Tópicos

Close binary stars

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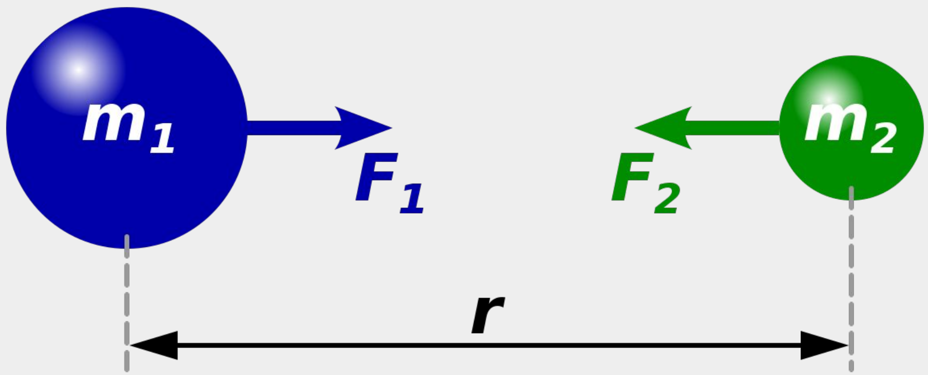
Lecture 5

25/08

Newton and Kepler

Newton's law of Universal gravitation

Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them:



Inverse-square law

$$F_1 = F_2 = G \frac{m_1 \times m_2}{r^2}$$

were G is the **universal gravitational constant**

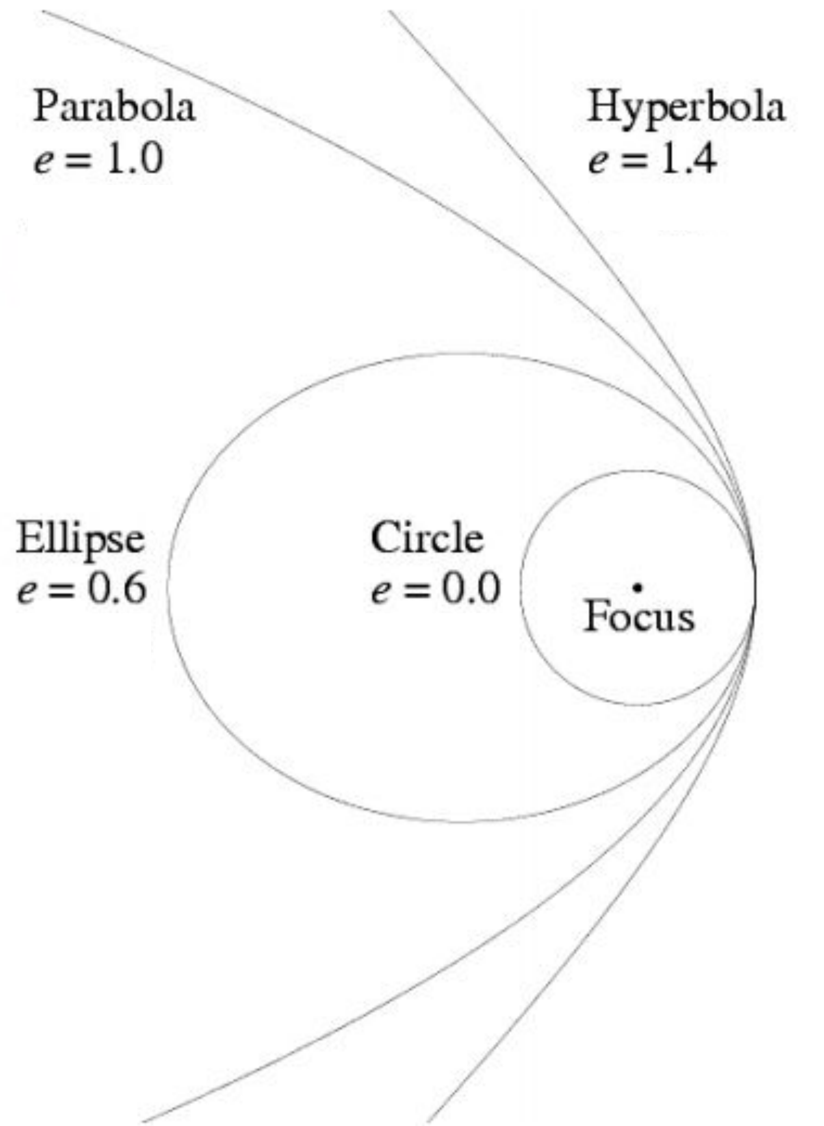
$$G = 6.673 \times 10^{-11} \text{ N m}^2/\text{kg}^2$$

Orbits in a Gravitational Potential

For a point mass (or spherically symmetric one), they are always the conic sections:

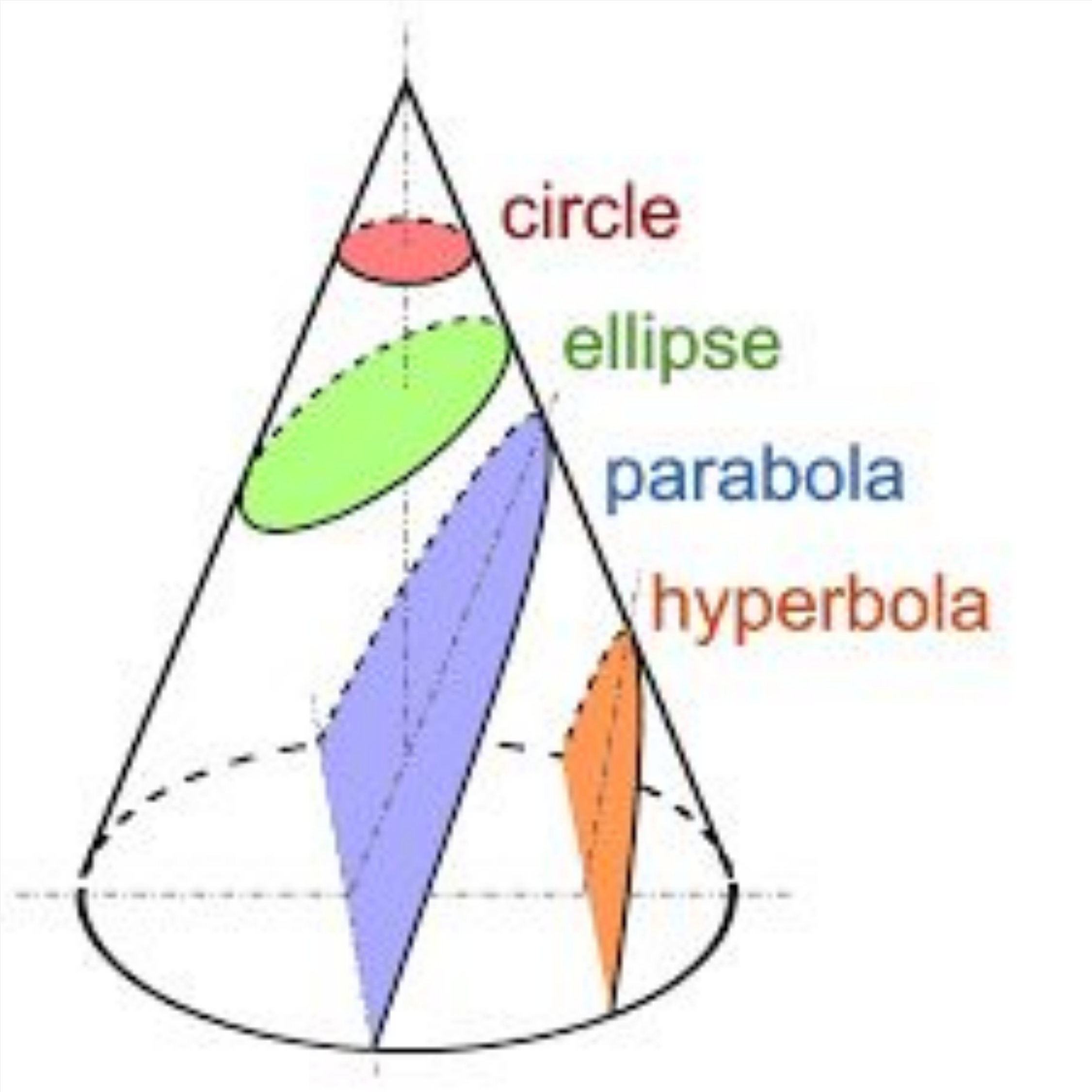
The shape depends on the sign of the total energy, $E_{tot} = E_{kin} - E_{pot}$:

$E_{tot} < 0 \rightarrow$ Ellipse
$E_{tot} = 0 \rightarrow$ Parabola
$E_{tot} > 0 \rightarrow$ Hyperbola



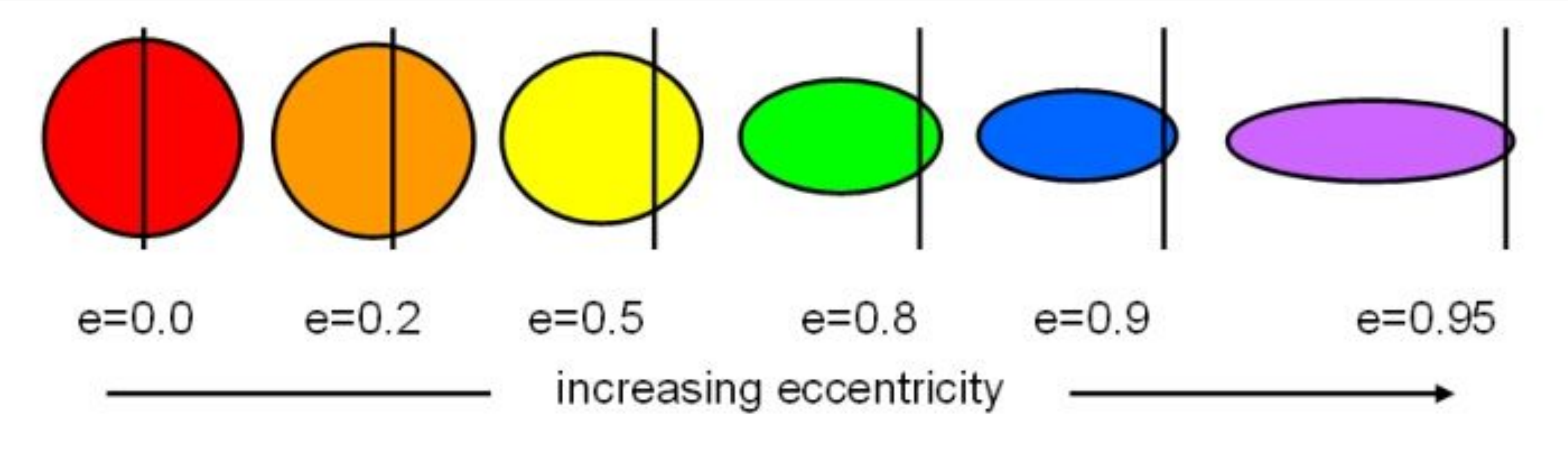
For the elliptical orbits, the eccentricity depends on the angular momentum: circular orbits have the maximum ang. mom. for a given energy

