot{\vec{r}} \tag{1}$$

Taylor expanding gives a force that opposes the motion:

$$F(v) = -(bv + cv^2) \tag{2}$$

where b and c are constants of the linear and quadratic drags respectively. The linear term of drag is dependent on the viscosity of the fluid, in this case the atmosphere while the quadratic term depends on the density:

$$b = 3\pi\eta D \tag{3}$$
$$c = \frac{\pi}{16}\rho D^2 \tag{4}$$

Both viscosity and density values are functions of altitude. At high speeds, the quadratic term quickly overtakes the linear contribution. Conversely, at slower speeds and when D is small, the linear term dominates.
This simulation examined variations of the differential equation that describes Felixs motion.

$$m\frac{dv_y}{dt} = mg - b(y)v - c(y)v^2 \tag{5}$$

Rewritten as an ODE this equation takes the form:

$$\frac{d^2y}{dt^2} - \frac{dy}{dt}y - (\frac{dy}{dt})^2 y = 0 \tag{6}$$

To solve this second order nonlinear differential equation numerically, the program utilized Eulers method. Eulers method approximates a value y(δ x) by adding a small δ x proportional to the rate of change. In general:

$$y_{n+1} = y_n + f(x_n, y_n)\delta x \tag{7}$$

For second order ODEs, we need to rewrite 6 as two separate first order ODEs and define:

$$v = \frac{dy}{dt} \tag{8}$$
$$\frac{d^2y}{dt^2} = \frac{dv}{dt} \tag{9}$$

The system of two 1st order ODEs is $\frac{dv}{dt} = f(y, v, t)$ and $\frac{dy}{dt} = v$. The model requires the initial conditions of at time $t = t_0 = 0$ that $y(t_0) = y_0 = 38970m$ and $v(t_0) = v_0 = 0$. The model then incrementally solves the system of equations. The model attempted a number of time steps (i.e. 10 s, 5s, 1s, 0.1s) to optimize convergence of the trajectories. Sample code: (full version of code can be seen in Appendix B).

```
for t=0:h:500

    vnew= vold + (g + b*vold +(c*(D^2)/m)*(vold^2))*h;
    ynew= yold - vold*h;
    if ynew<0
        break
    end
    if vnew>vterquad
        vnew=vterquad;
        v(t+1)=vnew;
    else
```

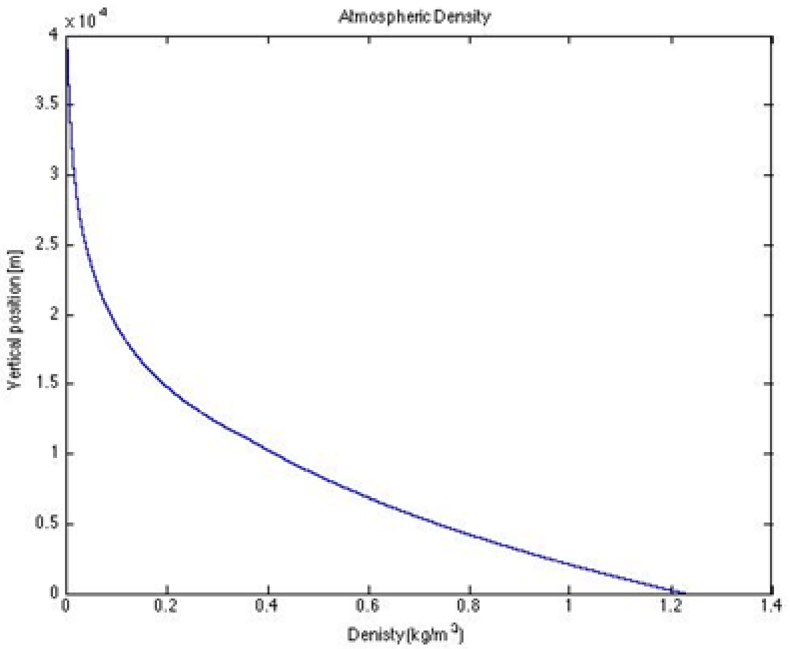

FIG. 1: Atmospheric Density

```
        v(t+1)=vnew;
    end

    y(t+1)= ynew;
    yold= ynew;
    vold= vnew;

    %t = t+h, done by for loop
end
```

The code iteratively constructs the trajectory of Felix. The model adjusts for constraints such as terminal velocity and how atmospheric variables change with height.

