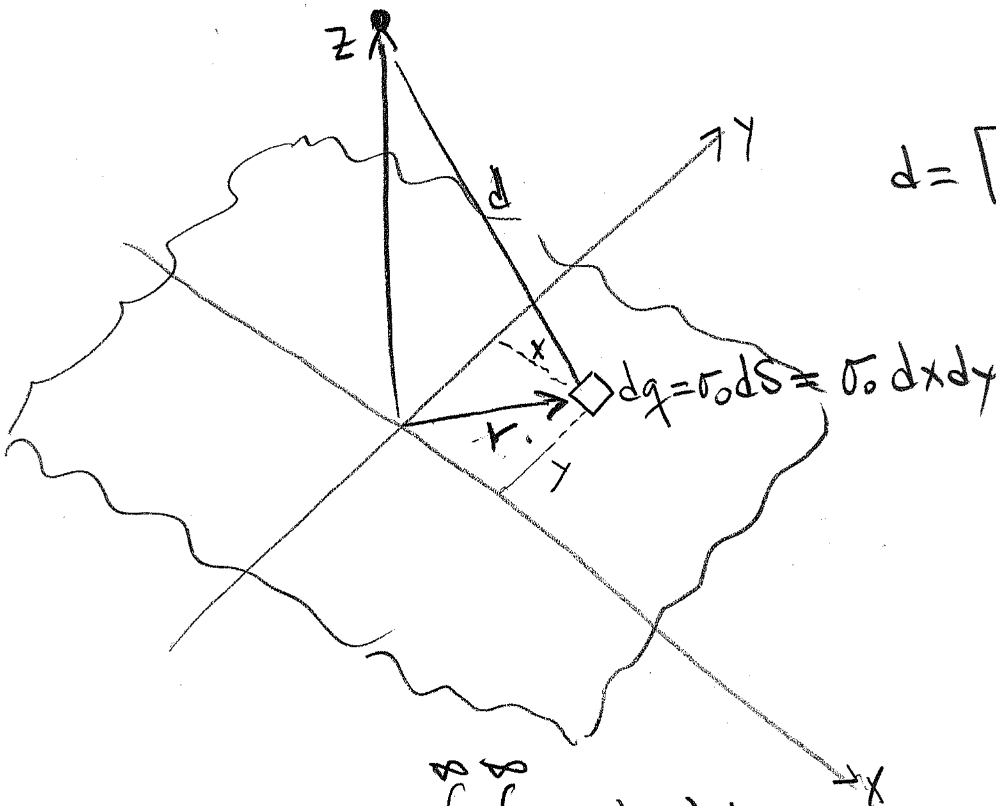
Ej Plano infinito con $\sigma_0 = cte$.



$$d = [x^2 + y^2 + z^2]^{1/2}$$

$$\phi(z) = \frac{\sigma_0}{4\pi\varepsilon_0} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{dx\,dy}{[x^2+y^2+z^2]^{1/2}}$$

$$\Downarrow$$

$$= \frac{\sigma_0}{4\pi\varepsilon_0} \int_{r=0}^{\infty}\int_{\theta=0}^{2\pi} \frac{r\,dr\,d\theta}{[r^2+z^2]^{1/2}}$$

$$= \frac{\sigma_0}{4\pi\varepsilon_0} \cdot 2\pi \int_0^{\infty} \frac{r\,dr}{[r^2+z^2]^{1/2}}$$

$$= \frac{\sigma_0}{2\varepsilon_0} \left[\sqrt{r^2+z^2}\right]_{r=0}^{\infty} \rightarrow \infty$$

Solución del dilema $\Longrightarrow$ Resolver integral indefinida y restar el resultado evaluado en z_0. (referencia).

$$\int \frac{r\,dr}{\sqrt{r^2+z^2}} = \sqrt{r^2+z^2}$$

$$\phi(z) - \phi(z_0) = \frac{\sigma_0}{2\varepsilon_0}\left[\sqrt{r^2+z^2} - \sqrt{r^2+z_0^2}\right]_{r=0}^{\infty}$$

$$= \frac{\sigma_0}{2\varepsilon_0}\left(0 - (z - z_0)\right)$$

$$\phi(z) - \phi(z_0) = -\frac{\sigma_0 z}{2\varepsilon_0} - \frac{\sigma_0 z_0}{2\varepsilon_0} = -\frac{\sigma_0}{2\varepsilon_0}(z - z_0)$$

$$\Downarrow$$

$$-\frac{d\phi(z)}{dz} = \frac{\sigma_0}{2\varepsilon_0} = E \quad \text{(Resultado conocido)}$$