Conic Sections



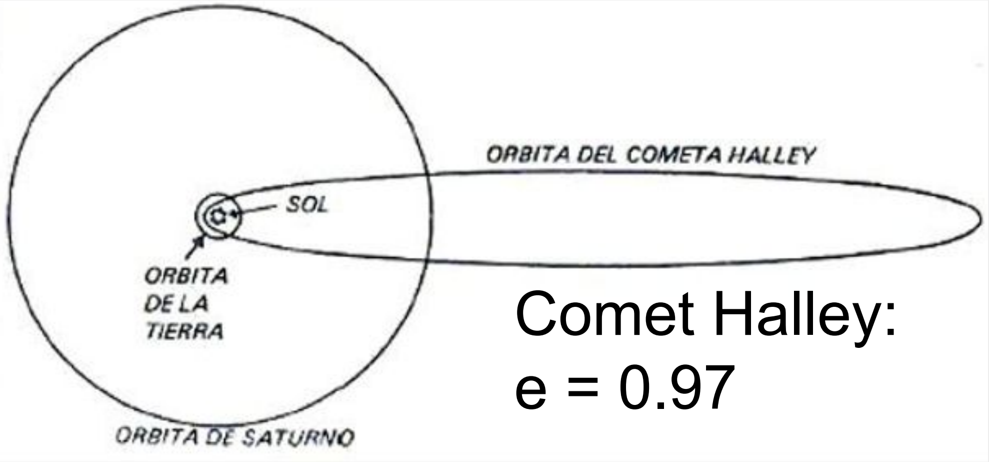
Bound orbits are ellipses (or circle if $e = 0$)

Eccentricity examples



Earth's orbit:
e = 0.0167

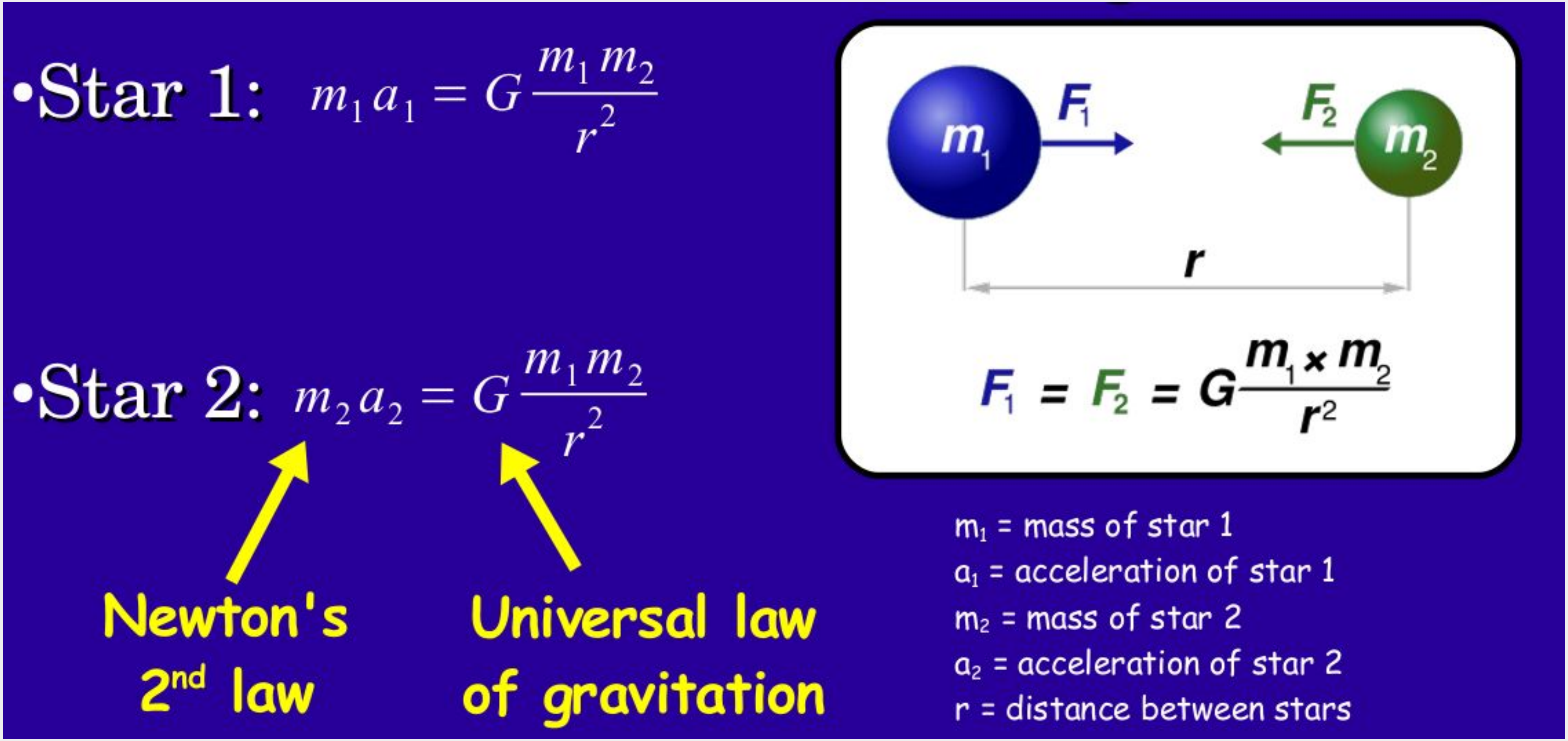
Pluto's orbit:
e = 0.248



Comet Halley:
e = 0.97

Binary stars

The motion of two stars about a common center of mass is governed by Newton's 2nd law of movement and Newton's universal law of gravitation



*caution, this a_1 and a_2 are for acceleration, not distance

Energy within a binary

Kinetic energy star 1: $$K_1 = \frac{1}{2} m_1 v_1^2$$

Kinetic energy star 2: $$K_2 = \frac{1}{2} m_2 v_2^2$$

Gravitational Potential energy: $$U = -\frac{G m_1 m_2}{r} \qquad (U_\infty = 0)$$

with r the position of one body with respect to the other one in an elliptical orbit.

Energy within a binary

Total energy:

$$E = \frac{1}{2}M_1\left|\dot{\mathbf{r}}_1\right|^2 + \frac{1}{2}M_2\left|\dot{\mathbf{r}}_2\right|^2 - \frac{GM_1M_2}{r}$$

bound orbits have negative total energy

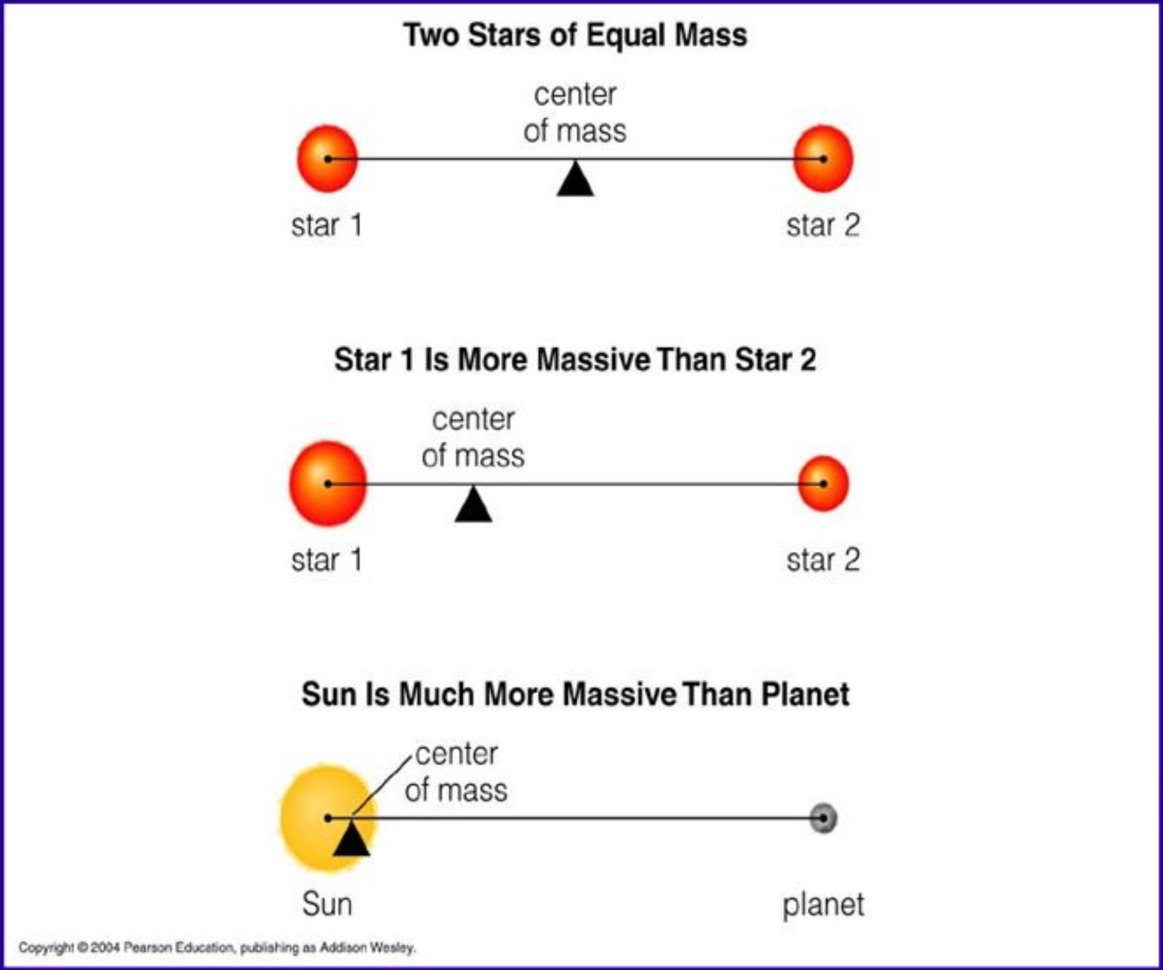
$$E = K_1 + K_2 + U < 0$$

Energy is also conserved!!

