**MA 1024. HOMEWORK 6**
**DUE: FRIDAY APRIL 25**

Assigned Problems:

*Section 13.5:* 14, 18, 26, 32, 34, 54,

*Section 13.6:* 2, 8, 12, 16, 22, 36, 42

Recommended Problems: (only hand in the subset listed above)

*Section 13.5:* 1 – 35, 53 – 55

*Section 13.6:* 1 – 46

MA 1024 HOMEWORK 6

13.5

(18) $m = \iint_R 1\, dA$ $\qquad 1 \le x \le e$, $\; 0 \le y \le \ln x$

so $$m = \int_1^e \int_0^{\ln x} dy\, dx = \int_1^e \ln x\, dx = \Big[x\ln x - x\Big]_1^e = (e\ln e - e) - (\ln 1 - 1)$$
$$= 1$$

$$M_x = \int_1^e \int_0^{\ln x} y\, dy\, dx = \int_1^e \frac{(\ln x)^2}{2}\, dx = \int_1^e \frac{x(\ln x)^2}{2} \cdot \frac{1}{x}\, dx$$

use integration by parts :) $u(x) = (\ln x)^2$, $v(x) = x$

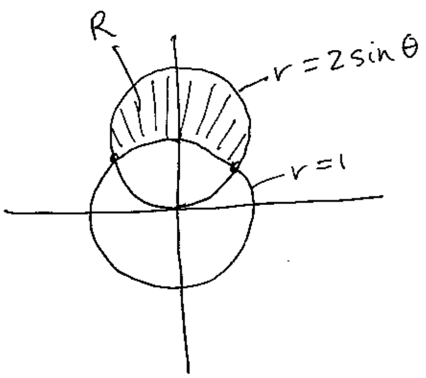
$$= \frac{x(\ln x)^2}{2} - 2\int_1^e \frac{\ln x}{2}\, dx = \left[\frac{x(\ln x)^2}{2} + x - x\ln x\right]_1^e$$

$$= \frac{e}{2} - 1$$

$$M_y = \int_1^e \int_0^{\ln x} x\, dy\, dx = \int_1^e x\ln x\, dx = \left[\frac{x^2 \ln x}{2}\right]_1^e - \int_1^e \frac{x}{2}\, dx$$ (int by parts again $u = \ln x$, $v = \frac{x^2}{2}$)

$$= \left[\frac{x^2\ln x}{2} - \frac{x^2}{4}\right]_1^e = \left(\frac{e^2}{2} - \frac{e^2}{4}\right) + \frac{1}{4} = \frac{1+e^2}{4}$$

(34) insider $r = 2\sin\theta$, outside $r = 1$, $\delta(x,y) = y$




the circles intersect when

$$2\sin\theta = 1$$
$$\sin\theta = \tfrac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}.$$

so $$\frac{\pi}{6} \le \theta \le \frac{5\pi}{6}$$
$$1 \le r \le 2\sin\theta.$$

to find $\quad I_0 = \iint_R r^2 \delta(x,y)\, dA$, note $\delta(x,y) = y$

$\Rightarrow \delta(r,\theta) = r\sin\theta$.

so $\quad I_0 = \int_{\pi/6}^{5\pi/6} \int_1^{2\sin\theta} r^4 \sin\theta \, dr\, d\theta$.

$$= \int_{\pi/6}^{5\pi/6} \left[\frac{r^5}{5}\sin\theta\right]_1^{2\sin\theta} d\theta$$

$$= \int_{\pi/6}^{5\pi/6} \frac{2^5}{5}\sin^6\theta \, \cancel{d\theta} - \frac{\sin\theta}{5} \, d\theta$$

$$= \frac{1}{5}\int_{\pi/6}^{5\pi/6} \left[32\sin^6\theta - \sin\theta\right] d\theta$$

I used a computer algebra system for this part :)

