Astronomía estelar

2024

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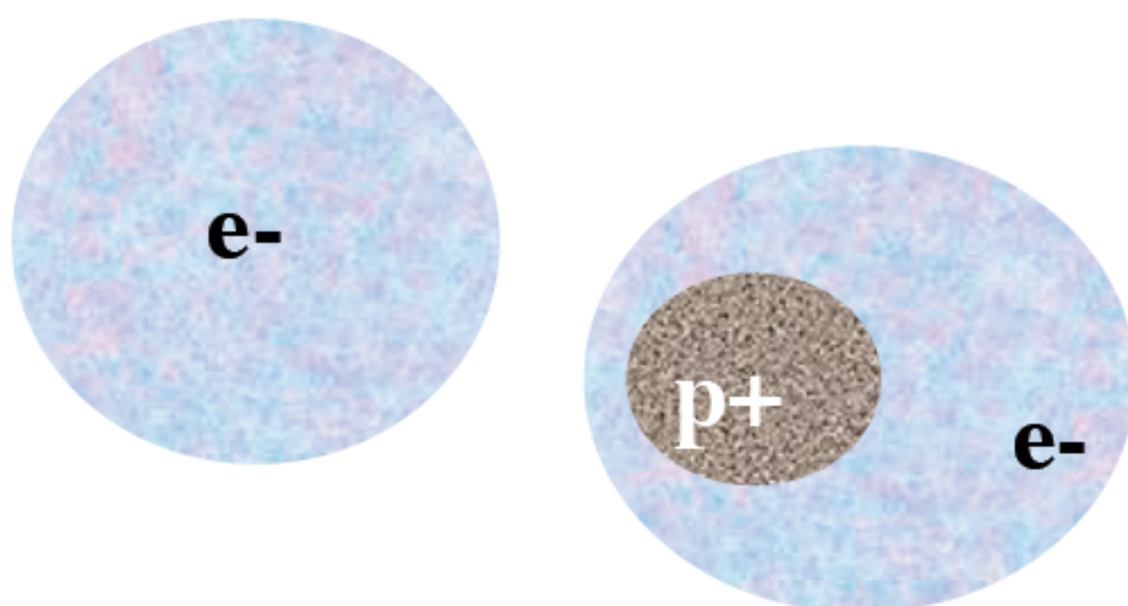
Clase 10

Atmósferas estelares III

The primary source of continuum opacity in the atmospheres of stars later than F0V is….



The binding energy of H^- is only 0.754 eV (1640 nm, in the infrared), so any photon with more energy than this can be absorbed by this b-f process. At longer wavelengths, also get f-f absorption.

…H^- opacity

$$\mathrm{H} + e^- \rightleftharpoons \mathrm{H}^- + \gamma .$$

In A and B-type stars, photoionization of hydrogen (b-f) and f-f absorption are main sources of opacity.

In O-type stars, most hydrogen is ionized, so electron scattering becomes more important and photoionization of helium contributes to the opacity.

$$\kappa_\lambda = \kappa_{\lambda,bb} + \kappa_{\lambda,bf} + \kappa_{\lambda,ff} + \kappa_{\lambda,es} + \kappa_{\lambda,H^-}$$

The total opacity in a star is the sum of individual opacities, with different opacity terms being more or less important in different types of stars.

Depends not only on wavelength, but also on composition, density and temperature

Rosseland Mean Opacity

- Averaged over all wavelengths
- Includes composition, density, temperature dependencies

$$\frac{1}{\langle\kappa\rangle} = \frac{\int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial B_\nu(T)}{\partial T} d\nu}{\int_0^\infty \frac{\partial B_\nu(T)}{\partial T} d\nu}$$

Weighting function depends on the rate that the bb spectrum changes with temperature

No analytical formulae for all of the contributions to b-b opacity by individual spectral lines, no Rosseland mean opacity for spectral lines.

However, approximations have been developed for the average b-f and f-f opacities:

$$\langle \kappa_{bf} \rangle = 4.34 \times 10^{21} \frac{g_{bf}}{t} Z(1+X) \frac{\rho}{T^{3.5}} m^2 kg^{-1}$$

$$\langle \kappa_{ff} \rangle = 3.68 \times 10^{18} g_{ff} (1-Z)(1+X) \frac{\rho}{T^{3.5}} m^2 kg^{-1}$$

Kramer's opacity Law

$$\kappa_o \rho / T^{3.5}$$

ρ Density [km m^{-3}]

T Temperature [K]

X Hydrogen (mass) abundance

Z Heavy metal (mass) abundance

X + Y + Z = 1

g Gaunt factor ~1 for visible and UV

t "guillotine" factor - after atom is ionized, the opacity cuts off (has values of 1 - 100)

$$X = \frac{\text{total mass of hydrogen}}{\text{total mass of gas}}$$

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Cross section for e- scattering is wavelength and temperature independent, so κ_{es} has simple form:

$$\overline{\kappa_{es}} = 0.02(1+X) \; m^2 kg^{-1}$$

Opacity from H$^-$ for Temp: 3000-6000K κ_{H^-} between 10^{-7} - 10^{-2}

$$\overline{\kappa_{H^-}} \approx 7.9 \times 10^{-34} (Z/0.02) \rho^{1/2} T^9 \; m^2 kg^{-1}$$

Total Rosseland mean opacity:

$$\overline{\kappa} = \overline{\kappa_{bb} + \kappa_{bf} + \kappa_{ff} + \kappa_{es} + \kappa_{H^-}}$$

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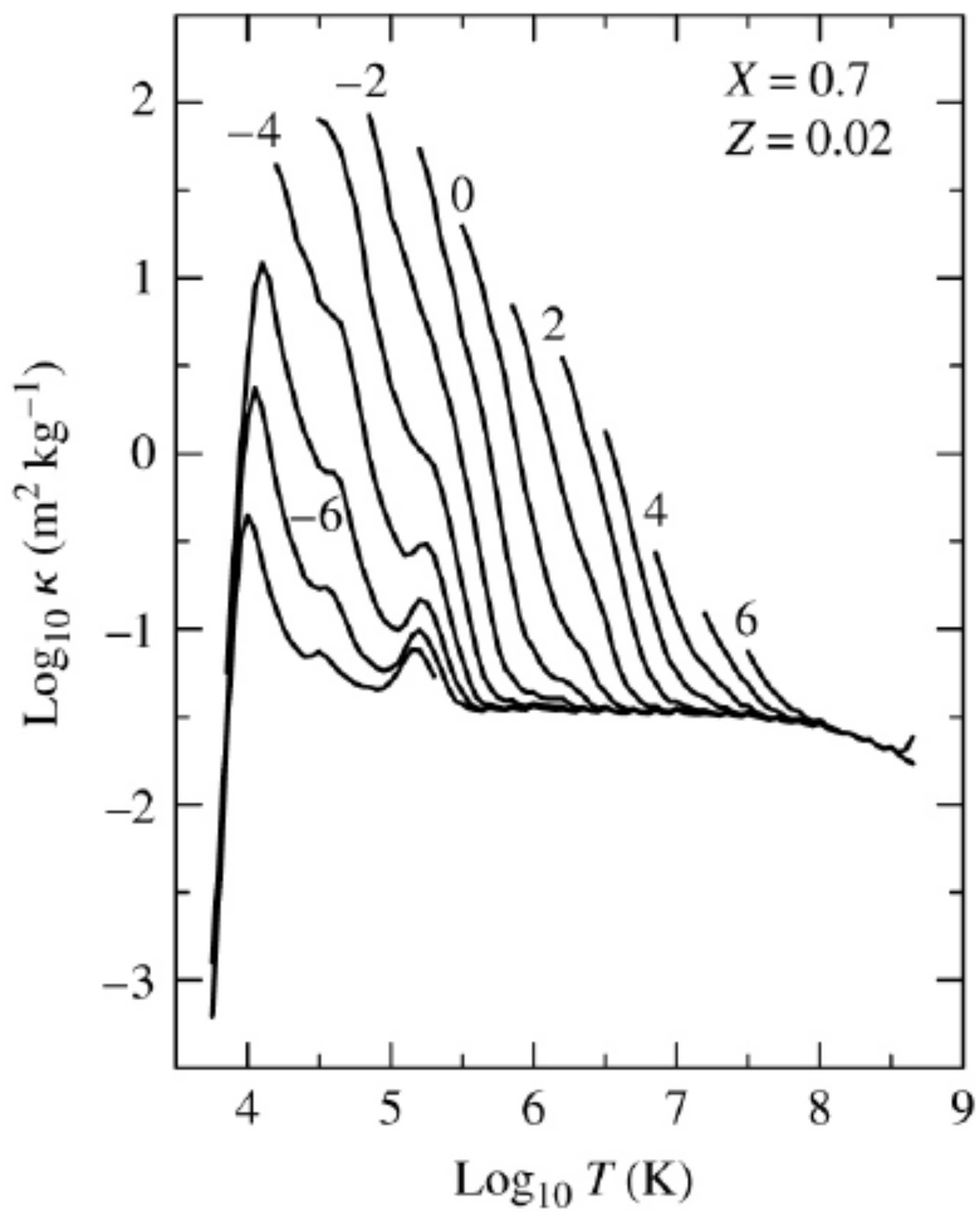


Opacity increases with increasing density at a given temperature.

At constant density it rises steeply from 5000K to 10,000K as the number of free electrons increases (ionization of H).

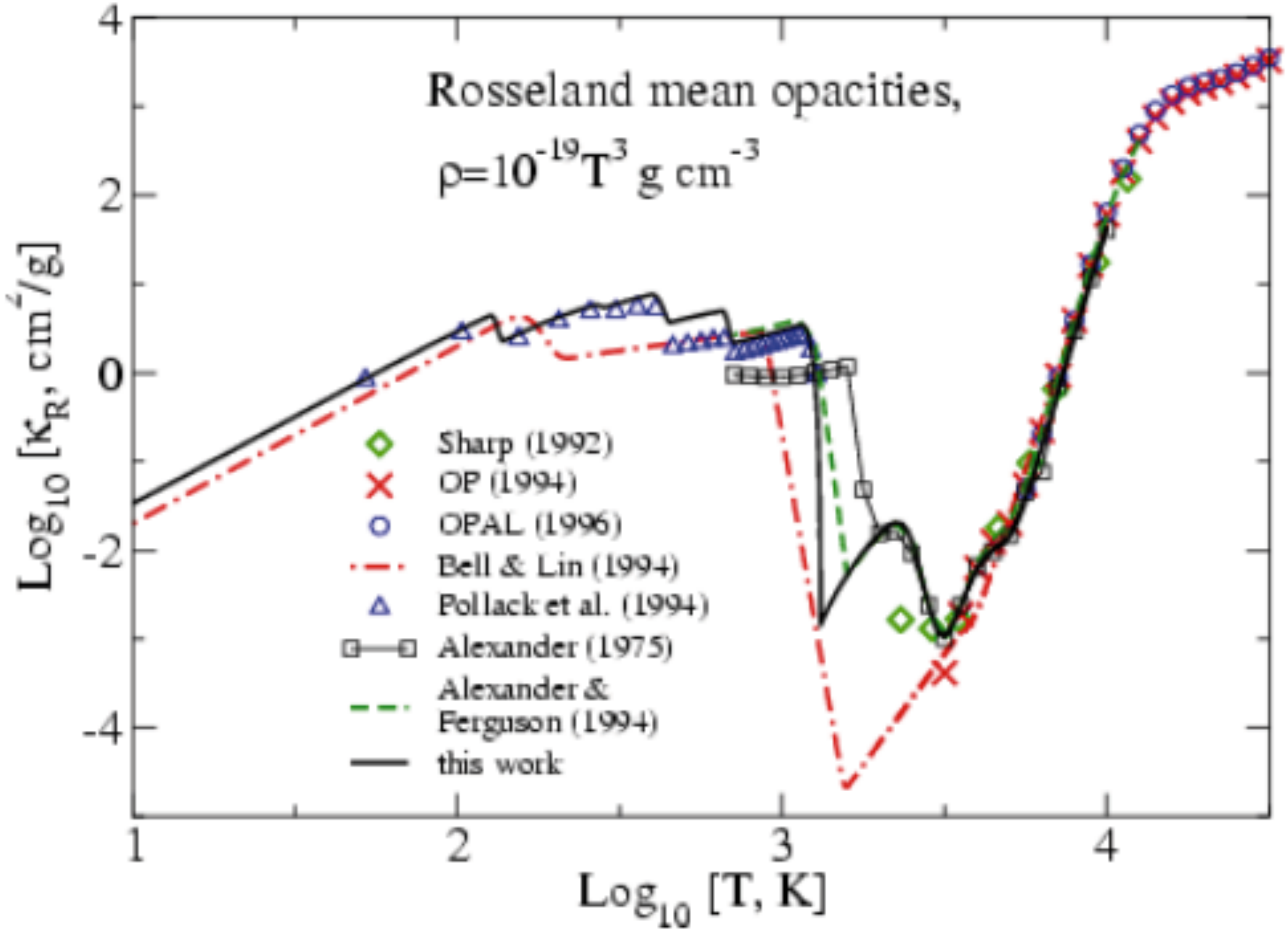
After the peak at 10000K, the decline in opacity follows Kramers law and is due to b-f and f-f absorption of photons

HeII loses it's second electron at about 40K - the increase in electrons causes a small bump in opacity.

