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Formulario Mecánica Intermedia

Licenciatura en Física.

Lacio California[1]

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ABSTRACT

Unidad 1: Coordenadas, Rotaciones, Análisis Vectorial, Análisis dimensional

Unidad 2: Mecánica clásica Newtoniana Leyes de Newton, Ecuaciones de movimiento, Teoremas de conservación, Energía y trabajo, Potenciales, Oscilaciones, Espacio de configuración y fases, Introducción al Lagrangiano

Unidad 3: Cálculo de variaciones Funcionales, Ecuación de Euler, Problemas clásicos del cálculo de variaciones.

1. Identidades diferenciales vectoriales

$\nabla = -\nabla'$
$\nabla \cdot \nabla\varphi = \nabla^2\varphi$
$\nabla \cdot \nabla \times \mathbf{F} = 0$
$\nabla \times \nabla\varphi = 0$
$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$
$\nabla(\varphi\psi) = (\nabla\varphi)\psi + \varphi\nabla\psi$
$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + \mathbf{F} \times (\nabla \times \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{G} \times (\nabla \times \mathbf{F})$
$\nabla \cdot (\varphi\mathbf{F}) = (\nabla\varphi) \cdot \mathbf{F} + \varphi\nabla \cdot \mathbf{F}$
$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = (\nabla \times \mathbf{F}) \cdot \mathbf{G} - (\nabla \times \mathbf{G}) \cdot \mathbf{F}$
$\nabla \times (\varphi\mathbf{F}) = (\nabla\varphi) \times \mathbf{F} + \varphi\nabla \times \mathbf{F}$
$\nabla \times (\mathbf{F} \times \mathbf{G}) = (\nabla \cdot \mathbf{G})\mathbf{F} - (\nabla \cdot \mathbf{F})\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

2. Teoremas Integrales

$\int_S \mathbf{n} \times \nabla\varphi \, da = \oint_C \varphi \, dl$
$\int_V \nabla\varphi \, dv = \oint_S \varphi\mathbf{n} \, da$
$\int_V \nabla \times \mathbf{F} \, dv = \oint_S \mathbf{n} \times \mathbf{F} \, da$
$\int_V (\nabla \cdot \mathbf{G} + \mathbf{G} \cdot \nabla)\mathbf{F} \, dv = \oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n}) \, da$
$\int_a^b \nabla\varphi \cdot d\mathbf{l} = \varphi\Big\vert_a^b$
$\int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, da = \oint_C \mathbf{F} \cdot d\mathbf{l}$
$\int_V \nabla \cdot \mathbf{F} \, dv = \oint_S \mathbf{F} \cdot \mathbf{n} \, da$

3. Identidades Trigonométricas

Básicas y Suma/Resta	Ángulo Doble y Medio	Suma a Producto; Producto a Suma
$\sin\theta = \frac{1}{\csc\theta}$	$\sin(2\theta) = 2\sin\theta\cos\theta$	$\sin\theta + \sin\beta = 2\sin\left(\frac{\theta+\beta}{2}\right)\cos\left(\frac{\theta-\beta}{2}\right)$
$\cos\theta = \frac{1}{\sec\theta}$	$\cos(2\theta) = \cos^2\theta - \sin^2\theta$	$\sin\theta - \sin\beta = 2\sin\left(\frac{\theta-\beta}{2}\right)\cos\left(\frac{\theta+\beta}{2}\right)$
$\csc\theta = \frac{1}{\sin\theta}$	$\cos(2\theta) = 1 - 2\sin^2\theta$	$\cos\theta + \cos\beta = 2\cos\left(\frac{\theta+\beta}{2}\right)\cos\left(\frac{\theta-\beta}{2}\right)$
$\sec\theta = \frac{1}{\cos\theta}$	$\cos(2\theta) = 2\cos^2\theta - 1$	$\cos\theta - \cos\beta = -2\sin\left(\frac{\theta+\beta}{2}\right)\sin\left(\frac{\theta-\beta}{2}\right)$
$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$	$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$	$\sin\theta\sin\beta = \frac{1}{2}[\cos(\theta-\beta) - \cos(\theta+\beta)]$
$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1}{\cot\theta}$	$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$	$\cos\theta\cos\beta = \frac{1}{2}[\cos(\theta-\beta) + \cos(\theta+\beta)]$
$\sin^2\theta + \cos^2\theta = 1$	$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$	$\sin\theta\cos\beta = \frac{1}{2}[\sin(\theta+\beta) + \sin(\theta-\beta)]$
$1 + \tan^2\theta = \sec^2\theta$	$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$	
$1 + \cot^2\theta = \csc^2\theta$	$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$	**Suplemento, complemento**
$\sin(\theta\pm\beta) = \sin\theta\cos\beta \pm \sin\beta\cos\theta$	$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$	$\sin(\pi\pm\theta) = \mp\sin\theta$
$\cos(\theta\pm\beta) = \cos\theta\cos\beta \mp \sin\theta\sin\beta$	**Par, impar**	$\cos(\pi\pm\theta) = -\cos\theta$
$\tan(\theta\pm\beta) = \frac{\tan\theta\pm\tan\beta}{1\mp\tan\theta\tan\beta}$	$\sin(-\theta) = -\sin\theta$	$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$
$\sin(ArcTan(y)) = \frac{y}{\sqrt{1+y^2}}$	$\cos(-\theta) = \cos\theta$	$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$
$\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$	$\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$	$\tan(\theta+\pi) = \tan(\theta)$
$\arccos(x) = \frac{\pi}{2} - \arcsin x$	$\sec(\arctan(\frac{L}{x})) = \frac{\sqrt{L^2+x^2}}{x}$	

4. Análisis Dimensional

Cantidad	Dimensión	MKS
Ángulo	adimensional	radián
Área	L^2	m^2
Volumen	L^3	m^3
Frecuencia	T^{-1}	m/s
Velocidad	LT^{-1}	m/s
Aceleración	LT^{-2}	m/s
Velocidad ángular	T^{-1}	rad/s
Aceleración ángular	T^{-2}	rad/s^2
Densidad	ML^{-3}	kg/m^3
Momentum	MLT^{-1}	$kg\ m/s$
Fuerza	MLT^{-2}	$kg\ m/s^2$
Trabajo, Energía	ML^2T^{-2}	$kg\ m^2/s^2 = J$
Torque	ML^2T^{-2}	$kg\ m^2/s^2$
Potencia	ML^2T^{-3}	$kg\ m^2/s^3 = W$
Presión	$ML^{-1}T^{-2}$	$kg\ m^{-1}/s^{-2} = Pa$
Carga eléctrica	IT	$Coulomb = C$

Table 1. Tabla de dimensiones.

