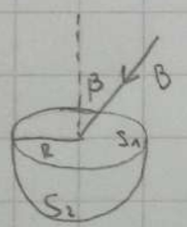
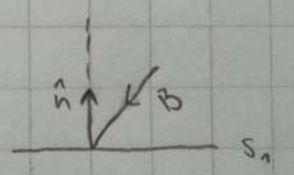
1) Calcular flujo en S_2

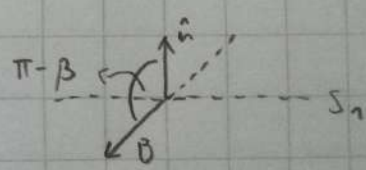


Maxwell: $\vec{\nabla}\cdot\vec{B} = 0 \iff \oint \vec{B}\cdot d\vec{a} = 0$

$$\therefore \int_{S_1} \vec{B}\cdot d\vec{a} + \int_{S_2} \vec{B}\cdot d\vec{a} = 0$$

obs:





Flujo en S_1?

$$\int_{S_1} \vec{B}\cdot d\vec{a} = \int_{S_1} B\,da\cos(\pi-\beta) =$$

$$\Phi_{m_1} = -B\pi R^2\cos(\beta)$$

$$\therefore \Phi_{m_2} = B\pi R^2\cos(\beta) \quad \square$$

2) Def: $I = \int \vec{J} \cdot d\vec{a}$; $\vec{J} = \frac{d\vec{I}}{da}$; $d\vec{a} = s\,ds\,d\phi\,(\pm\hat{k})$

(i) $I_0 = \int \vec{J}_1 \cdot d\vec{a} = \int_0^{2\pi}\int_0^{a} \alpha s \hat{k} \cdot s\,ds\,d\phi\,\hat{k} = 2\pi\alpha \frac{s^3}{3}\Big|_0^a = 2\pi\alpha\frac{a^3}{3} \Rightarrow \alpha = \frac{3 I_0}{2\pi a^3}$

Aquí, usamos $d\vec{a} \parallel \vec{J}$ por convención

(ii) $I_0 = \int \vec{J}_2 \cdot d\vec{a} = \int_0^{2\pi}\int_b^{c} -\beta\hat{k} \cdot s\,ds\,d\phi\,(-\hat{k}) = \beta 2\pi \frac{s^2}{2}\Big|_b^c = \pi\beta(c^2-b^2) \Rightarrow \beta = \frac{I_0}{\pi(c^2-b^2)}$

(iii) Ley de Ampere: $\oint_{TA} \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

a) $s<a$: $I_{enc} = \int_0^{2\pi}\int_0^{s} \alpha s' \hat{k} \cdot s'\,ds'\,d\phi = \alpha 2\pi \frac{s^3}{3} \Rightarrow B_1 = \frac{\mu_0 \alpha 2\pi}{2\pi s}\frac{s^3}{3} = \frac{\mu_0 \alpha s^2}{3}$

y por mano derecha: $\vec{B}_1 = B_1 \hat{\phi}$

b) $a<s<b$: $I_{enc} = I_0 \Rightarrow B_2 = \frac{I_0 \mu_0}{2\pi s} \wedge \vec{B}_2 = B_2\hat{\phi}$

c) $b < s < c$: $I_{enc} = I_0 - \int_0^{2\pi}\int_b^{s} \beta \hat{k} \cdot ds' s' \, d\phi \, \hat{k} = I_0 - \frac{2\pi\beta}{2}(s^2 - b^2)$

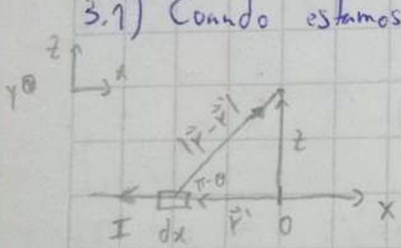
pero $\beta = \frac{I_0}{\pi(c^2-b^2)}$ $\Rightarrow$ $I_{enc} = I_0\left(1 - \frac{s^2-b^2}{c^2-b^2}\right) = I_0\left(\frac{c^2-s^2}{c^2-b^2}\right)$

$\therefore$ $B_3 = \frac{\mu_0 I_0}{2\pi s}\left(\frac{c^2-s^2}{c^2-b^2}\right) > 0$ # $[\mu_0] = T\,m/A$

d) $s > c$: $I_{enc} = 0$ $\Rightarrow$ $B_4 = 0$

3.1) Cuando estamos el plano de la corriente, usamos Biot escalar

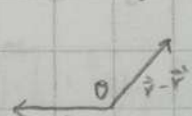
dir. corriente $\hat{i}$



$\vec{r} = z\hat{k}$; $\vec{r}' = x(-\hat{i})$; $|\vec{r}-\vec{r}'| = \sqrt{z^2+x^2}$; $d\vec{l} = dx(-\hat{i})$

$$dB = \frac{\mu_0 I}{4\pi}\,\frac{dx\,|\vec{r}-\vec{r}'|\sin(\theta)}{|\vec{r}-\vec{r}'|^3}$$

