Elision User Guide

Version 0.2 Build 201209100400

nominatim

Linguistically, an *elision* (i'liZHən) is the omission of sounds from an utterance to create a result that is (typically) easier to pronounce. Contractions are examples of elisions. Elision can also refer to omissions in books and films. Finally, it may mean the process of joining or merging together abstract ideas.

Contents

Chapte	er 1. Installation	6
1.1.	Running	6
1.2.	Building	6
1.3.	REPL	7
1.4.	Scala with Emacs, Notepad-Plus, and Others	7
1.5.	Scala with Eclipse	8
1.6.	Scala Code	8
Chapte	er 2. Interacting with the REPL	10
2.1.	Starting the REPL	10
2.2.	History	11
2.3.	Properties	12
2.4.	Binding	12
2.5.	Atom Display	13
2.6.	Automatic Rewriting	14
2.7.	Understanding Matching	15
2.8.	Understanding Parsing	16
2.9.	Reading Files	17
2.10	D. Bootstrapping	17
Chapte	er 3. Basic Atoms	18
3.1.		18
3.2.		18
3.3.	Special Symbols	20
3.4.	Variables	21
3.5.	Metavariables	22
3.6.	Lambdas	23
3.7.	Lambdas and Metavariables	24
Chapte	er 4. Collections	26
4.1.	The Basic Ordered Sequence	26
4.2.	Algebraic Properties	26
4.3.	Simple Collections	28
4.4.	Properties Specifications as Operators	28
4.5.	Long Property Specifications	29
Chapte	er 5. Operators	30
5.1.		30
5.2.	Operator Properties	33
5.3.		35
5.4.		36
5.5.	Operators, Symbols, and Naked Symbols	37
5.6.	Case Operators	38

CONTENTS 4

 5.7. Recursion 5.8. Summary: Case Operators 5.9. Documenting Operators 5.10. Closures 	42 42 43 44
Chapter 6. Special Forms 6.1. The Basic Special Form 6.2. Known Special Forms	45 45 46
Chapter 7. Matching and Rewriting 7.1. Bindings 7.2. Applying Bindings 7.3. Matching 7.4. Map Pairs 7.5. Variable Guards	48 48 48 49 50 51
Chapter 8. Rules and Strategies 8.1. Rewrite Rules and Guards 8.2. Rulesets 8.3. Strategies 8.4. The Map Strategy 8.5. Strategy Combinators	52 52 52 52 52 52 52
Chapter 9. Executors and Contexts	53
Appendix A. BasicAtom	54
Appendix B. Exceptions B.1. Elision Exceptions B.2. Java Exceptions and Throwables Appendix C. Reading Elision Files C.1. Processor	55 55 55 56 56
C.1. Processor C.2. Operators	57
Appendix D. Native Handlers	58
Appendix E. De Bruijn Indices	62
Appendix F. The Elision Directory Structure F.1. In the Distribution F.2. From the Build	63 63 64
Appendix G. The Elision Coding Guide G.1. Concepts G.2. Architectural Rules G.3. Scaladoc G.4. Comments G.5. Spacing G.6. Returning Appendix H. Grammar	66 66 66 67 67 68

CONTENTS 5

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The Elision Term Rewriter

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CHAPTER 1

Installation

This chapter explains how to build the Elision rewriter. Information about the rewriter can be obtained by visiting http://elision.github.com/. This page contains a frequently updated version, along with documentation links and a link to Elision's source.

Elision contains Parboiled, a parser expression grammar (PEG) library. See https://github.com/sirthias/parboiled/wiki for details, including the license agreement. The Elision REPL uses JLine, a Java library for handling console input. See https://github.com/jline/jline2 for details, including the license agreement. Note that Elision actually uses the modified version of JLine built for the Scala REPL and distributed with Scala.

1.1. Running

If you only want to run the Elision REPL, you have two choices.

- Make sure the Java 1.6 runtime is installed and download the large elision-all.jar. Run Elision with the command java -jar elision-all.jar. In some environments you can just double-click the jar file.
- Make sure both Java 1.6 and the Scala 2.9.1 (or later) distribution are installed. Download the (much smaller) elision.jar and run Elision with the command scala elision.jar.

1.2. Building

Elision requires Scala 2.9 or later to build. Be careful with later versions of Scala as the language is still changing quite a bit. Java 6 or later and Apache Ant 1.8 or later are also prerequisites.

- (1) Install the Java 6 (or later) SDK. Visit http://java.oracle.com to download the correct version for your platform.
- (2) Install Scala 2.9.1 (or later). Visit http://www.scala-lang.org to download the Scala distribution.
- (3) Set the environment variable SCALA_HOME to point to the root folder of your Scala installation (the folder that contains the bin and lib folders).
 - (a) On Windows platforms you should edit the environment variable settings under System, and add %SCALA_HOME%\bin to the Path environment variable. Open System Properties and click Advanced system settings. Then on the Advanced tab click Environment Variables... Under System Variables click New...

Variable name	Variable value
SCALA_HOME	C:\scala

You should replace C:\scala with the directory where you installed Scala. Next edit the value for Path carefully to add; %SCALA_HOME%\bin on the end.

(b) On MacOS X platforms you can edit the environment plist or follow the directions for UNIX below. You can do this from the Terminal prompt with open ~/.MacOSX/environment.plist. This will open the environment settings in the Property List Editor. Add a new Item at the top level.

 $^{^{1}\}mathrm{Actually}$ the filename will contain the build information, and will probably resemble elision-all-20120601.jar.

Key	Туре	Value	
SCALA_HOME	String	/usr/local/scala	

You should replace /usr/local/scala with the directory where you installed Scala. Next edit the value for PATH carefully to add: \$SCALA_HOME/bin on the end.

(c) On modern UNIX systems (including MacOS X) and most Linux systems you can create a /etc/profile.d/scala.sh file with the following content:

Add Scala

export SCALA_HOME=/usr/local/scala
PATH=\${PATH}:\$SCALA_HOME/bin

You should replace /usr/local/scala with the directory where you installed Scala.

(4) (Optional) It is recommended to get the Scala developer documentation package. After you have installed Scala, run the following command at the prompt. If you have installed Scala in a folder owned by root on Linux or UNIX, you will need to precede the command with sudo.

sbaz install scala-devel-docs

- (5) Install Apache Ant 1.8 (or later). Visit http://ant.apache.org to download the Ant distribution.
- (6) (Optional) If you want to use Eclipse, you should install Eclipse Indigo (or later).
- (7) To build the Elision jar file, cd to the root folder of the Elision distribution and run the command ant. This will build the jar file.
- (8) (Optional) If you want to build the API documentation, then run the command ant docs. To build both the jar files and documentation, use ant all. To see what other targets are available, use ant -p.

1.3. REPL

Elision comes with a read, evaluate, print loop interpreter (a REPL). You can start it in any of the following ways.

- Use the elision.sh script found in the root folder of the Elision distribution. This starts the REPL using the compiled class files in the bin folder, and is the best way to run it if you are using a continuous build system such as Eclipse. If you try to run the REPL this way while Eclipse is compiling, you will get missing class file errors. Just wait for Eclipse to finish compiling.
- To run the REPL from the jar file, execute the command scala elision.jar. You must have built the jar file to use this method, and must update it each time you make changes.
- To start the REPL from inside a program, invoke the ornl.elision.repl.ReplMain.run() method.

To exit from the REPL use :quit. To get help use help(). The REPL is the subject of chapter 2.

1.4. Scala with Emacs, Notepad-Plus, and Others

Most editors now come with a Scala mode installed. In case yours does not, you should look in the Scala distribution under misc/scala-tool-support. We will provide some instructions for Emacs here.

The Emacs mode works for Emacs 21.1or later, and does not work on XEmacs. Things change; consult the README file in the folder for current details. It is suggested you leave these files in place so they get magically updated when the next version of Scala is installed.

In your Emacs startup file (typically ~/.emacs on Linux or UNIX) add the following two lines.

```
(add-to-list 'load-path
  "/usr/local/scala/misc/scala-tool-support/emacs")
(require 'scala-mode-auto)
```

Adjust the second line as necessary to point to your Scala installation. Now restart Emacs, and you should get the Scala mode when you open a file ending with .scala. You can compile the .el files to .elc files using the provided Makefile, but be sure to run make each time you update.

1.5. Scala with Eclipse

The root folder of the Elision distribution contains an Eclipse project. Before you attempt to import it into Eclipse you should do the following.

• Install the Scala IDE. At present the version of the Scala IDE for Scala 2.9 can be found at the update site http://download.scala-ide.org/releases-29/stable/site. Visit http://scala-ide.org for the latest version, documentation, etc.

That is all that is required. You may optionally install the following helpful plugins.

- ShellEd is an excellent editor for working with shell scripts, but at present it only works with Eclipse Indigo. You can find out more at http://sourceforge.net/apps/trac/shelled. To install ShellEd you need to install components from the Linux Tools. The update site is http://download.eclipse.org/technology/linuxtools/update. Add this update site first, but do not install anything. The update site for ShellEd is https://downloads.sourceforge.net/project/shelled/shelled/ShellEd%202.0.1/update. After adding the update site for the Linux Tools, add this update site and then install ShellEd; it will resolve the dependencies and get the components from the Linux Tools that are actually required.
- Copyright Wizard can be used to automatically and quickly update the copyrights on all files and to make sure all new files have the correct copyright information. The distribution includes a configuration file for Copyright Wizard that will be automatically discovered if you install Copyright Wizard. Find out more at http://www.wdev91.com/?p=cpw. The update site is http://www.wdev91.com/update.

Once you have installed the Scala IDE plugin, do the following.

- (1) Unpack the Elision distribution into an Eclipse workspace, or create a workspace folder and place the Elision folder in it.
- (2) Start Eclipse. You will probably be prompted to run the Scala Setup Diagnostics. Click Yes to run them. Make sure you open Eclipse in the correct workspace containing the Elision distribution.
- (3) Go to the workbench. Right-click in the package explorer and choose New and then Project. In the New Project wizard expand Scala Wizards and select Scala Project. Click Next.
- (4) In the New Scala Project dialog enter Elision as the project name. If you have done everything correctly you should see a note at the bottom of the dialog telling you the wizard is about to automatically configure the project based on the existing source. This is what you want, so click Finish.
- (5) You may be asked if you want to switch to the Scala perspective. You do, so click Yes.

At this point Elision should automatically build, and you are ready to start working with the code or running the REPL using repl.sh.

1.6. Scala Code

Elision is written in Scala, and provides a Scala API. Discussions of the Scala API are highlighted in blue.

Scala. Elision's classes live in packages under ornl.elision. Throughout this document packages, classes, objects, traits, etc., will be discussed in reference to this top-level package. Thus core.BasicAtom refers to the fully-qualified class name (FQCN) ornl.elision.core.BasicAtom.

CHAPTER 2

Interacting with the REPL

The REPL is Elision's read, evaluate, print loop. It provides a way to interact directly with the rewriter.

2.1. Starting the REPL

You can start the REPL in any of the following ways.

- Use the elision.sh script found in the root folder of the Elision distribution. This starts the REPL using the compiled class files in the bin folder, and is the best way to run it if you are using a continuous build system such as Eclipse. If you try to run the REPL this way while Eclipse is compiling, you will get missing class file errors. Just wait for Eclipse to finish compiling.
- To run the REPL from the jar file, execute the command scala elision.jar. You must have built the jar file to use this method, and must update it each time you make changes.
- To start the REPL from inside a program, invoke the ornl.elision.repl.ReplMain.run() method. This is useful from the Scala prompt, assuming you have the correct CLASSPATH. You can use the run.sh script found in the Elision root to start Scala with the correct CLASSPATH.

All of these methods will display the Elision banner and present you with the REPL prompt e>. You can enter text to be evaluated as an Elision atom (described in later sections), interact with the history (using the up and down arrow keys), or edit the current line. Pressing Enter submits the line to the Elision parser for evaluation.

The most useful commands to know immediately are help() (to get help) and :quit (to quit). The REPL commands (with the exception of :quit and history recall) are actually Elision operators (see chapter 5). As such you must follow them with parentheses, otherwise the Elision REPL will interpret them as symbols. Try history and then try history().

The REPL reads zero or more atoms per line, evaluates them, and displays the output. Several atoms can be present on a single line; in that case the REPL displays each one.

¹It is permissible to put spaces between the operator name and the opening parenthesis: history (). This can lead to trouble. Suppose you want to have the symbol fred followed by a parenthesized expression: fred (5).\$x. Here the parentheses are necessary to prevent the parser from interpreting the dot as a decimal point. Unfortunately this will not work the way you might expect. The system will parse this as (fred(5)).\$x. To prevent this, put parentheses around fred: (fred) (5).\$x.

2.2. HISTORY 11

Atoms can span multiple lines. If a parenthesis, brace, bracket, or verbatim block² is open at the end of the line, then the REPL will prompt for additional input with the prompt >. The newlines are converted to spaces, except with verbatim blocks.

```
e> { operator
> #name=set
> #params=%ACI($P: BOOLEAN, $Q: BOOLEAN)
> #type=BOOLEAN
> }
$_rep10 = {: operator:SYMBOL { binds name -> set:SYMBOL
    params -> %ACI($P:BOOLEAN, $Q:BOOLEAN) type -> BOOLEAN } :}
e> ("Fred".
> " Flintstone")
$_rep11 = "Fred_Flintstone"
```

2.2. History

The Elision REPL saves a persistent history from session to session. This history can be viewed with the history() command.

```
e> history()
0: // New Session: Fri Jun 08 12:12:59 EDT 2012
1: // Running: 0.1, build 201206081150
2: {operator #name=test #params=%($x:INTEGER) #type=INTEGER}
3: {rule add($x,neg($x)) -> 0}
4: add(21,17,$x)
5: history()
Persistent history is found in: /home/userguy/.elision
```

It is possible to recall any prior history element by entering the history index after an exclamation mark.

```
e> !4
add(21,17,$x)
$_repl14 = add($x, 38)
e> bind($x, 2)
Bound $x
e> !4
add(21,17,$x)
$_repl15 = 40
e> history()
0: // New Session: Fri Jun 08 12:12:59 EDT 2012
```

```
"""This is the first line,
    and this is the second.
    Escapes and quotation marks are preserved.
    "See \n?" he said."""

Denotes the following string.

"This is the first line,\n and this is the second.\nEscapes and quotation marks are preserved.\"See \\n?\" he said."
```

²A verbatim block is text within triple double quotation marks: """..."". No escapes are interpreted, and spaces and newlines are maintained. Thus the verbatim block:

2.4. BINDING 12

```
1: // Running: 0.1, build 201206081150
2: {operator #name=test #params=%($x:INTEGER) #type=INTEGER}
3: {rule add($x,neg($x)) -> 0}
4: add(21,17,$x)
5: history()
6: add(21,17,$x)
7: bind($x, 2)
8: add(21,17,$x)
9: history()
Persistent history is found in: /home/userguy/.elision
```

2.3. Properties

The operation of the REPL, and of Elision itself, is configurable using *properties*. To see all properties defined in the current system, use the command listprops(). This lists each property name, gives a short description, and gives the *current* value of the property in parentheses.

To set the value of a property, use setprop(name, value), where name is the property name (a string) and the value is an atom of the correct type.

2.4. Binding

We can tell the Elision system that we want to replace the variable x with the value 7 wherever it appears. We say the variable x is bound to 7, and that the pair x and 7 is a binding, written x - 7. When we replace the variable x with 7 in some other term, we say we are rewriting the term with the binding. The REPL keeps a set of bindings that are used to immediately rewrite any atom after it is parsed and constructed. For instance, if the variable x is bound to 7, then entering x at the prompt results in the value 7. These bindings are stored in the REPL's context. Contexts are explained in chapter 9; for now you only need to know that the REPL creates and maintains a context for you.