4.1. Sistema de unidades naturales de Planck.

$C = 3x10^8\ [m/s] \ \Rightarrow\ LT^{-1}$ **Velocidad de la luz.**

$G = 6.67x10^{-11}\ [\frac{m^3}{kg\ s^2}] \ \Rightarrow\ L^3M^{-1}T^{-2}$ **Gravitación Universal**

$\hbar = 1.055x10^{-34}\ [J \cdot s] \ \Rightarrow\ L^2MT^{-1}$ **Cte. de Planck**

$K_B = 1.38x10^{-23}\ [J/K] \ \Rightarrow\ L^2MT^{-2}\theta^{-1}$ **Cte. de Boltzmann**

Para determinar las dimensiones de cualquier unidades:

$$C^{\alpha}G^{\beta}\hbar^{\gamma}K_B^{\delta}$$

$$1 \text{ uma} = 1.6605e - 27[kg]$$

velocidad angular y mov. angular:

$$\omega = \frac{L}{mr^2}$$

5. Vectores

5.1. Vectores unitarios

$$\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{\imath} \cdot \hat{\jmath} = \hat{\jmath} \cdot \hat{k} = \hat{k} \cdot \hat{\imath} = 0$$

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

$$\hat{\imath} \times \hat{\jmath} = \hat{k} \quad \hat{\jmath} \times \hat{k} = \hat{\imath} \quad \hat{k} \times \hat{\imath} = \hat{\jmath}$$

5.2. Base Ortonormal

Se cumple:

- **Ortogonalidad**:

  $\mathbf{e}_i \cdot \mathbf{e}_j = 0 \quad \text{si } i \neq j$

- **Normalización**:

  $|\mathbf{e}_i| = 1 \quad \Rightarrow \quad \mathbf{e}_i \cdot \mathbf{e}_i = 1$

En $\mathbb{R}^3$, base ortonormal es la base canónica:

$\mathbf{e}_1 = (1, 0, 0), \quad \mathbf{e}_2 = (0, 1, 0), \quad \mathbf{e}_3 = (0, 0, 1)$

5.3. Posición, desplazamiento y separación de vectores

$$\boldsymbol{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\hat{r} = \frac{\boldsymbol{r}}{r} = \frac{x\hat{\imath} + y\hat{\jmath} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$$

separación

$$\boldsymbol{r} = (x - x')\,\hat{\imath} + (y - y')\,\hat{\jmath} + (z - z')\,\hat{k}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\hat{r} = \frac{(x - x')\,\hat{\imath} + (y - y')\,\hat{\jmath} + (z - z')\,\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$$

Longitud:

$$|\boldsymbol{A}| = A = \left(A_1^2 + +A_2^2 + ...\right)^{1/2}$$

5.4. Producto punto

$$\boldsymbol{A} \cdot \boldsymbol{B} = |\boldsymbol{A}|\,|\boldsymbol{B}|\,cos\,\theta$$

$$\boldsymbol{A} \cdot \boldsymbol{B} = A_1B_1 + A_2B_2 + ...$$

$$|\boldsymbol{A}|^2 = \boldsymbol{A} \cdot \boldsymbol{A}$$

$$\boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{B} \cdot \boldsymbol{A}\text{Conmutativo}$$

$$(\boldsymbol{A} + \boldsymbol{B}) \cdot \boldsymbol{C} = \boldsymbol{A} \cdot \boldsymbol{C} + \boldsymbol{B} \cdot \boldsymbol{C}\text{Distributiva}$$

5.4.1. Proyección de vectores

$proj_{\boldsymbol{b}}\boldsymbol{a}$ Operador de proyección de $\boldsymbol{a}$ sobre una recta paralela $\boldsymbol{b}$

Proyección escalar:

$$a_1 = ||\boldsymbol{a}||cos\,\theta\ = \boldsymbol{a} \cdot \hat{b}\,, \qquad \theta{:}(\boldsymbol{a}, \boldsymbol{b})$$

Proyección vectorial en términos de vectores de entrada:

$$\boldsymbol{a_1} = (\boldsymbol{a} \cdot \hat{b})\hat{b} = \frac{\boldsymbol{a} \cdot \boldsymbol{b}}{||\boldsymbol{b}||}\frac{\boldsymbol{b}}{||\boldsymbol{b}||} = \frac{\boldsymbol{a} \cdot \boldsymbol{b}}{||\boldsymbol{b}||^2}\boldsymbol{b}$$

$$\boldsymbol{a_1} = a_1\hat{b} = (||\boldsymbol{a}||cos\,\theta\,)\hat{b} = (\boldsymbol{a} \cdot \hat{b})\hat{b}$$

5.4.2. Representación en filas y columnas

$$\boldsymbol{A} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} \rightarrow \boldsymbol{a}^T = (A_1, A_2, A_3)$$

$$\boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{A} \cdot \boldsymbol{b}^T = (A_1, A_2, A_3) \cdot \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = A_1 B_1 + A_2 B_2 + A_3 B_3$$

5.4.3. Ortogonalidad

$$\boldsymbol{A} \cdot \boldsymbol{B} = 0 \quad \perp$$

vectores de una base cartesiana son ortogonales

$$\hat{e}_i \cdot \hat{e}_j = \delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

