Tópicos

Close binary stars

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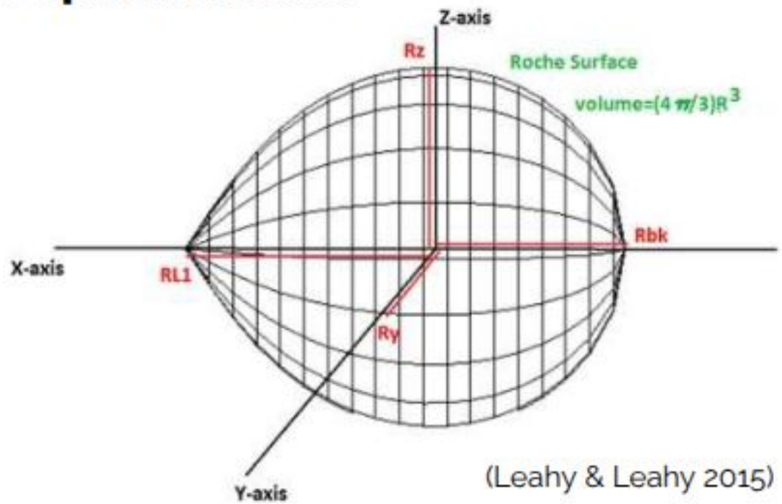
Lecture 7

01/09

Types of binaries based on RL

The Roche Model: Geometry

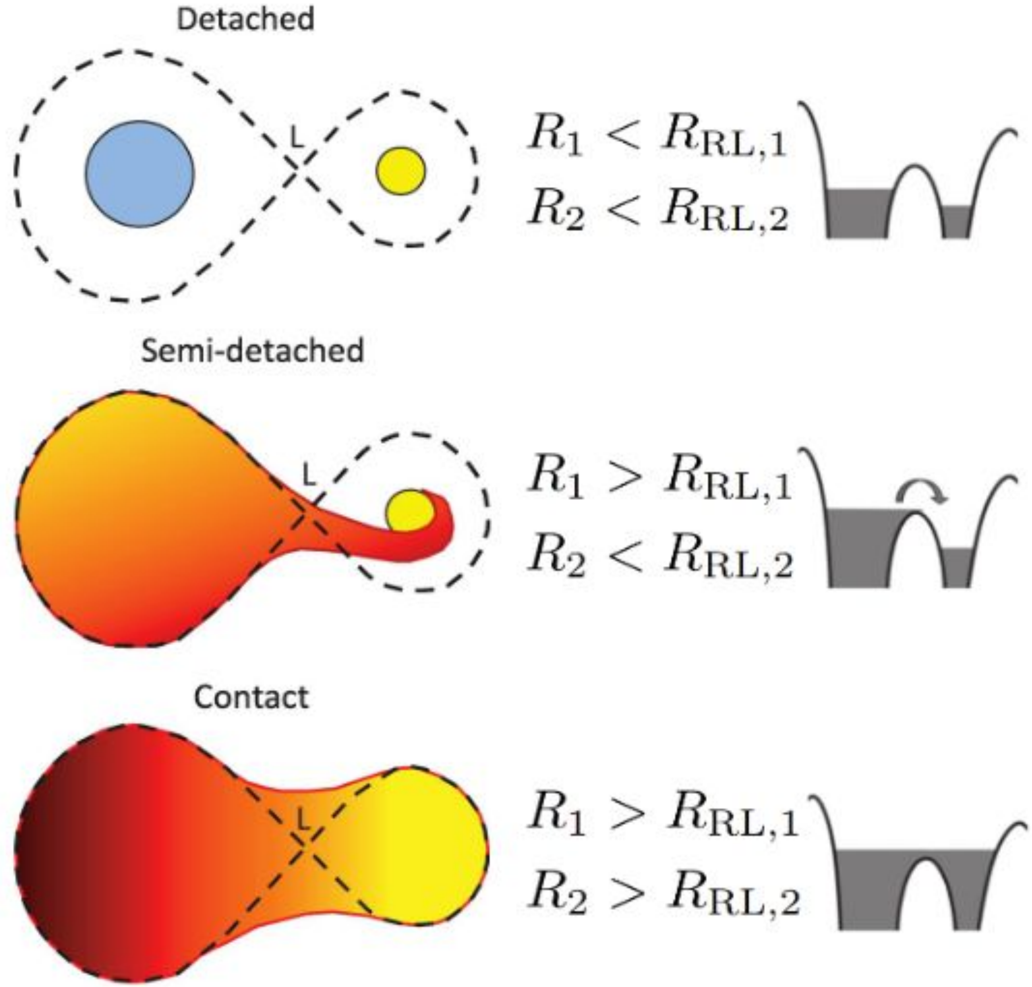
Non-spherical stars



(Leahy & Leahy 2015)

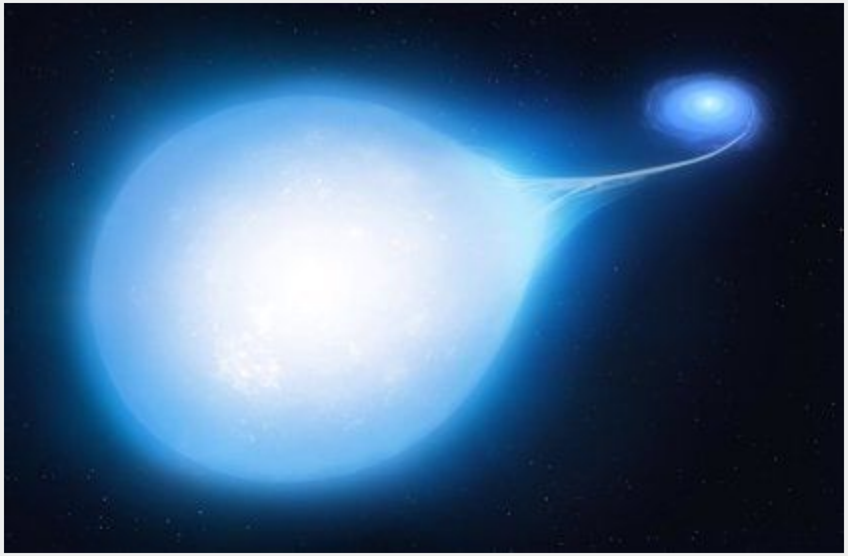
Roche radius (Eggleton 1983)

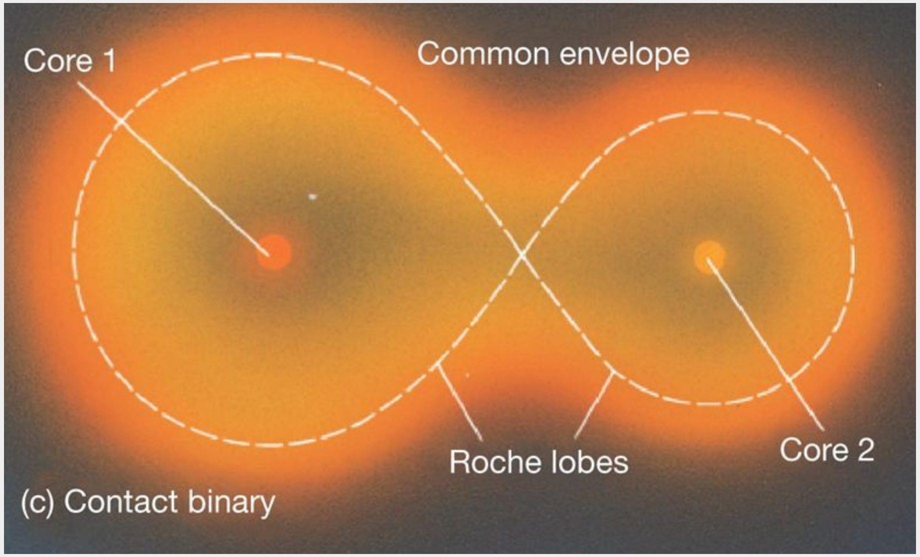
$$\frac{R_{\rm RL}}{a} = \frac{0.49\, q^{2/3}}{0.69\, q^{2/3} + \ln\left(1 + q^{1/3}\right)}$$



Adapted from Walker (2017) and Tauris & van den Heuvel (2023)

Mass transfer
Stable or Unstable?





