MA 1024. HOMEWORK 5
DUE: FRIDAY APRIL 18

Assigned Problems:

Section 13.2: 10, 12, 16, 22, 26, 30, 32

Section 13.3: 4, 8, 12, 18, 24, 28, 36

Section 13.4: 4, 10, 14, 16, 20, 28

Recommended Problems: (only hand in the subset listed above)

Section 13.2: 1 – 34

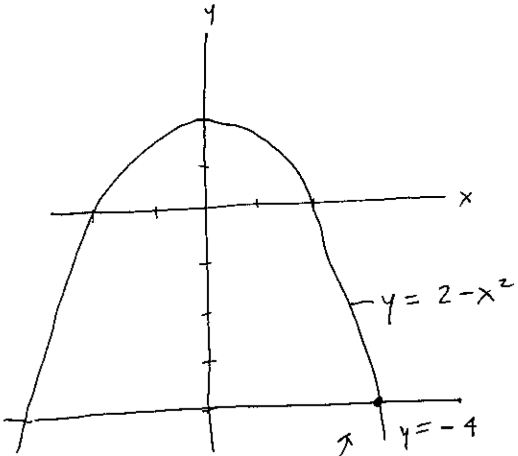
Section 13.3: 1 – 34

Section 13.4: 1 – 38

MA 1024 HOMEWORK 5

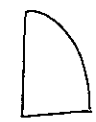
13-2

(16)



$$\iint_R f(x,y)\,dA = 2\int_0^{\sqrt{6}}\int_{-4}^{2-x^2} x^2\,dy\,dx$$

using symmetry. about y-axis



point of intersection:

$2 - x^2 = -4$

$x^2 = 6$

$x = \sqrt{6} \Rightarrow y = -4$

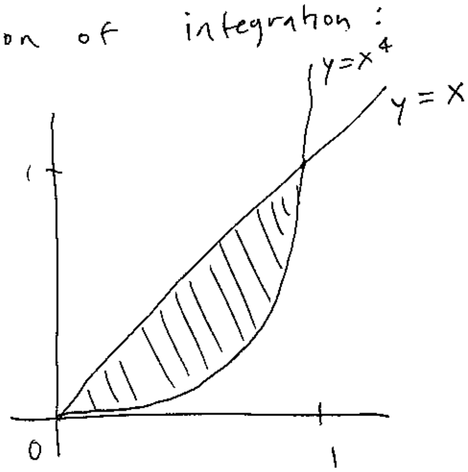
$$= 2\int_0^{\sqrt{6}} \left[x^2 y\right]_{-4}^{2-x^2} dx$$

$$= 2\int_0^{\sqrt{6}} x^2(2-x^2) - x^2(-4)\,dx = 2\int_0^{\sqrt{6}} 6x^2 - x^4\,dx$$

$$= 2\left[2x^3 - \frac{x^5}{5}\right]_0^{\sqrt{6}} = 2\left[2\cdot 6^{\frac{3}{2}} - \frac{6^{\frac{5}{2}}}{5}\right] = \frac{48}{5}\sqrt{6}$$

(26) $\displaystyle\int_0^1\int_{x^4}^{x} (x-1)\,dy\,dx.$

region of integration:



change order of integration:

$0 \le y \le 1$

$y \le x \le \sqrt[4]{y}$

So our integral becomes:

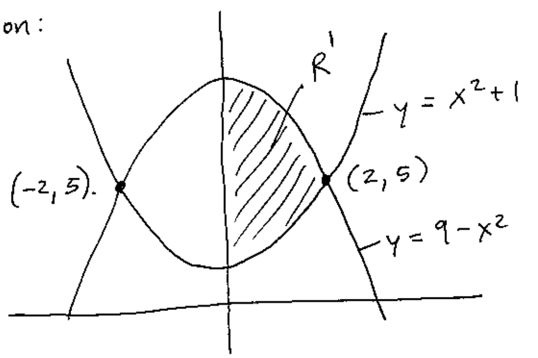
$$\int_0^1 \int_y^{\sqrt[4]{y}} (x-1)\,dx\,dy = \int_0^1 \left[\frac{x^2}{2} - x\right]_y^{\sqrt[4]{y}} dy$$

$$= \int_0^1 \frac{y^{\frac{1}{2}}}{2} - y^{\frac{1}{4}} - \frac{y^2}{2} + y \, dy = \left[\frac{y^{\frac{3}{2}}}{3} - \frac{4}{5} y^{\frac{5}{4}} - \frac{y^3}{6} + \frac{y^2}{2}\right]_0^1$$

$$= \frac{1}{3} - \frac{4}{5} - \frac{1}{6} + \frac{1}{2} = \frac{10 - 24 - 5 + 15}{30} = -\frac{2}{15}.$$

13.3

(8) region:



intersection point:

$$x^2+1 = 9-x^2$$
$$2x^2 = 8$$
$$x^2 = 4$$
$$x = \pm 2$$
$$\Rightarrow y = 5$$

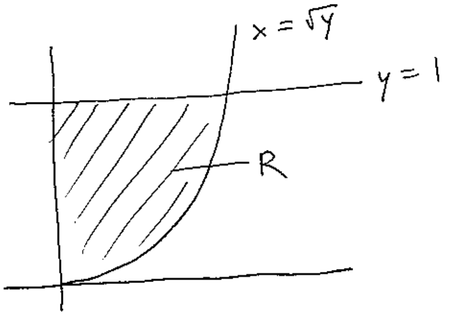
By symmetry, area is double the RHS of y axis:

$$\text{Area} = 2\iint_{R'} 1\,dA = 2\int_0^2 \int_{x^2+1}^{9-x^2} 1\,dy\,dx$$

$$= 2\int_0^2 \left[y\right]_{x^2+1}^{9-x^2} dx = 2\int_0^2 (9-x^2)-(x^2+1)\,dx = 2\int_0^2 8 - 2x^2\,dx$$

$$= 2\left[8x - \frac{2}{3}x^3\right]_0^2 = 2\left[16 - \frac{16}{3}\right] = 2\left(\frac{32}{3}\right) = \frac{64}{3}.$$

