— Dipolo magnético puntual —

Complemento I

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \vec{r}}{r^3}$$

$$\Downarrow$$

$$\vec{B} = \nabla \times \vec{A} \Rightarrow B_i = (\nabla \times \vec{A})_i$$

$$\Downarrow$$

$$B_i = \epsilon_{ijk} \frac{\partial}{\partial x_j} A_k \qquad \text{con } A_k = \frac{\mu_0}{4\pi} \frac{(\vec{m} \times \vec{r})_k}{r^3}$$

$$= \frac{\mu_0}{4\pi} \frac{\epsilon_{k\ell n} m_\ell x_n}{r^3}$$

$$\therefore B_i = \frac{\mu_0}{4\pi} m_\ell \epsilon_{ijk} \epsilon_{k\ell n} \frac{\partial}{\partial x_j}\left(\frac{x_n}{r^3}\right)$$

Donde $\frac{\partial}{\partial x_j}\left(\frac{x_n}{r^3}\right) = \frac{\delta_{jn}}{r^3} + x_n \frac{\partial}{\partial x_j}(r^{-3}) = \frac{\delta_{jn}}{r^3} - 3\frac{x_n}{r^4}\frac{\partial r}{\partial x_j}$

siendo $\frac{\partial r}{\partial x_j} = \frac{\partial (x_s x_s)^{1/2}}{\partial x_j} = \frac{1}{2} \cdot \frac{2 x_s}{r} \frac{\partial x_s}{\partial x_j} = \frac{x_s}{r} \delta_{sj} = \frac{x_j}{r}$

luego $\frac{\partial}{\partial x_j}\left(\frac{x_n}{r^3}\right) = \frac{\delta_{jn}}{r^3} - 3\frac{x_n x_j}{r^5}$ y $\epsilon_{klj}\epsilon_{ken} = \delta_{ie}\delta_{jn} - \delta_{je}\delta_{in}$ $\overline{\text{II}}$

$$\Downarrow$$

$$\therefore\ B_i = \frac{\mu_0}{4\pi} m_e \left(\delta_{ie}\delta_{jn} - \delta_{je}\delta_{in}\right)\left(\frac{\delta_{jn}}{r^3} - \frac{3x_n x_j}{r^5}\right)$$

$$= \frac{\mu_0}{4\pi}\left(m_i\delta_{jn} - m_j\delta_{in}\right)\left(\frac{\delta_{jn}}{r^3} - 3\frac{x_n x_j}{r^5}\right)$$

$$= \frac{\mu_0}{4\pi}\left(m_i\frac{\delta_{jj}}{r^3} - 3m_i\frac{x_j x_j}{r^5} - \frac{m_i}{r^3} + 3\frac{m_j x_j x_i}{r^5}\right)$$

$$B_i = \frac{\mu_0}{4\pi}\left(\cancel{\frac{3m_i}{r^3}} - \cancel{\frac{3m_i r^2}{r^5}} - \frac{m_i}{r^3} + \frac{3(\vec{m}\cdot\vec{r})x_i}{r^5}\right)$$

$$B_i = \frac{\mu_0}{4\pi}\left[\frac{3(\vec{m}\cdot\vec{r})}{r^5}x_i - \frac{m_i}{r^3}\right]$$

$$\Downarrow$$

Finally

$$\vec{B} = \frac{\mu_0}{4\pi}\left[\frac{3(\vec{m}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3}\right]$$