## Propiedades de las estrellas (repaso)

Clase 5

## Propiedades de las estrellas (repaso)

The stars are a major component of the Milky Way. Stars are ideal test particles which provide accurate positions, density distributions and motion information for the characterization of the Galactic potential and dynamical processes. In addition one can estimate for certain stars their age and/or their metallicity which provide further dynamical but also evolutionary information about the Milky Way system. On the other side the large scale Milky Way structure has a strong impact on the star formation which takes place in dense molecular clouds.

Stars can be characterized quite well by a few key parameters. The most basic quantities are L luminosity, R radius,  $T_{\rm eff}$  effective surface temperature, M mass, and  $\tau$  age. Another important parameter for galactic studies is the metallicity (e.g. Z). Further parameters are binarity and the corresponding binary parameters, stellar rotation, and magnetic fields. There exist several important relations between stellar parameters.

**Black-body laws:** For a sphere radiating like a black body there is according to the Stefan-Boltzmann law:

$$L = 4\pi R^2 \sigma T_{\text{eff}}^4 \,. \tag{2.1}$$

The Planck curve describes the spectral energy distribution of a black body

$$B_{T_{\text{eff}}}(\lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT_{\text{eff}}} - 1}.$$
 (2.2)

The wavelength spectrum has its maximum flux  $B_{\text{max}} = B_{T_{\text{eff}}}(\lambda_{\text{max}})$  according to Wien's law at

$$\lambda_{\text{max}} = \frac{2.9 \text{mm}}{T_{\text{eff}}[\text{K}]} \,. \tag{2.3}$$

For  $\lambda \gg \lambda_{\text{max}}$  the spectral energy distribution can be described by the Rayleigh-Jeans approximation:

$$B_{T_{\text{eff}}}(\lambda) \approx \frac{2c}{\lambda^4} k T_{\text{eff}} \,,$$
 (2.4)

## 2.2.1 Properties of main-sequence stars

Main sequence stars burn hydrogen to helium. This phase lasts about 90 % of the nuclear burning life time of a star. Therefore about 90 % of all stars are main sequence stars and their properties are therefore particularly relevant.

Mass-luminosity relations on the main sequence. The luminosity of main sequence stars is a strong function of mass which is described by a power law function with different exponent  $\alpha$  for different mass ranges:

$$\frac{L}{L_{\odot}} \approx a \left(\frac{M}{M_{\odot}}\right)^{\alpha},$$
 (2.5)

where 
$$\alpha = 2.3$$
,  $a = 0.23$  for  $M < 0.43 \, M_{\odot}$ ,  
 $\alpha = 4.0$ ,  $a = 1.0$  for  $0.43 \, M_{\odot} < M < 2 \, M_{\odot}$ ,  
 $\alpha = 3.5$ ,  $a = 1.5$  for  $2M_{\odot} < M < 20 \, M_{\odot}$ ,  
 $\alpha = 1.0$ ,  $a = 3200$  for  $M > 20 \, M_{\odot}$ .

Main sequence lifetime. The main sequence lifetime of star ends when about 10 % of all H is burnt to He. High mass stars have a much larger luminosity and therefore thy burn their fuel much faster than low mass stars. To first order one can write for example for higher mass stars

$$\tau_{\rm ms} \propto \frac{M}{L} \propto \frac{1}{M^{2.5}} \quad \text{for } 20 \, M > M_{\odot} > 2M_{\odot}$$
(2.6)

or for low mass stars

$$\tau_{\rm ms} \propto \frac{1}{M^{1.3}} \quad \text{for } M < 0.43 M_{\odot} \,.$$
(2.7)

Stellar parameters for main-sequence stars. The following table lists main sequence parameters for different spectral types of stars.

Table 2.1: Parameters for main sequence stars: mass, luminosity, radius, effective surface temperature and main sequence life time.

sp.type	O5 V	B0 V	$A0\mathrm{V}$	$G0\mathrm{V}$	M0V	M8 V
$M/{ m M}_{\odot}$	60	18	3.0	1.1	0.50	0.06
$L/{ m L}_{\odot}$	$8 \cdot 10^6$	$7 \cdot 10^4$	54	1.5	0.080	$1.2\cdot 10^{-3}$
$R/{ m R}_{\odot}$	12	7.5	2.5	1.1	0.50	0.10
$T_{\mathrm{eff}}$ [K]		30'000	9500	6000	3800	2600
$ au_{ m ms} \ [{ m yr}]$	$8 \cdot 10^5$	$4 \cdot 10^6$	$6 \cdot 10^{8}$	$7 \cdot 10^9$	$6 \cdot 10^{10}$	$5 \cdot 10^{11}$

The parameters given in Table 2.1 are only approximative. The given value allow to construct a  $\log T_{\rm eff} - \log L/L_{\odot}$  plot or a "theoretical Hertzsprung-Russel diagram". Detailed studies show that there are many subtle dependencies of the basic stellar parameters on e.g. age, metallicity, or rotation rate, but this is beyond the scope of this lecture.

