Problema 1: $S(T) = \frac{\alpha}{T}$

(a) Capacidad calorífica molar : $dQ = C\,dT$

Veamos que $dS = -\frac{\alpha}{T^2}\,dT \rightarrow \boxed{T\,dS = -\frac{\alpha}{T}\,dT}$

2º ley: $dQ = T\,dS \rightarrow \boxed{C = -\frac{\alpha}{T}}$

(b) $Q = \int_{T_1}^{T_2} C\,dT = \int_{T_1}^{T_2}\left(-\frac{\alpha}{T}\right)dT = -\alpha\int_{T_1}^{T_2}\frac{dT}{T} = -\alpha \ln\left(\frac{T_2}{T_1}\right)$

$\rightarrow \boxed{Q = \alpha \ln\left(\frac{T_1}{T_2}\right)}$

(c) $Q = \Delta U + W \rightarrow W = Q - \Delta U$

con $\Delta U = C_V \Delta T = C_V (T_2 - T_1)$, tenemos que

$$\boxed{W = \alpha \ln\left(\frac{T_1}{T_2}\right) + C_V (T_1 - T_2)}$$

Problema 2: $C_V = \left(\frac{\partial U}{\partial T}\right)_V$: C_V: calor específico molar a volumen constante

U: Energía interna por mol

V: Volumen molar,

$$\rightarrow \left(\frac{\partial C_V}{\partial V}\right)_T = \frac{\partial^2 U}{\partial V \partial T} = \frac{\partial}{\partial T}\left(\frac{\partial U}{\partial V}\right)_T \quad (*)$$

* Notamos que $dU = TdS - PdV = T\left(\frac{\partial S}{\partial T}\right)_V dT + \left\{T\left(\frac{\partial S}{\partial V}\right)_T - P\right\}dV$

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

$$\therefore \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - P.$$

Ahora, $\left(\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial T}\right)_V\right)_T = \left(\frac{\partial}{\partial T}\left(\frac{\partial U}{\partial V}\right)_T\right)_V$

$$\frac{\partial}{\partial V}\left(T\left(\frac{\partial S}{\partial T}\right)_V\right) = \frac{\partial}{\partial T}\left(T\left(\frac{\partial S}{\partial V}\right)_T - P\right)$$

$$\Rightarrow \boxed{\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V}$$

$$\rightarrow \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

$$\therefore \left(\frac{\partial C_V}{\partial V}\right)_T = \frac{\partial}{\partial T}\left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right] = \cancel{\left(\frac{\partial P}{\partial T}\right)_V} + \left(\frac{\partial^2 P}{\partial T^2}\right)_V - \cancel{\left(\frac{\partial P}{\partial T}\right)_V}$$

$$\Rightarrow \left(\frac{\partial C_V}{\partial V}\right)_T = \left(\frac{\partial^2 P}{\partial T^2}\right)_V$$

Para el gas de van der Waals: $P = \frac{RT}{V-b} - \frac{a}{V^2}$

$$\rightarrow \left(\frac{\partial^2 P}{\partial T^2}\right) = 0 \Rightarrow \boxed{\frac{\partial C_V}{\partial V} = 0} \Rightarrow$$ C_V no depende de V.

Energía interna: Teníamos

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV = C_V\, dT + \left\{ T \underbrace{\left(\frac{\partial P}{\partial T}\right)_V}_{\frac{R}{V-b}} - P \right\} dV$$

$$dU = C_V\, dT + \frac{a}{V^2}\, dV$$

$$\Rightarrow \boxed{U - U_0 = \int_{T_0}^{T} C_V\, dT - \frac{a}{V} + \frac{a}{V_0}} \qquad \begin{array}{l} U_0 : \text{cte.} \\ V_0 : \text{cte.} \end{array}$$

Entropía: Tenemos que

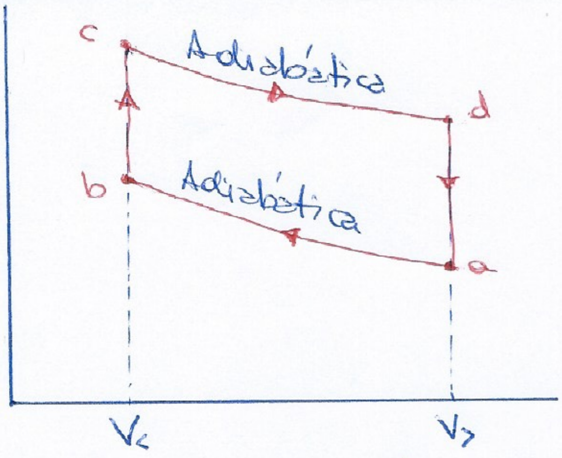
$$T\, dS = dU + P\, dV$$

$$= C_V\, dT + \cancel{\frac{a}{V^2} dV} + \frac{RT}{V-b} dV - \cancel{\frac{a}{V^2} dV}$$

$$\Rightarrow dS = \frac{C_V}{T} dT + \frac{R}{V-b} dV$$

$$\Rightarrow \boxed{S - S_0 = \int_{T_0}^{T} \frac{C_V}{T} dT + R \ln\left(\frac{V-b}{V_0-b}\right)}$$