$$\dot{E} = 0$$

Center of mass

The centre of mass of two stars is always in between the two stars on the line joining them, closer to the more massive star.



r_1 = distance of star 1 to CM

m_1 = mass of star 1

r_2 = distance of star 2 to CM

m_2 = mass of star 2

$r = r_1 + r_2$

$$\frac{r_1}{r_2} = \frac{m_2}{m_1}$$

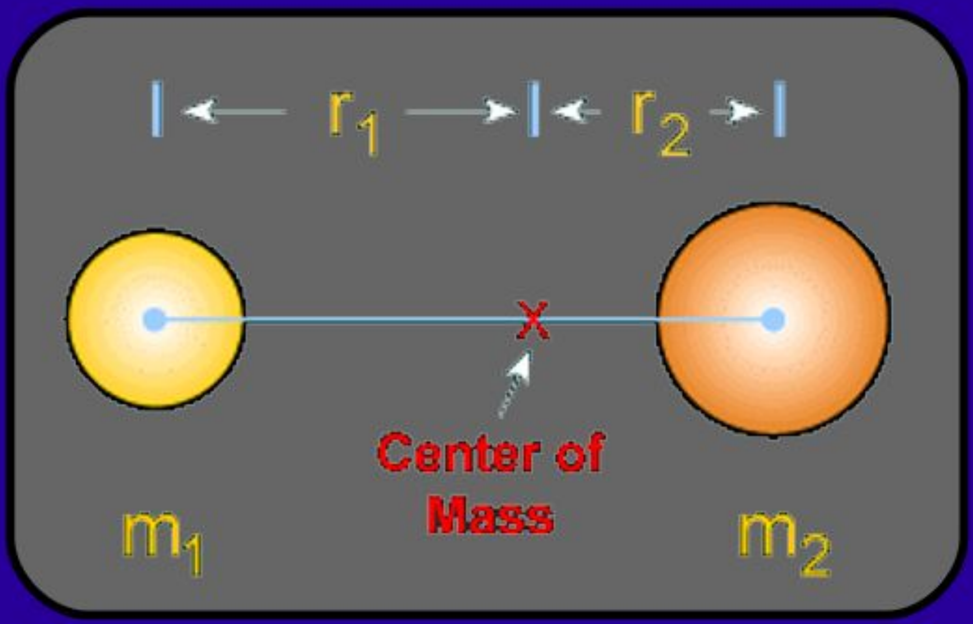
$$r_1 = \frac{m_2}{m_1 + m_2} r$$

$$r_2 = \frac{m_1}{m_1 + m_2} r$$

- **Both stars have the same orbital period P**

Binaries with circular orbits

The orbital speed (module of velocity) is constant for each star in circular orbits



- <u>Star 1</u>
- Orbital radius: r_1
- Orbital velocity:

$v_1 = 2\pi r_1/P$

- <u>Star 2</u>
- Orbital radius: r_2
- Orbital velocity:

$v_2 = 2\pi r_2/P$

$$\frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{m_2}{m_1}$$

Angular Momentum

Definition: **J** = **r** x **p**   (with **p** = m**v** = linear momentum)

$$\mathbf{J} = M_1\mathbf{r}_1 \times \dot{\mathbf{r}}_1 + M_2\mathbf{r}_2 \times \dot{\mathbf{r}}_2$$

it is conserved!!

$$\dot{\mathbf{J}} = \mathbf{0}$$

As it is a conserved vector, it implies that **r** and **v** are always restricted to the same plane.

***J** also denoted **L** in some books

Johannes Kepler

- 1571-1630
- German astronomer
- Best known for developing the laws of planetary motion.

Based on the observations of Tycho Brahe

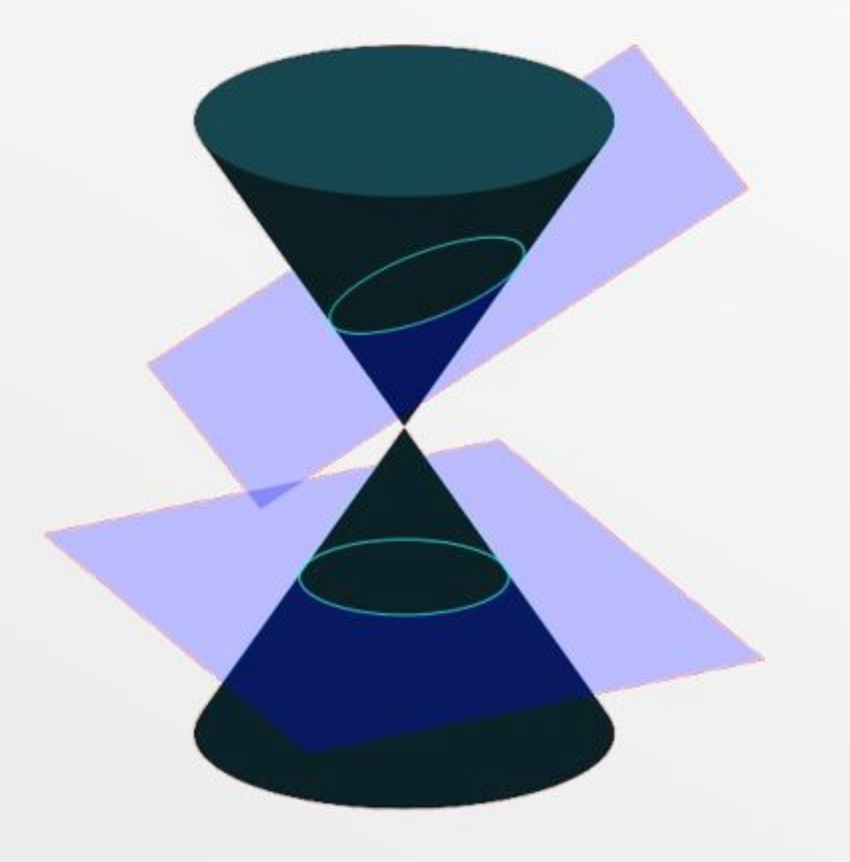
Kepler's laws of planetary motion follow from Newton's laws generalization to binary stars.

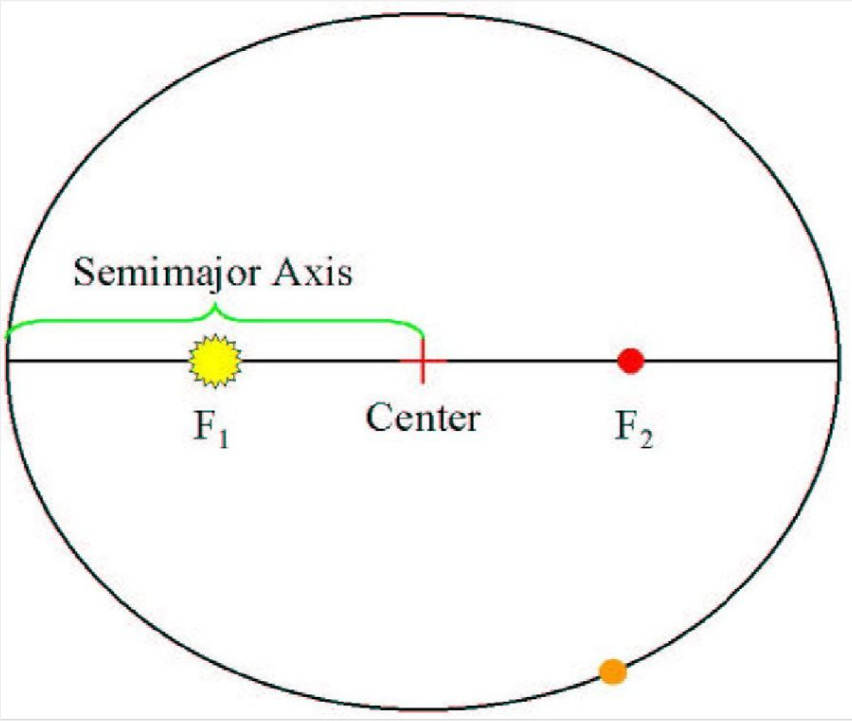
But Kepler lived before Newton!!!

Kepler's First Law

Elliptical orbits. Derived for planets around the sun.

Newton's generalization: Any object linked to another by the inverse-square law, with a total energy negative (bound), will move in elliptical orbit, with the center of mass in one of the focuses (the second focus is empty).



