Assignment 1

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1 Introduction

This assignment is based on a study of the biofuels supply chain that was made in Greece by Papapostolou et al. during 2006-2010. The problem we have been given is to optimize the profit from producing biofuels given data and constraints which will be stated later.

We were given data from three different seeds which displayed what they can produce as well as their demands. We were also given the contents of the different biofuels and their respective taxes.

Crop	Yield [t/ha]	Water demand [Ml/ha]	Oil content [l/kg]
Soybeans	2.6	5.0	0.178
Sunflower seeds	1.4	4.2	0.216
Cotton seeds	0.9	1.0	0.433

Table 1: Crops data

Product	Biodiesel [%]	Price [€/l]	Tax [%]
B5	5	1.43	20
B30	30	1.29	5
B100	100	1.16	0

Table 2: Products data

2 Model

Notation used in this assignment:

- B_i, $i \in I$ where I is the set of different biofuels (B5, B30, B100).
- X_j, $j \in J$ where J is the set of different seeds (soybeans, sunflower, cotton).
- $P_{i,n}$, $n \in [1, 2, 3]$ which contain the data from the products.

- $C_{j,k}$, $k \in [1, 2, 3]$ which contain the data from the crops
- p, which is the amount of petrol used. The cost for petrol is 1 €/l.
- m, which is the amount of methanol used. The cost for methanol is 1.5 €/l.
- $VegOil$, which is the amount of vegetable oil used.
- $UnrefBiodiesel$, which is the amount of biodiesel used before it is mixed with p,, i.e petrol diesel.

The function we wish to maximize is

$$max \quad z = \sum_{i \in I} B_i P_{j,2}(1 - P_{j,3}) \quad - p - 1.5m. \tag{1}$$

The explanation to this model can be given as a simple $revenue - cost$ situation. Our revenue is the amount of biofuel we manage to sell, minus the taxes. Our costs are the expenses for creating the biofuel, which is the petrol and methanol. The difference is our profit which is what we want to maximize.

To make the model easier to understand we introduce four new functions, $p(B_1, B_2, B_3)$, $UnrefBiodiesel(B_1, B_2, B_3)$, $VegOil(X_1, X_2, X_3)$, $m(B_1, B_2, B_3)$.

$$\begin{aligned} p(B_1, B_2, B_3) &= \sum_{i=1}^{3} B_i(1 - P_{i,1}) \\ UnrefBiodiesel(B_1, B_2, B_3) &= \sum_{i=1}^{3} B_i(P_{i,1}) \\ VegOil(X_1, X_2, X_3) &= 1\,000 \sum_{j=1}^{3} X_j C_{j,1} C_{j,3} \\ m(B_1, B_2, B_3) &= \frac{4}{3} \cdot \frac{1}{6} UnrefBiodiesel \end{aligned}$$

Our model is subjected to:

$$\sum_{j=1}^{3} X_j \leq 1\,600$$
$$\sum_{j=1}^{3} X_j C_{j,2} \leq 5\,000$$
$$\sum_{i=1}^{3} B_i \geq 280\,000$$
$$p \leq 150\,000$$
$$VegOil = \frac{4}{3} \cdot \frac{5}{6} UnrefBiodiesel$$
$$B_i,\ X_j \geq 0\,\forall\, i, j$$

In the model we are using four functions and six constraint. Firstly, the function, $p(B_1, B_2, B_3)$, is the amount of petrol used, as stated above. Secondly, we have the function $UnrefBiodiesel(B_1, B_2, B_3)$ witch is the amount of unrefined biodiesel, i.e. the biodiesel before it is mixed with petrol diesel. Thirdly, the function $VegOil(X_1, X_2, X_3)$ witch is the amount of vegetable oil, as stated above. And lastly, the function $m(B_1, B_2, B_3)$ is the amount of methanol used as stated above, but the calculation is more complicated compared to the functions before it. With some math, leaving this to the reader, we can conclude that unrefined biodiesel has $\frac{1}{6}$ methanol and $\frac{5}{6}$ vegetable oil, this is important later. We can also conclude with some math, also leaving this to the reader, that we will lose $\frac{3}{4}$ of methanol and vegetable oil when mixing them together. So when we want to know how much methanol and vegetable oil we are using we need to multiply the unrefined biodiesel with $\frac{4}{3}$ to compensate for the loss in material.