Stable
(semi-detached)

Dynamically unstable
(contact, meger, CE)

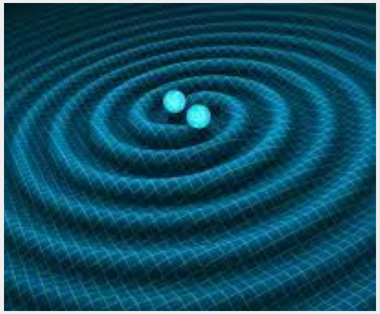
There is also dynamically stable but thermally “unstable” (bad name, as it is still stable but out of thermal equilibrium)

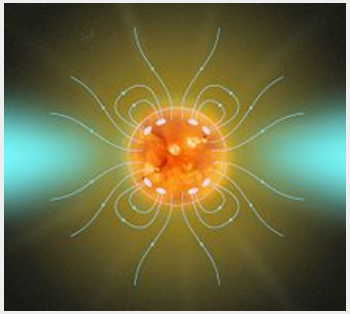
Mass transfer due to RLOF

Mass transfer occurs in close binary systems after one of the stars fills it's Roche lobe.

This can be triggered either by:

Expansion (due to stellar evolution, e.g. on red giant branch)

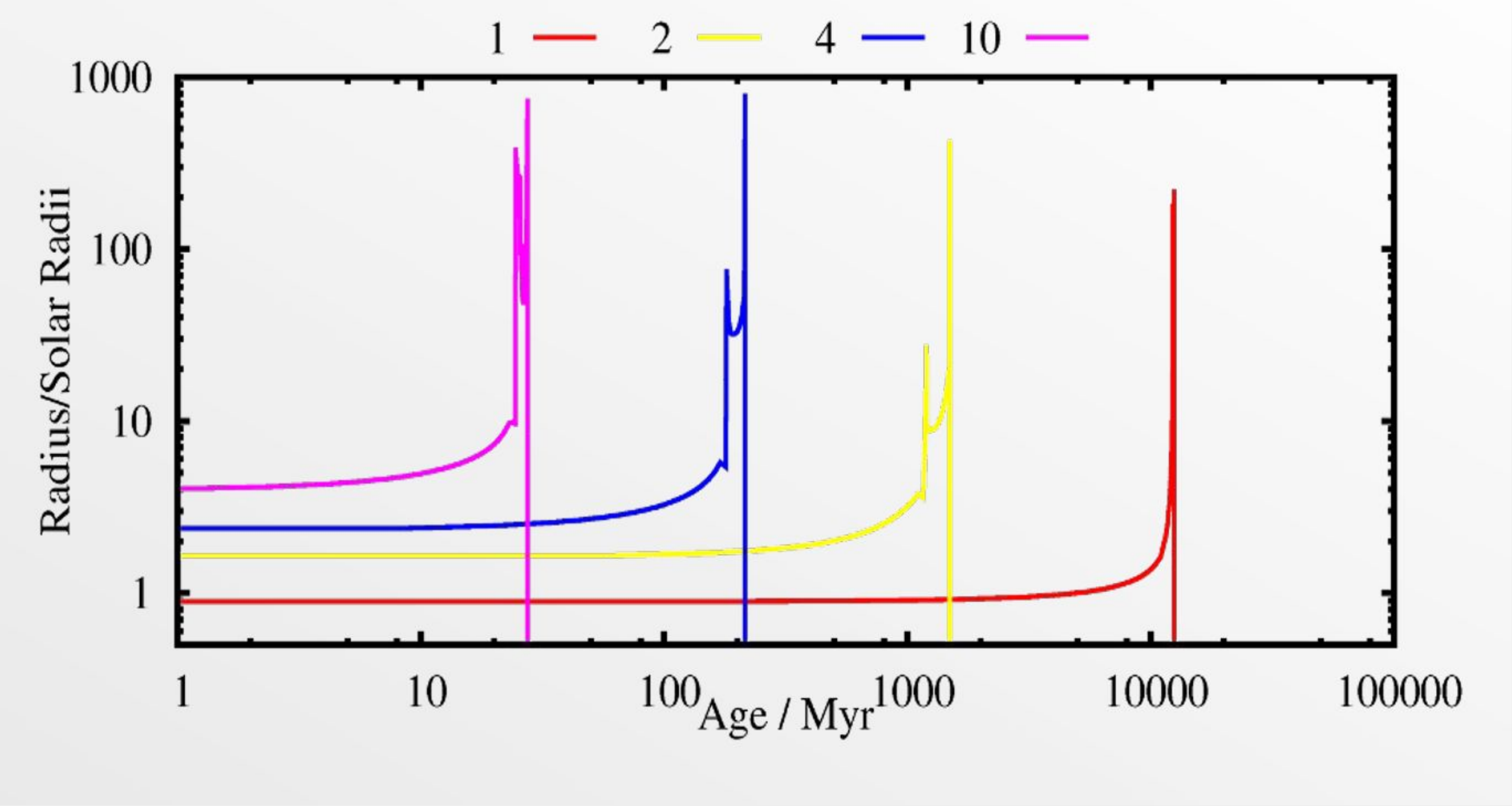




Roche lobe shrinking due to angular momentum loss causing contraction of the orbit (e.g. magnetic braking, gravitational radiation)

Radius expansion

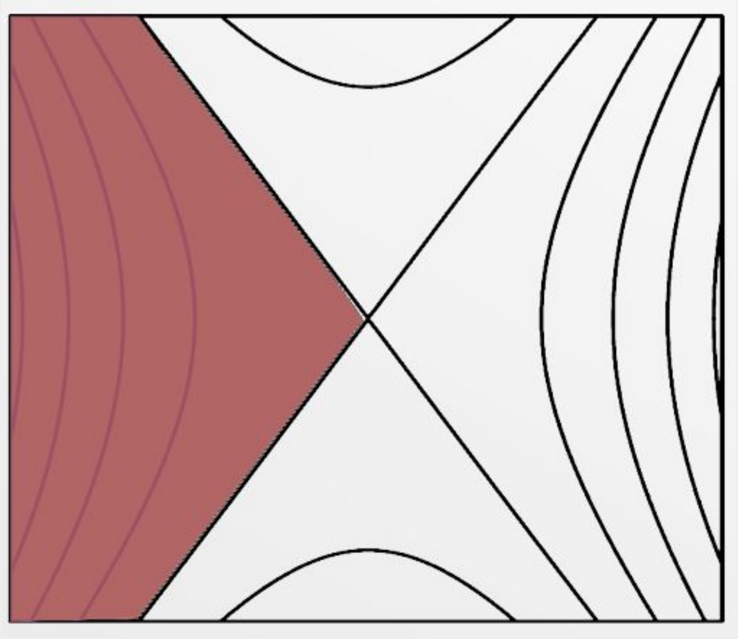
Example: expansion of a star during evolution, for different masses



Mass transfer due to RLOF

When mass is transferred from one star to the other (or is lost from the system) the angular momentum of the binary changes:
→ changes the orbital parameters (q, period, Roche-lobes size). The star's parameters also change (mass, luminosity, radius)

In particular, if one star fills its Roche lobe, then matter can freely escape from the surface through the inner Lagrange point L1 and will be captured by the other star.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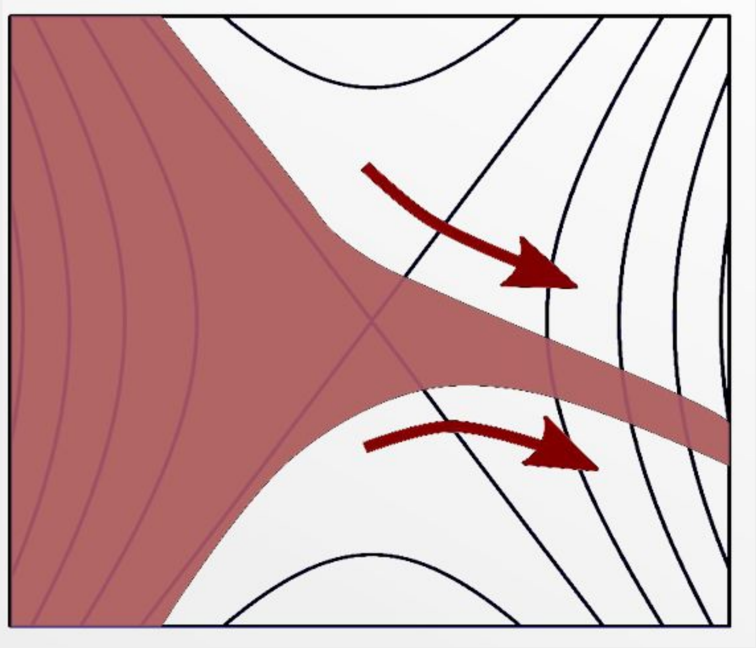


Stability of Mass Transfer

Stars reacts to mass loss/gain: contraction or expansion. Roche-lobes also contracts/expands.