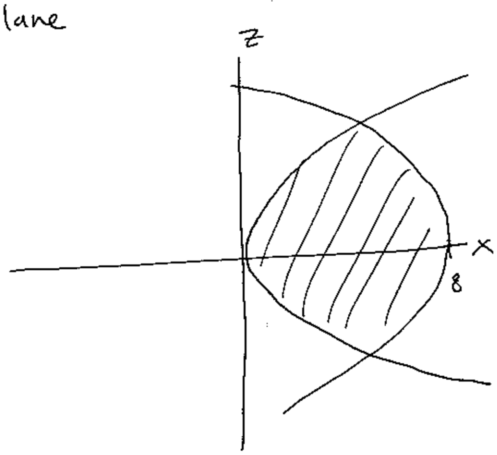
$$= \frac{1}{5}\left[10\theta - \frac{15}{2}\sin(2\theta) + \frac{3}{2}\sin(4\theta) - \frac{1}{6}\sin(6\theta) + \cos\theta\right]_{\pi/6}^{5\pi/6}.$$

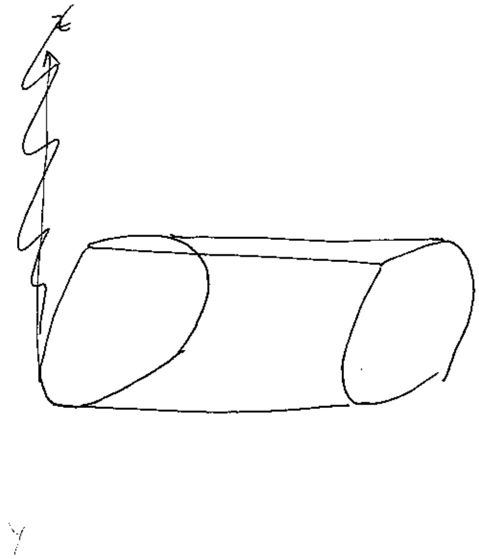
$$= \frac{4\pi - 3\sqrt{3}}{3}$$

B.6

(16) $x = z^2, \quad x = 8 - z^2, \quad y = -1, \quad y = -3.$

in x-~~z~~ plane




so $\quad z^2 \le x \le 8 - z^2$

$\quad -1 \le y \le -3$

to find limits of integration on $z$, let $z^2 = 8 - z^2$

$z^2 = 4$

$\Rightarrow z = \pm 2.$

so $V = \displaystyle\int_{-3}^{-1}\int_{-2}^{2}\int_{z^2}^{8-z^2} 1 \, dx\, dz\, dy = \int_{-3}^{-1}\int_{-2}^{2} 8 - 2z^2 \, dz\, dy$

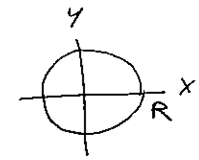
$= \displaystyle\int_{-3}^{-1} \left[ 8z - \frac{2}{3} z^3 \right]_{-2}^{2} dy = \int_{-3}^{-1} \frac{64}{3} \, dy = \frac{128}{3}.$

(22) $\delta = 1$, find centroid of $x^2+y^2+z^2 \le R^2$, $z \ge 0$.

This is symmetric wrt $x-y$, so $\bar{x} = \bar{y} = 0$.

Because density $= 1$, the mass is just the volume,

$$m = \frac{1}{2}\left(\frac{4}{3}\pi R^3\right) = \frac{2}{3}\pi R^3.$$




Let $R$ be the region $x^2+y^2 \le R^2$ in $x-y$ plane

Then

$$M_{xy} = \iint_R \int_0^{\sqrt{R^2-x^2-y^2}} z \, dz \, dA.$$

switch to polars: after 1st integral

$$M_{xy} = \iint_R \left[\frac{z^2}{2}\right]_0^{\sqrt{R^2-x^2-y^2}} dA = \iint_R \frac{R^2-x^2-y^2}{2} \, dA$$

$$= \int_0^{2\pi} \int_0^R \left(\frac{R^2-r^2}{2}\right) r \, dr \, d\theta$$

$$= \int_0^{2\pi} \left[\frac{R^2 r^2}{4} - \frac{r^4}{8}\right]_0^R d\theta = \int_0^{2\pi} \frac{R^4}{8} \, d\theta = \frac{R^4 \pi}{4}$$