The bind(v, a) command binds a variable v to an atom a. For instance, try bind(\$x,7) and $bind(\$y,\\$x.\%(\$x,\$x))$. Then try \$x and \$y at the prompt. Then try \$y.\$x. Variables can be rebound at any time, and there is no need to "unbind" them first.

```
e> bind($x,7)

Bound $x

e> bind($y,\$x.%($x,$x))

Bound $y
```

³This notation is called a map pair, and it is actually more general. It is explained in section §7.4.

```
e> $x
$_repl16 = 7
e> $y
$_repl17 = \$':1'.%($':1', $':1')
e> $y.$x
$_repl18 = %(7, 7)
```

The unbind(v) command removes any binding for the variable v. Try unbind(x) and then repeat x. No error is generated if a variable is not bound.

```
e> unbind($x)
Unbound $x
e> $y.$x
$_repl19 = %($x, $x)
```

If the setreplbinds property is true (which is the default), then every time an atom is evaluated by the REPL, the result is displayed and then bound to a new variable whose name starts with \$_repl followed by a number. These variables are displayed on the output. Try \$_repl1. Unlike the history, which is persistent between invocations of the REPL, these variables are re-started at each session.

To see all the bindings being used by the shell, enter the command showbinds().⁴

2.5. Atom Display

When you type an atom at the REPL prompt the system parses the atom and constructs it, which may involve evaluating operators. Otherwise the current bindings are applied to the atom, and the atom is then rewritten using any active rulesets. The result of this is then bound to the next \$repln variable.

You can see the atom after it is parsed and constructed, but before bindings are applied and rewriting is performed, by setting the **showprior** property to **true**.⁵

```
e> bind($x,10)
Bound $x
e> setprop("showprior", true)
' NO SHOW ':SYMBOL
e> add($x,$x,0)
add($x, $x)
$_repl20 = 20
e> setprop("showprior", false)
```

The atom as entered was add(x,x,0). During construction the identity 0 is removed, leaving the atom add(x,x), which is displayed. Then the bindings are applied, yielding add(10,10), which is rewritten to 20 and displayed as the final result.

Most REPL printing can be suppressed using the quiet(level) command. The level is one of the following values.

- O Show all output.
- 1 Suppress most output, but still show errors, warnings, and explicitly requested output.
- 2 Suppress warnings, but show errors and explicitly requested output.
- 3 Suppress warnings and errors, but still show explicitly requested output.
- 4 Suppress nearly everything.

⁴Note that any bindings to variables whose name starts with an underscore are suppressed in the output.

⁵This action results in the odd output 'NO SHOW ':SYMBOL. Elision expects every operator to yield an atom, but this makes no sense for operators executed only for their side effect. As such, these operators return a special atom typically hidden by the REPL. Enabling showprior causes the atom to be displayed.

Any quiet level except 0 changes the prompt to q>.

Scala. It is possible to display the Scala code to create the atom. This is enabled (and disabled) with the **showscala** property. The assumed context for the resulting code is that the content of the **core** package is imported.

This can be useful to discover how to construct an atom using the Scala API.

If the above seems excessively complex, don't panic. There are several helpful parts of the API, including implicit conversions, to make this all much simpler.

2.6. Automatic Rewriting

Once an atom is entered and bindings are applied, it is rewritten with the active rulesets. (Applying rules is the subject of chapter 8.)

Rulesets are initially disabled when declared. They can be *enabled* using the **enable(RS)** command, where **RS** is the name of the ruleset. Likewise, they can be *disabled* with the **disable(RS)** command.

```
e> decl.{rule fred -> james #rulesets RENAME}
ERROR: The ruleset RENAME has not been declared.
e> declare(RENAME)

Declared ruleset RENAME:SYMBOL.
e> decl.{rule fred -> james #rulesets RENAME}

Declared rule.
e> fred
$_rep18 = fred:SYMBOL
e> enable(RENAME)
e> fred
$_rep19 = james:SYMBOL
```

A limit is placed on the number of times a rule can be applied to rewrite an atom. This limit can be set via the setlimit(lim) command, where lim is the new limit. A limit of zero disables automatic rewriting, and a negative limit is equivalent to no limit. Automatic rewriting is on by default, but can be disabled (or re-enabled) altogether with the autorewrite property. It is enabled by default.

You can ask the system which rules may be applied to an atom, using the showrules(atom) command, where atom is the atom to test.

```
e> showrules(if (true, 6, 7))
{ RULE if(true, $thenpart, $elsepart) -> $thenpart rulesets IF level 2 }
{ RULE if(false, $thenpart, $elsepart) -> $elsepart rulesets IF level 2 }
```

2.7. Understanding Matching

To support better understanding of the matching process, it is possible to enable (or disable) tracing the matching process with the tracematch() command. When enabled considerable diagnostic information is printed for every match that is attempted.

Several things trigger the matching system. One is constructing an operator application, where arguments are matched against the formal parameters. Another is matching a lambda argument to the parameter. Of course, the most obvious way is by explicitly trying to match two atoms.

```
e> tracematch()
Match tracing is ON.
Sequence Matcher called:
   Patterns: $type
   Subjects: ANY
   Bindings: { binds }
TRYING (a55109e) in class ornl.elision.core.Variable:
 pattern: $type
 subject: ANY
 with: { binds }
SUCCESS (a55109e): { binds type -> ANY }
TRYING (f021a3c0) in class ornl.elision.core.INTEGER$:
 pattern: INTEGER
 subject: INTEGER
 with: { binds }
SUCCESS (f021a3c0): { binds }
SUCCESS (1412e2ce): \{ binds x -> 6 \}
prepl14 = 6
```

Reading this trace can be tricky. The sequence matcher tries to perform an in-order pairwise match of two sequences of atoms, given some set of bindings. For example, the first line indicates that the sequence matcher was called to match the pattern sequence \$domain, \$codomain against the subject sequence \$T, ANY with no prior bindings.

The lines that begin with TRYING and SUCCESS or FAILURE are linked using the hexadecimal number in parentheses. This number is actually a hash code for the match attempt and serves to link together the lines. In a large match attempts will often interleave.

Following a TRYING line the pattern, subject, and any bindings are printed. On SUCCESS we see the bindings that result in a successful match. On FAILURE we are told why the match failed, and the pattern and subject may be repeated.

In some cases there will be additional matching reports after the \$_repl result is shown. These are due to round-trip testing by the system. This is testing that determines whether or not an atom's representation can be successfully re-parsed to yield the original atom, and this output can safely be ignored. Look for the line beginning with \$_repl and ignore subsequent output. To disable round trip testing, set the property roundtrip to false.

```
e> {match 7}.$x

Sequence Matcher called:
    Patterns: $type
    Subjects: INTEGER
    Bindings: { binds }

TRYING (a9d51ff0) in class ornl.elision.core.Variable:
```

```
pattern: $type
  subject: INTEGER
  with: { binds }
SUCCESS (a9d51ff0): { binds type -> INTEGER }
Sequence Matcher called:
   Patterns: $domain,$codomain
   Subjects: ANY, BINDING
    Bindings: { binds }
TRYING (71407c54) in class ornl.elision.core.Variable:
  pattern: $domain
  subject: ANY
  with: { binds
SUCCESS (71407c54): { binds domain -> ANY }
TRYING (f6b11b01) in class ornl.elision.core.Variable:
  pattern: $codomain
  subject: BINDING
  with: { binds domain -> ANY }
SUCCESS (f6b11b01): { binds domain -> ANY codomain -> BINDING }
TRYING (c390faa3) in class ornl.elision.core.IntegerLiteral:
  pattern: 7
  subject: $x
  with: { binds }
TRYING (50a1946e) in class ornl.elision.core.INTEGER$:
  pattern: INTEGER
  subject: ANY
  with: { binds }
SUCCESS (50a1946e): { binds }
FAILURE (c390faa3): Literal pattern does not match subject.
  pattern: 7
  subject: $x
$_repl10 = NONE
```

For this simple example we see the inner attempt to match the types succeeds, and then the outer attempt to match the entire atoms fails because 7 is a literal pattern and it can only match another literal 7.

2.8. Understanding Parsing

The parser can also be traced, and this results in an enormous quantity of output. To enable (or disable) parser tracing, use the traceparse() command.

```
e> traceparse()
Parser tracing is ON.
e> 7
Starting new parsing run
a sequence of atoms/ZeroOrMore/AtomSeqPush1Action1, matched, cursor at 1:1 after ""
...(1)../ZeroOrMore/ZeroOrMore/Sequence/an atom/LApply/a simple atom/whitespace or
comments/FirstOf/OneOrMore/[ \n\r\t\f], failed, cursor at 1:1 after ""
...(9)../OneOrMore, failed, cursor at 1:1 after ""
...(8)../FirstOf/Sequence/"/*", failed, cursor at 1:1 after ""
...(9)../Sequence, failed, cursor at 1:1 after ""
```

```
..(9)../Sequence, failed, cursor at 1:1 after ""
..(8)../FirstOf, failed, cursor at 1:1 after ""
..(7)../whitespace or comments, matched, cursor at 1:1 after ""
...(6).../a simple atom/FirstOf/Sequence/'(', failed, cursor at 1:1 after ""
..(8)../Sequence, failed, cursor at 1:1 after ""
...(7).../FirstOf/a ruleset declaration/'{', failed, cursor at 1:1 after ""
..(8)../a ruleset declaration, failed, cursor at 1:1 after ""
..(7)../FirstOf/a rewrite rule/'{', failed, cursor at 1:1 after ""
..(8)../a rewrite rule, failed, cursor at 1:1 after ""
..(3)../Sequence, failed, cursor at 1:2 after "7"
..(2)../FirstOf/Sequence/"//", failed, cursor at 1:2 after "7"
..(3)../Sequence, failed, cursor at 1:2 after "7"
..(2)../FirstOf, failed, cursor at 1:2 after "7"
..(1)../whitespace or comments, matched, cursor at 1:2 after "7"
a sequence of atoms/EOI, matched, cursor at 1:3 after "7"
a sequence of atoms, matched, cursor at 1:3 after "7"
prepl14 = 7
```

Many, many lines are omitted from the above. The number shows the depth of the parse which, for this example, reaches 16. Note that, as with match tracing, the round trip testing results in additional output that can be ignored.

2.9. Reading Files

If you want to load several operator definitions and rules, you can put these in a file and then tell Elision to read the file. You do this via the read(filename) operator, where filename is the name of the file to load. Elision reads the contents of the file, processing bindings, operator definitions, rules, etc. Most output from this process is suppressed by first putting the REPL is quiet mode and suspending creating \$_repl bindings.

If the filename is absolute (that is, it fully specifies the location of the file), then the file is simply read. Otherwise Elision needs to find the file before it can read it. It does this by checking a search path. The path is specified by the path property. This is a string containing a list of folders to search. On Windows the folders are separated by semicolons (;), while on UNIX (including Mac OS X) and Linux they are separated by colons (:).

2.10. Bootstrapping

When the Elision REPL starts, it first reads its bootstrap files that declare all the "built in" operators and rules.⁶ It does this by locating and reading the file Boot.eli.

⁶This includes declaring the operator decl that is used to declare other operators and rules. How this is done is explained in appendix D.

CHAPTER 3

Basic Atoms

The objects manipulated by Elision are called *atoms*, and they exist in several forms.

- Literals that denote a simple, fixed value.
- Variables denote an unspecified atom, perhaps with type information.
- Lambdas denote simple unnamed functions of one parameter.
- Applies denote an ordered pair consisting of a left-hand element often called the *operator* and a right-hand element often called the *argument*.
- Operators denote a mapping.
- Rewriters transform an atom into a potentially new form, reporting whether this transformation succeeded.

Atoms can be combined to make composite atoms¹ in a variety of ways. These are discussed in the following sections and chapters.

3.1. Types

Every atom has an associated type, which is itself just another atom. Since every atom has a type, and every type is an atom, we have a potential infinite regress. To avoid this the special type universe atom, denoted ``TYPE, is its own type.\(^2\) The fact that atom a has type T is denoted by joining the atom and its type with a colon: a:T. The type colon is right associative, so a:B:C denotes a:(B:C). In fact, this is the only way the colon can work. Since an atom has exactly one type, (a:B):C is effectively meaningless, and generates an error in Elision. The term root type refers to any type whose own type is ``TYPE. For instance, STRING is a root type that denotes STRING: ``TYPE. If the symbol STRING is desired, it must be written STRING: SYMBOL.

Scala. Every atom is an instance of a class extending core.BasicAtom. All atoms are immutable after construction. The type of an atom must be specified at construction time, and can be accessed later via the public field theType. The type universe can be accessed in two ways: core.TypeUniverse is a singleton implementing the type universe, and it can be accessed via the alias 'TYPE' provided by the core package. Back ticks must be used since 'TYPE is not a valid Scala symbol.

3.2. Literals

Elision directly supports several kinds of literals. These are indivisible, simple-valued atoms.

3.2.1. Symbols. Symbols start with an underscore or letter, followed by any number of letters, digits, and underscores. The following are legal symbol literals.

A x = 21 Fred21

¹No, not molecules. This is not chemistry.

²As has been pointed out, "it's turtles all the way down." TYPE is the turtle that stands on its own head.

3.2. LITERALS 19

Table 1. Escape Codes

\f	Form Feed Character
\n	Newline
\r	Carriage Return
\t	Tab Character
\'	Back tick
\"	Double Quotation Mark
11	Backslash

Other characters may occur in symbols, but the entire symbol must be enclosed in back ticks, which are not themselves part of the symbol. Several escape sequences are interpreted; these are listed in table 1. Using the back tick notation the following are also legal symbol literals.

'1' '\$ 4' '\'Fred\'' '

Scala. Symbols are created via the core.Literal(sym) method, where sym is a Scala symbol. Take care! Scala symbols are automatically converted to Elision *variables*, not symbol literals, when used in a context expecting an Elision atom.

3.2.2. Strings. A string is a sequence of characters enclosed in double quotation marks, which are not themselves part of the symbol. As with back-tick-quoted symbols, escape codes are interpreted in the string. The escape codes are listed in table 1. The following are legal string literals.

"" "Fred" "'21,' she said." "\"Yes,\" he replied."

Elision also provides for verbatim blocks, which evaluate to strings. A verbatim block is indicated by any arbitrary text enclosed within triple quotation mark pairs. All characters are preserved as-is, and the result of evaluation is a string literal.

```
e> """

> This is a multiline string literal.

> Note that \ is left untouched."""

$_rep13 = "\nThis_is_ia_multiline_string_literal.\nNote_that_\\_is_left_untouched."
```

Finally, strings can be concatenated with the applicative dot. Thus "DJ "."Kousuke" is interpreted as "DJ Kousuke". This enables defining strings with variables, as with {binds last->"Kousuke"}.("DJ ".\$last), which also yields "DJ Kousuke".

Scala. Strings are created via the core.Literal(str) method, where str is a Scala string. Scala strings are automatically converted to Elision string literals as necessary. Thus it is possible to pass "\$" to a method expecting an Elision atom.

```
scala> "I'muaustring":BasicAtom
res0: ornl.elision.core.BasicAtom = StringLiteral(NamedRootType("STRING"),
"I'muaustring")
```

3.2.3. Numbers. Numbers may be expressed in binary, octal, decimal, or hexadecimal (the *radix*). Binary numbers are indicated by the prefix 0b, octal numbers are indicated by a leading 0, decimal numbers are indicated by a leading non-zero digit, and hexadecimal numbers are indicated by the prefix 0x. The prefixes are *not* case-sensitive.

Floating point numbers are also supported in binary, octal, decimal, and hexadecimal, and are indicated either by a decimal point, an exponent, or both. The exponent is typically indicated by an e, must be an integer, and may be in a different radix. Because e is a valid hexadecimal digit, p may also be used to indicate an exponent (and must be for hexadecimal numbers). Numbers are not case sensitive. If a floating point number consists of significand s, exponent e, and radix r, then the number denoted is given by the following equation.

$$s \times r^e$$

The following are legal number literals.

0	-1	05743645134635	-0x232
2.5	0b10.1	0x2.8	02.4
21e-6	0b10101e-0b110	0x15p-0x6	025e-06

Keep in mind that the radix of the significand and the exponent must both be specified.