Stability of mass transfer depends on several aspects:

● Donor response, depending on timescales of mass transfer:
- Change of radius R
- Change of Roche lobe radius R_L

● Accretor response: Timescale on which accretor can react, if it is able to remain in hydrostatic eq., if it also fills its own R_L …

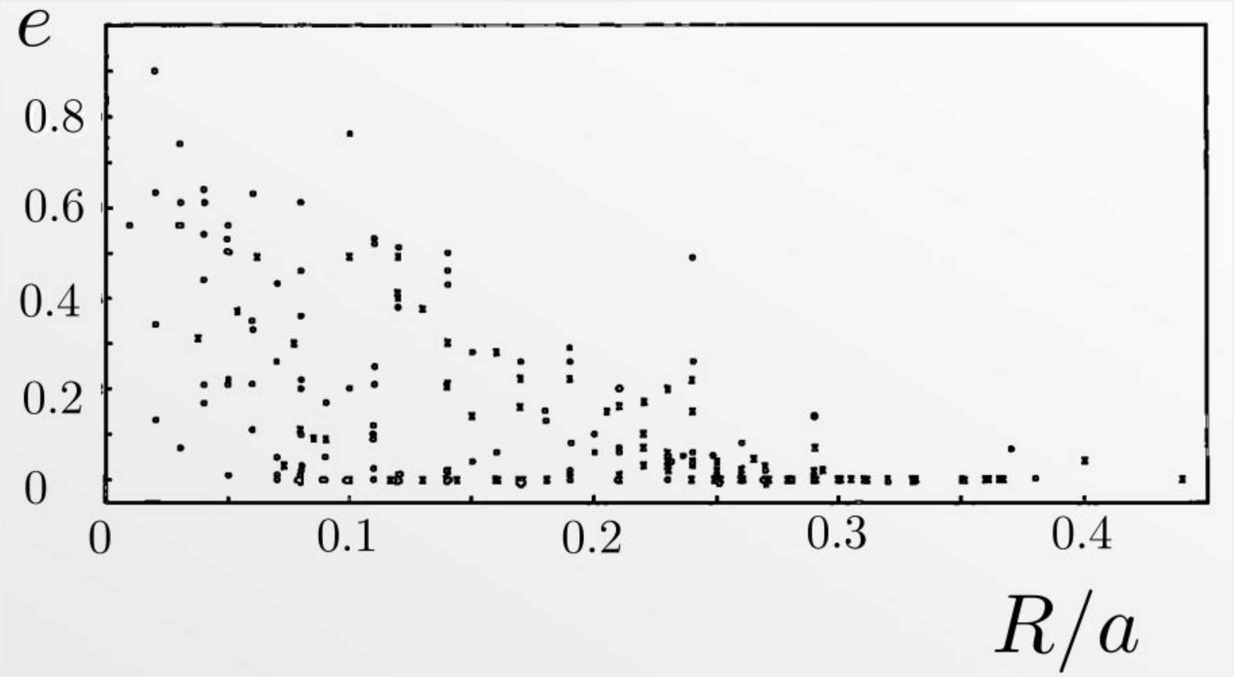
● Orbit response (period and orbital separation), increase or decrease? How fast?

● How conservative is it: is there mass lost from the system, taking away angular momentum?

Orbital Response to Mass Transfer

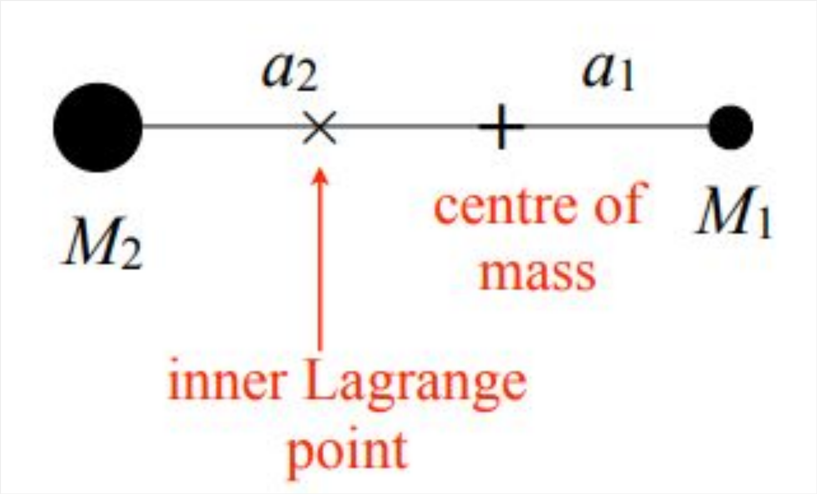
Let's see how the orbital distance (or period) changes when the stars transfer mass

We will assume **circular orbits**, because we can roughly assume that close binaries, at short term, evolve towards a circular configuration due to tidal forces (R close to a for the donor).



Angular Momentum

The binary angular momentum is $J = M_1 a_1^2 \Omega + M_2 a_2^2 \Omega,$



$$\Omega = 2\pi / P$$

$$a_1 = a \frac{M_2}{M_1 + M_2} \quad a_2 = a \frac{M_1}{M_1 + M_2}$$

$$J = \frac{M_1 M_2}{M_1 + M_2} a^2 \Omega. = \mu\, a^2\, \Omega$$

Note: Regardless of the sizes, in this figure, $M_1 > M_2 \longrightarrow a_1 < a_2$
CM closer to M_1 and L_1 closer to M_2

Note 2: For non-circular orbits $J = \mu\, a^2\, \Omega \sqrt{1 - e^2}.$

Fully conservative case: $\dot{M} = 0$ and $\dot{J} = 0$

Using $J = \frac{M_1 M_2}{M_1 + M_2} a^2 \Omega$ we get $\frac{\dot{J}}{J} = 2\frac{\dot{a}}{a} + \frac{\dot{\Omega}}{\Omega} + \frac{\dot{M}_1}{M_1} + \frac{\dot{M}_2}{M_2} - \frac{\dot{M}}{M}$

Conservative means that there is:

- **No mass loss** $\frac{d}{dt}M = \frac{d}{dt}(M_1 + M_2) = 0$ $\dot{M}_1 = -\dot{M}_2$

- **No angular momentum loss AML** (from the system) $\frac{dJ}{dt} = 0$

Note: not always true as in close binaries there are other sources of AML, but can be assumed to derive the response of the orbit due to mass transfer only.

Fully conservative case:

From
$$\frac{\dot{J}}{J} = 2\frac{\dot{a}}{a} + \frac{\dot{\Omega}}{\Omega} + \frac{\dot{M}_1}{M_1} + \frac{\dot{M}_2}{M_2} - \frac{\dot{M}}{M}$$

and using $\dot{M} = 0$ and $\dot{J} = 0$

we get
$$2\frac{\dot{a}}{a} + \frac{\dot{\Omega}}{\Omega} + \frac{\dot{M}_1}{M_1} + \frac{\dot{M}_2}{M_2} = 0.$$

We also have $\dot{M}_1 = -\dot{M}_2$

And what about $\frac{\dot{\Omega}}{\Omega}$ and $\frac{\dot{a}}{a}$?

$\Omega = 2\pi/P$ and using $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$ $\rightarrow$ $\frac{\dot{\Omega}}{\Omega} = -\frac{\dot{P}}{P}$

Applying the third Kepler Law: $a^3 = \frac{G\left(M_1 + M_2\right)}{4\pi^2} P^2$

$$\rightarrow \frac{\dot{a}}{a} = \frac{2}{3}\frac{\dot{P}}{P}$$

We obtain the change in period or orbital separation for fully conservative mass transfer

$$\frac{\dot{P}}{P} = \frac{3\dot{M}_1\left(M_1 - M_2\right)}{M_1 M_2}$$

$$\frac{\dot{a}}{a} = \frac{2\dot{M}_1\left(M_1 - M_2\right)}{M_1 M_2} = -\frac{2\dot{M}_1}{M_1}\left(1 - \frac{M_1}{M_2}\right)$$

For fully conservative case

$$\frac{\dot{a}}{a} = -\frac{2\dot{M}_1}{M_1}\left(1 - \frac{M_1}{M_2}\right)$$

>0, star 1 loses mass $\dot{M}_1 < 0$

→ Mass transfer from the more massive to the less massive star:
M1 > M2 → dP and da < 0 → orbit **shrinks** (P and a decrease)

$$\begin{array}{ll} \Rightarrow & \text{Roche lobe shrinks} \\ \Rightarrow & \text{more mass transferred} \\ \Rightarrow & \text{orbit shrinks more} \end{array}$$

(both due to mass loss and da)

can lead to a runaway mass transfer (depending on the reaction of R1 and the adjustment of the companion also)

For fully conservative mass transfer

$$\frac{\dot{a}}{a} = -\frac{2\dot{M}_1}{M_1}\left(1 - \frac{M_1}{M_2}\right)$$

>0, star 1 loses mass $\dot{M}_1 < 0$

→ Mass transfer from the less massive to the more massive star:

M1 < M2 → dP and da > 0 → orbit **widens** (P increases)

Mass transfer usually stops, unless we have
- a mechanism for AML (like GR or MB in CVs)
- or R1 is increasing a lot (e.g. when becoming a giant).

