Name: ______________________________ Monday, April 14, 2014

MA1024 D03 **Quiz 3**

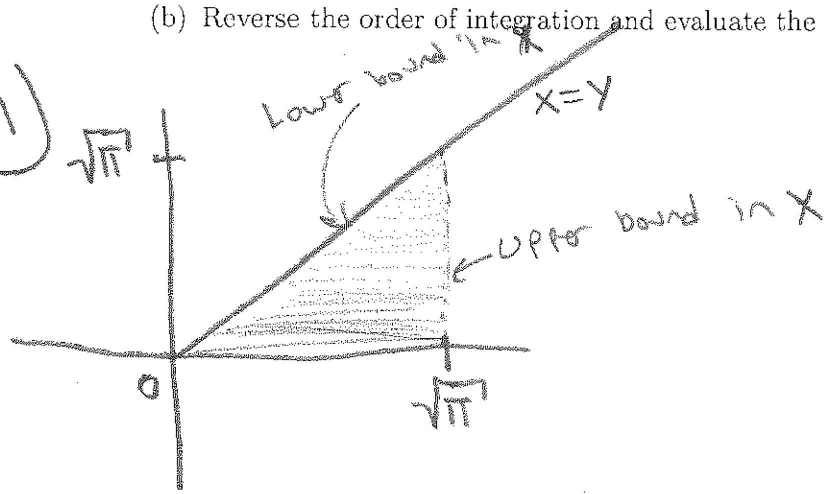
1. (**5 marks**) Consider the following integral:

$$\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \sin x^2 dx dy.$$

(a) Sketch the domain of integration.

(b) Reverse the order of integration and evaluate the resulting integral.

a)



b)

$u = x^2 \quad du = 2dx$

$$\int_0^{\sqrt{\pi}} \int_0^{x} \sin(x^2)\, dy\, dx = \int_0^{\sqrt{\pi}} x \sin(x^2)\, dx = \frac{1}{2}\int_0^{\pi} \sin u\, du$$

$$= \frac{1}{2}\left[-\cos u\right]_0^{\pi} = -\frac{1}{2}\left(\cos\pi - \cos(0)\right)$$

$$= -\frac{1}{2}(-1-1) = -\frac{1}{2}(-2) = \boxed{1}$$

This quiz has two sides!!!

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2. **(5 marks)** Evaluate the integral

$$\iint_R e^{x+y}\, dA$$

on the region $R = [0, \ln 2] \times [0, \ln 3]$ using iterated integrals in two different ways (e.g. first by integrating in the order "$dxdy$", and then "$dydx$").

$$\int_0^{\ln 2}\int_0^{\ln 3} e^{x+y}\, dy\, dx = \int_0^{\ln(2)}\int_0^{\ln(3)} e^x e^y\, dy\, dx = \int_0^{\ln(2)} e^x \int_0^{\ln(3)} e^y\, dy\, dx$$

$$= \int_0^{\ln(2)} e^x \left[e^y\right]\Big|_0^{\ln(3)} dx = \int_0^{\ln(2)} \left[e^{\ln(3)} - e^0\right] e^x\, dx$$

$$= (3-1)\int_0^{\ln(2)} e^x\, dx = 2\left(e^{\ln(2)} - e^0\right) = 2(2-1) = \boxed{2}$$

$$\int_0^{\ln(3)}\int_0^{\ln(2)} e^{x+y}\, dx\, dy = \int_0^{\ln(3)} e^y \int_0^{\ln(2)} e^x\, dx dy = \int_0^{\ln(3)} e^y\left(e^{\ln(2)} - e^0\right) dy$$

$$= (2-1)\int_0^{\ln(3)} e^y\, dy = e^{\ln(3)} - e^0 = 3 - 1 = \boxed{2}$$