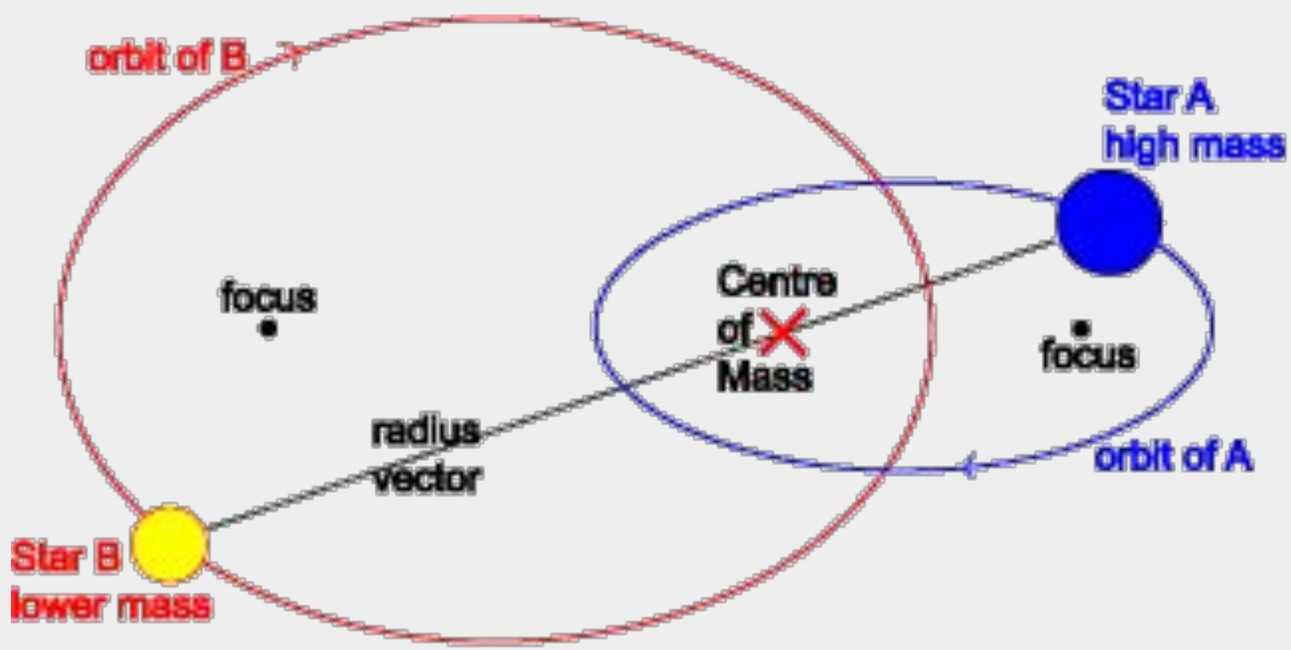


Orbit of binaries

Each star traces out the exact same-shaped conic section as the other (same eccentricity), but not same size! (except for $m_1 = m_2$)
That shape is determined by the total energy and angular momentum of the system.
The ratio of the dimensions of the two ellipses is the inverse of the ratio of their masses.

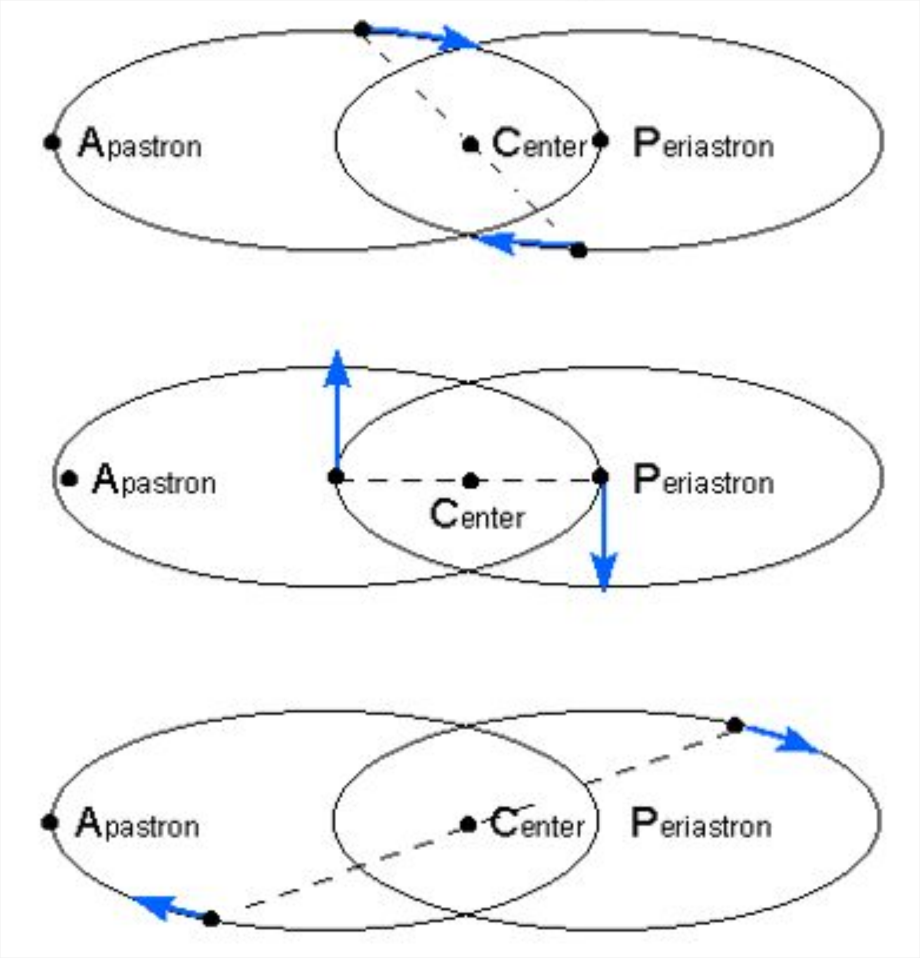
$$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \triangledown_2 / \triangledown_1$$

→ the less massive star moves faster and in a wider orbit

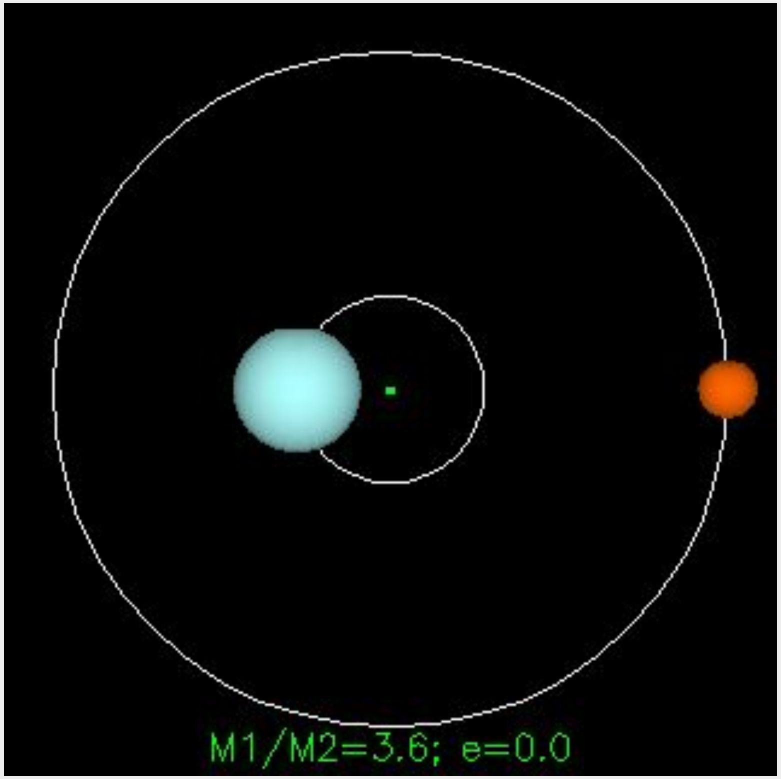


e.g. Elliptical orbits with different mass

Orbit of binaries
examples



Case for $m_1 = m_2$
($a_1 = a_2$, $v_1 = v_2$)

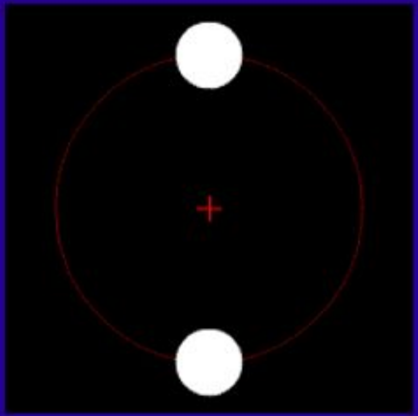


circular orbits don't cross (both centered at the CM)

What if M1 = M2?

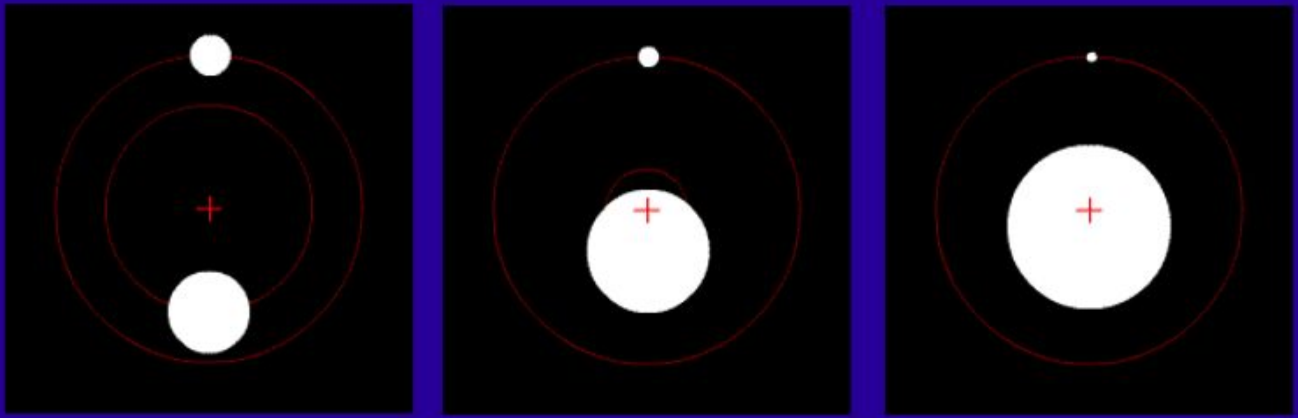
Binaries with circular orbits

•Two stars with equal masses:



Both stars are at the same distance from the centre of mass

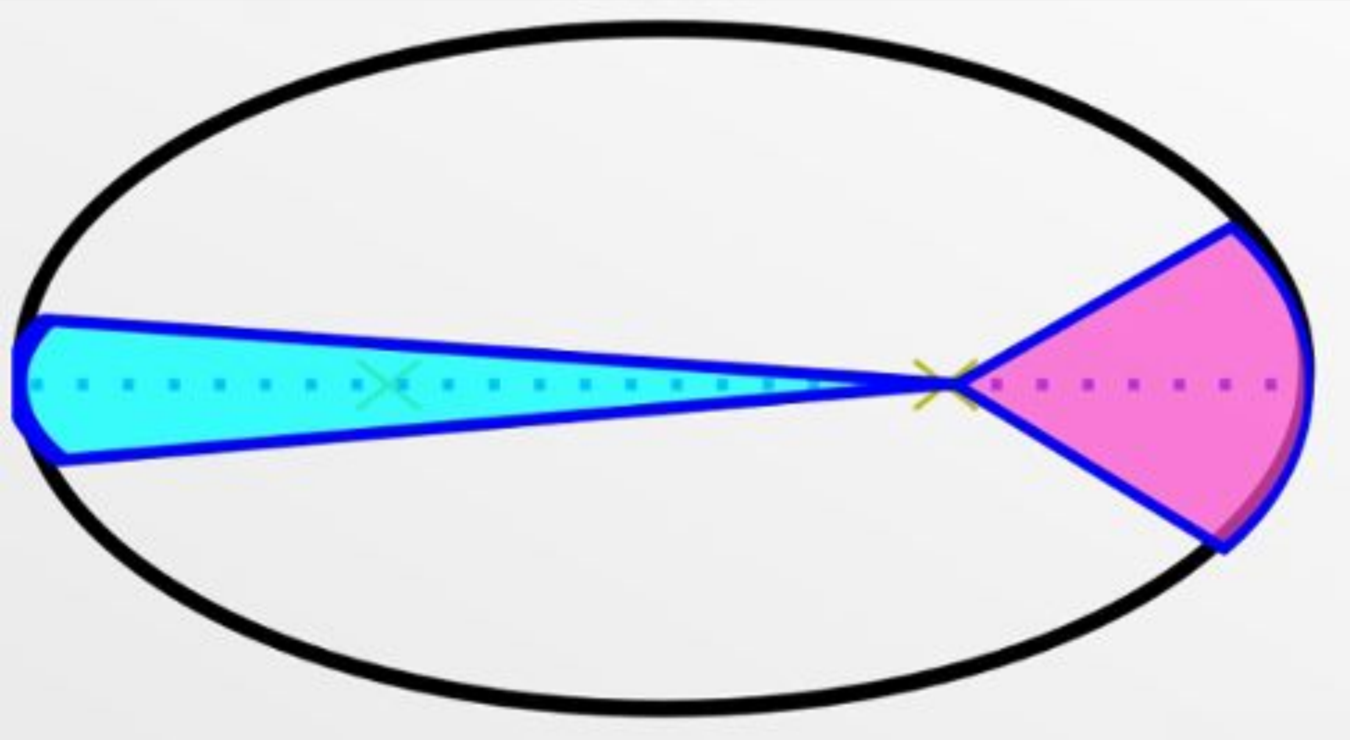
Two stars with increasingly different masses:



The heavier star is closer to the centre of mass

Kepler's Second Law

The position vector of a star in a binary with respect to the binary centre of mass sweeps out equal areas during equal intervals of time.



Star moves faster at periastron than at apastron

It is a consequence conservation of angular momentum **J**

It is a consequence conservation of angular momentum

$$\mathbf{J} = \mathbf{r} \times \mathbf{p} = m\,\mathbf{r} \times \mathbf{v}$$

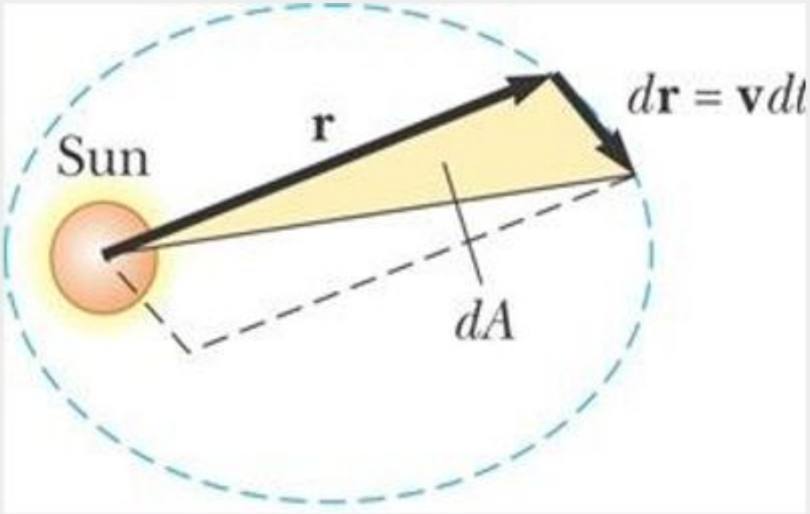
where $\mathbf{p} = m\mathbf{v}$ is the linear momentum.

$\mathbf{J}$ is a **constant vector** perpendicular to the radius vector $\mathbf{r}$ and the velocity $\mathbf{v}$ (rule of the right hand)

→ $\mathbf{r}$ and $\mathbf{v}$, at any moment, are restricted to the plane perpendicular to $\mathbf{J}$

Geometrically, we see that the radius **r** sweeps over an area dA in a time dt.
This area is equal to half the area of the parallelogram formed by vectors **r** and d**r** (small time, approximated by a right triangle)

As the planet or star displacement in a time dt is d**r** = **v**dt, we obtain:



dA = 1/2 |**r** x d**r**| = 1/2 |**r** x **v**|dt

as **J** = **r** x **p** = m **r** x **v**

dA = 1/2 |**J**/m| dt → dA/dt = |**J**|/2m

as |**J**| and m are constant, dA/dt = constant.

That is, in equal times, equal areas are swept.

Kepler's Third Law

The square of the orbital period of a binary star is proportional to the cube of the semimajor axis.

$$P^2 \propto a^3$$

This is independent of eccentricity.

Mean angular velocity $\omega = \frac{2\pi}{P}$

Mean tangential (orbital) velocities

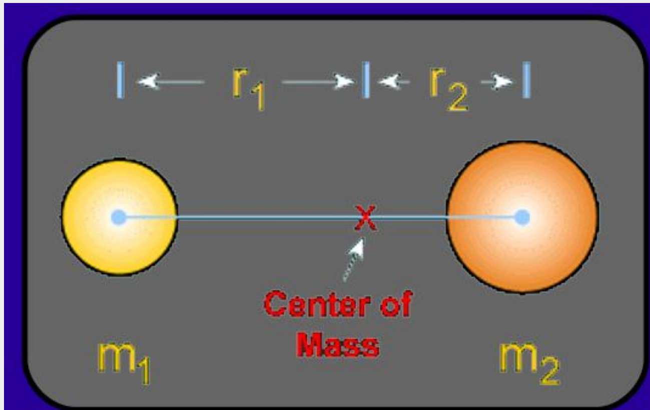
$$v_1 = 2\pi r_1/P \qquad v_2 = 2\pi r_2/P$$

Kepler's Third Law

Can be derived from Newton's 2nd law of movement (**F** = m**a**) and his universal law of gravitation.

Gravitational force provides the <u>centripetal acceleration</u> required to keep the stars in a bound (circular/elliptical) orbit.

For circular orbits:



- **E.g. star 1:**

$$F_1 = \frac{G m_1 m_2}{r^2} \qquad a_1 = \frac{v_1^2}{r_1}$$

Newton's 2nd law:

$$F_1 = m_1 a_1$$

$$G(m_1+m_2)P^2 = 4\pi^2 r^3$$

⇨ **Kepler's 3rd law**

$$r = r_1 + r_2$$

$$v_1 = 2\pi r_1/P$$

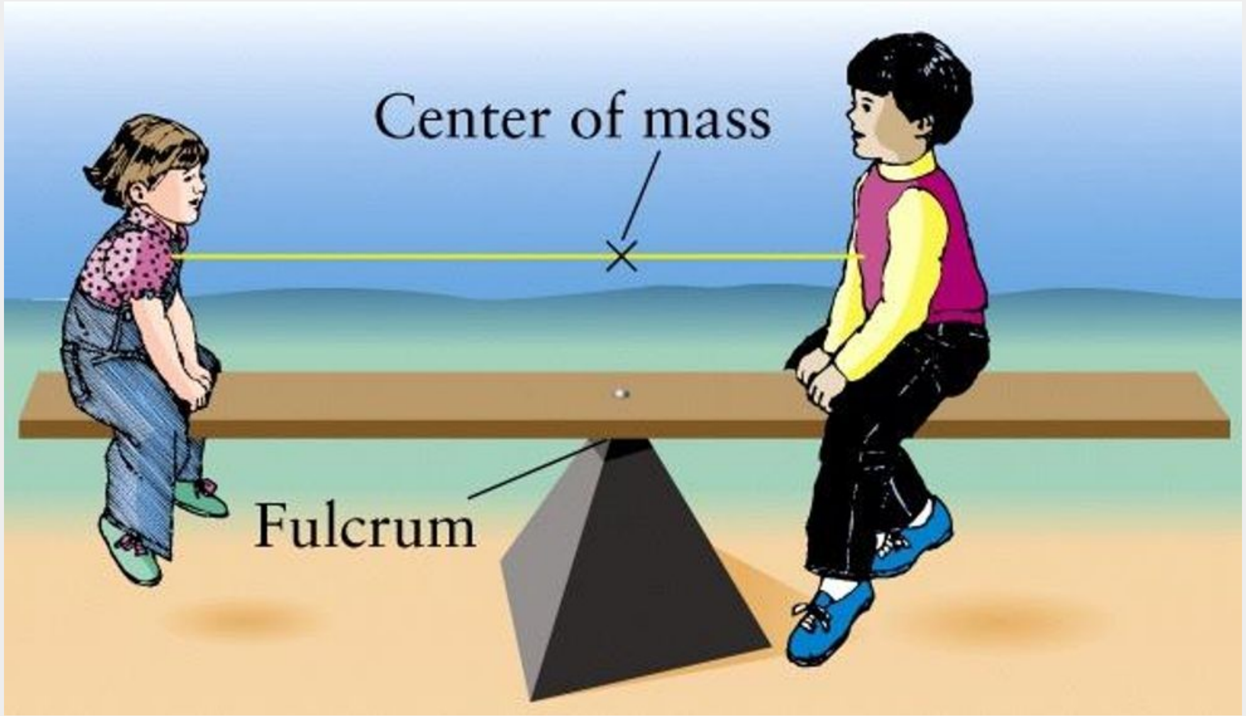
$$r_1 = \frac{m_2}{m_1+m_2} r$$

This can be extended to elliptical orbits replacing r by a

$$P^2 = \left(\frac{4\pi^2}{G(m_1+m_2)} a^3\right)$$

Using binaries to “weight” stars

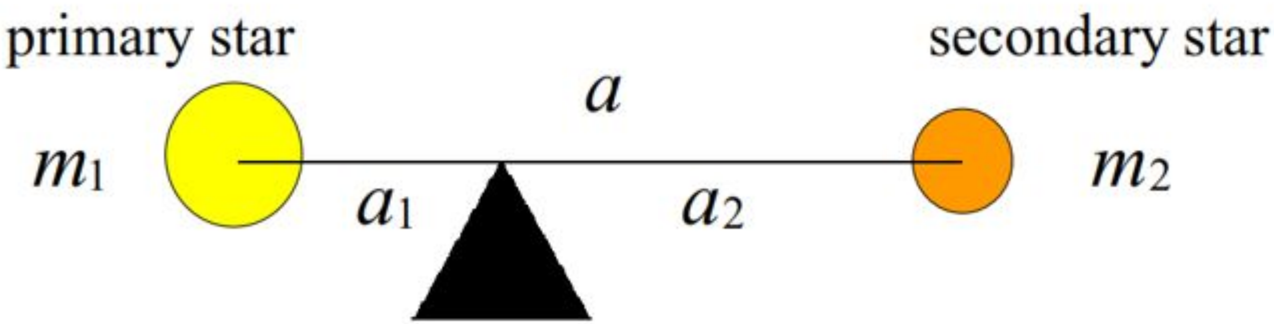
Binary stars are our fundamental source of information about the stellar masses. In particular those spectroscopic binaries which also are eclipsing systems, are the principal source of mass estimates to be compared with numerical models of stellar evolution.



For single stars, independent determination of the mass is impossible. All methods are indirect methods, that use models that have been calibrated via binaries.

Dynamical Mass Determination

Centre of Mass



The Centre-Of-Mass Formula is

$$a_1 m_1 = a_2 m_2$$

If orbital major axes (relative to centre of mass) or radial velocity amplitudes are known, then we know the ratio of masses:

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{v_{2r}}{v_{1r}}$$

Dynamical Mass Determination

If the period P and the radial velocities are known, Kepler third law can give us the sum of the masses (as a function of inclination)

$$P = \left(\frac{4\pi^2}{G(m_1 + m_2)} a^3 \right)^{1/2}$$

Using $a = a_1 + a_2$; $v_1 = 2\pi a_1/P$; $v_2 = 2\pi a_2/P$

$$(m_1 + m_2) = \frac{P}{2\pi G}(v_1 + v_2)^3 = \frac{P}{2\pi G}\left(\frac{v_{1r} + v_{2r}}{\sin i}\right)^3$$

where $v_{1r} = v_1 \sin i;\ v_{2r} = v_2 \sin i$ (for circular orbits)

are the maximum radial velocities (RVs), which can be measured.

Dynamical Mass Determination

If the period and the 2 radial velocities (v_{1r},v_{2r}) are known (SB2), we can derive the sum of the masses (still depending on i)

$$(m_1 + m_2) = \frac{P}{2\pi G} \left(\frac{v_{1r} + v_{2r}}{\sin i} \right)^3$$

and individual masses (still depending on the inclination i) combining this equation with

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{v_{2r}}{v_{1r}}$$

Dynamical Mass Determination

If only one radial velocity is known (SB1), a useful quantity is the **mass function**

From $\dfrac{m_1}{m_2} = \dfrac{a_2}{a_1} = \dfrac{v_{2r}}{v_{1r}}$ we replace the unknown $v_{2r} = \dfrac{m_1}{m_2} v_{1r}$

$$(m_1 + m_2) = \frac{P}{2\pi G}\left(\frac{v_{1r} + \frac{m_1}{m_2} v_{1r}}{\sin i}\right)^3$$

and the binary mass function is:

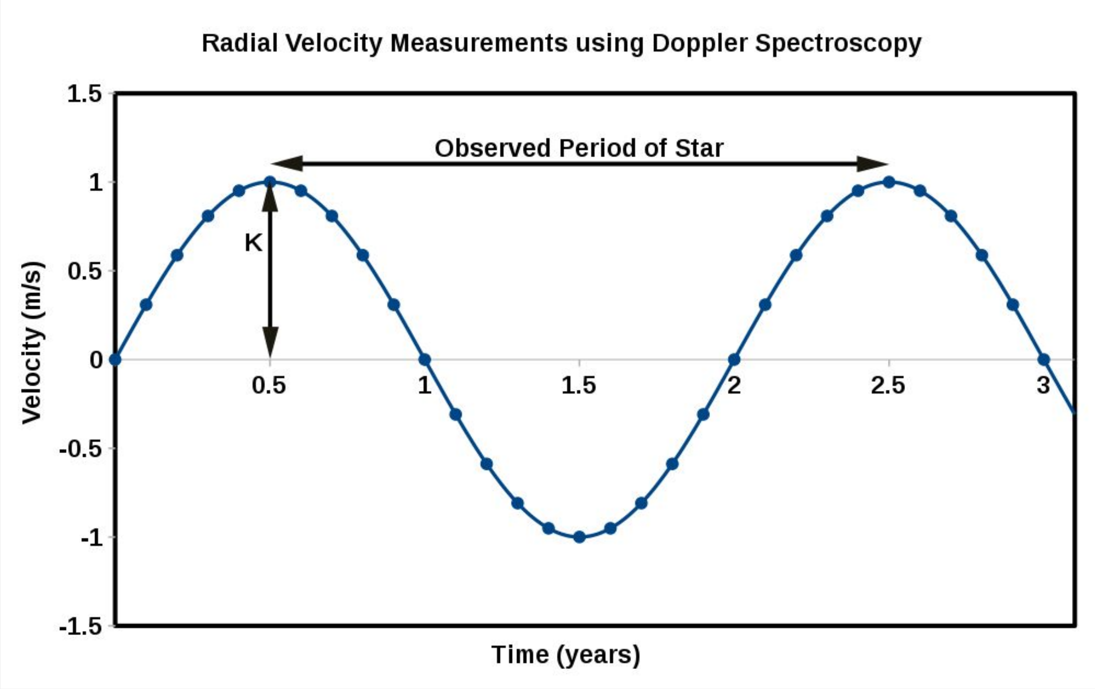
$$f(m_1, m_2) = \frac{v_{1r}^{\,3} P}{2\pi G} = \frac{(m_2 \sin i)^3}{(m_1 + m_2)^2}$$

which has units of mass

Mass Function

The maximum RV is often denoted K

$$f = \frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} = \frac{P_{\rm orb} K^3}{2\pi G}$$



→ If the orbital inclination *i* is also known, f allows to determine M_2/M_1. If we know (estimate) M_1 for example from a spectral fitting, we can estimate M_2

→ If *i* is not know, and we estimate M_1 the mass function allows us to estimate a lower limit for the mass M_2 of the unseen object (because $0 < \sin i < 1$)

Mass Function

→ The mass function we just saw is for circular orbits (common in short period binaries).

In an orbit with e > 0, the mass function is given by

$$f = \frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} = \frac{P_{\rm orb}\, K^3}{2\pi G}(1 - e^2)^{3/2}.$$

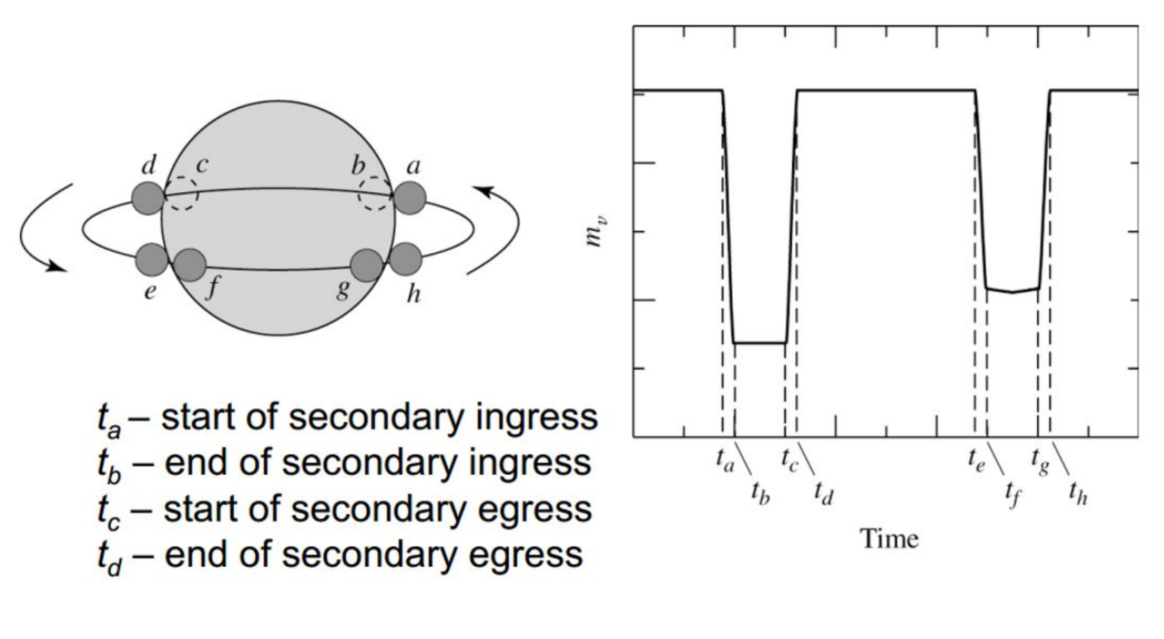
Determination of Radii and Teff's

In eclipsing binaries, the duration of eclipses and shape of light curve can be used to determine the radius of stars

$$R_1 = \frac{v_1 + v_2}{2}(t_b - t_a)$$

$$R_2 = \frac{v_1 + v_2}{2}(t_c - t_a)$$

*in this example $R_2 > R_1$, but the star 1 is brighter



The relative depth of primary (deepest) and secondary brightness minima of eclipses can be used to determine the ratio of effective temperatures of the stars:

$$\frac{F_0 - F_{prim}}{F_0 - F_{sec}} = \left(\frac{Teff_1}{Teff_2}\right)^4$$

Stay tuned to Maja's classes to learn more about how to derive orbital parameters from observations

