5.25) if B uniform; show $A(\vec{r}) = -\frac{1}{2}(\vec{r} \times \vec{B})$ works, i.e. $\vec{\nabla} \cdot \vec{A} = 0$
$\wedge$ $\vec{\nabla} \times \vec{A} = \vec{B}$

$$\vec{\nabla} \cdot \vec{A} = -\frac{1}{2} \vec{\nabla} \cdot (\vec{r} \times \vec{B}) = -\frac{1}{2} \left\{ \vec{B} \cdot (\vec{\nabla} \times \vec{r}) - \vec{r} \cdot (\vec{\nabla} \times \vec{B}) \right\} = -\frac{1}{2} \left\{ \vec{B} \cdot \vec{0} - \vec{r} \cdot \vec{0} \right\} = 0$$

$$\vec{\nabla} \times \vec{A} = -\frac{1}{2} \vec{\nabla} \times (\vec{r} \times \vec{B}) = -\frac{1}{2} \left\{ \vec{r}(\vec{\nabla} \cdot \vec{B}) - \vec{B}(\vec{\nabla} \cdot \vec{r}) + (\vec{B} \cdot \vec{\nabla})\vec{r} - (\vec{r} \cdot \vec{\nabla})\vec{B} \right\}$$

$$= -\frac{1}{2} \left\{ -3\vec{B} + \left[ B_x \frac{\partial}{\partial x} + B_y \frac{\partial}{\partial y} + B_z \frac{\partial}{\partial z} \right] (x\hat{i} + y\hat{j} + z\hat{k}) \right\}$$

$$= -\frac{1}{2} \left\{ -3\vec{B} + \vec{B} \right\} = \vec{B} \quad \square$$

Nov 16:

**1.2)** $\vec{\nabla}\cdot\vec{A} = 0$ Dem # $\vec{\nabla}$ : opera sobre la variables no primadas

$$\vec{\nabla}\cdot\frac{\mu_0}{4\pi}\int_V \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}\,dv' = \frac{\mu_0}{4\pi}\int_{V'}\vec{\nabla}\cdot\left[\frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}\right]dv'$$ # Ident: $\vec{\nabla}\cdot(f\vec{A}) = \vec{A}\cdot\vec{\nabla}f + f\vec{\nabla}\cdot\vec{A}$

$$\Rightarrow \vec{\nabla}\left(\frac{\vec{J}'}{|\vec{r}-\vec{r}'|}\right) = \vec{J}'\cdot\vec{\nabla}\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) + \frac{1}{|\vec{r}-\vec{r}'|}\underbrace{\vec{\nabla}\cdot\vec{J}'}_{0} = \vec{J}'\cdot -\vec{\nabla}'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)$$ # Queremos un $\vec{\nabla}'\cdot$ dentro de la int

$$\Rightarrow \vec{\nabla}'\cdot\left(\frac{\vec{J}'}{|\vec{r}-\vec{r}'|}\right) = \vec{J}'\cdot\vec{\nabla}'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) + \frac{1}{|\vec{r}-\vec{r}'|}\underbrace{\vec{\nabla}'\vec{J}'}_{0}$$ $\rightarrow$ por corrientes ctes, AKA Magnetostática

$$\Rightarrow \vec{\nabla}\cdot\vec{A} = -\frac{\mu_0}{4\pi}\int_V \vec{\nabla}'\cdot\left[\frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}\right]dv' \xrightarrow{\text{Teo. div}} \vec{\nabla}\cdot\vec{A} = \frac{-\mu_0}{4\pi}\int_{S(V)}\frac{\vec{J}(\vec{r}')\cdot d\vec{a}'}{|\vec{r}-\vec{r}'|}$$

Y como $\vec{J}$ está en el interior de $V$,
$\vec{J} = \vec{0} \quad \forall\, \vec{r}' \in S(V)$

$$\therefore\ \vec{\nabla}\cdot\vec{A} = 0 \qquad \square$$

b) $\vec{\nabla} \times \vec{A} = \vec{B}$ <u>dem</u> # $\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J} \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\, dv'$ Biot-Savart

$$\vec{\nabla} \times \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}\, dv' = \frac{\mu_0}{4\pi}\int \vec{\nabla} \times \left\{\frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}\right\} dv'$$ # Ident: $\vec{\nabla} \times (f\vec{A}) = \vec{\nabla} f \times \vec{A} + f\,\vec{\nabla} \times \vec{A}$

$$\Rightarrow \vec{\nabla} \times \left(\frac{\vec{J}'}{|\vec{r}-\vec{r}'|}\right) = \left(\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}'|}\right) \times \vec{J}(\vec{r}') + \frac{1}{|\vec{r}-\vec{r}'|}\underbrace{\vec{\nabla} \times \vec{J}(\vec{r}')}$$