Ionization of metals (Fe) cause the increase in opacity at about 10^5K

Flat floor: opacity due to electron scattering at high temperatures.

At cooler temperatures, the opacity source is dominated by molecules



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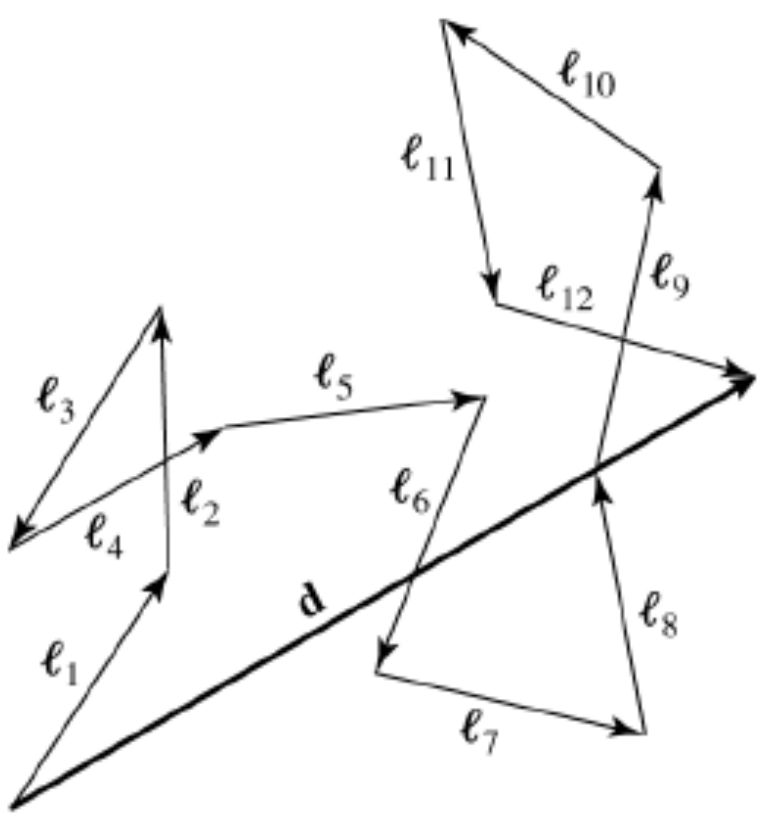
Radiative Transfer in Stars

Radiative transfer in a star is relatively inefficient. For a random walk, the displacement is related to the size of the steps (mfp) and the number of steps:



$$d = l\sqrt{N}$$

$$d = \tau\, l = l\sqrt{N}$$

$$\tau^2 = N$$

For $\tau_\lambda >> 1$, the average number of steps is roughly τ_λ^2. Near the surface, $\tau_\lambda = 2/3$, the average number of steps is 1. This is the definition of the photosphere.

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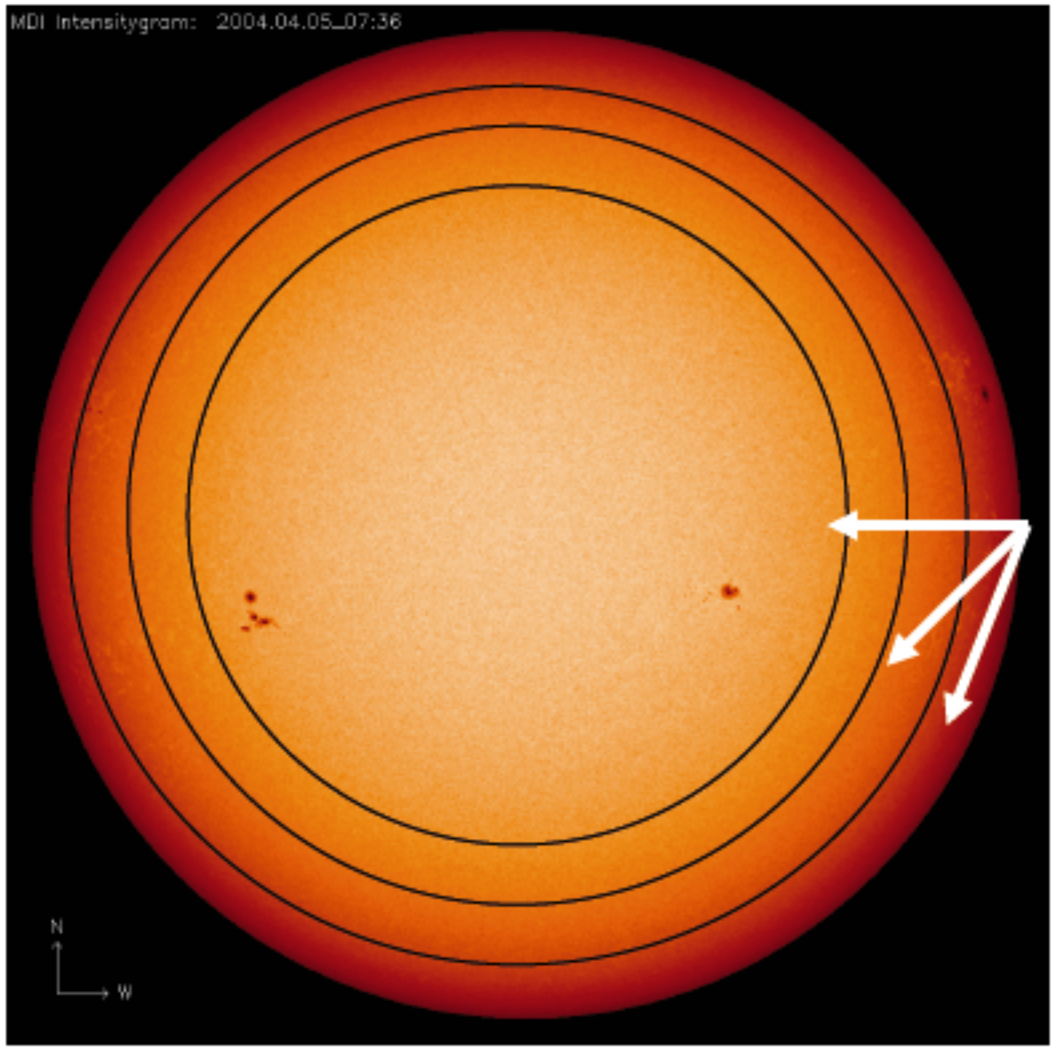

Looking into a star, we always see to an optical depth of $\tau_\lambda \sim 2/3$.

Optical depth is proportional to the path length, so looking toward the limb of a star, we get to an optical depth, $\tau_\lambda \sim 2/3$, at a cooler temperature than looking toward disk center.

The integrated spectrum of the Sun is spectral type G2V. However, the disk center of the Sun has the spectrum of a hotter F8V star!

This image is exaggerated for illustrative purposes - the thickness of the photosphere is about 0.1% of the solar radius.

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Pressure gradients: if you watch the fog roll in closely, you'll see that individual aerosol particles are swirling in all directions but the mass of fog still has a net bulk motion because of a pressure gradient.

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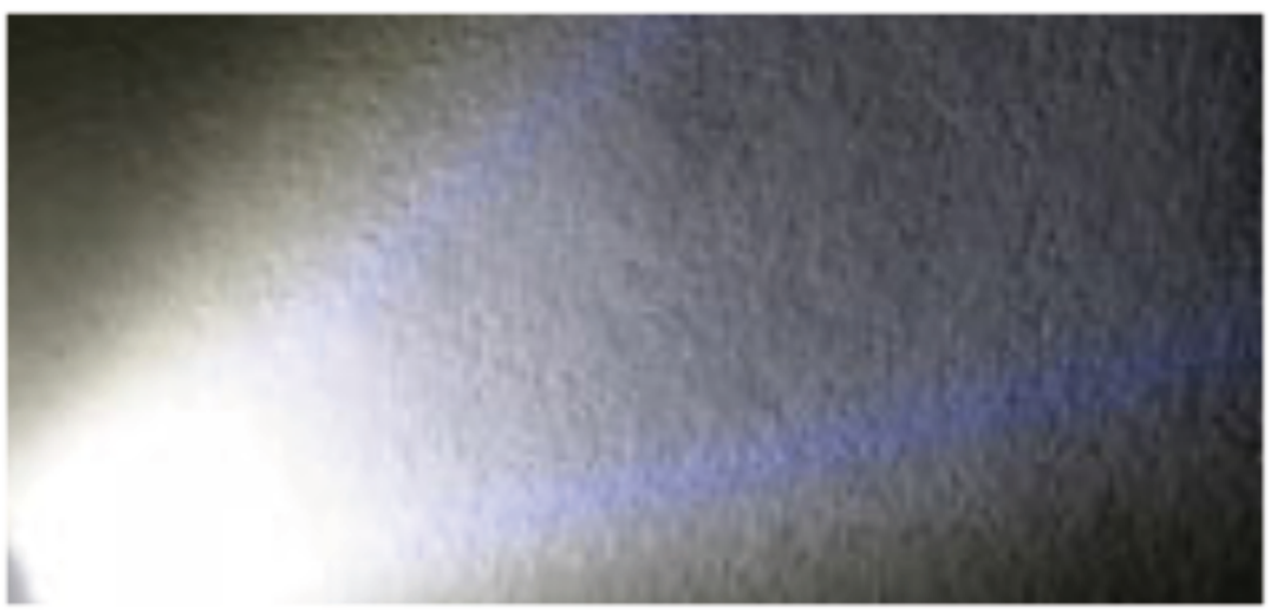
In the same way, the mfp of a photon near the center of the star is a fraction of a centimeter and a random-walk. But a gradient in the radiation pressure drives a slight net movement toward the surface of the star:

$$\frac{dP_{rad}}{dr} = \frac{\overline{\kappa}\rho}{c} F_{rad}$$



Next, we'll describe the emission and absorption processes that change the intensity of a ray of light with wavelength λ as it travels through a gas.

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Emission coefficient

For pure emission (no absorption or scattering) the increase in intensity of a beam is proportional to the path length, ds, and the density of the gas:

$$dI_\lambda = j_\lambda \rho \; ds$$

j_λ is the emission coefficient units of $\mathrm{m\ s^{-3}\ sr^{-1}}$

The total change in intensity is:

$$dI_\lambda = \underbrace{-\kappa_\lambda \rho \; I \; ds}_{\text{absorption}} + \underbrace{j_\lambda \rho \; ds}_{\text{emission}}$$

These competing processes determine how rapidly the intensity of light changes.

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$$-\frac{dI_\lambda}{\kappa_\lambda \rho \, ds} = I_\lambda - \frac{j_\lambda}{\kappa_\lambda}$$

Ratio of emission to absorption determines rate of Intensity change.

