When I talk to friends and family outside the STEM disciplines about what I do, I tend to fall back on a single principle: "Mathematics is what you do when your intuition fails you." In my mind, the purpose of mathematics is to provide a reliable method of organizing thoughts when the world around you and common sense seem at odds with each other. I achieve this in my teaching by helping students work through their preconceptions of topics we discuss, modeling good organizational practices, and tying our new abstract tools back into realistic problems.

To be able to convince my students of the power of math, I believe that it's important to first demonstrate how intuition can lead them astray. In Calculus I, after teaching my students the limit definition of the derivative, I ask them to think about the following questions: "Why do we use the limit definition? Can you come up with a definition for the derivative of a function at $x = a$ that does not involve limits?" After giving them a few minutes to think and share with their classmates, we come back together for a classroom discussion. Students are often afraid of giving the wrong answer, so I encourage them to guess wildly, and to shout out a word or phrase rather than trying to give a detailed response. Almost without fail, many of them say "tangent line!"

I then challenge them to think of a way to define a tangent line without using limits. Often, someone will bring up the idea that a tangent line touches the graph at exactly one point, and we'll go on to discuss familiar examples that don't fit that description, like straight lines, or the graph of sine. We also talk about the value of the limit definition as a way to anchor a bunch of seemingly-unrelated derivative rules to the concrete geometric concept of the tangent line. Many of my students have seen calculus in some form or another before, but it's only after examining their assumptions closely that they see the value of the concept of a limit.

Once my students understand the need for mathematical structure, it's my job to help them learn how to organize their thoughts in a reasonable way. One organizational tool I create for every class period is note-taking guides. This is particularly important when we talk about Newton's Method. Students start the class period by answering a couple of warm-up questions about our last class period, when we talked about linearization and how to use tangent lines to approximate the value of a function. We then get back together as a class and talk about how that compares to today's application of the tangent line as a way to find roots of a function, and the step-by-step approach we use for Newton's Method. Using note-taking guides allows me to show pre-made pictures for each step of Newton's Method, to give them a geometric interpretation of the calculations they'll be making later in the class period.

I've found that note-taking guides also help my students see technology as a useful tool, rather than a barrier in the way of their understanding. When we do our first example of Newton's Method, they have the calculator steps they'll need to use already printed in their notes, so we can focus on the calculus that makes it possible. We work an example together as a class, and I check in to make sure everyone is on the same page by walking around the room as they use their calculators to follow along. Once we've been successful as a group, they try another problem on their own, referring to their notes and asking their classmates for help if they get stuck.

If the motivation for learning mathematics is to resolve an apparent disparity between the world and intuition, at some point we must take our new organizational tools back into the real world. I have found that using technology in the classroom allows us to approach more realistic problems. In Calculus III, after my students learned about parameterizations in 3 dimensions, I break them up into small groups to do an activity in Mathematica. In the activity, they were asked to determine whether a rocket with a given parameterization would ever come within striking distance of a comet in orbit. Using Mathematica made it simple to do a relatively complex distance calculation, and I coded a quick animation of the paths of the objects over time that the students could run after they finished the problem. While we did many in-class activities in Mathematica that semester, that was one of the students' favorites because they could back up their mathematical understanding with a clear picture.

Helping students with mathematical thinking in the classroom is my job, but I've been fortunate to have a support structure outside of the classroom, in the form of teaching mentorship programs. When I taught College Algebra, Calculus I, Calculus II, and Calculus III, I had the benefit of one or more experienced teachers meeting with graduate instructors once a week to discuss teaching strategies. Facilitating group work has been something I've struggled with, but through discussions with my Calculus I mentors, I've gained a better understanding of the fine line between giving encouraging hints and not giving students space to make their own discoveries. The College Algebra mentorship program exposed me to materials I may have never chosen to use otherwise. In College Algebra, we used clickers for daily quizzes, which inspired me to create my own clicker questions for feedback during lectures in Basic Calculus. Because the mentor for College Algebra wanted to give students better homework questions, I had the opportunity to teach myself how to write custom, randomized questions in Perl for our online homework system.

I look forward to the opportunity to continue to grow as a teacher, and to share my passion for logical thinking with my students. Mathematics doesn't always make sense at first glance, but I am determined to help my students break down the assumptions that get in the way of their understanding and replace them with logical structures that will make irrefutable sense with time and practice.