↳ $\vec{\nabla}\times$ opera sobre $\vec{r}$, no $\vec{r}'$

# $\vec{\nabla}\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = \frac{-(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$

$$\therefore\ \vec{\nabla} \times \vec{A} = \frac{\mu_0}{4\pi}\int \frac{-(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} \times \vec{J}(\vec{r}') = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}') \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\, dv' \quad \square$$ # Antisimétrico

c) $\nabla^2 \vec{A} = -\mu_0 \vec{J}$ dem ($\sim$ Ley de Ampere)

$$\nabla^2 \vec{A} = \frac{\mu_0}{4\pi}\int \nabla^2\left[\frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}\right] dv'$$ # Ident: $\nabla^2(f\vec{A}) = f\nabla^2\vec{A} + \vec{A}\nabla^2 f$

# $\nabla^2[\,] = \frac{1}{|\vec{r}-\vec{r}'|}\underbrace{\nabla^2 \vec{J}(\vec{r}')}_{\vec{0}} + \vec{J}(\vec{r}')\nabla^2\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)$ # $\nabla^2\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = ?$

# Gauss: $\vec{\nabla}\cdot\vec{E} = \frac{\rho(\vec{r})}{\varepsilon_0}$, para una carga puntual en $\vec{r}'$: $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} q \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$

y $\rho(\vec{r}) = q\,\delta(\vec{r}-\vec{r}')$ ; $\frac{q}{4\pi\varepsilon_0}\vec{\nabla}\cdot\left[\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\right] = \frac{q}{\varepsilon_0}\delta(\vec{r}-\vec{r}')$

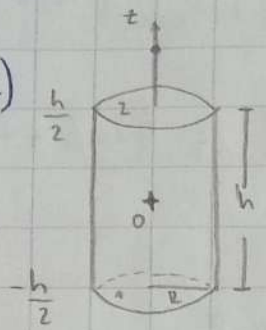
pero $\nabla\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = \frac{-(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$ ↳ $-\frac{q}{4\pi\varepsilon_0}\vec{\nabla}\cdot\left(\nabla\left[\frac{1}{|\vec{r}-\vec{r}'|}\right]\right) = \frac{q}{\varepsilon_0}\delta(\vec{r}-\vec{r}')$

$$\nabla^2\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = -4\pi\,\delta(\vec{r}-\vec{r}')$$

$$\Rightarrow \nabla^2\vec{A} = -\frac{\mu_0}{4\pi}\int J(\vec{r}')\,4\pi\,\delta(\vec{r}-\vec{r}')\, dv' = -\mu_0 \vec{J}(\vec{r}) \quad \square$$

1.1) See §2.5

2)




$\vec{M} = M_0 \hat{k}$ ; $\varphi_m(z) = ?$ ; $\begin{cases} \rho_M = 0 \\ \sigma_{M_1} = -M_0 \wedge \sigma_{M_2} = M_0 \end{cases}$

$$\varphi_m(z) = \frac{1}{4\pi} \oint_{S(V)} \frac{\sigma_M(\vec{r}')\, da'}{|\vec{r} - \vec{r}'|} + \frac{1}{4\pi} \oint_V \frac{\rho_M(\vec{r}')}{|\vec{r} - \vec{r}'|} dv'$$

$\vec{r} = z\hat{k}$ ; $\vec{r}_1' = -\frac{h}{2}\hat{k} + s\hat{s}$ ; $\vec{r}_2' = \frac{h}{2}\hat{k} + s\hat{s}$ ; $da' = 2\pi s\, ds$