#



$\sin(\theta) = \sin(\pi-\theta)$

$$dB = \frac{\mu_0 I}{4\pi}\,\frac{dx\; z}{|\vec{r}-\vec{r}'|^3} \quad /\int$$

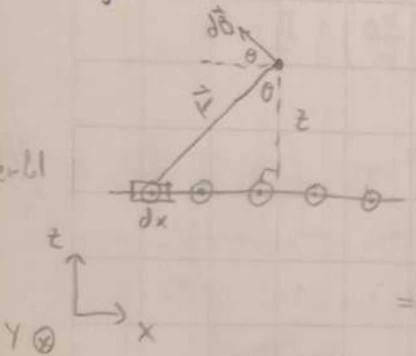
$$B = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{z\,dx}{(z^2+x^2)^{3/2}} = \frac{\mu_0 I}{2\pi}\, z\int_{0}^{\infty}\frac{dx}{(z^2+x^2)^{3/2}} = \frac{\mu_0 I}{2\pi z}$$

No necesariamente positivo I y z

Por mano derecha: $\vec{B} = B\hat{j}$

Ej: Plano de corriente (fuera del plano → descompongo el campo)

de aquí



$$d\vec{B} = -dB\cos(\theta)\,\hat{\imath} + dB\sin(\theta)\,\hat{k} \quad \wedge \quad dB = \frac{\mu_0}{2\pi}\frac{dI}{r}$$

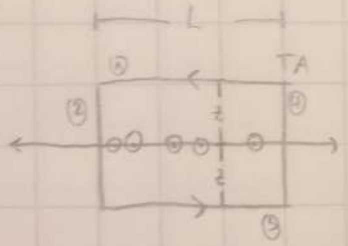
$$y \quad K = \frac{dI}{dx} \Rightarrow dI = K\,dx$$

$$\Rightarrow d\vec{B} = -\frac{\mu_0}{2\pi}\hat{\imath}\int_{-\infty}^{\infty}\frac{K\,dx}{\sqrt{z^2+x^2}}\overbrace{\frac{z}{\sqrt{z^2+x^2}}}^{\cos(\theta)} + \frac{\mu_0}{2\pi}K\hat{k}\underbrace{\int_{-\infty}^{\infty}\frac{dx}{\sqrt{z^2+x^2}}\overbrace{\frac{x}{\sqrt{z^2+x^2}}}^{\sin(\theta)}}_{\text{int impar se anula}}$$

$$\vec{B} = -\frac{\mu_0 K z}{2\pi}\hat{\imath}\underbrace{\int_{-\infty}^{\infty}\frac{dx}{(z^2+x^2)}}$$

$$\frac{\tan^{-1}\left(\frac{x}{z}\right)}{z}\Bigg|_{-\infty}^{\infty} = \frac{2}{z}\tan^{-1}\left(\frac{x}{z}\right)\Bigg|_{0}^{\infty} = \frac{\pi}{z} - 0 \quad \therefore \quad \boxed{\vec{B} = -\frac{\mu_0 K\hat{\imath}}{2}}$$

Ahora con ley de ampere



\# La TA cumple con $\vec{B} = \vec{0} \;\vee\; \vec{B} \parallel d\vec{l} \;\vee\; \vec{B} \perp d\vec{l}$

$$\oint_{TA} \vec{B} \cdot d\vec{l} = \int_1 + \underbrace{\int_2 + \int_3 + \int_4}_{0} = \int_1 \vec{B}_1 \cdot (-\hat{\imath})\, dx + \int_3 \vec{B}_3 \cdot \hat{\imath}\, dx$$

($\vec{B}_1 \parallel -\hat{\imath}$, $\vec{B}_3 \parallel \hat{\imath}$)

$$= 2 \int_1 B\, dx = 2BL$$

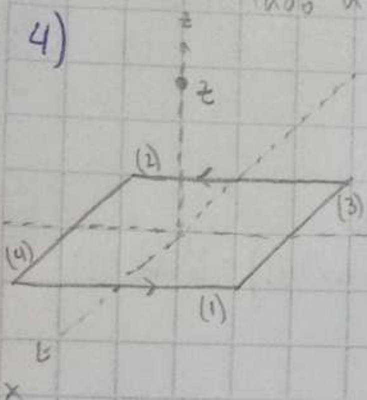
$$\oint_{TA} \vec{B} \cdot d\vec{l} = 2BL = \mu_0 I_{enc}$$

