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Discrete Math: HW 3

Problem set:

Section 1.8: 2, 8, 12, 22, 28, 36, 42
Section 2.1: 2, 6, 10, 18, 20, 22, 24, 38
Section 2.2: 2, 4, 12, 16, 18, 30, 36

Section 1.8:

2) $1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$
$8^3 = 512$
$9^3 = 729$
$10^3 = 1000$

Case (i): 1 can't be sum
Case (ii): $1+1 \neq 8$
Case (iii): $1+1$, $1+8$, $8+8 \neq 27$
Case (iv): $1+1$, $1+8$, $1+27$, $8+8$, $8+27$, $27+27 \neq 64$
Case (v): $1+1$, $1+8$, $1+27$, $8+8$, $8+27$, $1+64$, $8+64$, $27+64$, $64+64 \neq 125$

⋮

Case (ix): None of the previous sums add up to 729

∴ There are no positive perfect cubes less than 1000, that are the sum of the cube of two positive integers.

8) That integer is 1. Sum of set $A = \{1\}$ is 1. This proof was done via construction.

12)

Let $65^{1000} - 8^{2001} + 3^{177} = A$

[illegible]

22) given that

x is nonzero real $\left(\frac{x-1}{x}\right)^2 \geq 0$

Let x is nonzero real $= P$

Let $\left(\frac{x-1}{x}\right)^2 \geq 0 = Q$

Let $\left(\frac{x^2+1}{x^2}\right) \geq 2$

$P \to Q$ is true

$\left(\frac{x-1}{x}\right)^2 \geq 0$ for $x \neq 0$

$\left(\frac{x^2+1}{x^2}\right) \geq 2$ for -1 to 1

28)

$0^2 = 00$
$1^2 = 01$
$2^2 = 04$
$3^2 = 09$
$4^2 = 16$
$5^2 = 32$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$
$11^2 = 121$
$12^2 = 144$
$13^2 = 169$
$14^2 = 196$
$15^2 = 225$

Conjecture: The final digit of the square of an integer will be 1, 4, 9, 6, 5, or 0.

Case (i): The final digit of n is 1 or 9, the final digit of n^2 is the final digit of 1^2 or 9^2, namely 1.

Case (ii): The final digit of n is 2 or 8. The final digit of n^2 is the final digit of 2^2 or 8^2, namely 4.

Case (iii): The final digit of n is 3 or 7, the final digit of n^2 is the final digit of 3^2 or 7^2, namely 9.

Case (iv): The final digit of n is 4 or 6. The final digit of n^2 is the final digit of 4^2 or 6^2, namely 6.

Case (v): The final digit of n^2 is 5. The final digit of n^2 is the final digit of 5^2, namely 5.

Case (vi): The final digit of n^2 is 0. The final digit of 0^2 is the final digit of 0^2, namely 0.

36) Let $\frac{a}{b}$ be some rational number

Let $\frac{\sqrt{7}+a}{b}$ be some irrational number

Let $\frac{a}{b} < \frac{\sqrt{7}+a}{b}$

Let $\frac{\frac{1}{7}\sqrt{7}+a}{b}$ be some irrational number

$$\therefore \frac{a}{b} < \frac{\frac{1}{7}\sqrt{7}+a}{b} < \frac{\sqrt{7}+a}{b}$$

42) Checkerboard:

8x8 rect. w/ 64 squares

all four corners removed

60 spaces

c) Spaces covered $= 2k$

k = The number of dominoes

If 64 spaces covered $= 2k$

$k = 32$, Tiles evenly

(ii) If 63 spaces covered $= 2k$

$k = 31.5$, Does not tile

(iii) If 60 spaces covered $= 2k$

$k = 30$ Dominoes, Tiles evenly

Section 2.1:

2)
a) $P = \{x \mid 0, 3, 6, 9, 12\}$
b) $Q = \{x \mid -3, -2, -1, 0, 1, 2, 3\}$
c) $R = \{x \mid m, n, o, p\}$

