Astronomía estelar

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Clase 8
Atmósferas estelares

Chapter 9 Stellar Atmospheres: Radiation fields

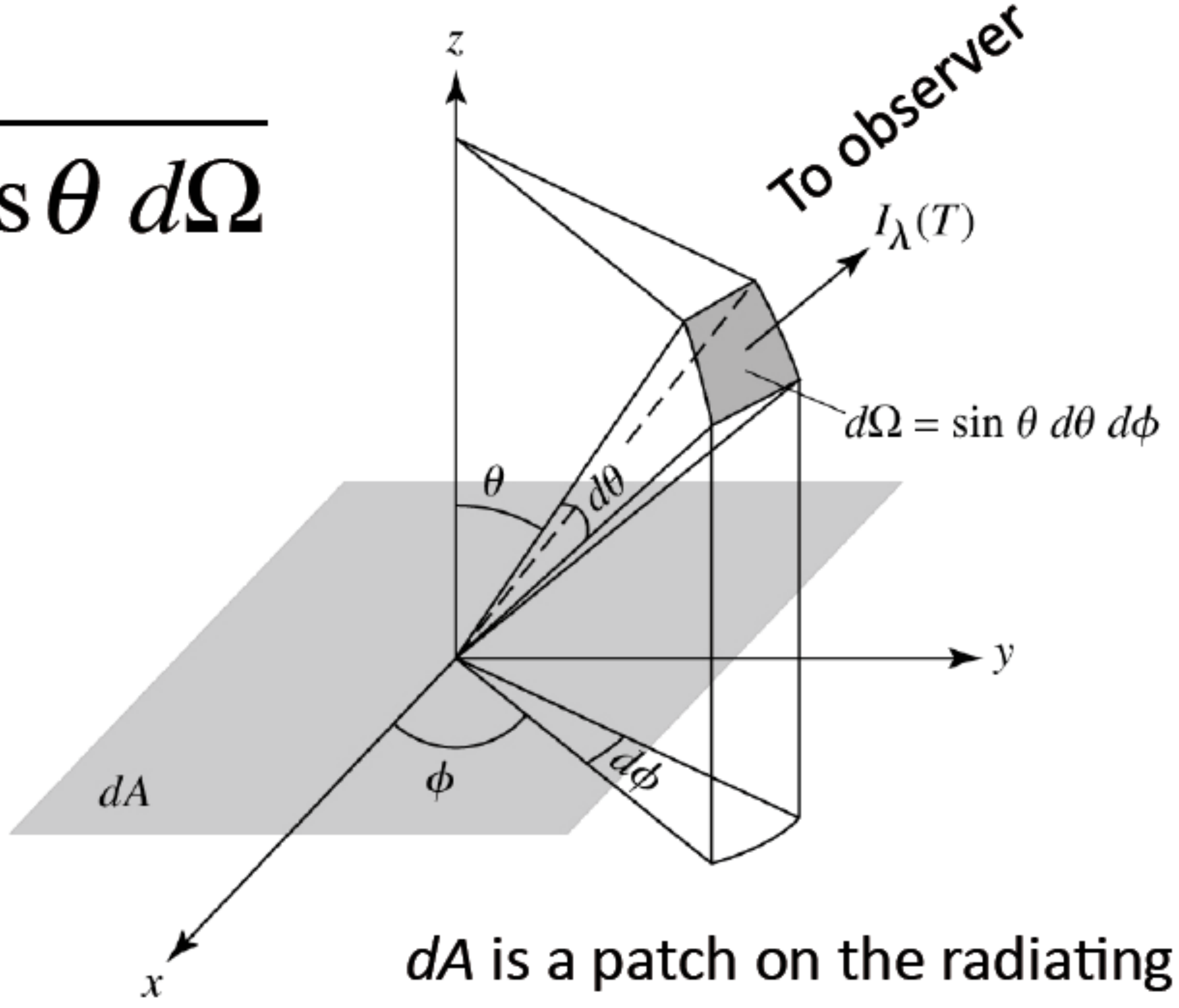
Let's jump ahead just a bit and look at some definitions.

The light we see emerging from a star comes from the outer layers. The temperature, density and composition of the outer layers determines the features of the stellar spectrum.

$$E_\lambda \, d\lambda = I_\lambda \, d\lambda \, dt \, dA \, \cos\theta \, d\Omega = I_\lambda \, d\lambda \, dt \, dA \, \cos\theta \, \sin\theta \, d\theta \, d\phi$$

$$I_\lambda = \frac{\partial I}{\partial \lambda} = \frac{E_\lambda d\lambda}{d\lambda \, dt \, dA \cos\theta \, d\Omega}$$

Specific intensity, $E_\lambda \, d\lambda$ Energy passing through a solid angle from a point on the surface, at a given time, in a direction θ, with wavelength between λ and $\lambda + d\lambda$