The number undergoes a conversion to place it in a consistent radix.³ This conversion is performed as follows. Let the number be i.fee, where i is the integer portion of the significand, f is the fractional portion of the significand, and e is the exponent. We note that i.f is in one radix, which we denote r, and the exponent may be in a different radix. If the length of f is |f|, then the entire significand is multiplied by $r^{|f|}$ to convert it to an integer. The exponent is then converted to radix r, yielding e', and |f| is subtracted from it.

$$n = i.f ee = (i.f)r^{|f|}e(e' - |f|)$$

Scala. Integers are created via the core.Literal(i) method, where i is a Scala BigInt. Floating point numbers can be created via the core.Literal(FLOAT, sig, exp, rad) method, where sig is the significand (a Scala BigInt), exp is the exponent (a Scala Int), and rad is the radix (one of 2, 8, 10, or 16).

Scala Int and BigInt are automatically converted to Elision integer literals as necessary. Thus it is possible to pass 17 to a method expecting an Elision atom.

If you have a floating point literal, you can obtain its value as a Scala float with toFloat and as a Scala double with toDouble. You can also convert it to radix two using toBinary. Because this changes the base of a floating point number you may lose precision.

At present Elision does not support the positive or negative infinity or the signaling or quiet NaN.

3.2.4. Booleans. Boolean literals come in two forms: true and false. Both have the type BOOLEAN. If you want an Elision symbol named true or false, you must override the type, and write true:SYMBOL, for instance.

Scala. The Boolean literals can be obtained as Literal.TRUE and Literal.FALSE. Scala Booleans are automatically converted to Elision Boolean literals as necessary.

3.3. Special Symbols

Certain symbols are interpreted differently from others. Three have already been discussed: ^TYPE denoting the type universe, true, and false. Others denote named root types. A root atom is an atom whose type is ^TYPE. table 2 lists the named root atoms declared by the current system.⁴

³This conversion takes place during parsing.

⁴It is possible to create new named root atoms. The following declares an operator that does just this. Give it a symbol, and that symbol becomes a named root type. For details about how this works, see appendix D.

21

	Table 2.	Special	Root	Type	Symbols
--	----------	---------	------	------	---------

Elision Symbol	Interpretation
ANY	The wildcard. The special atom denoting anything.
BINDING	The root type for all bindings.
BOOLEAN	The root type for all Booleans.
FLOAT	The root type for all floating point numbers.
INTEGER	The root type for all integer numbers.
NONE	The special atom denoting nothing.
OPREF	A root type for indicating that a symbol denotes an operator.
RSREF	A root type for indicating that a symbol denotes a ruleset.
STRATEGY	The root type for all strategies and rewrite rules.
STRING	The root type for all strings.
SYMBOL	The root type for all symbols.

These are typically used as types, so they are often called *root types*. Two of these: OPREF and RSREF exist to cause the system to "look up" a name in the current context.

The special atom ANY matches any atom, and any atom matches it, so it can be treated as a wildcard during matching, and is also the assumed type of otherwise untyped atoms. ANY is unusual; it violates the substitution principle (see chapter 7). ANY matches NONE, but NONE matches only itself.

Untyped symbols not in the above list have the assumed type SYMBOL. If you want an ordinary symbol counterpart to any of the above, you must explicitly ask for it. For example, if you want the ordinary symbol STRATEGY, you must use STRATEGY: SYMBOL.

Scala. All the symbols listed in table 2 are also Scala objects that extend core.NamedRootType. They can be used directly, so SYMBOL is a Scala value denoting the Elision SYMBOL type.

3.4. Variables

Variables are indicated by a leading dollar sign followed by the variable name, which is parsed as a symbol. The following are legal variables.

Variables are looked up by name, but matched by *both* name and type. This means that you should never use the same variable name with different types in a single atom, as it will break

```
decl.{! declare_rt($name: SYMBOL)
#description="Declare a new named root atom."
#detail="Declare $name to be a new named root atom."
#handler="""
    args match {
      case Args(SymbolLiteral(_, sym)) =>
            NamedRootType(sym.name)
      case _ => as_is
    }
""""}
```

matching in unusual ways.⁵ If no type is specified for a variable, Elision assigns it the special root type ANY.

A variable instance may have an associated guard that controls how it may be bound. This will be explained in chapter 7. Variables may also have zero or more labels attached to them. A *label* consists of a symbol preceded by an at symbol (@). An example is \$x @free @strict. Labels are used by the *map* strategy, explained in chapter 8.

Scala. Variables are created using core. Variable(typ,name,guard,labels), where typ is the variable type, name is the variable name, guard is an optional guard (it is true by default), and labels is an optional set of labels. Thus the Elision variable \$x:INTEGER can be created with Variable(INTEGER,"x"). Scala symbols are also automatically converted to variables of type ANY, so it is possible to pass a Scala symbol to methods expecting an atom. Thus \$x:ANY can also be written (depending on context) as 'x.

3.5. Metavariables

A variable x can be converted to a *meta* variable by writing it with an extra dollar sign: x. This term still denotes the variable x, but prevents evaluation of operators and other applications and rewrites. A *meta-term* is any term containing at least one metavariable. Metavariables may have guards and labels, just as ordinary variables do. The variable x and the metavariable x denote the same (unspecified) atom.

The function of metavariables is to delay evaluation of terms. For example, the term is_bindable(\$x) evaluates to true immediately, since the variable \$x is, in fact, bindable. However the term is_bindable(\$\$x) is preserved as-is until the metavariable \$\$x is rewritten to some other term. This rewriting is most easily performed using a lambda (see section §3.6).

```
e> is_bindable($x)
$_rep10 = true
e> is_bindable($$x)
$_rep11 = (is_bindable:OPREF.%($$x))
e> \$$x.is_bindable($$x)
$_rep12 = \$$':1'.(is_bindable:OPREF.%($$':1'))
e> \$$x.is_bindable($$x).5
$_rep13 = false
e> \$$x.is_bindable($$x).$y
$_rep14 = true
```

Metavariables are enormously useful, and should be used by convention in guards and certain other places where we want to defer evaluation.

Scala. Variables are created using core.MetaVariable(typ,name,guard,labels), where typ is the variable type, name is the variable name, guard is the optional guard (the default is true), and labels is an optional set of labels. Thus the Elision metavariable \$\$x:INTEGER can be created with MetaVariable(INTEGER,"x").

⁵This is not an attempt to be frustrating. It is a performance issue. Under the hood, bindings are stored in a hash table indexed by the interned variable name. This makes lookup faster than if we had to check for an equal name and type.

3.6. LAMBDAS 23

Table 3. Lambdas

Lambda Expression	Value
\\$x.(7).12	7
\\$x.\$x.12	12
\\$x.\\$y.\$x.12	\\$y.12

3.6. Lambdas

A lambda expression denotes a function with a single parameter, and consists of a variable called the *lambda parameter* and an atom called the *body*. The lambda is *evaluated* (or *curried*) by applying it to another atom, called the *argument*, and then replacing instances of the lambda parameter in the body with the argument. This is all much simpler than it sounds.

Lambdas are introduced by a backslash (\) followed by the parameter (a variable), then a dot (.), and then the body (an atom). The following is a lambda that denotes a constant function whose value is 7 no matter what argument is provided.

The following denotes the identity function that returns the atom it is applied to.⁶

The following is a lambda that creates constant functions.⁷ When applied to an argument, the result is a function that always returns that argument.

To apply a lambda to an atom, join the lambda and the atom with a dot (.). The lambda dot binds more tightly than the application dot. table 3 shows some examples of lambda applications and the results. The parentheses around the 7 in the first entry are required to prevent the system from seeing 7.12 as a floating point number.

If you enter \\$x.\\$y.\$x.12 at the Elision prompt, you will not see \\$y.12. What you will see is \\$':1'.12. The \$y has been replaced with the funny variable \$':1'. This variable is a DeBruijn index, and it is used to prevent lambdas from "capturing" variables. In practice you should not need to worry about DeBruijn indices; the system uses them when appropriate and keeps track of the details for you. If you are curious, please see appendix E.⁸

A lambda parameter is *matched* against the argument, and the result is used to *rewrite* the body. This is all explained in chapter 7, but in practice it means that if you specify the type of the parameter, then Elision will enforce this. Thus the lambda expression \\$x:INTEGER.\$x.7 will succeed with the value 7, while the lambda expression \\$x:INTEGER.\$x."Fred" will fail with an error. This also means that the type of an atom can be extracted using a lambda. The following lambda will extract the type of any atom it is applied to.

⁶This is sometimes called the *I combinator*.

⁷This also has a catchy name: it is the *K combinator*.

⁸For reasons now obvious, you should avoid variable names that start with colons. Elision uses these internally, and not just for DeBruijn indices, but also for "synthetic" rules and "synthetic" parameters created during matching. Partial rule completion is explained in chapter 8 and matching is explained in chapter 7.

Thus the expression \\$x:\\$T.\\$T.\"Fred" evaluates to STRING.

The applicative dot binds right to left. For this reason, multiple parameter lambda expressions may need to be parenthesized when applied. Consider the following example.

```
e> \$first.\$second.%($first,$second).(2).(1)

$_repl14 = %(2, 1)

e> \$first.(\$second.%($first,$second).2).(1)

$_repl15 = %(1, 2)
```

So what happened? The lambda dot binds more tightly than the applicative dot, and the applicative dot binds to the left, so in the first line the 2 become the argument to \frac{\\$second.\%(\\$first,\\$second). This is rewritten to \\$second.\%(\2,\\$second). Parentheses in the second line force 2 to be bound in the inner lambda first. This binding order was chosen so that arguments can, in the unparenthesized case, be given in the same order (left to right) as the parameters to which they bind.

Scala. To create a lambda use core.Lambda(param,body), where param is the lambda parameter and body is the lambda body. To apply a lambda to an argument, use core.Apply(lambda,arg), where lambda is the lambda expression and arg is the argument. The following shows how several lambda expressions can be created in Scala.

Lambda Expression	Scala Code
\\$x.7	Lambda('x,7)
\\$x.\$x	Lambda('x,'x)
\\$x.\\$y.\$x	Lambda('x,Lambda('y,'x))
\\$x:INTEGER.\$x.7	Apply(Lambda(Variable(INTEGER,"x"),'x),7)
\\$x:\$T.\$T."Fred"	<pre>Apply(Lambda(Variable('T,"x"),'T),"Fred")</pre>

3.7. Lambdas and Metavariables

It is often useful to use metavariables in lambdas. In fact, it is recommended to use a metavariable for the lambda parameter. Consider is_bindable(atom). This evaluates to true if the atom is bindable (that is, is a variable), and evaluates to false if it is not. Suppose I want a function to tell me whether an atom is bindable or not. I might try to write this as follows: \\$a.is_bindable(\$a). This will not work. The term is_bindable(\$x) immediately evaluates to true (since \$x is bindable), leaving the constant lambda \\$x.true. This is not what we want at all.

The solution is to use metavariables (see section §3.5). We write \\$\$x.is_bindable(\$\$x). This works as desired.

```
e> \$x.is_bindable($x).7

$_repl16 = true
e> \$$x.is_bindable($$x).7

$_repl15 = false
```

The first entry fails (because 7 is not bindable), but the second entry works. Once you decide to use a metavariable in one place in a lambda, however, you must use it everywhere. The reason is that the lambda variables get rewritten to De Bruijn indices (see appendix E) based on the lambda parameter. The following illustrates how this can fail.

```
e> \$x.is_bindable($$x).7
$_repl16 = true
e> \$$x.is_bindable($x).7
$_repl15 = true
```

In the first case a De Bruijn variable \$v\$ is created for \$x\$, and the body is rewritten to is_bindable(\$v) which is true. The second case falls into the problem discussed earlier, where is_bindable(\$x) is rewritten to true before De Bruijn substitution can even occur.

The simple rule is to use metavariables consistently in lambdas. If you use a metavariable in the body, you must also use a metavariable for the parameter.

CHAPTER 4

Collections

Elision provides a generalized collection type called, for lack of a better name, an *atom sequence*. In its "default" form it is an ordered list of atoms, but this can be changed in a variety of ways by specifying the algebraic properties of the collection.

4.1. The Basic Ordered Sequence

A basic ordered sequence of atoms is represented by a comma-separated list of atoms, enclosed in parentheses, and prefixed with a percent sign (%). This "looks like" a % operator applied to a list of atoms. For instance the list 1 followed by 2 can be represented with %(1,2). Empty sequences are permissible, too: %().

The collection can be heterogeneous. For instance %(1,"Fred") is perfectly fine. The type of a collection is deduced by looking at the types of the contained elements. If they all have the same type T, then the type is SEQ(T). If any has a different type, then the type of the collection is SEQ(ANY).

4.2. Algebraic Properties

Algebraic properties can be assigned to the sequence, and are specified starting with a percent sign. Actually, the leading percent sign for the basic ordered sequence is itself an algebraic properties specification, albeit a very simple one. Several properties are available, and the properties can even be "detached" from the sequence, matched, and rewritten. That is, an algebraic property specification is an atom on its own. Algebraic properties specifications look like operators, and even behave a bit like operators.

Each available property is described in more detail in the following subsections. Properties are specified by a single character, which is *not* case-sensitive, and can be negated by prefixing with an exclamation mark (!). Properties are combined by juxtaposing them, and omitting a property altogether leaves it unspecified. The following table is a quick reference to the properties.

Properties may be given an argument in square brackets. For absorbers and identities, this will be the appropriate atom. For associativity, commutativity, and idempotency, this is typically either a Boolean literal or a variable for matching. The sense of the Boolean represents whether the property is present, so A alone is the same as A[true], and !A is the same as A[false].

Property	Character	Can Be Negated	Requires an Argument
Associativity	A	Yes	No
Commutativity	C	Yes	No
Idempotency	I	Yes	No
Absorber	В	No	Yes
Identity	D	No	Yes

Table 1. Algebraic Properties

The order of properties is not significant, and conflicts are resolved with the last property specification winning. Thus %AA!A is the same as %!A. Spaces are only permitted in the arguments to a property (inside the square brackets).

Scala. Operator properties are specified by core.AlgProp. This class can be used directly, but it is typically easier to use its extensions: Associative(bool), Commutative(bool), Idempotent(bool), Absorber(atom), and Identity(atom). The first three take a Boolean value, and the last two take an atom. Join these with and. The following builds the properties specification for integer addition, which is associative, commutative, not idempotent, and has an identity of zero.

Associative(true) and Commutative(true) and Idempotent(false) and Identity(0) There is also an extension NoProps that indicates no properties. This is useful if you don't care to specify any properties. Note that leaving a property unspecified is not the same as negating it; it just leaves the property unspecified.

4.2.1. Associativity. An associative collection can have its elements grouped arbitrarily, provided the subgroups have the same algebraic properties specification. Associative collections are indicated with the symbol A. Thus we have %A(5,2,%A(3,4),1) is the same as %A(5,2,3,4,1). In fact, Elision will convert the former to the latter.

```
e> %A(5,2,%A(3,4),1)

$_repl0 = %A(5, 2, 3, 4, 1)

e> %A(5,2,%(3,4),1)

$_repl1 = %A(5, 2, %(3, 4), 1)
```

In the second line the properties specifications do not match, so Elision does not flatten the elements.