# Solo contribuyen las tapas

$|\vec{r} - \vec{r}_1'| = \sqrt{(z + h/2)^2 + s^2}$ ; $|\vec{r} - \vec{r}_2'| = \sqrt{(z - h/2)^2 + s^2}$

$$\Rightarrow \varphi_m(z) = \frac{1}{4\pi} \int_0^R \frac{-M_0\, 2\pi s\, ds}{\sqrt{z_+^2 + s^2}} + \frac{1}{4\pi} \int_0^R \frac{M_0\, 2\pi s\, ds}{\sqrt{z_-^2 + s^2}}$$

$$\# \int_0^R \frac{s\, ds}{\sqrt{a^2 + s^2}} = \frac{1}{2} \int_{u_1}^{u_2} \frac{du}{\sqrt{u}} = \sqrt{a^2 + s^2}\Big|_0^R = \sqrt{a^2 + R^2} - |a|$$

$$\varphi_m(z) = -\frac{M_0}{2}\left\{ \sqrt{z_+^2 + R^2} - |z_+| \right\} + \frac{M_0}{2}\left\{ \sqrt{z_-^2 + R^2} - |z_-| \right\}$$

i) Interior: $|z| < h/2 \rightarrow |z_+| = z_+ \wedge |z_-| = -z_-$

$$\Rightarrow \varphi_m(z) = -\frac{M_0}{2}\left\{ \sqrt{z_+^2 + R^2} - z - h/2 - \sqrt{z_-^2 + R^2} + h/2 - z \right\}$$

ii) Exterior: $|z| > l/2$ $\rightarrow$ $z > h/2$ : $|z_+| = z_+ \wedge |z_-| = z_-$ [arriba]

$\searrow$ $z < -h/2$ : $|z_+| = -z_+ \wedge |z_-| = -z_-$ [abajo]

2)

arriba) $\varphi_m(z) = -\frac{n_0}{2}\left\{\sqrt{z_+^2+R^2} - z_+ - \sqrt{z_-^2+R^2} + z_-\right\}$

$= \frac{n_0}{2}\left\{h + \sqrt{z_-^2+R^2} - \sqrt{z_+^2+R^2}\right\}$ , $z > h/2$

abajo) $\varphi_m(z) = \frac{-n_0}{2}\left\{\sqrt{z_+^2+R^2} + z_+ - \sqrt{z_-^2+R^2} - z_-\right\}$

$= \frac{n_0}{2}\left\{-h + \sqrt{z_-^2+R^2} - \sqrt{z_+^2+R^2}\right\}$ , $z < -h/2$

b) $\vec{B}(z) = \mu_0 \vec{n}(z) - \mu_0 \vec{\nabla}\varphi_m(z)$ , $\vec{G} \equiv \vec{\nabla}\varphi_m(z)$ , $^*$para $|z| < h/2$

$\vec{G} = \frac{n_0}{2}\left\{2 - \frac{1}{2}\left[z_+^2+R^2\right]^{-1/2}\cdot 2z_+ + \frac{1}{2}\left[z_-^2+R^2\right]^{-1/2}\cdot 2z_-\right\}\hat{k}$

interior : $|z| < h/2 \quad \rightarrow \quad \vec{B}_i(z) = \mu_0 n_0 \hat{k} - \mu_0 \vec{G}$

$\vec{B}_i(z) = \frac{\mu_0 n_0}{2}\left\{2 - 2 + z_+\left[z_+^2+R^2\right]^{-1/2} - z_-\left[z_-^2+R^2\right]^{-1/2}\right\}\hat{k}$

# $|\vec{B}_i(z)| > 0$ , $h/2 - z \geq 0$ # Interpretar dirección (sentido)

Exterior : $\vec{n}(\vec{r}) = \vec{0} \quad \Rightarrow \quad \vec{B}_e(z) = -\mu_0 \vec{\nabla}\varphi_m(z)$ , $\vec{G}_1 \equiv \vec{\nabla}\varphi_m(z)$ $^*z > h/2$

$\vec{G}_1 = \frac{n_0}{2}\left\{\frac{1}{2}\left[z_-^2+R^2\right]^{-1/2}\cdot 2z_- - \frac{1}{2}\left[z_+^2+R^2\right]^{-1/2}\cdot 2z_+\right\}$

$\Rightarrow \vec{B}_1(z) = \frac{\mu_0 n_0}{2}\left\{z_+[\;] - z_-[\;]\right\} \quad (= B_i(z))$

para $z < -h/2$, ocurre lo mismo

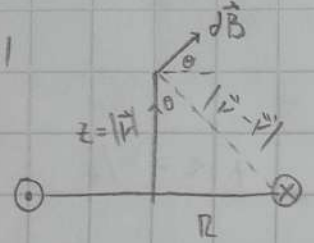
$\therefore \quad \vec{B}(z) = \frac{\mu_0 n_0}{2}\left\{(z+h/2)\left[(z+h/2)^2+R^2\right]^{-1/2} - (z-h/2)\left[(z+h/2)^2+R^2\right]^{-1/2}\right\}\hat{k}$

Interpretar direcciones ...

