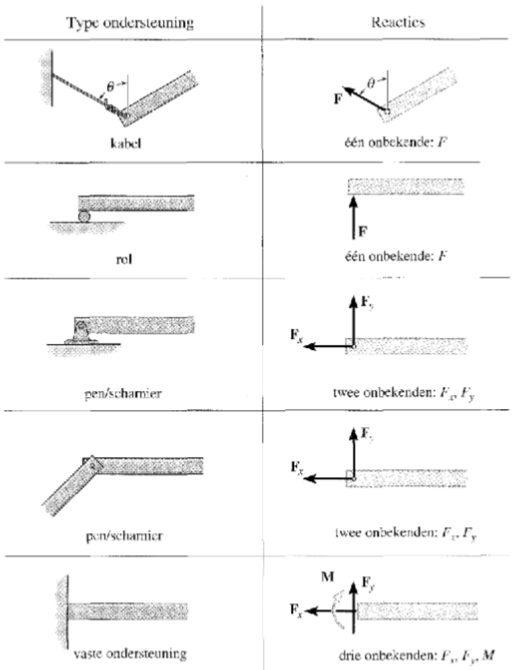
Inhoudsopgave

Hoofdstuk 1

Krachten, momenten, spanningne en rekken

1.1 STATICA EN EVENWICHT VAN CONSTRUCTIES

1.1.1 Types ondersteuningen



1.1.2 Evenwicht van een constructie

$$\sum \mathbf{F} = 0$$
$$\sum \mathbf{M}_O = 0$$

1.2 INTUÏtIEF BEGRIP VAN SPANNINGEN EN REKKEN

$$\sigma = \frac{F}{A_0}$$
$$\varepsilon = \frac{\Delta L}{L_0}$$
$$\sigma = E \cdot \varepsilon$$

1.3 SPANNINGEN

1.3.1 Definitie

De spanningsvector

$$\vec{\phi}^{(n)} = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \tag{1.1}$$

De normaalspanning

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A} \tag{1.2}$$

De schuifspanning

$$\tau = \lim_{\Delta A \to 0} \frac{\Delta F_t}{\Delta A} \tag{1.3}$$

De spanningstensor

$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \tag{1.4}$$

1.3.2 Verband tussen spanningsvector $\vec{\phi}^{(n)}$ en spanningstensor $[\sigma]$

Het verband tussen de spanningsvector en spanningstensor

$$\sigma_{ij} \cdot n_i = \phi_j^{(n)} \quad (i, j = x, y, z) \tag{1.5}$$

1.3.3 Vergelijkingen van het evenwicht

De vergelijkingen van het evenwicht

$$\begin{aligned} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + F_x = 0 \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + F_y = 0 \\ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + F_z = 0 \end{aligned} \tag{1.6}$$

Wet van de wederkerigheid der schuifspanningen

$$\begin{aligned} \tau_{xy} &= \tau_{yx} \\ \tau_{xz} &= \tau_{zx} \\ \tau_{yz} &= \tau_{zy} \end{aligned} \tag{1.7}$$

1.3.4 Transformatie van coördinaten en hoofdrichtingen

Transformatieregels

$$[\sigma'] = [a] \cdot [\sigma] \cdot [a]^\top \tag{1.8}$$

met

$$a_{rk} = \vec{e}'_r \cdot \vec{e}_k \quad r, k = x, y, z \tag{1.9}$$

Diagonaalsom van $[\sigma]$

$$\Sigma_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz} \tag{1.10}$$

Som van de hoofdminoren van $[\sigma]$

$$\Sigma_2 = \sigma_{xx}\sigma_{yy} + \sigma_{xx}\sigma_{zz} + \sigma_{yy}\sigma_{zz} - \tau_{xy}^2 - \tau_{xz}^2 - \tau_{yz}^2 \tag{1.11}$$

Determinant van $[\sigma]$

$$\Sigma_3 = \sigma_{xx}\sigma_{yy}\sigma_{zz} - \sigma_{xx}\tau_{yz}^2 - \sigma_{yy}\tau_{xz}^2 - \sigma_{zz}\tau_{xy}^2 + 2\tau_{xy}\tau_{xz}\tau_{yz} \tag{1.12}$$

1.3.5 Kromlijnige coördinaten

Cilindercoördinaten

De spanningstensor

$$[\sigma] = \begin{bmatrix} \sigma_{rr} & \tau_{x\theta} & \tau_{rz} \\ \tau_{r\theta} & \sigma_{\theta\theta} & \tau_{\theta z} \\ \tau_{rz} & \tau_{\theta z} & \sigma_{zz} \end{bmatrix} \tag{1.13}$$

De evenwichtsvergelijkingen

$$\begin{aligned} \frac{\partial\sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + \frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta} + \frac{\partial\tau_{r\theta}}{\partial z} + F_r = 0 \\ \frac{\partial\tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} + \frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta} + \frac{\partial\tau_{\theta z}}{\partial z} + F_\theta = 0 \\ \frac{\partial\tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} + \frac{1}{r}\frac{\partial\tau_{\theta z}}{\partial\theta} + \frac{\partial\sigma_{zz}}{\partial z} + F_z = 0 \end{aligned} \tag{1.14}$$