For fully conservative mass transfer

Summarizing, the rate of change in period and separation is determined by the mass lost by the donor and mass ratio.

$\mathbf{M_{donor} > M_{gainer}}$ → P and a decrease. Roche lobe of donor decreases, mass transfer is intensified (runaway), gainer spins up due to gain in mass and AM.

Continues until components have ~ equal masses →mass ratio inverts.

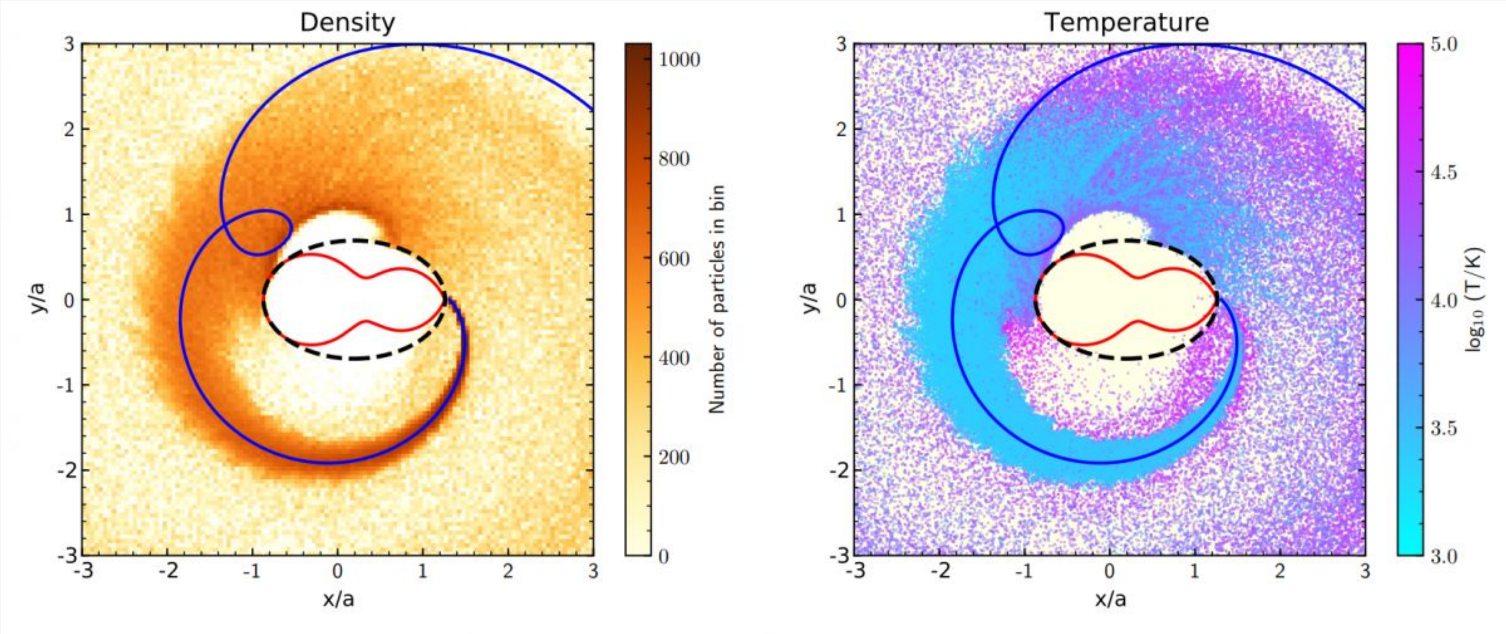
$\mathbf{M_{donor} < M_{gainer}}$ → P and a increased, mass transfer stops or is self regultaded by other sources of AML or by an increase in R.

But… life is not so simple. Usually mass transfer is not fully conservative.

Non-Conservative mass transfer

For example, If the acretor speeds up, it can start losing mass (or preventing accretion).

Example of mass lost through L2



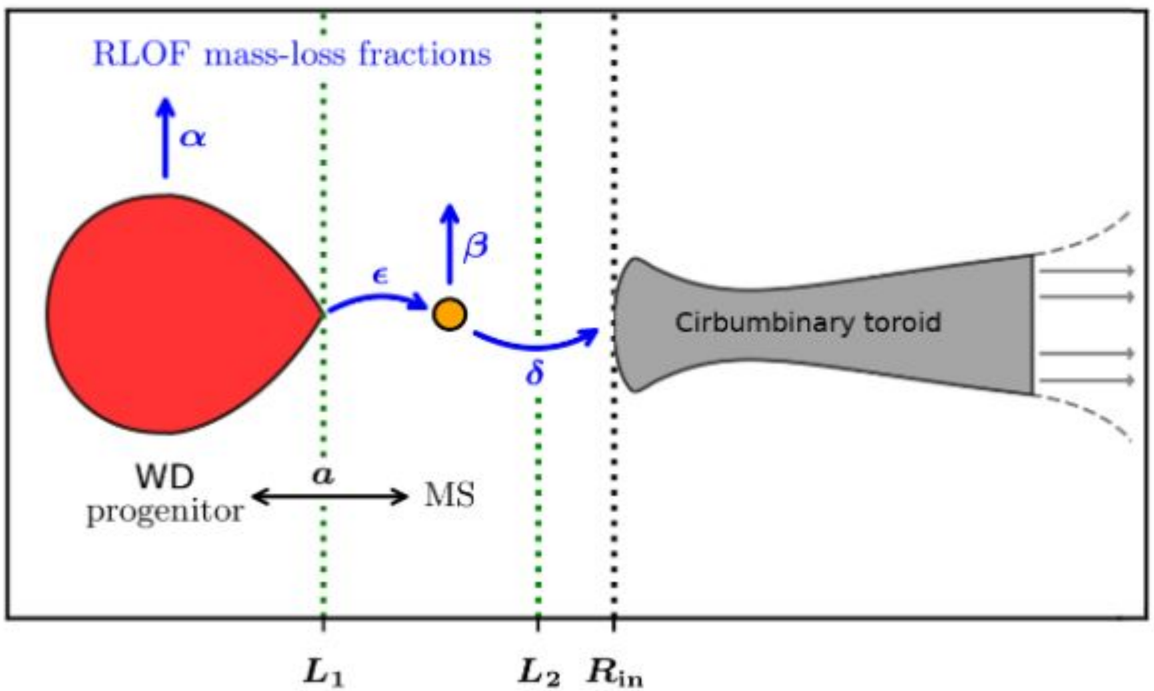
- Evolution is more complex
- Change with respect to conservative case depends on the fraction of mass that is lost.
- Effect on orbital distance depends on where the mass is lost from

Non-Conservative mass transfer

Mass can be lost from different locations

Mass Transfer efficiency

Formalism of Soberman et al. (1997; see also Tauris & van den Heuvel 2006)



Adapted from Vos & Vuckovic (2017)

- α: fraction of mass lost from the vicinity of the donor
- β: fraction of mass lost from the vicinity of the accretor
- δ: fraction of mass lost from a circumbinary toroid with radius $R_{toroid} = \gamma a^2$

The mass transfer efficiency:

$$\epsilon = 1 - \alpha - \beta - \delta$$

Usually simulations assume $\alpha = \delta = 0$

Non-Conservative mass transfer

Crefull, not always

In the case of non-conservative mass transfer, mass is lost from the system through the second Lagrangian point. In that case, the evolution of the orbit is more complex. Assume that a fraction β of the transferred mass leaves the system. In that case, we have

$$\dot{M} = \beta\,\dot{M}_1 \quad \text{and} \quad \dot{M}_2 = -\,(1\,-\,\beta)\;\dot{M}_1.$$

From

$$\frac{\dot{J}}{J} = \frac{1}{2}\frac{\dot{a}}{a} + \frac{\dot{M}_1}{M_1} + \frac{\dot{M}_2}{M_2} - \frac{1}{2}\frac{\dot{M}}{M},$$

we get

$$\frac{\dot{J}}{J} = \frac{1}{2}\frac{\dot{a}}{a} + \frac{\dot{M}_1}{M_1 M_2 M}\left[M_2\,M - (1\,-\,\beta)\,M_1 M - \frac{1}{2}\,\beta\,M_2 M_1\right]$$

and

$$\frac{3}{2}\frac{\dot{a}}{a} = \frac{\dot{P}}{P} = 3\,\frac{\dot{J}}{J} + \frac{3\,\dot{M}_1}{M_1 M_2 M}\left[(M_1\,-\,M_2) - \beta\,M_1\,(1\,-\,\frac{M_2}{2M})\right]$$

The second term on the right-hand side is less negative than in the case of conservative mass transfer and turns positive somewhat before the mass ratio inverts. The first term, however, is always negative and implies an increased reduction of the orbital separation compared to the conservative case.

this will depend on the location from where mass is lost

Donor/accretor response

Relevant timescales

To see the effect of mass transfer on the structure of the stars we need to consider 3 timescales:

• **Dynamical** timescale: for star to **re-establish hydrostatic equilibrium** after it is perturbed. It is given by the ratio of the radius of the star (R) and the average sound speed in the stellar matter.