3) Campo de una espira, Campo de un solenoide

(i) Descomponer el campo : $d\vec{B} = \underbrace{dB\cos(\theta)\,\hat{\imath}}_{\text{Se cancelan por simetría}} + dB\sin(\theta)\,\hat{\jmath}$

Perfil

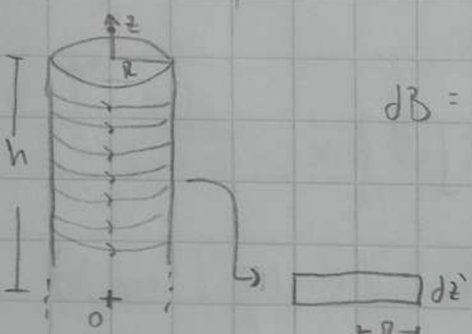



$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sqrt{z^2+R^2}}{(z^2+R^2)^{3/2}}\sin(\pi/2)$$

$$B(z) = \frac{\mu_0}{4\pi}\frac{I\,2\pi R}{(z^2+R^2)} = \frac{\mu_0}{2}\frac{IR}{(z^2+R^2)}$$

(ii) $$\vec{B}(z) = \frac{\mu_0}{2}\frac{IR}{(z^2+R^2)}\sin(\theta) = \frac{\mu_0 IR}{2(z^2+R^2)}\frac{R}{\sqrt{z^2+R^2}} = \frac{\mu_0 I}{2}\frac{R^2}{(z^2+R^2)^{3/2}}\hat{k} \quad //$$

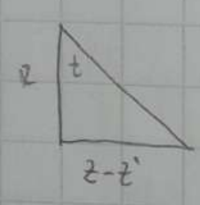
(iii) Sumar espiras




$$dB = \frac{\mu_0\,dI}{2}\frac{R^2}{\left[(z-z')^2+R^2\right]^{3/2}} \quad ; \quad dI' = In\,dz' \quad , \quad n = \frac{N}{L}$$

→ Porque la distribución ya no está solo en el origen

$$\vec{B}(z) = \frac{\mu_0 I n R^2}{2}\hat{k}\int_0^h \frac{dz'}{\left[R^2+(z-z')^2\right]^{3/2}}$$

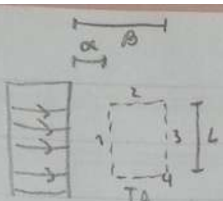



$$\tan(t) = \frac{z-z'}{R} \quad ; \quad dz' = -R\sec^2(t)\,dt$$

$$z' = z - R\tan(t) \quad ; \quad (z-z')^2 + R^2 = R^2\sec^2(t)$$

$$\int_{t_1}^{t_2} \frac{-R\sec^2(t)}{R^3\sec^3(t)}\,dt = -\frac{1}{R^2}\frac{z-z'}{\sqrt{(z-z')^2+R^2}}\Bigg|_0^h = \frac{1}{R^2}\left\{\frac{h-z}{\sqrt{(h-z)^2+R^2}} + \frac{z}{\sqrt{z^2+R^2}}\right\}$$

# hacer $\int_{-\infty}^{\infty}$ para el solenoide infinito $\qquad \forall z \in \mathbb{R}$



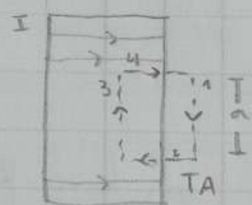

$$\oint_{TA} \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} = 0 \quad \int \rightarrow \quad B(\alpha)L - B(\beta)L = 0$$

$$\underbrace{\int_2 + \int_4}_{0} + \int_1 + \int_2 = 0 \qquad B(\alpha) = B(\beta) \quad \therefore B \text{ es cte fuera del Solenoide}$$