Proyección y cosenos directores (cos de los ángulos entre el $|v\rangle$ con los ejes):

$$A_x = Acos\,\alpha \quad A_y = Acos\,\beta \quad A_z = Acos\,\gamma$$

5.5. Producto Cruz

$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = (ABsin\,\theta\,)\,\hat{e}_c$$

$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{e}_x \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} + \hat{e}_y \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{e}_z \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$= \hat{e}_x \left(A_y B_z - A_z B_y\right) + \hat{e}_y \left(A_x B_z - A_z B_x\right) + \hat{e}_z \left(A_x B_y - A_y B_x\right)$$

$$C_i = \sum_{ij} \epsilon_{ijk} A_j B_k$$

$$\hat{e}_i \times \hat{e}_j = \sum_k \epsilon_{ijk} \hat{e}_k \quad \epsilon : Levi - Civita$$

$$\boldsymbol{A} \times \boldsymbol{B} = -\boldsymbol{B} \times \boldsymbol{A}$$

$$\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{C}) = \boldsymbol{A} \times \boldsymbol{B} + \boldsymbol{A} \times \boldsymbol{C}$$

$$\boldsymbol{A} \cdot (\boldsymbol{B} \times \boldsymbol{C}) = (\boldsymbol{A} \times \boldsymbol{B}) \cdot \boldsymbol{C} = (\boldsymbol{C} \times \boldsymbol{A}) \cdot \boldsymbol{B} = -\boldsymbol{A} \cdot (\boldsymbol{C} \times \boldsymbol{B})$$

$$\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = (\boldsymbol{A} \cdot \boldsymbol{C})\,\boldsymbol{B} - (\boldsymbol{A} \cdot \boldsymbol{B})\,\boldsymbol{C}$$

$$(\boldsymbol{u} \times \boldsymbol{v}) \times (\boldsymbol{w} \times \boldsymbol{x}) = [\boldsymbol{u} \cdot (\boldsymbol{v} \times \boldsymbol{x})]\boldsymbol{w} - [\boldsymbol{u} \cdot (\boldsymbol{v} \times \boldsymbol{w})]\boldsymbol{x}$$

5.5.1. Volumen paralelepípedo

altura: $h = \boldsymbol{A} \cdot \hat{n}$

area: $|\boldsymbol{B} \times \boldsymbol{C}| = BCsin\phi$

Volumen: $|\boldsymbol{B} \times \boldsymbol{C}|Acos\theta = |\boldsymbol{A} \cdot \boldsymbol{B} \times \boldsymbol{C}|$

6. Levi Civita y Delta de Kronecker

6.1. Delta de Kronecker

$$\delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

6.1.1. Linear Algebra

Identity matrix $n \times n$

$$I_{ij} = \delta_{ij}$$

Producto punto

$$\boldsymbol{A} \cdot \boldsymbol{B} = \sum_{i,j=1}^{n} A_i \delta_{ij} B_j = \sum_{i=1}^{n} A_i B_i$$

6.1.2. Propiedades

$$\sum_j \delta_{ij} a_j = a_i, \quad \sum_i \delta_{ij} a_i = a_j, \quad \sum_k \delta_{ik} \delta_{kj} = \delta_{ij}$$

$$\delta_{ij} = \delta_{ji}, \quad \text{Simétrico}$$

$$\delta_{ij} \delta jk = \delta_{ik}$$

6.1.3. Serie geométrica

$$\delta_{nm} = lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} e^{2\pi i \frac{k}{N}(n-m)}$$

6.1.4. Dirac Delta function

$$\int_{-\infty}^{\infty} \delta(x-y) f(x) dx = f(y)$$

6.1.5. Tensor (1,1)

$$\delta^i_j = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

i: Contravariante j: Covariante

6.1.6. Vectores base ortonormales

$$\hat{e}_i \cdot \hat{e}_j = \delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

6.2. Levi Civita

$$\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = 1$$

$$\epsilon_{321} = \epsilon_{213} = \epsilon_{132} = -1$$

$$\epsilon_{ijk} = \begin{cases} 1 & (i,j,k) \text{even permutation of } (1,2,3) \\ -1 & (i,j,k) \text{odd permutation of } (1,2,3) \\ 0 & ifi = j = k \end{cases}$$

6.2.1. Producto cruz

$$\hat{e}_i \times \hat{e}_j = \sum_{k=1}^{3} \epsilon_{ijk}\hat{e}_k$$

$$\hat{e}_i \times \hat{e}_j = \epsilon_{ijk}\hat{e}_k$$

6.2.2. Determinante

$$\epsilon_{ijk}\epsilon_{lmn} = \begin{vmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{vmatrix}$$

si $i = l$:

$$\epsilon_{ijk}\epsilon_{imn} = \delta_{jm}\delta_{jn} - \delta_{jn}\delta_{km}$$

7. Sistema de coordenadas

7.1. Cartesianas.

Posición:

$$\boldsymbol{r} = \boldsymbol{x} + \boldsymbol{y} + \boldsymbol{z} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

Posición paramétrica:

$$\boldsymbol{r(t)} = A\left(cos\ \omega t\hat{i} + sin\ \omega t\ \hat{j}\right)$$

$$x = Acos\ \omega t \quad y = Asin\ \omega t, \ \ \omega : velocidad\ ángular$$

Velocidad:

$$\mathbf{v} = \dot{\mathbf{r}} = \frac{\mathrm{d}}{\mathrm{d}t}\left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right) = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$$

Velocidad paramétrica:

$$\boldsymbol{v} = \dot{\boldsymbol{r}} = A\omega\left(-sin\ \omega t\ \hat{\imath} + cos\ \omega t\hat{\jmath}\right)$$

$$v^2 = \omega^2 A^2$$

Aceleración:

$$\mathbf{a} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}\right) = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$$

Aceleración paramétrica:

$$\boldsymbol{a} = -A\omega^2 cos\ \omega t\ \hat{\imath} - A\omega^2 sin\ \omega t\ \hat{\jmath} = -\omega^2\boldsymbol{r}$$