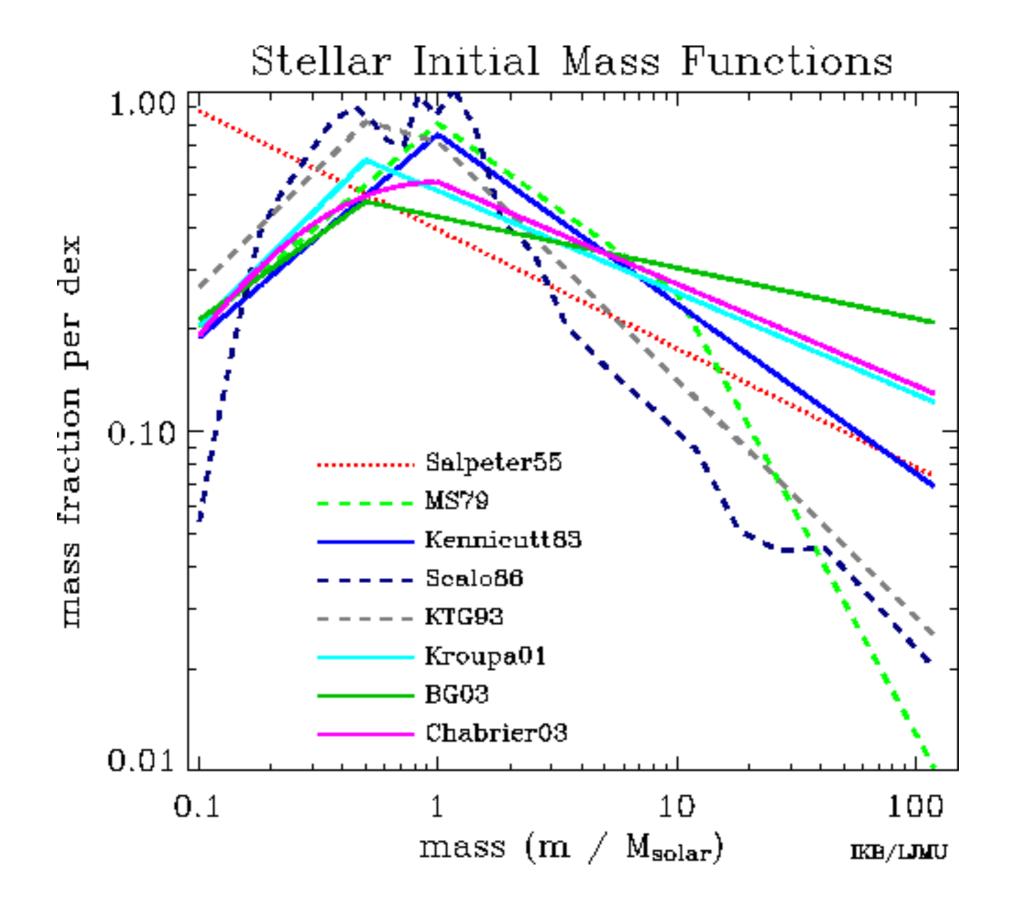
Initial mass function (IMF). The initial mass function describes the mass distribution  $N_S(M)$  of newly formed stars per mass bin  $\Delta M$ . This distribution is quite universal and it will be an important topic in the Chapter 5 on star formation. However, it is useful for the understanding of galactic stellar populations to introduce the IMF in this introductory chapter. The standard IMF (Salpeter 1955) can be described by a power law distribution

$$\frac{dN_S}{dM} \propto M^{-2.35} \quad \text{for} \quad M > 0.5 \,\text{M}_{\odot} \,.$$
 (2.8)

