Transformada de Laplace

1. $\mathcal{L}\{t^2\cos(at)\}$ R: $\frac{2s(s^2-3a^2)}{(s^2+a^2)^3}$

2. $\mathcal{L}\{\text{sen}^3(at)\}$ R: $\frac{6a^3}{(s^2+9a^2)(s^2+a^2)}$

3. $\mathcal{L}\{3e^{2t}+2\text{sen}^2(2t)\}$ R: $\frac{3s^3+64s-32}{s(s-2)(s^2+16)}$

4. $\mathcal{L}\{\text{senh}^2(3t)\}$ R: $\frac{18}{s^3-36s}$

5. $\mathcal{L}\{t^2e^{-t}\cos(2t)\}$ R: $\frac{2s^3+6s^2-18s-22}{(s^2+2s+5)^3}$

6. $\mathcal{L}\{te^{2t}\text{sen}^2(t)\}$ R: $\frac{6s^2-24s+32}{(s-2)^2(s^2-4s+8)^2}$

7. $\mathcal{L}\left\{\frac{\text{sen}(t)}{t}\right\}$ R: $\arctan\left(\frac{1}{s}\right)$

8. $\mathcal{L}^{-1}\left\{\frac{4s^2+3s+6}{s^3+2s^2}\right\}$ R: $3t+4e^{-2t}$

9. $\mathcal{L}^{-1}\left\{\frac{s^2+42s+9}{(s^2-9)^2}\right\}$ R: $e^{-3t}(4te^{6t}-3t)$

10. $\mathcal{L}^{-1}\left\{\frac{2s+3}{s^2-6s+13}\right\}$ R: $\frac{1}{2}e^{3t}(4\cos(2t)+9\text{sen}(2t))$

11. $\mathcal{L}^{-1}\left\{\frac{6s-3}{s^2+4s+13}\right\}$ R: $e^{-2t}(6\cos(3t)-5\text{sen}(3t))$

12. $\mathcal{L}^{-1}\left\{\frac{12}{s^3+8}\right\}$ R: $e^{-2t}-e^{t}\left(\sqrt{3}\text{sen}\left(\sqrt{3}\,t\right)-\cos\left(\sqrt{3}\,t\right)\right)$

13. $\mathcal{L}^{-1}\left\{\ln\left(\frac{s-2}{s+3}\right)\right\}$ R: $\frac{e^{-3t}-e^{2t}}{t}$

14. $\mathcal{L}^{-1}\left\{\ln\left(\frac{s^2+4}{s(s+1)}\right)\right\}$ R: $\frac{2\cos(2t)-e^{-t}-1}{t}$

15. $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s}\right\}$ R: $u(t-2)$

16. $\mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s-5}\right\}$ R: $e^{5t-15}u(t-3)$

17. $\mathcal{L}^{-1}\left\{\frac{8+e^{-s}}{s^2-16}\right\}$ R: $2\text{senh}(4t)+\frac{1}{4}\text{senh}(4t-4)u(t-1)$

18. $\mathcal{L}^{-1}\left\{\frac{se^{-5s}}{s^2+4}\right\}$ R: $\cos(2t-10)u(t-5)$

19. $\mathcal{L}^{-1}\left\{\frac{2(s+3)e^{-2s}}{(s^2+1)^2}\right\}$ R: $\big((6-3t)\cos(t-2)+(t+1)\text{sen}(t-2)\big)u(t-2)$

20. $y''+2y'-8y=0$; $y(0)=5$; $y'(0)=-2$ R: $y=3e^{2x}+2e^{-4x}$

21. $y'''-y''-6y'=0$; $y(0)=1$; $y'(0)=-1$; $y''(0)=2$ R: $y=\frac{1}{2}+\frac{1}{2}e^{2x}$

22. $y''+9y=x^3$; $y(0)=1$; $y'(0)=2$ R:$y=\cos(3x)+\frac{56}{81}\text{sen}(3x)+\frac{x^3}{9}-\frac{2x}{27}$

23. $y''-3y'+2y=3e^{-x}-10\cos(3x)$; $y(0)=0$; $y'(0)=0$
R: $y=\frac{7}{13}\cos(3x)+\frac{9}{13}\text{sen}(3x)+\frac{1}{2}e^{-x}$

24. $y'''-y''-6y'=0$; $y(0)=1$; $y'(0)=0$; $y''(0)=0$
R: $y=\frac{1}{16}(13+6x^2-4x+3\cos(2x)+2\text{sen}(2x))$

25. $y'''+y''=x+e^{-x}$; $y(0)=1$; $y'(0)=0$; $y''(0)=1$
R: $y=\frac{1}{6}[x^3-3x^2+18x-18+6e^{-x}(x+4)]$