The constrains are somewhat straight forward. Firstly we have that we can only use a maximum of 1 600 hectare, secondly that we can not use more than 5 000 Ml water. Thirdly we need to sell more than 280 000 liters of biofuel and fourthly that we can not buy more than 150 000 petrol diesel. Then fifthly that is a bit more complicated, but with the same explanation as for the function $m(B_1, B_2, B_3)$ the reader should be able to figure this out. Lastly we have the non-negative constraint.

How the product flows can be seen in figure 1.

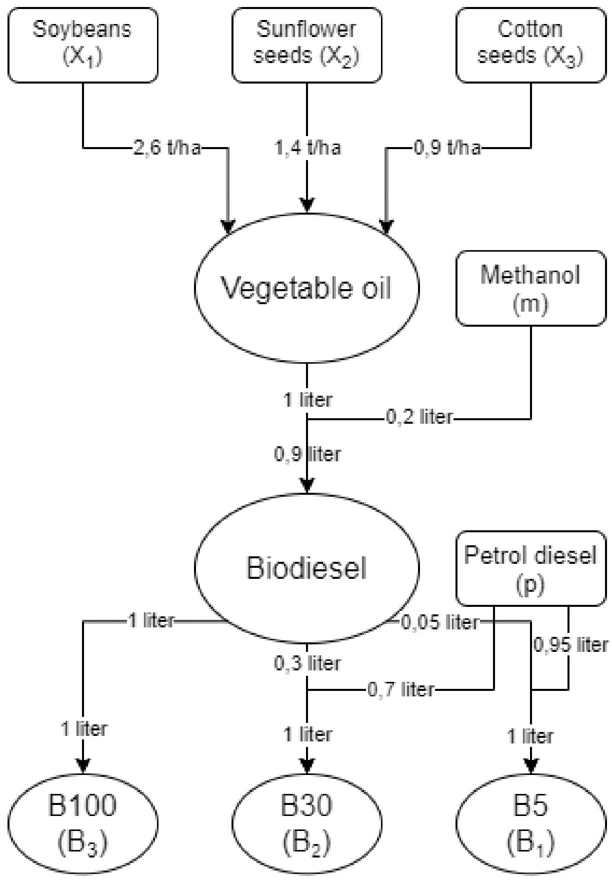

Figure 1: Here is the product flow over the optimization problem.

3 Results

The maximization of model 1 gives an optimal solution of $z = 548\ 163$€ when we sold 767 090 liter biofuel with a combined value of 903 859€. We where also require to buy methanol and petrol diesel for a combined value of 355 697€. The value of the optimal solution sounds plausible, because the income minus the expenses sums up to 548 162€ with a small rounding error. Not only that, but all the variables and constants where inside the set parameters. Table 3 shows the optimal value for all the variables and constants.

Name of var/cons	Value of var/cons
z ($Objective function$)	548 163 €
B5 (B_1)	0 liters
B30 (B_2)	214 286 liters
B100 (B_3)	552 804 liters
Methanol (m)	137 131 liters
Petrol diesel (p)	150 000 liters
Biodiesel ($--$)	617 090 liters
VegOil ($--$)	685 655 liters
Water in use ($--$)	5 000 liters
Area in use ($--$)	1 600 hectare
Soybeans (X_1)	850 hectare
Sunflower seeds (X_2)	0 hectare
Cotton seeds (X_3)	750 hectare

Table 3: This table shows the value for all the the variables and constants in the optimal solutions to the problem stated above.

