Universidad de Valparaíso CHILE

Facultad de Ciencias
Programa FOGEC
ÁLGEBRA I
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CLASE No. 11

Trigonometría: más ejercitación

Ejercicio 1

Resuelva la siguiente ecuación trigonométrica en [0,2π[:

$$2(1+\cos x) = \frac{1-sen\,x}{1-\cos x}$$

$$2(1-\cos x)(1+\cos x) = 1 - \operatorname{sen} x$$

$$2(1-\cos^2 x) + \operatorname{sen} x - 1 = 0$$

$$2\operatorname{sen}^2 x + \operatorname{sen} x - 1 = 0$$

Cambio de variable: $\boxed{y = \operatorname{sen} x} \Rightarrow 2y^2 + y - 1 = 0 \Rightarrow y = \frac{1}{2}$ o $y = -1$

$\Rightarrow$ $\operatorname{sen} x = \frac{1}{2} \Rightarrow x = 30^\circ$ o $x = 150^\circ$

$\operatorname{sen} x = -1 \Rightarrow x = 270^\circ$

$$\boxed{S = \{30^\circ, 150^\circ, 270^\circ\}}$$

Ejercicio 2

Resolver completamente en [0, 2π[la ecuación:

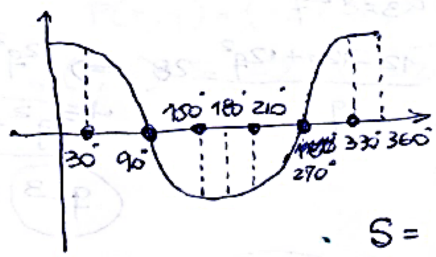
$$4\cos^3\alpha = 3\cos\alpha$$

Si $\cos\alpha \neq 0$: $4\cos^2\alpha \cdot \cancel{\cos\alpha} = 3\,\cancel{\cos\alpha}$

$$\cos^2\alpha = \frac{3}{4}$$

$$\boxed{\cos\alpha = \pm\frac{\sqrt{3}}{2}}$$

$\Rightarrow 4\cos^3\alpha = 3\cos\alpha \Leftrightarrow \cos\alpha = \pm\frac{\sqrt{3}}{2} \ \vee \ \cos\alpha = 0$



$\cos\alpha = 0 \Rightarrow \alpha = 90°, 270°$

$\cos\alpha = +\frac{\sqrt{3}}{2} \Rightarrow \alpha = 30°, 330°$

$\cos\alpha = -\frac{\sqrt{3}}{2} \Rightarrow \alpha = 150°, 210°$

$S = \{30°, 90°, 150°, 210°, 270°, 330°\}$

$$S = \underbrace{\left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}}_{6 \text{ soluciones}}$$

Ejercicio 3

Decidir la validez (discutiendo restricciones) de la siguiente identidad trigonométrica: $(1+tg^2x)(1-sen^2x)=1$

$$\left(1+\frac{sen^2\alpha}{cos^2\alpha}\right)\cdot cos^2\alpha = 1$$

$$\frac{cos^2\alpha+sen^2\alpha}{\cancel{cos^2\alpha}} = \cancel{cos^2\alpha} = 1$$

$1=1$ si $cos\,\alpha \neq 0$

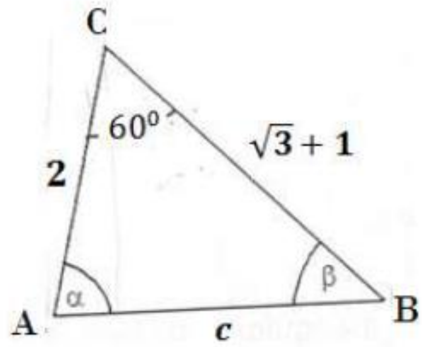
$\alpha \neq 90°, 270°, \frac{(2k+1)\pi}{2}$

$k=0,1,2,3\ldots$

$\boxed{\text{La igualdad es válida para } \forall\, \alpha \in \mathbb{R},\ \alpha \neq \frac{(2k+1)\pi}{2} \quad k \in \mathbb{N}_0}$

Ejercicio 4

Dado el triángulo acutángulo de la figura, encontrar las medidas del lado y los ángulos restantes.



Por teorema del coseno:

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos\gamma}$$

$$c^2 = 2^2 + (\sqrt{3}+1)^2 - \cancel{2}\cdot 2(\sqrt{3}+1)\cdot \underbrace{\frac{1}{\cancel{2}}}_{\cos 60^\circ}$$

$$c^2 = 4 + 3 + \cancel{2\sqrt{3}} + 1 - \cancel{2\sqrt{3}} - 2$$

$$c^2 = 6 \Rightarrow \boxed{c = \sqrt{6}}$$

Por teorema del seno:

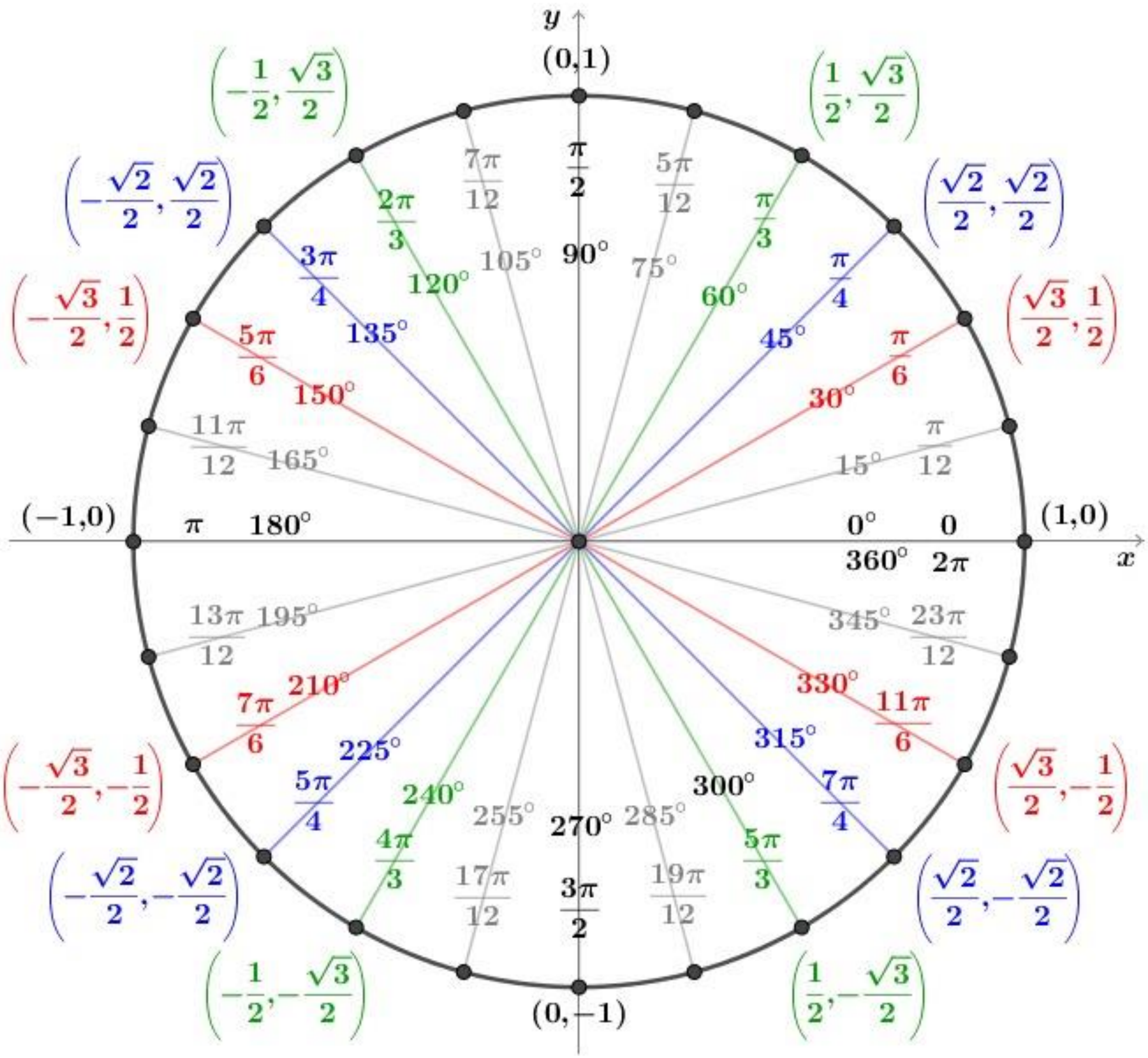
$$\boxed{\frac{\operatorname{sen}\beta}{b} = \frac{\operatorname{sen}\gamma}{c}} \Rightarrow \frac{\operatorname{sen}\beta}{2} = \frac{\operatorname{sen} 60^\circ}{\sqrt{6}} \Rightarrow$$

$$\Rightarrow \frac{\operatorname{sen}\beta}{\cancel{2}} = \frac{1}{\sqrt{6}}\,\frac{\sqrt{3}}{\cancel{2}} \Rightarrow \operatorname{sen}\beta = \frac{\sqrt{3}}{\sqrt{6}} \Rightarrow \operatorname{sen}\beta = \frac{1}{\sqrt{2}} \Rightarrow$$

$$\operatorname{sen}\beta = \frac{\sqrt{2}}{2} \text{ (racionalizando)} \Rightarrow \boxed{\beta = 45^\circ} \text{ (acutángulo)} \Rightarrow \alpha = 75^\circ$$

o

$$\boxed{\beta = 135^\circ} \ \underline{\text{NO}}$$



y
x
(0,1)
(0,−1)
(1,0)
(−1,0)
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$\frac{\pi}{2}$ 90°
$\frac{7\pi}{12}$ 105°
$\frac{5\pi}{12}$ 75°
$\frac{2\pi}{3}$ 120°
$\frac{\pi}{3}$ 60°
$\frac{3\pi}{4}$ 135°
$\frac{\pi}{4}$ 45°
$\frac{5\pi}{6}$ 150°
$\frac{\pi}{6}$ 30°
$\frac{11\pi}{12}$ 165°
$\frac{\pi}{12}$ 15°
π 180°
0° 0
360° 2π
$\frac{13\pi}{12}$ 195°
345° $\frac{23\pi}{12}$
$\frac{7\pi}{6}$ 210°
330° $\frac{11\pi}{6}$
$\frac{5\pi}{4}$ 225°
315° $\frac{7\pi}{4}$
240° $\frac{4\pi}{3}$
300° $\frac{5\pi}{3}$
255° $\frac{17\pi}{12}$
270° $\frac{3\pi}{2}$
285° $\frac{19\pi}{12}$