

## KDE: Ejemplo "V sini"

```
In[1]:= SetDirectory[NotebookDirectory[]]
          | establece direct... | directorio de cuaderno

Out[1]= /Users/michel/Library/CloudStorage/OneDrive-uv.cl/cursos/pregrado/Estadisticas/2024/
          Clases

In[2]:= vsini = Import["vsini.csv"] // Flatten;
          | importa           | aplana

In[3]:= Length@vsini
          | longitud

Out[3]= 11818

In[4]:= Select[vsini, ! NumberQ]
          | selecciona      | ¿número?

Out[4]= { }

In[5]:= Min@vsini
          | mínimo

Out[5]= 0

In[6]:= Max@vsini
          | máximo

Out[6]= 29

In[7]:= Count[vsini, 0]
          | conteo

Out[7]= 133

In[8]:= Histogram[vsini, Automatic, "PDF"]
          | histograma     | automático   | función

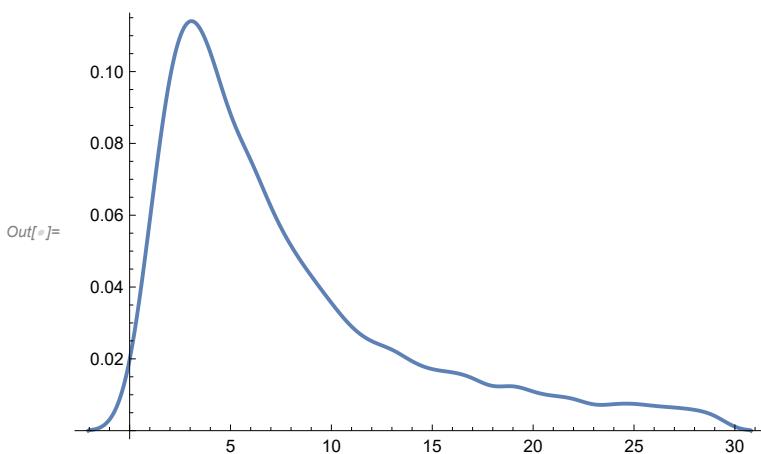
Out[8]=
```

A histogram titled "Histogram[vsini, Automatic, "PDF"]". The x-axis is labeled from 0 to 30 with major ticks every 5 units. The y-axis is labeled from 0.00 to 0.12 with major ticks every 0.02 units. The histogram consists of approximately 11818 bars. The distribution is unimodal and right-skewed, with the highest frequency occurring in the bin around 3, which reaches a height of about 0.12. The frequency decreases as the value increases, with most other bins having heights between 0.00 and 0.05.

Bin Range (x)	Frequency (y)
0-1	~0.055
1-2	~0.105
2-3	~0.120
3-4	~0.105
4-5	~0.085
5-6	~0.075
6-7	~0.060
7-8	~0.050
8-9	~0.045
9-10	~0.035
10-11	~0.030
11-12	~0.028
12-13	~0.025
13-14	~0.022
14-15	~0.020
15-16	~0.018
16-17	~0.017
17-18	~0.015
18-19	~0.014
19-20	~0.013
20-21	~0.012
21-22	~0.011
22-23	~0.010
23-24	~0.009
24-25	~0.008
25-26	~0.007
26-27	~0.006
27-28	~0.005
28-29	~0.004
29-30	~0.003