4 Sensitivity analysis

4.1 Petrol, area and water availability

The area could be reduced to 313 hectare and still meet the requirements for the problem. What we noticed when reducing the area availability is that more and more area was given to soybeans which has the best yield. We also saw that since we did not change the amount of petrol, the computer always wanted to make as much B30 as possible since it would be considered as waist otherwise. It was a significant drop in the objective value, which was expected since we could not produce as much oil as the original problem.

What we saw when changing the water supply is that we could go as low as having 371 Ml water while still staying feasible. When reducing water supply, more and more area was given to the cotton seeds since that had the lowest water demand. The objective value was decreased as well, because of the restriction of water combined with cost of petrol that did not change.

For the petrol, we saw that it was not forced to have petrol at all to stay feasible as B100 does not need any petrol and because it is not taxed, the objective function did not see as much of a decrease as the other two. We also observed that there never was any production of B5, possibly because of the high content of petrol combined with a high tax which deemed it not as profitable as the other two.

4.2 Marginal increments in petrol, area and water

When using marginal increments of the area availability we saw that our objective value increased with 276.3€, which made sense since we had more area to produce oil from. We still had the same amount of B30 produced since we had already used up all of our petrol, and B100 increased of the same reason. Our area went more and more to the cotton seeds since that had the least water demand as well as the most oil content. Since water is what is the restricting part of the oil production, it feels reasonable to use cotton.

Increments of water also resulted in an increase of 13.6€ to the objective value. It was not as big of an increase as the area increase, probably because the area allowed us to plant more seeds, but increasing the water leads to being able to choose what seed to use more freely. When increasing water availability, more of the area tends to being used for soybeans. Possibly because the area is now the restricting factor which means that we want to get as much out of it as possible. Soybeans has the highest yield which could be the reason to why that is the preferred crop. As before, the amount of B30 produced is unchanged because of reason mentioned above, which implies that B100 is increasing.

If we add more petrol, one can observe that our objective value increases by 0.2534 € for every increase in petrol. We gain more since we can produce more B30 which we can sell for a higher price than B100, as well as the total amount of biofuels produced increases. B5 is still not regarded as an option since the petrol content is too high combined with the tax. One reason to why the increase in petrol does not generate as much of a profit as increasing the area of crops is because petrol is regarded as a cost in our model. Even though one can produce more biofuels than before, you still need to pay money for the extra petrol whereas increasing the area is only regarded as profitable. The amount of vegetable oil produced stays the same since they are not related.

4.3 What needs to change to make sunflower seeds non-zero

We found that soybeans will be non-zero when:

- The yield of soybeans is increased to 2.075.
- The oil content is increased to 0.321.
- Combine the two above, i.e. increase both yield and oil at the same time.

When testing what data makes sunflower non-zero, one concluded that you cannot make soybeans or cotton "worse", since that would only result in transferring the area of one to the other.

4.4 How does the profit change if the price of petrol is increased to 1.2 €/l and what is the optimal solution then

When increasing the petrol price to 1.2 €/l the profit dropped with roughly 30 000€. The optimal solution remains the same. One reason to this could be that the it is still more profitable to use all the petrol that we have bought because of the higher selling price. The distribution of crops remains unchanged since the petrol and crops are independent. The distribution of crops should create as much vegetable oil as possible, no matter the amount of petrol.

4.5 Vary the taxes

Varying the taxes can be done in two different ways, one where all the taxes are increased at the same rate or when two are held constant and the other one is increasing or decreasing. Firstly, when the taxes are increasing with the same value, we can see that the revenue, or optimal solution, is decreasing linearly in figure 2 and that B_1 and B_3 stays the same to the point when the extra tax is equal to 19% in figure 3. In the same figure at $x = 19\%$, we can see that all the sold biofuel is shifted from B30 and B100 to only B100. Since the tax on B30 became to high, $5\% + 19\% = 24\%$.

