MA 2073 Midterm 1

February 3, 2014

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Name: _____

Student Number: _____

You have 50 minutes to complete this exam.

Examination rules and instructions:

- closed book
- no calculators or cell phones are permitted

Question	Mark
1	
2	
3	
4	
5	
6	
7	
Total	/40

Question 1 (8 marks)**.** TRUE FALSE: Circle T for true, or F for false. Each correct answer is worth 1 mark.

a) **T / F** Every matrix with 1's down its diagonal is invertible.

b) **T / F** If A is invertible, then A^{-1} and A^2 are invertible.

c) **T / F** Every set of four non-zero vectors in $\mathbb{R}^4$ is a basis.

d) **T / F** If the rows of A are independent, and the columns of A are independent, then A is a square matrix.

e) **T / F** The column space of A is the set of all solutions of $A\mathbf{x} = \mathbf{b}$.

f) **T / F** The vectors $\mathbf{b}$ that are *not* in the column space of A form a subspace.

g) **T / F** An $m \times n$ matrix has no more than n pivot variables.

h) **T / F** If A is not a square matrix, then the row vectors of A must be dependent.

Question 2 (6 marks)**.**

a) Let A be an $n \times n$ matrix. Define A^{-1}.

b) Use the definition of the inverse of a matrix to prove that an $n \times n$ matrix with a column of zeros cannot have an inverse.

Question 3 (6 marks)**.** Find the complete solution to the following system of equations:

$$x + y + z = 4$$

$$x - y + z = 4$$

Question 4 (4 marks)**.** What matrix E puts A into upper triangular form $EA = U$? Multiply by $E^{-1} = L$ to factor A into LU:

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 0 \end{bmatrix}$$

Question 5 (6 marks)**.**

a) Draw the row and column pictures for the equations $x - 2y = 0$, $x + y = 6$.

b) What is the nullspace of the matrix A?

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$$

Question 6 (5 marks)**.**

a) Is $\mathbb{R}^k$ a subspace of $\mathbb{R}^n$ when $k < n$? Why or why not?

b) Describe geometrically all subspaces of $\mathbb{R}^3$, and list their dimensions.

Question 7 (5 marks)**.**

a) Find a basis for the subspace of $\mathbb{R}^4$ consisting of all vectors whose components are equal.

b) The cosine space $\mathbf{F_3}$ contains all combinations

$$y(x) = A\cos x + B\cos 2x + C\cos 3x.$$

Find a basis for the subspace of $\mathbf{F_3}$ with $y(0) = 0$ (recall that $\cos(0) = 1$).

This exam has 5 pages (including the cover sheet) and 7 problems, worth a total of 40 marks.