Divide previous equation by $-\kappa_\lambda \rho \, ds$

$$S_\lambda = \frac{j_\lambda}{\kappa_\lambda}$$

The source function is the ratio of emission coefficient to the absorption coefficient. It describes how photons traveling with beam are removed and replaced.

$$-\frac{dI_\lambda}{\kappa_\lambda \rho \, ds} = I_\lambda - S_\lambda$$

This is the "Radiative Transfer Equation"

The source function, S_λ, has the same units as the intensity, W m^{-3} sr^{-1}.

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$$-\frac{dI_\lambda}{\kappa_\lambda \rho \, ds} = I_\lambda - S_\lambda$$ Radiative transfer equation

If the intensity of light does not vary, then $I_\lambda = S_\lambda$

If the intensity of light is greater than the source function, then $I_\lambda > S_\lambda$ and dI/ds is negative. The incoming (emission) photons can't keep pace with the loss (absorption).

If the intensity of light is less than the source function, then $I_\lambda < S_\lambda$ and dI/ds is positive and there are more incoming photons.

The intensity of the beam evolves to match the intensity of the local source function.

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The Special Case of Blackbody Radiation

LTE: $$I_\lambda = B_\lambda$$

Radiation field can be described by Planck function

Because $dI_\lambda/ds = 0$, $$S_\lambda = B_\lambda$$

Source field (, j_λ/κ_λ) can be described by Planck function

where $\tau_\lambda \gg 1$

at least τ_λ^2 steps to reach the surface

In summary, saying that $I_\lambda = B_\lambda$ is a statement that the radiation field is described by the Planck function, while $S_\lambda = B_\lambda$ describes the physical source of the radiation, j_λ/κ_λ, as one that produces blackbody radiation.

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Integrate:

$$-\frac{dI_\lambda}{\kappa\rho\ ds} = I_\lambda - S_\lambda$$

$$\int \frac{dI}{(I-S)} = -\int \kappa\rho\ ds$$

$$\ln(I-S) - \ln(I_o - S) = -\kappa\rho\ s$$

$$\ln\left(\frac{I-S}{I_o - S}\right) = -\kappa\rho\ s$$

$$\frac{I-S}{I_o - S} = e^{-\kappa\rho\ s}$$

$$I - S = (I_o - S)e^{-\kappa\rho\ s}$$

$$I = I_o e^{-\kappa\rho\ s} + S(1 - e^{-\kappa\rho\ s})$$

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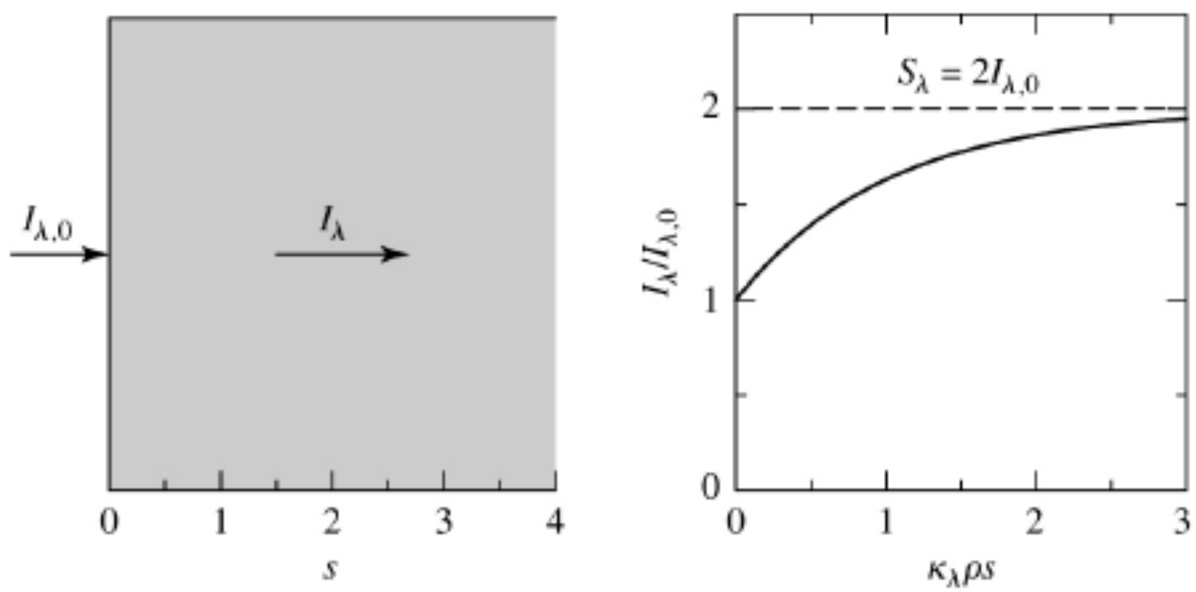

(a) (b)

To see how the intensity of a light ray tends to become equal to the local value of the source function, imagine a beam of light of initial intensity $I_{\lambda,0}$ at $s = 0$ entering a volume of gas of constant density, ϱ, that has a *constant* opacity, κ_λ, and a *constant* source function, S_λ.

$$I = I_o e^{-\kappa\rho\ s} + S(1 - e^{-\kappa\rho\ s})$$

The characteristic distance for this change to occur is $s = 1/\kappa_\lambda\varrho$, which is one photon mean free path

For: $S = 2I_o$

$$I = I_o\left(e^{-\kappa\rho s} + 2 - e^{-\kappa\rho s}\right) = 2I_o$$

Incoming specfic intensity tends toward the value of the source function!

The Assumption of a Plane-Parallel Atmosphere

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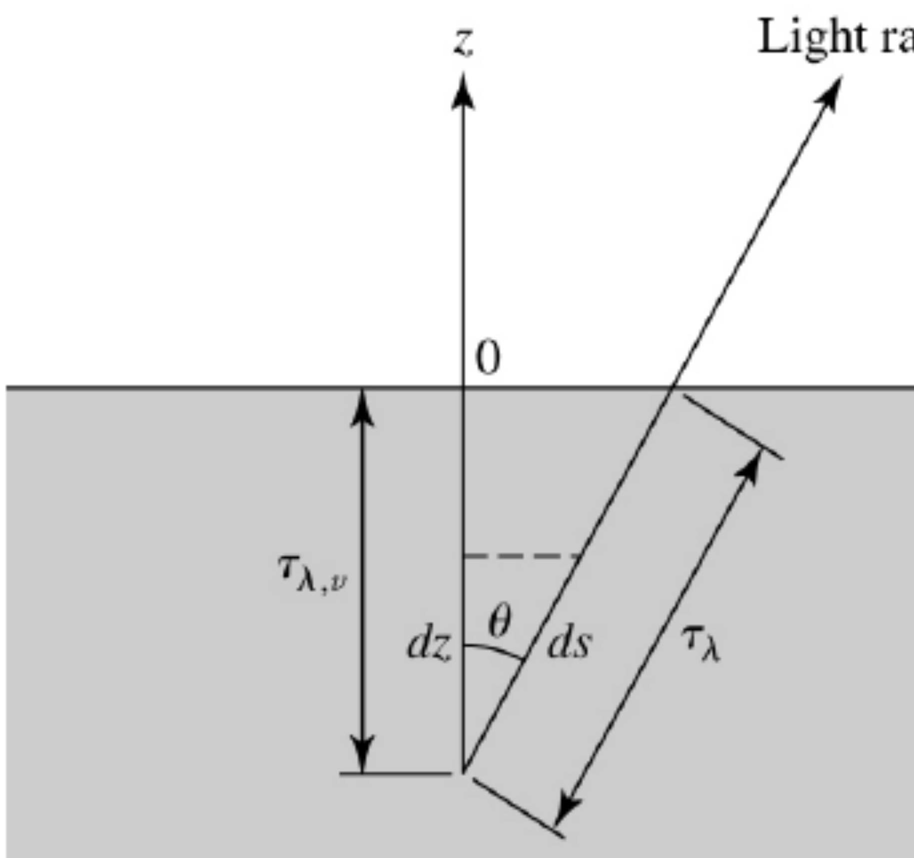

Rewrite the transfer equation in terms of the optical depth, τ_λ

$$d\tau_\lambda = -\kappa_\lambda \rho \, ds,$$

$$\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda$$

Assumption of plane parallel atmosphere

$$\cos\theta \frac{dI_\lambda}{d\tau_{\lambda,vert}} = I_\lambda - S_\lambda$$

$$\tau_{\lambda,vert}(z) = \int_z^0 \kappa_\lambda \rho \, dz = \int_s^0 \kappa_\lambda \rho \, \cos\theta \, ds$$

$$\tau_{\lambda,vert}(z) = \tau_\lambda \cos\theta$$

$$\tau_\lambda = \frac{\tau_{\lambda,vert}(z)}{\cos\theta}$$

Unfortunately, because the optical depth is measured along the path of the light ray, neither the optical depth nor the distance s in Eq. (34) corresponds to a unique geometric depth in the atmosphere. Consequently, the optical depth must be replaced by a meaningful measure of position.

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Gray atmosphere: λ-independent opacity

$$\kappa_\lambda \rightarrow \overline{\kappa}$$ **Rosseland opacity**

$$\tau_{\lambda,v} \rightarrow \tau_\lambda$$

$$\cos\theta \frac{dI}{d\tau_{vert}} = I - S$$

Integrating over all solid angles:

The source function, S, is independent of direction

$$\frac{d}{d\tau_{vert}} \int I\cos\theta \; d\Omega = \int I d\Omega - S \int d\Omega$$

$$\frac{dF_{rad}}{d\tau_v} = 4\pi\left(\langle I \rangle - S\right)$$

Gray, plane parallel atmosphere - what opacity case is this?

Because electron scattering is only relevant, gray opacity, not very realistic....

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A second important relation can be derived by multiplying through by $\cos\theta$:

$$\frac{d}{d\tau_{vert}} \int I \cos^2\theta \, d\Omega = \int I \cos\theta \, d\Omega - S \int \cos\theta \, d\Omega$$

$$\int \cos\theta \, d\Omega = \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} \cos\theta \, \sin\theta \, d\theta \, d\varphi = 0$$

$$\frac{dP_{rad}}{d\tau_v} = \frac{1}{c} F_{rad}$$

In spherical coordinates:

$$\frac{dP_{rad}}{dr} = \frac{\overline{\kappa}\rho}{c} F_{rad}$$

The radiative flux
is driven by differences in the
radiation pressure

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Equilibrium: no net energy subtracted from or added to the radiation field.

