MA 1024. HOMEWORK 3
DUE: FRIDAY APRIL 4

Assigned Problems:

Section 12.7: 4, 6, 16, 28, 30, 36, 48

Section 12.8: 8, 12, 16, 24, 32, 48, 50

~~*Section 12.10:* 2, 8, 10, 18, 24~~ Don't hand these in, but please note that you should know how to do this type of problem in preparation for the midterm. These problems will appear on Homework 4.

Recommended Problems: (only hand in the subset listed above)

Section 12.7: 1 – 37, 46, 47, 51

Section 12.8: 1 – 34, 45, 46, 51, 52

Section 12.10: 1 – 25

MA 1024 HOMEWORK 3.

12.7

(16) $p = f(v, w) \qquad v = v(x, y, z, t) \qquad w = w(x, y, z, t)$

$$\frac{\partial p}{\partial x} = \frac{\partial f}{\partial v}\frac{\partial v}{\partial x} + \frac{\partial f}{\partial w}\frac{\partial w}{\partial x} \qquad \frac{\partial p}{\partial z} = \frac{\partial f}{\partial v}\frac{\partial v}{\partial z} + \frac{\partial f}{\partial w}\frac{\partial w}{\partial z}$$

$$\frac{\partial p}{\partial y} = \frac{\partial f}{\partial v}\frac{\partial v}{\partial y} + \frac{\partial f}{\partial w}\frac{\partial w}{\partial y} \qquad \frac{\partial p}{\partial t} = \frac{\partial f}{\partial v}\frac{\partial v}{\partial t} + \frac{\partial f}{\partial w}\frac{\partial w}{\partial t}.$$

(30) eqn of plane tangent to

$x^2 + 2y^2 + 2z^2 = 14$ at $P(2, 1, -2)$

differentiate z implicitly wrt x, y.

first x: $\quad 2x + 4z\dfrac{\partial z}{\partial x} = 0$

then y: $\quad 4y + 4z\dfrac{\partial z}{\partial y} = 0.$

subs P: $\quad 4 + (-8)\dfrac{\partial z}{\partial x} = 0 \implies \dfrac{\partial z}{\partial x} = \dfrac{1}{2}$

$\quad 4 + (-8)\dfrac{\partial z}{\partial y} = 0 \implies \dfrac{\partial z}{\partial y} = \dfrac{1}{2}$

eqn of tangent plane:

$$z + 2 = \frac{1}{2}(x-2) + \frac{1}{2}(y-1)$$

$$0 = (x-2) + (y-1) - 2(z+2)$$

$$7 = x + y - 2z$$

(48) $w = f(u) + g(v) \qquad u = x - at \qquad v = x + at$

CLAIM: $$\frac{\partial^2 w}{\partial t^2} = a^2\frac{\partial^2 w}{\partial x^2}.$$

note $\dfrac{\partial w}{\partial u} = \dfrac{\partial f}{\partial u}$ $\qquad \dfrac{\partial w}{\partial v} = \dfrac{\partial g}{\partial v}.$

PROOF:

compute $\dfrac{\partial w}{\partial t} = \dfrac{\partial f}{\partial u}\dfrac{\partial u}{\partial t} + \dfrac{\partial g}{\partial v}\dfrac{\partial v}{\partial t}$, and $\dfrac{\partial u}{\partial t} = -a$

$\dfrac{\partial v}{\partial t} = a$

so $\quad \dfrac{\partial w}{\partial t} = f'(u)(-a) + g'(v)\,a$

then $\quad \dfrac{\partial^2 w}{\partial t^2} = \dfrac{\partial}{\partial t}\Big(f'(u)(-a) + g'(v)(a)\Big)$

but $f'(u)$ is a function of u, $g'(v)$ a function of v

so $\quad \dfrac{\partial^2 w}{\partial t^2} = -a\, f''(u)\dfrac{\partial u}{\partial t} + a\, g''(v)\dfrac{\partial v}{\partial t}$

$= a^2 f''(u) + a^2 g''(v).$

Similarly $\quad \dfrac{\partial w}{\partial x} = \dfrac{\partial f}{\partial u}\dfrac{\partial u}{\partial x} + \dfrac{\partial f}{\partial v}\dfrac{\partial v}{\partial x}$, and now $\dfrac{\partial u}{\partial x} = 1 = \dfrac{\partial v}{\partial x}$.

So $\quad \dfrac{\partial^2 w}{\partial x^2} = \dfrac{\partial}{\partial x}\big(f'(u) + g'(v)\big)$

$= f''(u) + g''(v)$

$= a^2 \dfrac{\partial^2 w}{\partial t^2}$, as desired.

12.8

(12) $f(x,y) = e^x \sin y \qquad P(0, \pi/4) \qquad \vec{v} = \langle 1, -1 \rangle.$

$\vec{u} = \dfrac{\vec{v}}{|\vec{v}|} = \left\langle \dfrac{1}{\sqrt{2}}, \dfrac{-1}{\sqrt{2}} \right\rangle.$

$\nabla f = \left\langle \dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y} \right\rangle = \left\langle e^x \sin y,\ e^x \cos y \right\rangle$

$\nabla f(0, \pi/4) = \left\langle \dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}} \right\rangle$

now $\quad D_{\vec{u}} f(0, \pi/4) = \nabla f(0, \pi/4) \cdot \left\langle \dfrac{1}{\sqrt{2}}, \dfrac{-1}{\sqrt{2}} \right\rangle$

$= \left\langle \dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}} \right\rangle \cdot \left\langle \dfrac{1}{\sqrt{2}}, \dfrac{-1}{\sqrt{2}} \right\rangle$

$= 0.$

(32) $3x^2 + 4y^2 + 5z^2 = 73 \qquad P(2,2,3)$ eqn of tangent plane?

$\nabla f(x,y,z) = \langle 6x, 8y, 10z \rangle$ is normal to point (x,y,z) on the surface.

at P,

$\nabla f(2,2,3) = \langle 12, 16, 30 \rangle$ so

$12(x-2) + 16(y-2) + 30(z-3) = 0$ is the plane tangent at P.

or $12x + 16y + 30z = 146$

(48) $w = 100 - x^2 - y^2 - z^2$.

a) rate of change of temp in direction $\vec{v} = \langle 3, -4, 12 \rangle$ at $P(3,-4,5)$.

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \left\langle \frac{3}{13}, \frac{-4}{13}, \frac{12}{13} \right\rangle$$

$$\nabla w = \langle -2x, -2y, -2z \rangle$$

$$\nabla w(3,-4,12) = \langle -6, 8, -10 \rangle$$

so $D_{\vec{u}} w = \nabla w \cdot \vec{u}$

$$= \langle -6, 8, -10 \rangle \cdot \left\langle \frac{3}{13}, \frac{-4}{13}, \frac{12}{13} \right\rangle$$

$$= \frac{-170}{13} \ {}^\circ C/m.$$

b) max directional derivative is $|\nabla w(P)| = \sqrt{6^2 + 8^2 + 10^2}$

$= \sqrt{200}$

$= 10\sqrt{2}$.

occurs in direction of gradient vect: $\langle -6, 8, -10 \rangle$.