Assignment 1

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1 Introduction

This assignment is based on a study of the biofuels supply chain that was made in Greece by Papapostolou et al. during 2006-2010. The problem we have been given is to optimize the profit from producing biofuels given data and constraints which will be stated later.

We were given data from three different seeds which displayed what they can produce as well as their demands. We were also given the contents of the different biofuels and their respective taxes.

Crop	Yield [t/ha]	Water demand [Ml/ha]	Oil content [l/kg]
Soybeans	2.6	5.0	0.178
Sunflower seeds	1.4	4.2	0.216
Cotton seeds	0.9	1.0	0.433

Table 1: Crops data

Product	Biodiesel [%]	Price [€/l]	Tax [%]
B5	5	1.43	20
B30	30	1.29	5
B100	100	1.16	0

Table 2: Products data

2 Model

Notation used in this assignment:

- B_i, $i \in I$ where I is the set of different biofuels (B5, B30, B100).
- X_j, $j \in J$ where J is the set of different seeds (soybeans, sunflower, cotton).
- $P_{i,n}$, $n \in [1, 2, 3]$ which contain the data from the products.

- $C_{j,k}$, $k \in [1,2,3]$ which contain the data from the crops
- p, which is the amount of petrol used. The cost for petrol is 1 €/l.
- m, which is the amount of methanol used. The cost for methanol is 1.5 €/l.
- $VegOil$, which is the amount of vegetable oil used.
- $UnrefBiodiesel$, which is the amount of biodiesel used before it is mixed with p,, i.e petrol diesel.

The function we wish to maximize is

$$max \quad z = \quad \sum_{i \in I} B_i P_{j,2}(1 - P_{j,3}) \quad - p - 1.5m. \tag{1}$$

The explanation to this model can be given as a simple $revenue - cost$ situation. Our revenue is the amount of biofuel we manage to sell, minus the taxes. Our costs are the expenses for creating the biofuel, which is the petrol and methanol. The difference is our profit which is what we want to maximize.

To make the model easier to understand we introduce four new functions, $p(B_1, B_2, B_3)$, $UnrefBiodiesel(B_1, B_2, B_3)$, $VegOil(X_1, X_2, X_3)$, $m(B_1, B_2, B_3)$.

$$\begin{aligned}
p(B_1, B_2, B_3) &= \sum_{i=1}^{3} B_i(1 - P_{i,1}) \\
UnrefBiodiesel(B_1, B_2, B_3) &= \sum_{i=1}^{3} B_i(P_{i,1}) \\
VegOil(X_1, X_2, X_3) &= 1\,000 \sum_{j=1}^{3} X_j C_{j,1} C_{j,3} \\
m(B_1, B_2, B_3) &= \frac{4}{3} \cdot \frac{1}{6} UnrefBiodiesel
\end{aligned}$$

Our model is subjected to:

$$
\begin{aligned}
\sum_{j=1}^{3} X_j &= 1\,600 \\
\sum_{j=1}^{3} X_j C_{j,2} &\leq 5\,000 \\
\sum_{i=1}^{3} B_i &\geq 280\,000 \\
p &\leq 150\,000 \\
VegOil &= \frac{4}{3} \cdot \frac{5}{6} UnrefBiodiesel \\
B_i,\ X_j &\geq 0 \,\forall\, i, j
\end{aligned}
$$

Lägg till appendix med modellen utan de tillagda hjälpfunktionerna.