6) $A = \{2, 4, 6\}$ $B = \{2, 6\}$ $C = \{4, 6\}$ $D = \{4, 6, 8\}$

10)
a) True
b) True
c) False
d) False
e) False
f) False
g) True

18)

$A = \{4, 5, 6\}$

$B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$\therefore A \subseteq B$

20) a) zero
b) zero / one
c) Two
d) Four

22) Yes, if $P(A) = \{\emptyset, \{x\}, \{y\}, \{x, y\}\}$

and $P(B) = \{\emptyset, \{x\}, \{y\}, \{x, y\}\}$

Then Set A = Set B

24) a) Not a power set
b) not a power set
c) Not a power set
d) A power set

38) $A \times B \neq B \times A$, unless $A = B$

$A = \{1, 2, 3\}$
$B = \{4, 5, 6\}$

$A \times B = \{(1,4), (2,4), (1,5), (2,5)\}$

$B \times A = \{(4,1), (5,1), (4,2), (5,1), (5,2)\}$

or

$A = \{1, 2\}$
$B = \{1, 2\}$

$A \times B = \{(1,2), (1,1), (1,2), (1,1)\}$

$B \times A = \{(1,2), (1,1), (1,2), (1,1)\}$

Section 2.2

2) a) $A \cup B$
b) $A \cup \overline{B}$
c) $A \cap B$
d) $\overline{A} \cap \overline{B}$

4) a) $A \cup B = \{a, b, c, d, e, f, g, h\}$
b) $A \cap B = \{a, b, c, d, e\}$
c)

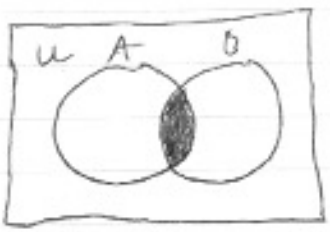
12)

A	B	$A \cap B$	$A \cup (A \cap B)$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

Logically equivalent

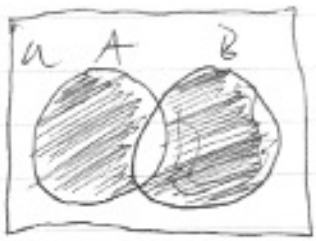
$A - B = A \cap \bar{B}$
$B \cap \bar{A}$

16) a)



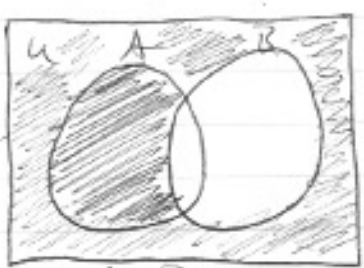
$(A \cap B) \subseteq A$

b)



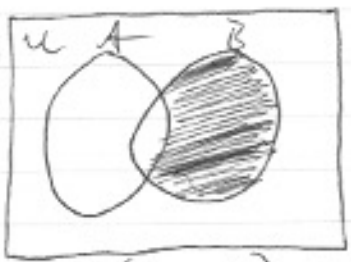
$A \subseteq (A \cup B)$

c)



$A - B \subseteq A$

d)



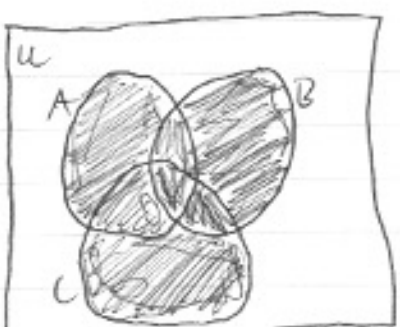
$A \cap (B - A) = \emptyset$

e)

A	B	$\bar{B}$	$\bar{A}$	$B \cap \bar{A}$ (B−A)	$A \cup (B \cap \bar{A})$	$A \cup B$
0	0	1	1	0	0	0
0	1	0	1	1	1	1
1	0	1	0	0	1	1
1	1	0	0	0	1	1

