Report Instructions - SF2980 Risk Management

HT25

Instructions

Objectives

The projects are intended as open ended exercises suitable for deeper investigation of some selected topics. Students will

- practice by applying the theory and methods on real examples
- develop their own statistical software functions in programs such as “R”, “MATLAB” and “Python” to solve the problems
- make qualitative and quantitative statements and conclusions about the risk management problems studied in the projects
- present the problems, relevant theory, results and conclusions in written reports
- present the problems, relevant theory, results and conclusions in oral presentations and organized discussions.

Format

- Students will work on the projects in groups of at most four people.
- Each project will be presented and evaluated in written form by handing in a Jupyter Notebook (part A) and a written report (part B). The deadline to submit the first project (Jupyter Notebook and written report) is November 19, 2025, at 23:59. The deadline to submit the second project (Jupyter Notebook and written report) is December 5, 2025 at 23:59.
- The projects will be presented orally at the presentation seminars on November 24, 2025, 10:15 - 12:00 (first project) and December 8, 2025, 10:15 - 12:00 (second project).

Grading

On each project you will be given a score in the range 0-25; the total score for the two reports is at most 50. A combined score of at least **25 pts is needed to pass** the assignments part of the course. Grades will be based on the following criteria.

- **Part A (7 pts)**
  - **Code:** A functioning code in R or Python (or any other programming language supported on Jupyter Notebook) producing results for the problems in Part A.
  - **Comments:** Adequate comments demonstrating knowledge and relevant interpretation of the results.
- **Part B (18 pts)**
  - **Objectives.** A clear description of the project and its objectives.
  - **Mathematical background.** A clear and concise presentation of the most relevant mathematical background.
  - **Results.** A clear and concise presentation of your solution and results. You may also add your own explorations/extensions that you find relevant.
  - **Summary.** A brief complete summary of your results.

Report Templates

For part A an example file in the form of a Jupyter Notebook including comments for Example 7.8 in the course book is provided in the Files section on the Canvas page, in folder "Projects".

For part B there is a report template in LaTeX available for download in the Files section on the Canvas page, in folder "Projects". Please use the sections outlined in the report template. If you are not able to use LaTeX to typeset your report you may use another software (e.g., Word, Pages, etc.)

Project I (A and B)

For the first project, due on **November 19, 2025 at 23:59**, you will present your analysis and results on the **mandatory project seminar on November 24, 2025, 10:15–12:00 in room K2**. For part A you need to submit a Jupyter Notebook file containing executable code in a programming language of your choice (e.g., Python 3 or R). The Jupyter Notebook must include comments on the interpretation of the results. If generative AI was used, the Jupyter Notebook should contain information about the prompt used and what modifications you needed to do in order to obtain a satisfactory code.

For part B, you need to submit a written report **typeset on a computer** (no handwriting) and handed in by uploading it on Canvas in PDF format. You do not have to supply your source code for part B (so you can write it in any programming language that you like, e.g., R, Matlab or Python).

Part A

Part A of the project consists in solving the problems A.1 and A.2 below, which are modified versions of Examples 7.10 and 7.11 in the course book. Source code in R for the two examples can be found at

`https://people.kth.se/ hult/riskbook.html`

You are encouraged to use a free generative AI service, such as Open AI's ChatGPT or Google Gemini, to help in producing the code for the two problems and for help with coding. For example, if you want to use parts of the R code from the examples 7.10 and 7.11, but you intend to write your code in Python, you may ask the AI tool to transcribe the example code from R to Python.

The notebook must contain the following.

- Clearly separated titles for the different parts of problems A.1 and A.2.
- A general comment stating how generative AI tools where used (what AI service, if any, and what prompt you used) and what modifications you needed to do in order to obtain a satisfactory code.
- Executable code for all the examples.
- Your own comments on the results/plots demonstrating your understanding on the concepts and key takeaways from the numerical experiments.

For Part A, you should submit an executable `.jpynb` file with your code and comments. An example file "Example 7-8.jpynb" is provided in Canvas under Files → Projects.

Problem A.1

(a) Let F be a $t_\nu(\mu, \sigma)$ distribution with $\nu = 5$ degrees of freedom, location parameter $\mu = 1$ and scale parameter $\sigma = 0.01$. Generate $n = 700$ samples from F.
Now suppose you do not know the true distribution from which the 700 samples were generated. Using only the 700 samples, construct an exact confidence interval with an approximate confidence level of 95% for the quantile $F^{-1}(0.975)$ using the exact method (Section 7.5.1 in the course book).

