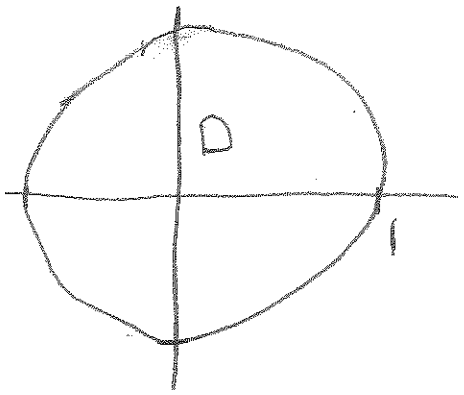
Name: _____ Monday, March 31, 2014

MA1024 D07 **Quiz 2**

1. **(6 marks)** Find the maximum and minimum values of $f(x,y) = 12x^2 + 13y^2$ on the disk $D = \{(x,y) : x^2 + y^2 \leq 1\}$.

1) Sketch Domain



1) Find Interior Critical Points

$f_x(x,y) = 24x = 0 \iff x = 0$

$f_y(x,y) = 26y = 0 \iff y = 0$

So $(0,0)$ is an interior critical point & a candidate for global max/min on D

2) Find boundary critical points. Boundary is given by $x^2+y^2=1$

so f on boundary is

$g(x) = f(x, y(x)) = 12x^2 + 13(1-x^2) = 13 + (12-13)x^2 = 13 - x^2$

so $g'(x) = -2x = 0 \iff x = 0$, when $x=0$ $y(0) = \pm\sqrt{1-0^2} = \pm 1$

so points $(0,1)$ and $(0,-1)$ are candidates

3) Check Corner points.

$(1,0)$ and $(-1,0)$

P	F(P)
(0,0)	0
(0,1)	13
(0,-1)	13

P	F(P)
(-1,0)	12
(1,0)	12

Thus max is 13 and occurs at $(0,\pm 1)$

And min is 0 and occurs at $(0,0)$

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This quiz has two sides!!!

2. **(4 marks)** Let

$$w = \frac{1}{u^2 + v^2},$$

where $u = \cos 2t$, and $v = \sin 2t$. Using the chain rule, find dw/dt.

$\frac{du}{dt} = (2)(-\sin(2t))$ $\qquad$ $\frac{dv}{dt} = (2)(\cos(2t))$

$\frac{\partial w}{\partial u} = -\frac{1}{(u^2+v^2)^2}(2u)$ $\qquad$ $\frac{\partial w}{\partial v} = -\frac{1}{(u^2+v^2)^2}2v$

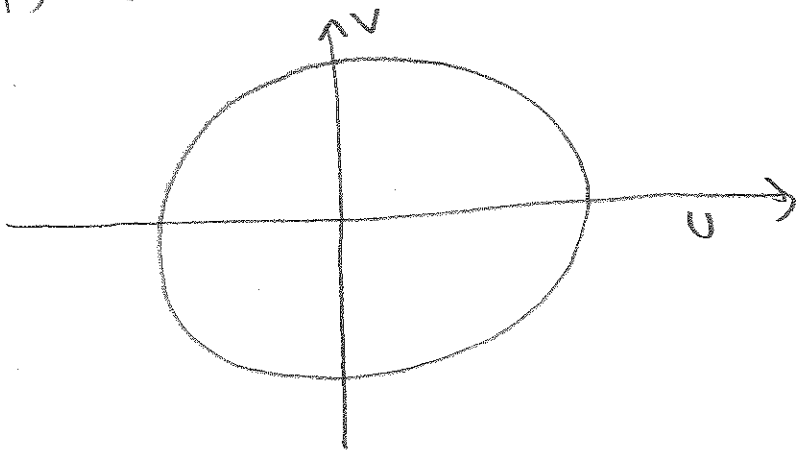
$$\frac{dw}{dt} = \frac{\partial w}{\partial u}\frac{du}{dt} + \frac{\partial w}{\partial v}\frac{dv}{dt}$$

$$= -\frac{4}{(\cos^2 2t + \sin^2 2t)^2}\left(\cos 2t(-\sin 2t) + \sin 2t(\cos 2t)\right)$$

$$\cos 2t(-\sin 2t) + \sin 2t(\cos 2t) \to 0$$

$$\boxed{\frac{dw}{dt} = -\frac{4}{1}\cdot(0) = 0}$$

What does this mean? In u-v plane $u = \cos 2t$, $v = \sin 2t$ is a circle for $t \in [0, \pi]$.



The contour lines of w are also circles, i.e., w does not vary along circles! $\frac{dw}{dt} = 0$