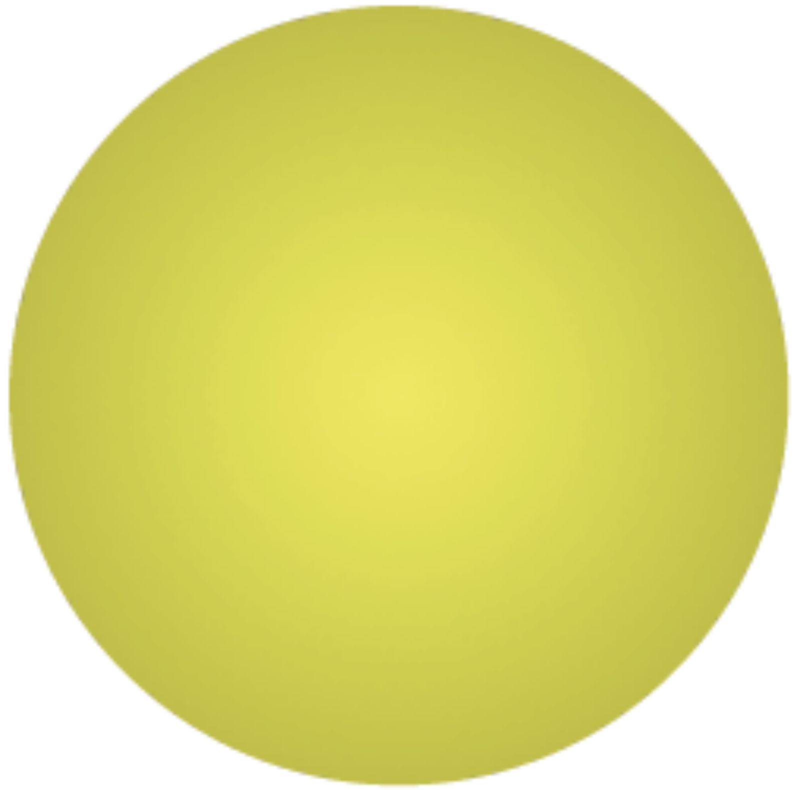


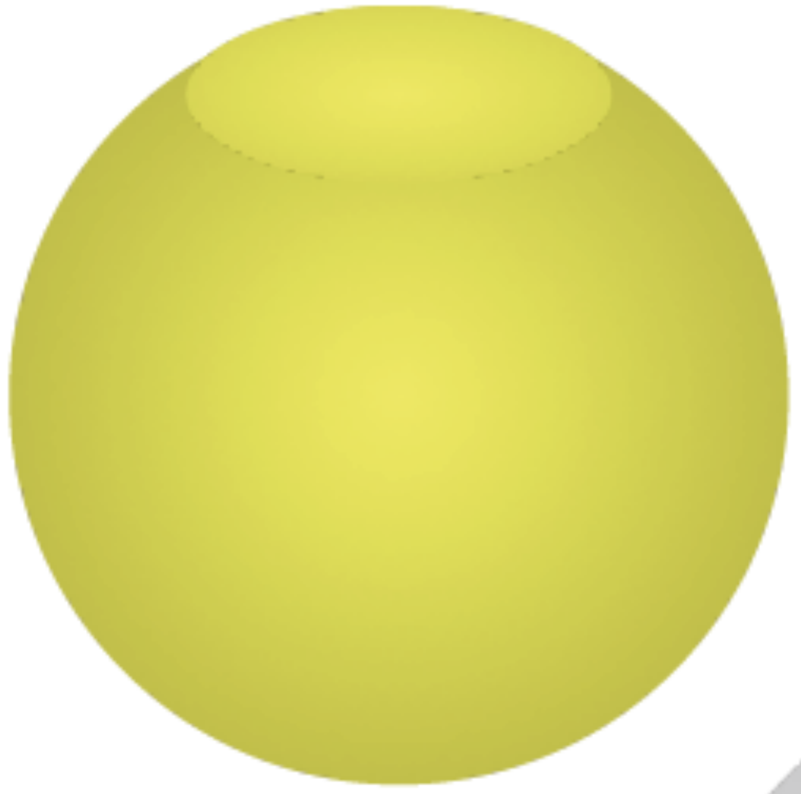
dA is a patch on the radiating surface of the star

$$\langle I_\lambda \rangle = \frac{1}{4\pi}\int I_\lambda \, d\Omega = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} I_\lambda \; \sin\theta \; d\theta \; d\phi$$

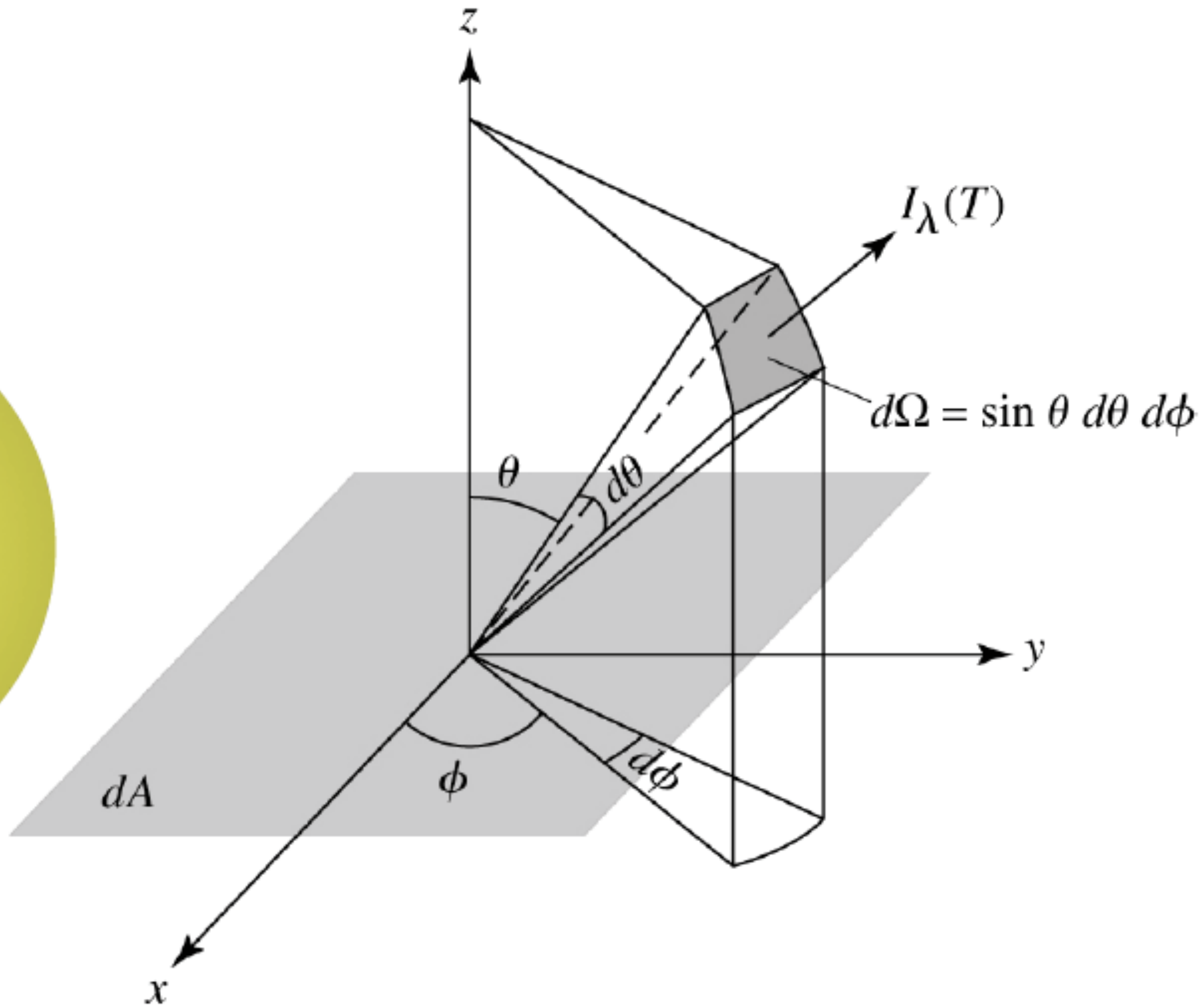
For an isotropic field (same intensity in all directions)

$$\langle I_\lambda \rangle = I_\lambda$$





isotropic



Chapter 9: Stellar Atmospheres

Blackbody radiation is isotropic. For blackbody radiation:

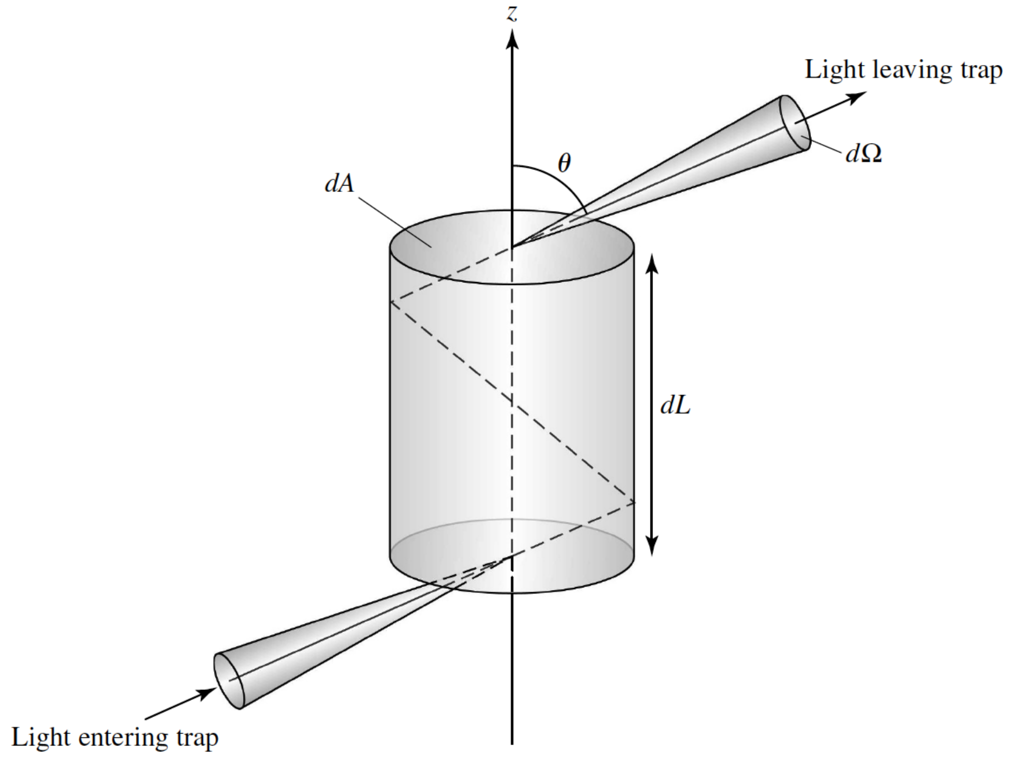
$$\langle I_\lambda \rangle = B_\lambda$$

Specific energy density with wavelength between λ and $\lambda + d\lambda$ is defined as:

$$u_\lambda d\lambda = \frac{1}{c} \int I_\lambda d\lambda \, d\Omega$$

$$E_\lambda \, d\lambda = I_\lambda \, d\lambda \, dt \, dA \cos\theta \, d\Omega = I_\lambda \, d\lambda \, dA \, d\Omega \frac{dL}{c}$$

$dA\,dL$ is just the volume of the trap



To determine how much energy is contained within the radiation field, we can use a "trap" consisting of a small cylinder of length dL, open at both ends, with perfectly reflecting walls inside; see Fig. Light entering the trap at one end travels and (possibly) bounces back and forth until it exits the other end of the trap. The energy inside the trap is the same as what would be present at that location if the trap were removed. The radiation that enters the trap at an angle θ travels through the trap in a time d t = d L/(c cos θ). Thus the amount of energy inside the trap with a wavelength between λ and λ + dλ that is due to the radiation that enters at angle θ is

Chapter 9: Stellar Atmospheres

For isotropic radiation:

$$u_\lambda d\lambda = \frac{4\pi}{c}\langle I_\lambda\rangle d\lambda$$

$$\begin{aligned} u_\lambda\, d\lambda &= \frac{1}{c}\int I_\lambda\, d\lambda\, d\Omega \\ &= \frac{1}{c}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} I_\lambda\, d\lambda\ \sin\theta\, d\theta\, d\phi \\ &= \frac{4\pi}{c}\langle I_\lambda\rangle\, d\lambda. \end{aligned}$$

For blackbody radiation:

$$\langle I_\lambda\rangle = B_\lambda \qquad B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$

Energy density in blackbody radiation for a characteristic wavelength:

$$\boxed{u_\lambda\, d\lambda = \frac{4\pi}{c}B_\lambda\, d\lambda = \frac{8\pi hc/\lambda^5}{e^{hc/\lambda kT}-1}\, d\lambda.}$$

Chapter 9: Stellar Atmospheres

Of course:

$$u = \int_0^\infty u_\lambda d\lambda = \int_0^\infty u_\nu d\nu$$

For blackbody radiation, $I_\lambda = B_\lambda$

$$u = \frac{4\pi}{c} \int_0^\infty B_\lambda(T)\; d\lambda = \frac{4\sigma\, T^4}{c} = aT^4$$

radiation constant

$$a = 4\sigma / c = 7.565767 \times 10^{-16} Jm^{-3} K^{-4}$$

Chapter 9: Stellar Atmospheres

Specific Radiative Flux:

$$F_\lambda d\lambda = \int I_\lambda d\lambda \cos\theta \, d\theta \, d\Omega$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} I_\lambda \cos\theta \sin\theta \, d\theta \, d\phi$$

$F\lambda$ $d\lambda$ is the *net* energy having a wavelength between λ and $\lambda + d\lambda$ that passes each second through a unit area in the direction of the z-axis:

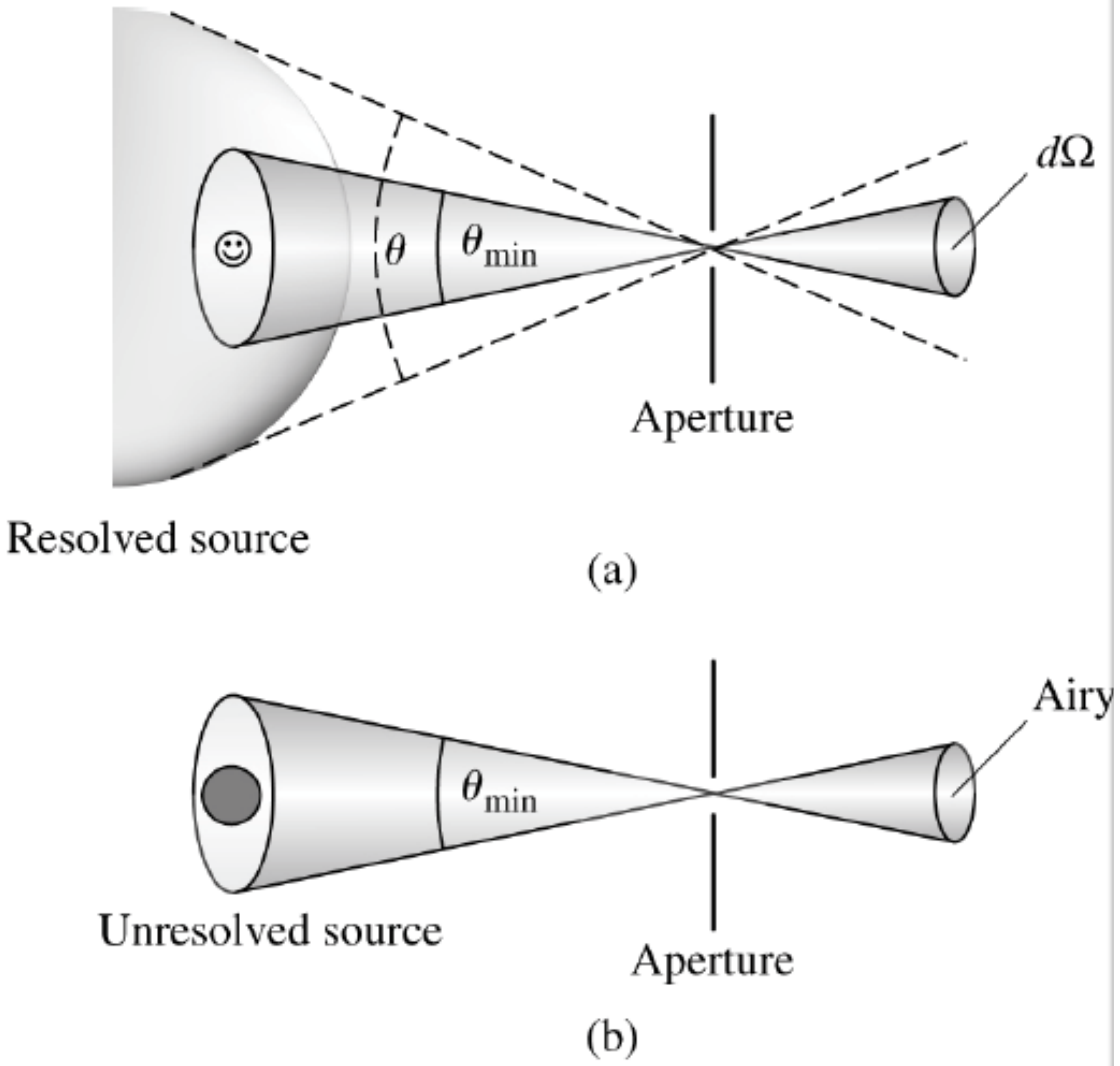
The factor of $\cos\theta$ determines the z-component of a light ray and allows the cancelation of oppositely directed rays. For an isotropic radiation field there is no net transport of energy, and so $F\lambda = 0$.

Chapter 9: Stellar Atmospheres

Both the radiative flux and the specific intensity measure light received from a celestial source.

When you point a photometer at a light source, which of these are you measuring?

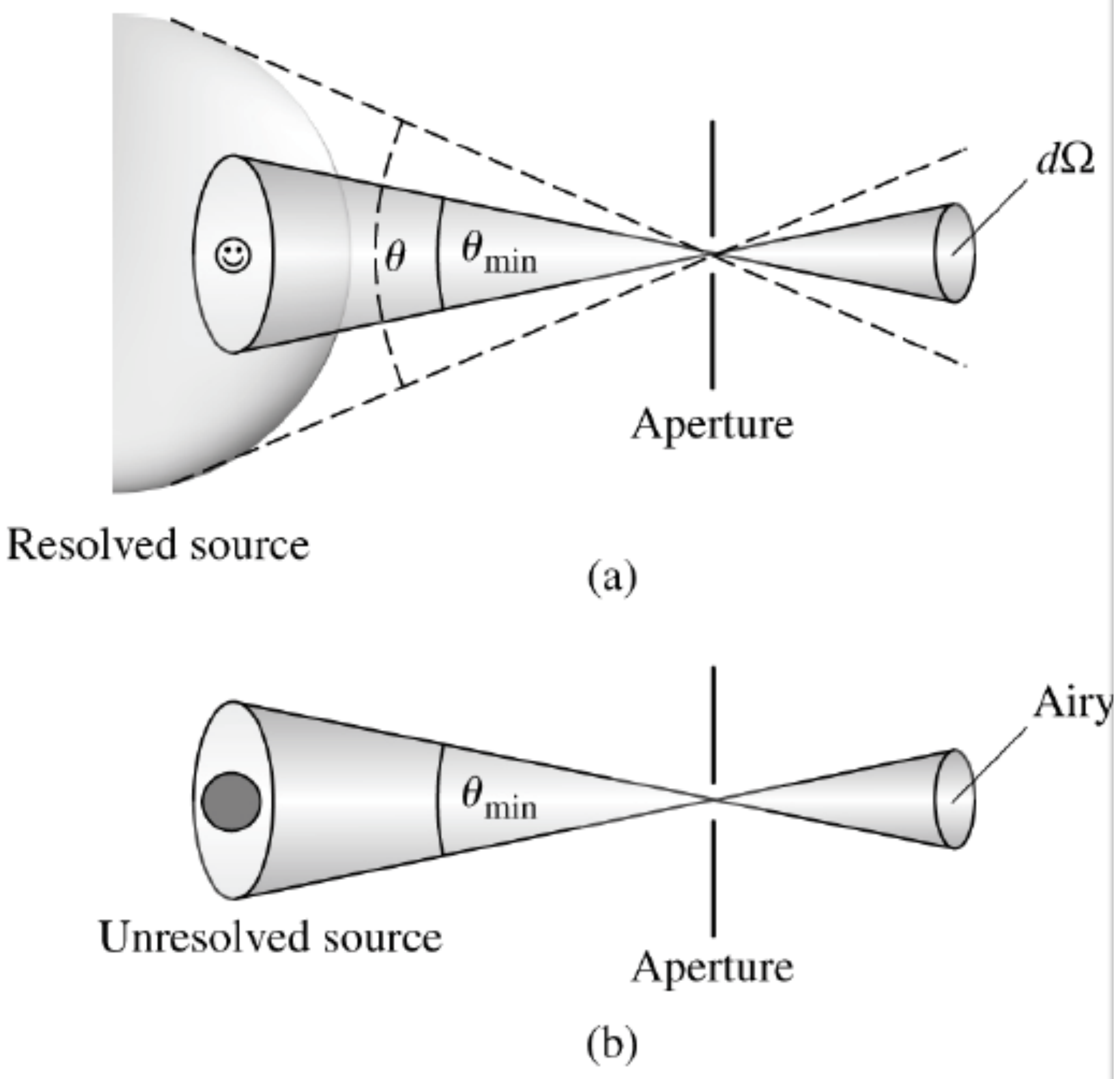
Chapter 9: Stellar Atmospheres



Both the radiative flux and the specific intensity describe the light received from a celestial source. Which of these quantities is actually measured by a telescope's photometer, pointed at the source of light. The answer depends on whether the source is resolved by the telescope. Top figure shows a source of light, uniform over its entire surface, that is resolved by the telescope; the angle θ subtended by the source as a whole is much larger than θ_{min}, the smallest angle resolvable according to Rayleigh's criterion. In this case, what is being measured is the *specific intensity*, the amount of energy per second passing through the aperture area into the solid angle Omega_{min} defined by θ_{min}. For example, at a wavelength of 501 nm, the measured value of the specific intensity at the center of the Sun's disk is