This relation is often given as a logarithmic power law of the form

$$\frac{dN_S}{d\log M} \propto M^{-1.35} \quad \text{because} \quad \frac{dN_S}{dM} = \frac{dN_S}{d\log M} \frac{d\log M}{dM} = \frac{1}{M} \frac{dN_S}{d\log M} \,.$$

This is equivalent to a linear fit with slope -1.35 in  $\log M$ - $\log N_S$  diagram (Figure 5.3). This law indicates, that the number of newly formed stars with a mass between 1 and 2  ${\rm M}_{\odot}$  is about 20 times larger than the stars with masses between 10 and 20  ${\rm M}_{\odot}$ . One may also say that twice as much gas from a star-forming cloud ends up in stars between 1 and 2  ${\rm M}_{\odot}$  when compared to stars with masses between 10 and 20  ${\rm M}_{\odot}$ . For low mass star the IMF power law has a steep cut-off for  $M < 0.5 \, M_{\odot}$  where the general law does not apply.



Discussion on main sequence stars. Luminosity, effective surface temperature, and the life time of main-sequence stars are very important for the interpretation of stellar populations. The following points can be made:

- high mass stars are born much less frequently than low mass stars,
- high mass stars, although rare, dominate the luminosity of a new-born population of stars (a young association or star cluster),
- high mass stars are blue stars and therefore a young population has a blue color,
- after some time (e.g. > 1 Gyr) the yellow-red low mass stars dominate the mainsequence population because all short-lived high-mass stars are gone,
- the total luminosity of a stellar population decreases steadily with age.

Measurements of magnitudes and colors. Photometric measurements are carried out typically in wavelength bands which are specific for each instrument used. As general photometric reference the Vega magnitude system is used.

All photometric measurements are related to the star Vega ( $\alpha$  Lyr) by the definition that Vega has an apparent magnitude of

$$m_{\lambda}(\text{Vega}) = 0.0^m \tag{2.9}$$

in all photometric bands in the wavelength region from about 150 nm to 15  $\mu$ m (UV - visual - IR range).

Photometric magnitude is a logarithmic quantity which relates the relative flux ratio of two measurements  $\ell_1$  and  $\ell_2$  by the relation

$$m_1 - m_2 = -2.5 \log \frac{\ell_1}{\ell_2} \,. \tag{2.10}$$

This means that a star with  $m_2 = 2.5^m$  is 10 times fainter than a star with  $m_1 = 0^m$ .

Apparent colors or color indices CI between to wavelength filters  $\lambda_1$  and  $\lambda_2$  are also quantified as magnitude difference

$$CI = m_{\lambda_1} - m_{\lambda_2} \,, \tag{2.11}$$

e.g. the color B–V is the difference between the standard Johnson blue filter and visual filter  $m_B - m_V$ . B–V is positive for a star which is more "red" than Vega and negative for a star which is more "blue". Colors for other filter pairs are defined according to the same principle.

Distances and interstellar extinction. The apparent magnitude m measured for stars must be converted in the next step into a absolute stellar magnitudes M and intrinsic stellar colors. For this one needs to take into account the distance of the star and the possible interstellar extinction.

The relation between the apparent flux  $f_{\lambda}$  and absolute flux  $F_{\lambda}$  of a star depends on the distance d and the interstellar extinction  $\tau_{\lambda}$ 

$$f_{\lambda}(d) = \frac{F_{\lambda}}{4\pi d^2} e^{-\tau_{\lambda}}. \tag{2.12}$$

This relation can be expressed in magnitudes. For this the absolute magnitude  $M_{\lambda}$  is introduced, which is the apparent magnitude of an object at a distance of 10 pc without interstellar extinction:

$$M_{\lambda} = m_{\lambda}(f_{\lambda}(10\,\mathrm{pc})). \tag{2.13}$$

For example, our sun has an absolute magnitude of  $M_V = +4.5^m$  in the visual band. Vega is at a distance of about 10 pc and therefore also the absolute magnitude of Vega is approximately

$$M(Vega) \approx 0^m$$
.

The general formula for the conversion of the apparent magnitude m of a star into absolute magnitudes M is given by the following formula:

$$m_{\lambda} = M_{\lambda} + 5\log d \left[ pc \right] - 5 + A_{\lambda}. \tag{2.14}$$

In this equation there are two terms:

- the distance modulus:  $5 \log d [pc] - 5$  which follows from

$$m_{\lambda} - M_{\lambda} = -2.5 \log \frac{f_{\lambda}(d)}{f_{\lambda}(10 \text{ pc})} = -2.5 \log \frac{(10 \text{ pc})^2}{(d \text{ [pc]})^2} = -(5 - 5 \log d \text{ [pc]}), \quad (2.15)$$

– and the interstellar extinction:  $A_{\lambda} \geq 0^m$ .