Problema 3:



- Trabajo neto entregado por el sistema $W = W_2 + W_1$

W_2: trabajo realizado por el gas ; W_1: trabajo realizado sobre el gas.

$$Q_{ab} = 0 = \Delta U_{ab} + W_1 \Rightarrow W_1 = -\Delta U_{ab} = -C_v(T_b - T_a)$$

$$Q_{cd} = 0 = \Delta U_{cd} + W_2 \Rightarrow W_2 = -\Delta U_{cd} = -C_v(T_d - T_c) = C_v(T_c - T_d)$$

$$\Rightarrow \boxed{W = C_v(T_c - T_d) - C_v(T_b - T_a) = C_v(T_a + T_c - T_b - T_d)}$$

Por otro lado, el gas recibe calor en el proceso bc

$$\boxed{Q = C_v(T_c - T_b)}$$

Así, la eficiencia es $\boxed{\eta = \frac{W}{Q} = \frac{T_a + T_c - T_b - T_d}{T_c - T_b} = 1 - \frac{T_d - T_a}{T_c - T_b}}$

En procesos adiabáticos: $TV^{\gamma-1} = cte$

$$\therefore \; T_a V_>^{\gamma-1} = T_b V_<^{\gamma-1} \quad \wedge \quad T_d V_>^{\gamma-1} = T_c V_<^{\gamma-1}$$

$$\Rightarrow \boxed{\frac{T_a}{T_b} = \frac{T_d}{T_c} = \left(\frac{V_<}{V_>}\right)^{\gamma-1} = \frac{T_d - T_a}{T_c - T_b}}$$

$\Rightarrow \eta = 1 - \left(\frac{V_<}{V_>}\right)^{\gamma-1}$, $V_> = 10 V_<$; Para nitrogeno: $\gamma = \frac{7}{5}$

$$\eta = 1 - 10^{-2/5} = 1 - 0{,}398 -$$

$$\boxed{\eta = 0{,}602}$$

Problema 4: Combinando el primer y segundo principio:

$$T\,dS = dU + P\,dV$$

$$\rightarrow dS = \frac{1}{T}dU + \frac{P}{T}dV$$

$$dS = \left[\frac{1}{T}\left(\frac{\partial U}{\partial V}\right)_T + \frac{P}{T}\right]dV + \frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_V dT$$

Ya que dS es diferencial exacta

$$\boxed{\left\{\frac{\partial}{\partial T}\left[\frac{1}{T}\left(\frac{\partial U}{\partial V}\right)_T + \frac{P}{T}\right]\right\}_V = \left\{\frac{\partial}{\partial V}\left[\frac{1}{T}\left(\frac{\partial U}{\partial T}\right)\right]\right\}_T}$$

$$* \left(\frac{\partial U}{\partial V}\right)_T = \frac{BT^m}{V} \quad ; \quad * \frac{P}{T} = \frac{AT^2}{V}$$

$$\rightarrow \boxed{\frac{\partial}{\partial T}\left[\frac{1}{T}\left(\frac{\partial U}{\partial V}\right)_T + \frac{P}{T}\right] = \frac{\partial}{\partial T}\left[\frac{BT^{m-1}}{V} + \frac{AT^2}{V}\right] = \frac{(m-1)BT^{m-2} + 2AT}{V}}$$

$$* \left(\frac{\partial U}{\partial T}\right)_V = mBT^{m-1}\,\mathrm{Ln}\left(\frac{V}{V_0}\right) + f'(T) \rightarrow \frac{1}{T}\left(\frac{\partial U}{\partial T}\right) = mBT^{m-2}\,\mathrm{Ln}\left(\frac{V}{V_0}\right) + \frac{f'}{T}$$

$$\boxed{\frac{\partial}{\partial V}\left[\frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_V\right]_T = \frac{mBT^{m-2}}{V}}$$

Luego,

$$(m-1)BT^{m-2} + 2AT = mBT^{m-2}$$

$$2AT = BT^{m-2}$$

$$\Rightarrow \boxed{B = 2A} \wedge \boxed{m = 3}$$