EECS 769 Optional Problems for Homework 1

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0.1 Three events E_1, E_2, and E_3 defined on the same space, have probabilities $P(E_1) = P(E_2) = P(E_3) = 1/4$. Let E_0 be the event that one or more of the events E_1, E_2 and E_3 occurs.

0.1.1 Find $P(E_0)$ when:

The events E_1, E_2, and E_3 are disjoint.

$$P(E_0) = P(E_1) + P(E_2) + P(E_3) = 1/4 + 1/4 + 1/4 = 3/4$$

The probability of one of the events occurring is equal to the sum of the individual probabilities of the disjoint events.

The events E_1, E_2, and E_3 are statistically independent.

$$P(E_0) = 1-(P(E_1 = 0)*P(E_2 = 0)*P(E_3 = 0)) = 1-(3/4)*(3/4)*(3/4) = 1-(27/64) = 37/64$$

If they are independent, the outcome of one event doesn't give any information on another event. This means that the probability that one or more events will occur is $1 - P(noeventsoccur)$

The events E_1, E_2, and E_3 are in fact names for the same event.

$$P(E_0) = 1/4$$

Since they are the same event, we know $P(E_0) = P(E_1)$ since if one event occurs, all events occur, and one event doesn't occur, no events occur. The events are completely dependent on each other.

0.1.2 Find the maximum value $P(E_0)$ can assume when:

Nothing is known about the independent or disjointness of E_1, E_2 and E_3.

If we know nothing about their independence or disjointedness, the maximum value that $P(E_0)$ can assume is 1. If $P(E_3)$ is always true when $P(E_1)$ and $P(E_2)$ are false then we could be assured that we would always have at least one of them be true. Each individual event could maintain their original probabilities of a single event occurring, but the combination could guarantee that at least one of them would always be true.

Is is known that E_1, E_2 and E_3 are pairwise independent. i.e., that the probability of realizing both E_i and E_j is $P(E_i)P(E_j), 1 \leq i \neq j \leq 3$ but nothing is known about the probability of realizing all three events together.

Again, $P(E_0)$ could feasibly take on the quantity one if $P(E_3)$ was fully dependent on the vector (E_1, E_2). It might be independent of the two outcomes individually, and still a function of the vector formed by the two values.

0.2 A dishonest gambler has a loaded die which turns up the number 1 with probability 2/3 and the numbers 2 to 6 with the probability 1/15 each. Unfortunately, he has left his loaded die in a box with two honest dice and cannot tell them apart.

0.2.1 He picks up one die (at random) from the box, rolls is once, and the number 1 appears. Conditional on this result, what is the probability that he picked up the loaded die?

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(loaded|1) = \frac{P(loaded) * \frac{2}{3}}{P(loaded) * \frac{2}{3} + P(fair) * \frac{1}{6}} = \frac{\frac{1}{3} * \frac{2}{3}}{\frac{1}{3} * \frac{2}{3} + \frac{2}{3} * \frac{1}{6}} = \frac{2}{3}$$

We know that if his choice of die was random the probability of him picking the loaded die is $\frac{1}{3}$ and the probability of the loaded die rolling a 1 is $\frac{2}{3}$ so we can use the property of conditional probability listed above to compute the probability of him picking the loaded die given that it rolled a one on the first try.

0.2.2 He now rolls the dice once more and it comes up 1 again. What is the possibility after this second rolling that he has picked up the loaded die?

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(loaded|1,1) = \frac{P(loaded) * (\frac{2}{3})^2}{P(loaded) * (\frac{2}{3})^2 + P(fair) * (\frac{1}{6})^2} = \frac{\frac{1}{3} * (\frac{2}{3})^2}{\frac{1}{3} * (\frac{2}{3})^2 + \frac{2}{3} * (\frac{1}{6})^2} = \frac{8}{9}$$

Again, we know the probability of either the unfair die or the fair die rolling two ones in a row. We just plug that into the aforementioned equation to work out the probability that we have the loaded die given that we rolled two ones in a row.