Bolcoördinaten

De spanningstensor

$$[\sigma] = \begin{bmatrix} \sigma_{rr} & \tau_{x\theta} & \tau_{r\phi} \\ \tau_{r\theta} & \sigma_{\theta\theta} & \tau_{\theta\phi} \\ \tau_{r\phi} & \tau_{\theta\phi} & \sigma_{\phi\phi} \end{bmatrix} \tag{1.15}$$

De evenwichtsvergelijkingen

$$\begin{aligned} \frac{\partial\sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta} + \frac{1}{r\sin\theta} + \frac{1}{r}\left(2\sigma_{rr} + \tau_{r\theta}\cot\theta - \sigma_{\theta\theta} - \sigma_{\phi\phi}\right) + F_r = 0 \\ \frac{\partial\tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta} + + \frac{1}{r\sin\theta}\frac{\partial\tau_{\theta\phi}}{\phi} + \frac{1}{r}\left(3\tau_{r\theta} + \sigma_{\theta\theta}\cot\theta - \sigma_{\phi\phi}\cot\theta\right) + F_\theta = 0 \\ \frac{\partial\tau_{r\phi}}{\partial r} + \frac{1}{r}\frac{\partial\tau_{\theta\phi}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial\sigma_{\phi\phi}}{\phi} + \frac{1}{r}\left(3\tau_{r\phi} + 2\tau_{\theta\phi}\cot\theta\right) + F_\phi = 0 \end{aligned} \tag{1.16}$$

1.4 REKKEN

1.4.1 Veralgemeende vervormingstoestand

Verband tussen rekken en verplaatsing

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial x} \\ \varepsilon_{yy} &= \frac{\partial v}{\partial y} \\ \varepsilon_{zz} &= \frac{\partial w}{\partial z} \\ \gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\ \gamma_{xz} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \\ \gamma_{yz} &= \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \end{aligned} \tag{1.17}$$

1.4.2 Transformatie van coördinaten en hoofdrichtingen

Transformatieregels

$$[\varepsilon'] = [a] \cdot [\varepsilon] \cdot [a]^\top \quad \text{met } a_{rk} = \vec{e}'_r \cdot \vec{e}_k \tag{1.18}$$

De rekstensor

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{xx} & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_{yy} & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix} \tag{1.19}$$

Diagonaalsom van $[\varepsilon]$

$$i_1 = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} \tag{1.20}$$

Som van de hoofdminoren van $[\varepsilon]$

$$I_2 = \varepsilon_{xx}\varepsilon_{yy} + \varepsilon_{xx}\varepsilon_{zz} + \varepsilon_{yy}\varepsilon_{zz} - \varepsilon_{xy}^2 - \varepsilon_{xz}^2 - \varepsilon_{yz}^2 \tag{1.21}$$

Determinant van $[\varepsilon]$

$$I_3 = \varepsilon_{xx}\varepsilon_{yy}\varepsilon_{zz} - \varepsilon_{xx}\varepsilon_{yz}^2 - \varepsilon_{yy}\varepsilon_{xz}^2 - \varepsilon_{zz}\varepsilon_{xy}^2 + 2\varepsilon_{xy}\varepsilon_{xz}\varepsilon_{yz} \tag{1.22}$$

1.4.3 Compatibiliteitsvoorwaarden

$$\begin{aligned}
\frac{\partial^2 \varepsilon_{xx}}{\partial y \partial z} &= \frac{\partial}{\partial x}\left(\frac{\partial \varepsilon_{xz}}{\partial y} + \frac{\partial \varepsilon_{xy}}{\partial z} - \frac{\partial \varepsilon_{yz}}{\partial x}\right) \\
\frac{\partial^2 \varepsilon_{yy}}{\partial x \partial z} &= \frac{\partial}{\partial y}\left(\frac{\partial \varepsilon_{xy}}{\partial z} + \frac{\partial \varepsilon_{yz}}{\partial x} - \frac{\partial \varepsilon_{xz}}{\partial y}\right) \\
\frac{\partial^2 \varepsilon_{zz}}{\partial x \partial y} &= \frac{\partial}{\partial z}\left(\frac{\partial \varepsilon_{yz}}{\partial x} + \frac{\partial \varepsilon_{xz}}{\partial y} - \frac{\partial \varepsilon_{xy}}{\partial z}\right) \\
\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y} &= \frac{1}{2}\left(\frac{\partial \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2}\right) \\
\frac{\partial^2 \varepsilon_{xz}}{\partial x \partial z} &= \frac{1}{2}\left(\frac{\partial \varepsilon_{zz}}{\partial x^2} + \frac{\partial^2 \varepsilon_{xx}}{\partial z^2}\right) \\
\frac{\partial^2 \varepsilon_{yz}}{\partial y \partial z} &= \frac{1}{2}\left(\frac{\partial \varepsilon_{yy}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial y^2}\right)
\end{aligned} \tag{1.23}$$