The radiative flux must have the same value at every level of the atmosphere, including the surface.

$$F_{rad} = const = F_{surface} = \sigma T^4$$

$$\therefore \frac{dF_{rad}}{d\tau_\nu} = 0$$

So, for the plane parallel gray atmosphere:

$$\cancelto{0}{\frac{dF_{rad}}{d\tau_\nu}} = 4\pi\left(\langle I \rangle - S\right)$$

$$\langle I \rangle = S$$

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Integrating the equation that relates
radiation pressure and radiative flux:

$$\frac{dP_{rad}}{dr} = \frac{\bar{\kappa}\rho}{c} F_{rad}$$

$$\int dP_{rad} = \frac{F_{rad}}{c} \int \bar{\kappa}\rho \, dr$$

$$\boxed{P_{rad} = \frac{1}{c} F_{rad} \tau_v + C}$$

Radiation pressure as a function of
vertical optical depth

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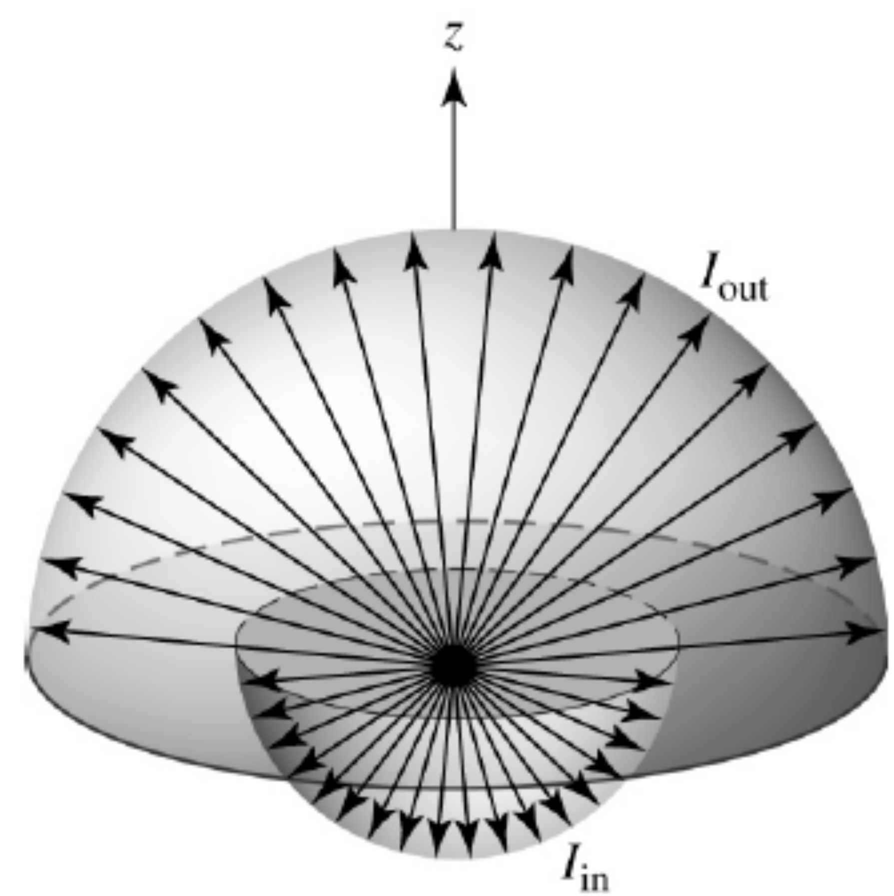
The Eddington Approximation: at every point in the atmosphere, there is some intensity in and some intensity out.

Both I_{in} and I_{out} vary with depth in the atmosphere and I_{in}=0 at the top of the atmosphere.



$$\langle I \rangle = \frac{1}{2}(I_{out} + I_{in})$$

$$F_{rad} = \pi(I_{out} - I_{in})$$

$$P_{rad} = \frac{2\pi}{3c}(I_{out} + I_{in}) = \frac{4\pi}{3c}\langle I \rangle = \frac{1}{c}F_{rad}\tau_v + \frac{2}{3c}F_{rad}$$

Boundary condition: I_{in}=0

$$\langle I(\tau_v = 0) \rangle = F_{rad}/2\pi$$

$$
\begin{aligned}
\langle I \rangle &= \frac{1}{4\pi}\left(\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} I_{\text{out}} \sin\theta \, d\theta \, d\phi + \int_{\phi=0}^{2\pi}\int_{\theta=\pi/2}^{\pi} I_{\text{in}} \sin\theta \, d\theta \, d\phi\right) \\
&= \frac{2\pi}{4\pi}\left(I_{\text{out}}\int_{\theta=0}^{\pi/2} \sin\theta \, d\theta + I_{\text{in}}\int_{\theta=\pi/2}^{\pi} \sin\theta \, d\theta\right) \\
&= \frac{1}{2}(I_{\text{out}} + I_{\text{in}}).
\end{aligned}
$$

$$
\begin{aligned}
F_{\text{rad}} &= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} I_{\text{out}} \cos\theta \sin\theta \, d\theta \, d\phi + \int_{\phi=0}^{2\pi}\int_{\theta=\pi/2}^{\pi} I_{\text{in}} \cos\theta \sin\theta \, d\theta \, d\phi \\
&= 2\pi\left(I_{\text{out}}\int_{\theta=0}^{\pi/2} \cos\theta \sin\theta \, d\theta + I_{\text{in}}\int_{\theta=\pi/2}^{\pi} \cos\theta \sin\theta \, d\theta\right) \\
&= \pi(I_{\text{out}} - I_{\text{in}}).
\end{aligned}
$$

$$
\begin{aligned}
P_{\text{rad}} &= \frac{1}{c}\left(\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} I_{\text{out}} \cos^2\theta \sin\theta \, d\theta \, d\phi + \int_{\phi=0}^{2\pi}\int_{\theta=\pi/2}^{\pi} I_{\text{in}} \cos^2\theta \sin\theta \, d\theta \, d\phi\right) \\
&= \frac{2\pi}{c}\left(I_{\text{out}}\int_{\theta=0}^{\pi/2} \cos^2\theta \sin\theta \, d\theta + I_{\text{in}}\int_{\theta=\pi/2}^{\pi} \cos^2\theta \sin\theta \, d\theta\right) \\
&= \frac{2\pi}{3c}(I_{\text{out}} + I_{\text{in}}) \\
&= \frac{4\pi}{3c}\langle I \rangle.
\end{aligned}
$$

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$$P_{\text{rad}} = \frac{1}{c} F_{\text{rad}} \tau_v + C$$

$$\frac{4\pi}{3}\langle I \rangle = F_{rad}\left(\tau_v + \frac{2}{3}\right)$$

$$\langle I \rangle = \frac{3\sigma}{4\pi} T_e^4 \left(\tau_v + \frac{2}{3}\right)$$

$$S = B = \frac{\sigma T^4}{\pi} \quad \text{for LTE}$$

$$\langle I \rangle = \frac{\sigma T^4}{\pi} = \frac{3\sigma}{4\pi} T_e^4 \left(\tau_v + \frac{2}{3}\right)$$

$$T^4 = \frac{3}{4} T_e^4 \left(\tau_v + \frac{2}{3}\right)$$

Temperature as a function of vertical optical depth in plane-parallel gray atmosphere

Where does $T=T_e$?

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$$T^4 = \frac{3}{4} T_e^4 \left(\tau_v + \frac{2}{3} \right)$$

$T = T_e$ at τ_v=2/3, not at τ_v=0

Thus, the surface of a star is at τ_v=2/3

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Spectral line profiles

$$\frac{F_c - F_\lambda}{F_c}$$ **Line depth**

One characterization of a line is equivalent width: the width of a box (zero to continuum) with the same area as the integrated area of the spectral line

$$W = \int \frac{F_c - F_\lambda}{F_c}\, d\lambda$$

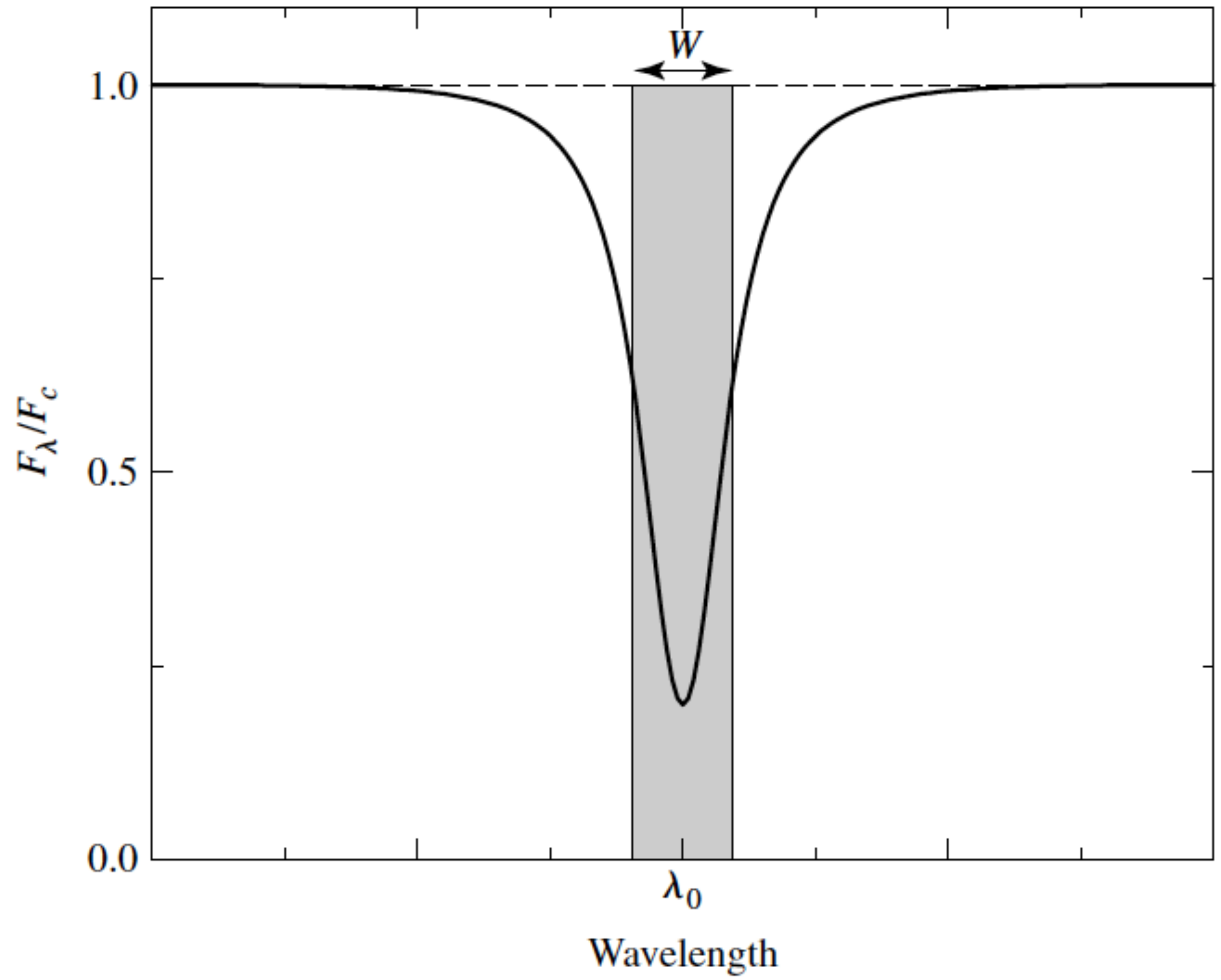

FIGURE 18 The profile of a typical spectral line.

