Problema 1:

(a) $S(T) = \alpha \frac{T}{T_0} + \beta \cos\left(\frac{\pi}{2}\frac{T}{T_0}\right)$

$\rightarrow dS = \left[\frac{\alpha}{T_0} - \frac{\beta\pi}{2T_0}\sin\left(\frac{\pi}{2}\frac{T}{T_0}\right)\right] dT$

$\therefore TdS = \left[\alpha\frac{T}{T_0} - \frac{\beta\pi}{2}\frac{T}{T_0}\sin\left(\frac{\pi}{2}\frac{T}{T_0}\right)\right] dT$

Ahora, sabemos que $đQ = CdT$: C: calor específico molar

y junto con $đQ = TdS$, Tenemos que

$$\boxed{C = \alpha\frac{T}{T_0} - \frac{\beta\pi}{2}\frac{T}{T_0}\sin\left(\frac{\pi}{2}\frac{T}{T_0}\right)}$$

Ahora, $C < 0 \Rightarrow \alpha\cancel{\frac{T}{T_0}} < \frac{\beta\pi}{2}\cancel{\frac{T}{T_0}}\sin\left(\frac{\pi}{2}\frac{T}{T_0}\right)$

$\frac{2\alpha}{\beta\pi} < \sin\left(\frac{\pi}{2}\frac{T}{T_0}\right) \Rightarrow \text{ArcSin}\left(\frac{2\alpha}{\beta\pi}\right) < \frac{\pi}{2}\frac{T}{T_0}$

$$\therefore \boxed{T > T_0 \cdot \frac{2}{\pi}\text{ArcSin}\left(\frac{2\alpha}{\beta\pi}\right)}$$

(b) $đQ = CdT \Rightarrow Q = \int_{T_1}^{T_2} CdT$

$Q = \frac{\alpha}{T_0}\int_{T_1}^{T_2} TdT - \frac{\beta\pi}{2T_0}\int_{T_1}^{T_2} T\sin\left(\frac{\pi}{2}\frac{T}{T_0}\right) dT$

- $I_1 = \int_{T_1}^{T_2} TdT = \frac{T^2}{2}\Big|_{T_1}^{T_2} = \frac{1}{2}\left(T_2^2 - T_1^2\right) = \frac{1}{2}\left(\frac{4T_0^2}{9} - \frac{T_0^2}{9}\right) = \frac{3T_0^2}{2\cdot 9}$

$$\boxed{I_1 = \frac{T_0^2}{6}}$$

$u = T \qquad dv = \sin\left(\frac{\pi T}{2T_0}\right)dT$

- $I_2 = \int_{T_1}^{T_2} T\sin\left(\frac{\pi}{2}\frac{T}{T_0}\right)dT \qquad du = dT \qquad v = -\frac{2T_0}{\pi}\cos\left(\frac{\pi}{2}\frac{T}{T_0}\right)$

$$I_2 = -\frac{2T_0 T}{\pi}\cos\left(\frac{\pi}{2}\frac{T}{T_0}\right)\Bigg|_{T_1}^{T_2} + \int_{T_1}^{T_2}\frac{2T_0}{\pi}\cos\left(\frac{\pi}{2}\frac{T}{T_0}\right)dT$$

$$I_2 = \frac{2T_0}{\pi}\left\{-T_2\cos\left(\frac{\pi}{2}\frac{T_2}{T_0}\right) + T_1\cos\left(\frac{\pi}{2}\frac{T_1}{T_0}\right) + \frac{2T_0}{\pi}\sin\left(\frac{\pi}{2}\frac{T_2}{T_0}\right) - \frac{2T_0}{\pi}\sin\left(\frac{\pi}{2}\frac{T_1}{T_0}\right)\right\}$$

$$I_2 = \frac{2T_0}{\pi}\left\{-\frac{2T_0}{3}\underbrace{\cos\left(\frac{\pi}{3}\right)}_{1/2} + \frac{T_0}{3}\underbrace{\cos\left(\frac{\pi}{6}\right)}_{\sqrt{3}/2} + \frac{2T_0}{\pi}\left[\underbrace{\sin\left(\frac{\pi}{3}\right)}_{\sqrt{3}/2} - \underbrace{\sin\left(\frac{\pi}{6}\right)}_{1/2}\right]\right\}$$

$$I_2 = \frac{2T_0}{\pi}\left\{-\frac{T_0}{3} + \frac{T_0\sqrt{3}}{6} + \frac{2T_0}{\pi}\left(\frac{\sqrt{3}-1}{2}\right)\right\} = \frac{2T_0^2}{\pi}\left\{\frac{\sqrt{3}-2}{6} + \frac{\sqrt{3}-1}{\pi}\right\}$$

