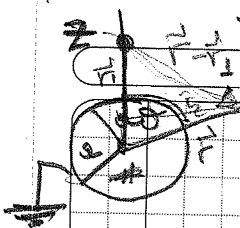


$\vec{r} - \vec{r}_1$ $\vec{r} = r\hat{r}$

$\vec{r}_1 = z\hat{k}$; $\vec{r}_2 = z'\hat{k}$ Fecha: $z = a$

$$\Phi = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{|\vec{r}-\vec{r}_1|} + \frac{q'}{|\vec{r}-\vec{r}_2|}\right]$$

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{\sqrt{r^2+z^2-2zr\cos\theta}} + \frac{q'}{\sqrt{r^2+z'^2-2z'r\cos\theta}}\right]$$

Ya que la superficie esférica está conectada a tierra

$$\Phi(R,\theta) = 0 \Rightarrow$$

$$\Rightarrow \frac{q}{\sqrt{R^2+z^2-2zR\cos\theta}} + \frac{q'}{\sqrt{R^2+z'^2-2z'R\cos\theta}} = 0$$

$$\frac{q/z}{\sqrt{1+\left(\frac{R}{z}\right)^2-2\left(\frac{R}{z}\right)\cos\theta}} = \frac{-q'/R}{\sqrt{1+\left(\frac{z'}{R}\right)^2-2\left(\frac{z'}{R}\right)\cos\theta}}$$

$$\Rightarrow \quad q' = -\left(\frac{R}{z}\right)q$$

$$\wedge \quad z' = \frac{R^2}{z}$$

$$\Phi(r,\theta) = \frac{q}{4\pi\epsilon_0}\left[(r^2+a^2-2ar\cos\theta)^{-1/2} - \frac{R/a}{\left[r^2+\left(\frac{R^2}{a}\right)^2-\frac{2R^2}{a}r\cos\theta\right]^{1/2}}\right]$$

$$\Phi(r,\theta) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r^2+a^2-2ar\cos\theta)^{1/2}} - \frac{R}{[a^2r^2+R^4-2aR^2r\cos\theta]^{1/2}}\right]$$

El campo eléctrico debido a la carga imagen es

Fecha:

$$\vec{E}'(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q'}{|\vec{r}-\vec{r}_2|^3}(\vec{r}-\vec{r}_2) \quad ; \quad \vec{r}_2 = \frac{R^2}{a}\hat{k} \ ; \ q' = -\frac{R}{a}q$$

$$\vec{E}'(\vec{r}) = -\frac{q}{4\pi\epsilon_0}\frac{R/a}{|\vec{r}-\vec{r}_2|^3}(\vec{r}-\vec{r}_2)$$

$\therefore$ La fuerza ejercida sobre q en $\vec{r} = \vec{r}_1$

$$\vec{F} = q\vec{E}'(\vec{r}_1) = -\frac{q^2}{4\pi\epsilon_0}\frac{R/a}{|\vec{r}_1-\vec{r}_2|^3}(\vec{r}_1-\vec{r}_2)$$

$$\vec{r}_1 - \vec{r}_2 = a\hat{k} - \frac{R^2}{a}\hat{k} = \frac{a^2-R^2}{a}\hat{k}$$

$$\Rightarrow \vec{F} = -\frac{q^2}{4\pi\epsilon_0}\frac{R/a}{\left(\frac{a^2-R^2}{a}\right)^3}\left(\frac{a^2-R^2}{a}\right)\hat{k} = \frac{q^2}{4\pi\epsilon_0}\frac{R/a}{\left(\frac{a^2-R^2}{a}\right)^2}(-\hat{k})$$

$$\Rightarrow \vec{F} = \frac{q^2}{4\pi\epsilon_0}\frac{Ra}{(a^2-R^2)^2}(-\hat{k})$$