Interacting binary stars

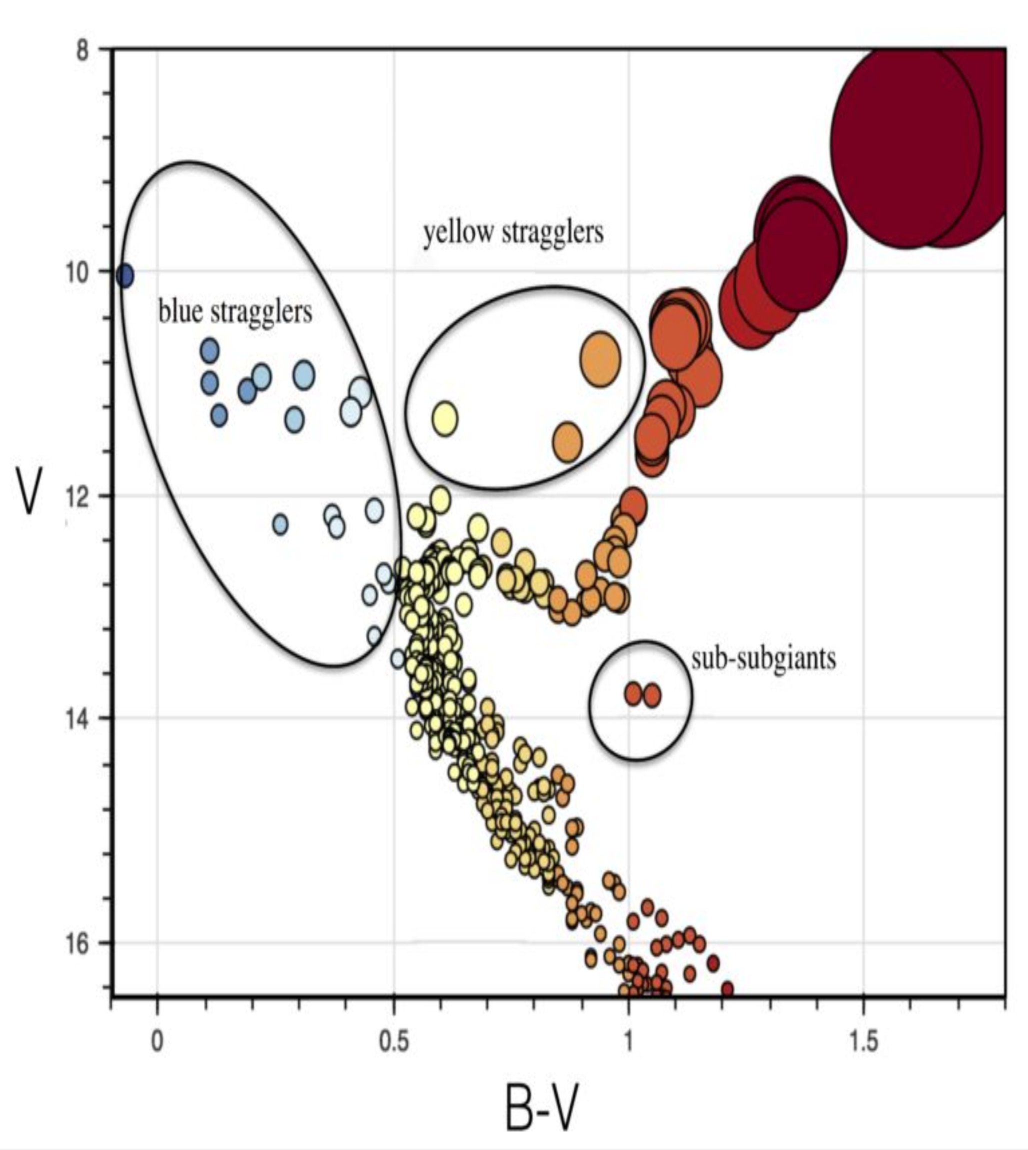
Interacting binaries

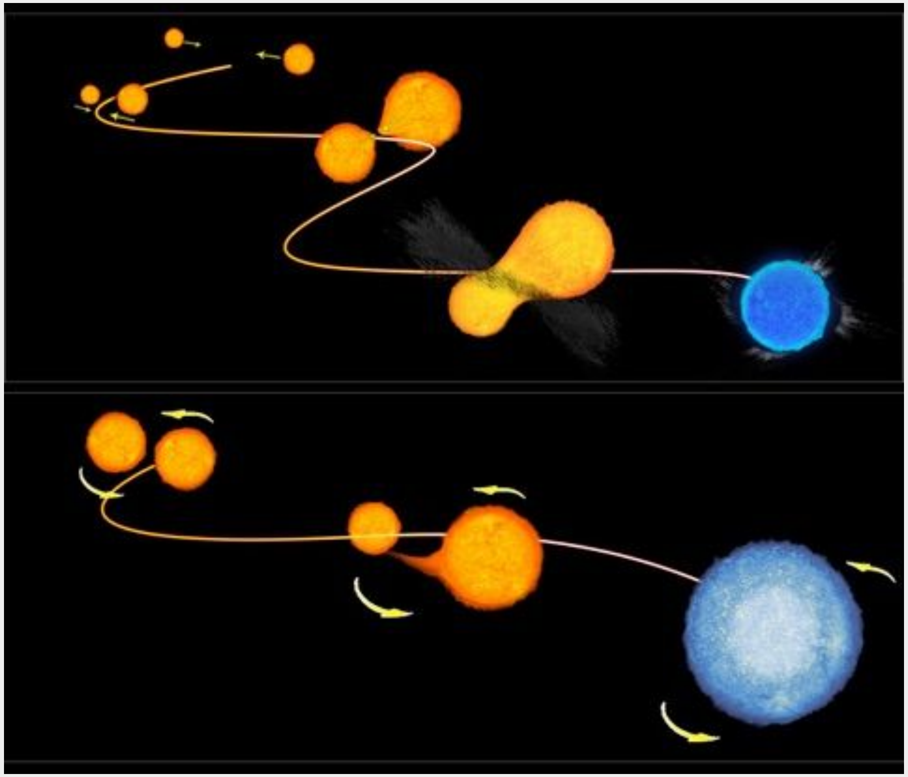
Wide binaries: if the two stars are far enough apart, they will have little or no effect on each other $\rightarrow$ Both stars evolve as isolated stars.

But the properties of some binary stars are inexplicable in terms of the ordinary evolution of isolated stars.

e.g, blue stragglers, algol, close compact binaries, hot subdwarfs...

Blue Stragglers
“rejuvenated stars”





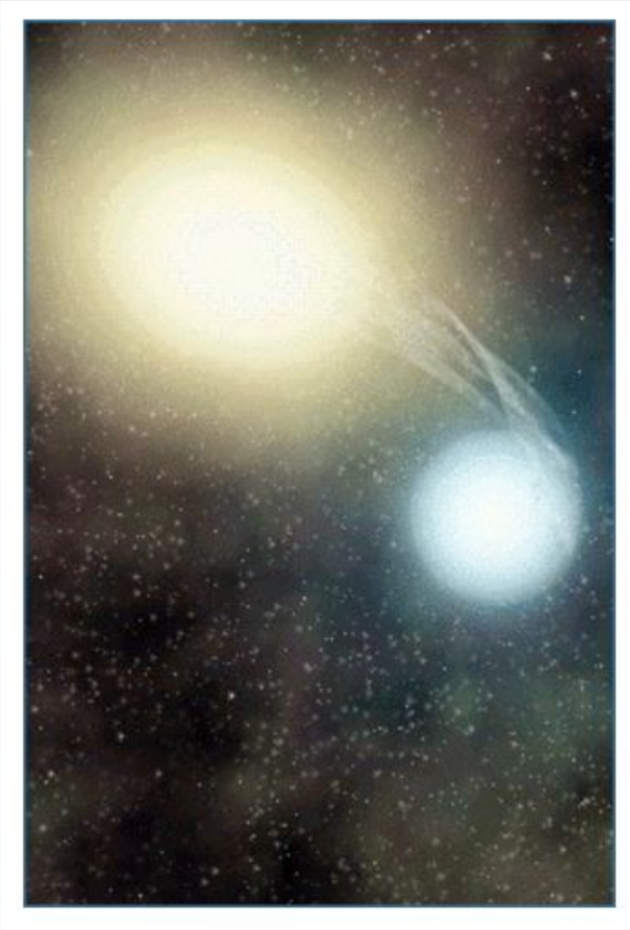
Up: collision model. Two low-mass stars in a overcrowded environment experience a head-on collision and merge. Form a new, single, hot (hence blue) and seemingly young star.

Down: the lower-mass object drains its heavier companion of hydrogen, that fuels its rebirth.

The Algol Paradox

e.g. Algol, consists of a main-sequence B star ($M = 3.5M_\odot$) and a giant K star ($M = 0.81M_\odot$).

How can the less massive star be further advanced in its evolution?