1.4.4 Kromlijnige coördinaten

Cilindercoördinaten

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{rr} & \frac{1}{2}\gamma_{r\theta} & \frac{1}{2}\gamma_{rz} \\ \frac{1}{2}\gamma_{r\theta} & \varepsilon_{\theta\theta} & \frac{1}{2}\gamma_{\theta z} \\ \frac{1}{2}\gamma_{rz} & \frac{1}{2}\gamma_{\theta z} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_r}{\partial r} & \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right) & \frac{1}{2}\left(\frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial r}\right) \\ \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right) & \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} & \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right) \\ \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) & \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right) & \frac{\partial u_z}{\partial z} \end{bmatrix} \tag{1.24}$$

Bolcoördinaten

$$\begin{aligned}
[\varepsilon] &= \begin{bmatrix} \varepsilon_{rr} & \frac{1}{2}\gamma_{r\theta} & \frac{1}{2}\gamma_{rz} \\ \frac{1}{2}\gamma_{r\theta} & \varepsilon_{\theta\theta} & \frac{1}{2}\gamma_{\theta\phi} \\ \frac{1}{2}\gamma_{r\phi} & \frac{1}{2}\gamma_{\theta\phi} & \varepsilon_{\phi\phi} \end{bmatrix} \\
&= \begin{bmatrix} \frac{\partial u_r}{\partial r} & \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right) & \frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial u_r}{\partial \phi} + \frac{\partial u_\phi}{\partial r} - \frac{u_\phi}{r}\right) \\ \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right) & \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} & \frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial u_\theta}{\partial \phi} + \frac{1}{r}\frac{\partial u_\phi}{\partial \theta} - \frac{u_\phi}{r}\cot\theta\right) \\ \frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial u_r}{\partial \phi} + \frac{\partial u_\phi}{\partial r} - \frac{u_\phi}{r}\right) & \frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial u_\theta}{\partial \phi} + \frac{1}{r}\frac{\partial u_\phi}{\partial \theta} - \frac{u_\phi}{r}\cot\theta\right) & \frac{1}{r\sin\theta} + \frac{u_r}{r} + \frac{u_\theta}{r}\cot\theta \end{bmatrix}
\end{aligned}$$

1.5 LINEAIR ELASTISCH MATERIAALGEDRAG

1.5.1 Wet van Hooke

Wet van Hooke

$$
\begin{aligned}
\varepsilon_{xx} &= \frac{1}{E}\left[\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right] \\
\varepsilon_{yy} &= \frac{1}{E}\left[\sigma_{yy} - \nu\left(\sigma_{xx} + \sigma_{zz}\right)\right] \\
\varepsilon_{zz} &= \frac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{xx} + \sigma_{yy}\right)\right] \\
\gamma_{xy} &= \frac{\tau_{xy}}{G} \\
\gamma_{xz} &= \frac{\tau_{xz}}{G} \\
\gamma_{yz} &= \frac{\tau_{yz}}{G}
\end{aligned}
\tag{1.25}
$$

Geïnverteerde wet van Hooke

$$
\begin{aligned}
\sigma_{xx} &= \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{xx} + \nu\left(\varepsilon_{yy} + \varepsilon_{zz}\right)\right] \\
\sigma_{yy} &= \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{yy} + \nu\left(\varepsilon_{xx} + \varepsilon_{zz}\right)\right] \\
\sigma_{zz} &= \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{zz} + \nu\left(\varepsilon_{xx} + \varepsilon_{yy}\right)\right] \\
\tau_{xy} &= G\gamma_{xy} \\
\tau_{xz} &= G\gamma_{xz} \\
\tau_{yz} &= G\gamma_{yz}
\end{aligned}
\tag{1.26}
$$

1.5.2 Bijzondere belastingsgevallen

Zuivere rek

(x-as volgens trekrichting)

$$
[\sigma] = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1.27}
$$

Zuivere afschuiving

(x-as volgens trekrichting)

$$
[\sigma] = \begin{bmatrix} 0 & 0 & \tau_{xz} \\ 0 & 0 & 0 \\ \tau_{xz} & 0 & 0 \end{bmatrix} \tag{1.28}
$$

Hydrostatische belasting

$$
[\sigma] = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} -|p| & 0 & 0 \\ 0 & -|p| & 0 \\ 0 & 0 & -|p| \end{bmatrix} \tag{1.29}
$$

Torsie of wringing

(x-as in de langsrichting)

$$
[\sigma] = \begin{bmatrix} 0 & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & 0 & 0 \\ \tau_{xz} & 0 & 0 \end{bmatrix} \tag{1.30}
$$

in polaire coördinaten

$$[\sigma] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \tau_{\theta z} \\ 0 & \tau_{\theta z} & 0 \end{bmatrix} \tag{1.31}$$