- **4.2.2.** Commutativity. A commutative collection can have its arguments re-ordered arbitrarily. Commutative collections are indicated with the symbol C. Thus C(1,2,3) is the same as C(3,2,1).
- **4.2.3. Idempotency.** An idempotent collection ignores element multiplicity. This is a complicated way of saying repeated elements do not matter. Idempotent collections are indicated with the symbol I. Thus %I(1,2,2,3) is the same as %I(1,2,3). In fact, Elision will discard repeated elements.

```
e> %I(1,2,2,3)
$_repl2 = %I(1, 2, 3)
```

4.2.4. Identity. A special element may be designated as the identity. Including the identity in the sequence does not change its meaning. In fact, Elision will discard instances of the identity from the collection. An identity is indicated by the character D followed by the identity element in square brackets. Thus %D(1,0,0,3) is the same as %(1,3).

```
e> %D[0](1,0,0,3)
$_repl3 = %D[0](1, 3)
```

4.2.5. Absorber. Similar to an identity, a special element may be designated as an absorber. If an absorber is present, the meaning is the same as if *only* the absorber were present. Because of this, if Elision finds an absorber it dicards everything else (including other instances of the absorber) from the collection. An absorber is indicated by the character B followed by the absorber element in square brackets. Thus %B[0](1,0,0,3) is the same as %B0.

```
e> %B[0](1,0,0,3)
$_rep14 = %B[0](0)
```

4.3. Simple Collections

Using the algebraic properties we can construct some common collections. First, the specification %CI denotes a set: a collection for which order and multiplicity are not significant. Sets within the set are preserved; if this is not desired, use %ACI.

```
e> %CI(4,4,5,Red,"Joe",Blue,4,%CI(Red,Blue))
$_rep15 = %CI(4, 5, Red:SYMBOL, "Joe", Blue:SYMBOL, %CI(Red:SYMBOL, Blue:SYMBOL))
e> %ACI(4,4,5,Red,"Joe",Blue,4,%ACI(Red,Blue))
$_rep16 = %ACI(4, 5, Red:SYMBOL, "Joe", Blue:SYMBOL)
```

If we omit idempotency or specify *not* idempotent with !I, then we have a multiset or bag, where multiple elements are preserved (though order still does no matter).

```
e> %C(4,4,5,Red,"Joe",Blue,4,%CI(Red,Blue))
$_rep17 = %C(4, 4, 5, Red:SYMBOL, "Joe", Blue:SYMBOL, 4, %CI(Red:SYMBOL, Blue:SYMBOL))
e> %AC(4,4,5,Red,"Joe",Blue,4,%AC(Red,Blue))
$_rep18 = %AC(4, 4, 5, Red:SYMBOL, "Joe", Blue:SYMBOL, 4, Red:SYMBOL, Blue:SYMBOL)
```

Omitting commutativity or specifying *not* commutative with !C results in a list. If we want lists to be flattened, we can specify associativity. Likewise, if we want only the first instance of an element, we can specify idempotency.

```
e> %(4,4,5,Red,"Joe",Blue,4,%(Red,Blue))

$_rep19 = %(4, 4, 5, Red:SYMBOL, "Joe", Blue:SYMBOL, 4, %(Red:SYMBOL, Blue:SYMBOL))

e> %A(4,4,5,Red,"Joe",Blue,4,%A(Red,Blue))

$_rep110 = %A(4, 4, 5, Red:SYMBOL, "Joe", Blue:SYMBOL, 4, Red:SYMBOL, Blue:SYMBOL)

e> %AI(4,4,5,Red,"Joe",Blue,4,%AI(Red,Blue))

$_rep111 = %AI(4, 5, Red:SYMBOL, "Joe", Blue:SYMBOL)
```

4.4. Properties Specifications as Operators

It was mentioned previously that the algebraic properties specification looks like an operator. In fact, it can function similarly to one, too. We use the applicative dot (.) to apply it to another collection. Specified properties *override* the properties of the argument. For instance, if %A is applied to %C, the result is %AC. Likewise, if %A is applied to %!AC, the result is again %AC.

```
e> %A.%C

$_repl11 = %AC

e> %A.%!AC

$_repl12 = %AC

e> %AD[0].%CD[2]

$_repl13 = %ACD[0]

e> (%A.%I).%(3,%AI(3),3)
```

¹Yes, you can designate the same atom as both an identity and an absorber, but why? You will get a collection consisting of exactly one instance of the absorber, as absorber processing takes precedence.

```
$_rep114 = %AI(3)
e> %A.%I(3,%I(3),3)

$_rep115 = %AI(3, %I(3))
e> %A.%I(3,%AI(3),3)

$_rep116 = %AI(3)
```

4.5. Long Property Specifications

A properties specification like %AC!ID[0] is terse almost to the point of being unreadable. If a properties specification contains no variables or other complex terms, it can be written in an alternate form, replacing the letters with the entire property name, and negation with not. The above specification can be written in the following form.

```
% associative, commutative, not idempotent, identity 0
```

The commas are required, and spaces are allowed. This form is much more readable, but also much longer. Note that absorbers are specified via the absorber keyword, followed by the absorber atom.

```
e> %associative.%commutative
$_repl1 = %AC
e> %associative.% not associative, commutative
$_repl2 = %AC
e> (%associative, identity 0).%commutative, identity 2
$_repl3 = %ACD[0]
e> (%associative.%idempotent).%(3,%associative,idempotent(3),3)
$_repl4 = %AI(3)
e> %associative.%idempotent(3,%idempotent(3),3)
$_repl5 = %AI(3, %I(3))
e> %associative.%idempotent(3,%idempotent, associative(3),3)
$_repl6 = %AI(3)
```

Operators

In general, operators denote mappings on atoms. However, Elision manipulates operators symbolically, so they should be regarded as mathematical functions or symbols, and not in the computer language sense. Elision supports the following kinds of operator.

- A symbolic operator is simply an atom that is manipulated by the system through rewriting. A Scala closure can be provided to construct instances of the operator, making the operator a native operator. This is how arithmetic operators, for example, are implemented.
- An *immediate* or *case* operator denotes a kind of macro. One or more patterns are provided, and arguments are matched against patterns, in order. The result of the match is used to construct a new atom, which is the "value" of the operator application. Case operators provide for a kind of controlled polymorphism, and can operate in ways that a symbolic operator cannot.

Operators must be declared in the current context to be used by name. This is done via the built-in decl operator.¹ This can be used in two ways: Either as decl.oper or as decl(oper), where oper is the operator to declare. The dot form is preferred in the system libraries.

Operator redeclaration is permissible, but results in a warning. Finally, operators are a special form (see chapter 6), so they may be specified in more than one way. In this chapter we present only one style of operator definition to simplify the discussion. Other forms are potentially more useful for matching and rewriting (see chapter 7).

```
e> setprop("autoop", false)
e> {operator #name=foo params=%($x)}
$_repl1 = {: operator:SYMBOL { binds name -> foo:SYMBOL params -> %($x) type -> ANY } :}
e> decl($_repl1)
Declared operator foo.
e> decl.$_repl1
WARNING: Redefining operator foo.
WARNING: Prior definition: {: operator:SYMBOL { binds name -> foo:SYMBOL params -> %($x) type -> ANY } :}
Declared operator foo.
e> foo:OPREF
$_repl2 = foo:OPREF
e> getop(foo:OPREF)
$_repl3 = {: operator:SYMBOL { binds name -> foo:SYMBOL params -> %($x) type -> ANY } :}
```

5.1. Symbolic Operators

A symbolic operator declaration must have the following elements.

- A name, specified as a symbol. For example, add.
- A *type*, which is the type of the fully-applied operator. For example, the type of the integer add operator is INTEGER.

¹Don't like this? Use the "autodefine" function of the REPL. Enable it by setting the autoop property to true.

• A parameter list, which specifies both the parameter types and also the operator's algebraic properties.

To specify a symbolic operator, give these three items in the following form:

```
{ operator #name=NAME #type=TYPE #params=PARAMS }
```

where NAME is the operator name, TYPE is the type of the fully-applied operator, and PARAMS is the parameter collection. The algebraic properties of this collection are the algebraic properties of the operator. If the #type is not declared, it is assumed to be ANY.

The following are some examples of common operators.

```
{ operator #name=iadd #type=INTEGER
    #params=%AC!ID[0]($x:INTEGER, $y:INTEGER) }
{ operator #name=imul #type=INTEGER
    #params=%AC!ID[1]B[0]($x:INTEGER, $y:INTEGER) }
{ operator #name=and #type=BOOLEAN
    #params=%ACID[true]B[false]($P:BOOLEAN, $Q:BOOLEAN) }
```

Scala. To create a definition of a symbolic operator, use

• core.TypedSymbolicOperator(name, type, parameters, description, detail)

where name is the operator's name (a string), type is the type of the fully-applied operator (an atom), parameters specifies both the operator's properties and the parameters and their types (it is an atom sequence), description is the short one-line description of the operator, and detail is the longer description of the operator and its use. The last two (description and detail) are optional. The following code creates the two examples given previously (the or and add operators).

Operator definitions should be installed in an instance of core.OperatorLibrary, typically found in an instance of core.Context. A context is automatically created and provided by the REPL, so if you are interacting with the REPL you do not need to worry about this. Contexts and operator libraries are explained in more detail in chapter 9. If you have a context instance, you can install a new operator by first getting the operator library from the context with operatorLibrary, and then invoking add(opdef) on the instance, where opdef is the operator definition.

For instance, if context is an instance of core.Context, then the operators defined above can be installed with the following lines of code.

```
// Install definitions.
context.operatorLibrary.add(orOp)
```

context.operatorLibrary.add(addOp)

The operator library is also where you register a closure for a symbolic operator. The closure must take an instance of core.ApplyData and return an instance of core.BasicAtom. More details on this are given in appendix D. You register the native handler using the core.OperatorLibrary.register(name, handler) instance method, where name is the (string) name of the operator, and handler is the closure described above. The following registers a native constructor for add, but does use some methods not explained just yet.

```
// Define add as a symbolic operator.
context.operatorLibrary.register("add",
  (data: ApplyData) => {
    // Accumulate the integer literals found.
var lits:BigInt = 0
// Accumulate other atoms found.
var other = IndexedSeq[BasicAtom]()
// Traverse the list and divide the atoms.
data.args.foreach {
  x \Rightarrow x match {
   case IntegerLiteral(_, value) => lits += value
    case => other :+= x
  }
// Now add the accumulated literals to the list.
other :+= Literal(INTEGER, lits)
// Construct and return a new operator application.
Apply(op, AtomSeq(NoProps, other), true)
```

It is the job of the core.Apply class to create operator (and other) applications. While this is explained in more detail in section §5.4, we note some important things here.

First, core.Apply understands how to use the operator to process absorbers and identities, etc., and this processing is performed *first*. Thus it is possible to attempt to construct a symbolic operator application with a native constructor and never have the native constructor get invoked. For instance, add(0) is immediately rewritten to 0, and the native constructor is never invoked.

Second, the native constructor may re-invoke core.Apply, as is done in the handler for add shown above. This can simplify the native constructor since it does not have to worry about identities, etc. In the add constructor, the result for lits might be zero. By re-invoking core.Apply it will get correctly processed.

This second case leads to a potential problem. The core.Apply processing will invoke the native constructor, and our native constructor invokes core.Apply. To prevent this recursion we pass true as the optional third argument to core.Apply to prevent re-invoking the native constructor. It is thus possible to write a native constructor that does some processing, lets itself get re-invoked, does more processing, etc., until it finally terminates the cycle.

The properties specified for the operator are interpreted a bit differently from how they are interpreted for collections, section §5.2 explains how they are interpreted.

Symbolic operators can be defined using a much more terse syntax. In this form the prototype is written directly, and the name, parameters, and type are pulled from it. For instance, the prototype imul(\$x:INTEGER, \$y:INTEGER):INTEGER specifies the name, parameters and their types, and the type of the fully-applied operator. Note that it does *not* specify the properties of the operator.

Those are optionally specified by following the prototype with **is** and then the algebraic properties specification. Use {!...} to enclose the entire operator definition. The three operators given previously can be written as follows using this new syntax.

```
{! iadd($x:INTEGER, $y:INTEGER):INTEGER is %AC!ID[0] }
{! imul($x:INTEGER, $y:INTEGER):INTEGER is %AC!ID[1]B[0] }
{! and($P:BOOLEAN, $Q:BOOLEAN):BOOLEAN is %ACID[true]B[false] }
```

If you use long property specifications (see section §4.5) then you are allowed to drop the leading %.

```
{! iadd($x:INTEGER, $y:INTEGER):INTEGER is
   associative, commutative, not idempotent, identity 0}
{! imul($x:INTEGER, $y:INTEGER):INTEGER is
   associative, commutative, not idempotent,
   identity 1, absorber 0}
{! and($P:BOOLEAN, $Q:BOOLEAN):BOOLEAN is
   associative, commutative, idempotent,
```

This is much more readable, but also much longer.

identity true, absorber false}

After specifying the prototype the description and details may be specified as usual. In fact, the type can be specified in the usual manner (with #type), if desired.

5.2. Operator Properties

As with collections, operators have associated algebraic properties; in fact, they are the same as those for sequences, and are specified in the same manner. For example, addition is associative, commutative, not idempotent, and has an identity of zero: %AC!ID[0]. Multiplication is associative, commutative, not idempotent, has an identity of one, and an absorber of zero: %d[1]b[0]!ica. Boolean logical or is associative, commutative, idempotent, has an identity of false and an absorber of true: %ACID[false]B[true].

Each property is described in more detail in the following subsections as it applies to operators. There are restrictions on the properties of operators that are not present for collections (specifically, associativity is required in many cases), and these are discussed in the following subsections.

5.2.1. Associativity. An associative operator f can have its arguments grouped arbitrarily. Thus f(a,b,c) = f(f(a,b),c) = f(a,f(b,c)). Note that any argument could be replaced with another application, so any number of arguments is possible.

Suppose we define operator f to have prototype f(a:A,b:B):F. Since we can replace either a or b with another application of f, we conclude that $F \subseteq A$ and $F \subseteq B$. We further note the following, by associativity.

$$f(f(c:A,d:B),b:B) = f(c:A,f(d:B,b:B))$$

Thus it must be the case that $B \subseteq A$. By a similar argument we conclude that $A \subseteq B$, and thus A = B. For these reasons we require that all parameters of an associative operator have the same type, and that the parameter type must be the same as the overall operator type.

```
e> { operator #name=bad1 #type=INTEGER
> #params=%A($x: INTEGER, $y: STRING) }
ERROR: The operator bad1 is marked as associative, but all parameters
do not have the same type, as required: %A($x:INTEGER, $y:STRING)
e> { operator #name=bad2 #type=STRING
> #params=%A($x: INTEGER, $y: INTEGER) }
ERROR: The operator bad2 is marked as associative, but the parameter
type (INTEGER) is not the same as the fully-applied type (STRING).
```

An associative operator must declare exactly two parameters.

```
e> { operator #name=bad3 #type=INTEGER
> #params=%A($x: INTEGER) }
ERROR: The operator bad3 is marked as associative, but does not have
exactly two parameters, as required: %A($x:INTEGER)
```

Associativity is required by idempotency, any identity, and any absorber.

5.2.2. Commutativity. A commutative (or abelian) operator f can have its arguments reordered arbitrarily. Thus f(a,b) = f(b,a). If we provide type information, we see the following: f(a:A,b:B) = f(b:B,a:A), and we conclude that $A \subseteq B$ and $B \subseteq A$, or A = B. We thus require that all arguments to a commutative operator have the same type, though this need not be the type of the fully-applied operator.

```
e> { operator #name=bad #type=STRING
> #params=%C($x: INTEGER, $y: STRING) }
ERROR: The operator bad is marked as commutative, but all parameters
do not have the same type, as required: %C($x:INTEGER, $y:STRING)
```

A commutative operator must declare at least two parameters.

5.2.3. Idempotency. An idempotent operator f has the property that f(a, a) = f(a), so repeated arguments are discarded. Because the length of the argument list can change, idempotent operators are required to be associative. An example of an idempotent operator is the Boolean inclusive or, since or(x, y, x) = or(x, y). Idempotency imposes no additional restrictions on an operator beyond those imposed by associativity, which is required.

