

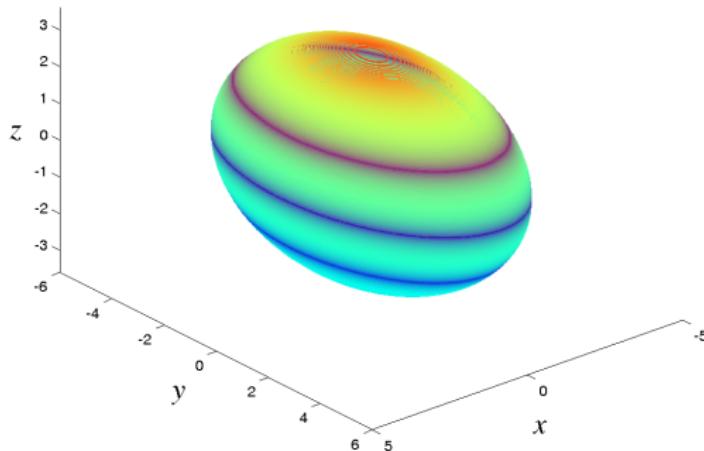
Geometria Analítica - Quádricas

Elizabeth Wegner Karas
Ademir Alves Ribeiro

Junho / 2006

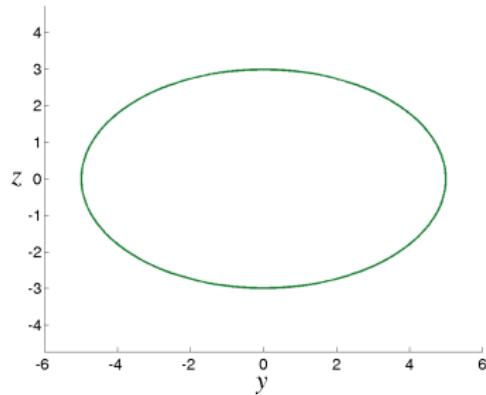
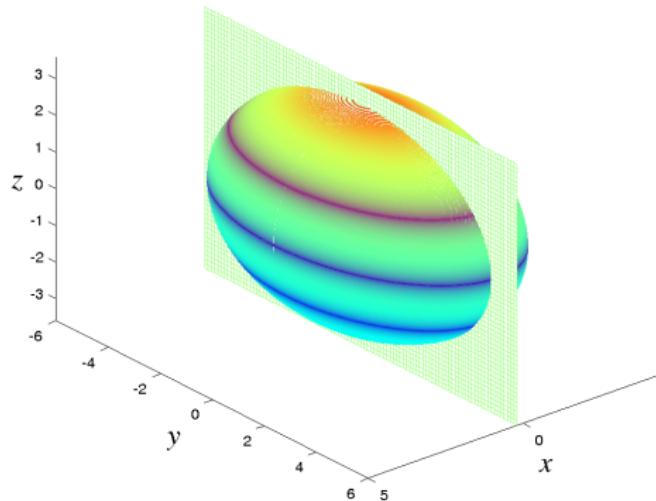
- 1 Elipsóide
- 2 Hiperbolóides
- 3 Parabolóides
- 4 Cilindros

Elipsóide: gráfico e equação



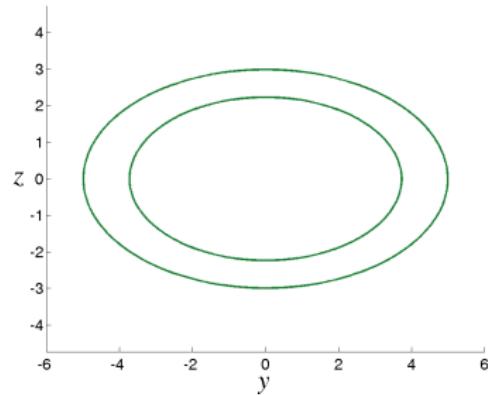
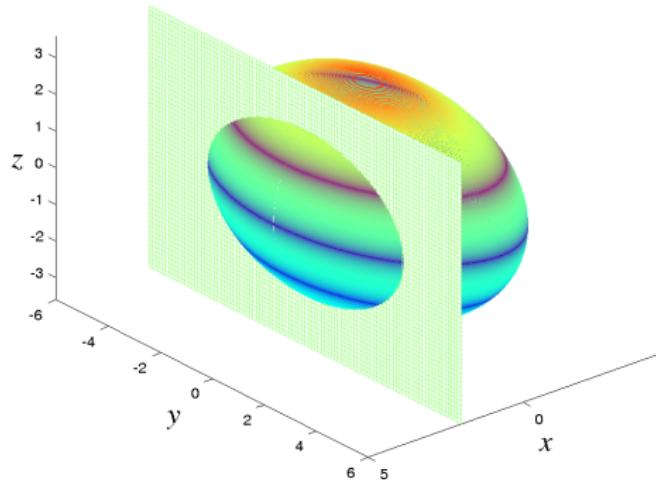
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Interseção com planos verticais: $x = k$



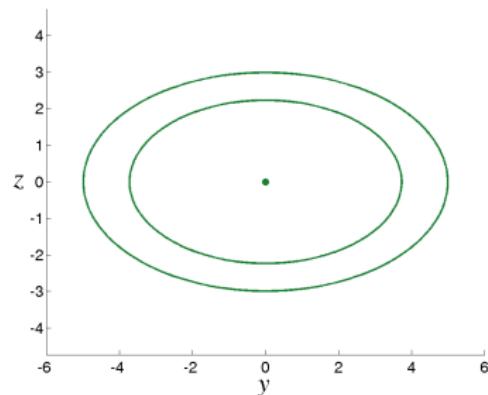
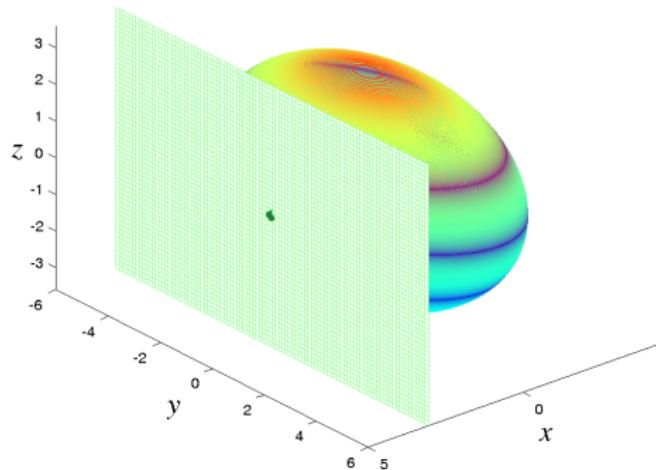
$$\begin{cases} \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 - \frac{k^2}{a^2} & : \text{ellipses } (-a < k < a). \\ x = k \end{cases}$$

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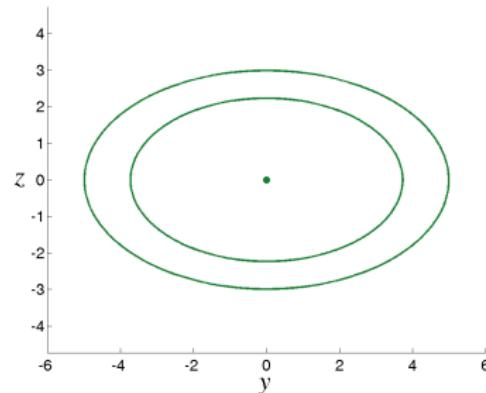
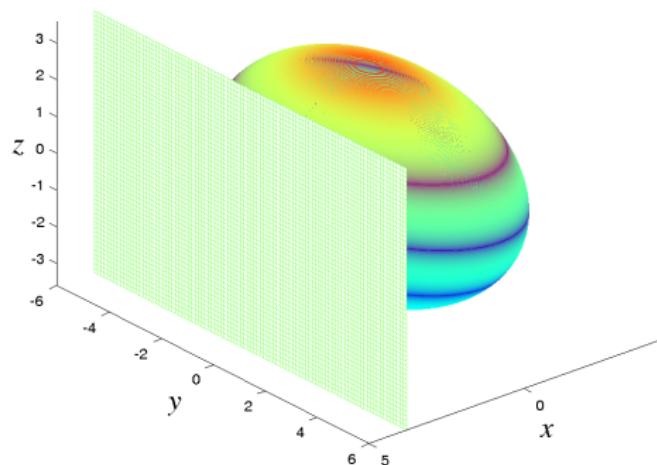
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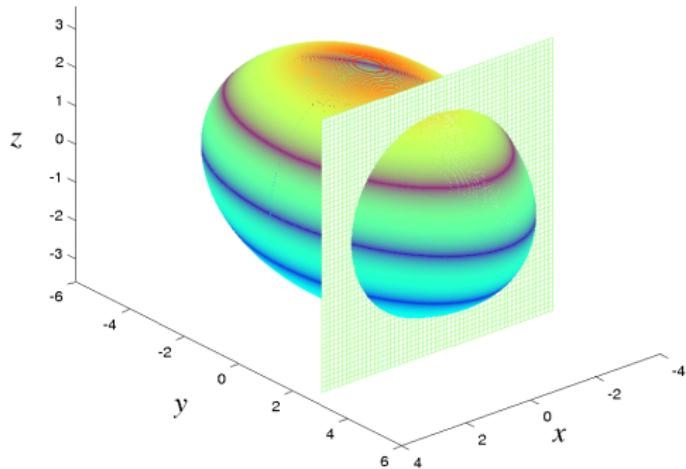
$$\left\{ \begin{array}{l} \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0 \\ x = k \end{array} \right. : \text{ um ponto } (k = -a \text{ ou } k = a).$$