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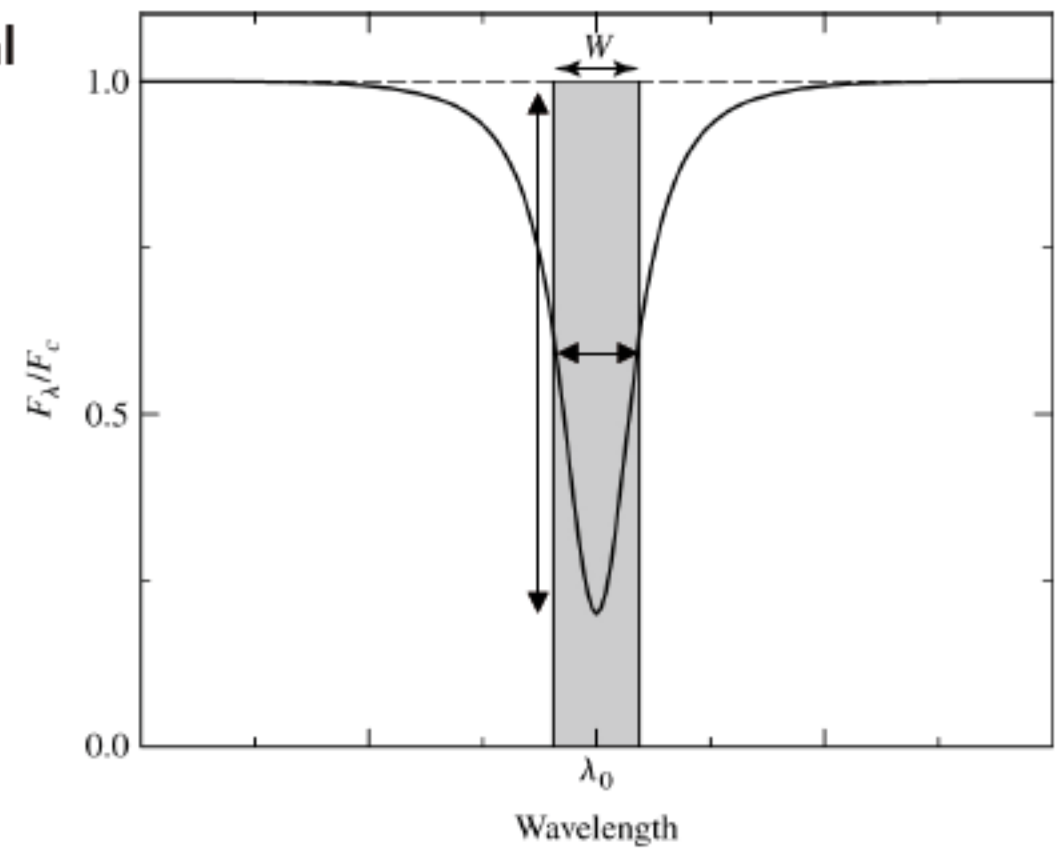
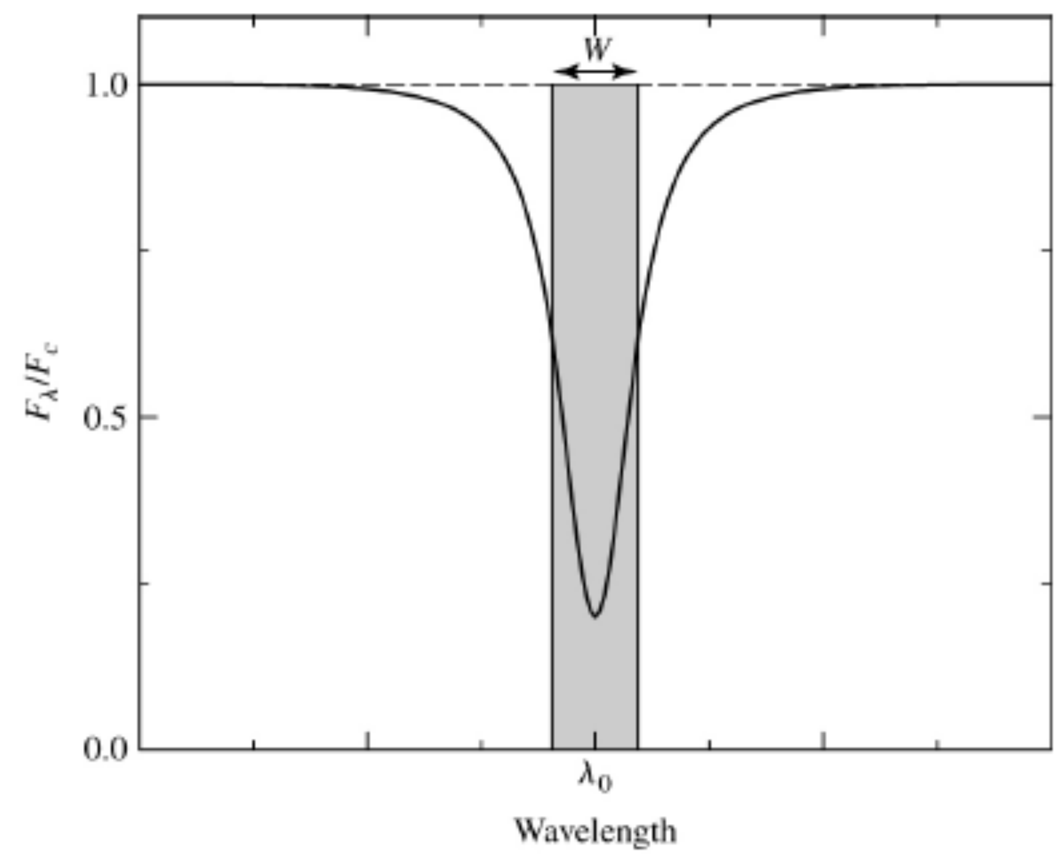
Spectral line profiles

Another way to characterize a spectral line is by the FWHM



Spectral line profiles

Spectral lines can be optically thin or optically thick. In the case shown here, the line is optically thin because the flux has not been completely blocked at any wavelength.



The opacity, κ_λ, is greatest in the line core. If the opacity increases, you don't see as deep into the star. Therefore, the line core forms in the higher (cooler) regions and the line wings form in deeper hotter regions.

At what optical depth does the continuum form?

$\tau_\lambda = 2/3$

Heisenberg's uncertainty principle, Eq. (5.20), relates ΔE, the uncertainty in the energy of an atomic orbital, to Δt, the time an electron occupies the orbital before making a downward transition:

$$\Delta E \approx \frac{\hbar}{\Delta t} = \frac{h}{2\pi \Delta t}.$$

When an electron makes a downward transition from an initial to a final orbital, the energy of the emitted photon is (Eq. 5.3)

$$E_{\text{photon}} = \frac{hc}{\lambda} = E_i - E_f.$$

The uncertainty in λ caused by an uncertainty in E_i is found by taking a derivative with respect to E_i (holding E_f constant):

$$-\frac{hc}{\lambda^2}\frac{d\lambda}{dE_i} = 1.$$

Writing $\Delta\lambda$ instead of $d\lambda$, ΔE_i instead of dE_i, and ignoring the minus sign yields the magnitude of the uncertainty,

$$\Delta\lambda = \frac{\lambda^2}{hc}\Delta E_i.$$

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Processes that broaden spectral lines

1. *Natural broadening*: **because of Heisenberg's uncertainty principle, an orbit cannot have a precise wavelength**

$$\Delta\lambda = \frac{\lambda^2}{2\pi c}\left(\frac{1}{\Delta t_i} + \frac{1}{\Delta t_f}\right)$$

Δt_i time in initial level

Δt_f time in final level

For $H\alpha$, λ =6563 A, this works out to be:

$$\boxed{\Delta\lambda \approx 4.57\times 10^{-14}\, m = 4.57\times 10^{-4}\, A}$$

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Processes that broaden spectral lines

2. *Doppler broadening*: atoms in a gas will have a M-B velocity distribution

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

Non-relativistic Doppler equation

$$v_{mp} = \sqrt{\frac{2kT}{m}}$$

Most probably velocity from MB distribution

$$\Delta\lambda \approx \frac{2\lambda}{c}\sqrt{\frac{2kT}{m}} = 0.427A$$

So, for H-atoms in the photosphere of the Sun (T=5777K), Doppler broadening is a factor of 1000 more important than natural broadening.

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Processes that broaden spectral lines

2. *Doppler broadening*: define the line width at half max

$$(\Delta\lambda)_{1/2} = \frac{2\lambda}{c}\sqrt{\frac{2kT\ln 2}{m}} = 0.427\times(0.83)A$$

Decreases exponentially moving away from line core because of fast falloff in MB distribution

Including larger scale turbulence:

$$(\Delta\lambda)_{1/2} = \frac{2\lambda}{c}\sqrt{\left(\frac{2kT}{m} + v_{turb}^2\right)\ln 2}$$

Turbulent term is important in atmospheres of giants and supergiants

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Processes that broaden spectral lines

3. *Pressure and collisional broadening*: **perturb atomic orbitals during collisions or by electric fields of large numbers of close encounters with ions.**

$$(\Delta\lambda)_{1/2} = \frac{\lambda^2}{\pi c}\frac{1}{\Delta t_o}$$

Similar format to natural broadening

$$\Delta t_o \approx \frac{l}{v} = \frac{1}{n\sigma}\frac{1}{\sqrt{2kT/m}}$$

Mean free path divided by v_{mp}

$$(\Delta\lambda)_{1/2} = \frac{\lambda^2}{c}\frac{n\sigma}{\pi}\sqrt{\frac{2kT}{m}}$$

Line width is proportional to the number density of atoms: this forms the basis for luminosity classes. The more luminous and tenuous giants have narrower lines than main sequence stars.

$$(\Delta\lambda)_{1/2} \approx 2.36\times 10^{-4}\,A$$

For solar type main sequence stars, comparable to natural broadening.

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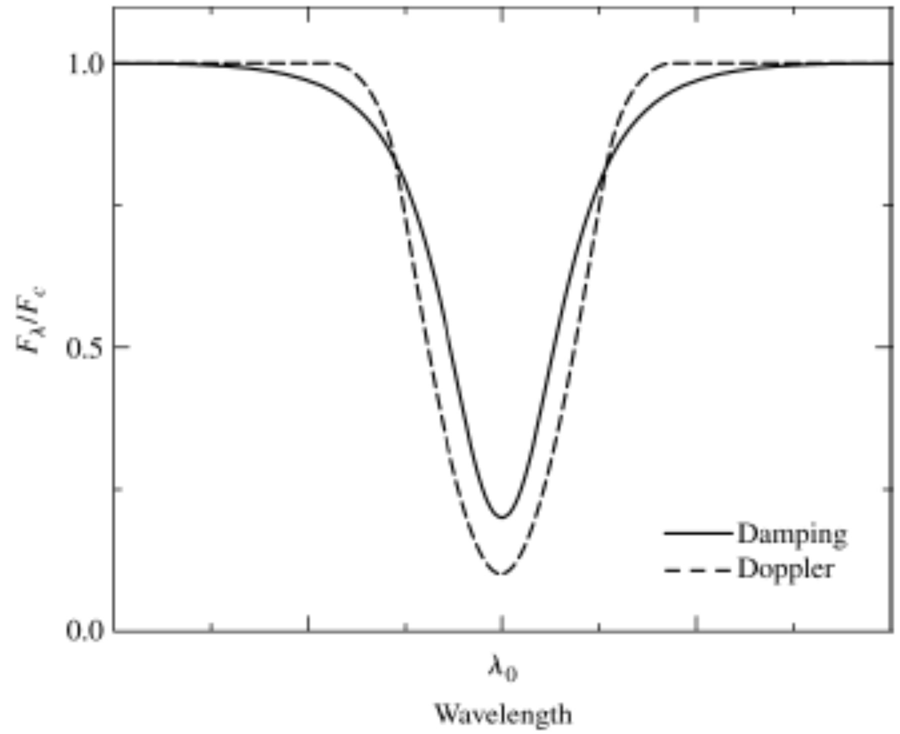
Processes that broaden spectral lines



A spectral line is broadened by composite of mechanisms. The total line profile is called a Voigt profile.