As an example, we implement multisets and sets using symbolic operators. A multiset allows repeated elements to be present.

```
e> def({ operator #name=mset #params=%AC($x,$y) })
e> mset(7, fred, "James", blue, 7, blue)
$_repl18 = mset(7, fred:SYMBOL, "James", blue:SYMBOL, 7, blue:SYMBOL)
```

A set does not allow repeated elements.

```
e> def({ operator #name=set #params=%ACI($x,$y) })
e> set(7, fred, "James", blue, 7, blue)
$_repl19 = set(7, fred:SYMBOL, "James", blue:SYMBOL)
```

Including idempotency in the list of properties allows a simple implementation of sets. Examples of idempotent operators are the logical or and and.

5.2.4. Identity. An operator f can have an associated named identity element e, with the property that f(x,e) = f(e,x) = f(x) = x. Instances of a named identity are discarded from the argument list. Because the length of the argument list can change, only associative operators can have a named identity.

Named identity elements include zero for addition, one for multiplication, false for Boolean inclusive or, and true for Boolean and.

Further, f() is regarded as an alternate name for a named identity e. The type of the identity must match the parameter type.

5.2.5. Absorber. An operator f can have an associated absorber element z, with the property that f(x,z) = f(z,x) = z. If an absorber is present in the argument list then the entire operator application is reduced to the absorber. Elision currently requires that operators with absorbers be associative, though this is not essential for any mathematical reason.

Absorber elements include zero for multiplication, true for Boolean inclusive or, and false for Boolean and. The type of the absorber must match the parameter type.

5.2.6. Construction. Atoms are transformed when entered, based on their properties, before any other processing, including a native handler, is performed. Suppose we have the following definitions.

We observe the following based on these definitions.

- foo(\$x:INTEGER, foo(\$y:INTEGER, \$z:INTEGER)) becomes foo(\$x:INTEGER, \$y:INTEGER, \$z:INTEGER) because associative applications are "flattened."
- foo(5,0,2) becomes foo(5,2) because identities are discarded.
- bar(true, false, \$x:BOOLEAN) becomes true because true is an absorber.
- \bullet foo(0,0,0) becomes 0 because discarding identities leaves foo(), which is equal to the identity 0.

Once this processing is complete, the system applies any native handler (see appendix D).

5.3. Operator Applications

Once an operator is defined, it can be applied to arguments. If the arguments match the parameters then the operator is fully applied and a new atom is created called an *apply*. Assuming the and, or, and typeof operators given in the previous section are defined, the following are all valid applications.

```
add(4,5,$x) or($x,false) typeof(6) typeof(add($x,9))
```

Because of the properties of the or operator, the second term or (\$x,false) immediately reduces to \$x. The last two reduce immediately to INTEGER.

Operator applications can be nested, provided types match. The following is an example.²

```
or(equal(add($x,21),64),equal(add($y,17),32))
```

Scala. Operator application is explained in section §5.4.

Before an operator can be applied, you must get an instance of that operator. If you have installed the operator definition in an operator library, you can retrieve it via either the get(name) instance method of core.OperatorLibrary, or via the apply(name) instance method, where name is the operator name. The get(name) method returns an optional core.OperatorRef, returning None if the operator is not known. The apply(name) method will will return an instance of core.OperatorRef, or throw an exception if the operator is not known.

²Note that this example is immediately reduced to false independent of any bindings for \$x and \$y because equal is evaluated with constructed, and thus prior to the result being rewritten with the current bindings. To prevent this, use metavariables (see section §3.5).

The operator itself can be obtained from the core. OperatorRef instance via its operator field. The following are two ways to obtain the add operator, given a context instance context.

```
context.operatorLibrary.get("add") match {
  case Some(opref) => // Do something with the operator opref.operator
  case None => // The operator is not known!
}
// The following throws an UndefinedOperatorException if
// add is not known.
context.operatorLibrary("add").operator
```

Of course, you probably don't need (or even want) the actual operator; a reference to the operator is sufficient.

Once you have the operator reference, you can apply it to arguments by just listing the arguments in parentheses after the reference, thus implicitly invoking the operator's (or reference's) apply method. You can also use the core.Apply class as explained in the next section. The following adds some numbers.

context.operatorLibrary("add")(2,2,9)

5.4. The Applicative Dot

Operator applications can be written in two forms: add(4,5,x) and add.%(4,5,x). This second form uses the *applicative dot* and a collection to apply an operator to arguments. The two forms are equivalent; they are alternative ways to represent the same atom. The first form is actually a shorthand notation for the second. Both operators and atom lists are themselves atoms, and can be manipulated by the system. In chapter 7 the applicative dot notation will turn out to be very useful.

While operators are atoms, if you just evaluate add by itself Elision will usually interpret this as a symbol of type SYMBOL. To force Elision to look this operator up, write add:OPREF. The OPREF is not actually the type of the operator, but just a hint to force the Elision parser to look up the operator in the parser's context. If the operator is not known, an error is reported. The built in operator getop can be used to get the actual operator from an operator reference.

The type of an operator³ is a mapping from some domain to some range, deduced from the operator prototype. This can be obtained using the lambda trick presented in section §3.6. With add defined in the current REPL session, try the following.

```
e> \$x:$T.$T.getop(add:OPREF)
repl0 = MAP(xx(INTEGER, INTEGER), INTEGER)
```

The type for the add operator is a mapping from pairs of integers to integers. The above expression parses correctly because the applicative dot binds the most weakly of all connectors; more weakly than the lambda dot, for instance, which binds more weakly than the type colon. Thus the expression is equal to the following.

```
(\$x:$T.$T).(getop(add:OPREF))
```

The applicative dot can be used to "glue" any two atoms together. The atom to the left of the dot is the *left-hand side* (*lhs*), while the atom to the right of the dot is the *right-hand side* (*rhs*). For instance the following are perfectly legal applications, though it is not entirely clear what the first and last examples "mean." The parentheses are used in the first example to prevent 7. from being interpreted as a floating point number; a space between the 7 and the dot would also work.

³There are two exceptions. The operators MAP and xx are used to represent the type of operators, so they get special treatment. Their type is ANY.

```
(7).x (\$x:$T.$T).getop(add:OPREF) add.x(5,6) $x.$y
```

In chapter 6 we will introduce several other uses of the applicative dot.

Scala. To create an apply using the applicative dot, use the core.Apply(lhs,rhs) constructor, where lhs is the left-hand side, and rhs is the right-hand side. Here are the Scala code equivalents for the apply examples given previously.

```
// (7).$x
Apply(7,'x)
// (\$x:$T.$T).getop(add:OPREF)
Apply(Lambda(Variable('T,"x"),'T),
    context.operatorLibrary("getop")(Literal(OPREF,'add)))
// add.%(5,6)
Apply(context.operatorLibrary("add"), AtomSeq(NoProps, 5, 6))
// $x.$y
Apply('x,'y)
```

5.5. Operators, Symbols, and Naked Symbols

If you type a symbol at the REPL prompt without type, this is regarded as a "naked" symbol, and is subject to special treatment. If the naked symbol appears on the left-hand side of an apply, the system will attempt to interpret it as an operator reference, and will look it up in the context. If it is not found, it will issue an error.

```
e> fred
$_repl14 = fred:SYMBOL
e> fred(5)
ERROR: The operator fred is not known.
e> fred. james
ERROR: The operator fred is not known.
```

To overcome this explicitly give the type of the symbol. Any type will suppress this behavior, but the most common (and obvious) type is SYMBOL. Enclosing the symbol in parentheses only helps with the simple operator application, and not with the applicative dot.

```
e> fred:SYMBOL.james
$_repl15 = (fred:SYMBOL.james:SYMBOL)
e> (fred)(5)
$_repl16 = fred:SYMBOL
$_repl17 = 5
e> (fred).5
ERROR: The operator fred is not known.
e> (fred:SYMBOL).5
$_repl18 = (fred:SYMBOL.5)
e> fred:SYMBOL(5)
ERROR: The operator SYMBOL is not known.
```

The last line above is included just to emphasize that types are themselves atoms, and are subject to the same parsing rules. Entering fred:SYMBOL:^TYPE(5) does, however, yield two atoms because ^TYPE is treated specially.

The opposite of this is to explicitly state that a symbol is an operator reference, using OPREF.

e> (fred:0PREF)(5)

ERROR: The operator fred is not known.

You may be wondering why Elision doesn't simply treat a naked symbol as a simple symbol when there is no operator with that name, so fred.5 would be accepted. The reason is that if an operator with name fred were later defined then the interpretation would be fundamentally changed (from a symbol to an operator), and the Elision design seeks to avoid that. This could "break" existing files of definitions and terms.

Scala. The special type OPREF is one place where the Scala API and REPL deviate. Consider the following code.

```
Apply(Literal(OPREF, 'fred), Literal(5)).toParseString
```

This yields the output "(fred:OPREF.5)," but this fails the "round trip" test; parsing this string results in an attempt to look up the fred operator in the current context.

This difficulty arises from the fact that there is no "universal" context. The REPL and the parser have a context where they can look up fred, but the overall API does not.

To prevent this future versions of Elision may do away with OPREF at the API level, and more rigorously dispose of it during parsing, but for now remember that OPREF is a piece of parser "magic."

5.6. Case Operators

A case operator declaration must have the following elements.

- A name, specified as a symbol. For example, sum.
- An optional *type*, which is the type of the fully-applied operator. If not specified, ANY is used.
- A case list, which specifies the alternatives for the operator.

To specify a case operator, give these three items in the following form:

```
{ operator #name=NAME #type=TYPE #cases CASES }
```

where NAME is the operator name, TYPE is the type of the fully-applied operator, and CASES is the comma-separated list of alternatives. Again, if the #type is not declared, it is assumed to be ANY.

A case definition defines an operator by explicitly giving the interpretation of the operator in terms of its argument. When the operator symbol appears on the left of an applicative dot (even when implicit) then each of the alternatives is considered, in order.

The alternatives can be any atom, but some atoms are special.

- If the atom is a rewriter (the "map pair" is the most common), then the rewrite is attempted. If it succeeds, then the result is the value of the operator application. If it fails, then the next alternative is tried.
- If the atom is an *applicable* (operators are the most common, but also algebraic property specifications), then the applicable is placed on the left hand side of the applicative dot, and the arguments on the right hand side. The result is the value of the operator and no further alternatives are considered.
- Otherwise the atom is the value of the operator.

The second and third cases are very simple, so we consider them first. The following creates an alias for the add operator.

```
e> def({operator #name='+' #cases add:OPREF})

Defined operator '+'.
e> '+'(5,4,3)

$_repl20 = 12
```

The following creates a constant function that yields the value one no matter what the arguments.

```
e> def({operator #name=one #cases 1})
Defined operator one.
e> one(4,5)
$_rep10 = 1
```

The first case uses a rewritable, which will be more fully explained in chapter 7. Here we consider a simple one: the *map pair*. A map pair consists of a pattern against which the argument list is matched, a text arrow (->), and an atom specifying the value of the operator when the pattern matches the argument list. It is advisable to use metavariables on the right-hand side, and we will do that by convention.

The following is a general definition using cases.

This definition allows adding both integers (via add) and strings (via the applicative dot). Note that the lists can specify properties to facilitate proper matching. Note also that sum(5, "Fred") results in an error, as this does not match any case. Likewise, sum() does not match any case, either. Suppose we want to allow the no-argument case but to leave it as sum(). We add an alternative with the form %() -> _. This matches the empty argument list. The value in this case is the underscore, which is a synonym for ANY, and is interpreted here to mean that the argument list should be left as it was at the start, yielding sum() as the final answer.

Suppose we never want an undefined case; when we do not match a given case, we want to accept and leave the operator application unchanged, perhaps for symbolic manipulation. We can use ANY as the pattern to catch this case. Consider the following alternate definition of add.

In the last line the lone underscore (a synonym for ANY) matches anything. Thus sum(5,"Fred") falls to this case and the result is sum:OPTYPE.%(5,"Fred"), or just sum(5,"Fred"), where construction halts. Note that we cannot directly refer to sum inside the definition since the operator is not yet defined.

This generality allows some rather unorthodox things to occur. Consider the following.

```
{operator #name=typeof #cases $x:$T -> $$T}
```

Now we have an operator that generates the type of anything on the right of an applicative dot.⁴

```
e> typeof.5
$_rep14 = INTEGER
```

In Elision, help is implemented as a case operator. The definition is potentially instructive. The help operator takes either no arguments (the "all" case) or a single operator reference. We can write this as follows.

⁴This is not perfect! The lambda trick described elsewhere is the best way to obtain the type of an atom.

```
def({operator #name=help #cases
    %($op: OPREF) -> _help_op($$op),
    _help_all: OPREF
})
```

Here _help_op and _help_all are operators that provide native handlers. The first case deals with entering an operator reference. The second case is an operator reference, which is an *applicable*, so Elision applies _help_all to the argument list, whatever it is. This results in some non-intuitive error messages.

```
e> help(5)
ERROR: Incorrect argument for operator _help_all at position 0: 5.
Sequences are not the same length.
```

To the end user this mentions a completely different operator, and complains about the length of the argument sequence.

Suppose instead we wanted to still perform the argument check (so we get a better error message). We can try to write it as follows.

```
def({operator #name=help #cases
    %($op: OPREF) -> _help_op($$op),
    %() -> _help_all()
})
```

This looks good, and the error messages are better, but now help() doesn't work!

```
e> help(5)
ERROR: Applied the operator help to an incorrect argument list: %(5)
e> help()
e>
```

We got no output. This is because the output of _help_all() is generated as a side-effect at construction time⁵ (when the native handler is invoked). Thus _help_all() has already been evaluated in the above.⁶

What we really want is to somehow delay applying the operator to the argument list. This could be done with metavariables, but we want to use the empty argument list here, so it isn't obvious how to do that since no variables are directly involved. The answer is that, after Elision applies one of the cases and gets a result, the result is then rewritten to replace any instances of the variable \$_ with the entire argument list. This means we can do the following. (Note that we use the metavariable \$\$_ by convention here.)

```
def({operator #name=help #cases
  %($op: OPREF) -> _help_op($$op),
  %() -> _help_all.$$_
})
```

This will work as expected. Note that this also means it is inadvisable to use the variable \$\$_ elsewhere.

Case operators are quite powerful, and can help avoid the need for native operators in many cases. For instance the Boolean not can be implemented as follows.

⁵Why is it done this way? Why not generate a string value? Then this definition would work, right? Well, not really. The string would be static, and we want to see help for new operators defined after we have defined help.

⁶So, what was the result of evaluating _help_all(), and why wasn't it printed? The result was the symbol _no_show, which is treated specially by the REPL. Whenever the result of evaluating an atom is _no_show all output is suppressed. This does not work from the prompt.

```
def({operator #name=not #type=BOOLEAN #cases
  true -> false,
  false -> true,
  $x:BOOLEAN -> _,
  %(true) -> false,
  %(false) -> true,
  %($x:BOOLEAN) -> _
})
```

This works, performs correct type checking, and is more general because it is also directly applicable to Boolean values.

Because case operators explicitly allow polymorphism their type cannot be correctly inferred as simply as it was for symbolic operators. For instance, what is the correct type for the help operator? For the fully-applied type, you have two options. You can figure out the correct type yourself and include it with #type=, or you can omit it and Elision will assign the operator the type ANY.

Scala. To create a definition of a case operator, use

• core.CaseOperator(name, type, cases, description, detail)

where name is the operator's name (a string), type is the type of the fully-applied operator (an atom), cases specifies the alternatives (it is an atom sequence), description is the short one-line description of the operator, and detail is the longer description of the operator and its use. The last two (description and detail) are optional. The following code creates the examples given previously (the sum operator).

```
import ornl.elision.core._
// Make the generic sum operator.
val List(x,y) = List("x","y").map(Variable(INTEGER,_))
val List(s,t) = List("s","t").map(Variable(STRING,_))
val List(mx,my) = List("x","y").map(MetaVariable(INTEGER,_))
val List(ms,mt) = List("s","t").map(MetaVariable(STRING,_))
val add = context.operatorLibrary("add")
val sumOp = CaseOperator("sum", ANY,
  AtomSeq(NoProps,
    MapPair(AtomSeq(NoProps,x), mx),
    MapPair(AtomSeq(NoProps,s), ms),
    MapPair(AtomSeq(Associative(true) and Commutative(true), x, y),
      add(mx,my)),
    MapPair(AtomSeq(Associative(true), s, t), Apply(ms, mt)),
    MapPair(ANY, ANY)),
  "Compute the generic polymorphic sum.",
  "This operator computes the polymorphic sum of its arguments.")
```

The last two strings provide documentation, as described in section §5.9. As with symbolic operators, the operator definition should be installed in an instance of core.OperatorLibrary, typically found in an instance of core.Context.