(b) Repeat (a) $k = 200$ times, that is, for each of the $k = 200$ times, generate 500 samples from F and determine an exact confidence interval with an approximate confidence level of 95% for $F^{-1}(0.975)$. Create a plot showing all 200 confidence intervals along with the true quantile. How many of the 200 intervals contain the true quantile $F^{-1}(0.975)$?

Problem A.2

Assume to have $n = 700$ historical observations $R_{-699}, \ldots, R_0$ of the daily returns of some financial instrument. Let V_0 and V_1 be, respectively, the current value and the value tomorrow of a long position in this financial instrument. The daily return on the risk-free bond is assumed to be 1. Let X denote the future net value of the position. Assume that $V_0 = 10$. We are interested in constructing confidence intervals for the quantity $\theta = \mathrm{VaR}_{0.025}(X)$.

(a) Assume that $R_1 \sim t_\nu(\mu, \sigma)$ with $\nu = 5$ degrees of freedom, location parameter $\mu = 1$ and scale parameter $\sigma = 0.01$. Determine θ (i.e., $\mathrm{VaR}_{0.025}(X)$). For $N = 1000$ times, generate $n = 100$ iid samples from $t_\nu(\mu, \sigma)$ and compute the corresponding empirical estimate $\hat{\theta} = \widehat{\mathrm{VaR}}_{0.025}(X)$. Plot the histogram of the $N = 1000$ values of $\theta - \hat{\theta}$ thus obtained.

(b) Generate $n = 700$ iid samples from $t_\nu(\mu, \sigma)$ and suppose these are observations of the daily returns $R_{-699}, \ldots, R_0$. Determine $\hat{\theta}_{\mathrm{obs}}$ and plot the histogram of $N = 1000$ bootstrap simulations of $\hat{\theta}_{\mathrm{obs}} - \hat{\theta}^*$. Compare the histogram of $\hat{\theta}_{\mathrm{obs}} - \hat{\theta}^*$ with the histogram of $\theta - \hat{\theta}$ generated in (a).

(c) Consider the same $n = 700$ iid samples from $t_\nu(\mu, \sigma)$ generated in (b) and the corresponding estimate $\hat{\theta}_{\mathrm{obs}}$. Plot the histogram of $N = 10000$ bootstrap simulations of $\hat{\theta}_{\mathrm{obs}} - \hat{\theta}^*$. Compare the two histograms form (b) and (c).

(d) Compute 100 exact confidence intervals of approximate confidence level of 95% using the exact method (Section 7.5.1 in the course book) and 100 approximate confidence intervals of confidence level of 95% with the nonparametric bootstrap method with $N = 1000$ (Section 7.5.1 in the course book). Plot the two sets of confidence intervals along with the exact value of θ computed in (a). Comment on the results.

Part B

Part B of the project consists of an analysis related to pension savings. Your analysis and solutions to the project questions will be presented in a written report to be submitted as a `.pdf` file. For part B you may use whatever programming language you like (e.g., R, Matlab or Python) and you do not have to submit your code with your report. A LaTeX-template for the report is provided on Canvas under Files → Projects.

Pension savings

Assignments (a) and (b) are almost identical to Project 8 at pages 228-229 in the book; assignments (c, d, e) are new.

Consider a yearly investment of $1,000$ dollars in long positions in a portfolio of stocks and a risk-free one-year, zero-coupon bond over a 30-year period. The yearly returns on the portfolio of stocks in year k is modeled as $R_k = e^{\mu+\sigma Z_k}$, where Z_k is standard normally distributed. The yearly returns are assumed to be independent. The yearly return on the risk-free bond is assumed to be $e^{0.01}$. Each year, after the investment of $1,000$ dollars has been added, the entire portfolio is rebalanced such that the fraction of the total portfolio value invested in the stocks at the beginning of year k is $p(1-c(k-1)/30)$, where $p, c \in [0,1]$.

(a) Determine a function f such that the value of the pension savings in 30 years can be expressed as $V_{30} = f(\mu, \sigma, p, c, Z_1, \dots, Z_{30})$.