Interseção com planos verticais: $x = k$



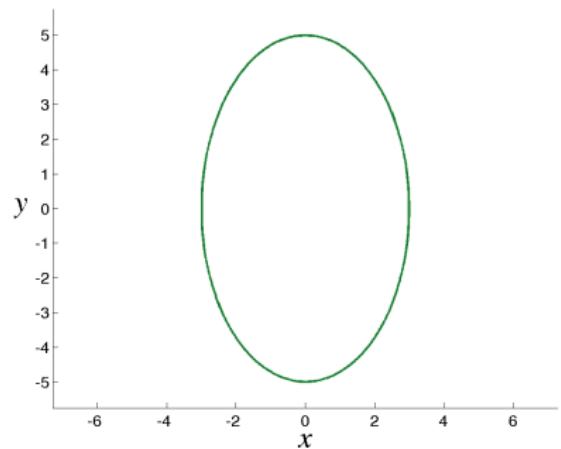
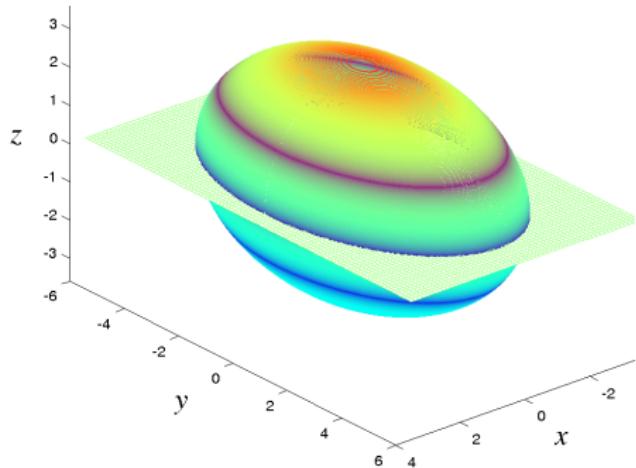
- O conjunto vazio quando $k = < -a \text{ ou } k > a$.

Interseção com planos verticais: $y = k$



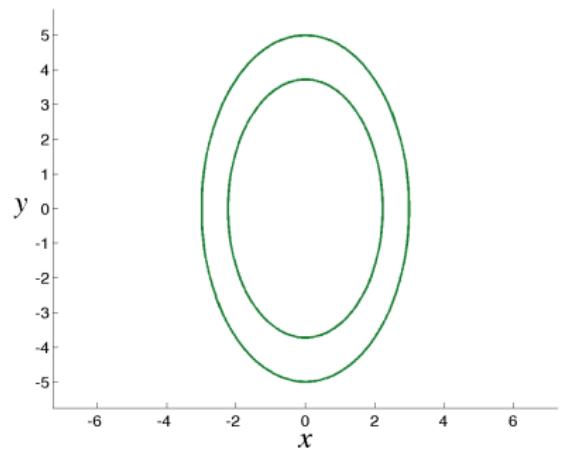
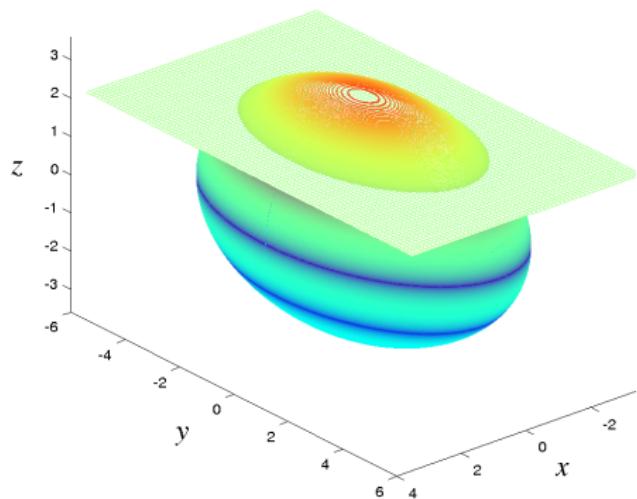
- Análogo ao caso anterior ($x \leftrightarrow y, a \leftrightarrow b$).

Interseção com planos horizontais: $z = k$



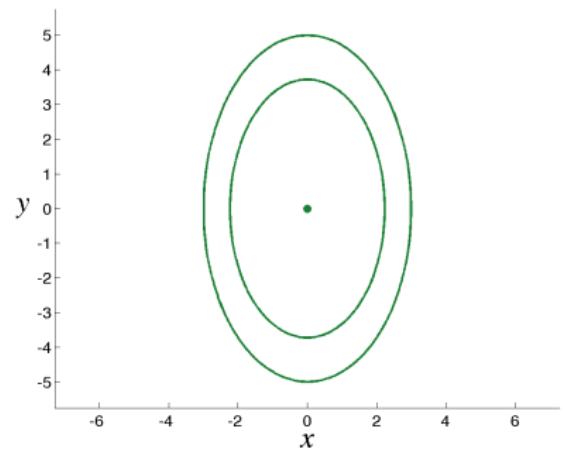
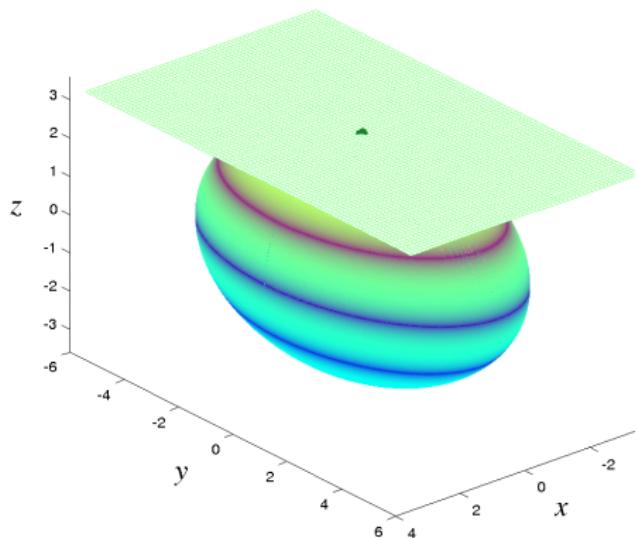
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \frac{k^2}{c^2} \\ z = k \end{array} : \text{ elipses } (-c < k < c). \right.$$

Interseção com planos horizontais: $z = k$



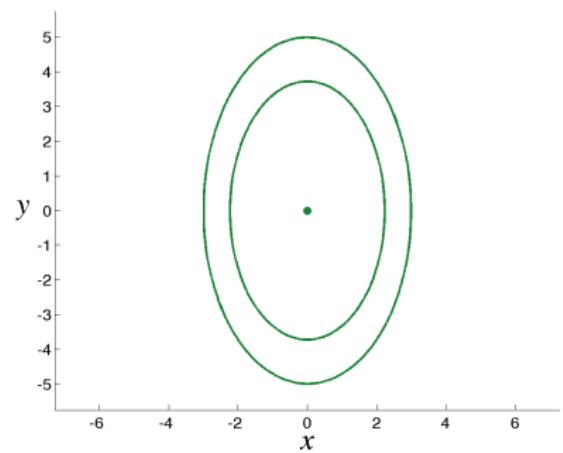
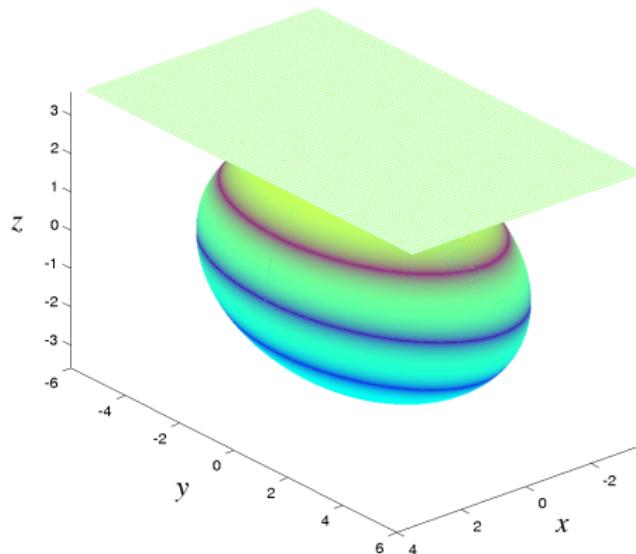
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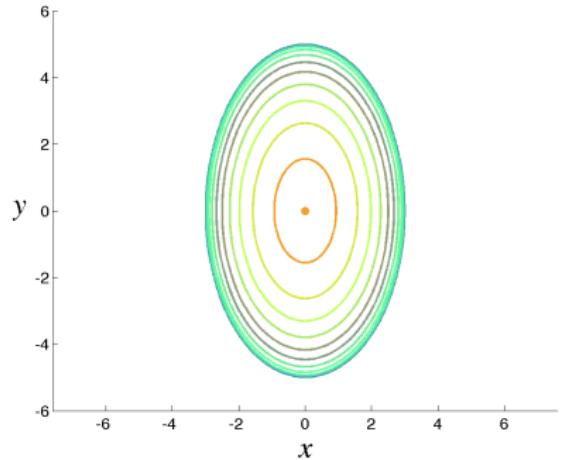
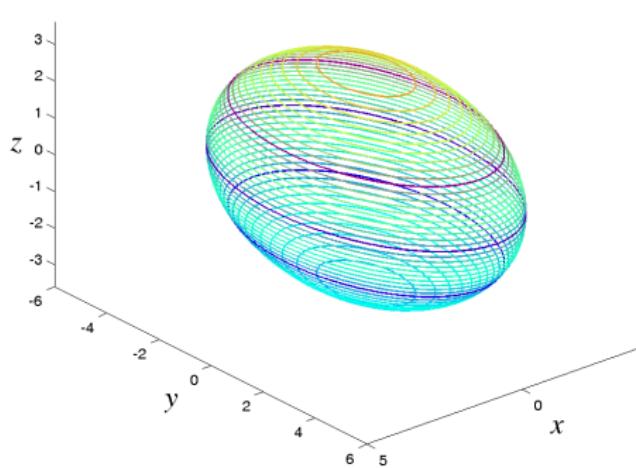
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 0 \\ z = k \end{array} \right. : \text{ um ponto } (k = -c \text{ ou } k = c).$$