In[1]:= **SmoothHistogram@vsini**

|histograma suave



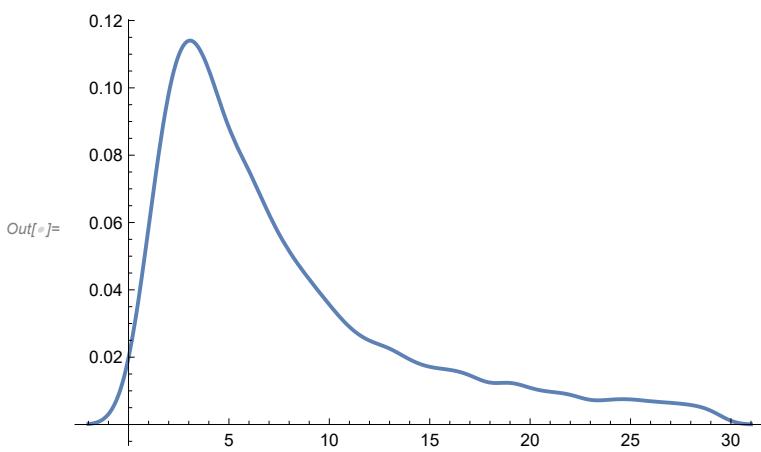
In[2]:= **dist0 = SmoothKernelDistribution@vsini**

|distribución de núcleo suave

Out[2]= **DataDistribution**[ Type: SmoothKernel  
Data points: 11818]

In[3]:= **grd0 = Plot[PDF[dist0, x], {x, -2, 31}]**

|repr...|función de densidad de probabilidad



In[4]:= **CDF[dist0, 0]**

|función de distribución acumulada

Out[4]= **0.0107542**

In[5]:= **NIntegrate[PDF[dist0, x], {x, -∞, 0}]**

|integra numé...|función de densidad de probabilidad

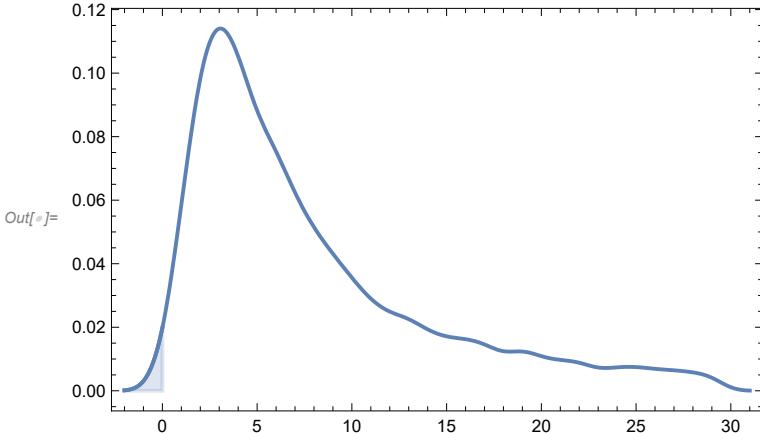
Out[5]= **0.0107542**

In[6]:= **Probability[x < 0, x ≈ dist0]**

|probabilidad

Out[6]= **0.0107542**

```
In[6]:= Show[Plot[PDF[dist0, x], {x, -2, 31}],  
  Plot[PDF[dist0, x], {x, -2, 0}, Filling -> Bottom, Frame -> True]  
Out[6]=
```

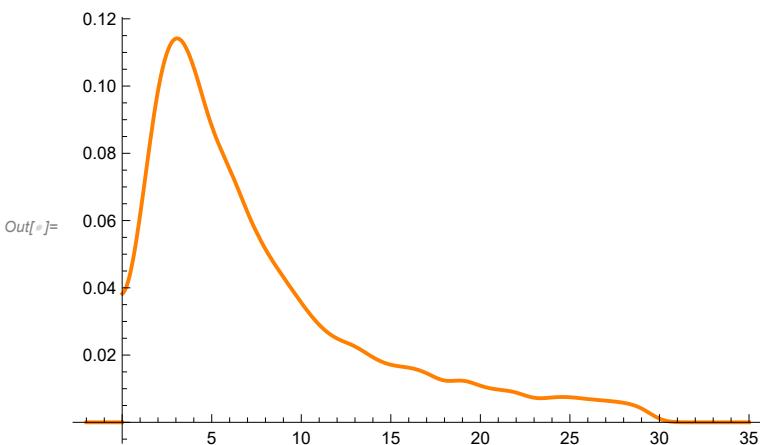


```
In[7]:= Probability[x ≥ 0, x ≈ dist0]
```

```
Out[7]= 0.989246
```

```
In[8]:= dist1 = SmoothKernelDistribution[v$ini, Automatic, {"Bounded", {0, 35}, "Gaussian"}];  
Out[8]=
```

```
In[9]:= grd1 = Plot[PDF[dist1, x], {x, -2, 35}, PlotStyle -> Orange]  
Out[9]=
```



```
In[10]:= Probability[x < 0, x ≈ dist1]
```

```
Out[10]= 0
```

```
In[11]:= Probability[x > 0, x ≈ dist1]
```

```
Out[11]= 1.
```