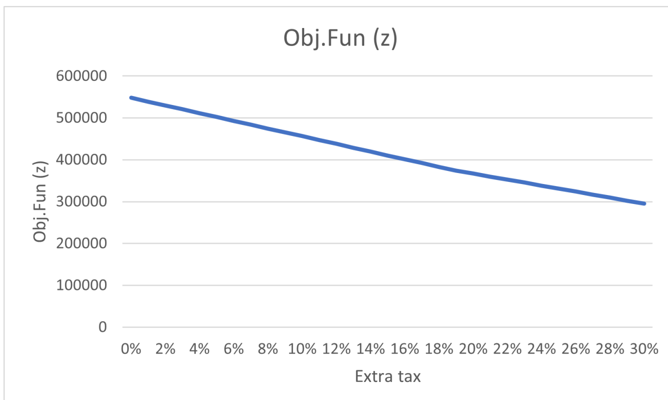

Figure 2: Figure showing the revenue or optimal solution on the y-axis and the extra tax on the x-axis when changing the tax, with the same amount, on B5, B30 and B100.

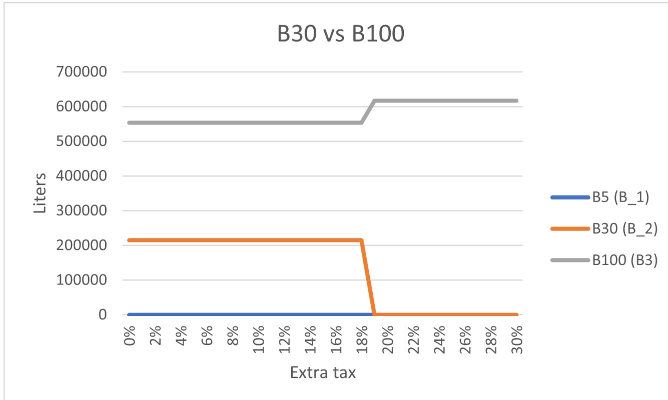

Figure 3: Figure showing the plot of B5, B30 and B10 on the y-axis and the extra tax on the x-axis when changing the tax, with the same amount, on B5, B30 and B100. An important point is when x = 19%.

On the other hand, when increasing only one tax at a time, we get different results. Firstly, increasing B5s tax does not change anything in the optimal solution because the tax on B5 is already too high. However decreasing B5s tax by 8% will change so that all biofuel is converted into B5 and B100 instead of B30 and B100, see figure 5. This will result in the revenue or optimal solution rising compared to the normal optimal solution.

When only rising the tax for B30 the results are similar to when the tax for B5 is decreasing. Although, instead of the revenue, or optimal solution, going up, it goes down, see figure 6. Furthermore figure 7 when showing the amount of B5, B30 and B100 we are producing, is very similar to the same figure 5 for B5 except that the important point, when all B30 changes to B5, are at 6% instead of -8%. Additionally, decreasing the tax for B30 will only yield a higher revenue or optimal solution.

Lastly, when only rising the tax for B100 nothing significant happens except that the revenue or optimal solution will be smaller and smaller until $x = 72\%$, in figure 9 where we stop producing 552 804 liters of B100 and only 65 714. The reason why we still are selling B100 is to fulfill the constrain that we need to sell over 280 000 liters of B5, B30 or B100 and we can not sell more B5 or B30 because we do not have enough petrol diesel. We can also see in figure 8 that the revenue or optimal solution will decline faster before $x = 72\%$ and slower after that. That is because before $x = 72\%$ we can still make some money from B100 but after $x = 72\%$ it will just be an expense to fulfill the 280 000 liter constrain.

