Tarea 3 (Solucionario)

Cálculo 3 (FOGEC)

12/07/2022

1.- (15 Puntos)

Calcular

$$\iint\limits_{R} (y + 3x)\, dA$$

donde $R = \{(x, y) \in \mathbb{R}^2 / 0 \leq x \leq 1 \wedge x^2 \leq y \leq x + 2\}$

2.- (15 Puntos)

Use integrales dobles para calcular el volumen del sólido acotado por los gráficos de las ecuaciones $z = \sqrt{x^2 + y^2}$; $z = 0$ y $x^2 + y^2 = 25$.

3.- (15 Puntos)

Use coordenadas cilíndricas para evaluar la integral

$$\iiint\limits_{E} z\, dV$$

donde E es la región cerrada por el paraboloide $z = x^2 + y^2$ y el plano $z = 4$

4.- (15 Puntos)

Use coordenadas esféricas para determinar

$$\iiint\limits_{E} e^{\sqrt{x^2+y^2+z^2}}\, dV$$

donde $E = \{(x, y, z) \in \mathbb{R}^3 \colon x^2 + y^2 + z^2 \leq 1\}$

Observación

Fecha para entregar tarea, martes 19 de julio en horario de clases.

1.-

$$\iint\limits_R (y+3x)\,dA = \int_0^1 \int_{x^2}^{x+2} (y+3x)\,dydx$$

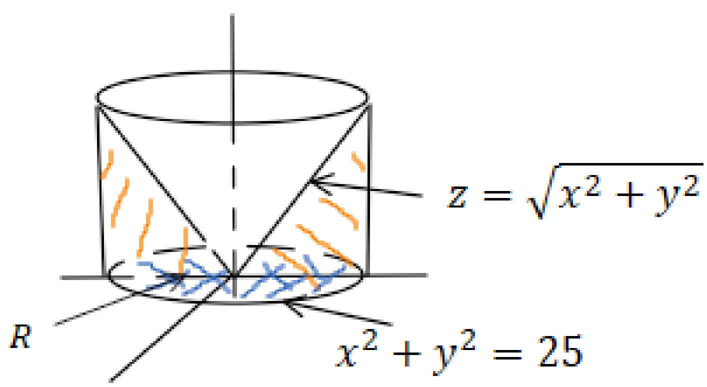
$$= \int_0^1 \left(\frac{y^2}{2} + 3xy\right)_{x^2}^{x+2} dx$$

$$= \int_0^1 \left[\frac{(x+2)^2}{2} + 3x(x+2) - \left(\frac{x^4}{2} + 3x^3\right)\right] dx$$

$$= \int_0^1 \left(\frac{7}{2}x^2 + 8x - \frac{x^4}{2} - 3x^3 + 2\right) dx$$

$$= \left(\frac{7}{6}x^3 + 4x^2 - \frac{x^5}{10} - \frac{3}{4}x^4 + 2x\right)_0^1 = \frac{379}{60}$$

2.-



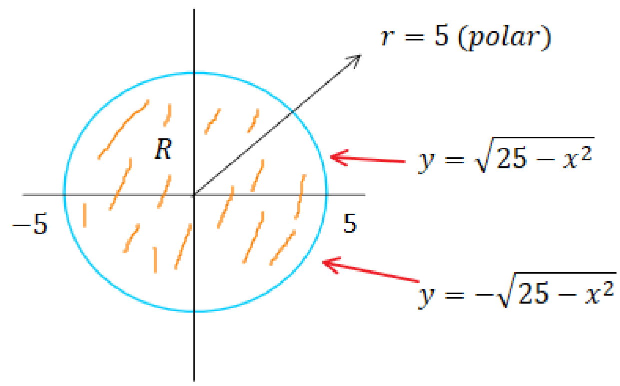
$$\begin{cases} z = \sqrt{x^2+y^2} \\ x^2 + y^2 = 25 \end{cases}$$

Entonces

$$z = \sqrt{25} = 5$$

$$V = \iint\limits_R z dV = \int_{-5}^{5} \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} \sqrt{x^2+y^2}\, dyd$$

Forma cartesiana y polar



$$V = \iint\limits_R z dV = 4 \int_0^5 \int_0^{\sqrt{25-x^2}} \sqrt{x^2 + y^2}\, dydx$$