Unlike symbolic operators, case operators should not be given native handlers.

5.7. Recursion

Within an operator definition it is often desirable to express recursion. Suppose we define a relation among types, where BYTE is considered a restriction of WORD, which is considered a restriction of DWORD, which is finally considered a restriction of the general BITSET. We want an operator is_restriction_of(\$T,\$U) to evaluate to true iff \$T is a restriction of \$U. We could try to write this as follows.

```
{ operator #name=is_restriction_of #type=BOOLEAN #cases
  %($T,$T) -> true,
  %(BYTE,$T) -> is_restriction_of(WORD,$$T),
  %(WORD,$T) -> is_restriction_of(DWORD,$$T),
  %(DWORD,$T) -> is_restriction_of(BITSET,$$T),
  _ -> false
}
```

The notion is that we promote types until the two arguments are equal, or we exhaust the cases. This will not parse because <code>is_restriction_of</code> is not yet defined. To get around this, we use the fact that after a case operator is applied the variable \$__ (two underscores) is rewritten to the operator. This means we can write the above as follows.

```
{operator #name=is_restriction_of #type=BOOLEAN #cases
  %($T,$T) -> true,
  %(BYTE,$T) -> $$__.%(WORD,$$T),
  %(WORD,$T) -> $$__.%(DWORD,$$T),
  %(DWORD,$T) -> $$__.%(BITSET,$$T),
  _ -> false
}
```

This works as we want.

5.8. Summary: Case Operators

We provide a quick summary of properties of case operators.

- Cases are evaluated in order.
- A case can be a strategy, such as a map pair. If so, then the strategy is applied to the argument. If it reports success, the result is the value of the operator (with substitutions). If the strategy reports failure, then subsequent cases are tried.
- A case can be an applicable, such as an operator reference or a lambda. If so, then the applicable is applied to the argument immediately and the result is the value of the operator (with substitutions). No further cases are tried.
- A case can be any other atom, such as a literal. If so, then that atom is the value of the operator (with substitutions). No further cases are tried.
- The variable \$_ is replaced with the argument list.
- The variable \$__ is replaced with the operator.
- If _ or ANY is the value of the operator, then the result is the operator applied to the arguments with no further expansion or modification.
- If no case matches the argument, then an error is generated.
- Case operators cannot have native handlers, but can use operators with native handlers in the cases.

It is possible to have unbounded recursion. Note that using _ or ANY as the result does not cause the operator to be re-evaluated, but using \$__ likely will. A simple example is {operator #name=recur #cases \$\$__.\$\$_}. This simply repeats the operator indefinitely. Elision will trap this and report that it has detected unbounded recursion (or at least recursion that exhausts the stack).

Recursion can, of course, be quite useful. The following shows how one can compute powers of two. (Here we use one operator, but to be more safe, we would typically use two: pow2 and _pow2. The first would be the "public" operator that takes a single argument, and the second would be a hidden operator with the cases that take two arguments. We have to declare _pow2 first in this case, so we can refer to it from pow2.)

5.9. Documenting Operators

Operators may include two additional elements. A short one-line description of the operator can be given via #description, while a longer description of the operator can be given via #detail. If these are included, they will be used by Elision's help system. The general style is to describe the operator in abstract terms in its #description and not mention parameters by name. Then in the #detail the parameters may be mentioned directly.

```
Cases:

(%($x:INTEGER) -> $$x)

(%($x:STRING) -> $$x)

(%AC($x:INTEGER, $y:INTEGER) -> add($$x, $$y))

(%A($x:STRING, $y:STRING) -> ($$x.$$y))

(ANY -> ANY)

This operator computes the polymorphic sum of its arguments.
```

5.10. Closures

Returning to the pow2 example given in section §5.8, we try the following.

Here we define the two-argument version of the operator first, then re-define the pow2 in what looks like a recursive form. In fact what has happened is that the second definition uses the *first* definition. When Elision creates a term using an operator reference, it captures and stores the operator's definition in the term. This means that re-defining known operators will not "break" the system. In short, Elision creates a small closure for every operator reference.

CHAPTER 6

Special Forms

Elision provides an atom called the *special form* that has some associated "syntactic sugar" and is widely used for many purposes. This short chapter describes this form.

6.1. The Basic Special Form

The special form is really just an ordered pair of atoms, and can be represented as {: atom atom :}, where the two atoms can be anything. Because of this structure, it is easy to match and rewrite special forms (see chapter 7). The first atom of a special form is called the *tag*, and the second atom is called the *content*. The tag is subject to special interpretation when it is a symbol. In this case Elision will perform a lookup and expect the content to have a specific structure. This representation is called the *two atom form*.

```
e> {: 5 6 :}
$_rep10 = {: 5 6 :}
e> {: $x $y :}
$_rep11 = {: $x $y :}
e> {: fred $y :}
$_rep12 = {: fred:SYMBOL $y :}
e> {: operator $y :}
ERROR: Form operator:SYMBOL expected bindings as content, but instead found: $y.
```

Many known special forms consist of a symbol as the tag, and bindings as the content. In fact, this is common enough that there is some "syntactic sugar" for representing this combination. In fact, this alternate form, the *short form*, was used to define operators in chapter 5. An opening brace is followed by a symbol tag, then zero or more atoms, and then zero or more *bind pairs*. There are two kinds of bind pairs:

- A hash mark (#) followed by a symbol s, an equal sign, and an atom a. This creates the binding s->a.
- A hash mark (#) followed by a symbol s, a space, and then a comma-separated list of atoms a1,a2,...,aN. This creates the binding s->%(a1,a2,...,aN).

Any number, including zero, bind pairs may be present. A list of atoms a1 a2 ... aN may be present before any bind pairs. If so, then the binding "->a1,a2,...,aN is created.

This is really easiest to show with a series of examples.

```
e> {sara}
$_rep13 = {: sara:SYMBOL { binds } :}
e> {sara 6}
$_rep14 = {: sara:SYMBOL { binds ''->%(6) } :}
e> {sara 6 7 8}
$_rep15 = {: sara:SYMBOL { binds ''->%(6, 7, 8) } :}
e> {sara #first="Sara" #last="Felton"}
$_rep16 = {: sara:SYMBOL { binds last -> "Felton" first -> "Sara" } :}
e> {sara #first="Sara" #last="Felton" #chars Zayre, Thorina}
$_rep17 = {: sara:SYMBOL { binds last -> "Felton" chars ->
```

Table 1. Known Special Forms

Tag	Meaning, Example, and "Two Atom" Form	
	Interpret the content as a list of bindings.	
binds	{binds x->21 y->19}	
	$\{: binds:SYMBOL \%(x->21, y->19) :\}$	
map	Map an atom as the left-hand side of an apply to every item in a	
	collection.	
	<pre>{map add:OPREF}</pre>	
	<pre>{: map:SYMBOL { binds " -> %(add:OPREF) } :}</pre>	
	Create an atom that matches other atoms against a pattern.	
match	{match \$op.\$arg }	
	<pre>{: match:SYMBOL { binds " -> %((\$op.\$arg)) } :}</pre>	
operator	Create an operator definition.	
	{!foo(\$x,\$y)}	
	{operator #name=foo #params=%(\$x,\$y)}	
	<pre>{: operator:SYMBOL { binds name -> foo:SYMBOL params -> %(\$x,</pre>	
	<pre>\$y) type -> ANY } :}</pre>	
	Define a rewrite rule.	
rule	<pre>{rule add(neg(\$x),\$y)->0 #rulesets MATH}</pre>	
	<pre>{: rule:SYMBOL { binds " -> %((add(neg(\$x), \$y) -> 0))</pre>	
	rulesets -> %(MATH:SYMBOL) } :}	

```
%(Zayre:SYMBOL, Thorina:SYMBOL) first -> "Sara" } :}
e> {sara #first="Sara" #last="Felton" #chars=%(Zayre, Thorina)}
$_rep18 = {: sara:SYMBOL { binds last -> "Felton" chars ->
%(Zayre:SYMBOL, Thorina:SYMBOL) first -> "Sara" } :}
```

So, if you want to bind key to a single value, you can use #key=value. If you want to bind key to a sequence of values, you can use #key v1, v2, ... Note that a binding is always created, even when it is empty, since the special form must always consist of exactly two atoms.

The purpose of the special form is to unify many different atoms that have varied structure, and provide a consistent way to represent them, to match them, and to rewrite them. As should be evident from the above example, the short form is usually much easier to enter and shorter than the two atom form. Any short form can be represented in the two atom form, but not all two atom forms can be represented in the short form.

6.2. Known Special Forms

There are several known special forms. Bindings are not treated the same as other special forms, simply because they are themselves used to define special forms. Thus the way bindings are interpreted is a bit different, as you can see from the first row of the table below. Note the additional syntactic sugar for specifying operators. This is described in section §5.1.

Scala. Creating a special form is quite easy. Use core.SpecialForm(tag, content), where tag is the tag, and content is the content. Invoking this method in the companion object will check the catalog of known special forms (shown in table 1). The following code creates each of the atoms shown in the table.

```
// Create a binding. There are easier ways to do this!
val binds = SpecialForm(
```

```
Literal('binds),
  AtomSeq(NoProps,
    MapPair(Literal('x), 21),
    MapPair(Literal('y), 19)))
// Create {map add:OPREF}.
val map = SpecialForm(
  Literal('map),
  Bindings(""->AtomSeq(NoProps, Literal(OPREF, "add"))))
// Create {match $op.$arg}.
val mat = SpecialForm(
 Literal('match),
  Bindings(""->AtomSeq(NoProps, Apply('op, 'arg))))
// Create {operator #name=foo #params=%($x,$y)}.
val foo = SpecialForm(
  Literal('operator),
  Bindings("name"->Literal('foo), "params"->AtomSeq(NoProps, 'x, 'y)))
// Create {rule add(neg($x),$y)->0 #rulesets MATH}.
val List(add, neg) = List("add", "neg").map(context.operatorLibrary(_))
val rule = SpecialForm(
  Literal('rule),
  Bindings(""->AtomSeq(NoProps, MapPair(add(neg('x),'y), 0)),
           "rulesets"->AtomSeq(NoProps, Literal('MATH))))
```

These examples show direct use of the core. SpecialForm object. Of course, each of the above has its own simpler, dedicated method of creating an instance. For example, the simpler method of creating core. Bindings instances is shown several times above. The following create the same special form instances.

```
// Create a binding.
val binds = Bindings("x"->21, "y"->19)

// Create {map add:OPREF}.
val map = MapStrategy(Literal(OPREF, 'add), AtomSeq(), AtomSeq())

// Create {match $op.$arg}.
val mat = MatchAtom(Apply('op, 'arg))

// Create {operator #name=foo #params=%($x,$y)}.
val foo = TypedSymbolicOperator("foo", ANY, AtomSeq(NoProps, 'x, 'y), "", "")

// Create {rule add(neg($x),$y)->0 #rulesets MATH}.
val List(add, neg) = List("add","neg").map(context.operatorLibrary(_))
val rule = RewriteRule(add(neg('x),'y), 0, Seq(), Set("MATH"))
```

There is yet one more way to create a special form. You can create and populate an instance of core.SpecialFormHolder. This is how the parser functions; it detects a special form, populates an instance of this object, and then passes it to an apply method that creates the appropriate special form. Since its primary use is to support parsers, it is not discussed here. Note that every special form instance, no matter how created, holds an instance of this class.

CHAPTER 7

Matching and Rewriting

Elision is a term rewriter; its primary purpose is matching and rewriting. This chapter provides an introduction to matching and rewriting, and to the components provided by Elision to perform these tasks.

7.1. Bindings

In Elision, a binding associates a variable, such as \$x with some atom, such as 5. We say that \$x is bound to the value 5. Variables are bound by name only; Elision does not differentiate between binding \$x:INTEGER and binding \$x:STRING. While this has the potential to cause trouble, in practice it should not, and results in a significant performance boost. Variable bindings are written in Elision as a bindings atom, which takes the form {binds n1->a1 n2->a2 ... }, where n1, n2, ..., are the variable names (without \$), and a1, a2, ..., are the respective values. The atom {binds} is legal, and represents the empty bindings set. Writing {binds x->5 x->6} is the same as {binds x->6}; the last binding of a variable wins.

Scala. Bindings behave like Scala Map objects. To create a set of bindings, write core.Bindings(s1->a1, s2->a2, ...), where s1, s2, ..., are strings that specify the variable names, and a1, a2, ..., are the atoms. It is legal to specify no atoms. The following are some examples.

Elision	Scala
{binds}	core.Bindings()
{binds x->5}	core.Bindings("x"->Literal(5))
{binds x->5 y->6}	<pre>core.Bindings("x"->Literal(5), "y"->Literal(6))</pre>

It is also possible to incrementally build up a binding. The following shows an example.

```
var binds = Bindings()
binds += ("x"->Literal(5))
binds += ("y"->Literal(6))
binds.toParseString
```

This code will generate the output $\{$ binds $x \rightarrow 5 y \rightarrow 6 \}$.

Bindings are immutable, and are created and discarded by the system during matching. Thus the core.Bindings object is explicitly kept "lightweight," and is not itself a core.BasicAtom. Instead, when an atom is needed (such as when we invoke toParseString in the above example) an instance of core.Bindings is automatically transformed into an instance of core.BindingsAtom. The latter is a core.BasicAtom. To see this transformation you can enter the above four lines to create binds, and then enter binds:BasicAtom, or BindingsAtom(binds). Likewise, an instance of core.BindingsAtom will be transformed into an instance of core.Bindings when necessary, enabling the use of iterators, etc.

7.2. Applying Bindings

Bindings are an Elision *applicable*, meaning that they behave like operators, and generate a new atom, when placed on the left-hand side of an applicative dot. When bindings are applied to an

7.3. MATCHING

atom, the atom is rewritten. Every instance of a bound variable in the atom is replaced with the bound value.

```
e> {binds x->5 y->6}.$x

$_rep10 = 5
e> {binds x->5 y->6}.($x.$y)

$_rep11 = ((5).6)
e> {binds x->5 y->6}.add($x,$y)

$_rep12 = 11
e> {binds x->5 y->6}.add($x,$y,neg($x))

$_rep13 = 6
e> {binds x->5}.{binds y->$x}

$_rep14 = { binds y -> 5 }
e> {binds x->5}.{binds y->$x}.$y

$_rep15 = 5
```

Scala. Bindings can be applied to atoms in two ways. First, they can be applied like any other applicable, using core.Apply. This causes the bindings to be converted to a core.BindingsAtom, which introduces overhead, but also prevents rewriting atoms that contain metavariables.

```
scala> Apply(Bindings("x"->Literal(5), "y"->Literal(6)),'x).toParseString res1:_{\square}String_{\square}=_{\square}5 scala>_{\square}Apply(Bindings("x"->Literal(5),_{\square}"y"->Literal(6)), _{\square}MetaVariable(ANY,"x")).toParseString res2:_{\square}String_{\square}=_{\square}({_{\square}binds_{\square}x_{\square}->_{\square}5_{\square}y_{\square}->_{\square}6_{\square}}.$$x)
```

Alternately, every atom has a rewrite(binds) method that accepts a set of bindings and applies the bindings to the atom, regardless of any metavariables. The return value of this method is a pair of the form (atom,flag), where atom is the rewritten atom, and flag is true iff any rewrites were actually performed.

```
scala> MetaVariable(ANY,"x").rewrite(Bindings("x"->Literal(5),
    "y"->Literal(6)))._1.toParseString
res4: String = 5
```

In this latter case the bindings are used directly, and no core.BindingsAtom is constructed.