(iv) Ley de Ampere : Asumiendo Solenoide infinito

$$\Rightarrow \left.\frac{z' - z}{\sqrt{(z'-z)^2 + R^2}}\right|_{-\infty}^{\infty} = 2 \qquad \therefore \quad \vec{B}(z) = \mu_0 n I \hat{k} \quad \forall z \in \mathbb{R}$$

↳ Campo en el eje




$$\oint_{TA} \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$\underbrace{\int_1 + \int_2 + \int_4}_{0} + \int_3 = \mu_0 I_{enc}$$

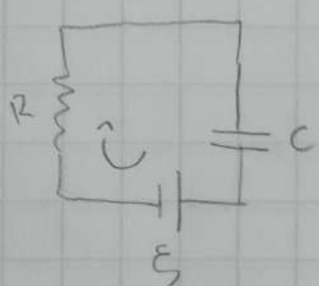
0 → el campo es nulo fuera y ortogonal a (3) y (4)

$$\Rightarrow \quad B a = \mu_0 n I a \qquad \# \ I_{enc} = NI = nIa \qquad \# \ N: \text{ número de espiras que entran en "a"}$$

$\vec{B} = \mu_0 n I \hat{k}$ , al interior

$\vec{B} = \vec{0}$ , al exterior

4)




$$q = C\xi\left(1 - e^{-t/RC}\right)$$

Kirchhoff: $\xi - RI - \frac{Q}{C} = 0 \quad / d/dt$

$$-R\dot{I} - \dot{Q} = 0 \Rightarrow -R\dot{I} - I = 0 \quad / \text{Separables}$$

# Al cargarse el condensador, la corriente disminuye

$$\Rightarrow \frac{dI}{I} = \frac{-1}{RC}dt \quad / \int$$

$$\ln(I/I_0) = -t/\tau \quad / \exp$$

$$I = I_0 e^{-t/\tau} \quad , \ I_0 = I(t=0) \ \text{Corriente máxima}$$

(i) Carga en $t \to \infty$ : $I(t\to\infty) = 0 \Rightarrow \xi - \frac{Q}{C} = 0 \Rightarrow Q = C\xi$ en $t \to \infty$

$$e^{-t/\tau} = 0.3 \quad /\ln \quad \rightarrow \quad t = \ln\left(\frac{10}{3}\right)\tau$$

$$-t/\tau = \ln\left(\frac{3}{10}\right)$$

(ii) $Q(t = 3ms) \equiv Q_2 \quad ; \quad Q = C\xi\left(1 - e^{-t/\tau}\right)$

$\Rightarrow R = ?$

$$\frac{Q}{C\xi} - 1 = -e^{-t/\tau} \qquad /-/\ln$$

$$\ln\left|1 - Q/C\xi\right| = -t/RC$$

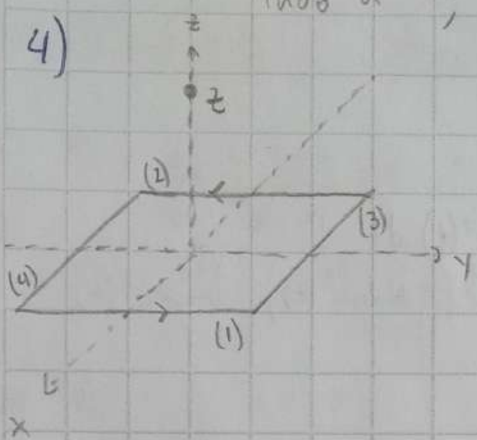
$$R = \frac{-t}{C}\frac{1}{\ln\left|1 - Q(t)/C\xi\right|}$$

# Evaluar en $t = 3ms$ para tener $R$

(iii) Hallar $t = ?$ donde la carga del capacitor es 70 % de la carga máxima

$$\frac{q}{C\xi} = 0.7 \quad \Rightarrow \quad 1 - e^{-t/\tau} = 0.7$$

$$t = \tau \ln(10/3) \ (s) \quad //$$

4) 


lado $a$ ; $\vec{B}(z) = \frac{\mu_0}{4\pi} \oint \frac{I\, d\vec{l} \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$ ; $\vec{r} = z\hat{k}$

$$|\vec{r} - \vec{r}\,'|^3 = \left[(\vec{r} - \vec{r}\,') \cdot (\vec{r} - \vec{r}\,')\right]^{3/2}$$

para (1) : $\left[\left(z\hat{k} - \frac{a}{2}\hat{i} - y'\hat{j}\right)\left(z\hat{k} - \frac{a}{2}\hat{i} - y'\hat{j}\right)\right]^{3/2}$