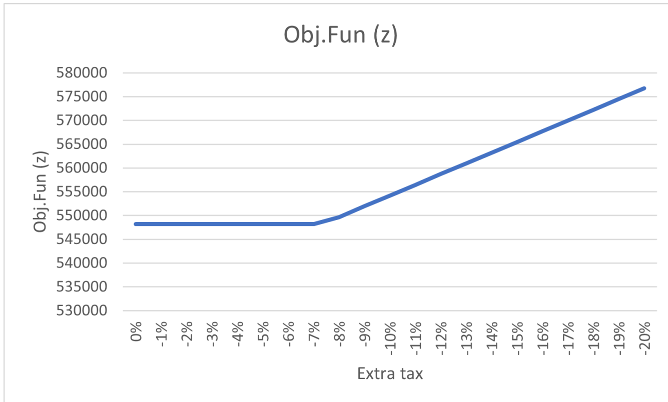

Figure 4: Figure showing the revenue or optimal solution on the y-axis and the extra tax on the x-axis when changing the tax on B5. An important point is when x = -8%.

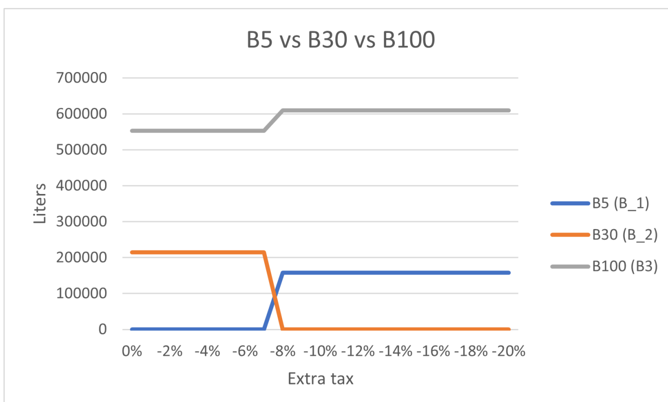

Figure 5: Figure showing the plot of B5, B30 and B10 on the y-axis and the extra tax on the x-axis when changing the tax on B5. An important point is when x = -8%.

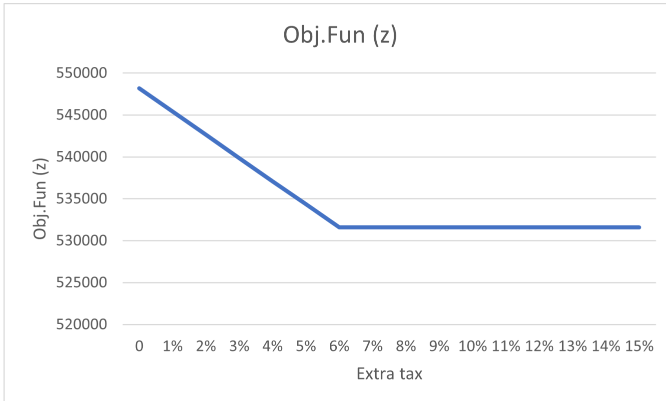

Figure 6: Figure showing the revenue or optimal solution on the y-axis and the extra tax on the x-axis when changing the tax on B30. An important point is when x = 6%.

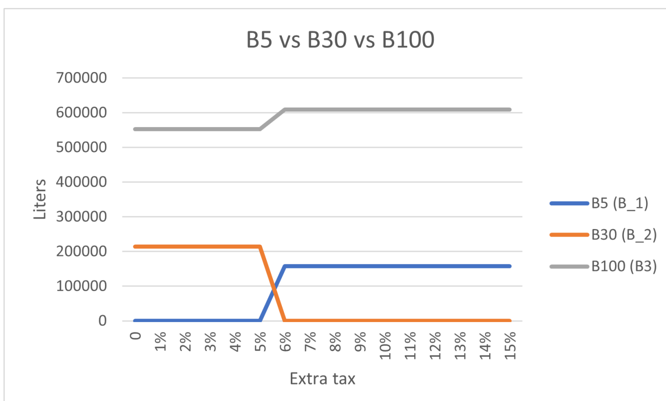

Figure 7: Figure showing the plot of B5, B30 and B10 on the y-axis and the extra tax on the x-axis when changing the tax on B30. An important point is when x = 6%. *Note to reader: This figure is very similar to figure 5, but they are not the same.*