1.5.3 Relaties tussen de elastische constanten

Verband tussen E, ν en G

$$G = \frac{E}{2(1+\nu)} \tag{1.32}$$

Volumeverandering en compressiemodulus

Volumerek of dilatatie

$$\varepsilon_{\mathrm{vol}} = \frac{\Delta V}{dV} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} \tag{1.33}$$

Compressiemodulus of volume-elasticiteitsmodulus

$$K = \frac{-|p|}{\frac{\Delta V}{dv}} = \frac{E}{3(1-2\nu)}, \quad \nu\frac{1}{2} \tag{1.34}$$

1.5.4 Kromlijnige coördinaten

Cilindercoördinaten

$$\begin{aligned}
\varepsilon_{rr} &= \frac{1}{E}\left[\sigma_{xx} - \nu\left(\sigma_{\theta\theta} + \sigma_{zz}\right)\right] \\
\varepsilon_{\theta\theta} &= \frac{1}{E}\left[\sigma_{\theta\theta} - \nu\left(\sigma_{xx} + \sigma_{zz}\right)\right] \\
\varepsilon_{zz} &= \frac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{xx} + \sigma_{\theta\theta}\right)\right] \\
\gamma_{x\theta} &= \frac{\tau_{x\theta}}{G} \\
\gamma_{xz} &= \frac{\tau_{rz}}{G} \\
\gamma_{\theta z} &= \frac{\tau_{\theta z}}{G}
\end{aligned} \tag{1.35}$$

1.6 OPLOSSING VAN HET LINEAIR ELASTISCH PROBLEEM

Gebruik *vergelijkingen van het evenwicht*, *vergelijkingen tussen rek en verplaatsing* en de wet van Hooke samen met randvoorwaarden om het linear statisch probleem op te lossen.

1.7 THERMISCHE SPANNINGEN

1.7.1 Vergelijkingen

$$\begin{aligned}
\varepsilon_{xx} &= \frac{1}{E}\left[\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right] + \alpha T \\
\varepsilon_{yy} &= \frac{1}{E}\left[\sigma_{yy} - \nu\left(\sigma_{xx} + \sigma_{zz}\right)\right] + \alpha T \\
\varepsilon_{zz} &= \frac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{xx} + \sigma_{yy}\right)\right] + \alpha T \\
\gamma_{xy} &= \frac{\tau_{xy}}{G} \\
\gamma_{xz} &= \frac{\tau_{xz}}{G} \\
\gamma_{yz} &= \frac{\tau_{yz}}{G}
\end{aligned} \tag{1.36}$$

Geïnverteerd

$$\begin{aligned}
\sigma_{xx} &= \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{xx} + \nu\left(\varepsilon_{yy}+\varepsilon_{zz}\right)\right] - \frac{E}{1-2\nu}\alpha T \\
\sigma_{yy} &= \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{yy} + \nu\left(\varepsilon_{xx}+\varepsilon_{zz}\right)\right] - \frac{E}{1-2\nu}\alpha T \\
\sigma_{zz} &= \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{zz} + \nu\left(\varepsilon_{xx}+\varepsilon_{yy}\right)\right] - \frac{E}{1-2\nu}\alpha T \\
\tau_{xy} &= G\gamma_{xy} \\
\tau_{xz} &= G\gamma_{xz} \\
\tau_{yz} &= G\gamma_{yz}
\end{aligned} \tag{1.37}$$

1.8 ARBEID EN ELASTISCHE ENERGIE

1.8.1 Arbeid van een kracht

Verichte arbeid

$$dU_{\text{uitw}} = F \cdot dx \tag{1.38}$$

Totale arbeid over afstand x

$$U_{\text{uitw}} = \int_0^x F(x) \cdot dx \tag{1.39}$$

1.8.2 Arbeid van een moment

Verichte arbeid

$$dU_{\text{uitw}} = M \cdot d\theta \tag{1.40}$$

Totale arbeid over hoekverdraaiing

$$U_{\text{uitw}} = \int_0^\theta M(\theta) \cdot \theta \tag{1.41}$$

1.8.3 Wet van behoud van mechanishce energie

$$U_{\text{uitw}} = U_{\text{inw}} \tag{1.42}$$

Hoofdstuk 2

Structureel gedrag

2.1 GEOMETRISCHE EIGENSCHAPPEN VAN DE DWARSDOORSNEDE

2.1.1 Opstellen vergelijkingen

Oppervlakte van de dwarsdoorsnede

$$A = \int \int dy' dz' \tag{2.1}$$

Statisch moment om de y'-as

$$S_{y'} = \int \int z' dy' dz' \tag{2.2}$$

Statisch moment om de z'-as

$$S_{z'} = \int \int y' dy' dz' \tag{2.3}$$

Ligging zwaartepunt

$$\begin{aligned} y'_o &= \frac{S_{z'}}{A} \\ z'_o &= \frac{S_{y'}}{A} \end{aligned} \tag{2.4}$$