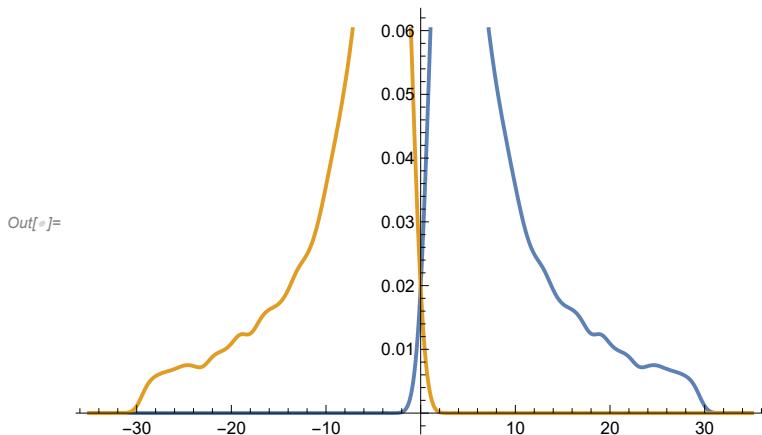
```
In[6]:= Show[grd0, grd1, Frame → True]
|muestra |marco |verdadε

Out[6]=
```

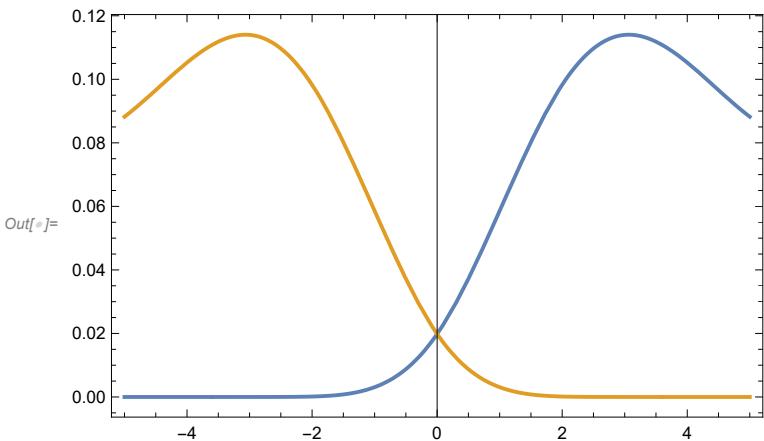
```
In[7]:= vsini2 = -vsini;
In[8]:= dist2 = SmoothKernelDistribution[volini, Automatic, "Gaussian"];
          |distribución de núcleo suave |automático

In[9]:= dist3 = SmoothKernelDistribution[volini2, Automatic, "Gaussian"];
          |distribución de núcleo suave |automático

In[10]:= Plot[{PDF[dist2, x], PDF[dist3, x]}, {x, -35, 35}]
          |repre... |función de densi... |función de densidad de probabilidad
```