The interstellar extinction is due to small  $< 1 \mu \text{m}$  interstellar dust particles. Their absorption is stronger in the blue than in the visual  $A_B > A_V$  and therefore the light is reddened. On average the following relation approximates quite well the extinction effect:

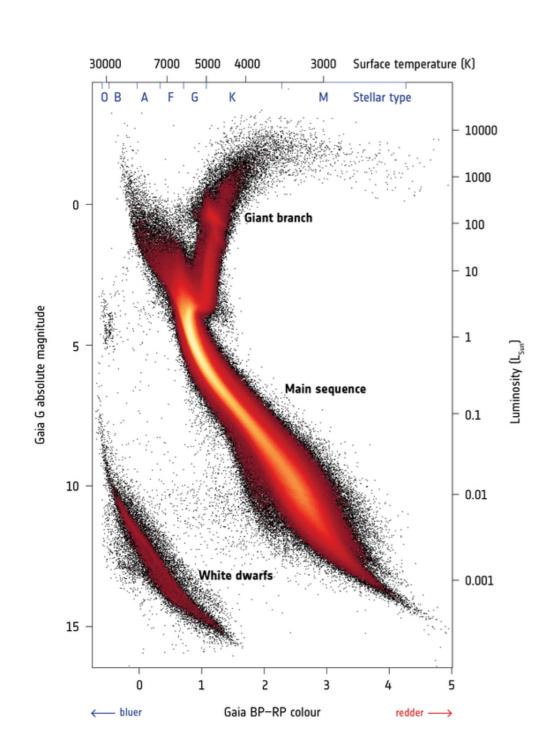
$$E_{\rm B-V} = A_B - A_V \approx 3.1 \, A_V \,.$$
 (2.16)

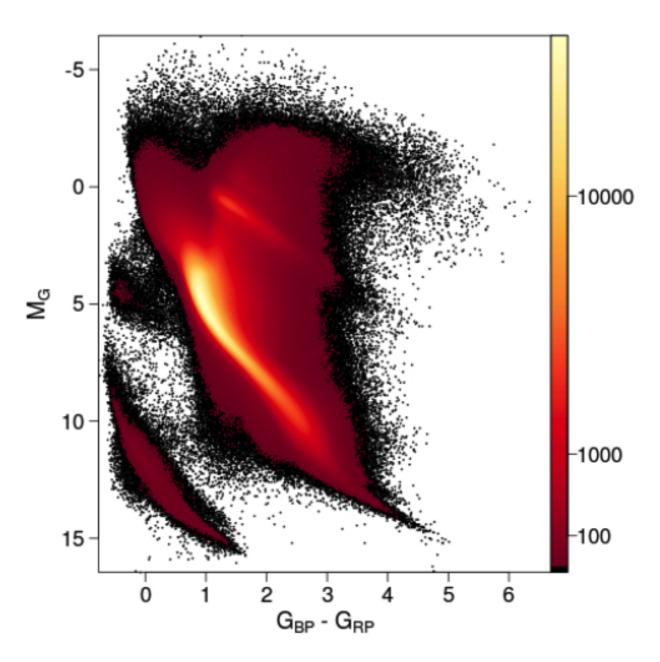
The color effect  $E_{\rm B-V}=A_B-A_V$  is according to this relation roughly proportional to the absolute extinction  $A_V$  and therefore one can use the reddening of a star as a measure for the extinction. The reddening follows from the measurements of the apparent color  $m_B-m_V$  for a star for which the intrinsic color  $M_B-M_V$  is known, for example from its spectral type. This method can also be applied to photometric measurements in other filters. Typically, the extinction is about  $A_V \approx 1.8^m/{\rm kpc}$  in the galactic disk and  $A_V < 0.2^m$  for extragalactic observations in the direction of the galactic poles.

HR-diagram for the stars in the solar neighborhood. HR-diagrams for nearby stars have two advantages:

- the distances d are well known from parallax measurements (to a precision of 10 %), and
- the interstellar extinction is small  $A_V < 0.2^m$  and can be neglected.