26. $ty''-y'=t^2$; $y(0)=0$ R: $y=2\frac{t^3}{3!}+c\frac{t^2}{2!}$

27. $ty''+2ty'+2y=0$; $y(0)=0$; $y'(0)=3$ R: $y=3te^{-2t}$

28. $t^2y''+2ty'+t^2y=0$; $y(0)=0$ R: $y=-c\frac{\text{sen}(t)}{t}$

Determine $\mathcal{L}\{f(t)\}$ si:

29. $f(t)=\begin{cases} t^2 & \text{si } t<3 \\ 2t & \text{si } t\geq 3 \end{cases}$ R: $\frac{2-2e^{-3s}}{s^3}-\frac{(3s+4)e^{-3s}}{s^2}$

30. $f(t)=\begin{cases} e^{3t} & \text{si } t<2 \\ 5 & \text{si } t\geq 2 \end{cases}$ R: $\frac{1}{s-3}-\frac{(5s-se^6-15)e^{-2s}}{s^2-3s}$

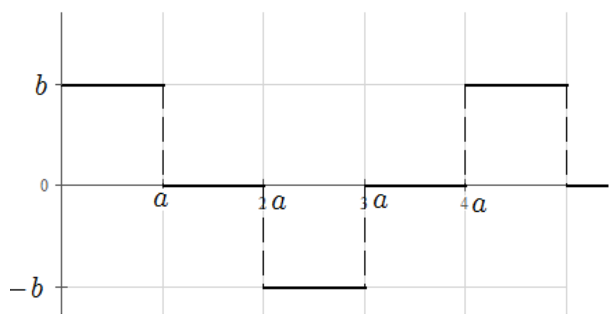
31. $f(t) = \begin{cases} 2t+1 & \text{si } t < 1 \\ 3t-4 & \text{si } 1 \leq t < 2 \\ 5t+2 & \text{si } t \geq 2 \end{cases}$ R: $\frac{1+(10s+2)e^{-2s}}{s} + \frac{2+(1-4s)e^{-s}}{s^2}$

32. $f(t) = \begin{cases} t^2 & \text{si } t < 2 \\ e^{2t} & \text{si } 2 \leq t < 4 \\ \cos(t) & \text{si } t \geq 4 \end{cases}$ R: $\frac{-2e^{-2s}(2s^2+2s+1+e^{2s})}{s^3} + \frac{e^{4-2s}-e^{8-4s}}{s-2} - \frac{e^{-4s}(\text{sen}(4)-s\cos(4))}{s^2+1}$

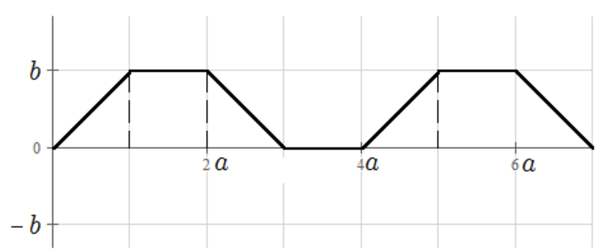
33. Halle la transformada de Laplace de las siguientes funciones periódicas:

a) 

R: $\frac{b(1-e^{-as})}{s(1-e^{-2as})}$

b) 

R: $\frac{b(1-e^{-as})}{s(1+e^{-2as})}$

c) 

R: $\frac{1-e^{-as}}{s^2(1+e^{-2as})}$

34. $y' - 6y = u(t-2)$; $y(0) = 0$ R: $\frac{1}{6}u(t-2)(e^{6t-12}-1)$

35. $y'' + 2y' + 2y = u(t-3)$; $y(0) = y'(0) = 0$

R: $\frac{1}{2}e^{3-t}u(t\text{-}3)(e^{t-3}+\text{sen}(3\text{-}t)\text{-}\cos(3\text{-}t))$

36. $y'' - 4y' + 4y = \delta(t-1)$; $y(0) = 0; y'(0) = 1$

R: $(t-1)u(t-1)e^{2t-2} + te^{2t}$

37. $y'' + 4y = f(t) + \delta(t-2\pi)$; $y(0)=0; y'(0)=1; f(t) = \begin{cases} 0 & \text{si } t<\pi \\ 1 & \text{si } \pi \le t < 2\pi \\ 0 & \text{si } t \ge 2\pi \end{cases}$

R: $u(t-\pi)[1-\cos(t-\pi)] + u(t-2\pi)[\operatorname{sen}(t-2\pi)+\cos(t-2\pi)-1] + \operatorname{sen}(t)$

38. $\displaystyle\int_0^\infty e^{-5t}\left[\int_0^t te^{3t}\operatorname{sen}(2t)dt\right]dt$ R: $\dfrac{1}{40}$