$= \left(z^2 + \frac{a^2}{4} + y'^2\right)^{3/2}$ ; $dl = dy$ # lo mismo para (2)

para (3) : $\left[\left(z\hat{k} - \frac{a}{2}\hat{j} - x'\hat{i}\right) \cdot \left(z\hat{k} - \frac{a}{2}\hat{j} - x'\hat{i}\right)\right]^{3/2}$

$= \left(z^2 + \frac{a^2}{4} + x'^2\right)^{3/2}$ ; $dl = dx$ # lo mismo para (4)

luego, resolvemos $d\vec{l} \times (\vec{r} - \vec{r}\,')$ para cada tramo # $d\vec{l} \parallel \vec{I}$

(1): $dy\,\hat{j} \times \left(z\hat{k} - \frac{a}{2}\hat{i} - y'\hat{j}\right) = \left(z\hat{i} + \frac{a}{2}\hat{k}\right) dy$

(2): $-dy\,\hat{j} \times \left(z\hat{k} + \frac{a}{2}\hat{i} - y'\hat{j}\right) = \left(-z\hat{i} + \frac{a}{2}\hat{k}\right) dy$

(3): $-dx\,\hat{i} \times \left(z\hat{k} - \frac{a}{2}\hat{j} - x'\hat{i}\right) = \left(z\hat{j} + \frac{a}{2}\hat{k}\right) dx$

(4): $dx\,\hat{i} \times \left(z\hat{k} + \frac{a}{2}\hat{j} - x'\hat{i}\right) = \left(-z\hat{j} + \frac{a}{2}\hat{k}\right) dx$

luego, $I_1 + I_2 = \int_1 \frac{\left(z\hat{i} + \frac{a}{2}\hat{k}\right) dy}{\left(z^2 + \frac{a^2}{4} + y'^2\right)^{3/2}} + \int_2 \frac{\left(-z\hat{i} + \frac{a}{2}\hat{k}\right) dy}{\left(z^2 + \frac{a^2}{4} + y'^2\right)^{3/2}}$

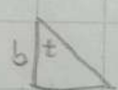
$= a\hat{k} \int_{-\frac{a}{2}}^{a/2} \frac{dy}{\left(z^2 + \frac{a^2}{4} + y'^2\right)^{3/2}}$

Pero como ya incluimos la dirección de $\vec{I}$ en $d\vec{l}$, los lim de int. son los mismos

e $I_3 + I_4 = a\hat{k} \int_{-\frac{a}{2}}^{a/2} \frac{dx}{\left(z^2 + \frac{a^2}{4} + x'^2\right)^{3/2}}$ $\therefore I_1 = I_2 = I_3 = I_4$

# $I_1' = \int_{-a/2}^{a/2} \frac{dx}{\left(z^2+\frac{a^2}{4}+x^2\right)^{3/2}}$ $\qquad b^2 \equiv z^2+\frac{a^2}{4}$

$\int \frac{dx}{(b^2+x^2)^{3/2}}$




$\tan(t) = \frac{x}{b}$, $dx = b\sec^2(t)\,dt$

$x^2+b^2 = b^2\left(1+\tan^2(t)\right) = b^2\sec^2(t)$

$= \int \frac{b\,s^2\,dt}{b^3 s^3} = \frac{1}{b^2}\sin(t) = \frac{1}{b^2}\frac{x}{\sqrt{x^2+b^2}}$

$I_1' = \frac{4}{4z^2+a^2}\left\{\frac{a/2}{\sqrt{\frac{a^2}{4}+b^2}} - \frac{-a/2}{\sqrt{\frac{a^2}{4}+b^2}}\right\} = \frac{4a}{4z^2+a^2}\frac{1}{\sqrt{z^2+\frac{a^2}{2}}}$

$\therefore\ \vec{B}(z) = \frac{\mu_0 I}{4\pi}\, 2a\hat{k} \cdot I_1'$ □