## → GAIA'S HERTZSPRUNG-RUSSELL DIAGRAM





**Fig. 1.** Full *Gaia* colour-magnitude diagram of sources with the filters described in Sect. 2.1 applied (65 921 112 stars). The colour scale represents the square root of the relative density of stars.

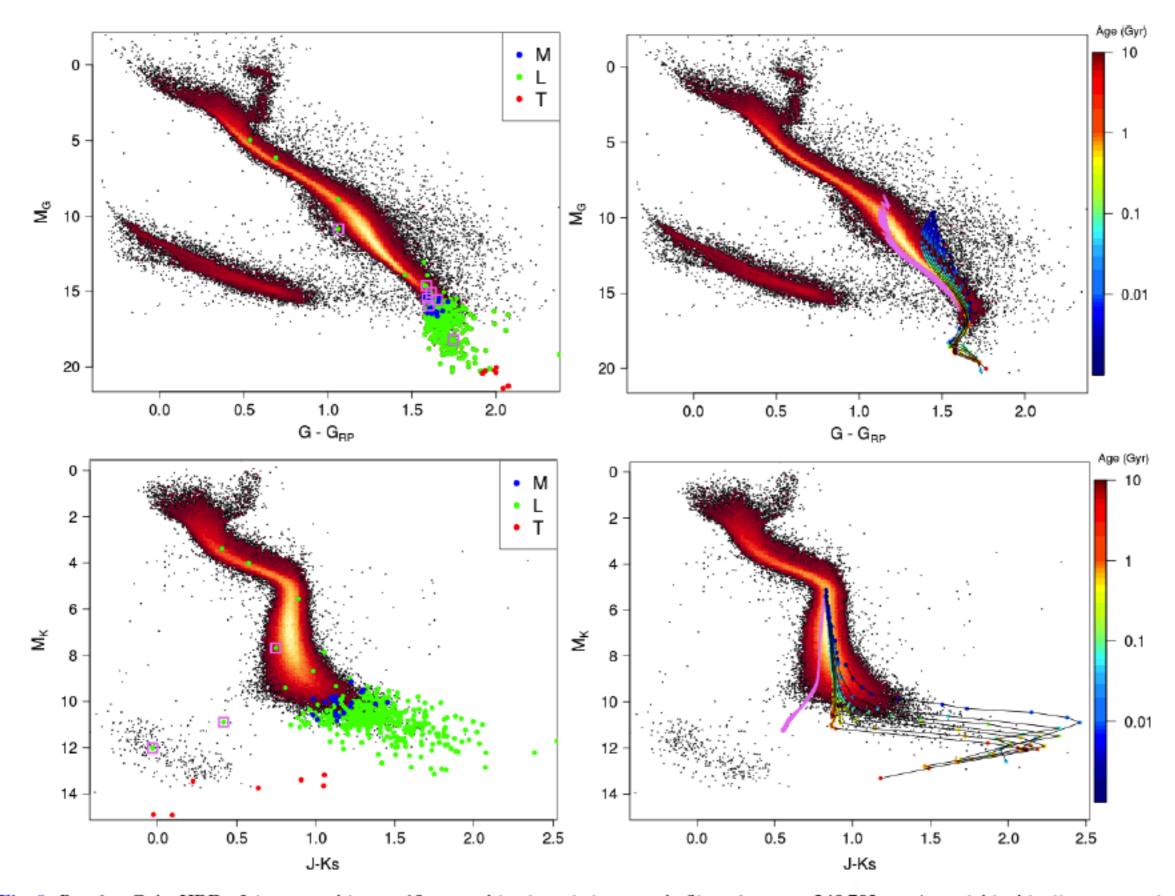


Fig. 9. Panel a: Gaia HRD of the stars with  $\varpi > 10$  mas with adapted photometric filters (see text, 240 703 stars) overlaid with all cross-matched GUCDS (Smart et al. 2017) stars with  $\sigma_{\varpi}/\varpi < 10\%$  in blue (M type), green (L type), and red (T type). Pink squares are added around stars with tangential velocity  $V_T > 200 \,\mathrm{km \, s^{-1}}$ . Panel b: BT-Settl tracks (Baraffe et al. 2015) of solar metallicity for masses from 0.01  $M_{\odot}$  to 0.08  $M_{\odot}$  in steps of 0.01 (the upper tracks correspond to lower masses) plus in pink the same tracks for [M/H] = -1.0. Panels c and d: same diagrams using the 2MASS colours.