Interseção com planos horizontais: $z = k$



- O conjunto vazio quando $k = < -c$ ou $k > c$.

Curvas de nível em \mathbb{R}^3 e em \mathbb{R}^2



- As curvas de nível são elipses para todo $k \in (-c, c)$, um ponto para $k = -c$ ou $k = c$ e o conjunto vazio para $k < -c$ ou $k > c$.

Equação geral de um hiperbolóide

Equação geral

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = \alpha$$

- $\alpha = 1$: hiperbolóide de uma folha
- $\alpha = -1$: hiperbolóide de duas folhas
- $\alpha = 0$: hiperbolóide degenerado (cone)

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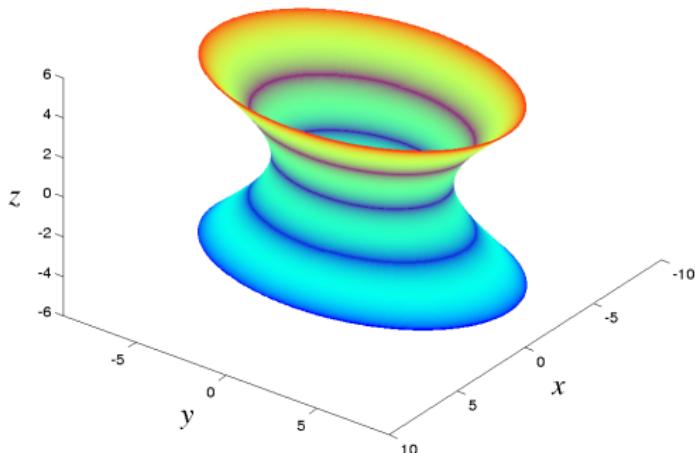
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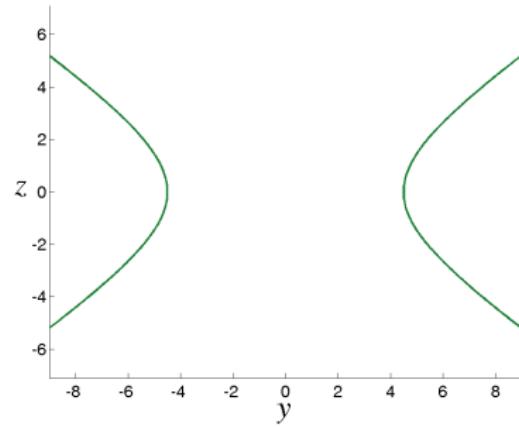
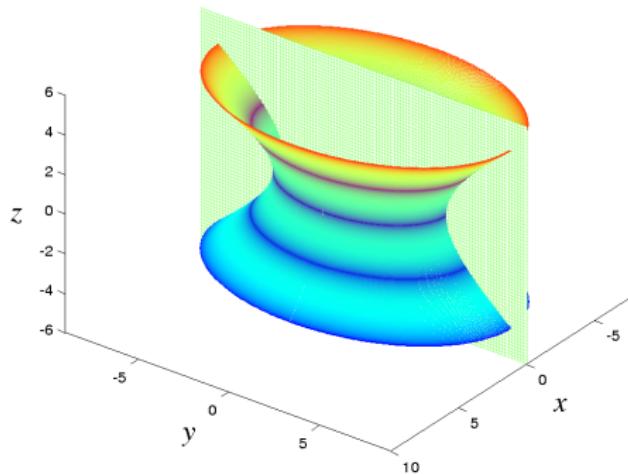
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Hiperbolóide de uma folha: gráfico e equação



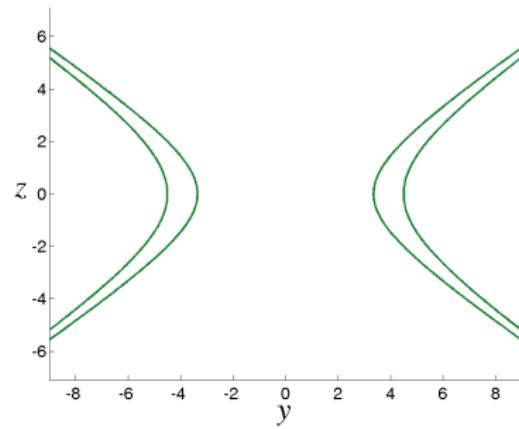
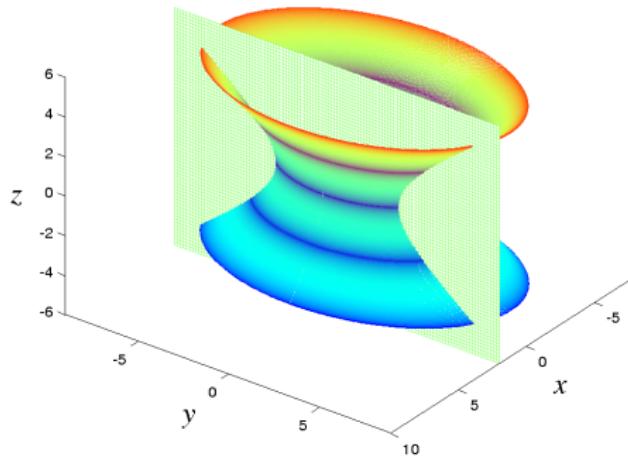
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Interseção com planos verticais: $x = k$



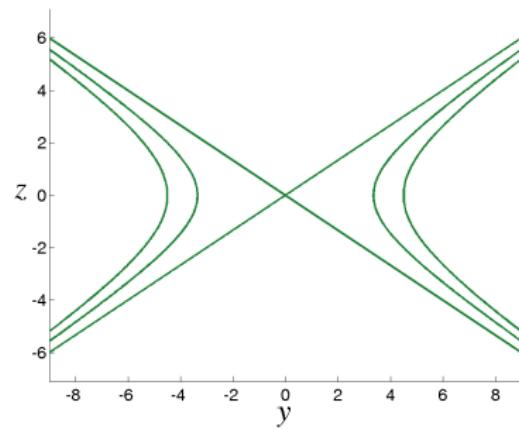
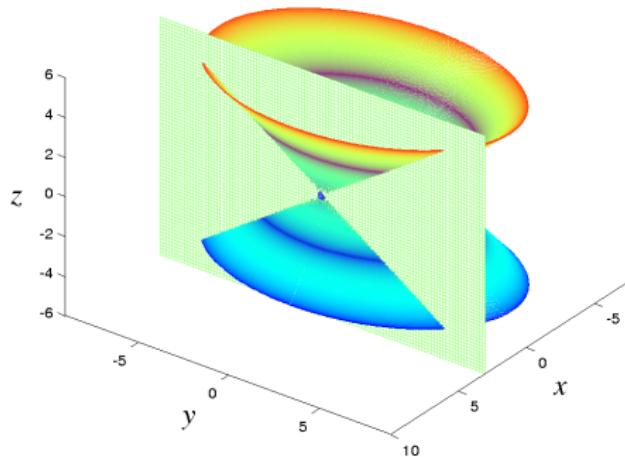
$$\left\{ \begin{array}{l} \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 - \frac{k^2}{a^2} \\ x = k \end{array} \right. : \text{ hipérboles } (-a < k < a).$$