Traagheidsmomenten van de doorsnede

$$\begin{aligned} I_{yy} &= \int \int z^2 dydz \\ I_{zz} &= \int \int y^2 dydz \\ I_{yz} &= -\int \int yzdydz \end{aligned} \tag{2.5}$$

Stelling van Steiner (parallel axis theorem)

$$\begin{aligned} I_{y'y'} &= I_{yy} + A \cdot (z'_o)^2 \\ I_{z'z'} &= I_{zz} + A \cdot (z'_o)^2 \\ I_{y'z'} &= I_{yz} - A \cdot y'_o \cdot z'_o \end{aligned} \tag{2.6}$$

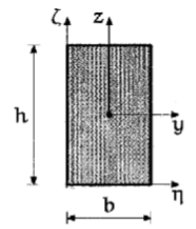
Rotatie van het assenstelsel

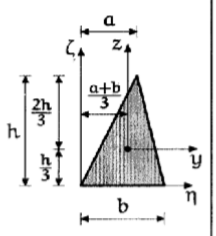
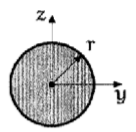
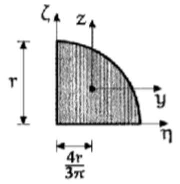
$$\begin{aligned} I_{y'y'} &= \cos^2 \alpha \cdot I_{yy} + \sin^2 \alpha \cdot I_{zz} + 2 \sin \alpha \cos \alpha \cdot I_{yz} \\ I_{z'z'} &= \sin^2 \alpha \cdot I_{yy} + \cos^2 \alpha \cdot I_{zz} - 2 \sin \alpha \cos \alpha \cdot I_{yz} \\ I_{y'z'} &= -\sin \alpha \cos \alpha \cdot I_{yy} + \sin \alpha \cos \alpha \cdot I_{zz} + \left(\cos^2 \alpha - \sin^2 \alpha\right) \cdot I_{yz} \end{aligned}$$

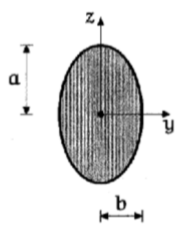
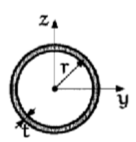
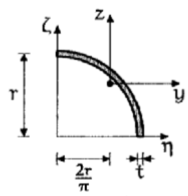
Voor de hoofdtraagheidsmomenten geldt

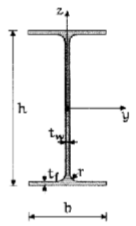
$$\tan 2\alpha = \frac{2I_{yz}}{I_{yy} - I_{zz}} \tag{2.7}$$

2.1.2 Praktische berekening

	$A = bh$	$S_\eta = \frac{bh^2}{2}$ $S_\zeta = \frac{hb^2}{2}$	$I_{yy} = \frac{bh^3}{12}$; $I_{zz} = \frac{hb^3}{12}$ $I_{\eta\eta} = \frac{bh^3}{3}$; $I_{\zeta\zeta} = \frac{hb^3}{3}$ $I_{\eta\zeta} = -\frac{b^2h^2}{4}$

	$A = \frac{bh}{2}$	$S_\eta = \frac{bh^2}{6}$ $S_\zeta = \frac{bh}{6}(a+b)$	$I_{yy} = \frac{bh^3}{36}$ $I_{zz} = \frac{bh}{36}(b^2 - ab + a^2)$ $I_{yz} = -\frac{bh^2}{72}(2a - b)$ $I_{\eta\eta} = \frac{bh^3}{12}$ $I_{\zeta\zeta} = \frac{bh}{12}(b^2 + ab + a^2)$ $I_{\eta\zeta} = -\frac{bh^2}{24}(2a + b)$
	$A = \pi r^2$		$I_{yy} = I_{zz} = \frac{\pi}{4} r^4$
	$A = \frac{\pi}{4} r^2$	$S_\eta = S_\zeta = \frac{1}{3} r^3$	$I_{\eta\eta} = I_{\zeta\zeta} = \frac{\pi}{16} r^4$ $I_{\eta\zeta} = -\frac{1}{8} r^4$

	$A = \pi ab$		$I_{yy} = \frac{\pi}{4} ba^3$ $I_{zz} = \frac{\pi}{4} ab^3$
	$A = 2\pi rt$		$I_{yy} = I_{zz} = \pi r^3 t$
	$A = \frac{\pi}{2} rt$	$S_\eta = S_\zeta = r^2 t$	$I_{\eta\eta} = I_{\zeta\zeta} = \frac{\pi}{4} r^3 t$ $I_{\eta\zeta} = -\frac{1}{2} r^3 t$