Simulate a sample of suitable size n from the distribution of $(Z_1, \dots, Z_{30})$ and use this sample to determine the empirical distribution F_n of V_{30} for a range of values of the parameters μ, σ, p, c.

(b) Set $\mu = 0.03$ and $\sigma = 0.2$ and investigate the effects on the empirical distribution $F_n(p,c)$ of V_{30} of varying p and c. Suggest a suitable criterion for selecting the optimal empirical distribution $F_n(p,c)$ and determine the optimizer (p,c). For the optimal portfolio you must report the mean $E[V_{30}]$ and the 1%-quantile: $F_{V_{30}}^{-1}(0.01)$.

(c) Sweden's pension system has transited from traditional life insurance to a system based on fund insurance. Roughly speaking fund insurance works as in (a) and (b) where the risky asset can be viewed as a portfolio selected by the customer. In traditional life insurance the individual participates in a collective where there is a guaranteed return as well as a potential for higher returns for the collective if the investments in the risky asset (portfolio) performs well. One version of traditional life insurance works as follows (there are some variations on this theme).

There is a guaranteed annual return $\bar{r}$ on all investments. The guaranteed return is typically slightly lower than the risk-free rate. The fund manager of the traditional life insurance policy then invests the necessary amount in the risk-free asset to cover the guarantee and the remaining surplus in the risky asset to generate a higher return. For the investor with a yearly investment of $1,000$ dollars, the guaranteed amount after k years is G_k where

$$
\begin{aligned}
G_0 &= 0, \\
G_k &= 1000 \sum_{j=1}^{k} e^{\bar{r} j}, \quad k \geq 1.
\end{aligned}
$$

Let $V_0 = 0$ and V_k, $k \geq 1$, be the value of the portfolio after k years. At the beginning of year $k+1$ the amount $G_{k+1}e^{-r}$ is invested in the bank account and $V_k + 1000 - G_{k+1}e^{-r}$ is invested in the risky asset.

Let $\bar{r} = 0.005$ and plot a histogram of V_{30} using the traditional life insurance strategy and compare it to your optimal solution in (b). You must also report the mean $E[V_{30}]$ and the 1%-quantile: $F_{V_{30}}^{-1}(0.01)$.

(d) It may be difficult to generate large profits using the strategy in (c) and a way to obtain larger profits is to have a leverage in the exposure to stocks. In this case it is assumed that the market will not experience a crash of more than, say, 50% over one year. At the beginning of year $k+1$, the surplus over the guarantee is invested in the stock with a leverage of 2 (the 2 is 1/0.5). That is, the amount $2(V_k + 1000 - G_{k+1}e^{-r})$ is invested in the stock and the remaining amount, $V_k + 1000 - 2(V_k + 1000 - G_{k+1}e^{-r})$ in the bond. This type of strategy is often referred to as a constant proportion portfolio insurance (CPPI).

Note that if the stock price falls by more than 50% during year $k+1$ the value V_{k+1} is less than the guarantee G_{k+1}. In practice one would monitor the stock price continuously and rebalance to make sure that this "never" happens. For this project you may assume that if you fall behind the guarantee, that is $V_{k+1} < G_{k+1}$, then you invest the future yearly investments in the bond until $V_{k+m} \geq G_{k+m}$ and then proceed according to the strategy, or until the end (whatever comes first).

Compare the histogram of V_{30} to the results in (b) and (c). You must also report the mean $E[V_{30}]$ and the 1%-quantile: $F_{V_{30}}^{-1}(0.01)$.

(e) In assignments (b - d) you have used a log-normal distribution for the annual returns of the portfolio of stocks. In this exercise you should replace the log-normal distribution by an empirical distribution based on monthly historical data and use the historical simulation approach. You may assume that monthly returns are independent, or use historical simulation based on, say, annual returns with overlapping segments of monthly returns. You must clearly specify your choice of historical simulation approach. You may select a portfolio consisting of one or several funds from the Swedish Pension System (PPM). You can find historical data from the PPM system at:

- `https://www.pensionsmyndigheten.se/nyheter-och-press/nyheter-fondtorg/statistik-om-premiepensionens-fonder`

You may also select your own portfolio of stocks or other funds. You may find historical data at

- `https://www.nasdaq.com/european-market-activity/shares`
- `https://www.nasdaq.com/european-market-activity/funds`

Repeat the analysis in (b, c, d) using your historical simulation approach and compare the results.