4. RESULTS

Just for a basic model, the code plotted the trajectory using the simple free fall model, ignoring factors such as atmosphere and drag. In this simple model of free fall, Felix would have hit the ground in a remarkable 80 seconds.

The model then calculated the density and viscosity of earths atmosphere. The density calculation depended on many parameters of the atmosphere as a function of height, including temperature and pressure. The density function is displayed in the **Figure 1**: [2]

Values for viscosity as a function of height were obtained from source [3]. Based on the initial assumptions and retaining both linear and quadratic terms of the drag equation, the model predicted the following outcomes:

δ t (s):	Terminal Velocity ($\frac{m}{s}$):	Time to Terminal Velocity (s):	Time to Parachute Pulled (s)
10	499.3	30	200
1	439.3	32	232
0.1	433.5	30.9	233.6

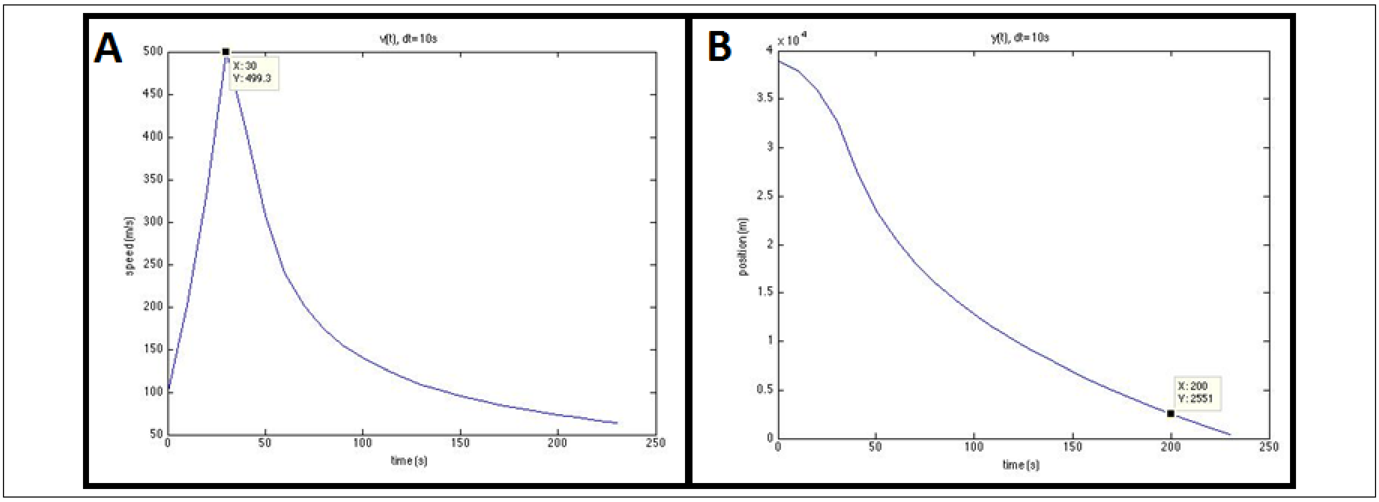

FIG. 2: 10 second time interval, y(t), v(t)