Momentum:

$$\boldsymbol{p} = m\dot{\mathbf{r}} = m\dot{x}\hat{\imath} + m\dot{y}\hat{\jmath} + m\dot{z}\hat{k} = p_x\hat{\imath} + p_y\hat{\jmath} + p_z\hat{z} = \sum_{i=1}^{3} m\dot{x}_i\hat{\mathbf{e}} = \sum_{i=1}^{3} p_i\hat{\mathbf{e}}_i$$

Momentum Ángular:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = L_x\hat{\imath} + L_y\hat{\jmath} + L_z\hat{k} = \epsilon_{ijk}r_jp_k = L_i$$

Torque:

$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F} = \frac{\mathrm{d}\boldsymbol{L}}{\mathrm{d}t} = \tau_x\hat{\imath} + \tau_y\hat{\jmath} + \tau_z\hat{k}$$

$$\Rightarrow \quad \tau = \hat{\imath}\left(yF_z - zF_y\right) + \hat{\jmath}\left(zF_x - xF_z\right) + \hat{k}\left(xF_y - yF_z\right)$$

7.2. Cilíndricas

Vectores unitarios:

$$\hat{s} = cos\ \phi\ \hat{\imath} + sin\ \phi\ \hat{\jmath}$$

$$\hat{\phi} = -sin\ \phi\ \hat{\imath} + cos\ \phi\ \hat{\jmath}$$

$$\hat{k} = \hat{k}$$

Posición:

$$x = scos\ \phi \quad y = ssin\ \phi$$

$$\boldsymbol{r} = s\hat{s} + z\hat{k}$$

$$\boldsymbol{s} = s\hat{s}$$

$$\boldsymbol{r} = \boldsymbol{s} + \boldsymbol{z} = s\hat{s} + z\hat{k} \qquad s = \sqrt{x^2 + y^2} \qquad \phi = arcTan\left(\frac{y}{x}\right)$$

Velocidad:

$$\dot{\mathbf{r}} = \dot{s}\hat{s} + s\dot{\phi}\hat{\phi} + \dot{z}\hat{k}$$

Velocidad Ángular:

$$\boldsymbol{\omega} = \dot{\phi}\hat{k}$$

Aceleración:

$$\boldsymbol{a} = \ddot{\boldsymbol{r}} = \left(\ddot{s} - s\dot{\phi}^2\right)\hat{s} + \left(s\ddot{\phi} + 2\dot{s}\dot{\phi}\right)\hat{\phi} + \ddot{z}\hat{k}$$

Momentum:

$$\boldsymbol{p} = m\dot{\boldsymbol{r}} = p_s\hat{s} + sp_\phi\hat{\phi} + p_z\hat{k}$$

Momentum Ángular:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = s^2 p_\phi\hat{k} + (zp_s - sp_z)\hat{\phi} - zp_\phi\hat{s}$$

Torque:

$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F} = \frac{\mathrm{d}\boldsymbol{L}}{\mathrm{d}t} = \dot{L}_s\hat{s} + \dot{L}_\phi\hat{\phi} + \dot{L}_z\hat{k}$$

7.3. Esféricas

Vector unitario:

$$\hat{r} = sin\ \theta\ cos\ \phi\ \hat{\imath} + sin\ \theta\ sin\ \phi\hat{\jmath} + cos\ \theta\ \hat{k}$$

$$\hat{\theta} = cos\ \theta\ cos\ \phi\ \hat{\imath} + cos\ \theta\ sin\ \phi\ \hat{\jmath} - sin\ \theta\ \hat{k}$$

$$\hat{\phi} = cos\ \phi\ \hat{\jmath} - sin\ \phi\ \hat{\imath}$$

Posición:

$$\boldsymbol{r} = r\hat{r} \quad cos\ \theta\ = \frac{z}{r} \ \Rightarrow\ z = rcos\ \theta \quad s = rsin\ \theta$$

$$x = scos\ \phi\ = rsin\ \theta\ cos\ \phi$$

$$y = ssin\ \phi\ = rsin\ \theta\ sin\ \phi$$

$$\Rightarrow \; \boldsymbol{r} = r\left(sin\,\theta\,cos\,\phi\,\hat{i} + sin\,\theta\,sin\,\phi\,\hat{j} - cos\,\theta\,\hat{k}\right) = r\hat{r}$$

$$r = \sqrt{x^2+y^2+z^2} \quad \theta = arcTan\left(\frac{\sqrt{x^2+y^2}}{z}\right) \quad \phi = arcTan\left(\frac{y}{x}\right)$$

Velocidad:

$$\boldsymbol{v} = \dot{\boldsymbol{r}} = \dot{r}\hat{r} + rsin\,\theta\,\dot{\phi}\hat{\phi} + r\dot{\theta}\hat{\theta}$$

Velocidad Ángular:

$$\boldsymbol{\omega} = \dot{\phi}cos\,\theta\,\hat{r} - \dot{\phi}sin\,\theta\,\hat{\theta} + \dot{\theta}\hat{\phi}$$

Aceleración:

$$\boldsymbol{a} = \left(\ddot{r} - r\dot{\theta}^2 - r\theta\dot{\phi}^2 sin^2\,\theta\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\theta - rsin\,\theta\,cos\,\theta\,\dot{\phi}^2\right)\hat{\theta} + \left(rsin\,\theta\,\ddot{\phi} + 2\dot{r}\dot{\phi}sin\,\theta + 2r\dot{\theta}\dot{\phi}cos\,\theta\right)\hat{\phi}$$

Momentum:

$$\boldsymbol{p} = m\dot{\boldsymbol{r}} = m\dot{r}\hat{r} + mr\dot{\phi}sin\,\theta\,\hat{\phi} + mr\dot{\theta}\hat{\theta} = p_r\hat{r} + p_\phi\hat{\phi} + p_\theta\hat{\theta}$$

Momentum Ángular:

$$\mathbf{L} = \mathbf{r}\times\mathbf{p} = rp_\phi\hat{\theta} - rp_\theta\hat{\phi} = L_\phi\hat{\theta} + L_\theta\hat{\phi}$$