39. $y = e^{-t} - 2\displaystyle\int_0^t \cos(t-u)y(u)du$ R: $e^{-t}(t-1)^2$

40. $y = t + \frac{1}{6}\displaystyle\int_0^t (t-u)^3 y(u)du$ R: $\dfrac{1}{4}e^t - \dfrac{1}{4}e^{-t} + \dfrac{1}{2}\operatorname{sen}(t)$

41. $y = \operatorname{sen}(t) - \displaystyle\int_0^t e^{t-u}y(u)du$ R: $\cos(t)+\operatorname{sen}(t)-1$

42. $y' + y + \operatorname{sen}(t) = \displaystyle\int_0^t \operatorname{sen}(t-u)y(u)du$; $y(0)=1$

R: $e^{-\frac{1}{2}t}\left(\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{\sqrt{3}}{3}\operatorname{sen}\left(\frac{\sqrt{3}}{2}t\right)\right)$

43. $y' - t = 2\displaystyle\int_0^t e^{t-u}y(u)du$; $y(0)=2$ R: $2e^{-t} + \dfrac{3}{4}e^{-2t} + \dfrac{1}{2}t - \dfrac{3}{4}$

44. $y'' + y' = \begin{cases} t+1 & \text{si } t<1 \\ 3-t & \text{si } t>1 \end{cases}$; $y(0) = -1; y'(0)=0$

R: $u(t-1)(4t+2e^{1-t}-t^2-5)-1$

45. $y'' + 4y = \begin{cases} \cos(2t) & \text{si } t<2\pi \\ 0 & \text{si } t>2\pi \end{cases}$; $y(0)=y'(0)=0$

R: $\frac{1}{4}\operatorname{sen}(2t)\big[t + (2\pi - t)u(t-2\pi)\big]$

46. $\begin{array}{ll} y' & = x \\ x' & = -y \end{array}$ $x(0)=1; y(0)=0$ R: $\begin{array}{ll} y(t) & = \operatorname{sen}(t) \\ x(t) & = \cos(t) \end{array}$

47. $\begin{array}{ll} y' & = x+1 \\ x' & = -y \end{array}$ $x(0)=1; y(0)=1$ R: $\begin{array}{ll} y(t) & = 2\cos(t)-\operatorname{sen}(t) \\ x(t) & = 2\operatorname{sen}(t)+\cos(t) \end{array}$

48. $\left.\begin{array}{ll} y' & = x + t \\ x' & = -y \end{array}\right|$

R: $\left.\begin{array}{ll} y(t) & = 1 - \cos(t) + c_2\cos(t) + c_1\text{sen}(t) \\ x(t) & = -t + \text{sen}(t) - c_2\text{sen}(t) + c_1\cos(t) \end{array}\right|$

49. $\left.\begin{array}{ll} y' & = 5x + 2y - t \\ x' & = 4x - 2y + e^t \end{array}\right|$ $x(0) = 1; y(0) = 0$

R: $\left.\begin{array}{ll} y(t) & = \frac{1}{54} + \frac{2}{9}t + \frac{5}{13}e^t - \frac{283}{702}e^{3t}\cos(3t) + \frac{437}{234}e^{3t}\text{sen}(3t) \\ x(t) & = \frac{1}{27} + \frac{1}{9}t - \frac{1}{13}e^t + \frac{8}{13}e^{3t}\text{sen}(3t) + \frac{365}{351}e^{3t}\cos(3t) \end{array}\right|$

50. $\left.\begin{array}{ll} x'' + x + y & = 0 \\ x' + y' & = 0 \end{array}\right|$ $x(0) = 0; x'(0) = 0; y(0) = 1$

R: $\left.\begin{array}{ll} y(t) & = t^2 - t + 1 \\ x(t) & = -t^2 + t + 1 \end{array}\right|$

51. $\left.\begin{array}{ll} x'' & = y + \text{sen}(t) \\ y'' & = -x' + \cos(t) \end{array}\right|$ $x(0) = 1; x'(0) = 0; y(0) = -1; y'(0) = -1$

R: $\left.\begin{array}{ll} y(t) & = -\cos(t) - \text{sen}(t) \\ x(t) & = \cos(t) \end{array}\right|$

52. $\left.\begin{array}{ll} x'' & = 2y + 2 \\ y'' & = -x + 5e^{2t} + 1 \end{array}\right|$ $x(0) = 2; x'(0) = 2; y(0) = 1$

R: $\left.\begin{array}{ll} y(t) & = -1 + 2e^{2t} \\ x(t) & = 1 + e^{2t} \end{array}\right|$