• **Thermal** timescale: for star to **re-establish thermal equilibrium** (i.e. on which a star reacts when energy loss and energy production are no longer in equilibrium), given by the ratio of the thermal energy content of the star and its luminosity.

• **Nuclear** timescale: on which a star uses its nuclear fuel (e.g. MS lifetime). Depends on core mass and composition, and luminosity.

Donor/accretor response

Relevant timescales

- **Dynamical** $\tau_{dyn} = \left(\frac{R^3}{GM^2}\right)^{1/2}$; $\tau_{dyn,\odot} = 19\ \mathrm{min}$
- **Thermal** $\tau_{th} = \frac{GM^2}{RL}$; $\tau_{th,\odot} = 10^7\ \mathrm{yr}$
- **Nuclear** $\tau_{nuc} = \frac{\eta M c^2}{L}$; $\tau_{nuc,\odot} = 10^{10}\ \mathrm{yr}$

Evolution of the donor

Stable or unstable mass transfer result in very different outcomes.

The donor's evolution depends on whether it is able to let his radius evolve (shrink or expand) together with its Roche lobe and whether this occurs at the expense of the conservation of thermal equilibrium in the star.

This is determined by the mass-radius exponents

$$\zeta \equiv \frac{d(\ln R)}{d(\ln M)} = \frac{M}{R}\,\frac{dR}{dM}$$

Mass radius exponents

$$\zeta \equiv \frac{d(\ln R)}{d(\ln M)} = \frac{M}{R}\,\frac{dR}{dM}$$

We should see the behaviour of the Roche Lobe, adiabatic (dynamical) and thermal radius-mass exponents:

Roche lobe, ζ_L –	Response of Roche lobe radius to mass loss
adiabatic, ζ_{ad} –	Response of the stellar radius to mass loss, while star retains hydrostatic equilibrium
thermal, ζ_{th} –	Response of the stellar radius to mass loss, while retaining thermal equilibrium

Stable mass transfer

$$\zeta_{th}, \zeta_{ad} > \zeta_L$$

The donor remains in thermal and hydrodynamic equilibrium.

Continuing mass transfer is driven either by nuclear evolution of the donor (expansion) or by orbital shrinkage due to angular momentum loss (AML).

In this case we say that **mass transfer occurs on the nuclear timescale of the donor, or on the timescale for angular momentum loss** (whichever is shorter).

E.g. in Cataclysmic Variables (CVs), mass transfer is stable, nuclear timescale of the donor is too long (typically low-mass MS star), so mass transfer proceeds due to AML (MB and GR) that slowly shrinks the orbit.

Thermal timescale mass transfer

$$\zeta_{ad} > \zeta_L > \zeta_{th}$$

Hydrostatic equilibrium is maintained, but thermal equilibrium is not.

Mass transfer is driven by thermal readjustment of the donor

Sometimes this is called ‘thermally unstable’ mass transfer, but this is misleading:

- despite the thermal disequilibrium of the donor, mass transfer is stable and self-regulating (not a runaway process).
- The radius excess adjusts itself to maintain the thermal-timescale mass loss rate, which implies $\delta R/R_L < 0.01$.
- Hence also in this case, the donor radius closely follows the Roche radius.

Dynamically unstable mass transfer

$$\zeta_{th}, \zeta_{ad} < \zeta_L$$

Roche lobe shrinking is too fast compared to the radius response.

The adiabatic response of the donor is unable to keep it within its Roche lobe (cannot reach hydrostatic equilibrium), and the radius excess ($\delta R/R_L$) is increasing → leads to ever-increasing mass-transfer rate.

This is an unstable, runaway situation, that has dramatic effects on the evolution of the binary, probably leading to a **common-envelope** situation and potentially a **merger**.

Dependence on donor’s envelope
Entropy profile

Difference in stability of mass transfer depends on the donor’s envelope structure.

Mainly it has to do with the envelope entropy (S) profile.

Let us assume that a star, initially with a mass m, loses an amount of mass δm at a high speed.

The loss of the external layers will not immediately alter the entropy profile. In other words, **at first instance the star reacts in an adiabatic way** on the mass loss.

If the donor gets the necessary time to restore the thermal equilibrium, it will adjust it’s entropy profile to match that of a star with mass m-δm.

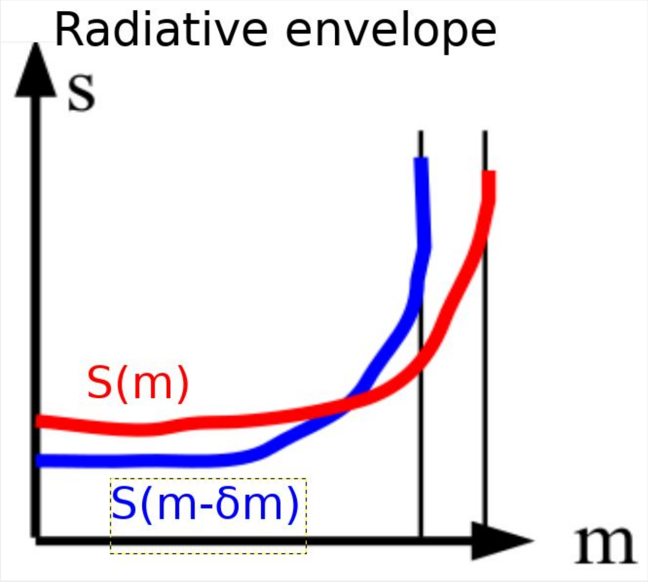
Dependence on donor's envelope
Radiative envelope

Consider a star in thermal equilibrium with mass m, consisting of a convective core + an extensive radiative envelope.

The entropy increases towards the stellar surface.

The exterior part of the entropy profile is less elevated than that of a comparison star with mass (m − δm).

When the donor loses a fraction δm of its mass at a high speed, it is cooler and less luminous than a star with mass (m - δm) in thermal equilibrium, so it becomes smaller (heating).

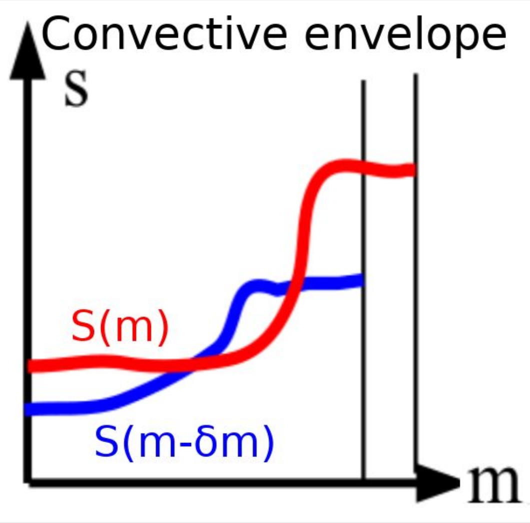


In the centre of the donor, the entropy profile is more elevated than the one of the comparison star. This elevated entropy, together with a reduced pressure to raise the upper layers, results in an expansion of the core. This way, the nuclear reactions are slowed down until the thermal equilibrium is reached.

Core expansion but envelope contraction → the radiative envelope allows the donor to **stay within its own** (reducing) **Roche lobe**.

Dependence on donor's envelope
Deep convective envelope

Stars with a <u>deep</u> convective envelope (like during the giant phases) have entropy profiles more complex, but the entropy is nearly constant in the outer layers.



When the star loses mass δm in a adiabatic way, its outer entropy is much more elevated than in the exterior layers of normal stars with mass $(m - \delta m)$.

The mass losing star is hotter and more luminous than the comparison star with $(m - \delta m)$.

Therefore **it expands** (cool)→ it is not able to stay within its reducing Roche lobe. As a result, the **mass loss is accelerated**.