Interseção com planos verticais: $x = k$



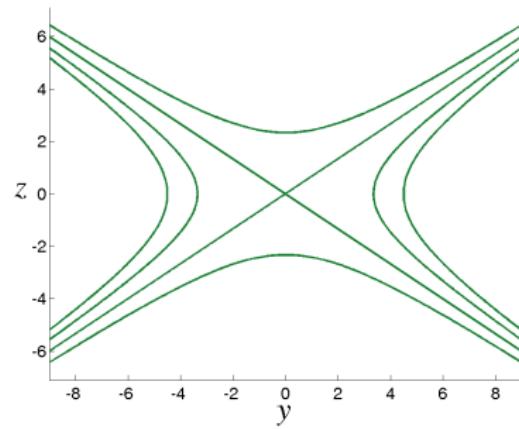
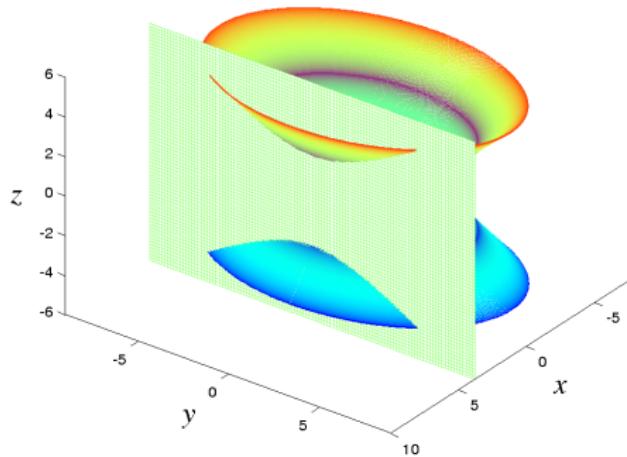
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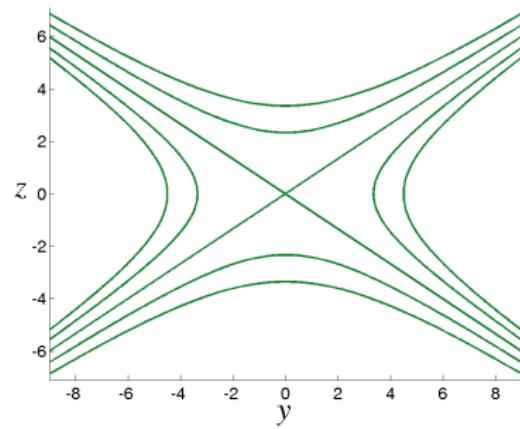
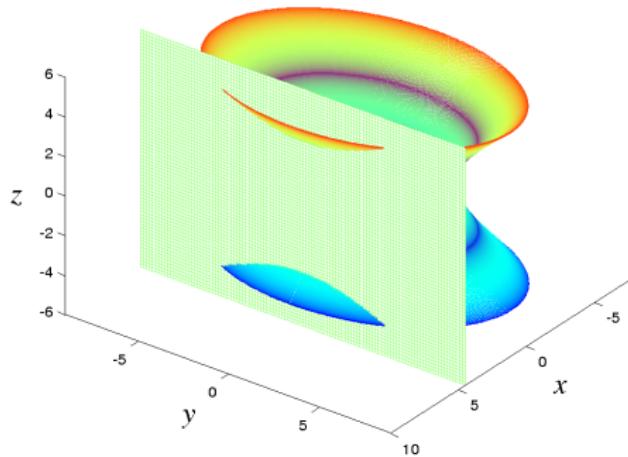
$$\left\{ \begin{array}{l} \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0 \\ x = k \end{array} \right. : \text{duas retas concorrentes } (k = -a \text{ ou } k = a).$$

Interseção com planos verticais: $x = k$



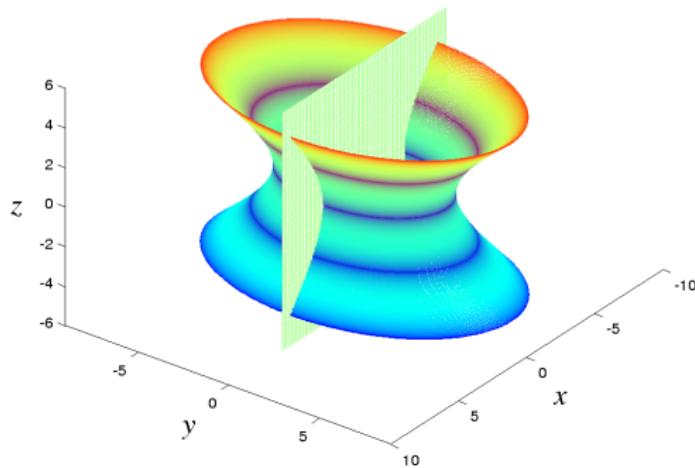
$$\begin{cases} -\frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{k^2}{a^2} - 1 \\ x = k \end{cases} : \text{ hipérboles } (k < -a \text{ ou } k > a).$$

Interseção com planos verticais: $x = k$



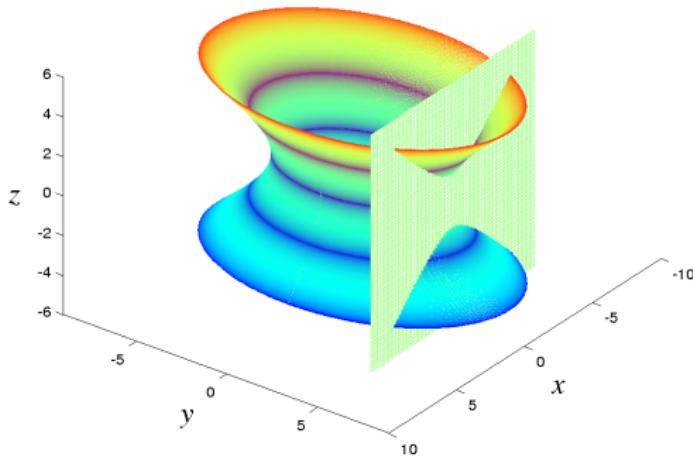
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Interseção com planos verticais: $y = k$



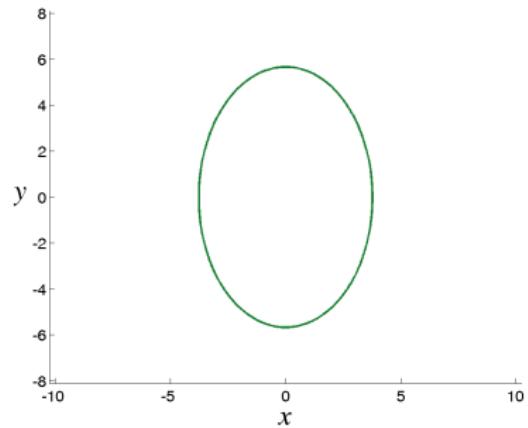
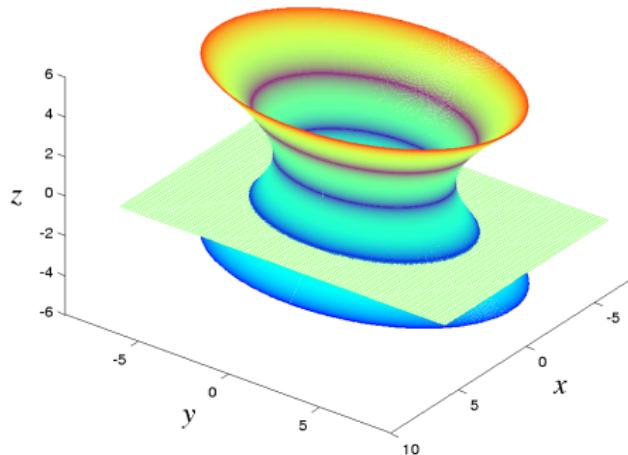
- Análogo ao caso anterior ($x \leftrightarrow y, a \leftrightarrow b$).

Interseção com planos verticais: $y = k$



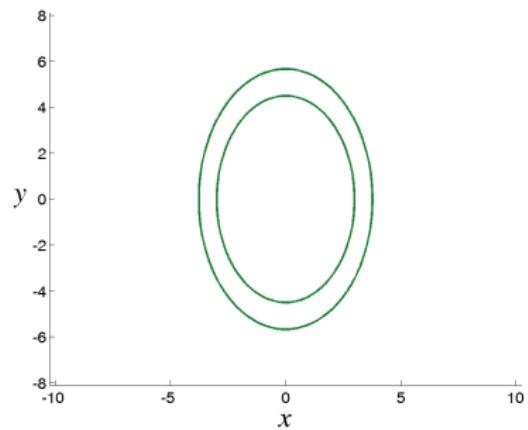
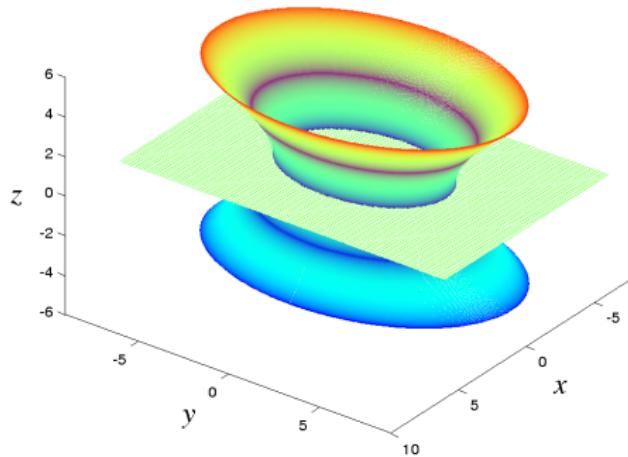
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Interseção com planos horizontais: $z = k$