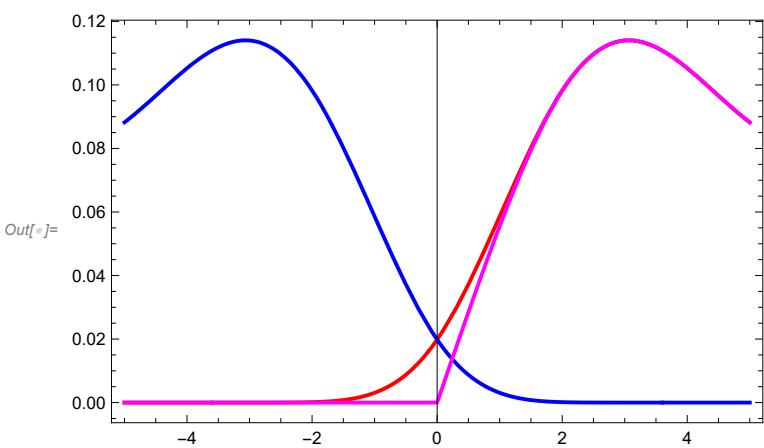


In[=] = Plot[{PDF[dist2, x], PDF[dist3, x]}, {x, -5, 5}, Frame → True]  
 [representación] [función de densidad] [función de densidad de probabilidad] [marco] [verdadero]

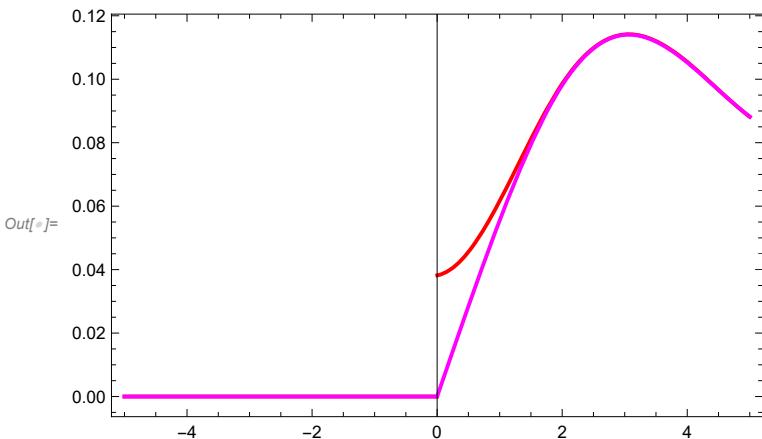


In[=] = dist4 = ProbabilityDistribution[PDF[dist2, x] - PDF[dist3, x], {x, 0, 35}];  
 [distribución de probabilidad] [función de densidad] [función de densidad de probabilidad]

In[=] = Plot[{PDF[dist2, x], PDF[dist3, x], PDF[dist4, x]},  
 [representación] [función de densidad] [función de densidad] [función de densidad de probabilidad]  
 {x, -5, 5}, PlotStyle → {Red, Blue, Magenta}, Frame → True]  
 [estilo de representación] [rojo] [azul] [magenta] [marco] [verdadero]



```
In[6]:= Plot[{PDF[dist1, x], PDF[dist4, x]}, {x, -5, 5}, PlotStyle -> {Red, Magenta}, Frame -> True]
[repre... función de densi... [función de densidad de probabilidad] [estilo de repre... [rojo] [magenta] [marco] [verdade]
```



```
In[7]:= cte = NIntegrate[PDF[dist4, x], {x, 0, 50}]
[integra numé... [función de densidad de probabilidad]
```

