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Discrete Math: Homework 2
ed. 7

Problem Set:
Section 1.5: 2, 4, 8
Section 1.6: 2, 4, 10
Section 1.7: 2, 4, 10, 18, 20

Section 1.5:

2)

a) $\exists x \forall y (xy = y)$

>>>> There is a real number x such that for every real number y, $(xy = y)$.

b) $\forall x \forall y (((x \geq 0) \wedge (y < 0)) \rightarrow (x - y > 0))$

>>>> For all real numbers x, for all real numbers y, $(((x \geq 0) \wedge (y < 0)) \rightarrow (x - y > 0))$

c) $\forall x \forall y \exists z (x = y + z)$

>>>> For all real numbers x and for all real numbers y there is a real number z such that $x = y + z$

4) next page

a) $\exists x \exists y P(x, y)$

there exist a real number ~~such~~ x ~~such~~ that a real number y

[x] consists of all students in class
[y] consists of all CS courses

4) a) $\exists x \exists y \, P(x,y)$

There exists a Student x, such that has taken a CS class

b) $\exists x \forall y \, Q(x,y)$

There exists a student x, such that they've taken all CS classes

c) $\forall x \exists y \, P(x,y)$

For all students x, there exists a CS at the school

d) $\exists y \forall x \, P(x,y)$

There exists a CS class y, such that All Students have taken class y.

e) $\forall y \exists x \, P(x,y)$

For all classes y, there exists a student x

f) $\forall x \forall y \, P(x,y)$

For all students x, they've taken All classes y

8) $Q(x, y)$ "Student x has been a contestant on quiz show y"

a) $\exists x \forall y Q(x, y)$

b) $\neg \exists x \forall y Q(x, y)$

c) $\exists x \exists y_{Jeopardy} \exists y_{wheel\ of\ fortune} Q(x, y_{Jeopardy}, y_{wheel\ of\ fortune})$

d) $\forall y \forall x Q(y, x)$

e) $\exists x \exists y_{Jeopardy} Q(x, y_{Jeopardy})$

Section 1.6

2) If, George does not have 8 legs = p
Then, he is not a spider = q
George is a spider = $\neg q$

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

* Modus Tollens (pg. 72)

We can conclude George has 8 legs, if the premises are true.

4)
a) Simplification
b) Disjunctive Syllogism
c) Modus Ponens
d) Addition
e) Hypothetical Syllogism

10)
a) If play hockey, then sore ($p \to q$)

~~use whirlpool~~

if sore, then use whirlpool ($q \to r$)

∴ You did not play hockey
Hypothetical Syllogism

b) If work, then sunny or partly sunny ($p \to q \vee r$)

worked last monday or last friday

∴ I worked last Monday
Modus tollens

c) ∴ Spiders are not insects
Contradiction

d) ∴ Homer is not a student and Maggie is.
modus ponens
modus tollens

e) $\boxed{\text{∴ cheeseburgers taste good} \quad \text{Modus tollens}}$

f) $\boxed{\text{∴ You see elephants running} \quad \text{Modus tollens} \quad \text{Modus Ponens}}$

Section 1.7

2) $\cancel{2k+2k=4k}$

If ~~x~~ is even and ~~y~~ is even, then $x+y=$ even

$P(x)$ $\quad P(y)$ $\quad Q(n)$ [written above a struck-out $x+y$]

$$\forall x \forall y (P(x) \land P(y) \rightarrow Q(n))$$ [with n written above a struck-out $x+y$]

$Q(n)$ is $x+y=$ even

$P(x)$ is x is even

$P(y)$ is y is even

$n = x+y$ where x is an even integer
and where y is an even integer

4) x is an even integer

$x = 2n$ where n is an integer

modus tollens ↓

$-x =$ add inverse of x

$\therefore -x = -2n$ or $2(-n)$

$q = -n$ where q is an integer

$-x = 2q$

$-x$ is even by definition

10) X and Y are both rational numbers

$X = \frac{a}{b}$ $Y = \frac{c}{d}$ by definition of rational

a, b, c, and d are integers and b and d do not equal zero

$XY = \left(\frac{a}{b}\right)\left(\frac{c}{d}\right)$ $XY = \left(\frac{ab}{cd}\right)$

ab and cd are integers

XY can be written as a quotient therefore a rational number