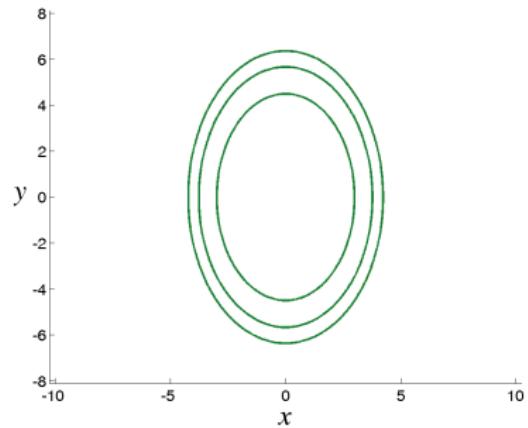
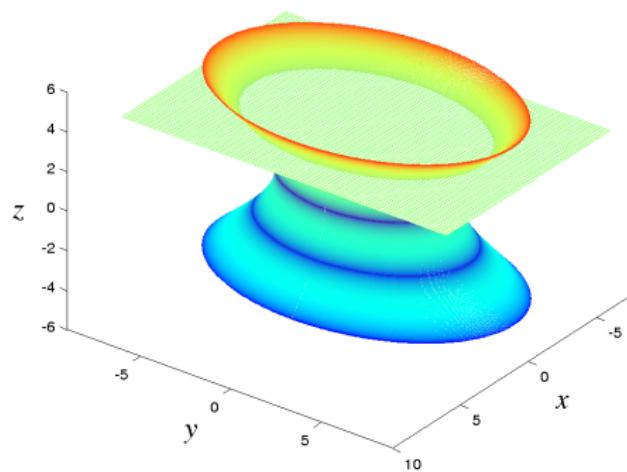
$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 + \frac{k^2}{c^2} \\ z = k \end{cases} : \text{ elipses } \forall k \in \mathbb{R}.$$

Interseção com planos horizontais: $z = k$



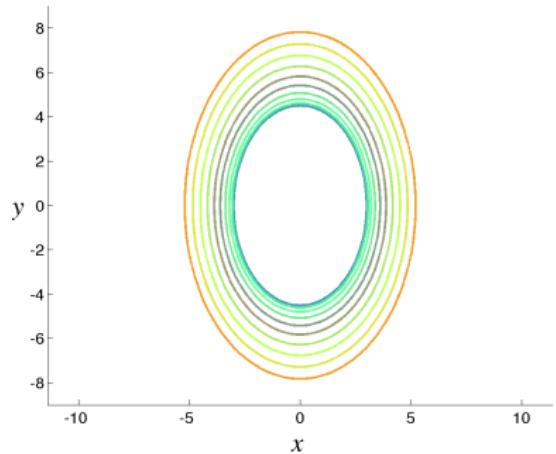
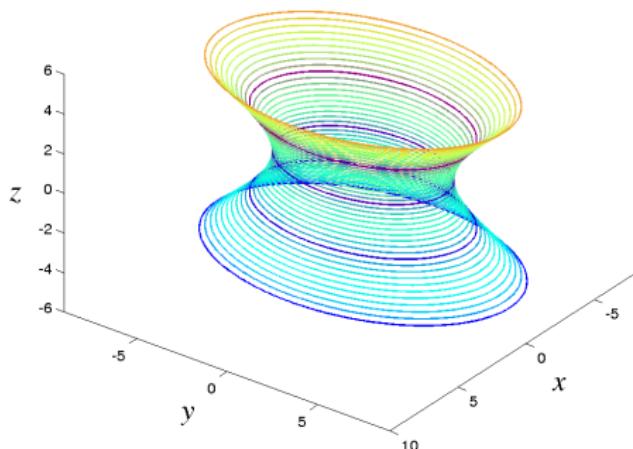
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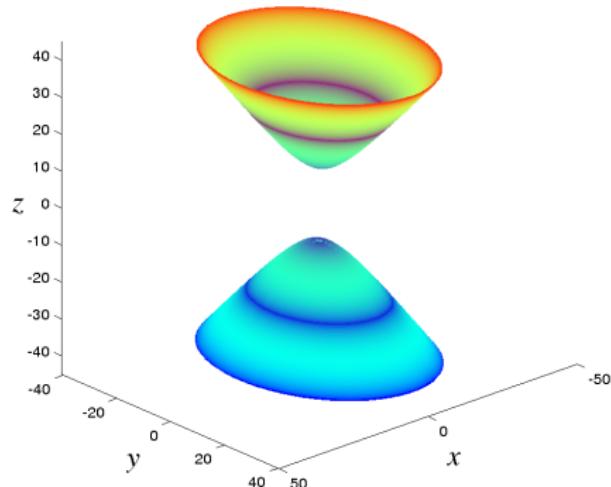
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Curvas de nível em \mathbb{R}^3 e em \mathbb{R}^2



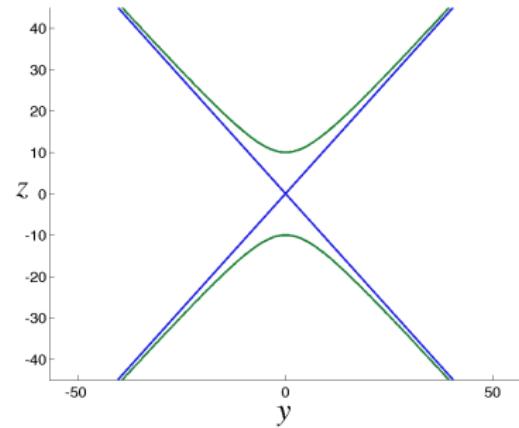
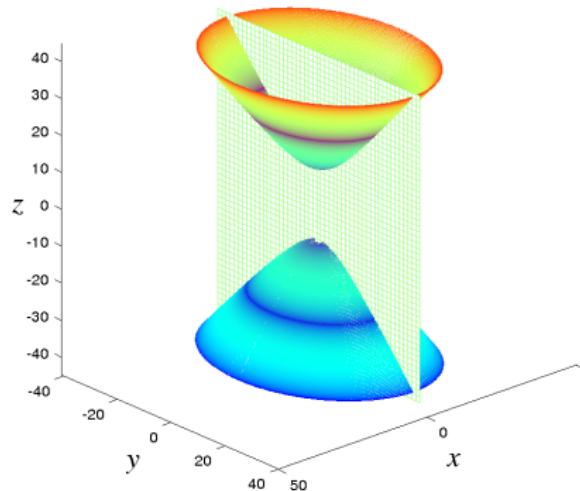
- As curvas de nível são elipses para todo $k \in \mathbb{R}$.

Hiperbolóide de duas folhas: gráfico e equação



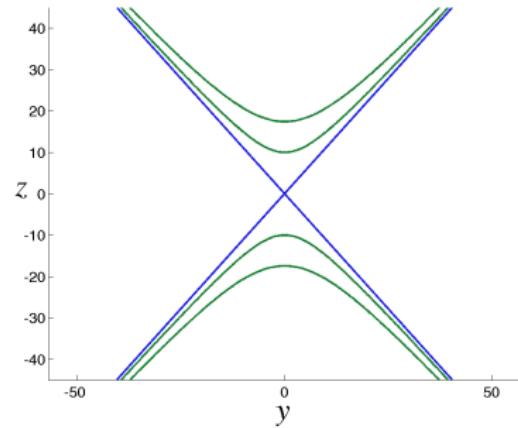
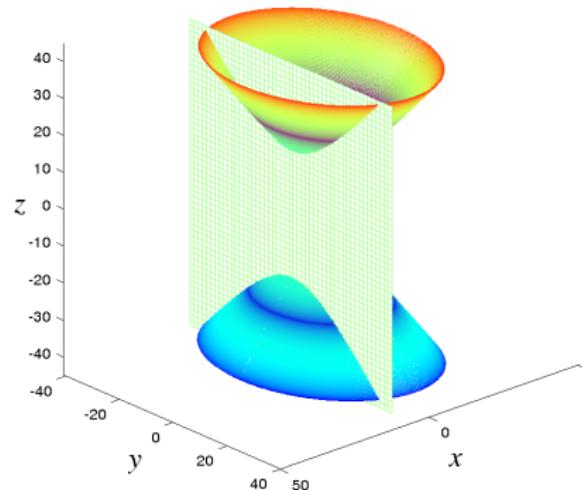
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Interseção com planos verticais: $x = k$