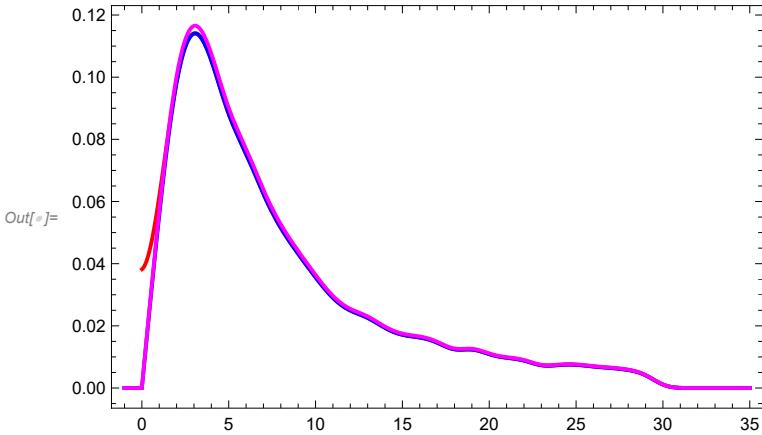
```
Out[7]= 0.978492
```

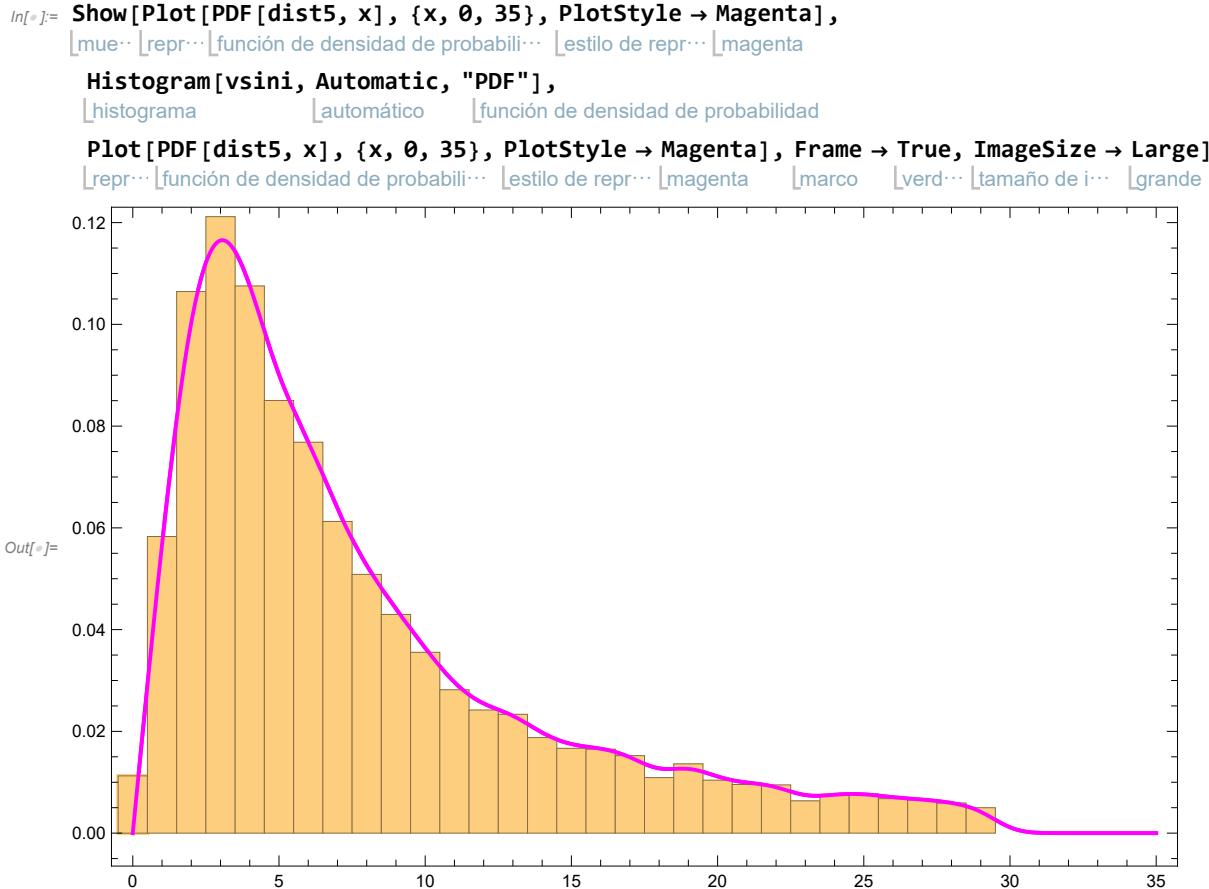
```
In[8]:= dist5 = ProbabilityDistribution[PDF[dist4, x] / cte, {x, 0, 35}];
[distribución de probabilidad] [función de densidad de probabilidad]
```

```
In[9]:= NIntegrate[PDF[dist5, x], {x, 0, 50}]
[integra numé... [función de densidad de probabilidad]
```

```
Out[9]= 1.
```

```
In[10]:= Plot[{PDF[dist1, x], PDF[dist4, x], PDF[dist5, x]}, {x, -1, 35}, PlotStyle -> {Red, Blue, Magenta}, Frame -> True]
[repre... función de densi... [función de densidad de probabilidad] [estilo de repre... [rojo] [azul] [magenta] [marco] [verdade]
```