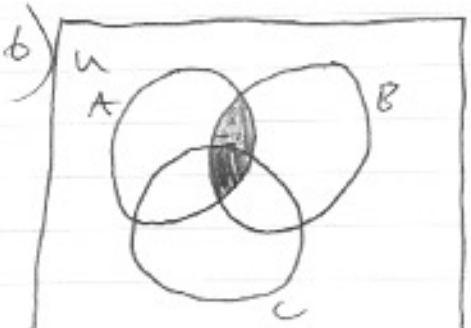
Logically equivalent

18) a)



$(A \cup B) \subseteq (A \cup B \cup C)$

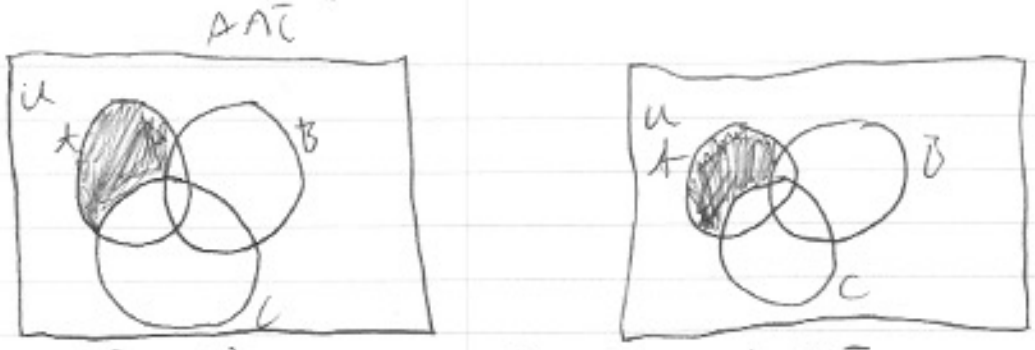
b)



$(A \cap B \cap C) \subseteq (A \cap B)$

$(A-B)-C = (A \cap \bar{B}) \cap \bar{C}$

c)



$(A \cap \bar{B}) \cap \bar{C}$ $\subseteq$ $A \cap \bar{C}$

d)

A-C = $A \cap \bar{C}$

B-B = $C \cap \bar{B}$

A	B	C	∅	$\bar{B}$	$\bar{C}$	$A \cap \bar{C}$	$C \cap \bar{B}$	$(A \cap \bar{C}) \cap (C \cap \bar{B})$
0	0	0	0	1	1	0	0	0
0	0	1	0	1	0	0	1	0
0	1	0	0	0	1	0	0	0
0	1	1	0	0	0	0	0	0
1	0	0	0	1	1	1	0	0
1	0	1	0	1	0	1	1	1
1	1	0	0	0	1	1	0	0
1	1	1	0	0	0	1	1	1

e)

B-A = $B \cap \bar{A}$

C-A = $C \cap \bar{A}$

(B∪C)-A = $(B \cup C) \cap \bar{A}$

A	B	C	$\bar{A}$	$B \cap \bar{A}$	$C \cap \bar{A}$	$(B \cap \bar{A}) \cup (C \cap \bar{A})$	B∪C	$(B \cup C) \cap \bar{A}$
0	0	0	1	0	0	0	0	0
0	0	1	1	0	1	1	1	1
0	1	0	1	1	0	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0	0
1	0	1	0	0	1	1	1	0
1	1	0	0	1	0	1	1	0
1	1	1	0	1	1	1	1	0

not equal

30)

a) ~~No~~ Yes
b) No
c) No

36)

A	B	$\bar{A}$	$\bar{B}$	$A \cap \bar{B}$	$B \cap \bar{A}$	$(A \cap \bar{B}) \cup (B \cap \bar{A})$	$A \oplus B$
0	0	1	1	0	0	0	0
0	1	1	0	0	1	1	1
1	0	0	1	1	0	1	1
1	1	0	0	0	0	0	0

$A - B =$
$A \cap B$
$B - A =$
$B \cap \bar{A}$

Logically equivalent