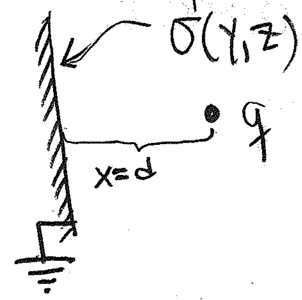
Carga total inducida en el plano

α_1

$$\sigma(y,z) = -\frac{q\,d}{2\pi\,(d^2+y^2+z^2)^{3/2}}$$



$$q_{\substack{\text{TOTAL}\\ \text{plano}}} = \int_{\text{PLANO}} \sigma(y,z)\,dS = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{q\,d}{2\pi\,(d^2+y^2+z^2)^{3/2}}\,dy\,dz$$

$$= -\frac{q\,d}{2\pi}\,4\int_0^{\infty}\int_0^{\infty} \frac{1}{(d^2+y^2+z^2)^{3/2}}\,dy\,dz$$

$$= -\frac{2\,q\,d}{\pi}\int_0^{\infty}\int_0^{\infty} \frac{1}{(d^2+y^2+z^2)^{3/2}}\,dy\,dz$$

Usando MoB:

$$\frac{1}{(d^2+y^2+z^2)^{3/2}} = \sum_n\sum_m\sum_\ell \phi_n\,\phi_m\,\phi_\ell\; d^{2n}\,y^{2m}\,z^{2\ell}\,\frac{\langle \frac{3}{2}+n+m+\ell\rangle}{\Gamma(3/2)}$$

$$\text{donde}\quad \phi_j = \frac{(-1)^j}{\Gamma(j+1)}$$

reemplazando en la integral

$$q_{\substack{\text{TOTAL}\\ \text{PLANO}}} = -\frac{2\,q\,d}{\pi}\sum_{n,m,\ell} \phi_{n,m,\ell}\; d^{2n}\,\frac{\langle \frac{3}{2}+n+m+\ell\rangle}{\Gamma(3/2)}\;\underbrace{\int_0^{\infty} y^{2m}\,dy}_{\langle 2m+1\rangle}\;\underbrace{\int_0^{\infty} z^{2\ell}\,dz}_{\langle 2\ell+1\rangle}$$

Luego la serie de brackets equivalente de la integral es:

$$q_{\substack{TOTAL\\PLANO}} = -\frac{2qd}{\pi\Gamma(\frac{3}{2})}\sum_{n,m,l}\phi_{n,m,l}\, d^{2n}\langle 3/2+n+m+l\rangle\langle 2m+1\rangle\langle 2l+1\rangle$$

$$= \frac{-2qd}{\pi\Gamma(3/2)}\frac{d^{2n}\,\Gamma(-n)\Gamma(-m)\Gamma(-l)}{4}\Bigg|_{\substack{m=-1/2\\ l=-1/2\\ n=-1/2}}$$

$$= -\frac{2qd}{\pi\Gamma(3/2)}\, d^{-1}\frac{\Gamma(1/2)^{3}}{4} \quad ; \text{ con } \Gamma(1/2)=\sqrt{\pi},\ \Gamma(3/2)=\frac{\sqrt{\pi}}{2}$$

$$q_{\substack{TOTAL\\PLANO}} = -\frac{\cancel{2}q\cancel{d}}{\cancel{\pi\frac{\sqrt{\pi}}{2}}}\cdot\frac{\cancel{\pi\sqrt{\pi}}}{\cancel{4d}} = -q$$

¡La carga inducida en el plano es $(-q)$ porque todas las líneas de fuerza desde q llegan al plano!