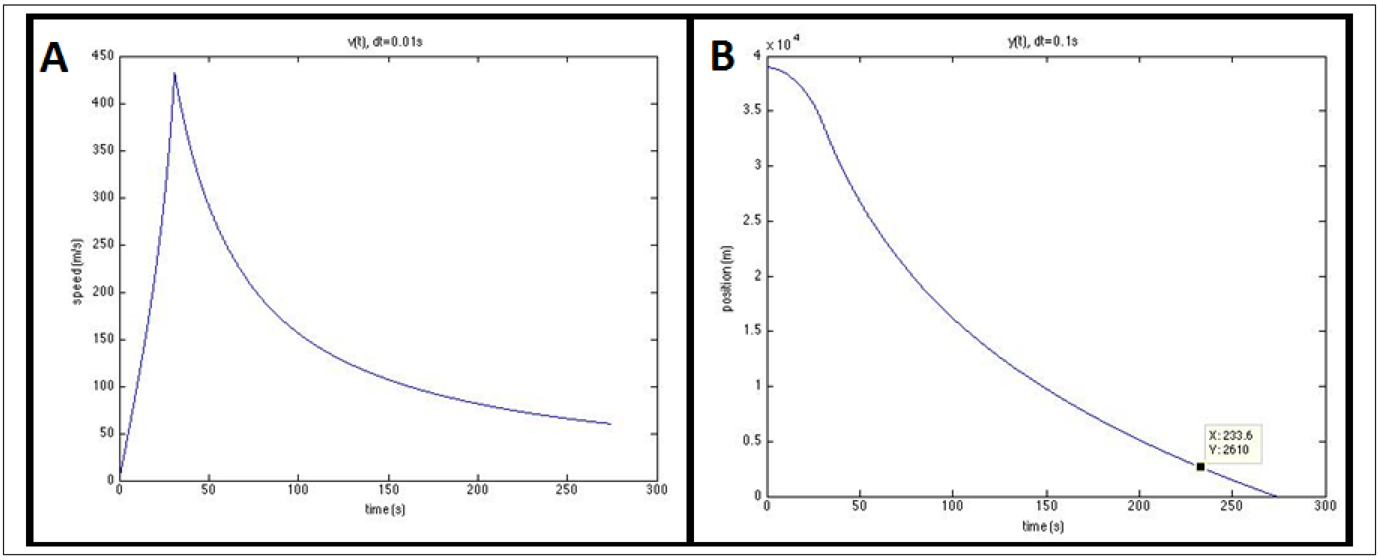

FIG. 3: 0.1 second time interval, y(t), v(t)

Figure 2 depicts the height and velocity functions computed at a 10s time interval. This is compared to the second pair of graphs (Figure 3) computed at a 0.1 s time interval.

The models quickly converge. The time interval of ten seconds is too high and produces skewed results. There is a significant change in terminal velocity from each time step.

The measurements made during Felixs jump are recorded in the scientific data review released by the Red Bull Stratos team.

Exit Altitude (m):	Max Velocity ($\frac{m}{s}$):	Vertical Distance (no parachute) (m):
38969	377.1	36403

He pulls his drogue parachute at approximately 2600m above the earth 260s into the dive. The total time until touch down was 9 minutes and 18 seconds. Felix did experience some spinning for about 13 seconds; however, he was able to position himself to fall correctly and avoid deploying the emergency safety parachute system.

The simulation code was then altered to reflect other physical parameters previously ignored. In reality, the linear term would not have much of an effect. The linear term of the drag equation pertains primarily to small particles, moving at slower speeds. Only the quadratic term contributes in the final simulation. Also, according to the weight specifications given in the scientific review, we re-estimate the total weight to be 110 kg. We also adjust the diameter to be 1.3 m. Finally, the simulation used smaller time steps for more accurate calculations. The results using these parameters are listed in Table 3:

