Report Electromagnetism I

FDTD Simulation of Lossless Transmission Lines

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Abstract
Short review of the report.

1 A bit traveling through a lossless transmission line

1.1 General behavior of a bit traveling through a transmission line

This section aims to predict, and thus validate, the behavior of the numerical simulation of the voltage in a LTL. A rough analytical approach is used, since it gives more insight in the physical nature of the problem.

Rearranging the telegrapher's equations for a LTL yields

$$\frac{\partial^2 \hat{v}(z,t)}{\partial z^2} - k^2 \frac{\partial^2 \hat{v}(z,t)}{\partial t^2} = 0, \quad k = \frac{1}{c} = \sqrt{LC} = \text{cte}. \tag{1}$$

This means that $\hat{v}$ satisfies the wave equation and thus can be written as a superposition of two voltage waves traveling in opposite directions with constant speed c. That is,

$$\hat{v}(z,t) = \underbrace{\hat{v}^+(z-ct)}_{\text{forward wave}} + \underbrace{\hat{v}^-(z+ct)}_{\text{backward wave}}. \tag{2}$$

Realising that the voltage and current are related;

$$\hat{i}(z,t) = \frac{1}{R_c}(\hat{v}^+(z,t) - \hat{v}^-(z,t)), \tag{3}$$

with R_c a constant that depends on the characteristics (e.g. geometry) of the TL, and is hence called the characteristic impedance. Although it has the units of resistance (Ω) it is not reactive; there is no energy dissipation. The characteristic impedance can be interpreted as a scale for (1) the current to voltage amplitude and (2) the dissipation of energy to the load or generator (see (REF)). In further sections, R_c is assumed to be finite and constant.

1.2 Reflections and amplitude dampening

In section (REF) the wave behavior of the voltage and current is discovered. To this end, the analysis of voltage waves at the boundaries for specific values of the generator resistance R_g and load resistance R_L is discussed in this section.

Our first question concerns the entrance of the bit to the TL: the simulation starts at $t = 0$ and the generator $\hat{e}_g$ produces a bit with amplitude V_0. Before the bit enters the TL, it meets the generator resistance R_g and thus energy will be dissipated i.e. the voltage amplitude is dampened. This can be clarified by expressing Kirchoff's Voltage Law at the origin ($z = 0$) of the TL;

$$\hat{e}_g(t) = (R_g + R_c)\hat{i}(0,t) \tag{4}$$

$$\Rightarrow \hat{i}(0,t) = \frac{1}{R_g + R_c}\hat{e}_g(t) = \frac{1}{R_c}(\hat{v}^+(0,t) - \hat{v}^-(0,t)) \tag{5}$$

$$\Rightarrow \hat{v}(0,t) = \hat{v}^+(0,t) = \kappa \hat{e}_g(0,t). \tag{6}$$

The amplitude V_0 of generated bit is reduced by a factor $\kappa = \frac{R_c}{R_g + R_c}$. Here, the role of the characteristic impedance becomes clear; at the beginning of the TL, the bit 'sees' it as a resistance, but does not dissipate energy to it. Therefore the name input impedance. More interestingly, when $R_g = 0$, $\kappa = 1$ and the bit amplitude remains unchanged; the bit freely enters the TL since it does not meet resistance Figure (REF).