\# $I_{enc} = \int K\, dl = KL$

$$\therefore B = \frac{\mu_0 K}{2}$$

\# Ampere solo nos da el módulo

4)



lado a ; $\vec{B}(z) = \frac{\mu_0}{4\pi}\oint \frac{I\, d\vec{l} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$; $\vec{r} = z\hat{k}$

$$|\vec{r} - \vec{r}'|^3 = \left[(\vec{r} - \vec{r}')\cdot(\vec{r} - \vec{r}')\right]^{3/2}$$

para (1) : $\left[\left(z\hat{k} - \frac{a}{2}\hat{i} - y'\hat{j}\right)\cdot\left(z\hat{k} - \frac{a}{2}\hat{i} - y'\hat{j}\right)\right]^{3/2}$

$= \left(z^2 + \frac{a^2}{4} + y'^2\right)^{3/2}$; $dl = dy$ # lo mismo para (2)

para (3) : $\left[\left(z\hat{k} - \frac{a}{2}\hat{j} - x'\hat{i}\right)\cdot\left(z\hat{k} - \frac{a}{2}\hat{j} - x'\hat{i}\right)\right]^{3/2}$

$= \left(z^2 + \frac{a^2}{4} + x'^2\right)^{3/2}$; $dl = dx$ # lo mismo para (4)

luego, resolvemos $d\vec{l} \times (\vec{r} - \vec{r}')$ para cada tramo # $d\vec{l} \parallel \vec{I}$

(1) : $dy\,\hat{j} \times \left(z\hat{k} - \frac{a}{2}\hat{i} - y'\hat{j}\right) = \left(z\hat{i} + \frac{a}{2}\hat{k}\right) dy$

(2) : $-dy\,\hat{j} \times \left(z\hat{k} + \frac{a}{2}\hat{i} - y'\hat{j}\right) = \left(-z\hat{i} + \frac{a}{2}\hat{k}\right) dy$

(3) : $-dx\,\hat{i} \times \left(z\hat{k} - \frac{a}{2}\hat{j} - x'\hat{i}\right) = \left(z\hat{j} + \frac{a}{2}\hat{k}\right) dx$

(4) : $dx\,\hat{i} \times \left(z\hat{k} + \frac{a}{2}\hat{j} - x'\hat{i}\right) = \left(-z\hat{j} + \frac{a}{2}\hat{k}\right) dx$

luego, $I_1 + I_2 = \int_1 \frac{\left(z\hat{i} + \frac{a}{2}\hat{k}\right) dy}{\left(z^2 + \frac{a^2}{4} + y'^2\right)^{3/2}} + \int_2 \frac{\left(-z\hat{i} + \frac{a}{2}\hat{k}\right) dy}{\left(z^2 + \frac{a^2}{4} + y'^2\right)^{3/2}}$

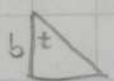
$= a\hat{k}\int_{-\frac{a}{2}}^{a/2} \frac{dy}{\left(z^2 + \frac{a^2}{4} + y'^2\right)^{3/2}}$

Pero como ya incluimos la dirección de $\vec{I}$ en $d\vec{l}$, los lim de int. son los mismos

e $I_3 + I_4 = a\hat{k}\int_{-\frac{a}{2}}^{a/2} \frac{dx}{\left(z^2 + \frac{a^2}{4} + x'^2\right)^{3/2}}$ $\therefore\ I_1 = I_2 = I_3 = I_4$

$I_1' = \int_{-a/2}^{a/2} \frac{dx}{\left(z^2+\frac{a^2}{4}+x'^2\right)^{3/2}}$ $\qquad b^2 \equiv z^2 + \frac{a^2}{4}$

$\int \frac{dx}{(b^2+x^2)^{3/2}}$



$\tan(t) = \frac{x}{b} \quad , \quad dx = b\sec^2(t)\,dt$

$x^2+b^2 = b^2\left(1+\tan^2(t)\right) = b^2\sec^2(t)$

$= \int \frac{b\, s^2\, dt}{b^3 s^3} = \frac{1}{b^2}\sin(t) = \frac{1}{b^2}\frac{x}{\sqrt{x^2+b^2}}$

$I_1' = \frac{4}{4z^2+a^2}\left\{\frac{a/2}{\sqrt{\frac{a^2}{4}+b^2}} - \frac{-a/2}{\sqrt{\frac{a^2}{4}+b^2}}\right\} = \frac{4a}{4z^2+a^2}\frac{1}{\sqrt{z^2+\frac{a^2}{2}}}$

$\therefore \vec{B}(z) = \frac{\mu_0 I}{4\pi}\, 2a\hat{k} \cdot I_1' \quad \square$