EECS 731: Quiz 1

2016-09-08

Name: ______________________________________

1. (3 points) Prove $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

$$
\begin{aligned}
&\text{Let } x \in A \cup (B \cap C) \\
&\implies x \in A \text{ or } x \in B \cap C \\
&\implies x \in A \text{ or } \{x \in B \text{ and } x \in C\} \\
&\implies x \in A \text{ or } x \in B \text{ and } x \in A \text{ or } x \in C \\
&\implies x \in A \cup B \text{ and } x \in A \cup C \\
&\implies x \in (A \cup B) \cap (A \cup C) \\
&\implies A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \text{ (1 point)}
\end{aligned}
$$

by a similar argument we can show that

$$(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C) \text{ (1 point)}$$

Recall if $A \subseteq B$ and $B \subseteq A$, then $A = B$. Thus $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$(1 point)

2. (2 points) Define conditional probability and Bayes' Theorem.

Solution: Conditional Probability (1 point):

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Bayes' Rule (1 point):

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Bayes' Theorem (bonus point):

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{A_j \in A} P(B|A_j)P(A_j)}$$

3. (5 points) 2% of women have lung cancer. 85% of lung tests for cancer correctly detect cancer when the woman has cancer. 10 % of lung cancer test detect cancer when it's not there. A woman receives a positive test result. What are the chances she actually has cancer?

Solution:

$$
\begin{aligned}
P(\text{ cancer }) &= .02 & P(\text{ positive test } | \text{ cancer }) &= .85 \\
P(\text{ not cancer }) &= .98 & P(\text{ positive test } | \text{ not cancer }) &= .10
\end{aligned}
$$

$$P(\text{ cancer } | \text{ positive test }) = \frac{P(\text{ positive test } | \text{ cancer })P(\text{ cancer })}{P(\text{ positive test })}$$

$$P(\text{positive test}) = P(\text{positive test}|\text{cancer})P(\text{cancer}) + P(\text{positive test}|\text{not cancer})P(\text{not cancer})$$

$$P(\text{ cancer } | \text{ positive test }) = \frac{.85 * .02}{(.85 * .02) + (.1 * .98)} = \frac{.017}{.115} = 14.78\%$$