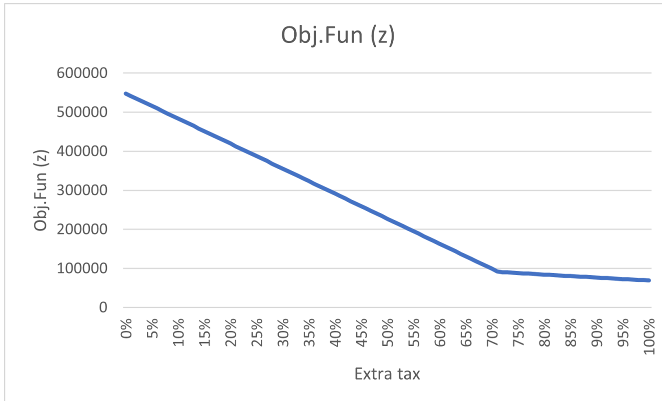

Figure 8: Figure showing the revenue or optimal solution on the y-axis and the extra tax on the x-axis when changing the tax on B100. An important point is when x = 72%.

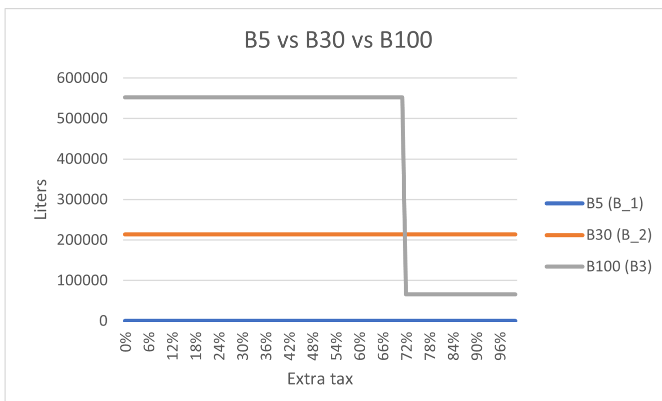

Figure 9: Figure showing the plot of B5, B30 and B10 on the y-axis and the extra tax on the x-axis when changing the tax on B100. An important point is when x = 72%.

4.6 Vary the water demand of the seeds

If we vary the water demand of the crops with a variable $\delta \in [-0.4, 0.4]$ we can plot the results in a coordinate system, 10, with the optimal value z as y-axis and δ as the x-axis. Negative δ means that it is rainy and the crops need less water and vice versa.

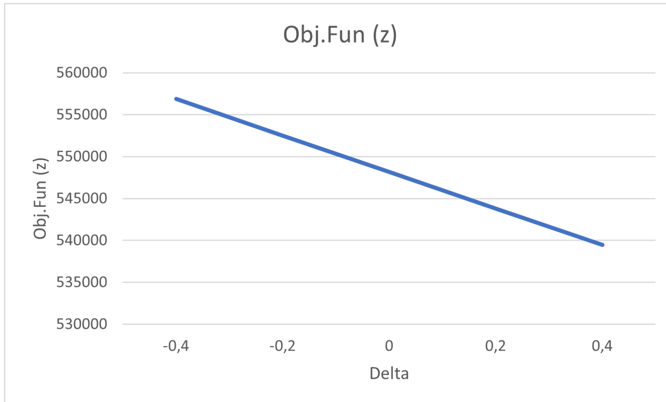

Figure 10: Figure showing the plot of the revenue or optimal value on the y-axis and δ, i.e. the change in water demand for all crops.

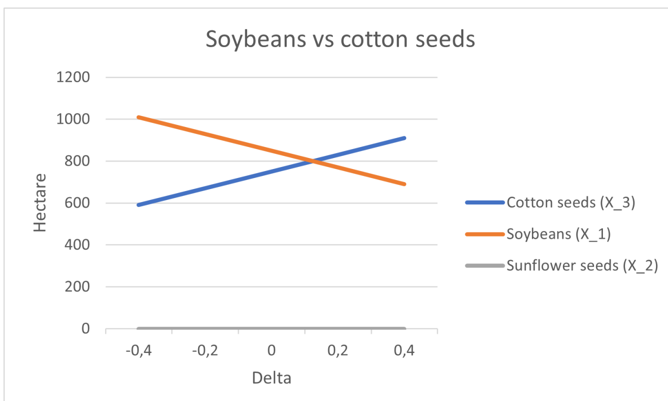

Figure 11: Figure showing the plot of the amount of hectare a crop is using on the y-axis and δ, i.e. the change in water demand for all crops.