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Ejercicio : de la sección "A rule-of-thumb bandwidth estimator" de la página web  
["https://en.wikipedia.org/wiki/Kernel\\_density\\_estimation"](https://en.wikipedia.org/wiki/Kernel_density_estimation)

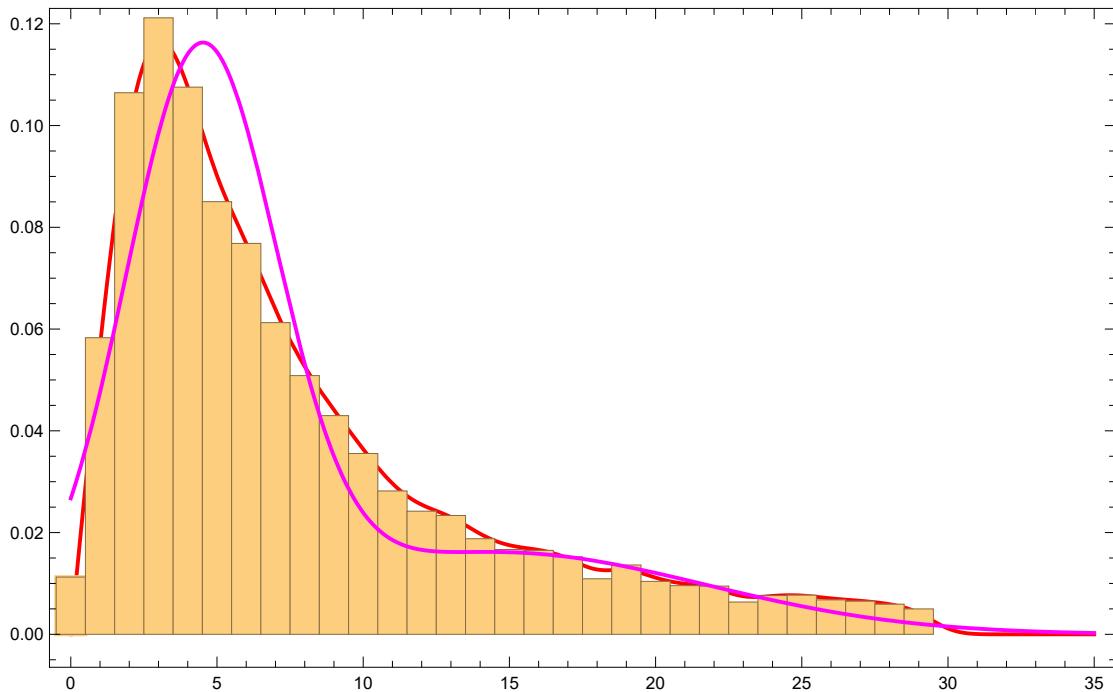
---

A partir de qué distribución provienen los datos de "V sin i"