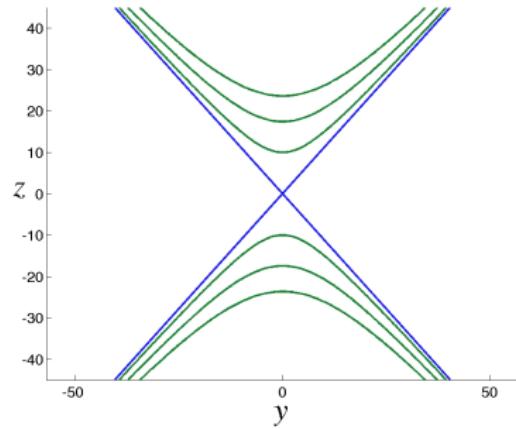
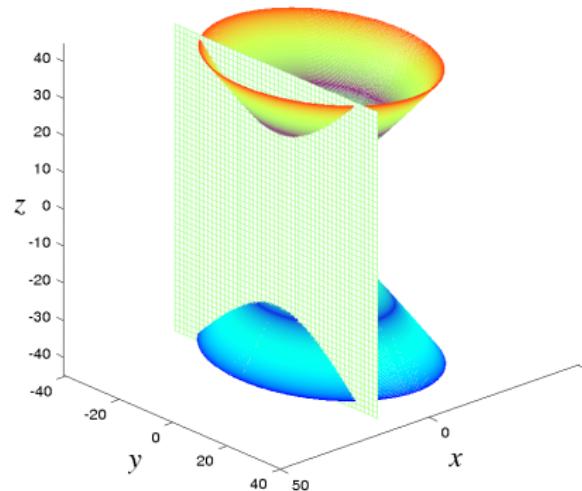
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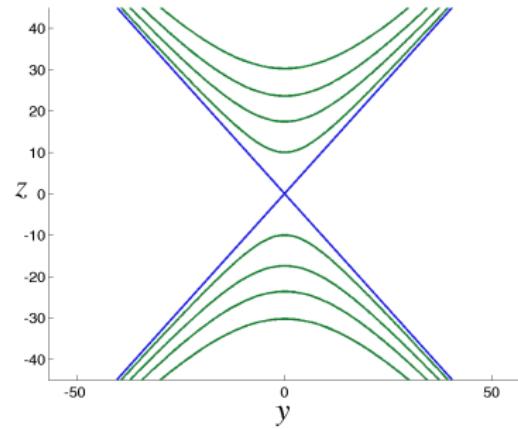
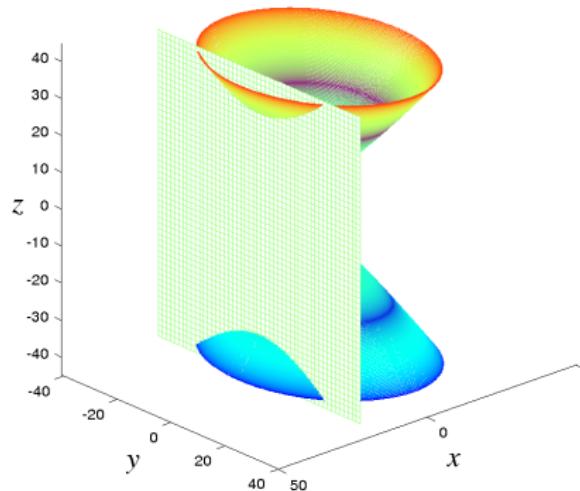
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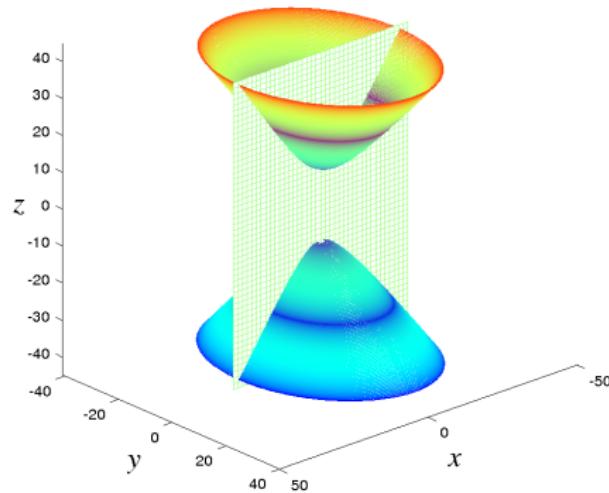
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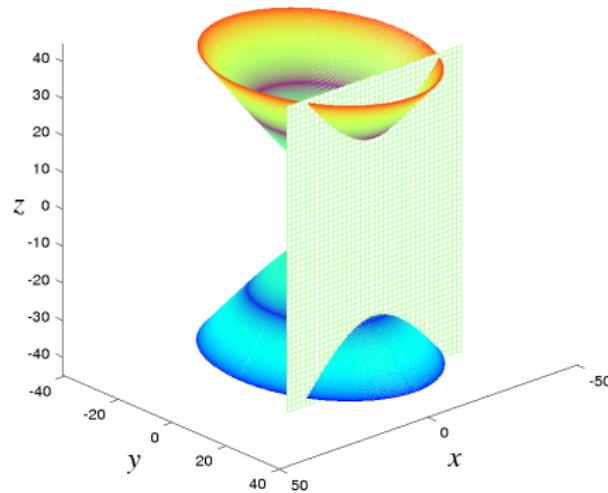
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Interseção com planos verticais: $y = k$



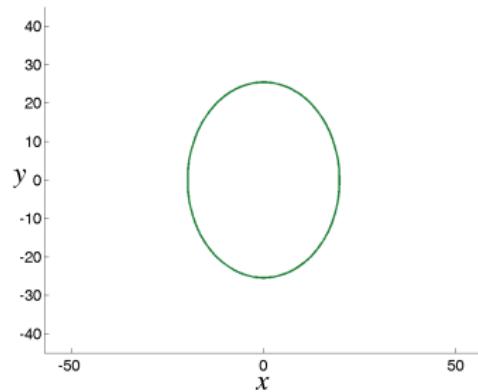
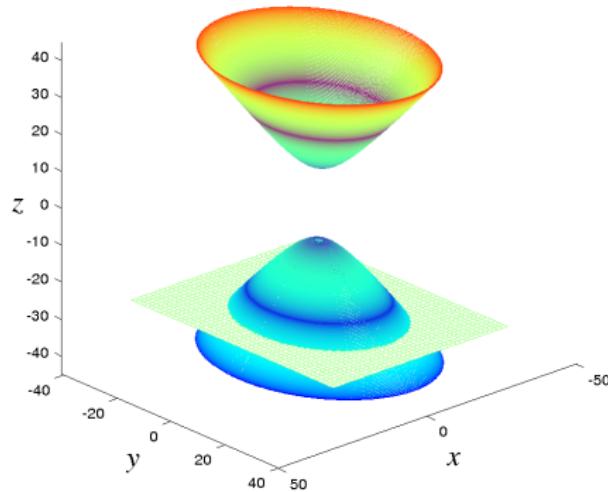
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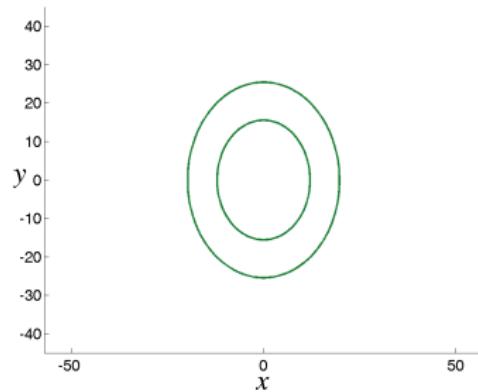
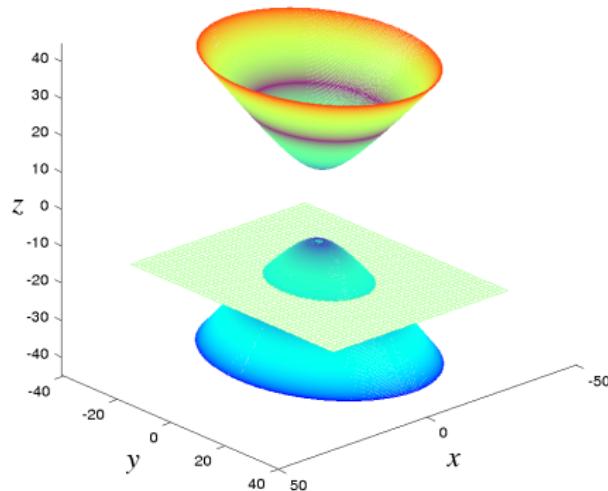
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Interseção com planos horizontais: $z = k$



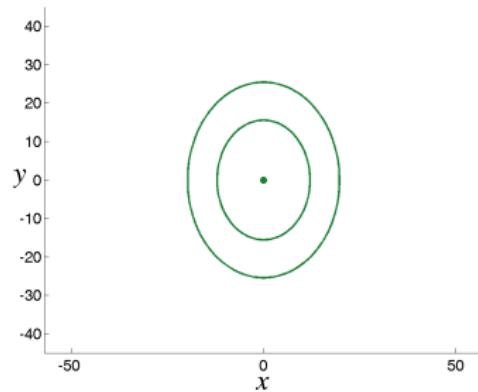
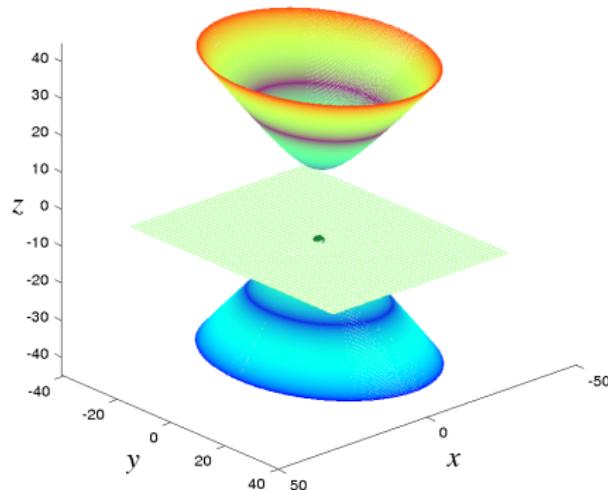
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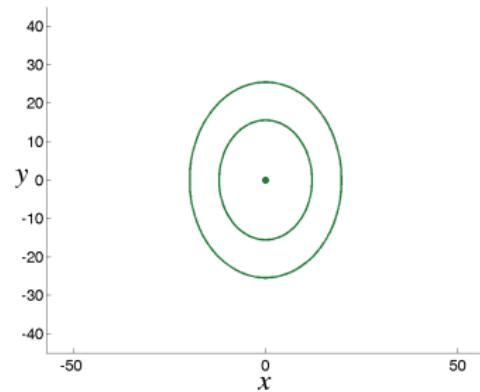
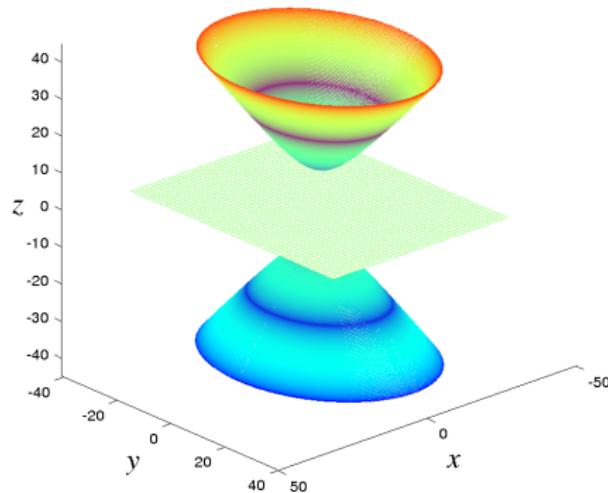
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Interseção com planos horizontais: $z = k$