Torque:

$$\boldsymbol{\tau} = \boldsymbol{r}\times\boldsymbol{F} = \frac{d\boldsymbol{L}}{dt} = rm\ddot{\phi}\hat{\theta} + rm\ddot{\theta}\hat{\phi}$$

7.4. No ortogonales

$$\boldsymbol{A} = \alpha_1^*\boldsymbol{b}_1 + \alpha_1^*\boldsymbol{b}_2 + \alpha_3^*\boldsymbol{b}_3 \quad \alpha_i^* : \text{Componentes contravariantes}$$

$\boldsymbol{b}_i$: vectores base covariante

$$\boldsymbol{A} = \alpha_1\boldsymbol{b}^1 + \alpha_2\boldsymbol{b}^2 + \alpha_3\boldsymbol{b}^3 \quad \alpha_i^* : \text{Componentes covariantes}$$

$\boldsymbol{b}^i$: vectores base contravariante

$$V_i = \boldsymbol{V}\cdot\boldsymbol{a_i} \quad \text{componentes covariantes}$$

$$V^i = \boldsymbol{V}\cdot\boldsymbol{a^i} \quad \text{componentes contravariantes}$$

$$g_{ij} = \boldsymbol{a_i}\cdot\boldsymbol{a}_j, \qquad g^{ij} = \boldsymbol{a}^i\cdot\boldsymbol{a}^j, \qquad g_{ki}g^{ij} = \delta_i^j$$

Soluciones:

$$\alpha_1^* = \frac{\begin{vmatrix} A_x & A_y & A_z \\ b_{2x} & b_{2y} & b_{2z} \\ b_{3x} & b_{3y} & b_{3z} \end{vmatrix}}{\begin{vmatrix} b_{1x} & b_{1y} & b_{1z} \\ b_{2x} & b_{2y} & b_{2z} \\ b_{3x} & b_{3y} & b_{3z} \end{vmatrix}} = \frac{\boldsymbol{A}\cdot\boldsymbol{b}_2\times\boldsymbol{b}_3}{\boldsymbol{b}_1\cdot\boldsymbol{b}_2\times\boldsymbol{b}_3} = \boldsymbol{A}\cdot\boldsymbol{b}^1$$

$$\alpha_2^* = \frac{\begin{vmatrix} A_x & A_y & A_z \\ b_{3x} & b_{3y} & b_{3z} \\ b_{1x} & b_{1y} & b_{1z} \end{vmatrix}}{\begin{vmatrix} b_{1x} & b_{1y} & b_{1z} \\ b_{2x} & b_{2y} & b_{2z} \\ b_{3x} & b_{3y} & b_{3z} \end{vmatrix}} = \frac{\boldsymbol{A}\cdot\boldsymbol{b}_3\times\boldsymbol{b}_1}{\boldsymbol{b}_1\cdot\boldsymbol{b}_2\times\boldsymbol{b}_3} = \boldsymbol{A}\cdot\boldsymbol{b}^2$$

$$\alpha_3^* = \frac{\begin{vmatrix} A_x & A_y & A_z \\ b_{1x} & b_{1y} & b_{1z} \\ b_{2x} & b_{2y} & b_{2z} \end{vmatrix}}{\begin{vmatrix} b_{1x} & b_{1y} & b_{1z} \\ b_{2x} & b_{2y} & b_{2z} \\ b_{3x} & b_{3y} & b_{3z} \end{vmatrix}} = \frac{\boldsymbol{A}\cdot\boldsymbol{b}_1\times\boldsymbol{b}_2}{\boldsymbol{b}_1\cdot\boldsymbol{b}_2\times\boldsymbol{b}_3} = \boldsymbol{A}\cdot\boldsymbol{b}^3$$

$$\boldsymbol{b}^1 = \frac{\boldsymbol{b}_2\times\boldsymbol{b}_3}{\boldsymbol{b}_1\cdot\boldsymbol{b}_2\times\boldsymbol{b}_3}$$

$$\boldsymbol{b}^2 = \frac{\boldsymbol{b}_3\times\boldsymbol{b}_1}{\boldsymbol{b}_1\cdot\boldsymbol{b}_2\times\boldsymbol{b}_3}$$

$$\boldsymbol{b}^3 = \frac{\boldsymbol{b}_1\times\boldsymbol{b}_2}{\boldsymbol{b}_1\cdot\boldsymbol{b}_2\times\boldsymbol{b}_3}$$

7.4.1. Conjunto recíproco de vectores base

$$\boldsymbol{b}^1\cdot\boldsymbol{b}_1 = \boldsymbol{b}^2\cdot\boldsymbol{b}_2 = \boldsymbol{b}^3\cdot\boldsymbol{b}_3 = 1$$

$$\boldsymbol{a}^i\cdot\boldsymbol{a}_j = \delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

8. Ecuaciones cinemática

aceleracion cte

$$v = \frac{x}{t}$$

$$v = v_0 + at$$

$$x = v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2ax$$

caída libre

$$v_y = v_{0y} - gt$$

$$y = v_{0y}t - \frac{1}{2}gt^2$$

$$v_f^2 = v_0^2 + 2gh$$

2d:

$$\boldsymbol{v} = v_x\hat{i} + v_y\hat{j}$$

$$v_x = vcos\theta\,, \quad v_y = vsin\theta\,, \quad v^2 = v_x^2 + v_y^2$$

$$\theta = ArcTan\left(\frac{v_y}{v_x}\right)$$

mov circular : $\ddot{r} = 0$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$$

$$\omega = \frac{v}{r} \Rightarrow v = \omega r$$
$$\omega_f = \omega_o \pm \alpha t$$

$$\theta = \omega_0 t \pm \frac{1}{2}\alpha t^2$$

$$\omega_f^2 = \omega_o^2 \pm 2\alpha\theta$$

$$a_t = \alpha r = \frac{v}{t}$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$L = \theta r$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$\boldsymbol{a} = \boldsymbol{a}_t + \boldsymbol{a}_c$$

