Eli Sobylak
Discrete Math

1-Mar-15
$\boxed{WS:10}$

WS 10
7.2
1) a)

$a_n = a_{n-1} + 6a_{n-2}$ for $n \geq 2$
$a_0 = 3$
$a_1 = 6$

LHE:

$a_n = C_1 a_{n-1} + C_2 a_{n-2}$
$\underbrace{C_1 = 1 \qquad C_2 = 6}_{\text{constants}}$

$a_n = r^n \qquad r^n = C_1 r^{n-1} + C_2 r^{n-2}$

$\frac{r^n}{r^{n-k}} = \frac{r^{n-1}}{r^{n-k}} + \frac{6r^{n-2}}{r^{n-k}}$

$r^k = r^{k-1} + 6r^{k-2}$
$r^2 = r^1 + 6r^0$
$r^2 - r - 6 = 0$
$(r+2)(r-3) = 0$
$\underbrace{r_1 = -2 \qquad r_2 = 3}_{\text{roots}}$

$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$
$= \alpha_1(-2)^n + \alpha_2(3)^n$

$a_0 = 3 = \alpha_1(-2)^0 + \alpha_2(3)^0$
$3 = \alpha_1 + \alpha_2$
$-\alpha_1 + 3 = \alpha_2$
$\cancel{\alpha_1 - 3 = -(\alpha_2)}$

$a_1 = 6 = \alpha_1(-2)^1 + \alpha_2(3)^1$
$6 = \alpha_1(-2)^1 + \alpha_2(3)^1$
$6 = \alpha_1(-2)^1 + -\alpha_1 + 3(3)^1$

$6 = 3\alpha_1 + (3 - \alpha_1)(-2)$
$\frac{12}{5} = \alpha_1 \qquad \frac{3}{5} = \alpha_2$

8.2 S: 10

6) $a_n = 7a_{n-1} - 10a_{n-2}$ for $n \geq 2$

$a_0 = 2$
$a_1 = 1$

LHE: $a_n = c_1 a_{n-1} + c_2 a_{n-2}$

$\underbrace{c_1 = 7 \qquad c_2 = -10}$

$a_n = r^n$ $\qquad r^n = c_1 r^{n-1} + c_2 r^{n-2}$

$$\frac{r^n}{r^{n-k}} = \frac{7r^{n-1}}{r^{n-k}} + \frac{(-10)r^{n-2}}{r^{n-k}}$$

$$r^k = 7r^{k-1} + (-10)r^{k-2}$$

$$r^2 = 7r^1 + (-10)r^0$$

$$r^2 = 7r - 10$$

$$r^2 - 7r + 10 = 0$$

$$(r-2)(r-5) = 0$$

$$\underbrace{r_1 = 2 \qquad r_2 = 5}$$

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$$
$$= \alpha_1 (2)^n + \alpha_2 (5)^n$$

$a_0 = 2 = \alpha_1 (2)^0 + \alpha_2 (5)^0$

$2 = \alpha_1 + \alpha_2$

$\cancel{\alpha_1 = 2 - \alpha_2}$

$\cancel{\alpha_1 = \frac{2}{\alpha_2}}$

$\alpha_1 = 2 - \alpha_2$

$\alpha_1 = 2 - (-1)$

$\underbrace{\alpha_1 = 3}$

$a_1 = 1 = \alpha_1 (2)^1 + \alpha_2 (5)^1$

$1 = 2\alpha_1 + 5\alpha_2$

$\cancel{1 = 2(\frac{2}{\alpha_2}) + 5\alpha_2}$

$1 = 2(2 - \alpha_2) + 5\alpha_2$

$1 = 4 - 2\alpha_2 + 5\alpha_2$

$1 = 4 + 3\alpha_2$
$-4 \quad -4$

$\frac{-3}{3} = \frac{3\alpha_2}{3}$ $\qquad \underbrace{-1 = \alpha_2}$

$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$

$a_n = 3 \cdot 2^n + (-1)(5)^n$

$a_n = 3 \cdot 2^n - 5^n$

$a_0 = 2$
$a_1 = 1$

$a_2 = 3 \cdot 2^2 - 5^2$

$a_2 = 12 - 25$

$a_2 = -13$

2. $L_n = (1/2)L_{n-1} + (1/2)L_{n-2}$

a) $L_n = \frac{L_{n-1}}{2} + \frac{L_{n-2}}{2}$

it is just the average of the last two years

$a_n = c_1 a_{n-1} + c_2 a_{n-2}$

$a_n = (1/2)L_{n-1} + (1/2)L_{n-2}$

$a_n = r^n$

$\frac{r^n}{r^{n-k}} = \frac{(1/2)r^{n-1}}{r^{n-k}} + \frac{(1/2)r^{n-2}}{r^{n-k}}$

$r^k = 1/2 r^{k-1} + 1/2 r^{k-2}$

$r^2 = 1/2 r^1 + 1/2 r^0$

$r^2 = 1/2 r + 1/2$

$r^2 - 1/2 r - 1/2 = 0$

$(r - 1/2)(r+1) = 0$

$r_1 = -1/2 \quad r_2 = +1$

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$$
$$a_n = \alpha_1(-1/2)^n + \alpha_2(1)^n$$

$a_0 = 100{,}000 = \alpha_1(-1/2)^0 + \alpha_2(1)^0$

$100{,}000 = \alpha_1 + \alpha_2$ $\qquad \alpha_1 = 100{,}000 - 700{,}000$

$\alpha_1 = 100{,}000 - \alpha_2$ $\qquad \alpha_1 = -600{,}000$

$a_1 = 300{,}000 = \alpha_1(-1/2)^1 + \alpha_2(1)^1$

$300{,}000 = -\frac{1}{2}\alpha_1 + \alpha_2$

$300{,}000 = -\frac{1}{2}(100{,}000 - \alpha_2) + \alpha_2$

$300{,}000 = -50{,}000 + \frac{1}{2}\alpha_2$

$+50{,}000 \qquad +50{,}000$

$2(350{,}000) = (1/2\,\alpha_2)2$

$700{,}000 = \alpha_2$

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$$
$$a_n = -600{,}000(-1/2)^n + 700{,}000^n$$
$$a_n = -600{,}000(-1/2)^n + 700{,}000^n$$

7.3

1) The relation is

$a(n) = 2a(n/2) + 2$

$a(1) = 0$

$a(128) = 2a(64) + 2 = \boxed{254}$

$a(64) = 2a(32) + 2 = 126$

$a(32) = 2a(16) + 2 = 62$

$a(16) = 2a(8) + 2 = 30$

$a(8) = 2a(4) + 2 = 14$

$a(4) = 2a(2) + 2 = 6$

$a(2) = 2a(1) + 2 = 2$

$= 2(0) + 2 = 2$

2)

a) Suppose $f(n) = 2f(n/2) + 3$
when n is even and $f(1) = 1$

$$f(2) = 2f(1) + 3$$
$$= 2(1) + 3$$
$$= \boxed{5}$$

$f(8) = 2f(4) + 3 = \boxed{29}$

$$f(4) = 2f(2) + 3$$
$$= 2(5) + 3 = 13$$

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