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Discrete Math

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HW 7

Problem Set:
Section 4.1: 4, 10, 32, 44
Section 4.2: 4, 8, 14
Section 4.3: 4, 8, 24, 44

Section 4.3

4) a) $39 = 3 \cdot 13$

b) $81 = 3 \cdot 27$
$= 3 \cdot 3 \cdot 9$
$= 3 \cdot 3 \cdot 3 \cdot 3$
$81 = 3^4$

c) $101 = 101$

d) $1001 = 7 \cdot 11 \cdot 13$

e) $289 = 17 \cdot 17$
$289 = 17^2$

f) $899 = 29 \cdot 31$

10) If $2^m + 1$ is odd prime

$m = 2^n$; non-negative n

$x^m + 1 = (x^n + 1)(x^{n(t-1)} - x^{n(t-2)} +$
$\ldots - x^n + 1)$; $m = nt$ t is odd

32) a) gcd(12,18)

$18 = 12 \cdot 1 + 6$

$12 = 6 \cdot 2$

gcd(12,18) = 2

b) gcd(100,101)

$101 = 100 \cdot 1 + 1$

$100 = 1 \cdot 100$

gcd(100,101) = 100

c) gcd(123, 277)

$277 = 123 \cdot 2 + 31$

$123 = 31 \cdot 3 + 30$

$31 = 30 \cdot 1 + 1$

$30 = 1 \cdot 30$

gcd(123, 277) = 30

d) gcd(1529, 14039)

$14039 = 1529 \cdot 9 + 278$

$1529 = 278 \cdot 5 + 139$

$278 = 139 \cdot 2$

gcd(1529, 14039) = 139

e) gcd(1529, 14038)

$14038 = 1529 \cdot 9 + 277$

$1529 = 277 \cdot 5 + 144$

$277 = 144 \cdot 1 + 133$

$144 = 133 \cdot 1 + 11$

$133 = 11 \cdot 12 + 1$

$11 = 1 \cdot 10 + 1$
$1 = 1 \cdot 1$
$gcd(1529, 14038) = 1$

f) $gcd(11111, 111111)$
$111111 = 11111 \cdot 10 + 1$
$11111 = 1 \cdot 11111$
$gcd(11111, 111111) = 11111$

44)

Section 4.2

4) a) $11011_2 = 1 \cdot 2^0 + 1 \cdot 2^1 + 1 \cdot 2^3 + 1 \cdot 2^4$
$27_{10} = 7 \cdot 10^0 + 2 \cdot 10^1$

b) $10\ 1011\ 0101_2 = 1 \cdot 2^0 + 1 \cdot 2^2 + 1 \cdot 2^4 + 1 \cdot 2^5 + 1 \cdot 2^7 + 1 \cdot 2^9$
$691_{10} = 1 \cdot 10^0 + 9 \cdot 10^1 + 6 \cdot 10^2$

c) $11\ 1011\ 1110_2 = 1 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 1 \cdot 2^5 + 1 \cdot 2^7 + 1 \cdot 2^8 + 1 \cdot 2^9$
$958_{10} = 8 \cdot 10^0 + 5 \cdot 10^1 + 9 \cdot 10^2$

d) $111\ 1100\ 0001\ 1111 = 1 \cdot 2^0 + 1 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 1 \cdot 2^{10} + 1 \cdot 2^{11} + 1 \cdot 2^{12} + 1 \cdot 2^{13} + 1 \cdot 2^{14}$
$31775_{10} = 5 \cdot 10^0 + 7 \cdot 10^1 + 7 \cdot 10^2 + 1 \cdot 10^3 + 3 \cdot 10^4$

8) BADFACED$_{16}$ to binary

31 30 29 28   27 26 25 24   23 22 21 20   19 18 17 16   15 14 13 12   11 10 9 8   7 6 5 4   3 2 1 0

$1011 \ 1010 \ 1101 \ 1111 \ 1010 \ 1100 \ 1110 \ 1101_2$

$= 1\cdot 2^0 + 1\cdot 2^2 + 1\cdot 2^3 + 1\cdot 2^5 + 1\cdot 2^6 + 1\cdot 2^7 + 1\cdot 2^{10} + 1\cdot 2^{11} +$
$1\cdot 2^{13} + 1\cdot 2^{15} + 1\cdot 2^{16} + 1\cdot 2^{17} + 1\cdot 2^{18} + 1\cdot 2^{19} + 1\cdot 2^{20} +$
$1\cdot 2^{22} + 1\cdot 2^{23} + 1\cdot 2^{25} + 1\cdot 2^{27} + 1\cdot 2^{28} + 1\cdot 2^{29} + 1\cdot 2^{31}$

T4)

Section 4.1

4) $p =$

8) This is false. For example

$6 \mid 12$ is true

$6 \mid 3\cdot 4$ is true

but $6 \nmid 3$ and $6 \nmid 4$

32) a) $(19^2 \bmod 41) \bmod 9$

$\boxed{33 \bmod 9 = 6}$

b) $(32^3 \bmod 13)^2 \bmod 11$

$\boxed{100 \bmod 11 = 1}$

c) $(7^3 \bmod 23)^2 \bmod 31$

$\boxed{441 \bmod 31 = 7}$

d) $(21^2 \bmod 15)^3 \bmod 22$

$\boxed{216 \bmod 22 = 18}$

44) $a, b, c \in \mathbb{Z}_m$

$$a(b+c) \equiv a(b+c)$$
$$\equiv a(b+c)$$
$$\equiv (ab+ac)$$
$$\equiv (ab)+(ac)$$
$$\equiv (a)(b)+(a)(c)$$

Since distributive property holds over addition.