$$\therefore Q = \frac{\alpha}{T_0}\cdot I_1 - \frac{\beta\pi}{2T_0}\cdot I_2 = \frac{\alpha}{\cancel{T_0}}\cdot\frac{T_0^{\cancel{2}}}{6} - \frac{\beta\cancel{\pi}}{\cancel{2T_0}}\cdot\frac{\cancel{2}T_0^{\cancel{2}}}{\cancel{\pi}}\left\{\frac{\sqrt{3}-2}{6} + \frac{\sqrt{3}-1}{\pi}\right\}$$

$$\boxed{Q = T_0\left\{\frac{\alpha}{6} - \beta\left(\frac{\sqrt{3}-2}{6} + \frac{\sqrt{3}-1}{\pi}\right)\right\}}$$

(c) Ya que $Q = \Delta U - W \to W = \Delta U - Q$

$$W = C_V(T_2 - T_1) - T_0\left[\frac{\alpha}{6} - \beta\left(\frac{\sqrt{3}-2}{6} + \frac{\sqrt{3}-1}{\pi}\right)\right]$$

$$W = C_V\left(\frac{2T_0}{3} - \frac{T_0}{3}\right) - T_0\left[\frac{\alpha}{6} - \beta\left(\frac{\sqrt{3}-2}{6} + \frac{\sqrt{3}-1}{\pi}\right)\right]$$

$$\boxed{W = T_0\left[\frac{C_V}{3} - \frac{\alpha}{6} + \beta\left(\frac{\sqrt{3}-2}{6} + \frac{\sqrt{3}-1}{\pi}\right)\right]}$$

Problema 2 : (a) Partamos combinando el 1° y 2° principio de la termodinámica: $TdS = dU + PdV$, o bien,

$$dS = \frac{1}{T}dU + \frac{P}{T}dV$$

si $U = U(T,V) \longrightarrow dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$

$$\therefore \quad dS = \frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_V dT + \left[\frac{1}{T}\left(\frac{\partial U}{\partial V}\right)_T + \frac{P}{T}\right]dV$$

Entonces,

$$\left(\frac{\partial S}{\partial T}\right)_V = \frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_V$$

$\wedge$

$$\left(\frac{\partial S}{\partial V}\right)_T = \frac{1}{T}\left(\frac{\partial U}{\partial V}\right)_T + \frac{P}{T}.$$

Ahora, ya que dS es diferencial exacta,

$$\frac{\partial^2 S}{\partial T \partial V} = \frac{\partial^2 S}{\partial V \partial T}$$

$$\frac{\partial}{\partial T}\left[\frac{1}{T}\left(\frac{\partial U}{\partial V}\right)_T + \frac{P}{T}\right] = \frac{\partial}{\partial V}\left[\frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_V\right]$$

$$-\frac{1}{T^2}\left(\frac{\partial U}{\partial V}\right)_T + \cancel{\frac{1}{T}\frac{\partial}{\partial T}\left(\frac{\partial U}{\partial V}\right)_T} + \frac{\partial}{\partial T}\left(\frac{P}{T}\right) = \cancel{\frac{1}{T}\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial T}\right)_V}$$

$$\Rightarrow \left(\frac{\partial U}{\partial V}\right)_T = T^2 \frac{\partial}{\partial T}\left(\frac{P}{T}\right)$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T^2\left\{\frac{1}{T}\left(\frac{\partial P}{\partial T}\right)_V - \frac{P}{T^2}\right\}$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

Ya que $P = f(V)T$, entonces

$$\left(\frac{\partial U}{\partial V}\right)_T = Tf(V) - f(V)T = 0$$

De esta manera, U no depende de V.

(b) $P = \frac{1}{3}u(T)$; $u = \frac{U}{V}$ $\Rightarrow$ $P = \frac{U}{3V}$ $\wedge$ $U = Vu$

Usamos $$\underbrace{\left(\frac{\partial U}{\partial V}\right)_T}_{u(T)} = T \underbrace{\left(\frac{\partial P}{\partial T}\right)_V}_{\frac{1}{3}\frac{du}{dT}} - \underbrace{P}_{\frac{1}{3}u(T)}$$

$$u(T) = \frac{T}{3}\frac{du}{dT} - \frac{1}{3}u(T)$$

$$\frac{4}{3}u(T) = \frac{T}{3}\frac{du}{dT} \Rightarrow \frac{du}{u} = 4\frac{dT}{T}$$

$$\Rightarrow \ln u = \ln T^4 + \underbrace{\ln C}_{cte.}$$

$$\Rightarrow \boxed{u(T) = C T^4} \leftarrow \text{Gas de fotones.}$$

Problema 3 : Ec. de estado: $\boxed{\mathcal{E} = \mathcal{E}_0 - \alpha(T - T_0)}$