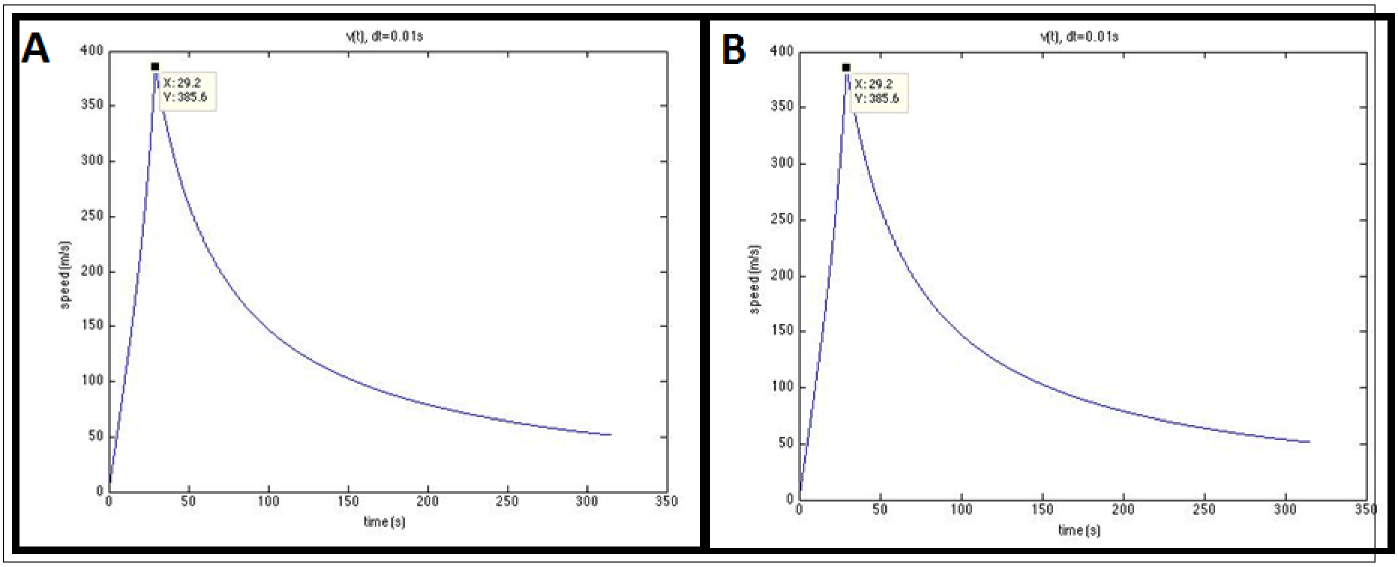

FIG. 4: 0.01 time interval, y(t), v(t)

δ t (s):	Terminal Velocity ($\frac{m}{s}$):	Time to Terminal Velocity (s):	Time to Parachute Pulled (s)
0.1	386.2	29.2	268.2
0.01	385.62	29.2	268.3
0.001	385.59	29.2	268.3

5. CONCLUSIONS

Overall, the final simulation did model Felixs jump accurately to an extent. The simulation predicted terminal velocity (377 m/s) within a 2.3 percent error. The model also predicted the time to parachute deployment (260 seconds) within 3.2 percent error. However the time until the terminal velocity was reached was more significantly off. The scientific report states that, according to the instruments in Felixs suit, he reached terminal velocity about 50 seconds after jumping. The simulation estimated he reached terminal velocity 30 seconds after jumping.

These discrepancies occur because the simulation was a relatively simple model of what is in reality a complex dynamical system. The shape of the object has a significant effect on how the object moves through the fluid and experiences drag. Felix, an experienced skydiver, also maneuvered himself and adjusted his position throughout the dive. We estimated Felix as a symmetrical sphere (as physicists tend to do). The weight approximation was estimated based on he scientific reports recordings of the individual components weight. The actual team had a scientist monitoring the weather and how it would affect Felixs jump, something this simulation did not account for. Finally, it is likely that Felix experienced some turbulence during his jump. This chaotic flow can create hard to predict variations in modeling drag force.

6. BIBLIOGRAPHY

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