Donor response
summary

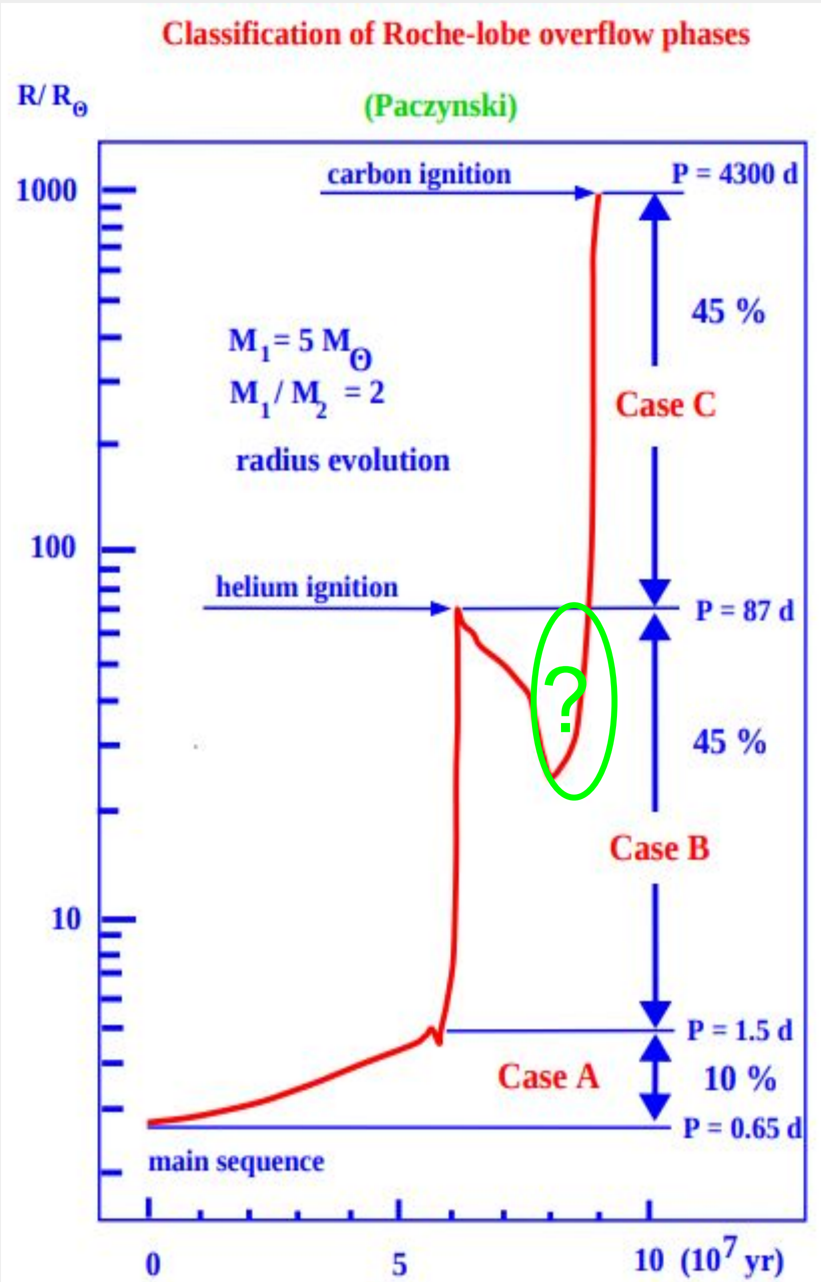
Initial response to mass loss: dynamical

Donor with radiative envelope (e.g. some MS, core-He burning)
→ shrink with mass loss
● Loss of external layer does not change entropy profile drastically.
● Star reacts adiabatically on the mass loss (contracting to reach hydrodynamical equilibrium).
● Donor has time to adjust and reach thermal equilibrium.

Donor with deep convective envelope (e.g. giant phases)
→ expand with mass loss
● Donor becomes hotter and more luminous
● Cannot stay within its shrinking Roche lobe
● Extreme mass loss on dynamical time scale

Later response: thermal, nuclear

Cases of Mass Transfer (via RLOF)
depending on the donor's evolutionary phase



According to Kippenhahn & Weigert (1967, Z. Ap., 65, 251):

Case A: donor in core H burning phase (MS)

Case B: donor in fast shell H burning phase (subgiant or RGB)

Case C: Donor after core He ignition (HB or AGB):

Why case C cannot occur at ?

Figure 1.1 The evolution of the radius of a 5 $M_\odot$ star as a function of its lifetime to illustrate the ranges in radius and orbital period for the different cases of RLOF phases, as indicated, assuming a 2 $M_\odot$ companion.

Gainer (accreting star) response

The reaction of the accretor is also very important

Typically, much more drastic changes than for donor.
Again depends on entropy profiles, but now mass is increasing

● Gainers with **radiative envelope** → **expand** while the **core contracts**: thermal readjustment if mass transfer timescale shorter than its KH timescale.

● Gainers with **convective envelope** → **shrink on dynamical time scale**.

It might be **rejuvenated**: gain of mass implies a growing of the core's fresh fuel from outer layers that reach the burning zone.

Gainer (accreting star) response

The gainer also has to deal with the gain in angular momentum → **spin-up** (thick accretion disk, hot spot?)

If it accreetes a lot of mass ($\Delta M/M \gtrsim 0.1$) it can spin up beyond breakup.

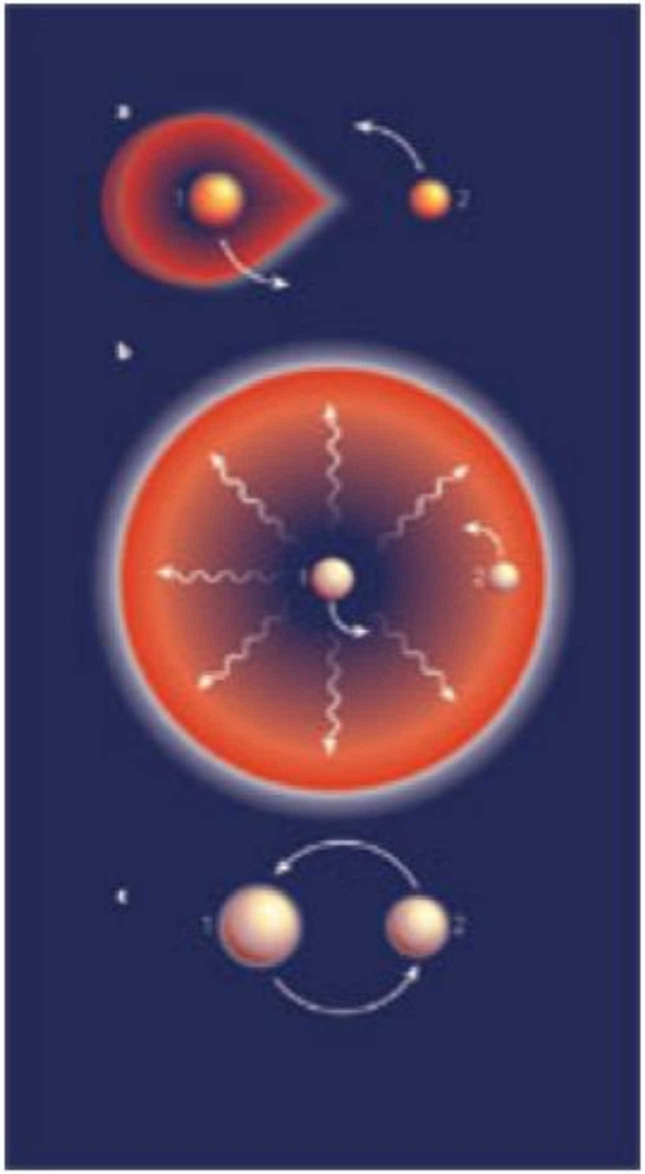
Luminosity of accretion may exceed Eddington $$L \sim \frac{GM\dot{M}}{R} > L_{\mathrm{Edd}}$$

It might also experience nuclear burning on the surface (if it is a WD it can experience a novae or even SN Ia).

For high mass transfer rates, the binary might become a contact binary → Common envelope evolution

Dynamically unstable mass transfer

- In the case of runaway mass transfer, we can get a **spiral-in**. The donor's envelope engulfs both stars, since the Roche lobe of the donor has shrunk so much $\Rightarrow$ forms a common envelope around both stars.
- Both stars feel a frictional drag $\Rightarrow$ energy is extracted from orbit and imparted to the envelope. The two stars spiral together, and the envelope is ejected. if they don't merge
- Angular momentum is no longer conserved



Response of the Roche radius

The Roche Lobe size depends on the orbital separation a (we saw it might increase or decrease) and also on the mass ratio q.