$$\Rightarrow \frac{d\mathcal{E}}{dT} = -\alpha$$

La energía interna:

$$U = U_0 + \left(\mathcal{E} - T\frac{d\mathcal{E}}{dT}\right)Z + C_Z(T - T_0)$$

$$\therefore U = U_0 + (\mathcal{E}_0 - \cancel{\alpha T} + \alpha T_0 + \cancel{\alpha T})Z + C_Z(T - T_0)$$

$$U = U_0 + (\mathcal{E}_0 + \alpha T_0)Z + C_Z(T - T_0)$$

$$U = U_0 + (\mathcal{E}_0 + \alpha T_0)Z + \frac{C_Z}{\alpha}(\mathcal{E}_0 - \mathcal{E})$$

$$\Rightarrow dU = (\mathcal{E}_0 + \alpha T_0)\,dZ - \frac{C_Z}{\alpha}\,d\mathcal{E}$$

Luego, $đQ = dU - \mathcal{E}\,dZ$

$$\Rightarrow \boxed{đQ = (\mathcal{E}_0 - \mathcal{E} + \alpha T_0)\,dZ - \frac{C_Z}{\alpha}\,d\mathcal{E}} \quad (*)$$

(a) Para representar un ciclo de Carnot necesitamos determinar las curvas adiabáticas e isotérmicas.

De la EdE,

$$\mathcal{E} = \mathcal{E}_0 + \alpha T_0 - \alpha T$$

Si $T_f < T_c \Rightarrow \mathcal{E}_f > \mathcal{E}_c$, donde $\mathcal{E}_f = (\mathcal{E}_0 + \alpha T_0) - \alpha T_f$

$\mathcal{E}_c = (\mathcal{E}_0 + \alpha T_0) - \alpha T_c$

Las adiabáticas las obtenemos de (*), $đQ = 0$,

$$(\mathcal{E}_0 - \mathcal{E} + \alpha T_0)\,dZ - \frac{C_Z}{\alpha}\,d\mathcal{E} = 0$$

$$\rightarrow \quad \frac{\alpha}{C_Z}\,dZ = \frac{d\mathcal{E}}{(\mathcal{E}_0 - \mathcal{E} + \alpha T_0)}$$

$$\frac{\alpha}{C_Z}\int_{Z_i}^{Z} dZ = \int_{\mathcal{E}_i}^{\mathcal{E}} \frac{d\mathcal{E}}{\mathcal{E}_0 - \mathcal{E} + \alpha T_0}$$

$$\frac{\alpha}{C_Z}(Z - Z_i) = \ln\left(\frac{\mathcal{E}_0 - \mathcal{E}_i + \alpha T_0}{\mathcal{E}_0 - \mathcal{E} + \alpha T_0}\right)$$

$$\frac{\mathcal{E}_0 - \mathcal{E} + \alpha T_0}{\mathcal{E}_0 - \mathcal{E}_i + \alpha T_0} = e^{-\frac{\alpha}{C_Z}(Z - Z_i)} = \frac{\mathcal{E}_0 - \mathcal{E}_i + \alpha T_0 + \mathcal{E}_i - \mathcal{E}}{\mathcal{E}_0 - \mathcal{E}_i + \alpha T_0} =$$

$$= 1 + \frac{(\mathcal{E}_i - \mathcal{E})}{\mathcal{E}_0 - \mathcal{E}_i + \alpha T_0}$$

$$e^{-\frac{\alpha}{c_2}(z-z_i)} - 1 = \frac{\mathcal{E}_i - \mathcal{E}}{\mathcal{E}_0 - \mathcal{E}_i + \alpha T_0} = -\frac{\mathcal{E} - \mathcal{E}_i}{\mathcal{E}_0 - \mathcal{E}_i + \alpha T_0}$$

$$\therefore \boxed{\mathcal{E}(z) = \mathcal{E}_i + (\mathcal{E}_0 - \mathcal{E}_i + \alpha T_0)\left(1 - e^{-\frac{\alpha}{c_2}(z-z_i)}\right)}$$