(b) Densidad de carga inducida:

$$\sigma(\theta) = -\epsilon_0 \left.\frac{\partial \Phi}{\partial n}\right|_R = -\epsilon_0 \left.\frac{\partial \Phi}{\partial r}\right|_{r=R}$$

$$\sigma(\theta) = -\frac{q}{4\pi}\left\{\frac{\partial}{\partial r}\left[(r^2+a^2-2ar\cos\theta)^{-1/2} - R(a^2r^2+R^4-2R^2ar\cos\theta)^{-1/2}\right]\right\}_R$$

$$\sigma(\theta) = -\frac{q}{4\pi}\left\{-\frac{1}{\cancel{2}}(\ )^{-3/2}\cdot(\cancel{2}r-\cancel{2}a\cos\theta) + \frac{R}{\cancel{2}}(\ )^{-3/2}(\cancel{2}a^2r-\cancel{2}aR^2\cos\theta)\right\}$$

$$\sigma(\theta) = -\frac{q}{4\pi}\left\{\frac{R(a^2R-aR^2\cos\theta)}{(a^2R^2+R^4-2aR^3\cos\theta)^{3/2}} - \frac{R-a\cos\theta}{(R^2+a^2-2aR\cos\theta)^{3/2}}\right\}$$

$$\sigma(\theta) = -\frac{q}{4\pi}\left\{\frac{\cancel{R^3}(a^2/R - a\cos\theta)}{\cancel{R^3}(a^2+R^2-2aR\cos\theta)^{3/2}} - \frac{R-a\cos\theta}{(a^2+R^2-2aR\cos\theta)^{3/2}}\right\}$$

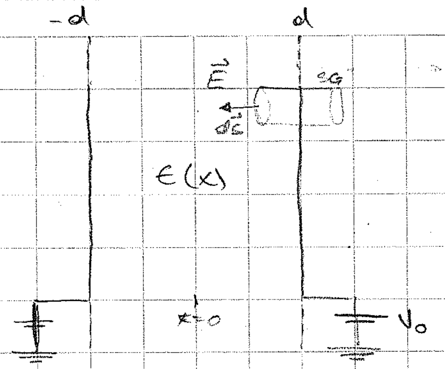
Fecha:

$$\sigma(\theta) = -\frac{q}{4\pi}\left\{\frac{\frac{a^2}{R} - \cancel{a\cos\theta} - R + \cancel{a\cos\theta}}{(a^2+R^2-2aR\cos\theta)^{3/2}}\right\}$$

$$\Rightarrow \sigma(\theta) = -\frac{q}{4\pi R}\frac{a^2-R^2}{(a^2+R^2-2aR\cos\theta)^{3/2}}$$

$$Q = \int \sigma(\theta)\, da = 2\pi R^2 \int_0^\pi \sigma(\theta)\sin\theta\, d\theta$$

$$Q = -\frac{qR}{2}\cdot(a^2-R^2)\int_0^\pi \frac{\sin\theta\, d\theta}{(a^2+R^2-2aR\cos\theta)^{3/2}}$$



$$\epsilon(x) = \frac{4\epsilon_0}{\left(\frac{x}{d}\right)^2+1}$$

$$\vec{E} \text{ en } |x| > d = 0$$

$$\oint_{SG} \vec{D}\cdot d\vec{a} = Q_{libre} = \sigma A$$

$$\epsilon(x)E\cdot \cancel{A} = \sigma \cancel{A} \Rightarrow E = \frac{\sigma}{\epsilon(x)} \Rightarrow \vec{E} = -\frac{\sigma}{\epsilon(x)}\hat{\imath}$$

$$\boxed{\vec{E}(x) = -\frac{\sigma}{4\epsilon_0}\left[\left(\frac{x}{d}\right)^2+1\right]\hat{\imath}}$$

El potencial:

$$V(x) = -\int_{-d}^{x}\vec{E}\cdot d\vec{l} \quad ; \quad d\vec{l} = dx\,\hat{\imath}$$

$$V(x) = \frac{\sigma}{4\epsilon_0}\int_{-d}^{x}\left[\left(\frac{x}{d}\right)^2+1\right]dx = \frac{\sigma}{4\epsilon_0}\left[\frac{x^3}{3d^2}+x\right]_{-d}^{x}$$

$$V(x) = \frac{\sigma}{4\epsilon_0}\left\{\frac{x^3}{3d^2}+x+\frac{d^3}{3d^2}+d\right\} = \frac{\sigma}{4\epsilon_0}\left\{\frac{x^3}{3d^2}+x+\frac{4d}{3}\right\}$$

$$\boxed{V(x) = \frac{\sigma}{4\epsilon_0}\left\{\frac{x^3}{3d^2}+x\right\}+\frac{\sigma d}{3\epsilon_0}}$$

