MA 1024. HOMEWORK 1
DUE: FRIDAY MARCH 21

Assigned Problems:

2 2 2

Section 12.2: 4, 10, 14, 20, 24, 26, 34, 36, 40, 50, 54, 56

Section 12.3: 2, 4, 12, 22, 26, 28, 34, 36, 38, 51, 54

2 2 2 3

Recommended Problems: (only hand in the subset listed above)

Section 12.2: 1 – 58

Section 12.3: 1 – 40

MA 1024 HOMEWORK 1

12.2

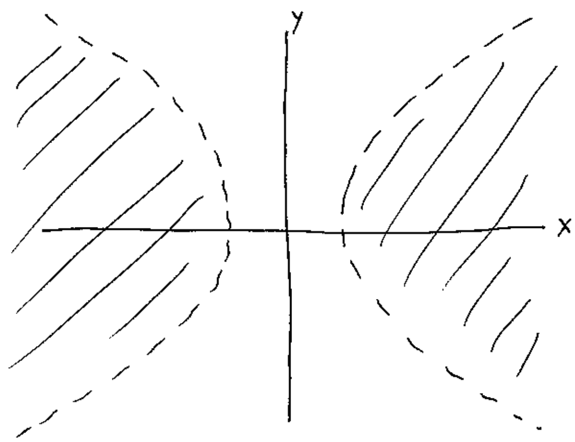
(10) $f(x,y) = \ln(x^2 - y^2 - 1)$

recall $\ln(t)$ is only defined for $t > 0$

so domain of $f(x,y)$ is $x^2 - y^2 - 1 > 0$

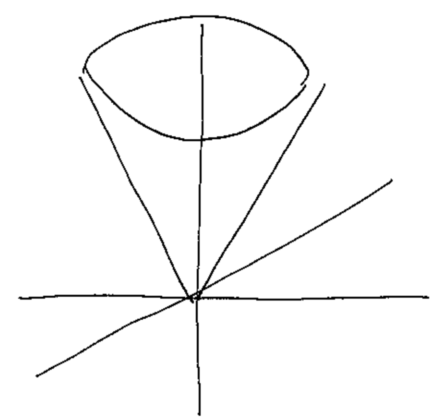
$$x^2 - y^2 > 1$$

The graph of $x^2 - y^2 = 1$ is a hyperbola:



← Domain of $f(x,y)$ is shaded region, not including the boundary.

(24) $f(x,y) = \sqrt{x^2 + y^2}$



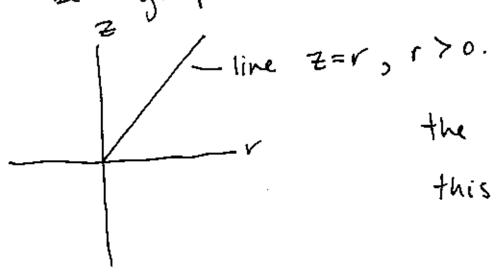
let $z = f(x,y)$. Then $z^2 = x^2 + y^2$

this is the graph of a cone:

(to see this, let $r = \sqrt{x^2+y^2}$

then our function $f(x,y) = z = r$

can be graphed:



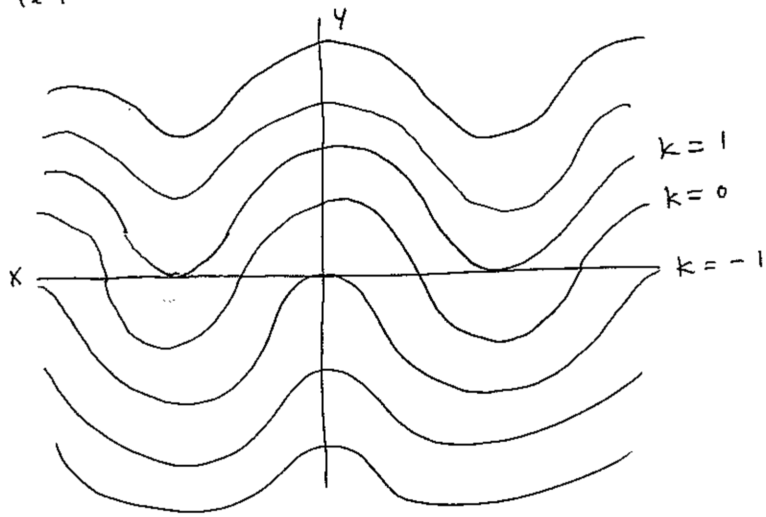
the cone is obtained by revolving this around the z-axis.

(36) level curves of $f(x,y) = y - \cos x$.

let $k = y - \cos x$

$y = \cos x + k$

now let $k = 0, \pm 1, \pm 2, \ldots$ etc.



12-3

(4) $f(x,y) = \dfrac{x+y}{1+xy}$ is the quotient of the continuous functions

$x+y$: cts as sum of cts fcns

$1+xy$: cts as " " " "

and xy is cts as the product of cts functions.

The quotient is continuous everywhere that it is defined, i.e $1 + xy \neq 0$

or $xy \neq -1$

hence $\lim_{(x,y)\to(0,0)} f(x,y) = \dfrac{0+0}{1+0\cdot 0} = \dfrac{0}{1} = 0.$

(since $0 \cdot 0 \neq -1$)

(28) $\displaystyle\lim_{(x,y)\to(0,0)} \sin\left(\ln(1+x+y)\right)$

the sum & composition of continuous functions are continuous where defined.

Hence our limit becomes

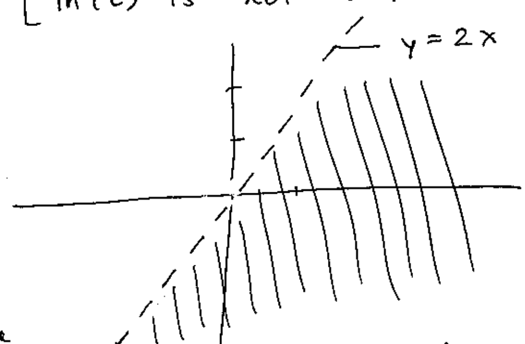
$$\sin\left(\ln(1+0+0)\right) = \sin\left(\ln(1)\right) = \sin(0) = 0.$$

(34) $f(x,y) = \ln(2x-y)$

To find the largest set ~~of~~ of points in the xy-plane where the function is defined, we need

$2x - y > 0$ [$\ln(t)$ is not defined for $t \leq 0$].

or $y < 2x$



This is the region of the plane under (not including) the line $y = 2x$

(51) Let $f(x,y) = \dfrac{2x^2 y}{x^4 + y^2}$

a) CLAIM: $f(x,y) \to 0$ as $(x,y) \to (0,0)$ along any & every straight line through the origin.

PROOF: A line through the origin has the form $y = mx$.

when $m \neq 0$,

$$f(x, mx) = \frac{2mx^3}{x^4 + m^2x^2} = \frac{2mx}{x^2 + m^2}$$

so $\lim_{(x,y)\to(0,0)} f(x, mx) = \frac{0}{m^2} = 0.$

when $m=0$, the line is vertical, and we approach along y-axis.

$\lim_{(x,y)\to(0,0)} f(0,y) = \frac{0}{y^2}$, which still approaches 0.

b) CLAIM: $f(x,y) \longrightarrow 1$ as $(x,y) \to (0,0)$ along the parabola $y = x^2$.

PROOF:

$$f(x, x^2) = \frac{2x^4}{x^2 + x^2} = \frac{\cancel{2}x^4}{2x^4} = 1.$$

hence the limit does not exist.