$$I501 = 4.03 \times 10^{13} \text{ W m}^{-3}\text{sr}^{-1}.$$

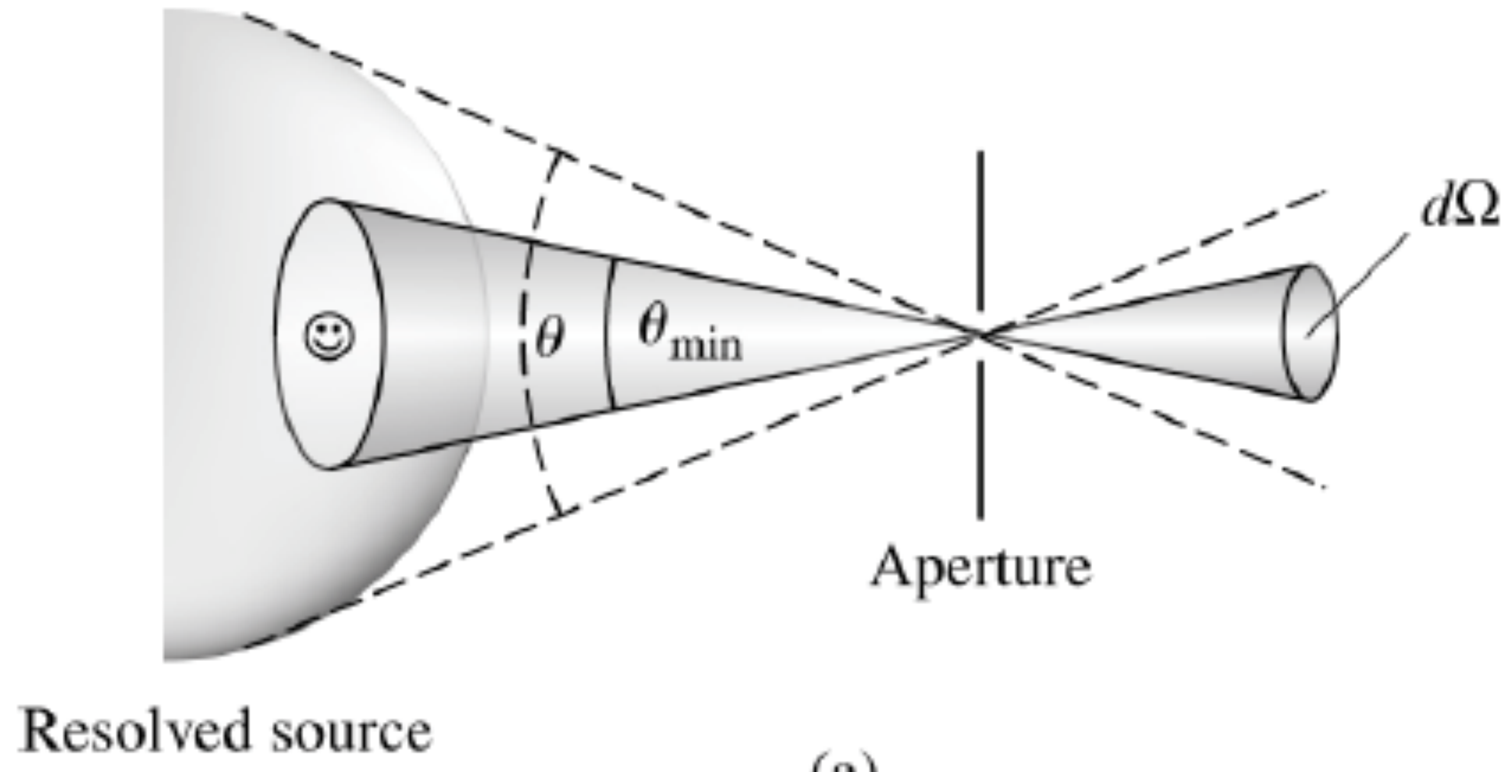
Now imagine that the source is moved twice as far away. According to the inverse square law for light, there will be only $(1/2)^2 = 1/4$ as much energy received from each square meter of the source. If the source is still resolved, however, then the amount of source area that contributes energy to the solid angle Omega_{min} has increased by a factor of 4, resulting in the same amount of energy reaching each square meter of the detector. The specific intensity of light rays from the source is thus measured to be constant.



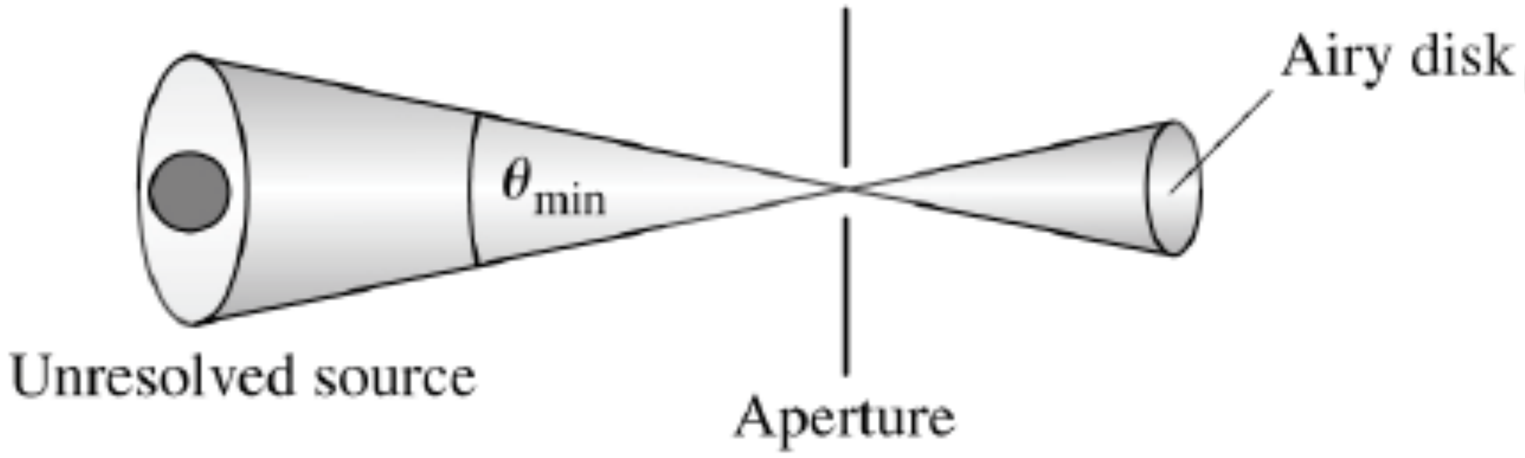
However, it is the *radiative flux* that is measured for an unresolved source. As the source recedes farther and farther, it will eventually subtend an angle θ smaller than θmin, and it can no longer be resolved by the telescope. When θ < θmin, the energy received from the entire source will disperse throughout the diffraction pattern (the Airy disk and rings) determined by the telescope's aperture. Because the light arriving at the detector leaves the surface of the source at all angles [see Bottom Fig.], the detector is effectively integrating the specific intensity over all directions. This is just the definition of the radiative flux, Eq. down. As the distance *r* to the source increases further, the amount of energy falling within the Airy disk (and consequently the value of the radiative flux) decreases as $1/r^2$, as expected.

