

# MA1024 D14 - D03 - Quiz #1 Solutions

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## Problem 1

Our problem is to find and sketch the domain of the function  $f(x, y) = \ln(x^2 + y^2 - 4)$ . We know that  $\ln$  is defined for positive values only (i.e.,  $\ln(t)$  then  $t > 0$ ), thus, we must have that

$$x^2 + y^2 - 4 > 0,$$

i.e.,  $x^2 + y^2 > 2^2$ . So the domain of  $f(x, y)$  is all points outside of and not including the circle of radius 2.

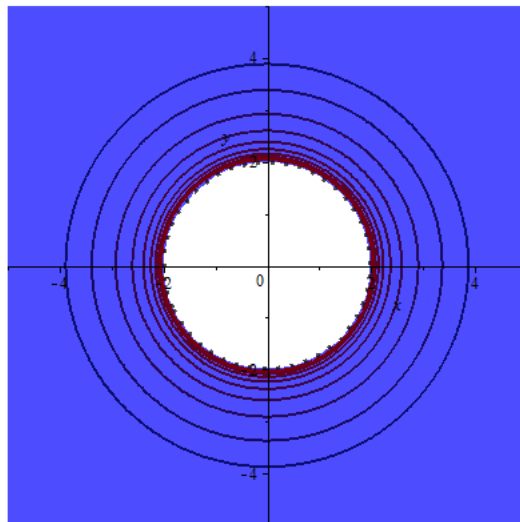


Figure 1: Allowed domain is shaded in blue.

The function itself is shaped like the usual  $\ln$  function shifted over by 2 and rotated about the  $z$ -axis. Notice that the level curves of the function are circles of varying radii.

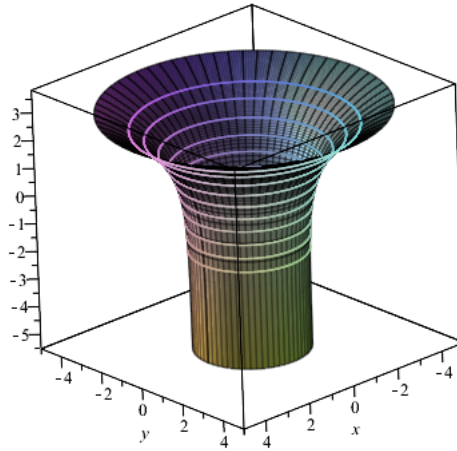


Figure 2: 3D plot of surface.

## Problem 2

We wish to find the following limit (if it exists),

$$\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2}{x^2 + 2y^2}.$$

As a first guess, we will try examining the limit along lines of the form  $y = mx$ . If the resulting value depends on  $m$  then we know that the limit **does not** exist.

$$\begin{aligned} \lim_{(x,y) \rightarrow (0,0)} \frac{3x^2}{x^2 + 2y^2} &= \lim_{x \rightarrow 0} \frac{3x^2}{x^2 + 2(mx)^2} \\ &= \lim_{x \rightarrow 0} \frac{x^2}{x^2} \left( \frac{3}{1 + 2m^2} \right) \\ &= \left( \frac{3}{1 + 2m^2} \right) \end{aligned}$$

Thus, because the limit depends on  $m$ , this limit depends on the path of approach. We conclude that the function is discontinuous at the point  $(0,0)$  and the resulting limit does not exist.

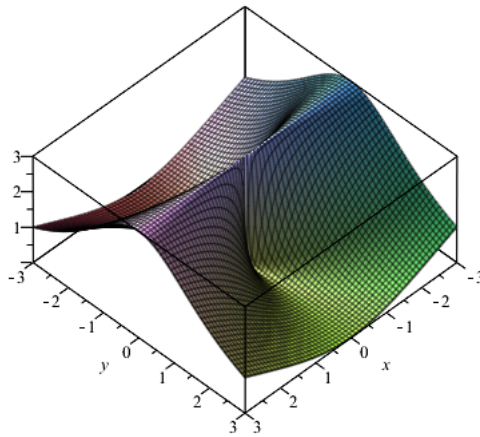


Figure 3: Plot of the function  $f(x, y) = \frac{3x^2}{x^2 + 2y^2}$ .

### Problem 3

We need to find the partial derivatives of the function  $f(x, y) = xy e^{4y}$ .

$$f_x(x, y) = (1)ye^{4y} = ye^{4y}$$

$$f_y(x, y) = x(1)e^{4y} + xye^{4y}(4) = xe^{4y}(1 + 4y)$$

$$f_{xy}(x, y) = e^{4y}(1 + 4y)$$

$$f_{yx}(x, y) = e^{4y}(1 + 4y)$$