Doppler cores

Damping wings

Calculation of a line profile described by a "curve of growth" and depends on:

Temperature

Density

Opacity

Element abundance

Quantum mechanical probabilities (oscillator strengths, or f-values)

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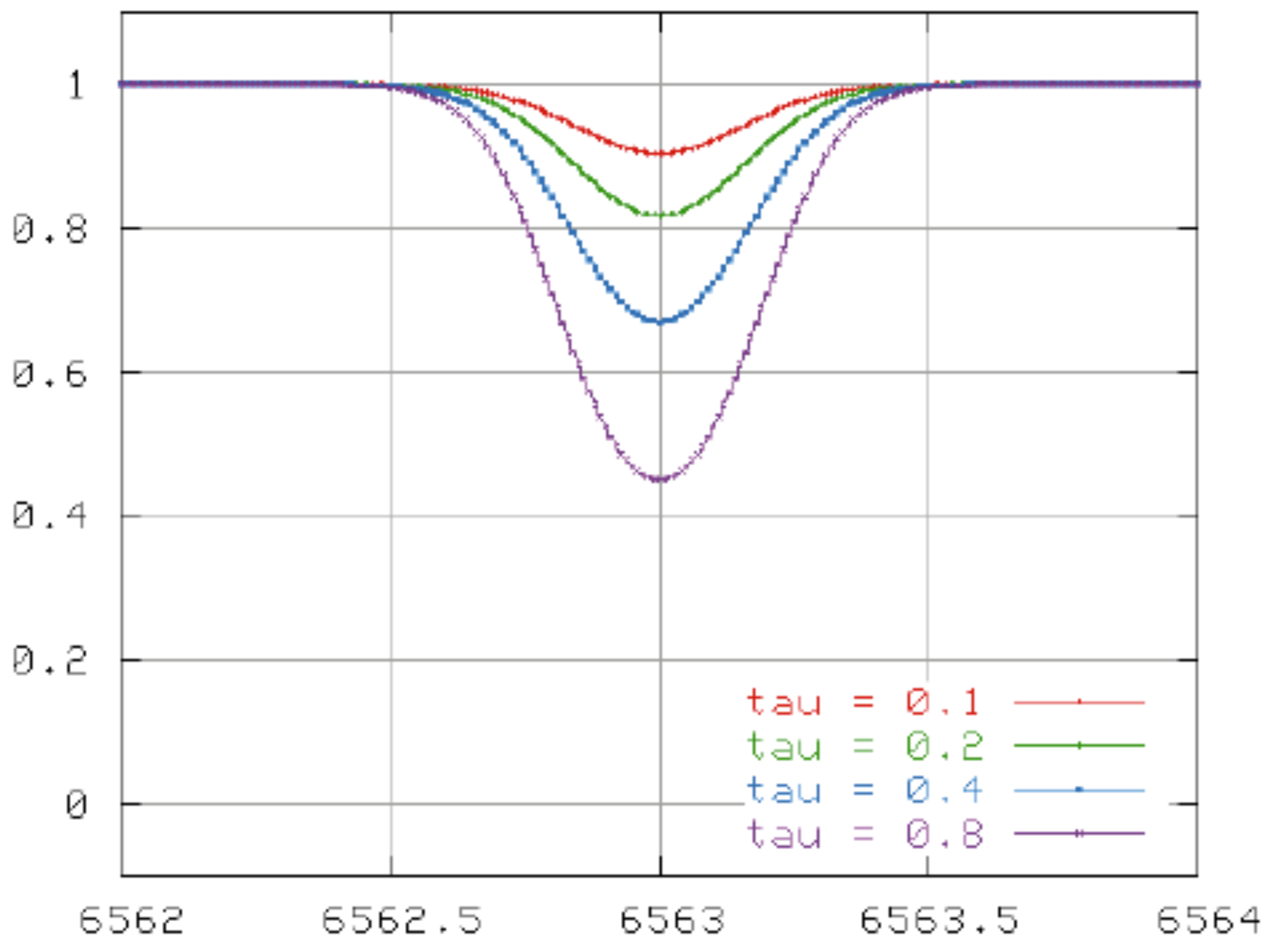

Case 1: thermal (Doppler) broadening

At τ =0.1, most photons escape before being scattered or absorbed.

Moving to greater optical depths, more photons are absorbed.

$$\tau = -\kappa\rho \; ds$$

So the optical depth could be increasing because the density of absorbers or scatterers is increasing or because the absorption coefficient is increasing.

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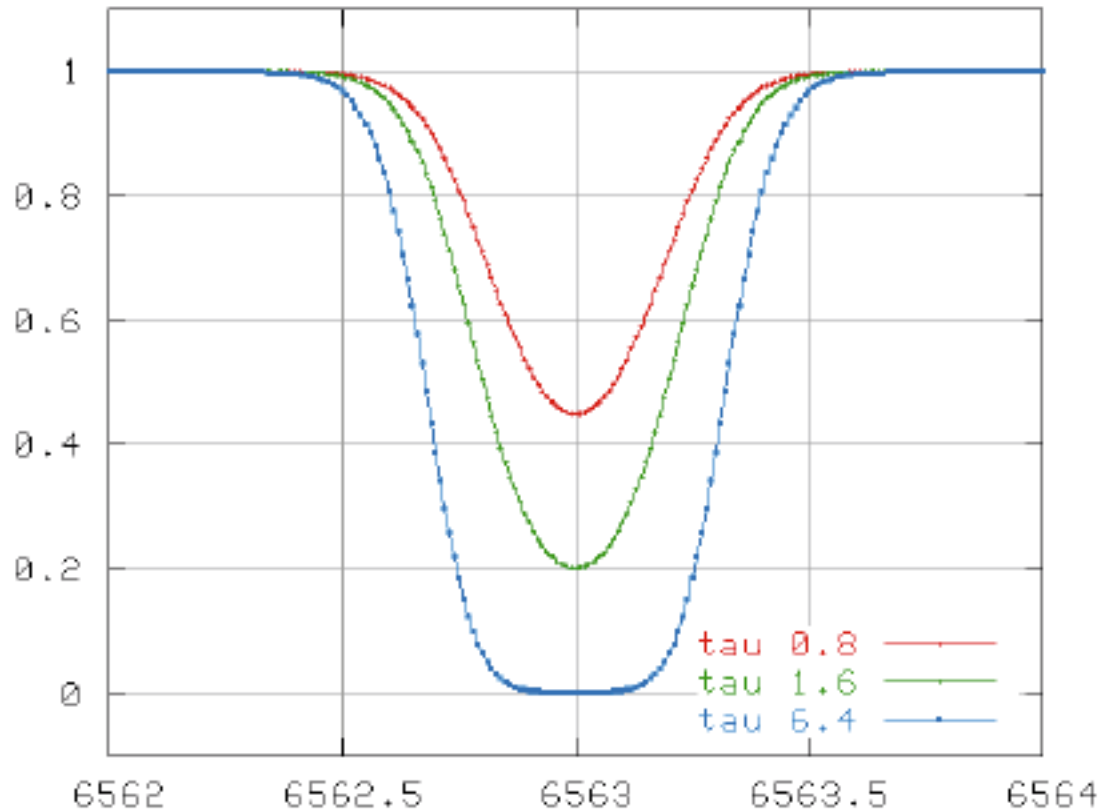

Case 1: thermal (Doppler) broadening

At first, doubling the optical depth doubles the line equivalent width. There is a linear growth between the abundance of atoms and the strength of the line.

But, as the line becomes optically thick, the equivalent width no longer increases linearly with the particle abundance.

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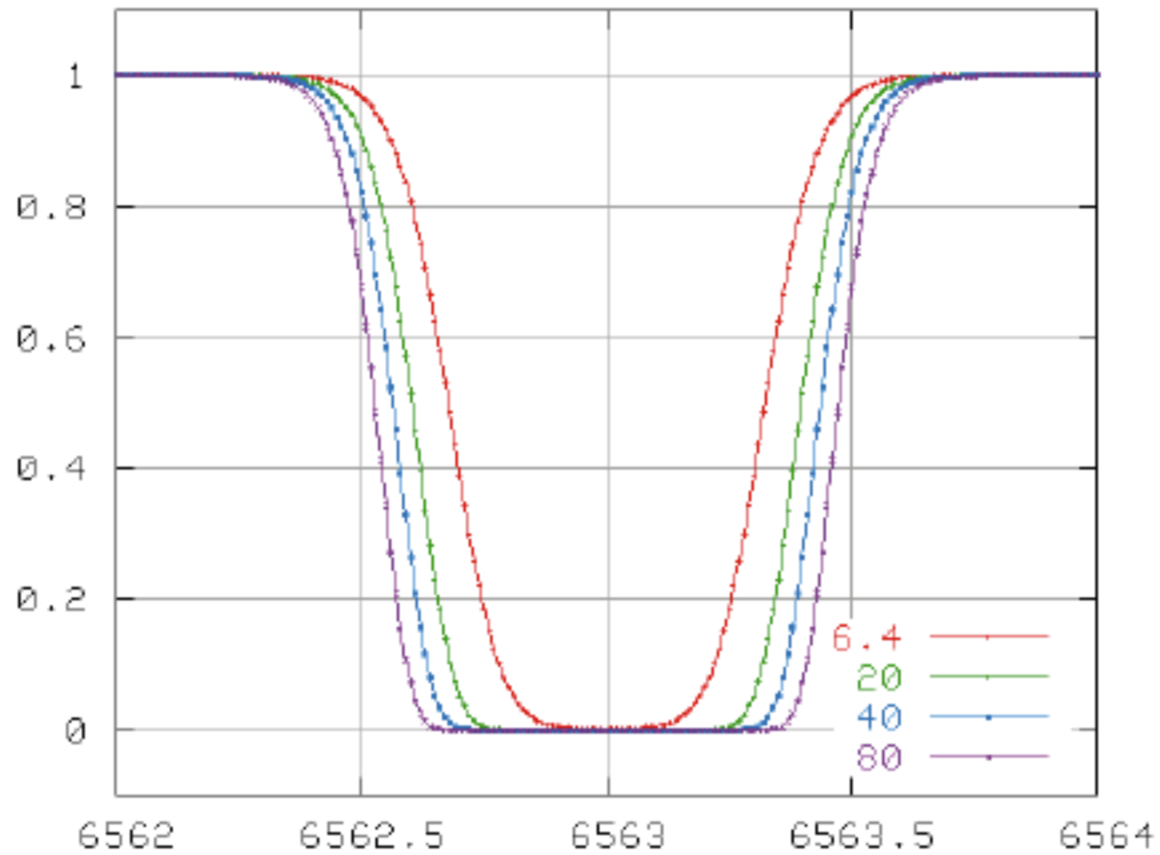
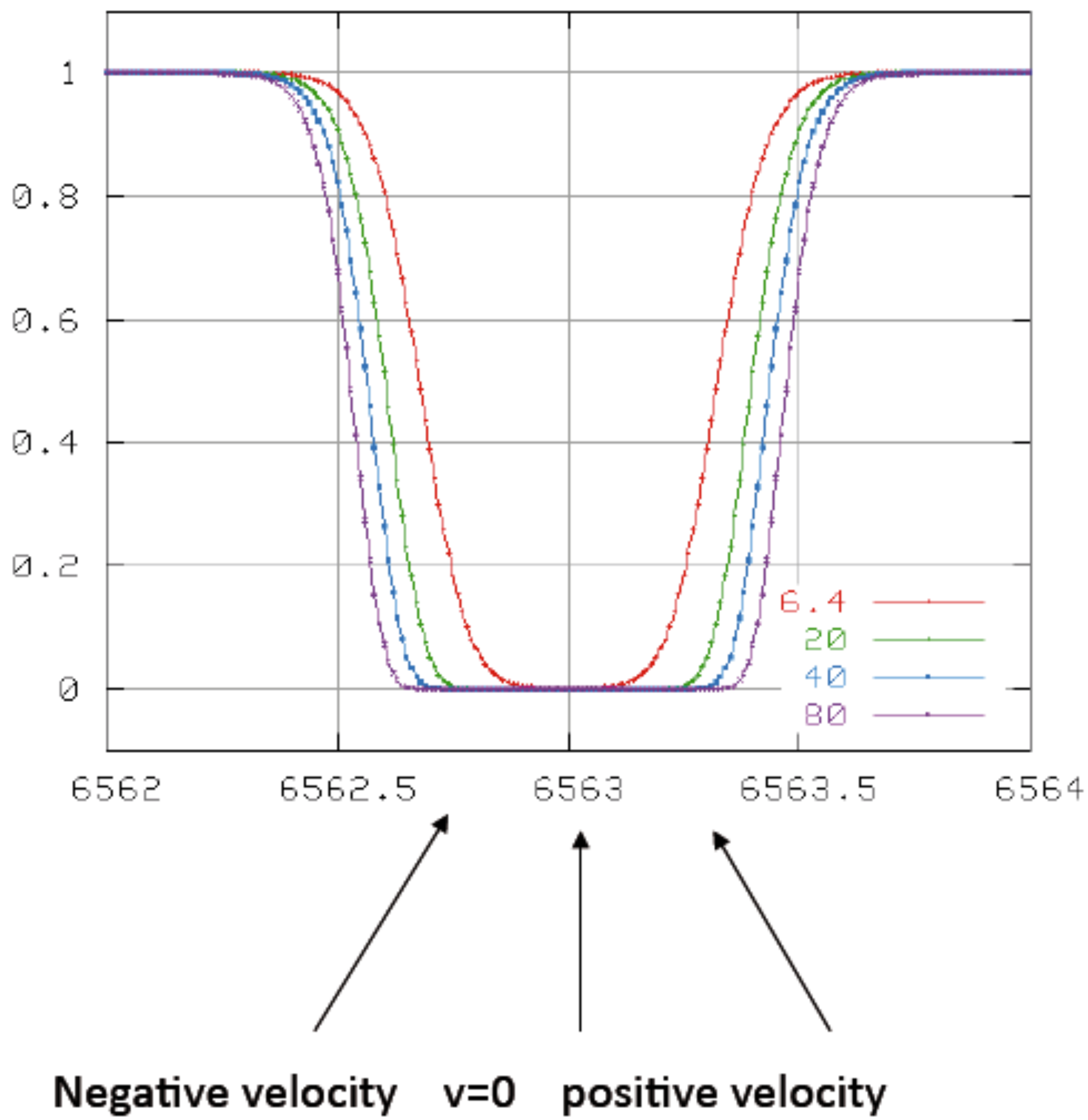


Case 1: thermal (Doppler) broadening

For lines that are very optically thick, the increase in equivalent width is very slow because the only way to increase the line width is for the wings to grow.