Frecuencia ciclotrón:

$$\omega = \frac{qB}{m}$$

Relación entre K y V para una partícula acelerada por potencial:

$$K = qV$$

Ecuación de trayectoria:

$$y = x\tan\theta - \frac{gx^2}{2v_0^2\cos^2\theta}$$

9. Mecánica Newtoniana

1 ley: Inercia

2 ley:

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt} = m\boldsymbol{a}, \quad \boldsymbol{p} = m\boldsymbol{v}$$

3 ley:

$$\boldsymbol{F}_{12} = -\boldsymbol{F}_{21}$$

4 ley:

$$\boldsymbol{F} = -G\frac{m_1 m_2}{|\boldsymbol{r_2} - \boldsymbol{r}_1|^3}(\boldsymbol{r_2} - \boldsymbol{r_1})$$

$$\boldsymbol{F} = \sum_{j=1}^{m} \boldsymbol{F}_j$$

9.1. *Fuerzas Conservativas*

$$\boldsymbol{F} = -\boldsymbol{\nabla} U$$

$$W_{A\to B} = \int_A^B \boldsymbol{F}\cdot d\boldsymbol{r} = -\Delta U$$

$$E = \frac{1}{2}m\boldsymbol{v}^2 + U = T + U$$

Corolario: $\boldsymbol{\nabla}\times\boldsymbol{F} = 0$, $\oint \boldsymbol{F}\cdot d\boldsymbol{r} = 0$

9.1.1. Fuerzas Centrales Conservativas

$$\boldsymbol{F} = f(r)\hat{r} = -\boldsymbol{\nabla} U = \frac{dU}{dr}\hat{r}$$

Satisface: $\boldsymbol{\tau} = \boldsymbol{r}\times\boldsymbol{F} = 0$ mov. π azimutal $\boldsymbol{r} = s\boldsymbol{s}$

$$E = \frac{1}{2}m(\dot{s}^2 + s^2\dot{\phi}^2) + U(s)$$

$$\boldsymbol{L} = L_\phi\hat{k} = ms^2\dot{\phi} \Rightarrow \quad L = ms^2\dot{\phi} \quad \Rightarrow \quad \dot{\phi} = \frac{L}{ms^2}$$

$$v = s\dot{\phi} \quad \Rightarrow \quad L = msv$$

$$E = \frac{1}{2}m\dot{s}^2 + \frac{L^2}{2ms^2} + U(s)$$

Energía Potencial efectiva: $U_{ef} = \frac{L^2}{2ms^2} + U(s)$
Energía Potencial Centrífuga: $U_{centri} = \frac{L^2}{2ms^2}$
Energía Potencial física: $U(s)$

10. Teorema de Binet

$$r = r(\phi)$$

$$F\left(\frac{1}{u}\right) = -\frac{L^2}{m}(u^2u'' + u^3)$$

$$\frac{d^2u}{d\phi^2} + u = -\frac{m}{L^2}\frac{d}{du}V(\frac{1}{u})$$

11. Potenciales Conservativos

Puntos de Retorno: $E = U(x)$

11.1. *Equilibrio estable e inestable*

$$\frac{dU}{dx} = 0$$

Taylor:

$$f(x) = \sum_{n=0}^{\infty}\frac{1}{n!}f^{(n)}(x_0)[x - x_0]^n$$

$$f^{(n)}(x_0) = \frac{d^n f(x)}{dx^n}|_{x=x_0}$$

$$U(x) = U(x_0) + \frac{1}{2!}U^2(x_0)(x - x_0)^2 + ...$$

cond. de extremo:

$$f'(x_0) = 0$$

cond. de mínimo:

$$f''(x_0) > 0, \text{mín local en } x_0$$

cond. de máx:

$$f''(x_0) < 0, \text{máx local en } x_0$$

Periodo de pequeñas oscilaciones:

$$T_{osc} = 2\pi\sqrt{\frac{m}{k}}$$

donde k:

$$k = \frac{d^2V_{eff}}{dr}|_{r=a}$$

12. Cálculo de Variaciones

Longitud de una curva:

$$l = \int_A^B ds = \int_A^B \sqrt{dx^2 + dy^2} = \int_A^B \sqrt{1 + \left(\frac{dx}{dy}\right)^2}$$

Tiempo recorrido:

$$T = \int_0^T dt = \int_A^B \frac{dt}{ds} ds = \int_A^B \frac{ds}{v}$$

Sistema Conservativo:

$$v = \sqrt{2g(y_a - y)} \quad \Rightarrow \quad T = \int_A^B \frac{\sqrt{1 + y'^2}}{\sqrt{2g(y_a - y)}}, \text{Integral funcional}$$

12.1. *Ecuación de Euler-Lagrange*

Acción:

$$\delta I = \int_a^b \left[\partial_y \Phi - \frac{d}{dx}(\partial_{y'}\Phi)\right] \delta y dx = 0, \ \Phi{=}\Phi(y, y', t)$$

$$\Rightarrow \frac{d}{dx}\partial_{y'}\Phi - \partial_y \Phi = 0, \quad \text{EcuacióN E-L}$$

12.2. *Principio de Hamilton, Mínima Acción*

$$\mathcal{L} = \mathcal{L}(\{q\}, \{\dot{q}\}, t), \quad \{q\} = q_1, ..., q_n, \quad \{\dot{q}\} = \dot{q}_1, ..., \dot{q}_n$$

$\{q\}_j$: posiciones generalizadas

$\left\{\dot{q}_j\right\}$: velocidades generalizadas

$\mathcal{L}$: Lagrangiano

Ecuaciones de Euler-Lagrange:

$$\frac{d}{dt}\partial_{\dot{q}_j}\mathcal{L} - \partial_{q_j}\mathcal{L} = 0$$

Para Sistemas Conservativos:

$$\mathcal{L} = T - U$$

12.3. *Leyes de Conservación*

Uniformidad del tiempo:

$$E = \sum_{j=1} \partial_{\dot{q}_j}\mathcal{L}\dot{q}_j - \mathcal{L} : cte, \quad \text{Energía del sistema cerrado}$$