7.3. Matching

The basic principle of matching in Elision can be explained as follows: Let s be an atom called the *subject*, and let p be an atom called the *pattern*. Then we say the subject s matches the pattern p iff there is at least one decidable set of bindings s such that s and s are is one exception to this general rule: ANY. For instance, the subject 5 matches the pattern ANY, but there is obviously no set of bindings that can transform ANY into 5, as ANY contains no variables.

In other words a subject matches a pattern if either the pattern is ANY, or if parts of the subject can be bound to variables in the pattern such that the subject is equal to the rewritten pattern. This is somewhat complicated to explain, but rather easy to show. table 1 gives some examples.

Note that it is possible to match two terms without having to bind any variables, resulting in an empty binding. As the last case table 1 indicates, it is also possible to have many possible bindings. In this last case $\{binds x->add(\$b,\$c)\}$ and $\{binds x->add(\$a,\$b) y->\$c\}$ are both acceptable matches, since add is associative. However add is also commutative, so additionally we permit $\{binds x->\$b y->add(\$a,\$c)\}$, $\{binds x->\$c y->add(\$a,\$b)\}$, $\{binds x->add(\$b,\$c)\}$

Pattern Subject Outcome 5 5 Match with {binds} ANY 5 Match with {binds} 5 ANY Match with {binds} 5 Match with {binds x - > 5} \$x 5 No matches \$x

No matches

Multiple matches

joe:SYMBOL.5

add(\$a,\$b,\$c)

(5).6

(6).6

\$x.5

\$x.\$x

\$x.\$x

add(x, y)

Match with {binds x->joe:SYMBOL}

Match with {binds x->6}

Table 1. Examples of Matching

y->\$a}, and {binds x->add(\$a,\$c) y->\$b}. The outcome of a match attempt can be no match, a single match, or many matches.

Elision provides a simple way to test matching. The *match atom* is an applicable (so it works like an operator) that has the form {match pattern}, where pattern is the pattern to attempt to match. The following examples reproduce the matches from table 1. The result will either be a binding if the subject matches the pattern, or NONE if it does not. In the case of multiple matches, only the first is returned.

```
e> {match 5}
$_rep10 = {: match:SYMBOL { binds '' -> %(5) } :}
e> {match 5}.5
$_repl1 = { binds }
e> {match ANY}.5
$_repl2 = { binds }
e> {match 5}.ANY
prepl3 = {binds}
e > \{match \$x\}.5
repl4 = {binds x -> 5}
e> {match 5}.$x
$_rep15 = NONE
e> {match $x.5}.(joe:SYMBOL.5)
$_rep16 = { binds x -> joe:SYMBOL }
e > \{ match \ \$x. \$x \}. ((5).6)
$ rep17 = NONE
e > \{ match \ \$x. \$x \}. ((6).6)
= {\text{binds } x \rightarrow 6}
e> {match add($x,$y)}.add($a,$b,$c)
prepl9 = { binds x -> $a y -> add($b, $c) }
```

7.4. Map Pairs

We can use the result of a match to rewrite another atom. The following is one simple example.

```
e> {match add($x,$y,neg($x))}.add($a,neg($a),19)
$_repl10 = { binds x -> $a y -> 19 }
e> {match add($x,$y,neg($x))}.add($a,neg($a),19).$y
$_repl11 = 19
```

Remember that the applicative dot is left-associative, so the bindings are created first, and then used to rewrite the atom \$y\$, yielding the result 19. Of course this trick won't work so well when NONE is returned. We can build a solution using lambdas, but it will be a bit tedious. Elision provides a simpler way: the map pair. The map pair has the form pattern -> newatom, where pattern is the pattern to match, and newatom is the atom to rewrite if a match is obtained. The map pair is a rewriter, meaning that it yields a specialized binding of the form {binds atom->newatom flag->theflag}, where newatom is the result and theflag indicates whether the rewriter succeeded in some sense. For the map pair, theflag is true iff the pattern match succeeded.

The following is an example.

```
e> (add($x,neg($x),$y)->$y).add($a,neg($a),19)

$_repl12 = { binds flag -> true atom -> 19 }

e> (add($x,neg($x),$y)->$y).add($a,neg($a))

$_repl13 = { binds flag -> false atom -> add($a, neg($a)) }

e> (add($x,neg($x),$y)->$y).add($a,neg($a)).$atom

$_repl14 = add($a, neg($a))
```

In the first case the match succeeds. In the second, it fails. Note that when the match fails the result is the unmodified subject.

7.5. Variable Guards

Variables can be matched based on type information.

```
e> {match $x:INTEGER}. "Korra"

$_rep10 = NONE
e> {match $x:INTEGER}.5

$_rep11 = { binds x -> 5 }
```

This is often sufficient, but sometimes we would like to have greater control over the matching process. For example, we might like to match only a non-zero integer. We can do this with a variable guard. This is an atom that is attached to a variable using braces. The atom is rewritten with the proposed variable binding, and if it evaluates to true, the binding succeeds. Otherwise the entire match is rejected.

```
e> {match $x{not(equal($$x,0))}:INTEGER}.5
$_rep12 = { binds x -> 5 }
e> {match $x{not(equal($$x,0))}:INTEGER}.0
$_rep13 = NONE
```

Using metavariables in the guards prevents trouble, and we adopt it as a convention. If we did not use it, equal(\$x,0) would immediately evaluate to false, which would be negated to true, making the guard {true}. This would always succeed, which is not what we want at all.

We can use this to gain some control when there are many possible matches. Consider the following.

```
e> decl.{! foo($x,$y) is %AC}

Defined operator foo.
e> {match foo($x:INTEGER, $y)}.foo("Jim",5,"Tim",0)

$_repl2 = { binds x -> foo("Jim", 5) y -> foo("Tim", 0) }
e> {match foo($x{equal($$x,0)}:INTEGER, $y)}.foo("Jim",5,"Tim",0)

$_repl3 = { binds x -> 0 y -> foo("Jim", 5, "Tim") }
```

Variable guards are sufficient in most cases, but there are still cases in which you want a guard that involves several variables. How to do this will be discussed in chapter 8.

CHAPTER 8

Rules and Strategies

Basic matching and rewriting were introduced in chapter 7. This chapter will explain how to manage rewriting on a much larger scale, creating more general rewrite rules, managing them with rulesets, and controlling the process through the use of strategies and the build-in strategy combinators.

8.1. Rewrite Rules and Guards

A rewrite rule generalizes the notion of a map pair a bit by introducing additional guards.

8.2. Rulesets

8.3. Strategies

In Elision a strategy is an atom S that, when applied to another atom A yields two things: a potentially new atom A' and a Boolean value f. These items are packaged in a binding of the form $\{\text{bind atom-}>A' \text{ flag-}>f\}$. Any atom that satisfies this requirement is a strategy.

There are already several examples of strategies. Map pairs and rewrite rules both satisfy this requirement, and are considered strategies. The following is a simple strategy that takes an atom and returns that atom with the flag true.

This illustrates some points about strategies. First, the names of strategy operators start with s_. Second, strategies are given the type STRATEGY, even though the result is a binding.

8.4. The Map Strategy

8.5. Strategy Combinators

CHAPTER 9

Executors and Contexts

Rewrite rules, operator definitions, and bindings need to be made persistent during a session. This means we need some form of mutable data store to hold these. That general storage is called the *context*, and is explored in more detail in this chapter. Also, Elision code must be parsed to obtain atoms, and that is the job of an *executor*.

Executors are provided by classes that implement the core. Executor trait, and contexts are instances of core. Context. Every executor instance also holds an instance of a context.

Elision atoms are immutable, so

APPENDIX A

${\bf Basic Atom}$

APPENDIX B

Exceptions

Elision attempts to handle exceptions rationally, and to *never* allow the system to fail completely. Once Elision is up and the REPL is running, exceptions are trapped and an attempt is made to dispose of them appropriately. The following sections describe how Elision handles exceptions.

B.1. Elision Exceptions

Elision uses a number of exceptions to signal conditions internally. These are usually serious conditions, from the point of view of the rewriter, but do *not* represent complete failure. That is, it should be possible to continue after one of these exceptions. All such exceptions extend core.ElisionException and provide a human-readable string as the message.

- core.ArgumentListException: An error was detected in the argument list for an operator. This is typically a type check error, or the wrong number of arguments, as with add(5,"Tim").
- core.LambdaUnboundedRecursionException: The stack was exhausted during evaluation of a lambda expression. This typically represents unbounded recursion.
- core.LambdaVariableMismatchException: The argument provided to a lambda does not match the lambda parameter. This is typically a type error, as with \\$x:INTEGER.\$x."Not an integer.".
- core.NativeHandlerException: A native handler for an operator could not be parsed, or represented incomplete input (missing a closing parenthesis or brace, for instance). For example, {! foo() #handler=" for(atom <- args) { "}.</pre>
- core.OperatorDefinitionException: An error occurred in evaluating an operator definition. Most likely your properties and prototype do not agree, as with {! foo(\$x:INTEGER):INTEGER is associative}.
- core.OperatorRedefinitionException: An attempt was made to re-define an operator, but the operator library does not allow this. Under normal circumstances (i.e., using the REPL), redefinition is allowed with a warning.
- core.SpecialFormException: A structural error was discovered interpreting a special form. Typically this means that the content was missing an required bind pair, as with {operator #type=INTEGER}.

B.2. Java Exceptions and Throwables

Other exceptions are handled differently by Elision.

APPENDIX C

Reading Elision Files

Elision provides several mechanisms to read files.

- The REPL. You can pipe input to the REPL, or you can use the read and read_once commands at the prompt.
- The read(filename: String) method of parse.Processor.
- The elc (Elision compiler) command that reads input, and then generates the Scala code to re-create the resulting context.

These methods all work essentially the same way: input is consumed from some source, parsed into a sequence of atoms, and each atom is then handled in some appropriate manner. All this work is generalized by the parse.Processor class. The following sections discuss how to use these to create additional facilities.

C.1. Processor

If you want to process Elision files you should begin with parse. Processor. This class provides a context, along with several methods to read data.

Internally, all text is ultimately processed by the execute(String) method. The flow of this method is as follows.

- (1) Parse the text, obtaining a sequence of parse. AtomParser. AstNode instances.
- (2) For each node, do the following for each registered instance of parse.Processor.Handler, in the order registered.
 - (a) Pass the current node to the handleNode(node: AtomParser.AstNode) method. This method must return an Option[AtomParser.AstNode]. If the returned value is None, then the node is discarded. If the returned value is Some(node), then the returned node becomes the current node. A simple default implementation of handleNode is:

def handleNode(node: AtomParser.AstNode) = Some(node)

- (b) Pass the node to all listening actors via actor ! node.
- (c) Convert the node into an atom by invoking the node's interpret method.
- (d) Pass any current atom to the handleAtom(atom: BasicAtom) method. This method is responsible for performing any specialized processing of the atom post-construction (as the atom was already constructed during execution of the interpret method in handleAtom), and should return an optional atom. If None is returned then the atom is discarded and the system proceeds to the next node. If the return value is Some(atom),

Table 1. Methods to Read Atoms

Method	Use
read(filename: String)	Read the content of the specified file.
read(stream: Source)	Read the content of the provided stream.
execute(text: String)	Read from the provided string.

then the returned atom becomes the current atom. A simple default implementation of handleAtom is:

def handleAtom(atom: BasicAtom) = Some(atom)

(e) Pass the atom to all listening actors via actor! atom.

Other events can be passed to actors. Whenever a new file is provided for processing, a FileStart object is sent to the actors. When processing of the file is complete, a FileEnd is sent. When a new stream is provided for processing, a StreamStart is sent to all actors. When a stream is exhausted, a StreamEnd is sent to all actors. The flow of messages thus looks as follows, if a valid filename is provided to the read method.

FileStart
StreamStart
AtomParser.AstNode
BasicAtom
AtomParser.AstNode
BasicAtom
AtomParser.AstNode
BasicAtom
...
AtomParser.AstNode
BasicAtom

BasicAtom StreamEnd FileEnd

C.2. Operators

The

APPENDIX D

Native Handlers

This appendix describes how to create native handlers for symbolic operators. Native handlers cannot be created for case operators, as they don't really make sense. If you want a native handler for a case operator, create a symbolic operator, provide it with a native handler, and include it as a case. The help operator described in section §5.6 is such an example.

A short description of creating native handlers is given in the Scala block of 5.3. This is suitable for creating them programmatically, but Elision provides support for providing the definition more directly.

Consider the operator mod(\$b,\$d) that computes the remainder when \$b is divided by \$d. For example, mod(5,2) is 1, while mod(4,2) is 0. First we give the definition of the operator, without a native handler.

```
{! mod($b: INTEGER, $d: INTEGER): INTEGER}
```

The definition specifies two parameters: \$b and \$d, both integers.

A native handler consists of a Scala closure. It's argument is an instance of core.ApplyData, and the handler is expected to return an instance of core.BasicAtom. There are several fields and methods of core.ApplyData that can be useful in writing a native handler.

- The field exec provides access to an instance of core.Executor. This is a class that provides an parse(text) method, where text is a string to be parsed as a sequence of atoms. The core.Executor instance also provides access to a core.Context instance, and to a core.Console instance.
- The field context is the context provided by the executor. It provides access to the operator and rule libraries and to the current bindings.
- The field console is the console provided by the executor. It has methods error(msg) and warn(msg) to generate errors and warnings, and emit(msg) and emitln(msg) to write arbitrary text to the console.
- The field _no_show is a special symbol that is used by certain interactive systems, such as the REPL, to indicate that no value should be displayed as the result of the handler. This is the common return value for operators that are executed solely for their side effect. For example, this is the value returned by the help operator. This should be checked by object identity, not equality, so that it is possible to use _no_show as a symbol normally.
- The method as_is indicates that the result should be the application of the operator to the argument list without any further processing. This is commonly used as the return value for an operator in cases where no special processing is required.
- The field op is the instance of core. Symbolic Operator being applied.
- The field args is an instance of core. AtomSeq that contains the arguments to the operator, in order.
- The field binds is an instance of core.Bindings that binds each parameter to its corresponding argument. For associative operators the parameters bound are synthetic, and are named \$':1', \$':2', ...

Remember that strings and numbers are automatically converted to atoms. To take apart the argument list, a special extractor core.Args is provided. This is a sequence extractor for core.AtomSeq instances. Given all this, we can write the native handler for mod as follows.

```
def modHandler(data: ApplyData) = data.args match {
  case Args(IntegerLiteral(_, b), IntegerLiteral(_, d) => b mod d
  case _ => as_is
}
```

Now if **context** is the context where the operator is defined we can register the handler as follows. Note that we convert the function into a closure and pass the closure.

```
context.operatorLibrary.register("mod", modHandler _)
```

It is possible to declare a native handler for an operator from the REPL. This is done by adding a #handler bind pair to the definition of the operator. The value for #handler must be a string literal. The string literal executes in the following context.

```
import ornl.elision.core._
def _handler(_data: ApplyData): BasicAtom = {
  import _data._
  import ApplyData._
  // The #handler value is placed here.
}
```

The result is a function whose closure is stored as the native handler for the operator. Note that the operator need not be declared in any context; it is possible to use this to execute arbitrary Scala code. Consider the following.

```
e> {! _() #handler=""" "hello world!" """}.%()
$_rep10 = "hello_world!"
e> {! _() #handler=""" println("hello world!") ; _no_show """}.%()
hello world!
```

Returning to the closure for the mod operator, we can define it as follows.

As with #detail, if the first character is the pipe symbol (|), then the margin is stripped. Note that handler does not need to refer to _data.args, as everything in _data is imported. If other classes are required, you may add an import statement to obtain them.