Profiel	afmetingen (mm)					massa	A	I_{yy}	W_y	I_{zz}	W_z
	h	b	t_w	t_f	r	(kg/m)	(mm²)	(10^4mm⁴)	(10^3mm³)	(10^4mm⁴)	(10^3mm³)
IPE 80	80	46	3.8	5.2	5	6.0	764.3	80.1	20.0	8.49	3.69
IPE 100	100	55	4.1	5.7	7	8.26	1032	171	34.2	15.9	5.79
IPE 120	120	64	4.4	6.3	7	10.4	1321	318	53.0	27.7	8.65
IPE 140	140	73	4.7	6.9	7	12.9	1643	541	77.3	44.9	12.3
IPE 160	160	82	5.0	7.4	9	15.8	2009	869	109	68.3	16.7
IPE 180	180	91	5.3	8.0	9	18.8	2395	1317	146	101	22.2
IPE 200	200	100	5.6	8.5	12	22.4	2848	1943	194	142	28.5
IPE 220	220	110	5.9	9.2	12	26.2	3337	2771	252	205	37.3
IPE 240	240	120	6.2	9.8	15	30.7	3912	3891	324	284	47.3
IPE 270	270	135	6.6	10.2	15	36.1	4595	5790	429	420	62.2
IPE 300	300	150	7.1	10.7	15	42.2	5381	8356	557	604	80.5
IPE 330	330	160	7.5	11.5	18	49.15	6261	11770	713	788	98.5
IPE 360	360	170	8.0	12.7	18	57.1	7273	16270	904	1043	123
IPE 400	400	180	8.6	13.5	21	66.3	8446	23130	1156	1318	146
IPE 450	450	190	9.4	14.6	21	77.6	9882	33740	1500	1676	176
IPE 500	500	200	10.2	16.0	21	90.7	11550	48200	1928	2142	214
IPE 550	550	210	11.1	17.2	24	105.5	13440	67120	2441	2668	254
IPE 600	600	220	12.0	19.0	24	122.6	15600	92080	3069	3387	308

met de weerstandsmomenten gedefinieerd als

$$W_y = \frac{2 \cdot I_{yy}}{h}$$
$$W_z = \frac{2 \cdot I_{zz}}{b}$$

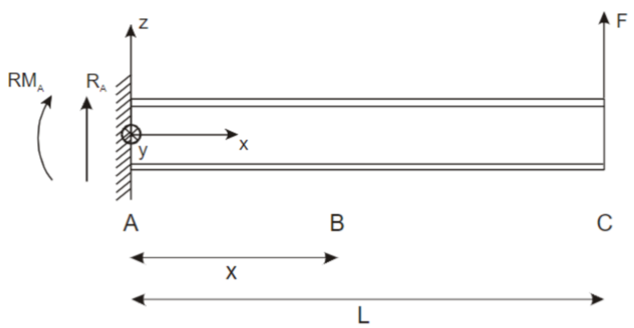
2.2 NORMAALKRACHT, BUIGEND MOMENT EN DWARSKRACHT

2.2.1 Verband tussen q, V en M

$$q = \frac{-dV}{dx}$$
$$V = \frac{dM}{dx}$$
$$q = \frac{-d^2M}{dx^2} \tag{2.8}$$

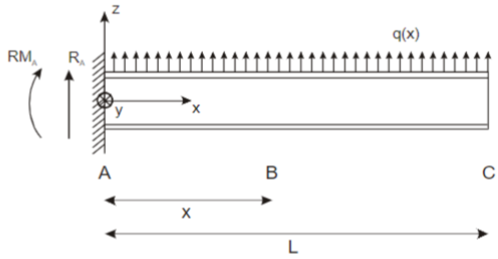
2.2.2 Enkele referentiegevallen

Ingeklemde balk met puntlast



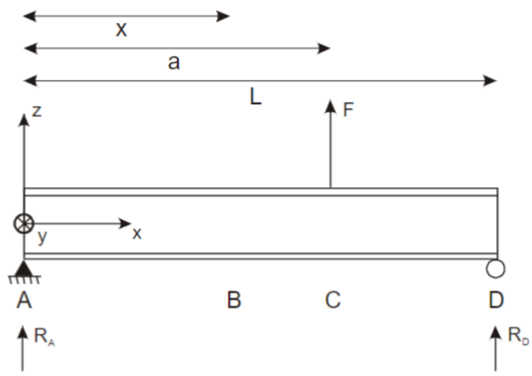
$$V = F$$
$$M = -F \cdot (L - x)$$

Ingeklemde balk met verdeelde belasting



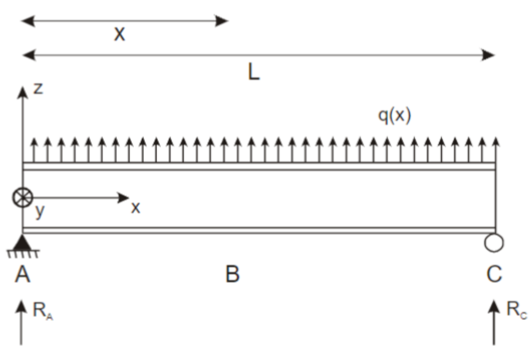
$$V = q \cdot (L - x)$$
$$M = \frac{-q \cdot (L-x)^2}{2}$$

