Name: _____   Monday, April 14, 2014
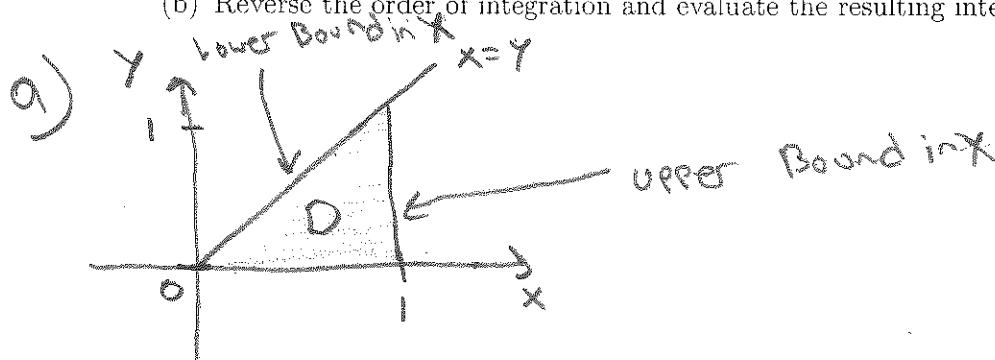
$$\boxed{\text{MA1024 D07} \quad \textbf{Quiz 3}}$$

1. (**5 marks**) Consider the following integral:

$$\int_0^1 \int_y^1 e^{-x^2}\,dx\,dy.$$

(a) Sketch the domain of integration.

(b) Reverse the order of integration and evaluate the resulting integral.

a)

b)

$$\int_0^1 \int_0^x e^{-x^2}\,dy\,dx = \int_0^1 x e^{-x^2}\,dx = -\frac{1}{2}\int_0^{-1} e^u\,du$$

$$u = -x^2 \quad du = -2x\,dx$$

$$= -\frac{1}{2}\left[e^{-1} - e^0\right] = \frac{1}{2}\left(1 - \frac{1}{e}\right) = \boxed{\frac{e-1}{2e}}$$

This quiz has two sides!!! $\boxed{/10}$

2. (**5 marks**) Evaluate the integral

$$\iint_R 2xy - 3y^2 \ dA$$

on the region $R = [-1, 1] \times [-2, 2]$ using iterated integrals in two different ways (e.g. first by integrating in the order "$dxdy$", and then "$dydx$").

1)

$$\int_{-1}^{1} \int_{-2}^{2} \{ 2xY - 3Y^2 \} \ dY \ dx = \int_{-1}^{1} [xY^2 - Y^3] \Big|_{-2}^{2} \ dx$$

$$= -\int_{-1}^{1} -\left(2^3 - (-2)^3 \right) \ dx$$

$$= 2^4 (1 - (-1)) = 2^5 \quad \checkmark$$

$$\int_{-2}^{2} \int_{-1}^{1} \{ 2xY - 3Y^2 \} \ dx \ dY = \int_{-2}^{2} [x^2 Y - 3Y^2 x] \Big|_{-1}^{1} \ dY$$

$$= \int_{-2}^{2} -3Y^2 (1 - (-1)) \ dY$$

$$= -2 \cdot 3 \int_{-2}^{2} Y^2 \ dY = 2 \cdot 3 \left[\frac{1}{3} Y^3 \right] \Big|_{-2}^{2}$$

$$= 2 \cdot \left(2^3 - (-2)^3 \right) = 2 \cdot 2^4 = 2^5 \quad \checkmark$$