$$F_\lambda \, d\lambda = \int I_\lambda \, d\lambda \, \cos\theta \, d\Omega = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} I_\lambda \, d\lambda \, \cos\theta \, \sin\theta \, d\theta \, d\phi.$$

Chapter 9 Stellar Atmospheres: Radiation fields



(a)



(b)

Resolved source: specific intensity, $\boxed{I_\lambda}$

$$\langle I_\lambda \rangle = \frac{1}{4\pi}\int I_\lambda \, d\Omega = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} I_\lambda \; \sin\theta \, d\theta \, d\phi$$

Question: Does $\boxed{I_\lambda}$ decrease as $1/r^2$?

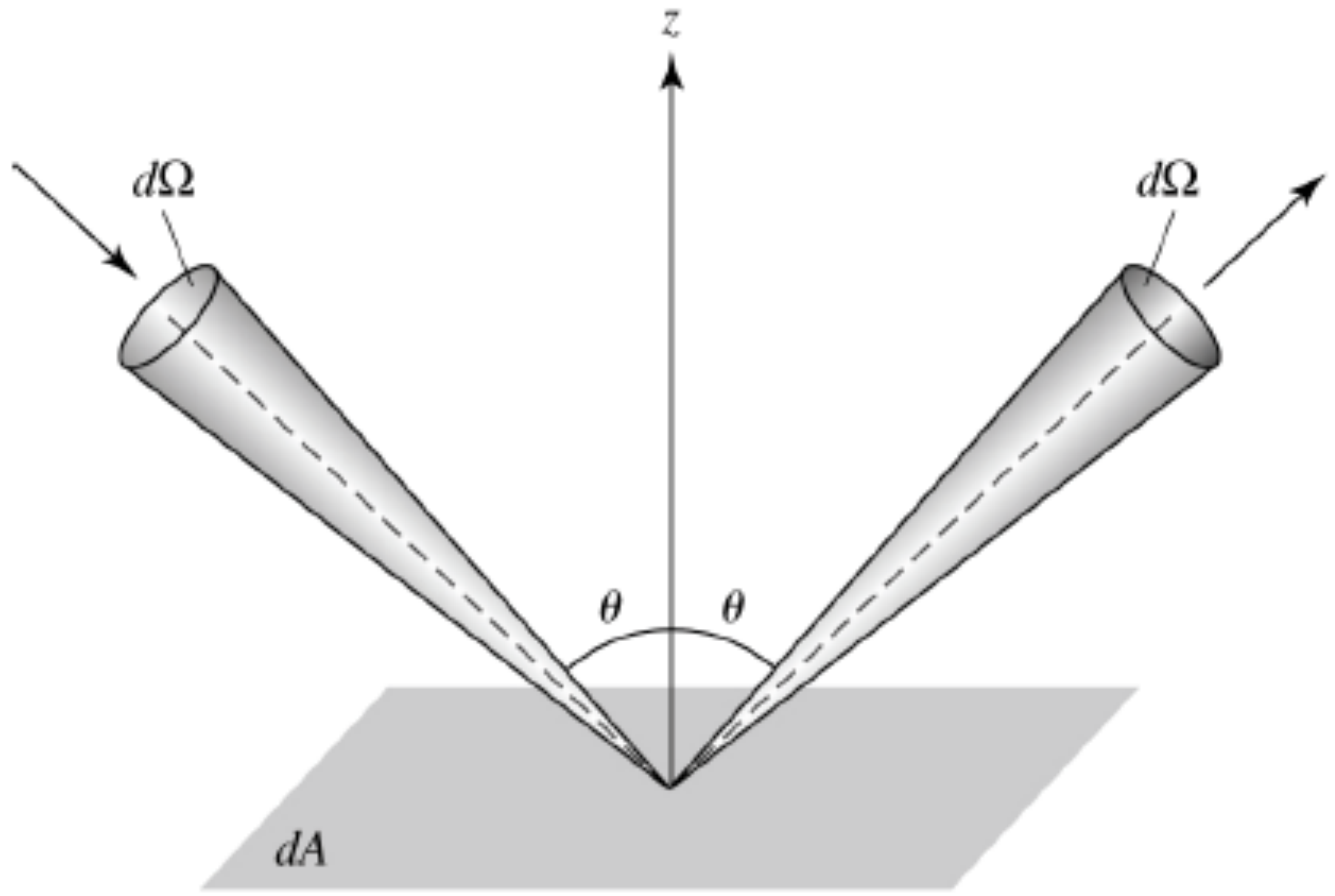
Not necessarily - more energy may be intercepted as the solid angle grows

Unresolved source: radiative flux, $\boxed{F_\lambda}$

$$F_\lambda d\lambda = \int I_\lambda d\lambda \cos\theta \, d\theta \, d\Omega$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} I_\lambda \cos\theta \sin\theta \, d\theta \, d\phi$$

Chapter 9: Stellar Atmospheres



Radiation pressure

Case 1: Photons incident on a perfectly reflecting surface

Momentum: p = E/c

$$
\begin{aligned}
dp_\lambda\, d\lambda &= \left[(p_\lambda)_{\mathrm{final},z} - (p_\lambda)_{\mathrm{initial},z}\right] d\lambda \\
&= \left[\frac{E_\lambda \cos\theta}{c} - \left(-\frac{E_\lambda \cos\theta}{c}\right)\right] d\lambda \\
&= \frac{2\,E_\lambda \cos\theta}{c}\, d\lambda \\
&= \frac{2}{c}\, I_\lambda\, d\lambda\, dt\, dA\, \cos^2\theta\, d\Omega,
\end{aligned}
$$