Paczynski (1967) derived a simple equation for R_L (accurate to 2% when $M_1/M_2 < 0.8$)

$$R_{L1} = 0.46a\left(\frac{M_1}{M}\right)^{1/3}$$

Using this equation and assuming that the total mass M is constant (conservative mass transfer) we can derive the rate of change in R_{L1}

$$\frac{\dot{R_{L1}}}{R_{L1}} = \frac{\dot{a}}{a} + \frac{1}{3}\frac{\dot{M_1}}{M_1}$$

For the conservative mass transfer (mass and angular momentum conserved) we had

$$\frac{\dot{a}}{a} = \frac{2\dot{M_1}\left(M_1 - M_2\right)}{M_1 M_2}$$

So we get

$$\frac{\dot{R_{L1}}}{R_{L1}} = 2\frac{\dot{M_1}}{M_1}\left(q - \frac{5}{6}\right) \longrightarrow q_{crit} = \frac{5}{6}$$

<0 (star 1 is losing mass)

Response of the Roche radius

For conservative mass transfer

- q < 5/6
- conservative mass transfer makes Roche lobe expand.
  - cuts off mass transfer
- Mass transfer if
- 1) star expands
  - nuclear evolution
- 2) angular momentum lost
  - winds
  - gravitational radiation
- donor star fills Roche lobe

- q > 5/6
- Roche lobe shrinks down around the star, stripping it down.
- Rapid (dynamical)
- violent
- rare because very fast
- must occur (more massive stars evolve first)

But careful: this is derived from Paczynski equation for R_L, valid only for $q < 0.8$!!

In general we should use

$$\frac{R_{\rm L}}{a} \approx \frac{0.49\, q^{2/3}}{0.6\, q^{2/3} + \ln(1 + q^{1/3})}$$

Summary RLOF mass transfer

- **Mass transfer occurs when**
  - star expands to fill Roche-lobe
    - due to stellar evolution
  - orbit, and thus Roche-lobe, shrinks till $R_* < R_L$
    - due to angular momentum loss
    - e.g. magnetic braking, gravitational radiation

- **Three cases**
  - Case A: mass transfer while donor is on main sequence
  - Case B: donor star is in (or evolving to) Red Giant phase
  - Case C: SuperGiant phase

- **Mass transfer changes mass ratio**
  - changes Roche-lobe sizes
  - can drive further mass transfer

Consequences of Mass Transfer
Donor

End Points Reached ONLY in Binary Evolution.

The primary star can become:

He WD – Core remnant following case A or B mass transfer; i.e., loss of envelope before He ignition.

sdB/sdO star – Core remnant from case B mass transfer. Envelope is lost but core He burning is reached → hydrogen-exhausted helium burning star with at most a small hydrogen-rich envelope. Later becomes a C/O WD or a hybrid He/C/O WD.

Consequences of Mass Transfer
Accretor

Mass accretion will also change the structure of the accreting star.

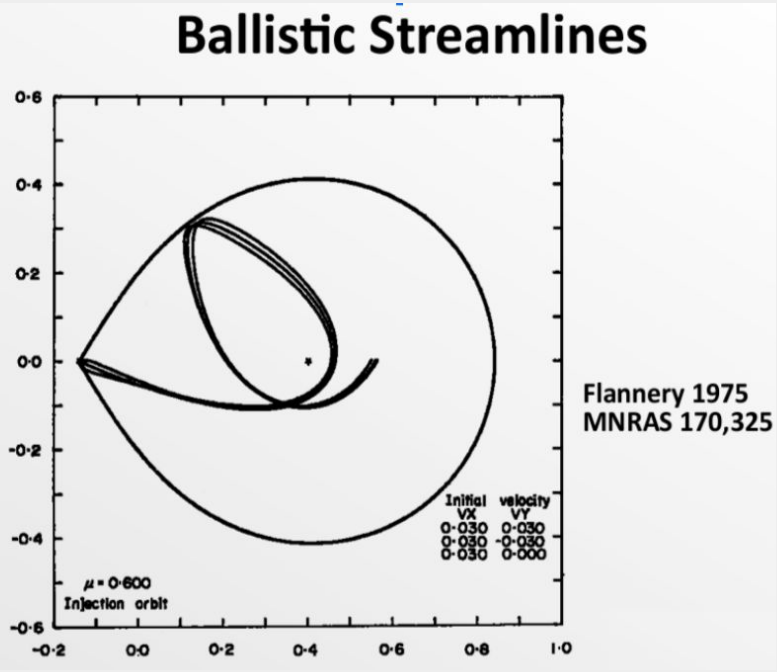
✦ If it is still on the MS, the accretor tends to be **rejuvenated,** i.e, H is mixed into the core**.** Behave like a more massive normal main-sequence star.
Could explain blue stragglers and young stars in old stellar populations.

✦ If it has already left the MS, its evolution can be drastically altered, and the star may never evolve to become a red supergiant, but explode as a blue supergiant (if it is a massive star)

Extra material: Ballistic trajectory

For stable mass transfer: The gas particles that flow through L1 to the Roche lobe of the companion behave as if they leak from the stellar atmosphere of the donor to the vacuum (because the companion is not yet filling its Roche lobe)
→ Equal the speed of sound in the atmosphere of the donor.

After passing through L1, the gas is highly accelerated towards the companion, reaching a supersonic velocity. This will cause the gas to follow a **ballistic trajectory**.



Flannery 1975
MNRAS 170,325

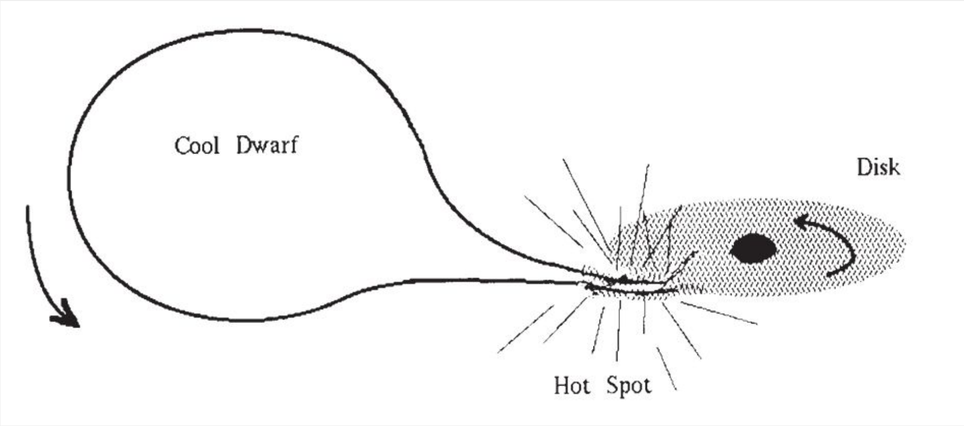
Extra material: Ballistic trajectory

Two scenarios can occur as the gas stream approximates star 2:

1) Supersonic (ballistic) flow through L_1 forms a disc and hot spot

The incoming gas stream is too far away from the stellar surface of star 2 and it is not captured at once. It moves supersonically around the star. The stream evolves to a state of minimal energy (a circular ring around star 2). The material in the gas ring radiates energy: some of the particles move closer towards star 2 while some others will move away due to conservation of angular momentum → an **accretion disk** is formed around star 2.

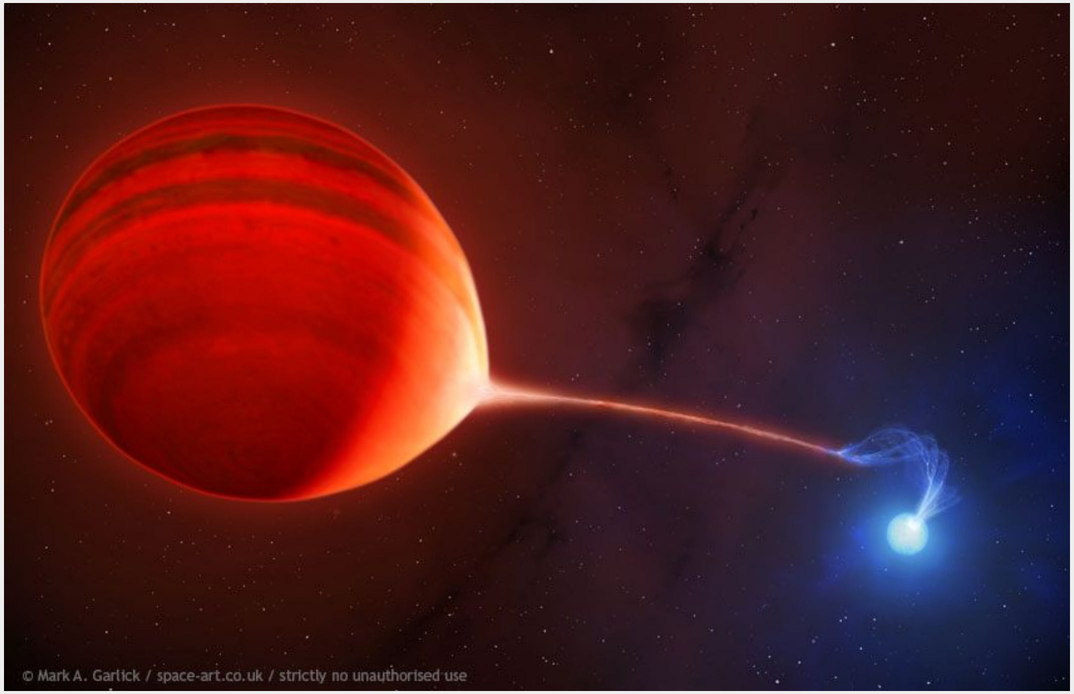
The matter which is coming from the donor through L1 causes the formation of a so-called **hot spot** at the exterior of the accretion disk, which causes energetic radiation that can be detected as a UV excess (or at even shorter wavelengths according to the heating that emerges) .