Balk op twee steunpunten met puntlast



$$V = \begin{cases} \frac{F \cdot (L-a)}{L} & , x < a \\ \frac{-F \cdot a}{L} & , x > a \end{cases}$$
$$M = \begin{cases} \frac{F \cdot x \cdot (L-a)}{L} & , x < a \\ \frac{F \cdot a \cdot (L-x)}{L} & , x > a \end{cases}$$

Balk op twee steunpunten met verdeelde belasting



$$V = q \cdot \left(\frac{L}{2} - x \right)$$
$$M = \frac{q \cdot x \cdot (L-x)}{2}$$

2.3 VERBAND TUSSEN SNEDEKRACHTEN EN SPANNINGEN

2.3.1 Spanningen t.g.v. normaalkracht N

De spanning t.g.v. de normaalkracht N

$$\sigma_{xx} = \frac{N}{A} \tag{2.9}$$

en de bijhorende rek

$$\varepsilon_{xx} = \frac{N}{E \cdot A} \tag{2.10}$$

2.3.2 Spanningen t.g.v. buigend moment M

$$\sigma_{xx} = \frac{M \cdot z}{I_{yy}} \tag{2.11}$$

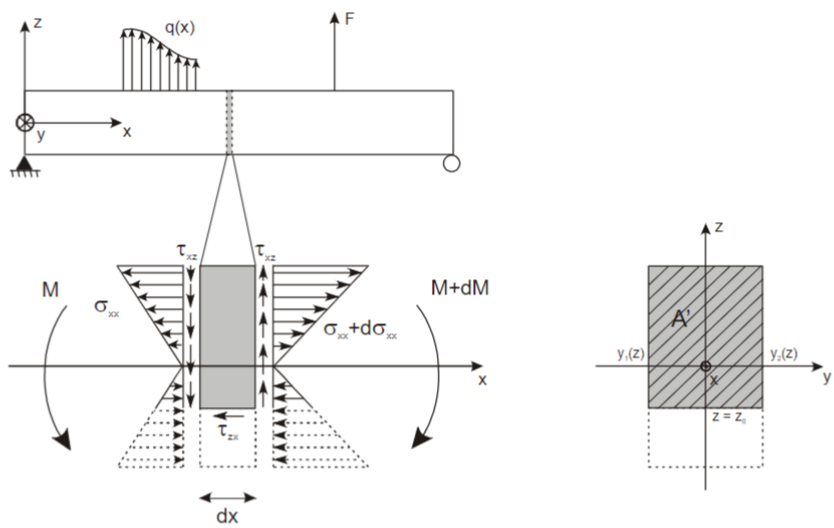
Kromtestraal

$$\frac{1}{R} = \frac{-M}{E \cdot I_{yy}} \tag{2.12}$$

2.3.3 Spanningen t.g.v. dwarskracht V

Formule van Jourawski

$$\tau_{xz}(z) = \frac{V}{I_{yy}} \cdot \frac{S_y(z)}{[y_2(z) - y_1(z)]} \tag{2.13}$$



2.4 VERPLAATSINGEN

2.4.1 Verplaatsingen t.g.v. de normaalkracht N

$$\Delta L = \int_0^L \frac{N}{A \cdot E} dx \tag{2.14}$$

2.4.2 Verplaatsingen t.g.v. het buigend moment M

$$\frac{d^2u}{dx^2} = \frac{-M}{E \cdot I_{yy}} \tag{2.15}$$

$$q = \frac{-dV}{dx} = \frac{-d^2M}{dx^2} = -EI_{yy}\frac{d^3\alpha}{dx^3} = EI_{yy}\frac{d^4u}{dx^4} \tag{2.16}$$

Ingeklemde balk met puntlast

$$u(x) = \frac{F}{E \cdot I_{yy}} \cdot \left(L \cdot \frac{x^2}{2} - \frac{x^3}{6}\right) \tag{2.17}$$

Ingeklemde balk met verdeelde belasting

$$u(x) = \frac{q}{E \cdot I_{yy}} \cdot \left[\frac{(L-x)^4}{24} + \frac{L^3}{6} \cdot x - \frac{L^4}{24} \right] \tag{2.18}$$

Balk op twee steunpunten met puntlast

$$u(x) = \begin{cases} \frac{F}{E \cdot I_{yy}} \cdot \left[-\frac{(L-a)}{L} \cdot \frac{x^3}{6} + \frac{a \cdot (L-a) \cdot (2L-*a)}{6L} \cdot x \right] & , x < a \\ \frac{F}{E \cdot I_{yy}} \cdot \left[-\frac{a \cdot (L-x)^3}{6L} - \frac{a \cdot (L-a) \cdot (L+a)}{6L} \cdot x + \frac{a \cdot (L-a) \cdot (L+a)}{6} \right] & , x > a \end{cases} \tag{2.19}$$