Por simetría. pero conviene hacer cambio de coordenadas a polares

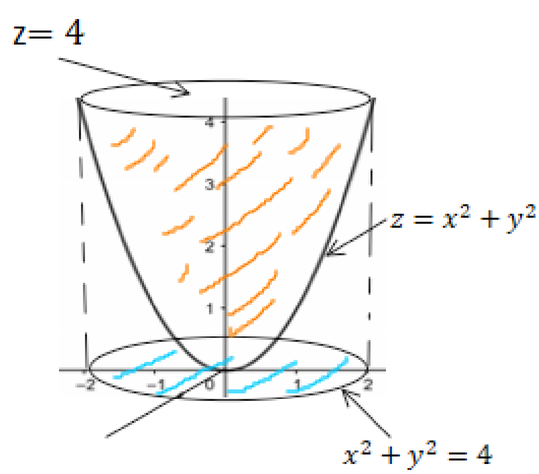
$$V = \iint\limits_R z dV = 4 \int_0^5 \int_0^{\sqrt{25-x^2}} \sqrt{x^2 + y^2}\, dydx$$

$$= 4 \int_0^{\pi/2} \int_0^5 r(rdrd\theta)$$

$$= 4 \int_0^{\pi/2} \int_0^5 r^2 drd\theta = 4 \int_0^{\pi/2} \frac{125}{3} d\theta$$

$$= \frac{500}{3} [\theta]_0^{\pi/2} = \frac{500}{3} \left(\frac{\pi}{2} - 0\right) = \frac{250\pi}{3}\ u^3$$

3.-



$$V = \underset{R}{\iiint} z\,dydx = \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} z\,dzdydx$$

$$= \int_{0}^{2\pi}\int_{0}^{2}\int_{r^2}^{4} z\,r\,dzdrd\theta$$

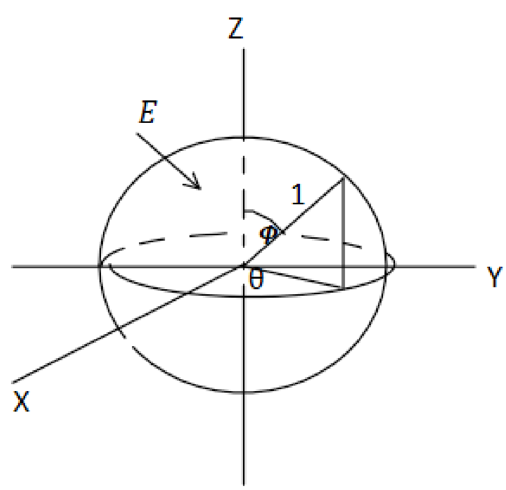
$$= \int_{0}^{2\pi}\int_{0}^{2} r\left[\frac{z^2}{2}\right]_{r^2}^{4} drd\theta = \int_{0}^{2\pi}\int_{0}^{2} r\left(8-\frac{r^4}{2}\right)drd\theta$$

$$= \int_{0}^{2\pi}\int_{0}^{2}\left(8r-\frac{r^5}{2}\right)drd\theta = \int_{0}^{2\pi}\left[4r^2-\frac{r^6}{12}\right]_{0}^{2} d\theta$$

$$= \int_{0}^{2\pi}\left(16-\frac{16}{3}\right)d\theta = \int_{0}^{2\pi}\left(\frac{48-16}{3}\right)d\theta$$

$$= \int_{0}^{2\pi}\left(\frac{32}{3}\right)d\theta = \frac{32}{3}\int_{0}^{2\pi} d\theta = \frac{32}{3}[\theta]_{0}^{2\pi} = \frac{64\pi}{3}$$

4.-



$$x^2 + y^2 + z^2 = r^2$$

$$dV = r^2 sen\,\theta\; dr\; d\phi\; d\theta$$

$$0 \le r \le 1$$

$$0 \le \phi \le \pi$$

$$0 \le \theta \le 2\pi$$

$$\iiint_E e^{\sqrt{x^2+y^2+z^2}}\, dV$$

$$= \int_0^{2\pi} \int_0^{\pi} \int_0^{1} e^r r^2 sen\, \phi\, dr\, d\phi\, d\theta$$

$$= \int_0^{2\pi} \int_0^{\pi} sen\, \phi \left(\int_0^{1} e^r r^2\, dr \right) d\phi\, d\theta$$

Ahora

$$\int_0^{1} e^r r^2\, dr \ = (r^2 e^r - 2re^r + 2e^r)_0^1$$

$$= e - 2e + 2e - 2 = e - 2$$

Luego

$$\iiint_E e^{\sqrt{x^2+y^2+z^2}}\, dV$$

$$= \int_0^{2\pi} \int_0^{\pi} sen\, \phi (e-2) d\phi\, d\theta$$

$$= \int_0^{2\pi} (e-2) \int_0^{\pi} sen\, \phi\ \ d\phi\, d\theta$$

$$= \int_0^{2\pi} (e-2)\, [-\cos\phi]_0^{\pi}\, d\theta$$

$$= (e-2) \int_0^{2\pi} \big(-(\cos\pi - \cos 0)\big)\ d\theta$$

$$= (e-2) \int_0^{2\pi} \big(-(-1-1)\big)\ d\theta$$

$$= 2(e-2) \int_0^{2\pi} d\theta$$

$$= 2(e-2)[\theta]_0^{2\pi}$$

$$= 2(e-2)\ (2\pi) = 4\pi(e-2)$$