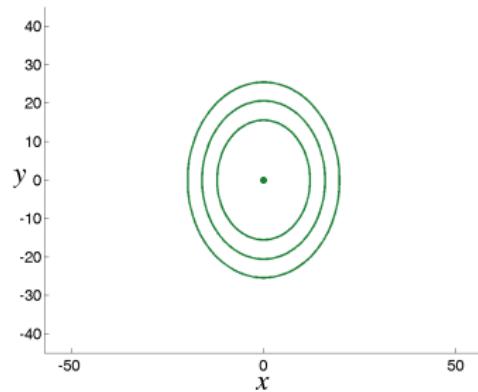
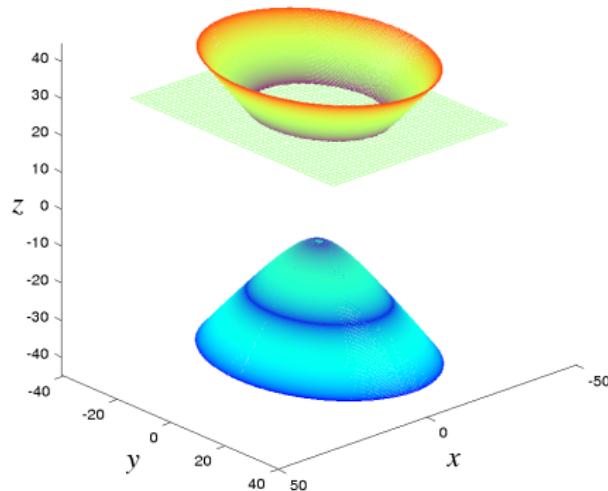
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 0 \\ z = k \end{array} \right. : \text{ um ponto } (k = -c \text{ ou } k = c).$$

Interseção com planos horizontais: $z = k$



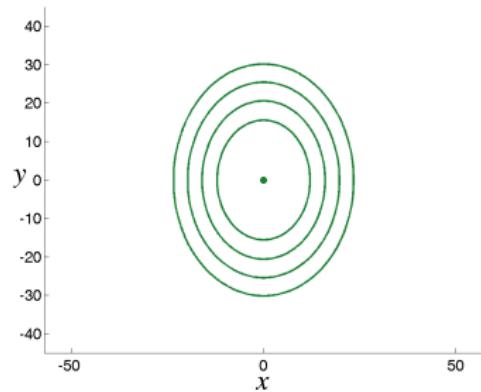
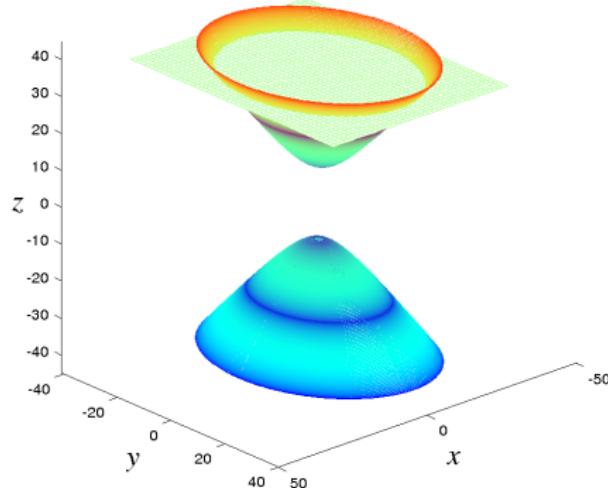
- O conjunto vazio quando $-c < k < c$.

Interseção com planos horizontais: $z = k$



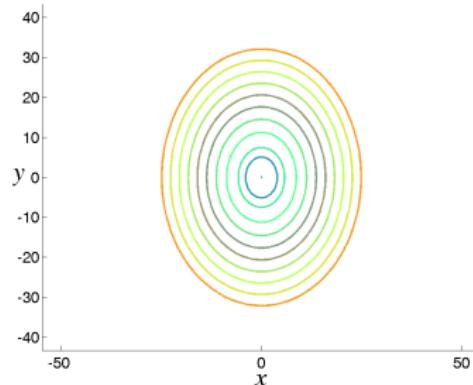
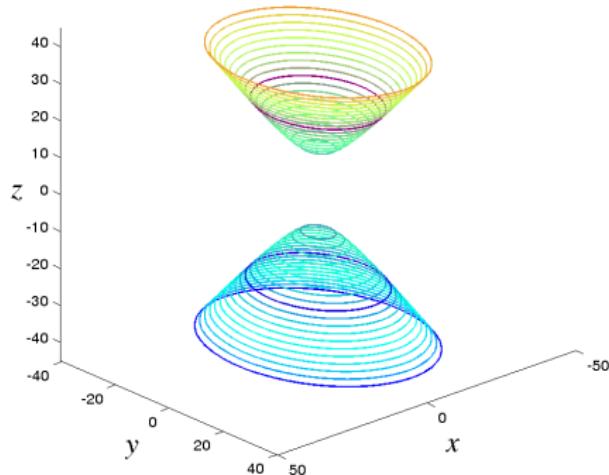
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{k^2}{c^2} - 1 \\ z = k \end{array} : \text{ elipses } (k < -c \text{ ou } k > c). \right.$$

Interseção com planos horizontais: $z = k$



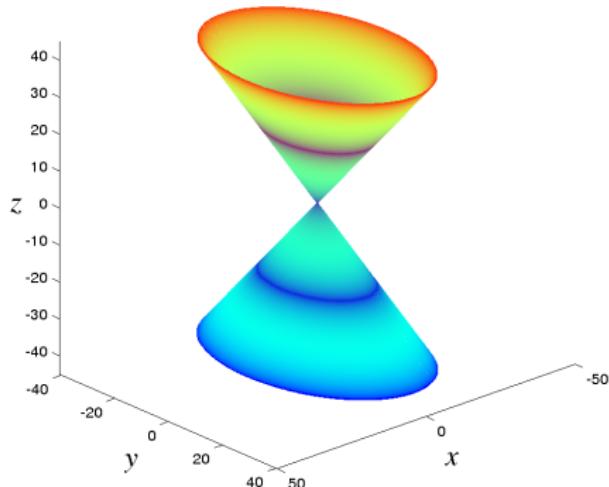
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{k^2}{c^2} - 1 \\ z = k \end{array} : \text{ elipses } (k < -c \text{ ou } k > c). \right.$$

Curvas de nível em \mathbb{R}^3 e em \mathbb{R}^2



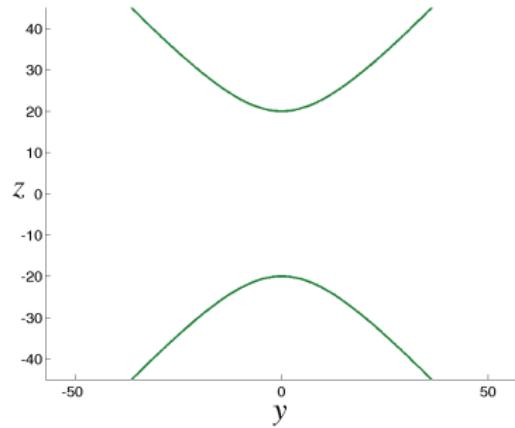
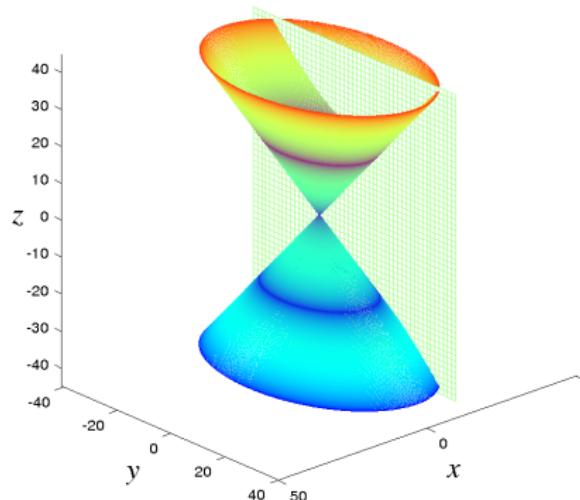
- As curvas de nível são elipses para todo $k < -c$ ou $k > c$, um ponto para $k = -c$ ou $k = c$ e o conjunto vazio para $k \in (-c, c)$.

