MA 1024. HOMEWORK 2
DUE: FRIDAY MARCH 28

Assigned Problems:

Section 12.4: 4, 6, 20, 26, 30, 32, 38, 42, 74

Section 12.5: 8, 12, 16, 28, 36, 42

Section 12.6: 8, 10, 18, 22, 28, 32

Recommended Problems: (only hand in the subset listed above)

Section 12.4: 1 – 50, 63, 71, 73

Section 12.5: 1 – 28, 35, 36, 37, 39, 42, 53,

Section 12.6: 1 – 36

MA 1024 HOMEWORK 2

12.4

(38) $z = e^{-x^2-y^2}$ $\qquad P = (0,0,1)$

$$\frac{\partial z}{\partial x} = -2x\, e^{-x^2-y^2} \qquad \frac{\partial z}{\partial x}(0,0) = 0$$

$$\frac{\partial z}{\partial y} = -2y\, e^{-x^2-y^2} \qquad \frac{\partial z}{\partial y}(0,0) = 0$$

hence the plane tangent to P is horizontal.

The equation of the plane is $z = 1$.

12.5

(8) where is tangent plane horizontal?

$$z = x^4 + y^3 - 3y$$

$$\frac{\partial z}{\partial x} = 4x^3 \qquad \frac{\partial z}{\partial x} = 0 \iff x = 0$$

$$\frac{\partial z}{\partial y} = 3y^2 - 3 = 3(y^2-1) \qquad \frac{\partial z}{\partial y} = 0 \iff y = \pm 1.$$

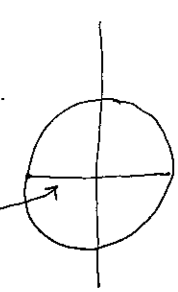
So z has two horizontal tangent planes, one at $(0,1,-2)$ and one at $(0,-1,2)$.

(28) $f(x,y) = xy^2 \qquad R = \{(x,y) : x^2+y^2 \leq 3\}$

interior critical pts:

$$\left.\begin{aligned} f_x(x,y) &= y^2 \\ f_y(x,y) &= 2xy \end{aligned}\right\}$$ when $y=0$, both partials are zero. (for any value of x).

this corresponds to x-axis contained in the circle



but note the value of $f(x,y)$ is 0 on this line of critical points. ~~However,~~

boundary pts:

On the boundary, $x^2+y^2=3$.

Write $y^2 = 3-x^2$

Then on the boundary, our function $f(x,y)$ becomes:

$$f(x,y) = g(x) = x(3-x^2) = 3x - x^3$$

so the critical points of $g(x)$ occur when $g'(x)=0$,

$$g'(x) = 3-3x^2$$
$$= 3(1-x^2)$$

and $g'(x)=0 \iff x = \pm 1$.

On the boundary, these correspond to the points

$x^2+y^2 = 3$

$y^2 = 2$

$y = \pm\sqrt{2}$.

Note the domain of $g(x)$ is

$-\sqrt{3} \le x \le \sqrt{3}$

So we check the endpts here too

Now compare all the values!

$f(x,0) = 0$

$f(1,\sqrt{2}) = 2$ MAXIMA

$f(1,-\sqrt{2}) = 2$

$f(-1,\sqrt{2}) = -2$ MINIMA

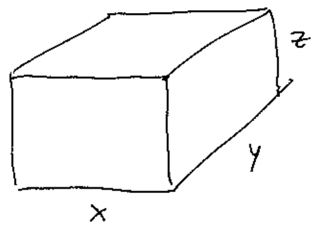
$f(-1,-\sqrt{2}) = -2$

$f(-\sqrt{3},0) = 0$

$f(\sqrt{3},0) = 0$

42 Box volume $12m^3 = xyz$

assume material for bottom costs $2K$, where K is the cost of the material for top + sides.



$$C(x,y) = \cancel{(\text{cost of bottom})\,xy + (\text{cost of top})\,xy +}$$

$$= \underbrace{(2K)\,xy}_{\text{bottom}} + \underbrace{Kxy}_{\text{top}} + 2K\,yz + 2K\,xz.$$

$$= (3K)\,xy + 2K\,y\left(\frac{12m^3}{xy}\right) + 2K\,x\left(\frac{12m^3}{xy}\right)$$

$$= 3Kxy + \frac{24K}{x} + \frac{24k}{y}, \quad x>0,\ y>0.$$

Now minimize:

$C_x(x,y) = 3Ky - \dfrac{24K}{x^2}$ ← multiply by x

$C_y(x,y) = 3Kx - \dfrac{24K}{y^2}$ ← multiply by y

$$3Kxy = \frac{24K}{x} = \frac{24k}{y} \implies x = y.$$

substitute into $C_x(x,y) = 0$

$$3Kx - \frac{24K}{x^2} = 0$$

$$3x^3 = 24$$

$$x^3 = 8$$

$x = 2$, and hence $y = 2$ too.

This is a global minimum for $C(x,y)$.

The corresponding value for z is $z = 3$.

Total cost will be

$$C(2,2) = 3K(4) + \frac{24k}{2} + \frac{24k}{2}$$

$$= 12k + 12k + 12k = 36K.$$

12.6

(10) $w = xy e^{uv}$

$$dw = \frac{\partial w}{\partial x} dx + \frac{\partial w}{\partial y} dy + \frac{\partial w}{\partial u} du + \frac{\partial w}{\partial v} dv$$

$$\frac{\partial w}{\partial x} = y e^{uv} \qquad \frac{\partial w}{\partial u} = x y e^{uv} v$$

$$\frac{\partial w}{\partial y} = x e^{uv} \qquad \frac{\partial w}{\partial v} = x y e^{uv} u.$$

(18) $f(x,y) = \sqrt{x^2 - y^2}$ $\qquad P(13, 5) \qquad Q(13.2, 4.9)$

let $z = f(x,y)$

$$dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

$$\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2 - y^2}} \qquad \frac{\partial f}{\partial y} = \frac{-y}{\sqrt{x^2 - y^2}}$$

so $$dz = \frac{x dx - y dy}{\sqrt{x^2 - y^2}}$$

let $x = 13 \qquad dx = 0.2$

$y = 5 \qquad dy = -0.1$

so $$f(13.2, 4.9) \cong f(13, 5) + \frac{13(0.2) - 5(-0.1)}{\sqrt{13^2 - 5^2}}$$

$$= 12 + \frac{31}{120}$$

$$= \frac{1471}{120} \approx 12.2583\overline{3}$$

calculating "real value" $f(13.2, 4.9) = 12.25683482$

(pretty close!)

(32) want to estimate the x-coord corresponding to y-coord 3.9 on the curve

$$4x^4 + 4y^4 = 17x^2y^2.$$

let $f(x,y) = 4x^4 + 4y^4 - 17x^2y^2$. We are interested in the curve $f(x,y)=0$.

$$\frac{\partial f}{\partial x} = 16x^3 - 34xy^2$$

$$\frac{\partial f}{\partial y} = 16y^3 - 34x^2y.$$

$$df = (16x^3 - 34xy^2)dx + (16y^3 - 34x^2y)\,dy$$

let $x=2$, $y=4$, $dy = -0.1$.

Solve for dx in the expression $df=0$.

$$(128 - 34\cdot 36)dx + (1024 - 34\cdot 16)(-0.1) = 0$$

$$\rightsquigarrow dx = -\frac{1}{20} = -0.05.$$

So the x-coord is $2+dx = 1.95$.