The wings of the line are from Doppler shifted atoms.

Chapter 9: Stellar Atmospheres



Case 1: thermal (Doppler) broadening

$$v = \frac{\Delta\lambda}{\lambda} c$$

At $\Delta\lambda$ = 0.5 A and λ=6563 A:

$$v = 22800 \ m/s$$

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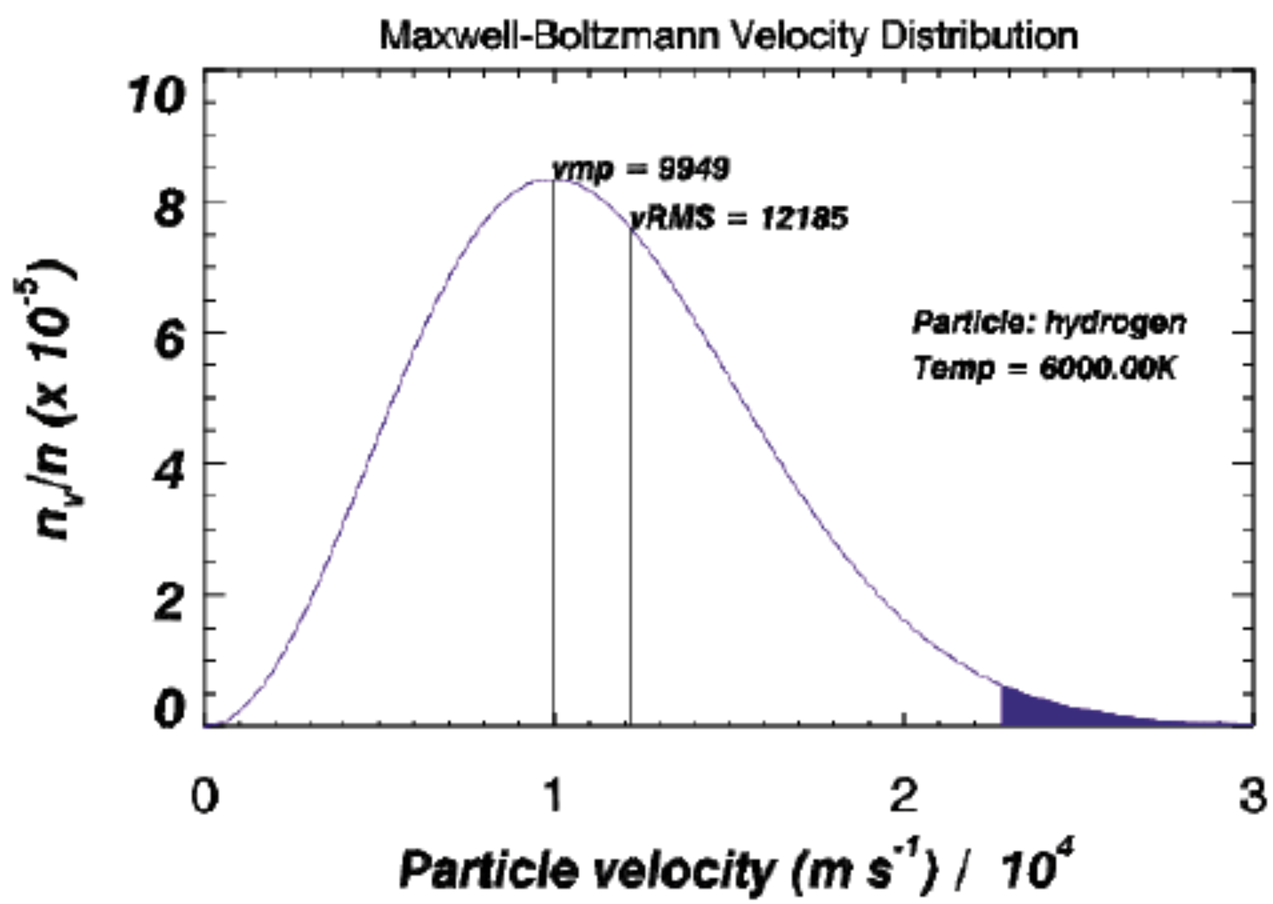
Spectral line profiles



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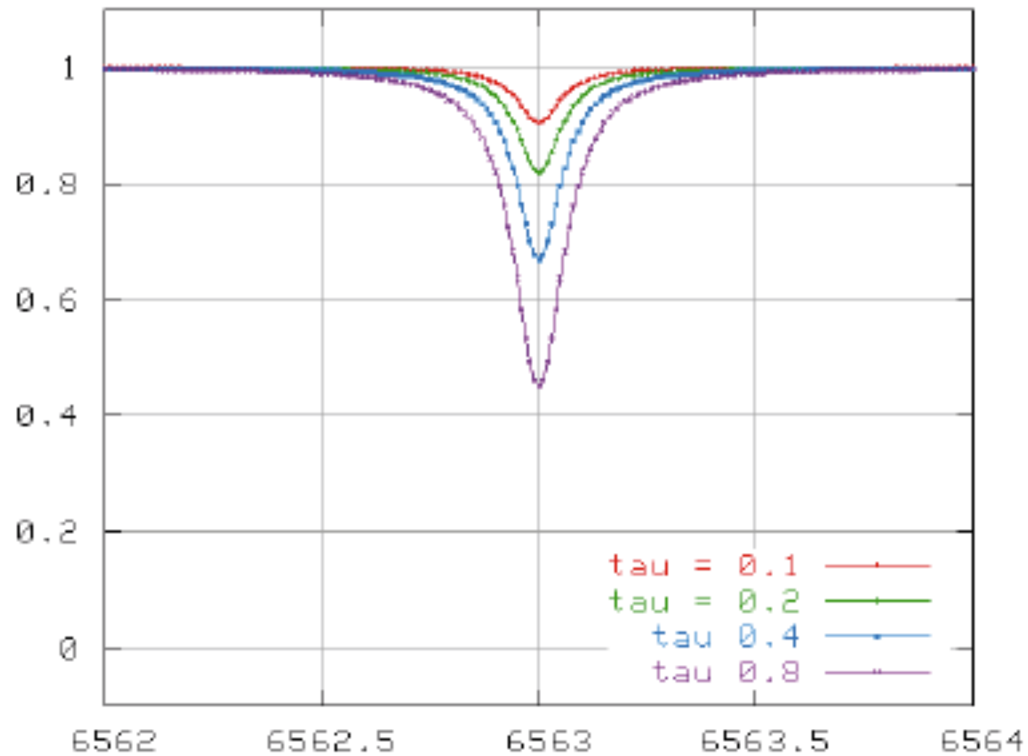

Case 2: pressure broadening

At large distances from the line center:

$$\tau \propto \frac{1}{(\lambda - \lambda_o)^2}$$

No exponential, so the line grows more slowly than for thermal broadening and the lines have wider wings.

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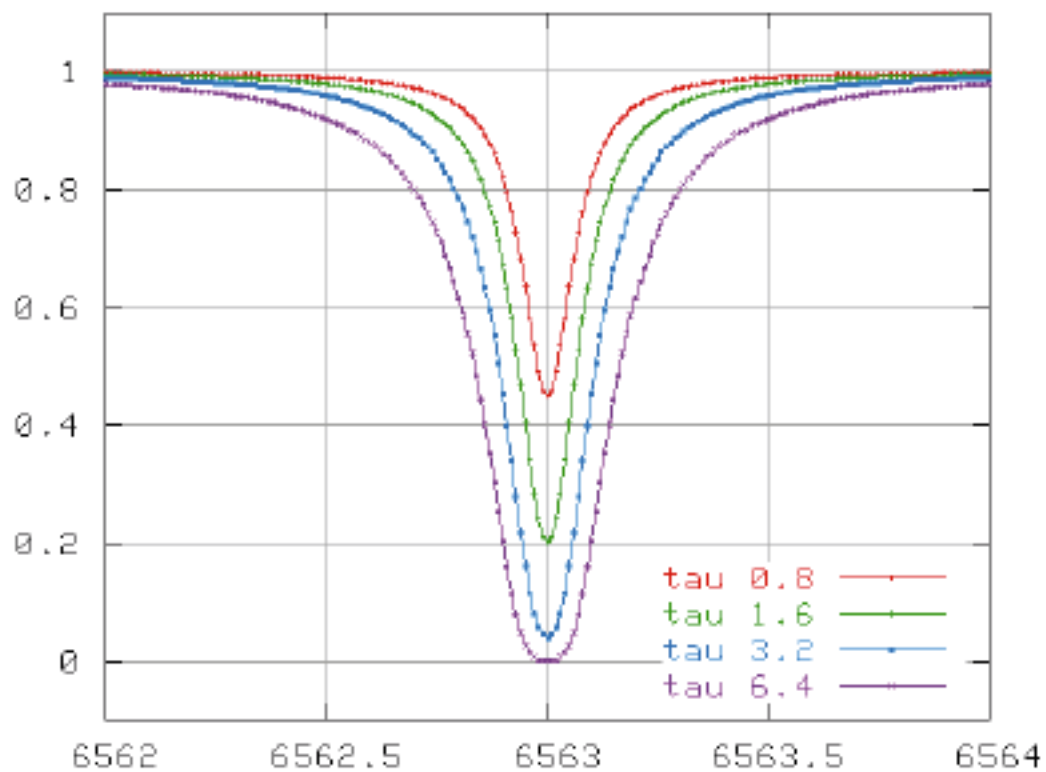

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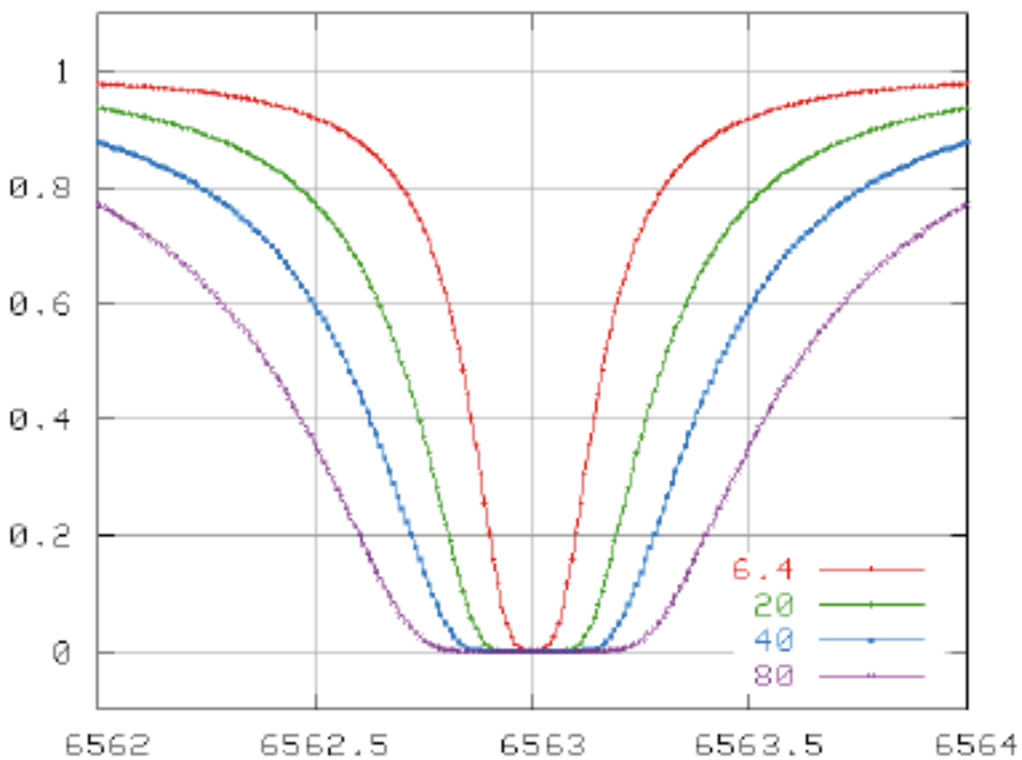