(*) Sistema campo exterior tipo conservativo:

$$\mathcal{L} = T(q, \dot{q}) - U(q)$$

12.3.1. Teoremas de funciones homogéneas de Euler

$$f(\lambda x, \lambda y) = \lambda f(x, y)$$

$$E = T + U$$

12.3.2. **Uniformidad del espacio**

Invarianza: $\boldsymbol{r}_a = \boldsymbol{r} + \boldsymbol{\epsilon} \rightarrow \ \delta\boldsymbol{r} = \boldsymbol{\epsilon}$

Momentum:

$$\boldsymbol{P}_a \equiv \partial_{\boldsymbol{v}_a}\mathcal{L} \quad \Rightarrow \quad \boldsymbol{P}_a = \sum_a \boldsymbol{P}_a = \sum_a \partial_{\boldsymbol{v}_a}\mathcal{L} = cte$$

12.3.3. **Coordenadas cartesianas**

$$\mathcal{L} = T(\dot{q}) - U(q)$$

$$\Rightarrow \partial_{q_j}\mathcal{L} = \partial_{q_j}U = -F_j; \quad \boldsymbol{F}_a = \partial_{\boldsymbol{r}_a}U$$

$$\nabla = \partial_{\boldsymbol{r}_a}$$

$$p_i = \partial_{\dot{q}_j}\mathcal{L}, \quad \text{Momentum generalizado}$$

$$F_i = \partial_{q_j}\mathcal{L}, \quad \text{Fuerzas generalizada}$$

12.3.4. **Coordenadas cíclicas**

coor. que no aparece en $\mathcal{L}$

$$\mathcal{L}(\{q\}, \{\dot{q}\})$$

$$F_n = \partial_{q_n}\mathcal{L} = 0$$

$p_n = \partial_{\dot{q}_n}\mathcal{L} : cte$

12.4. *Fuerza de Ligadura y multiplicadores de Lagrange*

12.4.1. **Ligaduras Holonómicas**

:

$$f_j(\{q\}, t) = 0, \quad j = 1, ..., k$$

Integrables:

$$df_j = \sum_{i=1}^{n}(\partial_{q_i})dq_i = 0$$

Ligaduras → Fuerza de Reacción

12.5. *Coordenadas generalizadas*

$n = 3N$ g-l mov. en k lig. hol. con $n - k$ g-l ind.

Forma geneérica de $x_n = x_n(\{q\}, t)$

$$dx_i = \sum_{j=1}^{s}\left(\partial_{q_j}x_i\right)dq_j + \partial_t x_i, \quad i = 1, ..., n$$

12.6. *Desplazamiento virtual*

des. inf. o intneo de coor.

$$\delta x_i = \sum_{\sigma=1} \partial_{q_\sigma}x_i\delta q_\sigma$$

12.7. Principio de D'Alembert

$$\dot{p}_i = F_i^{(a)} + R_i \begin{cases} F_i^{(a)}: & \text{Fuerzas aplicadas} \\ R_i: & \text{Fuerza de reaccion} \end{cases}$$

$$\Rightarrow \sum_i \left(F_i^{(a)} + R_i - \dot{p}_i\right)\delta x_i = 0$$

12.8. Trabajo virtual

$$\delta W = \sum_{i=1}^{n} F_i \delta x_i = \sum_{\sigma=1}^{s}\left(\sum_{i=1}^{n} F_i \partial_{q_\sigma} x_i\right)\delta q_\sigma$$

$$\Rightarrow Q_\sigma = \sum_{i=1}^{n} F_i \partial_{q_\sigma} x_i, \quad \textbf{Fuerza Generalizada}$$

$$\Rightarrow \delta W = \sum_{\sigma=1}^{s} Q_\sigma \delta q_\sigma$$

Velocidad de las partículas:

$$\dot{x}_i = \dot{x}_i(\{q\}, \{\dot{q}\}, t)$$

$$\frac{dx_i}{dt} \equiv \dot{x}_i = \sum_{\sigma=1}^{s}\left(\partial_{q_\sigma} x_i\right)\dot{q}_\sigma + \partial_t x_i, \quad i = 1, ..., n,$$

Relación lineal:

$$\partial_{\dot{q}_\sigma} \dot{x}_i = \partial_{q_\sigma} x_i$$

Energía cinética T del sistema:

$$T = \sum_i m_i \dot{x}_i^2$$

Principio de D'Alembert:

$$\sum_\sigma \left(\frac{d}{dt}\partial_{\dot{q}_\sigma} T - \partial_{q_\sigma} T - Q_\sigma\right)\delta q_\sigma = 0, \quad \{q_\sigma\} : ind$$

Lagrange eq:

$$\frac{d}{dt}\partial_{\dot{q}_\sigma} T - \partial_{q_\sigma} T = Q_\sigma, \qquad \sigma = 1, ..., n-k$$

12.8.1. Fuerzas Conservativas:

$$U(\{x\}, t) = U(\{q\}, t), \quad \{q\} = q_1, ..., q_s$$

Fuerza aplicada:

$$Q_\sigma = -\partial_{q_\sigma} U$$

Lagrange eq:

$$\frac{d}{dt}\partial_{\dot{q}_\sigma}\mathcal{L} - \partial_{q_\sigma}\mathcal{L} = 0$$

12.8.2. Lagrange Multiplier

$$\frac{d}{dt}\partial_{\dot{q}_\sigma}\mathcal{L} - \partial_{q_\sigma}\mathcal{L} = \sum_{j=1}^{k} \lambda_j \partial_{q_\sigma} f_j$$

Fuerzas de Reacción:

$$Q_\sigma^r = \sum_{j=1}^{k} \lambda_j \partial_{q_\sigma} f_j$$

13. Scattering

Potencial gravitatorio:

$$V(r) = -\frac{GMm}{r}$$

Para órbitas hiperbólicas:

$$\theta = \pi - 2\psi \quad \psi = \arccos(\frac{1}{\epsilon})$$

Ángulo de deflexión órbitas hiperbólicas:

$$\theta = 2\arccos\frac{1}{\epsilon}$$

$$\epsilon = \left\{1 + \left(\frac{bv_\infty^2}{GM}\right)^2\right\}^{1/2}$$

Dispersión atractiva y repulsiva:

$$\chi = |\pi - 2\phi_0|$$

$$\phi_0 = \int_{r_{min}}^{\infty} \frac{\frac{L}{r^2}}{\sqrt{2m(E-V) - \frac{L^2}{r^2}}}$$