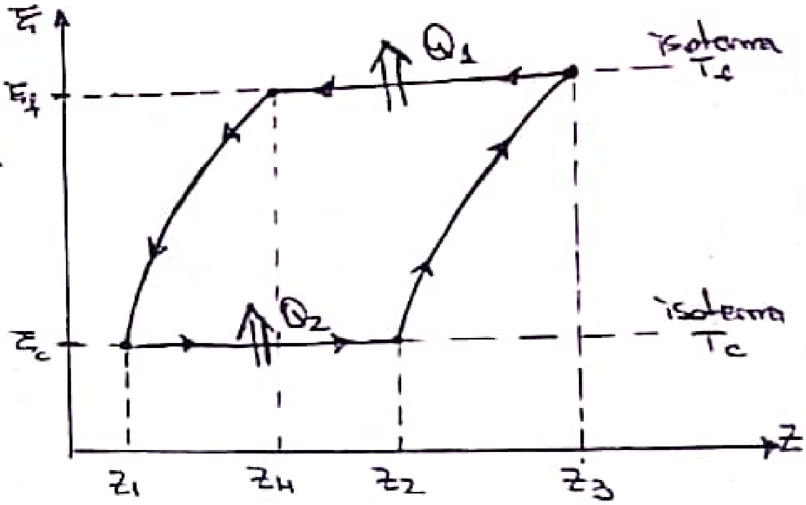
ciclo de Carnot:

i) Carga isotérmica a T_c

ii) Carga adiabática; la temperatura disminuye desde T_c hasta T_f

iii) Descarga isotermica a T_f

iv) Descarga adiabatica; la temperatura aumenta desde T_f hasta T_c.



a) En la carga isotérmica, $\mathcal{E} = \mathcal{E}_c \rightarrow d\mathcal{E} = 0$

$$\therefore \boxed{Q_2 = (\mathcal{E}_0 - \mathcal{E}_c + \alpha T_0)(z_2 - z_1) > 0}$$

$$\rightarrow \boxed{Q_2 = (\mathcal{E}_0 - \mathcal{E}_c + \alpha T_0)\,\Delta z_c} \quad \boxed{\Delta z_c = z_2 - z_1} : \text{ Carga}$$

2) En la descarga isotérmica, $\mathcal{E} = \mathcal{E}_f \rightarrow d\mathcal{E} = 0$

$$\therefore \boxed{Q_1 = (\mathcal{E}_0 - \mathcal{E}_f + \alpha T_0)(z_4 - z_3) < 0}$$

$$\rightarrow \boxed{|Q_1| = (\mathcal{E}_0 - \mathcal{E}_f + \alpha T_0)\,\Delta z_D} \quad \boxed{\Delta z_D = z_3 - z_4} : \text{ Descarga}$$

b) - La eficiencia $\eta = 1 - \frac{|Q_1|}{Q_2} = 1 - \frac{(\mathcal{E}_0 - \mathcal{E}_f + \alpha T_0)}{(\mathcal{E}_0 - \mathcal{E}_c + \alpha T_0)} \frac{\Delta z_D}{\Delta z_c}$

Pero, de la EdE

$$\alpha T = \mathcal{E}_0 - \mathcal{E} + \alpha T_0 \Rightarrow \alpha T_f = \mathcal{E}_0 - \mathcal{E}_f + \alpha T_0$$
$$\alpha T_c = \mathcal{E}_0 - \mathcal{E}_c + \alpha T_0$$

Luego, $\eta = 1 - \frac{\cancel{\alpha} T_f}{\cancel{\alpha} T_c} \cdot \frac{\Delta z_D}{\Delta z_c} = 1 - \frac{T_f}{T_c} \cdot \frac{\Delta z_D}{\Delta z_c}$

Ya que es un ciclo de Carnot, $\eta = \eta_c = 1 - T_f/T_c$, y así

$$\boxed{\Delta z_D = \Delta z_c} \;//$$

Escaneado con CamScanner

Problema 4 $\quad C_v = 3R \cdot \frac{4\pi^4}{5}\left(\frac{T}{\Theta}\right)^3$

$$T\,dS = m\,C_v\,dT + \underbrace{P\,dV}_{=0} \quad \text{(Proceso isocórico)}$$

$$\therefore\ dS = 3mR \cdot \frac{4\pi^4}{5} \frac{T^2}{\Theta^3}\,dT$$

$$\int_{S_1}^{S_2} dS = \frac{mR \cdot 4\pi^4}{5\,\Theta^3} \cdot \int_{T_1}^{T_2} 3T^2\,dT$$

$$\frac{\Delta S}{R} = \frac{4\pi^4 \cdot m}{5}\left(\frac{T_2^3 - T_1^3}{\Theta^3}\right)$$

Datos: $T_1 = 10\,K$, $T_2 = 330\,K$; $\Theta = 2230K$; $m = 1.2\,g$.

Peso atómico : $12\left(\frac{g}{mol}\right) \Rightarrow$ $\left.\begin{matrix} 12\,g = 1\,mol \\ 1.2\,g = m \end{matrix}\right\} \Rightarrow m = 0.1\,mol.$

$$\therefore\ \boxed{\frac{\Delta S}{R} \simeq 0.03\ mol}$$