Minimos Cuadrados

```
In[7]:= dist2N = FindDistribution[vSini, TargetFunctions -> "Continuous"]  
  | encuentra distribución | funciones objetivo |  
Out[7]= MixtureDistribution[{0.712421, 0.287579},  
  {NormalDistribution[4.45085, 2.57368], NormalDistribution[14.5821, 7.11049]}]
```

```
In[=]:= Show[Plot[PDF[dist5, x], {x, 0, 35}, PlotStyle -> Red], Histogram[v$ini, Automatic, "PDF"],
  Plot[PDF[dist2N, x], {x, 0, 35}, PlotStyle -> Magenta], Frame -> True, ImageSize -> Large]
Out[=]=
```



```
In[=]:= modelo = MixtureDistribution[{a1, a2},
  {NormalDistribution[\mu1, \sigma1], NormalDistribution[\mu2, \sigma2]}]
Out[=]= MixtureDistribution[{a1, a2}, {NormalDistribution[\mu1, \sigma1], NormalDistribution[\mu2, \sigma2]}]
```

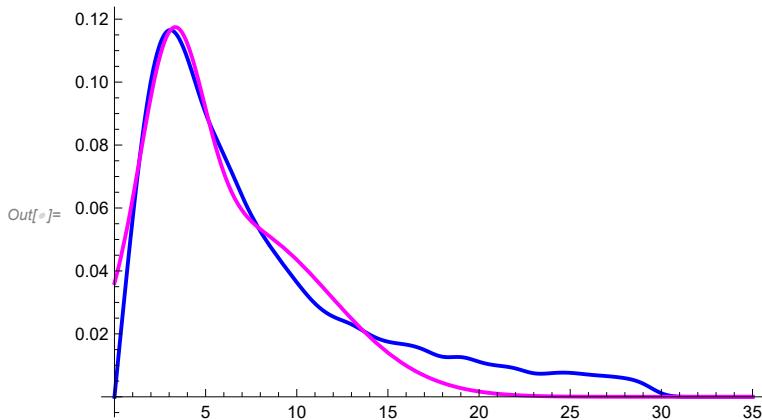
```
In[=]:= mincuad =
  ParallelSum[(PDF[dist5, v$ini[[k]]] - PDF[modelo, v$ini[[k]]])^2, {k, 1, Length@v$ini}];
  Out[=]=
```

```
In[=]:= solMC = NMinimize[mincuad, {{a1, 0.1, .9}, {a2, 0.1, .9}, {\mu1, 4., 5.},
  {\sigma1, 2., 3.}, {\mu2, 12., 17.}, {\sigma2, 5., 9.}}, Method -> "SimulatedAnnealing"]
Out[=]= {0.470675,
```

{ $a_1 \rightarrow 0.507909$ ,  $a_2 \rightarrow 1.061$ ,  $\mu_1 \rightarrow 3.11463$ ,  $\sigma_1 \rightarrow 1.70988$ ,  $\mu_2 \rightarrow 6.75482$ ,  $\sigma_2 \rightarrow 5.03249$ }

```
Plot[{PDF[dist5, x], PDF[modelo, x] /. solMC[[2]]}, {x, 0, 35}, PlotStyle -> {Blue, Magenta}]
```

[representación de la función de densidad de probabilidad] [función de densidad de probabilidad] [estilo de representación] [azul] [magenta]