Recall:

$$\boxed{E_\lambda = I_\lambda dt\ dA \cos\theta\ d\Omega}$$

Chapter 9: Stellar Atmospheres

$$dp_\lambda d\lambda = \frac{2}{c} I_\lambda d\lambda \; dt \; dA \; \cos^2\theta \; d\Omega$$

$$\frac{dp/dt}{dA} = \frac{F}{dA} = \frac{2}{c} I_\lambda d\lambda \cos^2\theta \; d\Omega$$

Force per unit Area = pressure

Case 1: Reflection

$$P_{rad,\lambda} d\lambda = \frac{2}{c} \int_{hemisphere} I_\lambda d\lambda \; \cos^2\theta \; d\theta \; d\Omega$$

$$= \frac{2}{c} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_\lambda d\lambda \; \cos^2\theta \sin\theta \; d\theta \; d\phi$$

Chapter 9: Stellar Atmospheres

Case 2: Transmission - lose factor of 2 because no change in momentum on reflection

$$P_{rad,\lambda} d\lambda = \frac{1}{c} \int_{sphere} I_\lambda d\lambda \, \cos^2\theta \, d\theta \, d\Omega$$

$$= \frac{1}{c} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} I_\lambda d\lambda \, \cos^2\theta \sin\theta \, d\theta \, d\phi$$

$$= \frac{4\pi}{3c} I_\lambda d\lambda$$

For blackbody radiation:

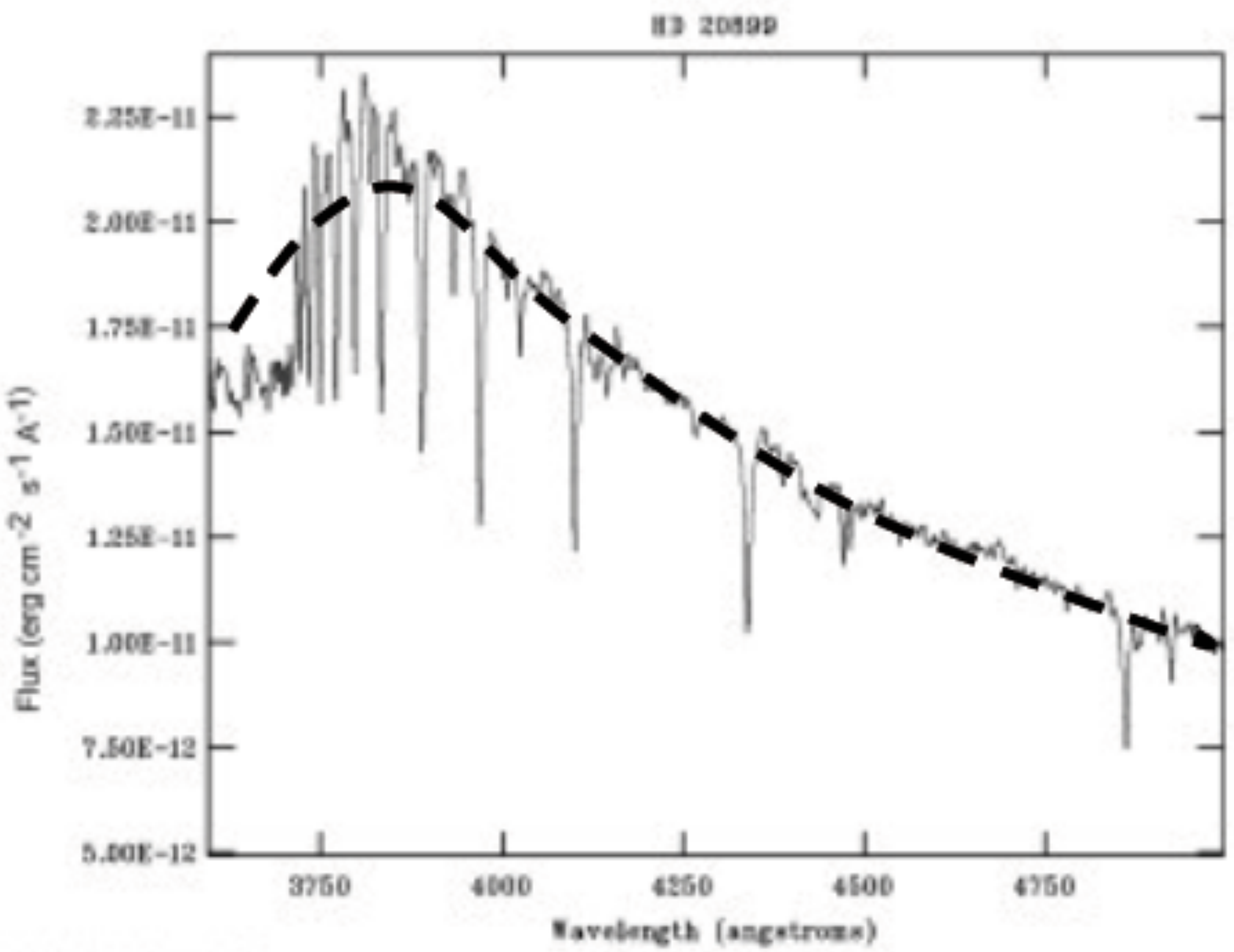
$$P_{rad} = \frac{4\pi}{3c} \int_0^\infty B_\lambda(T) d\lambda = \frac{4\sigma T^4}{3c} = \frac{1}{3} a T^4$$

$$\boxed{P_{rad} = \frac{1}{3} u}$$

For comparison, the radiation pressure of an ideal monatomic gas is 2/3 its energy density

Just as the pressure of a gas exists throughout the volume of the gas and not just at the container walls, the radiation pressure of a "photon gas" exists everywhere in the radiation field. Imagine removing the reflecting surface dA in the Fig. and replacing it with a mathematical surface. Leading factor of 2 (which originated in the change in momentum upon reflection of the photons) is removed!

Chapter 9 Stellar Atmospheres: Opacity



We approximate a stellar photosphere as a blackbody (e.g, dashed curve) but the radiation actually deviates substantially from this because absorption lines remove light from the "continuum" (a contraction of "continuous spectrum").

Removal of flux from absorption lines is called ***line blanketing***. In other parts of the spectrum (e.g., the ultraviolet) there may be contribution to flux at particular wavelengths from emission.