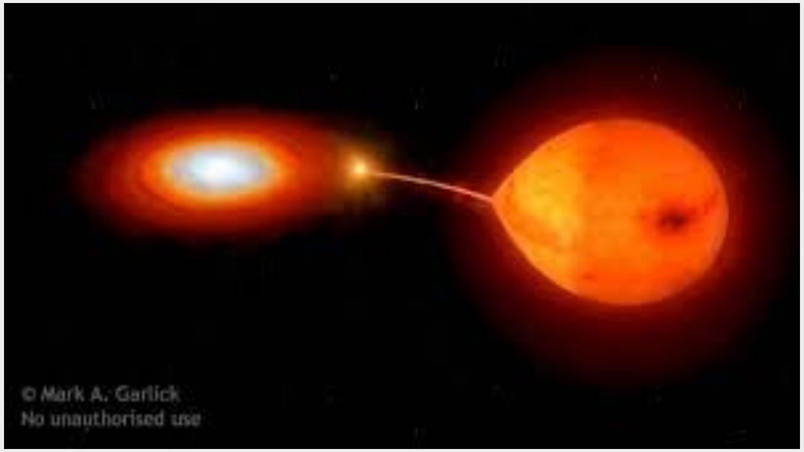
Mass transfer!
(the giant was originally the more massive)

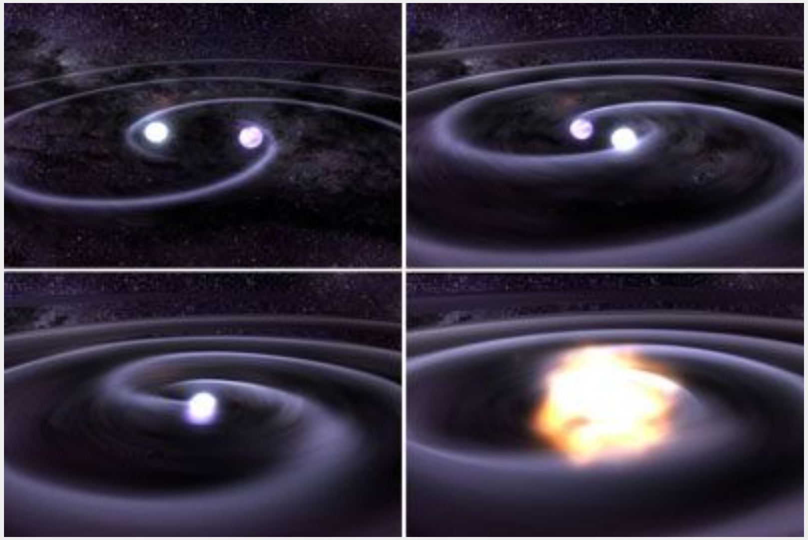
Close compact binaries

Many binaries containing at least one compact stellar remnants (e.g. white dwarf, neutron star) have orbital periods of hours or less

→ orbital separation $a < R_\odot$

e.g. CVs Close double WDs

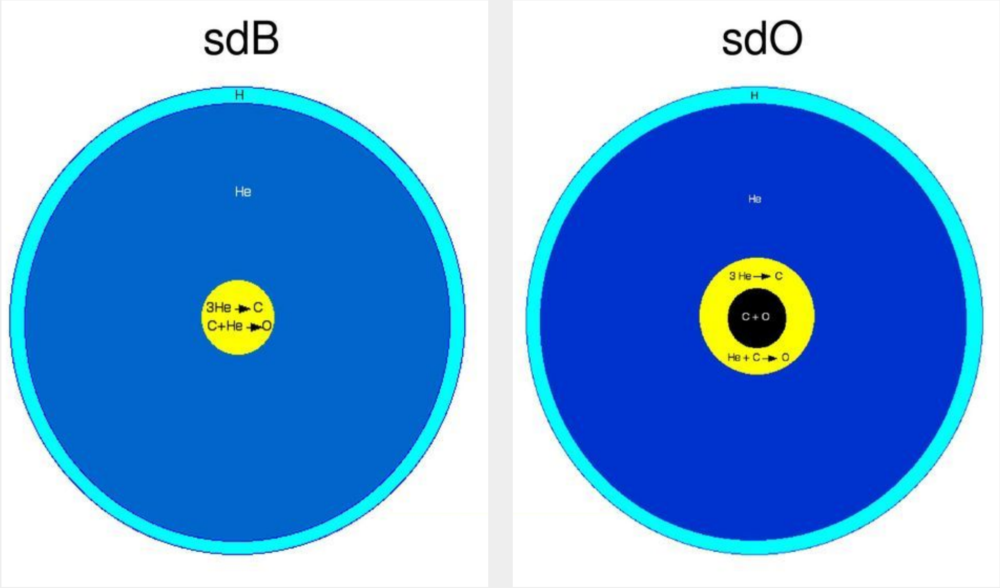


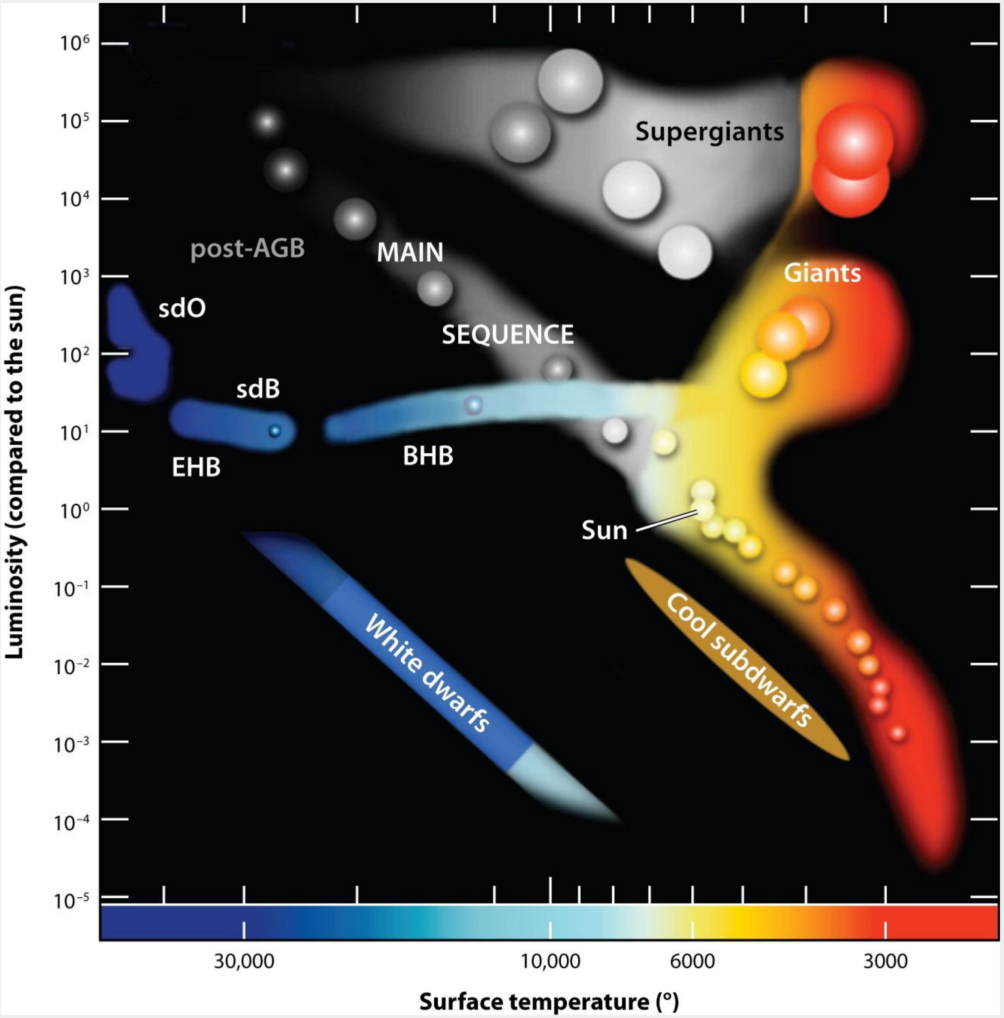


Must have been interactions between the progenitors during the evolution.

Naked helium burning stars

A hot subdwarf star is a star burning helium in its core (sdB) or in a shell (sdO), but that has almost completely lost its hydrogen envelope. They are located on the extreme HB





How did they loose the envelope?
→ Binaries!

Interacting binaries

Resolution of these paradoxical situations is often mass transfer between the components of a close binary.

By definition, **close (or interacting) binaries** are binary systems in which some significant interaction (other than simple inverse square law gravitational attraction between point masses) takes place.

The interaction may be:
- Mass transfer (Roche lobe filling or wind driven mass transfer).
- Tidal (distortion of one or both components through the combination of gravitational and centrifugal effects).
- Radiative (as in the heating of the face of one component by a hot companion).

Range of binary orbits (MS+MS)

Log Period Nominal [Actual]	Period[2] days/years	Orbital Radius[2] Solar Radii/AUs	Mean Eccen-tricity	Category Label (Note)
–1 [–0.63]	*0.23/0.0006*	*2.0/0.009*	0.00	CONTACT *(smallest orbit before stars merge)*
0 [0.01]	*1.02/0.0028*	*5.4/0.025*	0.05	
1 [0.91]	*8.2/0.022*	*22/0.10*	0.23	INTERACTING
2 [1.96]	*91/0.25*	*108/0.50*	0.33	*(Venus R = 0.72 AU, limit of corotation)*
3 [3.01]	*1021/2.8*	2.50	0.41	CLOSE *(asteroid belt R = 2.8 AU)*
4 [3.91]	22	10	0.50	*(Saturn R = 9.6 AU)*
5 [4.96]	250	50	0.52	MEDIAN *(Kuiper Belt R = ~50 AU)*
6 [6.01]	2800	250	0.62	*(Heliosphere R = ~120 AU)*
7 [6.91]	22,000	1000	0.65	WIDE[3] *(longest period of solved orbits)*
8 [7.96]	250,000	5000	.	*(all identified as CPM pairs)*
9 [9.01]	2,800,000	25,000	.	FRAGILE[4] *(widest confirmed = ~75,000 AU)*
11 [10.7]	150,000,000	350,000	.	tidal radius *(comoving but not bound)*

1: P to a calculated for a binary of 2 solar mass stars (M1 + M2 = $2M_\odot$)
2: P and a rounded for simplicity
3: Approximately the largest orbital radius of a permanent system
4: Likely to be disrupted by passing stars or giant molecular clouds within the lifetime of the components

Next class: tides, Roche Geometry