MA1024 D14 - D07 - Quiz #1 Solutions

William C. Sanguinet

March 30, 2014

Problem 1

Our problem is to find and sketch the domain of the function $f(x, y) = \sqrt{x^2 + y^2 - 4}$. We know that $\sqrt{}$ is defined for values greater than or equal to zero (i.e., $\sqrt{t}$ then $t \geq 0$), thus, we must have that

$$x^2 + y^2 - 4 \geq 0,$$

i.e., $x^2 + y^2 \geq 2^2$. So the domain of $f(x, y)$ is all points outside of and not including the circle of radius 2.

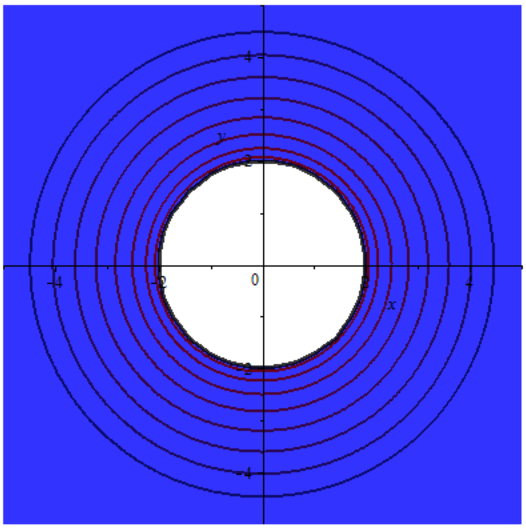

Figure 1: Allowed domain is shaded in blue.

The function itself is shaped like the usual $\sqrt{}$ function shifted over by 2 and rotated about the z-axis. Notice that the level curves of the function are circles of varying radii.

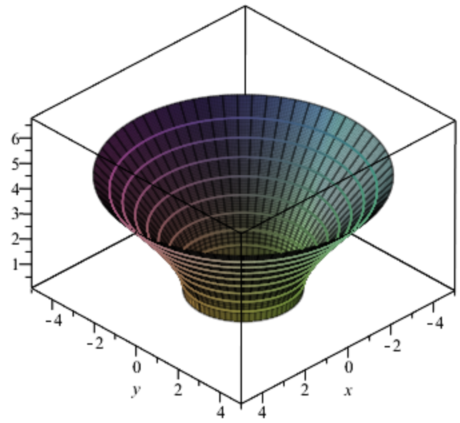

Figure 2: 3D plot of surface.

Problem 2

We wish to find the following limit (if it exists),

$$\lim_{(x,y)\to(0,0)} \frac{2x^2}{5x^2+4y^2}.$$

As a first guess, we will try examining the limit along lines of the form $y = mx$. If the resulting value depends on m then we know that the limit **does not** exist.

$$\begin{aligned}\lim_{(x,y)\to(0,0)} \frac{2x^2}{5x^2+4y^2} &= \lim_{x\to 0} \frac{2x^2}{5x^2+4(mx)^2} \\ &= \lim_{x\to 0} \frac{x^2}{x^2}\left(\frac{2}{5+4m^2}\right) \\ &= \left(\frac{2}{5+4m^2}\right)\end{aligned}$$

Thus, because the limit depends on m, this limit depends on the path of approach. We conclude that the function is discontinuous at the point $(0,0)$ and the resulting limit does not exist. For visualization of this, see the following figure.

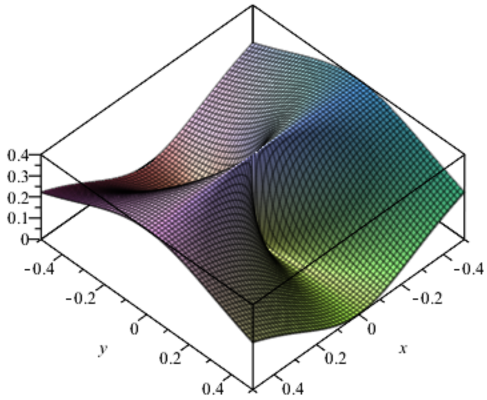

Figure 3: Plot of the function $f(x, y) = \frac{3x^2}{x^2+2y^2}$.

Problem 3

We need to find the partial derivatives of the function $f(x, y) = -3\sin(2x + y) + \cos(x - y)$, these are as follows,

$$\begin{aligned} f_x(x, y) &= -3\cos(2x + y)(2) - 9\sin(x - y)(1) = -6\cos(2x + y) - 9\sin(x - y), \\ f_y(x, y) &= -3\cos(2x + y)(1) - 9\sin(x - y)(-1) = -3\cos(2x + y) + 9\sin(x - y), \\ f_{xy}(x, y) &= 6\cos(2x + y) + 9\sin(x - y), \\ f_{yx}(x, y) &= 6\cos(2x + y) + 9\sin(x - y). \end{aligned}$$