Chapter 9: Stellar Atmospheres

Effective temperature: from the Stefan-Boltzmann law characterizes the temperature at a particular depth in the star and is widely used as the global descriptive temperature

$$L = 4\pi R^2 \sigma T^4$$

Excitation temperature: defined by the Boltzmann equation

$$\frac{N_b}{N_a} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$

Ionization temperature: defined by the Saha equation

$$\frac{N_{i+1}}{N_i} = \frac{2kTZ_{i+1}}{P_e Z_i} \left(\frac{2\pi \ m_e kT}{h^2} \right)^{3/2} e^{-\chi_i / kT}$$

Kinetic temperature: from the Maxwell-Boltzmann velocity distribution

$$n_v dv = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{\frac{-mv^2}{2kT}} 4\pi v^2 dv$$

Color temperature: fitting a Planck function to the continuum

Chapter 9: Stellar Atmospheres

Thermodynamic equilibrium

Every process (absorption of photons) is balanced by an inverse process (emission of photons).

Local thermodynamic equilibrium (LTE)

A star can hardly be in thermodynamic equilibrium. There is an outward flow of energy with temperature T=T(R). However, if the distance where temperature changes is large compared to the m.f.p. of particles and photons, then there is LTE in that region.

What is the temperature scale height?

How does it compare to the average distances for atoms between collisions?

Chapter 9: Stellar Atmospheres

Temperature Scale Height

According to model solar atmospheres (specifies temperature, density etc as a function of optical depth or radius) the temperature near the photosphere changes from 5580K to 5790K over a distance of 25 km. Scale height, H_T, is given by:

$$H_T \equiv \frac{T}{dT/dr} = \frac{5685K}{(5790K - 5580K)/25.0km} = 677km$$

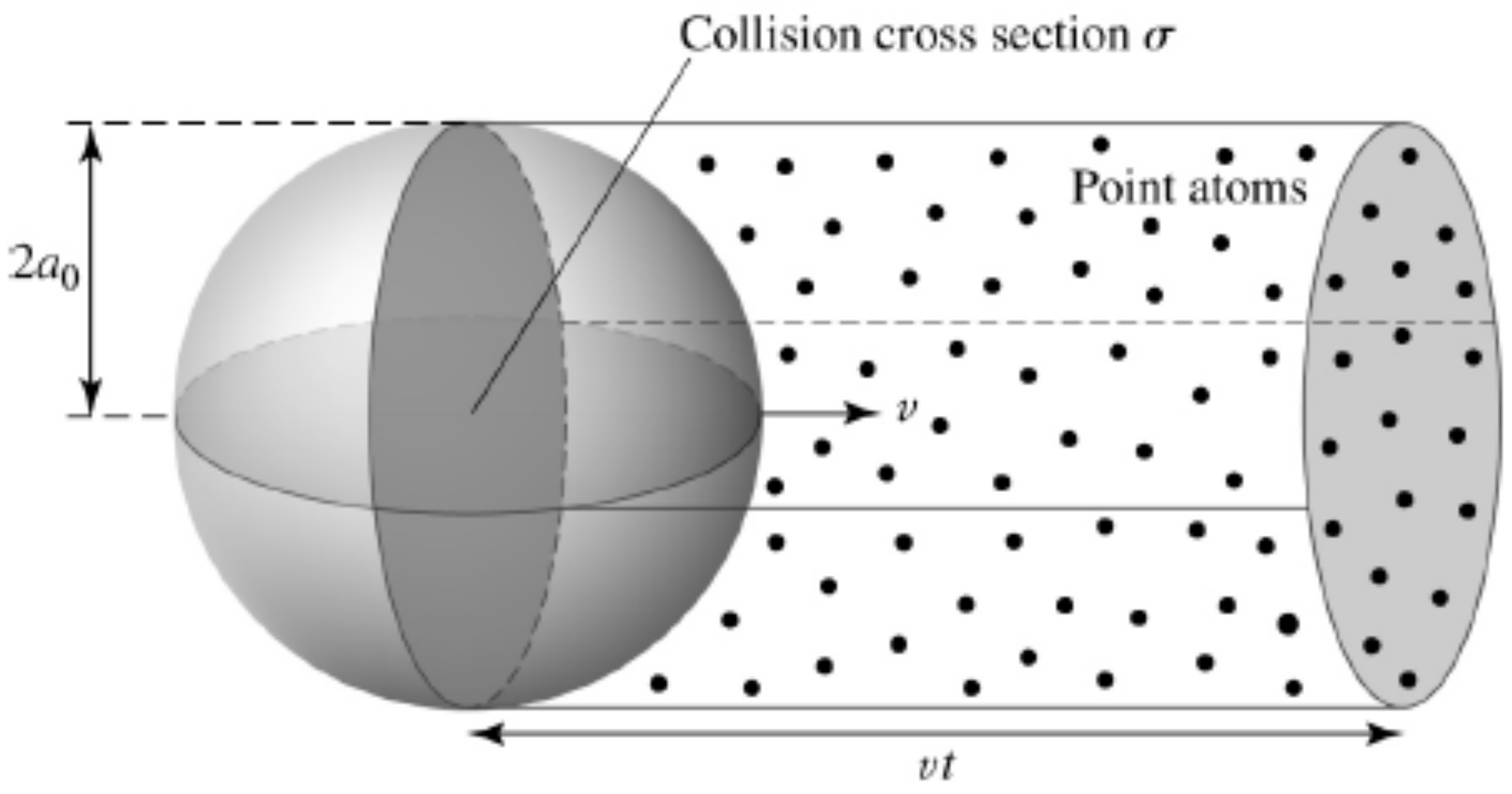
Distance between atomic collisions

According to model solar atmospheres the density of the photosphere in this same temperature region is about:

$$\rho = 2.1 \times 10^{-4}\, km\ m^{-3}$$

and it consists primarily of *neutral H atoms* in the ground state (as you'd expect, now that you know about the Boltzmann eqna and Saha eqn!)

$$n = \frac{\rho}{m_H} = 1.25 \times 10^{23}\, m^{-3}$$



Two atoms collide if their centers pass within two Bohr radii

Equivalently: imagine a single atom with a radius of $2a_o$ moving with speed *v* through a collection of points that represent the centers of other H atoms. The m.f.p. is the distance traveled divided by the number of atoms encountered (number density times the volume swept out).

volume V are $nV = n\sigma vt$ point

$$l_{mfp} = \frac{vt}{nV} = \frac{vt}{n\sigma vt} = \frac{1}{n\sigma} = 2.27\times10^{-4}\,m$$ **mean free path**

$$n = 1.25\times10^{23}\,m^{-3}$$

$$\sigma = \pi(2a_o)^2 = 3.52\times10^{-20}\,m^2$$

Chapter 9: Stellar Atmospheres

In the photosphere:

Temperature scale height: 677 km = 677,000 m

Distance between atomic collisions: 2.27×10^{-4} m

The temperature scale height exceeds the mfp of H atoms by factor of 3 billion. LTE seems like a reasonable assumption in the solar photosphere, but stay tuned…