Extra material: Ballistic trajectory

Two scenarios can occur as the gas stream approximates star 2:

2) Direct impact with the stellar surface (e.g. in the presence of strong magnetic fields). In this case, the energy gained by the gas stream as a result of the attraction of star 2, is dissipated in a shock at the stellar surface of star 2.



e.g. polars (magnetic CVs)

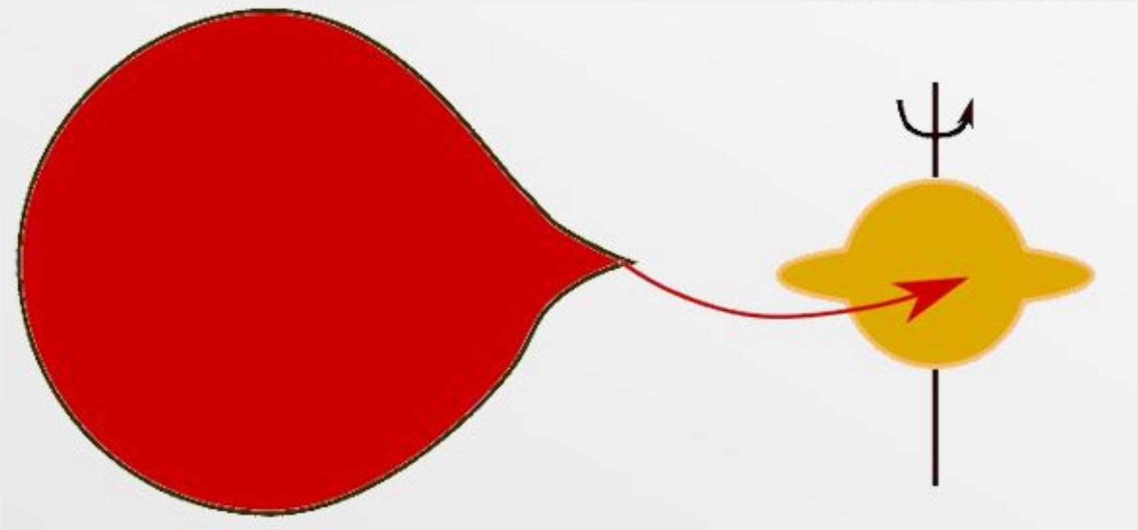
Extra material: Spin up and break up

Keplerian disk: obey's Kepler's laws of motion due to the dominance of a massive body at its centre. The velocity of the material a distance R from the central star of mass M is

$$v = \sqrt{\frac{GM}{R}}$$

Accretion from a Keplerian disc limits accretion if angular momentum is conserved → the accretor is spinning up.

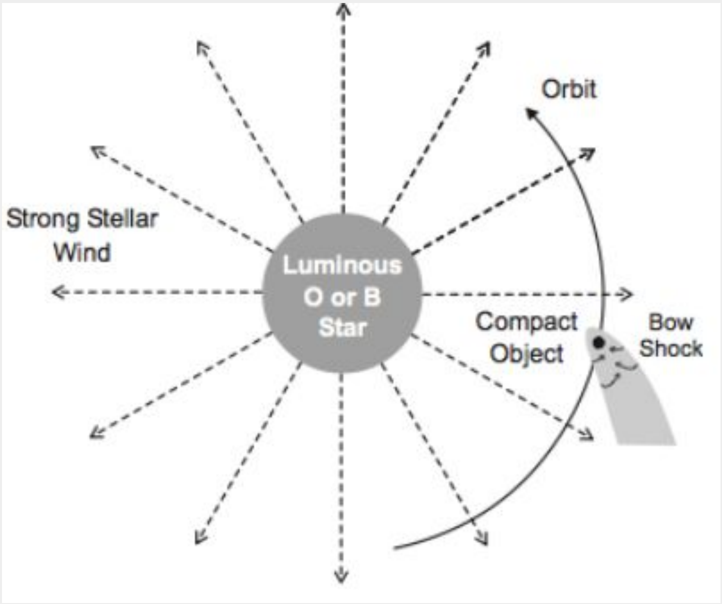
If $\gtrsim$ 10% of mass is accreted, star breaks up

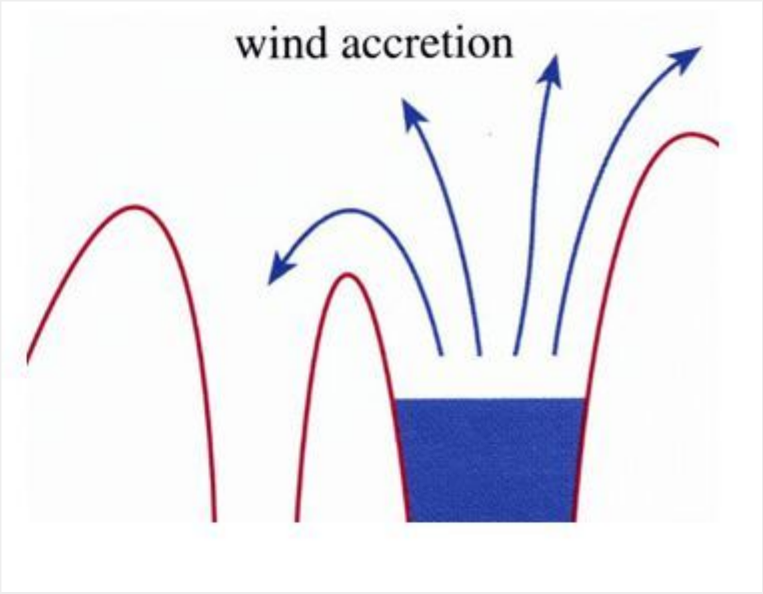


- Unless angular mom. can be removed …
- Tides? Outflow?

Extra material: Wind driven mass transfer

● Wind-driven accretion is understood much less well than accretion by Roche-lobe overflow.
● Particularly important for binaries containing massive stars (O and B stars).
● Only a tiny fraction (0.01–0.1%) of the matter in the wind is accreted onto the companion, unlike RL overflow (close to 100%).
● Angular momentum loss implies increase of orbital separation
→ Mass transfer is slowed down





Extra material: Wind driven mass transfer

For binary systems involving massive components, one also has to take into account the mass loss by a radiation-driven wind. The same is true for lower mass binaries which undergo a dust-driven wind. Such winds also affect the orbital elements of the binary. Assume that one of the components undergoes mass loss $\dot{M}_1$. In that case, there is a loss of angular momentum:

$$\dot{J} = \dot{M}_1\, a_1^2\, \Omega.$$

We have

$$\frac{\dot{J}}{J} = \frac{\dot{M}_1\, a_1^2\, \Omega}{(a^2\, \Omega\, M_1\, M_2)/M} = \dot{M}_1\, \frac{M_2}{M_1 M}$$

and also

$$\frac{\dot{J}}{J} = \frac{2}{3}\left(\frac{\dot{M}_1}{M} - 2\,\frac{\dot{\Omega}}{\Omega}\right) + \frac{\dot{\Omega}}{\Omega} + \frac{\dot{M}_1}{M_1} - \frac{\dot{M}_1}{M} = -\frac{1}{3}\,\frac{\dot{M}_1}{M} + \frac{1}{3}\,\frac{\dot{P}}{P} + \frac{\dot{M}_1}{M_1}$$

such that

$$\frac{3}{2}\,\frac{\dot{a}}{a} = \frac{\dot{P}}{P} = -\,\frac{2\,\dot{M}_1}{M}.$$

In this case, both the orbital period and the separation *increase* such that the mass transfer is slowed down.

Extra material: Wind driven mass transfer

● We will not necessarily get an accretion disk in wind-driven systems; (the angular momentum of the transferred particles makes it less probable than in Roche-lobe overflow).

● However, some wind-driven systems do show evidence of accretion disks.

Losing Angular Momentum Adiabatically...

Stellar Winds

Fast wind from a hot star

Carries specific angular momentum of loser

$$\frac{a_f}{a_i} = \frac{M_i}{M_f}$$

Slow wind from low-mass star (convective and magnetized)

Particles flow along large lever arm