From figure 10 we can see that the revenue, or optimal solution, is decreasing from 556 864€ to 539 461€. This is because when the plants need less water, at $\delta = -0.4$, we can plant more soybeans, which gives us a higher oil content per hectare, but normally costs more water. However, when the plants need more water, at $\delta = 0,4$, we need to plant more cotton seeds instead. Cotton seeds need less water to plant but are not as lucrative as soybeans. Therefore, we can produce more vegetable oil when the crops need less water because we have a higher oil harvest compared to when the crops need more water. More vegetable oil equals more biofuel, which equals a higher optimal solution, and vice versa.

4.7 Environmental friendly objective function

At this moment the objective function is to maximize the profit for selling as much biofuel as possible and that is a good thing for the seller, but not for the environment. That is because of the use of petrol diesel and also methanol. A better objective function would be to minimize the use of petrol diesel and methanol and add a new constant, $environmental\ effect$. That constant will be used to multiply the petrol diesel and methanol used because methanol and petrol diesel does not have the same effect on the environment. One example can be seen in equation 2, where the environmental effect for petrol diesel and methanol is set to 2 and 0.5 respectively. This will give us a new objective function that will have values in the new name *environmental credits*.

$$min \quad z_{env} = 2p + 0.5m \tag{2}$$

The optimal values did not come as a surprise in table 4 and gave us that we sold the absolute minimum amount of 280 000 liter biofuel for a profit of 231 467.

Name of var/cons	Value of var/cons
z ($Objectivefunction$)	31 111 environmental credits
B5 (B_1)	0 liters
B30 (B_2)	0 liters
B100 (B_3)	280 000 liters
Methanol (m)	62 222 liters
Petrol diesel (p)	0 liters
Biodiesel ($--$)	280 000 liters
VegOil ($--$)	31 111 liters
Water in use ($--$)	3 361 liters
Area in use ($--$)	672 hectare
Soybeans (X_1)	672 hectare
Sunflower seeds (X_2)	0 hectare
Cotton seeds (X_3)	0 hectare

Table 4: This table shows the value for all the the variables and constants in the optimal solutions to the problem stated above.

On the other hand, if we are a company and are trying to make the largest profit while being as environmentally friendly as possible, we need to change the objective function even more. One way is to take $z - z_e nv$ (equation 1 - 2) and use that as the new objective function. We can use the new value, *environmental credits*, as a sort of "cost" for polluting the environment. So the new objective function can look like

$$max \quad z_{envProf} = \quad \Big(\sum_{i \in I} B_i P_{j,2}(1 - P_{j,3}) \quad - p - 1.5m\Big) - (2p + 0.5m). \tag{3}$$

The optimal solution for equation 3 can be seen in figure 5.

Name of var/cons	Value of var/cons
z ($Objective function$)	441 561 environmental credits
Profit ($--$)	501 127 €
B5 (B_1)	0 liters
B30 (B_2)	0 liters
B100 (B_3)	617 090 liters
Methanol (m)	137 131 liters
Petrol diesel (p)	0 liters
Biodiesel ($--$)	617 090 liters
VegOil ($--$)	685 655 liters
Water in use ($--$)	5 000 liters
Area in use ($--$)	1 600 hectare
Soybeans (X_1)	850 hectare
Sunflower seeds (X_2)	0 hectare
Cotton seeds (X_3)	750 hectare

Table 5: This table shows the value for all the the variables and constants in the optimal solutions to the problem stated above.