1) $N=1$

$\Rightarrow \left(\frac{\partial U}{\partial V}\right)_T = 0$; $dU = T\,dS - p\,dV = C_V dT + \left[T\left(\frac{\partial p}{\partial T}\right)_V - p\right]dV$

Enunciado

Significa que $T\left(\frac{\partial p}{\partial T}\right)_V = p$ (i) y $\left(\frac{\partial U}{\partial T}\right)_V = C_V$

U Independiente de V

Otra: $\left(\frac{\partial H}{\partial p}\right)_T = 0$; $dU = T\,dS + V\,dp = C_p dT + \left[V - T\left(\frac{\partial V}{\partial T}\right)_p\right]dp$

$\Downarrow$

$\Rightarrow \quad V = T\left(\frac{\partial V}{\partial T}\right)_p$ (ii)

de (i) y (ii) al integrar:

(i) $p = T\,a(V)$

(ii) $V = b(p)\,T$

si dividimos:

$$p\,b(p) = V a(V)$$

Para que se cumpla la igualdad:

$$p\,b(p) = V\,a(V) = Cte = R$$

Pq hay Variables diferentes a cada lado.
Reemplazando en (i) o (ii) queda.

$PV = RT$, Independiente de $C_V(T)$

Entropía.

Cualquier formula TdS

$$TdS = C_V(T)\,dT + T\left(\frac{\partial P}{\partial T}\right)_V dV = C_V\,dT + \frac{RT}{V}\,dV$$

En principio no podemos suponer $C_V = cte$, pq, estamos demostrando que es un gas Ideal, necesitamos ser lo más general posible.

Entonces

$$S = S_0 + \int_{T_0}^{T} C_V \frac{dT}{T} + R\ln\left(\frac{V}{V_0}\right)$$

$$U = \int_{T_0}^{T} C_V\,dT$$

En un sistema que depende de N, sabemos que S es Homogenea entonces

$$S(T, \lambda V, \lambda N) = \lambda S(T, V, N)$$

Si usamos $\lambda = \frac{1}{N}$

$$\to N S\left(T, \frac{V}{N}, 1\right)$$

Entonces para la ecuación de estado

$$\boxed{P\frac{V}{N} = RT}$$

Gas ideal $\Rightarrow$ $C_V = cte$, experimentalmente.

$$S = s_0 N + N C_V \ln\left(\frac{T}{T_0}\right) + \nu R \ln\left(\frac{V}{V_0 N}\right)$$

$$U = U_0 N + N C_V T$$

despejando T de S

$$U = N c_v T_0 \, e^{\left[\frac{S}{N c_v} - \frac{s_0}{c_v} - \frac{R}{c_v} \ln\left(\frac{V}{N v_0}\right)\right]}$$

$$\rightarrow S(U,V,N) = N s_0 + N c_v \ln\left[\frac{U}{N c_v T_0}\right] + RN \ln\left(\frac{V}{N v_0}\right)$$

con $c_v = mR$ y $U = N_0 c_v T_0$ y $v_0 = \frac{V_0}{N_0}$

$$S = N s_0 + NR \ln\left[\frac{V}{V_0}\left(\frac{U}{U_0}\right)^m \left(\frac{N_0}{N}\right)^{m+1}\right]$$

2) $dW = p_1 dV_1 + p_2 dV_2$

Proceso Isotermico

$$dW = -(dA_1 + dA_2)$$

$$A = U - TS = N c_v T + u_0 N + NT\left[s_0 + c_v \ln\left(\frac{T}{T_0}\right) + R \ln\left(\frac{V}{N v_0}\right)\right]$$

a N y T ctes:

$$\Delta A = -NTR \ln\left(\frac{V_i}{V_f}\right)$$

$\therefore$

$$W = NTR \ln\left(\frac{V_{f1} V_{f2}}{V_{i1} V_{i2}}\right)$$

3) $F(T,l) = -K(T)\left[L - L_0(T)\right]$

Veamos la expansión termica.

$$dL = \underbrace{\left(\frac{\partial L}{\partial T}\right)_F}_{\text{Expansión Termica.}} dT + \left(\frac{\partial L}{\partial F}\right)_T dF$$

$$dL = \left(\frac{K' F}{K^2} + L_0'\right) dT + \cancel{\left(\frac{\partial L}{\partial F}\right)_T}^{\frac{1}{K}} dF$$

Condiciones para que sea 0

$F \approx 0$ y L_0 no depende de T.

↳ $L \approx L_0$

es decir estiramientos cortos:

$$F(T,L) = -K(T)\left[\underbrace{L - L_0}_{\xi}\right]$$

$$\boxed{dU = T dS - F d\xi}$$

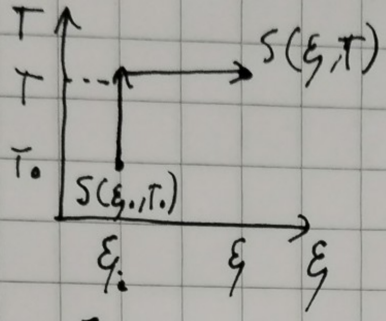
$$T dS = C_\xi dT + T\left(\frac{\partial F}{\partial T}\right)_\xi d\xi = C_\xi dT - T K' \xi d\xi$$

Sabemos $\left(\frac{\partial C_\xi}{\partial \xi}\right)_T = -T \xi K''$

$$C_\xi = -\frac{T}{2}\xi^2 K''(T) + g(T)$$

Para calcular S ~~xxxx~~ podemos "inventar" um processo que nos convenga, pq, nos podemos integrar C_ξ, pq que depende de ξ.

Supomos



$$\Delta S = \underbrace{\int_{T_0}^{T} C_\xi \frac{dT}{T} - \underbrace{\int_{\xi_0}^{\xi_0} T\xi K' d\xi}_{\text{se anula}}}_{\xi = cte} + \underbrace{\underbrace{\int_{T}^{T} C_\xi \frac{dT}{T}}_{\text{se anula}} - \int_{\xi_0}^{\xi} \xi K' d\xi}_{\text{Isotermico}}$$

$$S = S_0 + \int_{T_0}^{T} C_\xi \frac{dT}{T} - \int_{\xi_0}^{\xi} \xi K' d\xi \quad ; \text{ Podemos usar } \xi_0 = 0$$

$$S = S_0 + \int_{T_0}^{T} C_\xi \frac{dT}{T} - \frac{1}{2}\xi^2 K'(T)$$

$$U = U_0 + \int_{T_0}^{T} C_\xi \frac{dT}{T} + \frac{1}{2}\left[K(T) - T K'(T)\right]\xi^2$$

definimos $U(T,0) = U_0 + \int_{T_0}^{T} C_\xi \, dT$ y $S(T,0) = S_0 + \int_{T_0}^{T} C_\xi \frac{dT}{T}$

$$A = U - TS = A(T,0) + \frac{1}{2} K(T)\xi^2$$