Balk op twee steunpunten met verdeelde belasting

$$u(x) = \frac{q}{E \cdot I_{yy}} \cdot \left[\frac{1}{24} \cdot x^4 - \frac{L}{12} \cdot x^3 + \frac{L^3}{24} \cdot x \right] \tag{2.20}$$

2.5 SINGULARITEITSFUNCTIES

Voor de verdeelde belasting $q(x)$

$$\forall n \in \mathbb{N} : \quad \langle x-a \rangle^n = \left| \begin{array}{cc} 0 & , x < a \\ (x-a)^n & , x > a \end{array} \right. \tag{2.21}$$

met volgende regels

$$\begin{gathered} \frac{d}{dx} \langle x-a \rangle^n = n \cdot \langle x-a \rangle^{n-1} \\ \int \langle x-a \rangle^n dx = \frac{\langle x-a \rangle^{n+1}}{n+1} + C \end{gathered} \tag{2.22}$$

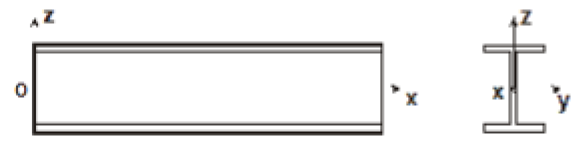
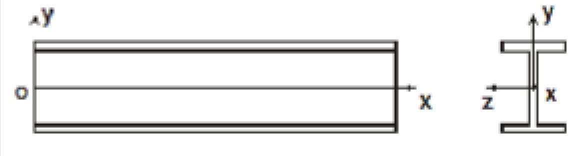
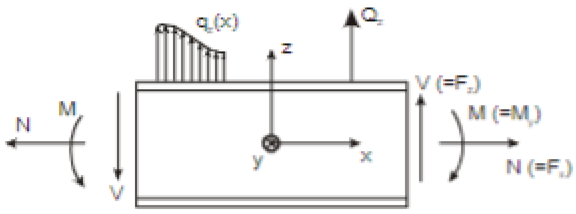
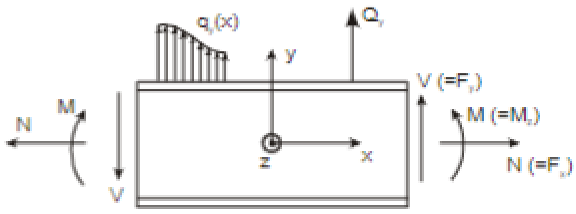
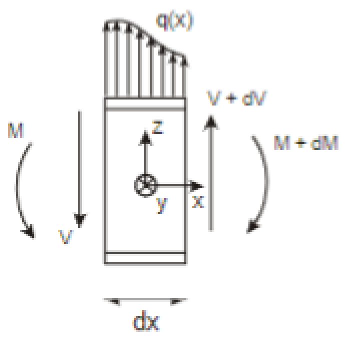
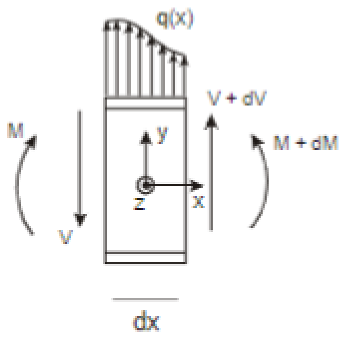
En voor geconcentreerde puntlasten F of koppes K

$$\begin{gathered} q = F \cdot \langle x-a \rangle^{-1} = \left| \begin{array}{ll} 0 & , x < a \\ F & , x > a \end{array} \right. \\ q = K \cdot \langle x-a \rangle^{-2} = \left| \begin{array}{ll} 0 & , x < a \\ K & , x > a \end{array} \right. \end{gathered} \tag{2.23}$$

met rekenregels $\forall n \in \mathbb{N}$

$$\begin{gathered} \frac{d}{dx} \langle x-a \rangle^{-n} = n \cdot \langle x-a \rangle^{-n-1} \\ \int \langle x-a \rangle^{-n} dx = \frac{\langle x-a \rangle^{n+1}}{-n+1} \end{gathered} \tag{2.24}$$

2.6 INVLOED VAN DE KEUZE VAN HET ASSENSTELSEL

<table>
<tr><th>Keuze in deze cursus</th><th>Alternatieve keuze</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>$$q = \frac{-dV}{dx}$$ $$V = \frac{dM}{dx}$$ $$q = \frac{-d^2M}{dx^2}$$</td><td>$$q = \frac{-dV}{dx}$$ $$V = \frac{-dM}{dx}$$ $$q = \frac{d^2M}{dx^2}$$</td></tr>
<tr><td>Positief koppel K: $q = K \cdot \langle x - a \rangle^{-2}$</td><td>Positief koppel K: $q = -K \cdot \langle x - a \rangle^{-2}$</td></tr>
</table>

Hoofdstuk 3

Tweedimensionale problemen