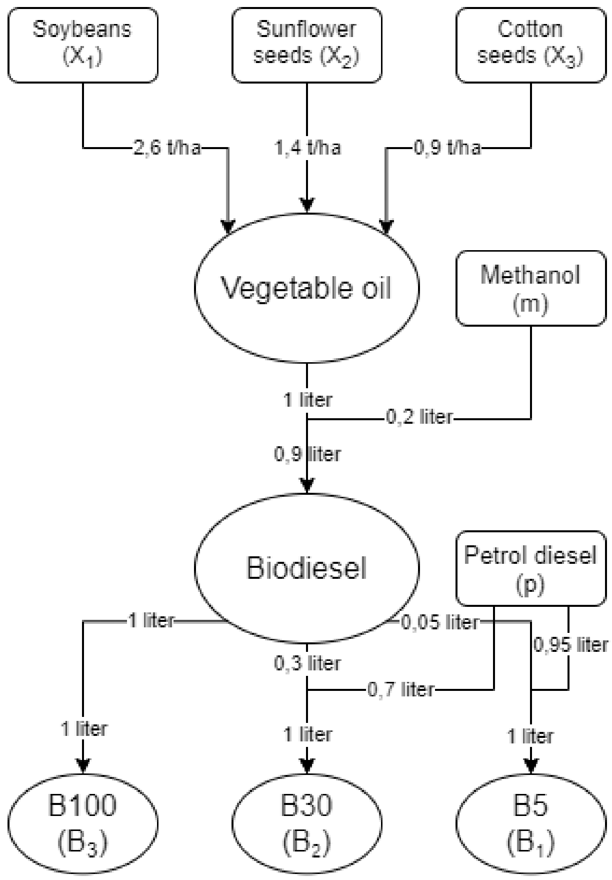

Figure 1: WRITE SOMETHING HERE.

3 Results

The maximization of model 1 gives an optimal solution of $z = 548\ 163$€ when we sold 767 089.5 liter biofuel with a combined value of 903 859€. We where also require to buy methanol and petrol diesel for a combined value of 355 697€. The value of the optimal solution sounds plausible, because the income minus the expenses sums up to 548 162€ with a small rounding error. Not only that, but all the variables and constants where inside the set parameters. Table 3 shows the optimal value for all the variables and constants.

Name of var/cons	Value of var/cons
z ($Objective function$)	548 163 €
B5 (B_1)	0 liters
B30 (B_2)	214 286 liters
B100 (B_3)	552 804 liters
Methanol (m)	137 131 liters
Petrol diesel (p)	150 000 liters
Biodiesel ($--$)	617 090 liters
VegOil ($--$)	685 655 liters
Water in use ($--$)	5 000 liters
Area in use ($--$)	1 600 hectare
Soybeans (X_1)	850 hectare
Sunflower seeds (X_2)	0 hectare
Cotton seeds (X_3)	750 hectare

Table 3: This table shows the value for all the the variables and constants in the optimal solutions to the problem stated above.

4 Sensitivity analysis

4.1 Petrol, area and water availability

The area could be reduced to 313 hectare and still meet the requirements for the problem. What we noticed when reducing the area availability is that more and more area was given to soybeans which has the best yield. We also saw that since we did not change the amount of petrol, the computer always wanted to make as much $B30$ as possible since it would be considered as waist otherwise. It was a significant drop in the objective value, which was expected since we could not produce as much oil as the original problem.

What we saw when changing the water supply is that we could go as low as having 371 Ml water while still staying feasible. When reducing water supply, more and more area was given to the cotton seeds since that had the lowest water demand

4.2 Marginal increments in petrol, area and water

4.3 What needs to change to make sunflower seeds non-zero

4.4 How does the profit change if the price of petrol is increased to 1.2 €/l and what is the optimal solution then

4.5 Vary the taxes

4.6 Vary the water demand of the seeds

If we vary the water demand of the crops with a variable $\delta \in [-0.4, 0.4]$ we can plot the results in a coordinate system, 2, with the optimal value z as y-axis and δ as the x-axis. Negative δ means that it is rainy and the crops need less water and vice versa.

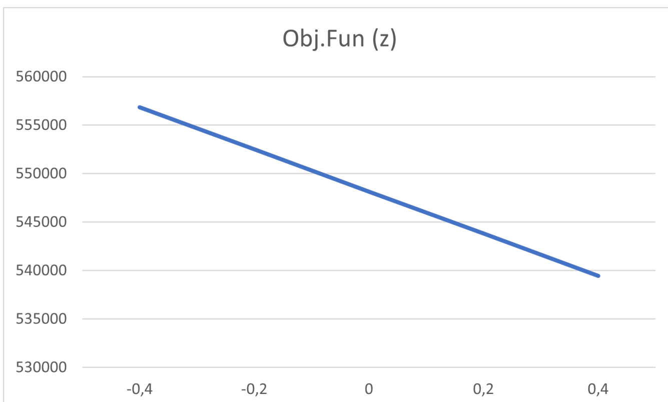

Figure 2: WRITE SOMETHING HERE.