Así, podemos calcular σ considerando que $V(d) = V_0$

$$V_0 = \frac{\sigma}{4\epsilon_0}\left\{\frac{d}{3}+d\right\}+\frac{\sigma d}{3\epsilon_0} = \frac{2\sigma d}{3\epsilon_0}$$

$$\Rightarrow \boxed{\sigma = \frac{3\epsilon_0 V_0}{2d}} \quad \text{en } x = d$$

$$\wedge \quad \boxed{\sigma = -\frac{3\epsilon_0 V_0}{2d}} \quad \text{en } x = -d$$

El vector polarización

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \frac{4\epsilon_0}{\left(\frac{x}{d}\right)^2 + 1} \vec{E}$$

$$\Rightarrow \vec{P} = \vec{E}\epsilon_0 \left\{ \frac{4}{\left(\frac{x}{d}\right)^2+1} - 1 \right\} = \epsilon_0 \vec{E} \left\{ \frac{4 - \left(\frac{x}{d}\right)^2 - 1}{\left(\frac{x}{d}\right)^2 + 1} \right\}$$

$$\vec{P} = \left(\frac{3 - \left(\frac{x}{d}\right)^2}{1 + \left(\frac{x}{d}\right)^2} \right) \epsilon_0 \vec{E} = \left(\frac{3 - \left(\frac{x}{d}\right)^2}{\cancel{1 + \left(\frac{x}{d}\right)^2}} \right) \cancel{\epsilon_0} \cdot \frac{(-\sigma)}{4\cancel{\epsilon_0}} \cancel{\left(\left(\frac{x}{d}\right)^2 + 1\right)} \hat{\imath}$$

$$\vec{P} = -\sigma \frac{\left(3 - \left(\frac{x}{d}\right)^2\right)}{4} \hat{\imath} = -\frac{3\epsilon_0 V_0}{2d} \cdot \frac{\left(3 - \left(\frac{x}{d}\right)^2\right)}{4} \hat{\imath}$$

$$\boxed{\vec{P} = -\frac{3\epsilon_0 V_0}{8d} \left[3 - \left(\frac{x}{d}\right)^2\right] \hat{\imath}}$$

las cargas de Polarización

$$\rho_p = -\vec{\nabla}\cdot\vec{P} = \frac{3\epsilon_0 V_0}{8d} \frac{d}{dx} \left[3 - \left(\frac{x}{d}\right)^2\right]$$

$$\boxed{\rho_p = -\frac{3\epsilon_0 V_0}{8d} \cdot \frac{2x}{d^2} = -\frac{3\epsilon_0 V_0}{4d^3} x}$$

$$\sigma_{P_d} = \vec{P}\cdot\hat{n} = \vec{P}\cdot(-\hat{\imath}) = \frac{3\epsilon_0 V_0}{4d} = \frac{\sigma}{2}$$

$$\sigma_{P_{-d}} = \vec{P}\cdot\hat{n} = \vec{P}\cdot(\hat{\imath}) = -\frac{3\epsilon_0 V_0}{4d} = -\frac{\sigma}{2}$$

la energía: $W = \frac{1}{2} C V_0^2$

$C = ?$

$$\sigma = \frac{Q}{A} = \frac{3\epsilon_0 V_0}{2d} \qquad C = \frac{Q}{V_0} = \frac{3\epsilon_0 A}{2d}$$

$$U = \frac{1}{2} C\cdot V_0^2 = \frac{3\epsilon_0 A V_0^2}{4d}$$

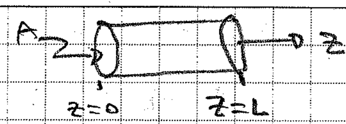
la fuerza entre las placas $\vec{F} = \frac{1}{2} Q \vec{E}$

$$\vec{F} = \frac{1}{2}\sigma\cdot A\cdot\left(\frac{-\sigma}{4\epsilon_0}\right)\left(\left(\frac{d}{d}\right)^2 + 1\right)\hat{\imath}$$

$$\vec{F} = \frac{\sigma^2 A}{4\epsilon_0}\hat{\imath} = \frac{9\epsilon_0^2 V_0^2 A}{4\epsilon_0\cdot 4d^2} = \frac{9\epsilon_0 V_0^2 A}{16 d^2}$$