Paramétro de impacto:

$$\phi_0 = \int_{r_{min}}^{\infty} \frac{\frac{b}{r^2}}{\sqrt{1 - \frac{b^2}{r^2} - \frac{U(r)}{E}}}$$

$$\phi_0 = b\int_0^{u_m} \frac{du}{\sqrt{1 - \frac{V}{E} - u^2 b^2}}$$

$$d\Omega = 2\pi \sin\theta d\theta$$

Sección transversal:

$$d\sigma = 2\pi b \left|\frac{db}{d\theta}\right| d\theta$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Sección transversal total:

$$\sigma_T = \int d\Omega \left(\frac{d\sigma}{d\Omega}\right)$$

13.1. Rutherford Scattering

$$V = \frac{\alpha}{r} \quad \alpha = \frac{1}{4\pi\epsilon_0} q_1 q_2$$

Formula de Rutherford:

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{\alpha}{2mv_0}\right)^2 \frac{1}{\sin^4(\frac{\chi}{2})}$$

13.2. Scattering by a Hard Sphere

$$\sigma_T = \pi b_{max}^2$$

$$l = mvb$$

14. Oscilaciones pequeñas

Energía cinética:

$$T = \frac{1}{2} I\dot{\theta}^2$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$\sin\theta \approx \theta$$

$$q = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

$$q^T = \begin{pmatrix} q_1 & q_2 \end{pmatrix}$$

$$\bar{T} = \frac{1}{2} \dot{q}^T \mathcal{M} \dot{q}$$

$$\bar{K} = \frac{1}{2} q^T \mathcal{K} q$$

$$\bar{T} = \frac{1}{2} \left(A\dot{q}_1^2 + B\dot{q}_1\dot{q}_2 + C\dot{q}_1\dot{q}_2 + D\dot{q}_2{}^2 \right)$$

$$\left(\mathcal{K} - \omega_n^2 \mathcal{M}\right) |\boldsymbol{v}_n\rangle = 0$$

$$\det(\mathcal{K} - \omega_n^2 \mathcal{M}) = 0$$

$$\langle \boldsymbol{u} | \mathcal{M} | \boldsymbol{v} \rangle = \boldsymbol{u}^T \mathcal{M} \boldsymbol{v}$$

$$\langle \boldsymbol{u} | \mathcal{M} | \boldsymbol{u} \rangle = \langle \boldsymbol{v} \rangle_{\mathcal{M}}^2$$

$$\mathcal{A} \equiv \begin{pmatrix} v^{(1)} & v^{(2)} & \dots & v^{(n)} \end{pmatrix}$$

$$\mathcal{L} = \frac{1}{2} \dot{q}^T \mathcal{M} \dot{q} - \frac{1}{2} q^T \mathcal{K} q$$

$$\mathcal{L}' = \frac{1}{2} \dot{Q}^T \mathcal{M}' \dot{Q} - \frac{1}{2} Q^T \mathcal{K}' Q$$

$$Q = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = \mathcal{A}^T \mathcal{M} q$$

$$\mathcal{M}' = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \mathcal{M}' = \mathcal{A}^T \mathcal{M} \mathcal{A}$$

$$\mathcal{K}' = \begin{pmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{pmatrix} \Rightarrow \mathcal{K}' = \mathcal{A}^T \mathcal{K} \mathcal{A}$$

NOETHER

15. Hamilton

$$\partial_{\dot{q}_j} \mathcal{L} = p_j \quad \text{Momento generalizado}$$

$$p = A\dot{q} \Rightarrow \begin{pmatrix} p_\theta \\ p_\phi \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \dot{\phi} \end{pmatrix}$$

$$A^{-1} = \frac{1}{det(A)} \begin{pmatrix} D & -C \\ -B & A \end{pmatrix}$$

$$E = T + V = H \quad \text{conservation}$$

$$H = \sum_j p_j \dot{q}_j - \mathcal{L}$$

Ecuaciones de movimiento:

$$\dot{p}_j = -\partial_{q_j} H \quad \dot{q}_j = \partial_{p_j} H$$

Relatividad:

$$p_j = \gamma m \dot{r}_j \,, \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad E = \gamma m c^2$$

$$E^2 = p^2c^2 + m^2c^4 \quad p^2 = p_r^2 + \frac{p_\theta^2}{r^2} + \frac{p_\phi^2}{r^2 sin^2\theta}$$

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$$

$$\boldsymbol{E} = -\boldsymbol{\nabla}\Phi - \frac{1}{c}\partial_t \boldsymbol{A}$$

$$\frac{dA}{dt} = \dot{x}\partial_x A + \dot{y}\partial_y A + \dot{z}\partial_z A$$

Parentesis de Poisson:

$$\{f, g\} = \sum_{j=1}^{n} \left(\partial_{q_j} f \partial_{p_j} g - \partial_{q_j} g \partial_{p_j} f \right)$$

Propiedades:

$$\{f, c\} = 0$$

$$\{f + g, h\} = \{f, h\} + \{g, h\}$$

$$\{fg, h\} = f\{g, h\} + g\{f, h\}$$

$$\partial_t \{f, g\} = \{\partial_t f, g\} + \{f, \partial_t g\}$$

$$\left\{q_i, p_j\right\} = \delta_{ij}, \quad \left\{q_i, q_j\right\} = \left\{p_i, p_j\right\} = 0$$

$$\left\{p_i, q_j\right\} = -\delta_{ij}$$

$$\{f, \{g, h\}\} + \{h, \{f, g\}\} + \{g, \{h, f\}\} = 0$$

$$\{fg, hk\} = f\{g, h\}k + fh\{g, k\} + gk\{f, h\} + gh\{f, k\}$$