

Formularium - Biomechanics

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Chapter 1

BioFluidMechanics

1.1 section one

first block of the lectures

Chapter 2

Soft Tissue Engineering

2.1 Stress & strain in large deformations

2.1.1 Kinematics

- Mapping function

$$\vec{x} = \varphi(\vec{X}, t) \quad (2.1)$$

- Displacement function

$$\vec{x} = \vec{X} + \underline{U}(\vec{X}, t) \quad (2.2)$$

- The deformation gradient tensor

$$d\vec{x} = \mathbf{F}(X, t)d\vec{X} \Rightarrow \mathbf{F} = \frac{\partial \vec{x}}{\partial \vec{X}} = \frac{\partial \varphi}{\partial \vec{X}} \quad (2.3)$$

- Volume transformation

$$dv = \det(\mathbf{F})dV = JdV \quad (2.4)$$

- Surface transformation: Nanson's Formula

$$\vec{n}ds = J\mathbf{F}^{-\top}\vec{N}dS \quad (2.5)$$

- Polar decomposition

$$\mathbf{F} = \mathbf{R}\mathbf{U} = \mathbf{V}\mathbf{R} \quad (2.6)$$

$$\Rightarrow \mathbf{U}^2 = \mathbf{F}^\top \mathbf{F} = \mathbf{C} \quad (2.7)$$

$$\Rightarrow \mathbf{V}^2 = \mathbf{F}\mathbf{F}^\top = \mathbf{B} \quad (2.8)$$

2.1.2 Strain

- Nominal strain, Biot strain or Engineering strain

$$\mathbf{e} = \mathbf{U} - \mathbf{I} \quad (2.9)$$

- Green-Lagrange strain

$$\mathbf{E} = \frac{1}{2}(\mathbf{F}^\top \mathbf{F} - \mathbf{I}) = \frac{1}{2}(\mathbf{U}^2 - \mathbf{I}) \quad (2.10)$$

- Euler-Almansi strain

$$\mathbf{A} = \frac{1}{2}(\mathbf{I} - \mathbf{V}^{-2}) \quad (2.11)$$

- True strain or logarithmic strain

$$\varepsilon = \ln(\mathbf{U}) \quad (2.12)$$

2.1.3 Stress

- True stress or Cauchy stress

$$\boldsymbol{\sigma}^\top = \frac{d\vec{f}}{\vec{n}dS} \quad (2.13)$$

- 1PK: first Piola Kirchoff stress (Nominal stress or Engineering stress)

$$\mathbf{P} = \frac{d\vec{f}}{\vec{N}dS_0} \quad (2.14)$$

- 2PK: second Piola Kirchoff stress

$$\mathbf{S}^\top = \frac{\mathbf{F}^{-1}d\vec{f}}{\vec{N}dS_0} \quad (2.15)$$

- Switching between stresses:

	$\boldsymbol{\sigma}$	\mathbf{P}	\mathbf{S}
$\boldsymbol{\sigma}$	$\boldsymbol{\sigma}$	$J^{-1}\mathbf{P}\mathbf{F}^\top$	$J^{-1}\mathbf{F}\mathbf{S}\mathbf{F}^\top$
\mathbf{P}	$J\boldsymbol{\sigma}\mathbf{F}^\top$	\mathbf{P}	$\mathbf{F}\mathbf{S}$
\mathbf{S}	$J\mathbf{F}^{-1}\boldsymbol{\sigma}\mathbf{F}^{-\top}$	$\mathbf{F}^{-1}\mathbf{P}$	\mathbf{S}

2.2 Constitutive modeling

2.3 Mechanical testing & parameter fitting

2.3.1 Uniaxial tensile testing

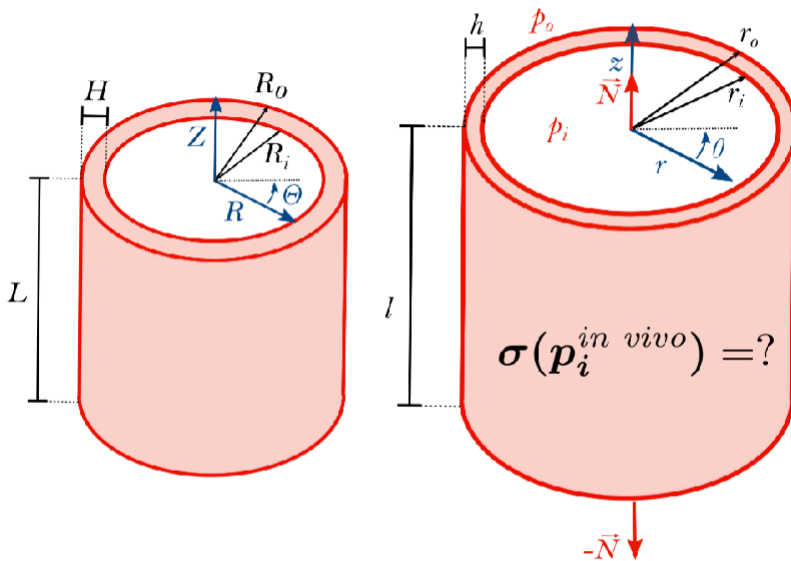
2.3.2 Biaxial tensile testing

2.3.3 Extension-inflation testing

2.3.4 Strain mapping

2.4 In vivo wall stress estimation

2.4.1 Laplace's law for thin-walled tubes



The deformation gradient tensor is given as

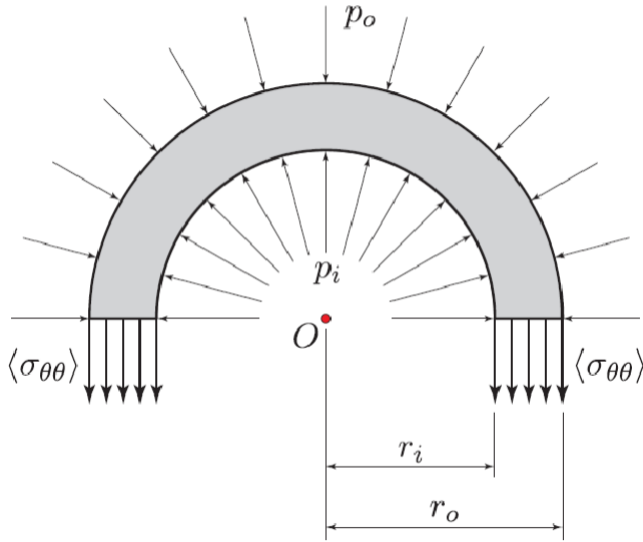
$$\mathbf{F} = \begin{bmatrix} 1/(\lambda_\theta \lambda_z) & 0 & 0 \\ 0 & \lambda_\theta & 0 \\ 0 & 0 & \lambda_z \end{bmatrix} \quad (2.16)$$

with corresponding stresses

$$\langle \sigma_{rr} \rangle = -\frac{P}{2} \quad (2.17)$$

$$\langle \sigma_{\theta\theta} \rangle = \frac{P}{h/r_i} \quad (2.18)$$

$$\langle \sigma_{zz} \rangle = \frac{||\vec{N}||}{\pi(r_o^2 - r_i^2)} \quad (2.19)$$



2.4.2 Analytical analysis for thick-walled cylinders

2.4.3 The Finite Element Method

2.5 Other biomechanical applications