MA 1024. HOMEWORK 7
DUE: FRIDAY MAY 2

Assigned Problems:

Section 13.7: 2, 6, 14, 22, 30, 34

Section 13.8: 2, 6, 10, 12, 18

Recommended Problems: (only hand in the subset listed above)

Section 13.7: 1 – 44

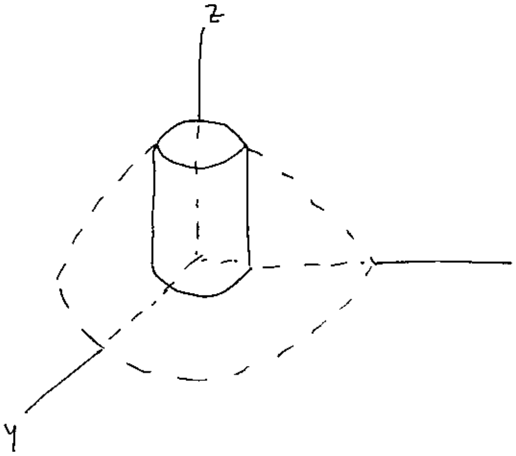
Section 13.8: 1 – 24

13.7 (6) centroid of the region bounded by

$x^2+y^2+z^2=4$ and

$x^2+y^2=1$ and

$z>0$



need mass. Use cylindricals

$$\text{mass} = \int_0^{2\pi}\int_0^1\int_0^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^1 r\sqrt{4-r^2}\,dr\,d\theta$$

let $u=4-r^2$ $\quad du=-2r\,d\cancel{x}$

$$= \int_0^{2\pi}\int_{r=0}^{r=1} -2\sqrt{u}\,du\,d\theta = \int_0^{2\pi}\left[-\frac{4}{3}\cancel{a}(4-r^2)^{\frac{3}{2}}\right]_0^1 d\theta$$

$$= 2\pi\left(-\frac{4}{3}\left(3^{\frac{3}{2}}-4^{\frac{3}{2}}\right)\right) = \frac{2}{3}\pi\left(8-2\sqrt{3}\right)$$

Centroid must have $\bar{x}=\bar{y}=0$ so

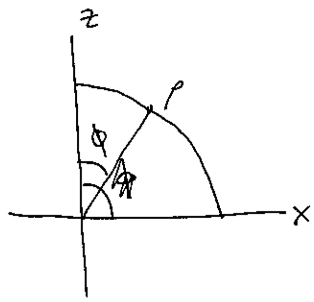
$$\bar{z} = \frac{1}{m}\iiint z\,dz\,dx\,dy = \frac{1}{m}\int_0^{2\pi}\int_0^1\int_0^{\sqrt{4-r^2}} rz\,dz\,dr\,d\theta$$

$$= \frac{1}{m}\int_0^{2\pi}\int_0^1 \frac{1}{2}r(4-r^2)\,dr\,d\theta = \frac{1}{2m}\int_0^{2\pi} 2r^2-\frac{r^4}{4}\Big]_0^1 d\theta$$

$$= \frac{1}{2m}\cdot 2\pi\cdot\left(\frac{7}{4}\right) = \frac{7\pi}{4}\cdot\frac{3}{2\pi\left(8-2\sqrt{3}\right)} = \frac{21}{8\left(8-2\sqrt{3}\right)}$$

(22) $x^2 + y^2 + z^2 \leq a^2, \qquad z > 0 \qquad \delta(x, y, z) = kz$

mass



$0 \leq \theta \leq 2\pi$

$0 \leq \phi \leq \frac{\pi}{2}$

$0 \leq \rho \leq a.$

so $\quad \text{mass} = \displaystyle\int_0^{2\pi}\int_0^{\pi/2}\int_0^{a} k\rho^3 \sin\phi \cos\phi \, d\rho \, d\phi \, d\theta$

$$= \frac{k}{4}\int_0^{2\pi}\int_0^{\pi/2} a^4 \sin\phi \cos\phi \, d\phi \, d\theta$$

$$= \frac{ka^4}{4}\int_0^{2\pi} \left.\frac{\sin^2\phi}{2}\right]_0^{\pi/2} d\theta = \frac{ka^4}{4} \cdot \frac{1}{2} \cdot 2\pi = \frac{k\pi a^4}{4}$$

Now $\bar{x} = \bar{y} = 0$, so need only compute $\bar{z}$

$$\bar{z} = \int_0^{2\pi}\int_0^{\pi/2}\int_0^{a} k\rho^4 \sin\phi \cos^2\phi \, d\rho \, d\phi \, d\theta$$

$$= \frac{ka^5}{5}\int_0^{2\pi}\int_0^{\pi/2} \sin\phi \cos^2\phi \, d\phi \, d\theta = \frac{ka^5}{5}\int_0^{2\pi}\left[-\frac{\cos^3\theta}{3}\right]_0^{\pi/2} d\theta$$

$$= \frac{ka^5}{5} \cdot 2\pi \cdot \frac{1}{3} = \frac{2}{15} k\pi a^5$$

so centroid is $\left(0, 0, \frac{2}{15} k\pi a^5\right)$.

13.8

(10) area cut from $z = x^2 - y^2$ by cylinder $x^2 + y^2 = 4$

$$A = \iint_R \sqrt{1 + z_x^2 + z_y^2}\, dx\,dy = \iint_R \sqrt{1 + 4x^2 + 4y^2}$$

R → cylinder

switch to polars, integrate over $0 \le \theta \le 2\pi$
$0 \le r \le 2$

$$= \int_0^{2\pi} \int_0^2 \sqrt{1+4r^2}\; r\,dr\,d\theta$$

note we need this b/c we started in rectangular coords and then convert.

let $u = 1 + 4r^2$

$$\frac{du}{dr} = 8r$$

$$A = \frac{1}{8} \int_0^{2\pi} \int_{r=0}^{r=2} u^{\frac{1}{2}}\, du\, d\theta = \frac{1}{12} \int_0^{2\pi} \left(1 + 4r^2\right)^{\frac{3}{2}} \Big]_0^2 \, d\theta$$

$$= \frac{1}{12} \cdot 2\pi \cdot \left(17^{\frac{3}{2}} - 1\right)$$

$$= \frac{\pi}{6}\left(17\sqrt{17} - 1\right)$$