Maxima verosimilitud

```
In[1]:= modelo = MixtureDistribution[{a1, a2},
  {NormalDistribution[\mu1, \sigma1], NormalDistribution[\mu2, \sigma2]}]
  [distribución mezcla] [distribución normal] [distribución normal]

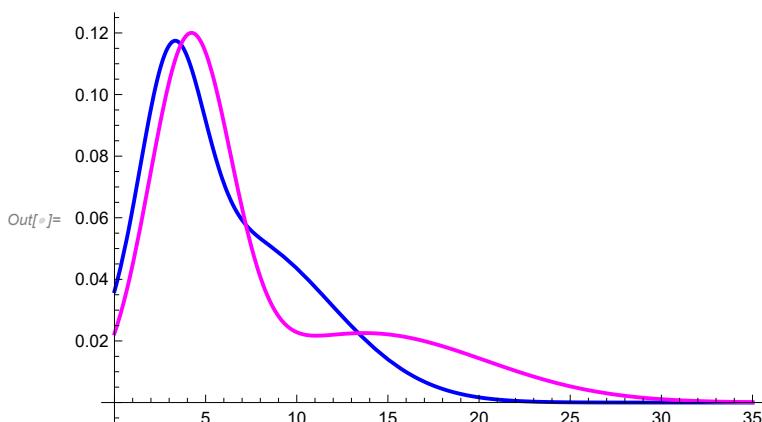
In[2]:= loglik = LogLikelihood[modelo, vsini];
  [verosimilitud logarítmica]

In[3]:= solML = NMaximize[{Re@loglik, a1 > 0, a2 > 0}, {{a1, 0.1, .9}, {a2, 0.1, .9}, {\mu1, 4., 5.},
  [\sigma1, 2., 3.], {\mu2, 12., 17.}, {\sigma2, 5., 9.}], Method -> "SimulatedAnnealing"]
  [maximiza automáticamente] [parte real] [método]

Out[3]= {-35690.2,
  {a1 -> 1.57467, a2 -> 0.937125, \mu1 -> 4.1516, \sigma1 -> 2.23115, \mu2 -> 13.7308, \sigma2 -> 6.60134}}
```

```
In[4]:= Plot[{PDF[modelo, x] /. solMC[[2]], PDF[modelo, x] /. solML[[2]]},
  [representación de la función de densidad de probabilidad] [función de densidad de probabilidad]
  {x, 0, 35}, PlotStyle -> {Blue, Magenta}]
```

[estilo de representación] [azul] [magenta]



```
In[1]:= solML = NMaximize[{Re@loglik, a1 > 0, a2 > 0}, {{a1, 0.1, .9}, {a2, 0.1, .9}, 
  {μ1, 4., 5.}, {σ1, 2., 3.}, {μ2, 12., 17.}, {σ2, 5., 9.}}, Method → "NelderMead"]
  maximiza ap... [parte real]
  método

Out[1]= {-35 690.2,
  {a1 → 0.57266, a2 → 0.340804, μ1 → 4.1516, σ1 → 2.23115, μ2 → 13.7308, σ2 → 6.60134} }

In[2]:= solML = NMaximize[{Re@loglik, a1 > 0, a2 > 0}, {{a1, 0.1, .9}, {a2, 0.1, .9}, {μ1, 4., 5.}, 
  {σ1, 2., 3.}, {μ2, 12., 17.}, {σ2, 5., 9.}}, Method → "DifferentialEvolution"]
  maximiza ap... [parte real]
  método

... NMaximize: The function value Indeterminate is not a number at {a1, a2, μ1, μ2, σ1, σ2} =
  {0., 0., 4.2913, 13.8269, 2.24668, 6.53956}.

Out[2]= {-35 691.,
  {a1 → 1.01073, a2 → 0.585392, μ1 → 4.13945, σ1 → 2.22881, μ2 → 13.8353, σ2 → 6.57494} }

In[3]:= solML = NMaximize[{Re@loglik, a1 > 0, a2 > 0, σ1 > 0, σ2 > 0}, {{a1, 0.1, .9}, {a2, 0.1, .9}, 
  {μ1, 4., 5.}, {σ1, 2., 3.}, {μ2, 12., 17.}, {σ2, 5., 9.}}, Method → "RandomSearch"]
  maximiza ap... [parte real]
  método

Out[3]= {-35 690.2,
  {a1 → 0.76704, a2 → 0.456484, μ1 → 4.1516, σ1 → 2.23115, μ2 → 13.7308, σ2 → 6.60134} }
```