Fecha:

3) a) $\eta(z) = \eta_0 \cos\left(\frac{\pi z}{4L}\right)$



$A = \pi a^2$

$$dR = \frac{\eta(z)\,dz}{A}$$

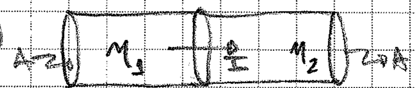
$$\int_0^R dR' = \frac{\eta_0}{\pi a^2}\int_0^L \cos\left(\frac{\pi z}{4L}\right) dz$$

$$R = \frac{\eta_0}{\pi a^2}\left[\frac{\sin\left(\frac{\pi z}{4L}\right)}{\pi/4L}\right]_0^L =$$

$$R = \frac{\eta_0}{\pi a^2}\cdot\frac{4L}{\pi}\sin\left(\frac{\pi}{4}\right) = \frac{\overset{2}{\cancel{4}}L\eta_0}{\pi^2 a^2}\cdot\frac{\sqrt{2}}{\cancel{2}}$$

$$\boxed{R = \frac{2\sqrt{2}\,L\,\eta_0}{(\pi a)^2}}$$

b)



Ya que las secciones son iguales a A, tenemos que

$$J = \frac{I}{A} \Rightarrow I = J\cdot A$$

Si los conductores son óhmicos, entonces

$$\vec{J} = g\vec{E} \Rightarrow \vec{E} = \frac{\vec{J}}{g} = \eta\vec{J}$$

Ya que el campo (normal) es discontinuo se tiene que

$$(E_2\hat{n} - E_1\hat{n})\cdot\hat{n} = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$\eta_2 J - \eta_1 J = (\eta_2 - \eta_1)J = \frac{Q}{\epsilon_0 \cancel{A}} = (\eta_2-\eta_1)\frac{I}{\cancel{A}}$$

$$\Rightarrow \boxed{Q = \epsilon_0 I(\eta_2 - \eta_1)}$$

4.- a) La carga en el condensador

$$Q(t) = Q_0\left(1 - e^{-t/\tau}\right) \; ; \; Q_0 = C\xi$$

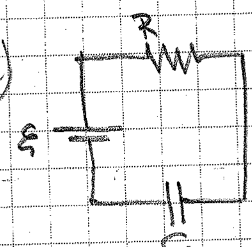
$$Q(0) = 0$$

b) El voltaje en el condensador

$$V(t) = \frac{Q(t)}{C} \underset{t\to\infty}{\Rightarrow} V \to \xi$$

(Polaridad inversa)

c)



$$R = R_1 + \left(\frac{R_2 R_3}{R_2 + R_3}\right) = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2 + R_3}$$

La corriente en el circuito es

$$I = I_0\, e^{-t/\tau} \; ; \; I_0 = \frac{\xi}{R}$$

La corriente máxima que circula es I_0 y ésta es la misma que circula por R_1

$$\Rightarrow \boxed{I_0 = \frac{(R_2 + R_3)}{R_1R_2 + R_1R_3 + R_2R_3} \cdot \xi}$$

d) La corriente que circula por R_2 y R_3 es

$$I_2 = \frac{R_3}{R_2 + R_3}\, I \quad \wedge \quad I_3 = \frac{R_2}{R_2 + R_3}\, I$$

La máxima potencia disipada será en $t=0$

$$P(t=0) = P_0 = I_3^2 R_3 = \frac{R_2^2 R_3}{(R_2 + R_3)^2}\, I_0^2$$

$$P_0 = \frac{R_2^2 R_3}{(R_2+R_3)^2} \cdot \frac{\xi^2}{R^2} = \frac{R_2^2 R_3 \xi^2}{\cancel{(R_2+R_3)^2}} \cdot \frac{\cancel{(R_2+R_3)^2}}{(R_1R_2 + R_1R_3 + R_2R_3)^2}$$