Some examples follow. First we write a "hello world" operator.

```
{! foo() #handler=""" "hello world!" """}
```

Next we create a much more complex operator that evaluates its string argument. If the argument evaluates to one or more atoms, then the last atom is returned. Alternately we could return a collection of atoms. This makes use of the executor, console, and some of the special fields (like _no_show and as_is).

```
{! evaluate($text: STRING)
    #handler="""|import ornl.elision.core._
```

Finally, we will use a few language features to create a _decl operator that can be used to declare other operators and rules.

```
// Create the def operator.
{! _decl($item)
#handler=
11 11 11
 args(0) match {
    case op: Operator =>
      context.operatorLibrary.add(op)
      console.emitln("Declared operator " + toESymbol(op.name) + ".")
      _no_show
    case rule: RewriteRule =>
      context.ruleLibrary.add(rule)
      console.emitln("Declared rule.")
      _no_show
    case _ =>
      as is
 }
11 11 11
#evenmeta=true
#description="Declare a rule or operator in the current context."
#detail=
"Given a rewrite rule, add the rewrite rule to the rule library in the ".
"current context. Given an operator, add the operator to the operator ".
"library in the current context."
```

There is one field in the above that requires some explanation. The #evenmeta=true forces evaluation of this operator even when the argument is a metaterm. Since both operator definitions and rewrite rules typically contain metaterms, this is essential. This feature breaks the general model of "metaterms prevent evaluation," so its use is not recommended. In fact, it only exists to address cases like the above.

An interesting question is how to get _decl itself declared without the use of another operator. That is, how could we "bootstrap" the system? Since _decl is capable of declaring operators, we could use it to declare itself. We do this with a lambda.

```
\$op.($op.%($op)).{! _decl($item) ... }
```

This yields $_\mathtt{decl}$ applied to an argument list containing itself, and thus results in the operator declaring itself, as we want.

APPENDIX E

De Bruijn Indices

Elision converts lambdas into a representation using a form of De Bruijn indices. This appendix describes how they are implemented in Elision.

A "proper" De Bruijn index indicates the the number of binds that are in scope. Thus the lambda $\lambda x.\lambda y.x$ should correctly produce the De Bruijn term $\lambda\lambda 2$, as the body binds to the outermost lambda. Elision does not actually use proper De Bruijn indices, but uses a very similar concept somewhere in between symbolic names and De Bruijn indices.

In Elision every atom has an associated De Bruijn index. Literals and (most) variables have index zero. Other terms have De Bruijn index equal to the maximum index of their parts.

Suppose we are given a lambda term such as \\$x.add(\$x,\$y). The body contains no lambdas or De Bruijn indices, so it has De Bruijn index zero. The lambda parameter also contains no lambdas or De Bruijn indices, so it also has De Bruijn index zero. Thus the maximum De Bruijn index of the parts is zero. Since we are constructing a lambda, we increment the maximum index, obtaining one. The system constructs a special variable \$':1' representing the De Bruijn index, and then rewrites lambda parameter and body with \$x -> \$':1', obtaining the result \\$':1'.add(\$':1', \$y).

Next consider \$x.\$y.\$x. First \$y.\$x is constructed, yielding \$':1'.\$x, whose Dr Bruijn index (assigned by Elision) is one. Next we construct \$x.(\$':1'.\$x). The De Bruijn index of the parameter is zero, and of the body is one, so the De Bruijn index to use for the new expression is \$':2', and we rewrite with $$x \to $':2'$. The trick is that the rewrite actually causes us to re-build the lambdas. Rewriting the body \$':1'.\$x results in a new lambda whose body is \$':2' with De Bruijn index of two, and thus the entire lambda constructs a new

APPENDIX F

The Elision Directory Structure

The Elision repository contains several files and folders. This appendix explains what these items are, where to find them, and where to put stuff if you want to contribute. The description here contains items that are *not* part of the built distribution, since many of these are not required by the distribution. This description also contains items that are *not* found in the repository, but are created during the build process.

F.1. In the Distribution

The folders described in the following subsections are part of the Elision distribution, created by the dist target in build.xml.

F.1.1. The Root Folder. The root of the distribution is reserved for a few files.

- The README.txt file that describes the distribution and how to build it and start the REPL. You should only modify this if you need to correct information or document a new build.
- The CHANGES.txt file that describes changes from one version to the next.
- The LICENSE.txt file that contains the license for Elision.
- The build.xml file that is used by Ant to compile the system and generate the API documentation. You should only modify this if you need to change how the system is built. This is not needed if you are just adding new source files or third-party libraries.

There are scripts present in this folder.

- The elision.sh script sets up the CLASSPATH and invokes the Elision REPL. This is a Bash script that is only useful on UNIX, Mac OS X, and Linux machines, or on Windows machines with Cygwin.¹
- The run.sh script sets up the CLASSPATH and then starts the Scala interpreter. This is a Bash script that is only useful on UNIX, Mac OS X, and Linux machines, or on Windows machines with Cygwin.

In general do not place any other items in the root folder. Instead, put them in the appropriate subfolder.

F.1.2. The etc Folder. This folder is reserved for the following items.

- The config.xml file that provides the template for the system's runtime configuration, and the elision_configuration.dtd that describes the format of the configuration file. You should not need to modify either of these unless you are modifying the ornl.elision.Version object.
- Files that provide Elision support for editors or IDE's, such as Emacs², Eclipse³, and Sublime⁴. See the editors folder.
- Miscellaneous files, such as properties files for use in Eclipse, if desired.

¹Elision is developed on Mac OS X and Linux platforms, and only occasionally tested on Windows, so YMMV.

²http://www.gnu.org/software/emacs/

³http://www.eclipse.org/

⁴http://www.sublimetext.com/2

If you have a file or files you need to add, and have no other place to put it, this is the place it belongs.

- F.1.3. The doc Folder. This folder contains the Elision documentation, other than the README.txt file. Specifically, it contains the following.
 - This file, elision.pdf.
 - The elision.lyx file. This is a LyX⁵ source file that is converted into elision.pdf, and contains the user and developer documentation you are reading right now.
 - The makedoc.sh script that will compile the API documentation without requiring you to have Ant. This file runs the Scaladoc command and puts the documentation in the api subfolder, the same as the build.xml script does. This is a Bash script, and should work on UNIX and Mac OS X, Linux, and possibly on Windows under Cygwin.
 - The index.html file used by the Elision web site.
 - If the API documentation has been built, either by running the makedoc.sh script or by running the Ant build and specifying the doc target, then there will be an api folder containing the API documentation. Point your browser to doc/api/index.html to view the documentation.

Other documentation files should be put in this folder.

F.1.4. The lib Folder. This folder contains third party code necessary to compile and run the system. There is a simple rule for this folder: The build and the Bash scripts will search this folder for any jar files, and will add all such files found to the class path.

If you need to use third party code in Elision, and you want to contribute back to the main distribution, you must first clear the inclusion of the third-party code with the maintainers to make sure there are no licensing conflicts, and then you should place the distribution in this folder. This allows for consistent and rational versioning. If the user were required to get and install the third party libraries, they might obtain incompatible versions.

F.1.5. The src Folder. This folder contains the source code for Elision, organized into folders that mirror the package hierarchy. All Elision code should be placed in a package rooted under ornl.elision. This is where you will place new source files, as well as any extra files required by Scaladoc. The one exception is the src/bootstrap folder that holds the Elision source for bootstrapping the system.

F.2. From the Build

The folders described in the following subsections are created by the build process.

- **F.2.1.** The bin Folder. This folder is created by the build process and contains the compiled class files and other elements that should be in the class path. In fact, the executable jar file is created by compressing this folder and adding a manifest, as specified in the build.xml file. Anything that should be in the class path should be placed in this folder.
- **F.2.2.** The latest Folder. This folder is created by the build process and contains the most recently built executable elision.jar and elision-all.jar files. Any jar files in the third-party library folder (lib) are included in these files, and elision-all.jar also includes some of the Scala distribution's jar files. Nothing else should be present in this folder.

⁵LyX is an editor that is backed by L^AT_EX. If you want to edit this document, you need LyX. Fortunately it runs on almost every platform out there. Visit http://lyx.org for more information and links to the latest version.

F.2.3. Date-Stamped Folder(s). Every time the system is built, it creates a folder whose name is the current date, in YYYYMMDD format. This folder contains two items: the same elision.jar file found in the latest folder, and a jar file containing the content of the src folder at the time of the build. Because elision-all.jar is so large, it is not included. These folders can be deleted if they are not needed. Nothing else should be present in these folders.

APPENDIX G

The Elision Coding Guide

This chapter describes the general style of coding to use for contributions to Elision.

G.1. Concepts

The following definition is due to David Parnas.

Definition. One code entity A uses another entity B iff the correct functioning of A depends on the availability of a correct implementation of B.

The "uses" relation is not the same as "invokes." A may invoke code in B without using B in the manner described above. For example, an operating system invokes programs, but the functioning of the operating system should not depend on the correct functioning of the programs.

Problems arise when there are loops in the uses relation. That is, A uses B and B uses A. Then neither A nor B work until both work. This condition is also referred to as a tangle. These are bad because nothing works until everything works. The code becomes very brittle, and changes "ripple" through the system in ways that are hard to trace.

Rule: Avoid loops in the use hierarchy.

G.2. Architectural Rules

Rule: All code must be in packages under ornl.elision.

G.3. Scaladoc

In general it is recommended to follow the Scaladoc guidelines found at http://docs.scala-lang.org/style/scaladoc.html. We make some revisions here.

Rule: The Elision style is to use Java-style documentation comments (first asterisks align and comment starts on second line), and not the style recommended for Scaladoc (second asterisks align and comments start on first line). There is better tool support for this style at present.

```
/** This is the style from the Scaladoc page.
  * Please don't use it. Most formatters don't
  * know what to do with it.
  */

/**
  * This is the common "Javadoc" style. Do use it.
  */
```

Rule: Document every public and protected method and field using Scaladoc. It is not necessary to document private methods and fields, but it is probably advisable.

Rule: Documentation may be omitted when a method or field overrides or implements a method from a parent class. In some cases you may wish to replace the textual documentation, but may omit the documentation of parameters and/or the return value.

G.5. SPACING 67

G.4. Comments

Comment your code by recording design decisions, your rationale for choices, and descriptions of how complicated code sections work. Essentially record what you will want to know when you have to understand the code five years later, or when someone new to the project has to make changes and not break anything. If there is an assumptions in your code, you should record them.

Rule: Write complete sentences. Capitalize and punctuate appropriately, and make a reasonable attempt to spell correctly.

Rule: Use single-line comments for short comments. For long explanations, use the /*...*/ style.

Rule: Use comment blocks to delineate logical sections of large classes. These take the following form:

Don't comment every line, and don't just re-state what the line does. Assume the person reading the code (probably you) will know how to read the code. Use comments to explain what is going on. The following commentary is useless.

```
// get atoms
val atoms = atomset.filter(!_.isBindable)
// loop over atoms
for (atom <- atoms) {
    // print atom
    println(atom.toParseString)
}</pre>
```

This commentary is just a more verbose version of the above commentary.

```
// Get the unbindable atoms.
val atoms = atomset.filter(!_.isBindable)
// Print all the atoms.
for (atom <- atoms) {
    // Print the next atom on its own line.
    println(atom.toParseString)
}</pre>
```

This is actually the recommended style. Keep it simple.

```
// Print all unbindables.
val atoms = atomset.filter(!_.isBindable)
for (atom <- atoms) {
   println(atom.toParseString)
}</pre>
```

G.5. Spacing

Rule: Use two-space increments for indentation. If possible, set your editor to use spaces instead of tab characters, and do not use tab characters.

Rule: Skip a blank line between any Scaladoc documented items, and between classes no matter whether they are documented or not.

Rule: Break lines at 80 characters (or just slightly over).

G.6. Returning

APPENDIX H

Grammar

The grammar for Elision is described in extended Backus-Naur form (EBNF) in this appendix. Terminals are indicated with text in fixed font, boxed, as terminal. Elision is normally case-sensitive, but some terminals are case-insensitive. These are indicated with a double box, as terminal. Nonterminals are indicated with italics, as nonterminal. Character ranges in unicode are indicated by a dash, so any single nonzero digit is 1-9. Productions are indicated with ::=. Parentheses indicate grouping. If an item may appear zero or more times, this is indicated with a suffixed asterisk, as $(atom)^*$. Optional items are indicated with a suffixed question mark, as $(atom)^*$.

Though not explicitly in the grammar, comments are allowed in two forms. Single line comments starting with \[\frac{1}{2} \] and ending with a newline, and multi-line comments starting with \[\frac{1}{2} \] and ending with \[\frac{*}{2} \].

```
atom\text{-seq} ::= (atom)^*
\mathbf{atom} ::= first-atom \longrightarrow first-atom
       | first-atom ( \underline{ } \underline{ first-atom} ) |^*
first-atom ::= [(] atom [)]
         special-form
         lambda
         apply
         typed-list
         alg-prop
         variable
         operator-symbol
         literal
         number
          ^TYPE
special-form ::= alternate-operator-def
```

H. GRAMMAR 70

```
\textbf{alternate-operator-def} ::= \boxed{ \{! \ SYMBOL \ ( \ atom-sequence \ ) \ (:first-atom)? \ (sf-bound-atom \ | \ atom-sequence \ ) } 
\mathbf{sf}-bound-atom ::= \boxed{\#}SYMBOL \boxed{=} atom
lambda ::= \bigvee variable \underline{\quad} first-atom
apply ::= SYMBOL ( atom-sequence )
\mathbf{alg\text{-}prop} ::= \textit{ alg\text{-}prop\text{-}short }
      | % alg-prop-long
I ([ atom ] )?
        B [ atom ]
alg-prop-long := (|absorber| atom)
       | | identity | atom
        (not)? associative
        (not)? commutative
       (not)? |idempotent|)*
\mathbf{variable} ::= (\$ \mid \$\$) \ SYMBOL (\{ \} \ atom \})? ([ : first-atom)? (@ SYMBOL)*
operator-symbol ::= SYMBOL :: | OPTYPE |
```

H. GRAMMAR 71

```
literal ::= SYMBOL \mid STRING
 number ::= any-number ( : first-atom )?
 any-number ::= hex-number | hex-number | binary-number | decimal-number | octal-number
\mathbf{hex\text{-}number} ::= hex\text{-}integer \ (\cdot \underline{\quad} hex\text{-}digit)? \ (\cdot \underline{\quad} \underline{\quad} exponent)?
\mathbf{binary}\text{-}\mathbf{number} ::= \mathit{binary}\text{-}\mathit{integer} \ (\cdot \underline{\quad \cdot } \mathit{binary}\text{-}\mathit{digit})? \ (\cdot (\underline{\quad } \underline{\quad } 
\mathbf{decimal-number} ::= \mathit{decimal-integer} \ (\cdot \underline{\quad} \cdot \mathit{decimal-digit})? \ (\cdot (\boxed{\mathbb{E}} \mid \boxed{\mathbb{P}}) \cdot \mathit{exponent})?
\mathbf{octal\text{-}number} ::= \mathit{octal\text{-}integer} \ (\cdot \ \  \cdot \  \cdot \  \  \, \mathit{octal\text{-}digit})? \ (\cdot (\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ )?
exponent ::= ( - | + )? ·any-integer
 any-integer ::= hex-integer \mid hex-integer \mid binary-integer \mid decimal-integer \mid octal-integer
\mathbf{hex\text{-}integer} ::= \boxed{\texttt{OX}} \cdot (hex\text{-}digit)^*
binary-integer ::= \boxed{0B} \cdot (\boxed{0} \mid \boxed{1})^*
decimal-integer ::= \boxed{1} \boxed{9} \cdot (digit)^*
\mathbf{octal\text{-}integer} ::= \boxed{0} \cdot (\boxed{0} - \boxed{7})^*
\mathbf{hex}-\mathbf{digit} ::= A F | digit
escape ::= \" | \" | \' | \n | \r | \t | \\
```

H. GRAMMAR 72

 $\mathbf{letter} ::= \boxed{ \texttt{A-Z} }$

digit ::= 0-9

 $\mathbf{ANY} ::= Any \ character \ other \ than \ \boxed{\ }$