(Very large optical depths)

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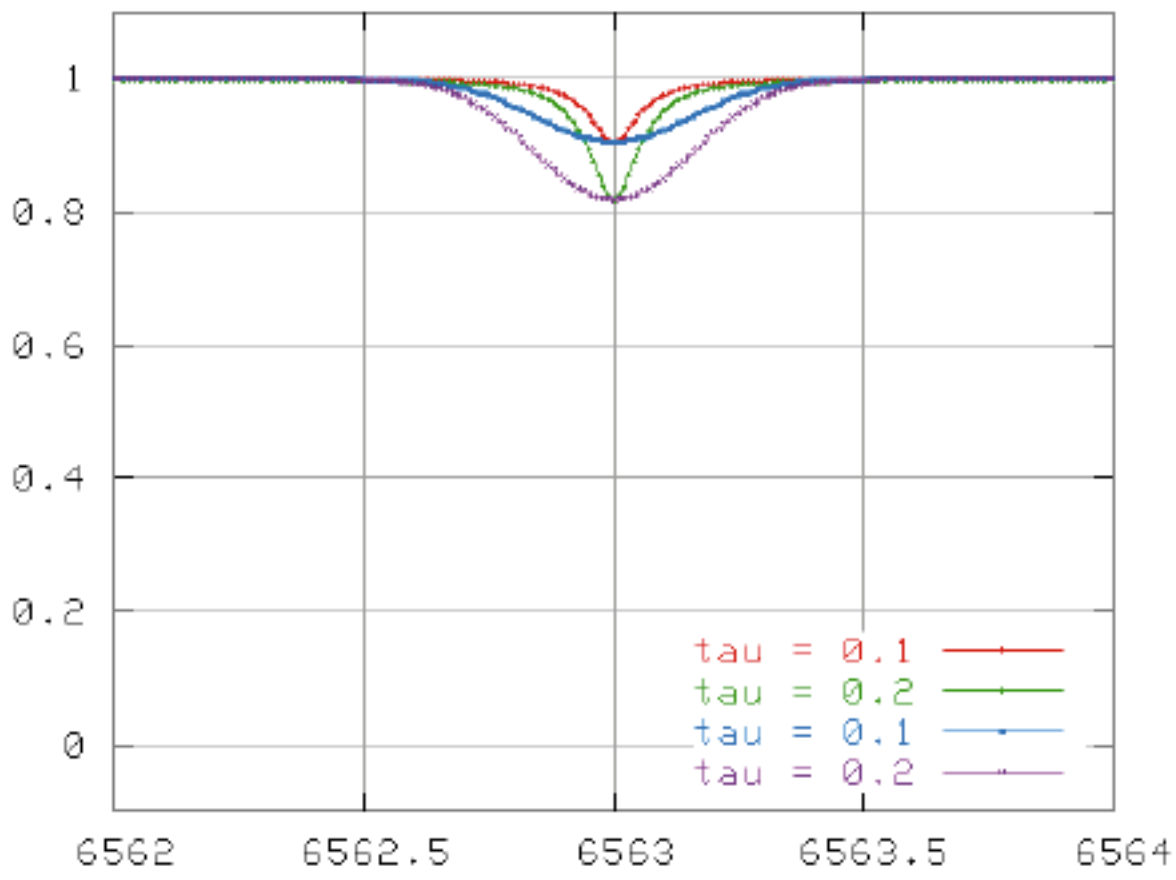

Compare thermally broadened lines (blue, purple) with collisionally broadened lines (green, red) at small optical depths.

Significantly different line profiles!

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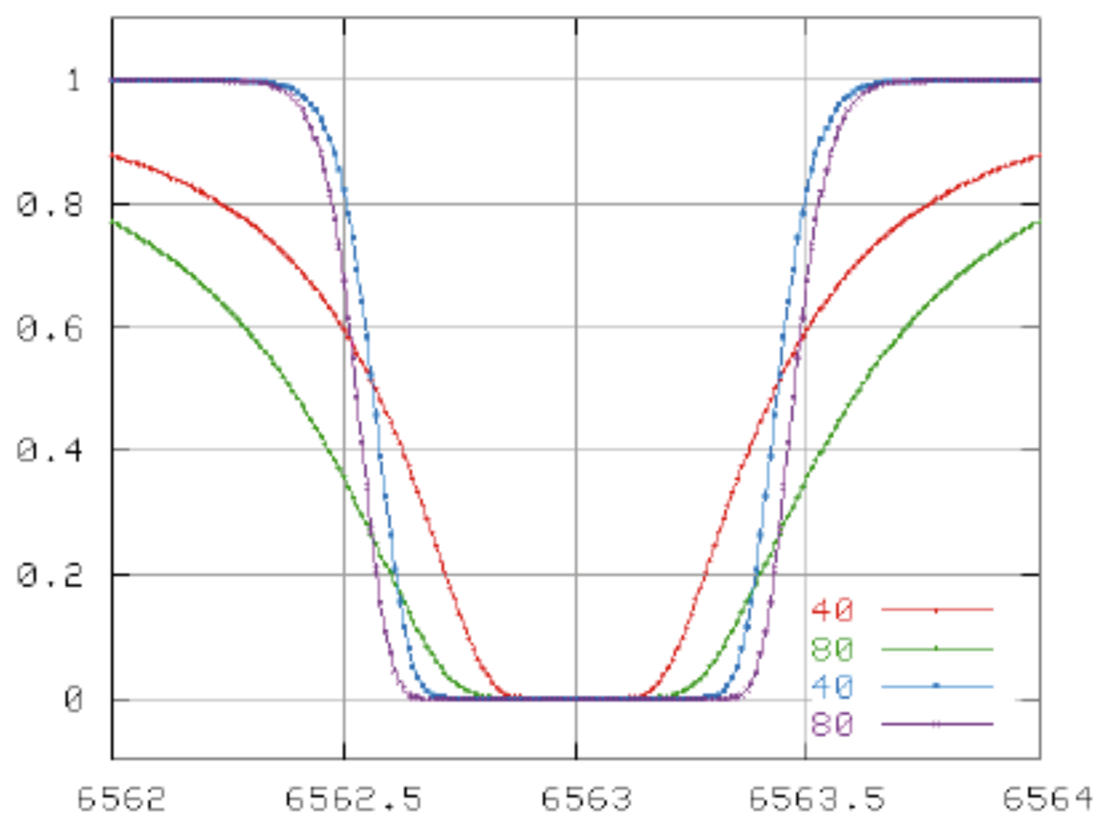
Spectral line profiles



Compare thermally and collisionally broadened lines at large optical depths.

EW of Doppler-broadened lines shows a non-linear response once saturated.

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Spectral line profiles

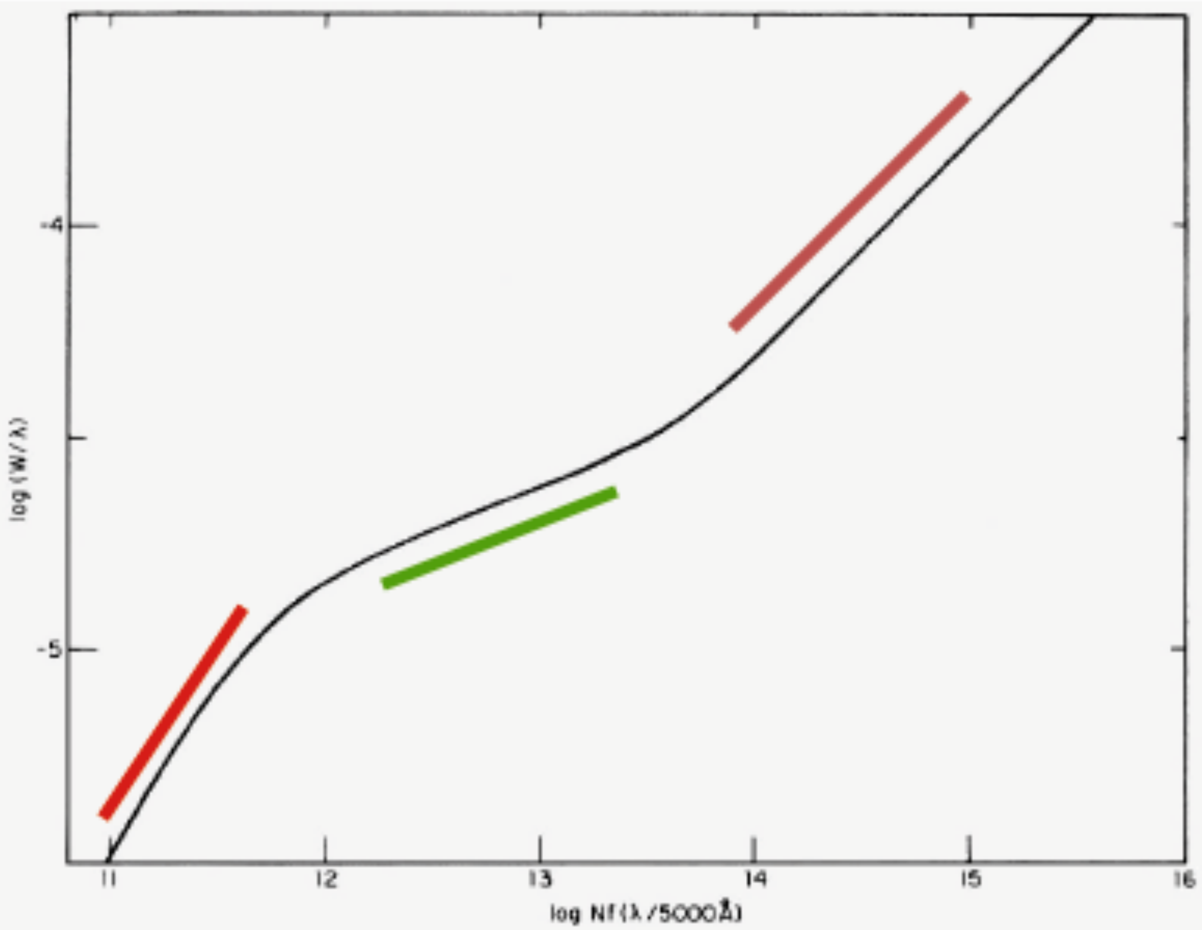

Figure 9.22 A general curve of growth for the Sun. (Figure from Aller, *Atoms, Stars, and Nebulae*, Revised Edition, Harvard University Press, Cambridge, MA, 1971.)

Curve of growth

$W \propto N$ when there are few absorbers, the optical depth is small

$W \propto \sqrt{\ln N}$ after the line saturates (when $\tau > 5$ or so), the Doppler wings barely change as the number of atoms grow

$W \propto \sqrt{N}$ the wings due to collisional broadening overwhelm the Doppler wings, even though the collisional term is much smaller near the line center