(18) region of integration:



$$V = \iint_R 2x + y \, dA$$

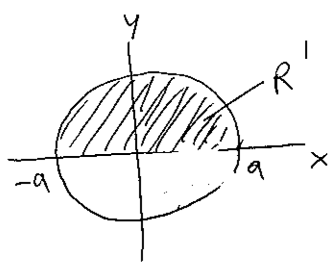
$$V = \int_0^1 \int_0^{\sqrt{y}} 2x + y \, dx \, dy \overset{\text{OR}}{=} \int_0^1 \int_{x^2}^1 2x + y \, dy \, dx$$

$$= \int_0^1 \left[2xy + \frac{y^2}{2} \right]_{x^2}^1 dx$$

$$= \int_0^1 2x + \tfrac{1}{2} - 2x^3 - \frac{x^4}{2} \, dx = \left[x^2 + \tfrac{1}{2}x - \tfrac{1}{2}x^4 - \frac{x^5}{10} \right]_0^1$$

$$= 1 + \tfrac{1}{2} - \tfrac{1}{2} - \tfrac{1}{10} = \frac{9}{10}$$

(36) ~~area~~ region of integration



By symmetry we can integrate over ½-circle since the plane $z = x + h$ is symmetric WRT x-axis. $\rightarrow$ $-a \le x \le a$, $0 \le y \le \sqrt{a^2 - x^2}$

$$V = 2\iint_{R'} x + h \, dA = 2 \int_{-a}^{a} \int_0^{\sqrt{a^2-y^2}} x + h \, dy \, dx$$

$$= 2 \int_{-a}^{a} \left[xy + hy \right]_0^{\sqrt{a^2-x^2}} dx = \underbrace{2 \int_{-a}^{a} x (a^2 - x^2)^{\frac{1}{2}} dx}_{1} + \underbrace{2 \int_{-a}^{a} h (a^2 - x^2) \, dx}_{2}$$

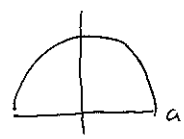
TERM ①

let $u = a^2 - x^2$

$\frac{du}{dx} = -2x$, so $$2\int_{-a}^{a} x\left(a^2-x^2\right)^{\frac{1}{2}} dx = 2\int_{x=-a}^{x=a} x\, u^{\frac{1}{2}} \frac{-1}{2x}\, du$$

$$= \left[\frac{2}{3} u^{\frac{3}{2}}\right]_{x=-a}^{x=a} = \left[\frac{2}{3}\left(a^2-x^2\right)^{\frac{3}{2}}\right]_{x=-a}^{x=a} = 0.$$

TERM ②

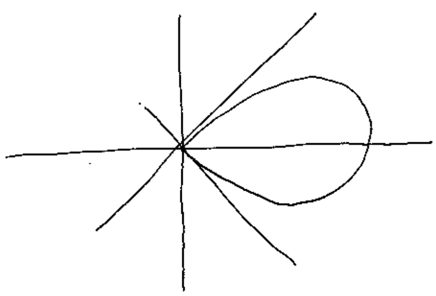
$$2h\int_{-a}^{a} a^2 - x^2\, dx$$ but this is just the area of the semicircle  of radius a.

area $= \frac{1}{2}\pi a^2$

so term ② is $2h\left(\frac{1}{2}\pi a^2\right) = h\pi a^2$, as desired.

13.4

④ area bounded by one loop of $r = 2\cos 2\theta$



take loop with $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

then $0 \le r \le 2\cos 2\theta$

and area $= \displaystyle\int_{-\pi/4}^{\pi/4}\int_{0}^{2\cos 2\theta} r\, dr\, d\theta$.

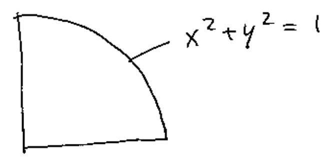
$$= \int_{-\pi/4}^{\pi/4}\left[\frac{r^2}{2}\right]_0^{2\cos 2\theta} d\theta = \int_{-\pi/4}^{\pi/4} 2\cos^2 2\theta\, d\theta = \int_{-\pi/4}^{\pi/4} \cos 4\theta + 1\, d\theta$$

$$= \left[\frac{1}{4}\sin 4\theta + \theta\right]_{-\pi/4}^{\pi/4} = \frac{\pi}{2}.$$

(14) convert to polars:

$\sqrt{4-x^2-y^2}$ becomes $\sqrt{4-r^2}$

the region



is $0 \le \theta \le \pi/2$

$0 \le r \le 1$

so the integral is

$$\int_0^{\frac{\pi}{2}} \int_0^1 \frac{1}{\sqrt{4-r^2}} \, r \, dr \, d\theta$$

substituting: $u = 4-r^2$

$\frac{du}{dr} = -2r$

$$= -\frac{1}{2} \int_0^{\frac{\pi}{2}} \int_{r=0}^{r=1} \frac{1}{\sqrt{u}} \, du \, d\theta = -\frac{1}{2} \int_0^{\frac{\pi}{2}} \left(2\sqrt{u}\right)_{r=0}^{r=1} d\theta = -\int_0^{\frac{\pi}{2}} \left[\sqrt{4-r^2}\right]_0^1 d\theta$$

$$= \left(2-\sqrt{3}\right)\frac{\pi}{2}.$$