Cone: gráfico e equação



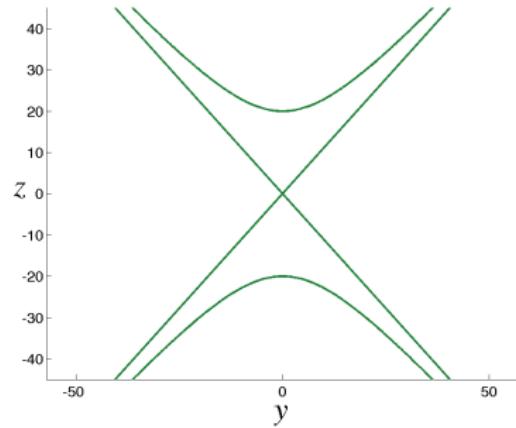
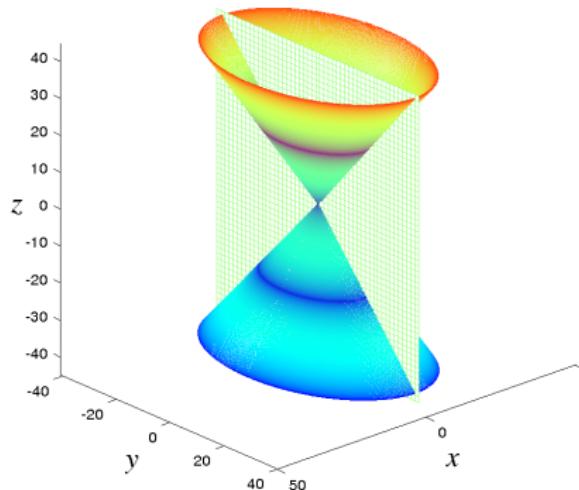
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

Interseção com planos verticais: $x = k$



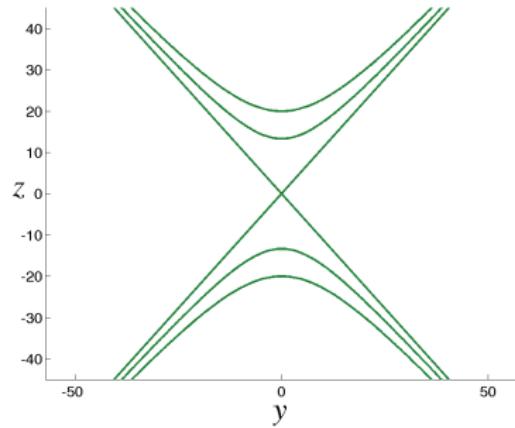
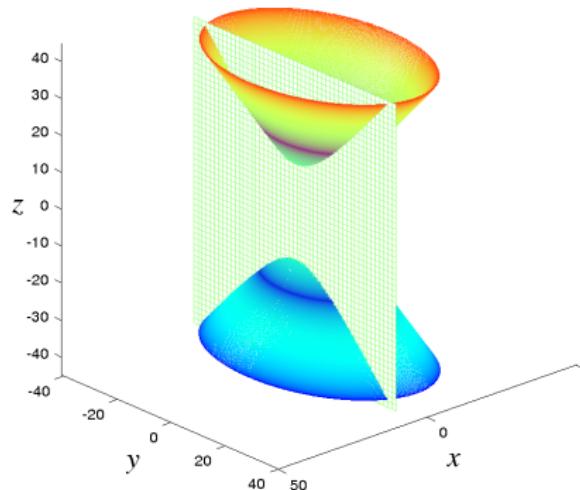
$$\left\{ \begin{array}{l} -\frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{k^2}{a^2} \\ x = k \end{array} : \text{ hipérboles } \forall k \neq 0. \right.$$

Interseção com planos verticais: $x = k$



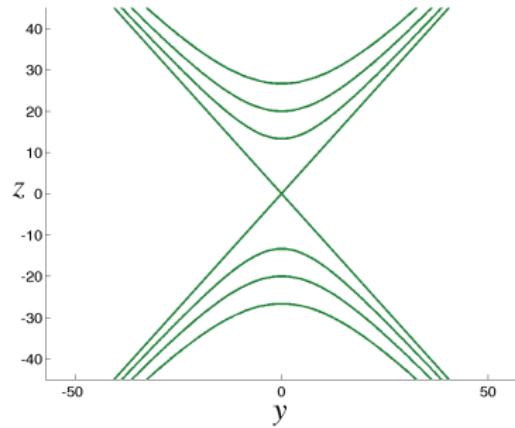
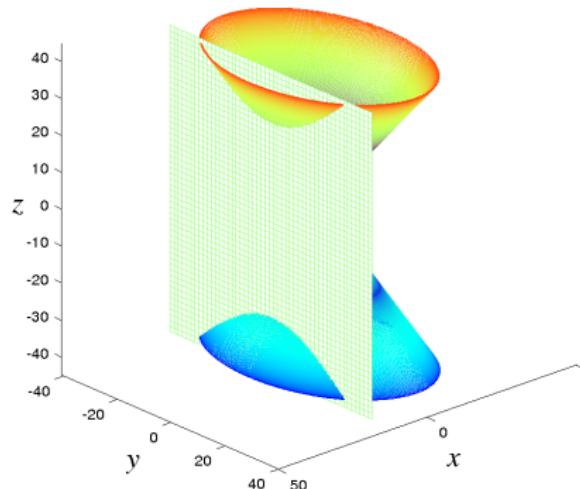
$$\left\{ \begin{array}{l} \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0 \\ x = 0 \end{array} \right. : \text{duas retas concorrentes } (k = 0).$$

Interseção com planos verticais: $x = k$



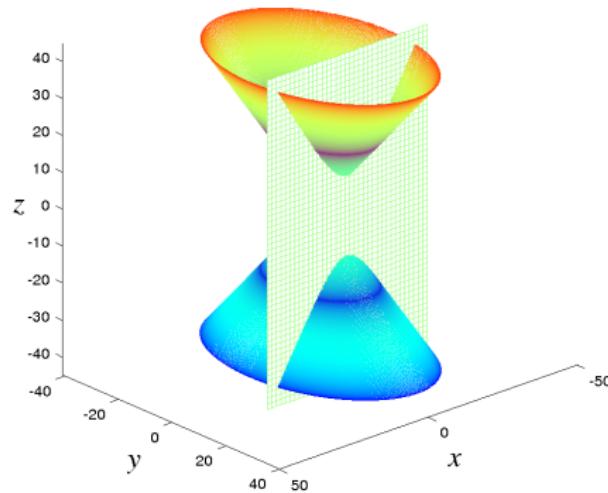
$$\left\{ \begin{array}{l} -\frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{k^2}{a^2} \\ x = k \end{array} \right. : \text{ hipérboles } \forall k \neq 0.$$

Interseção com planos verticais: $x = k$



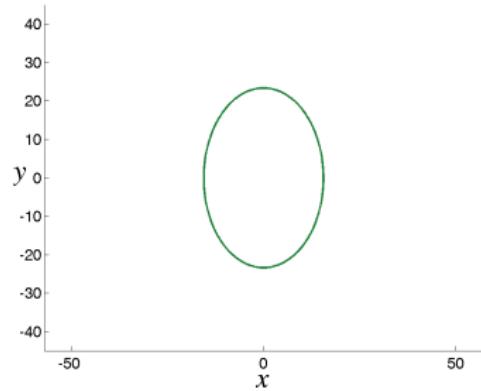
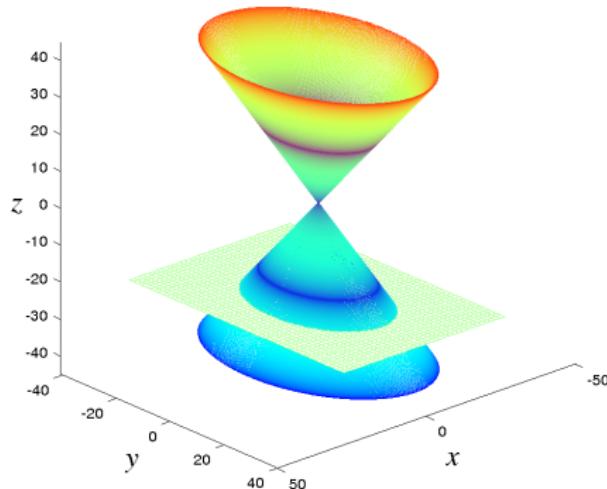
$$\left\{ \begin{array}{l} -\frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{k^2}{a^2} \\ x = k \end{array} \right. : \text{ hipérboles } \forall k \neq 0.$$

Interseção com planos verticais: $y = k$



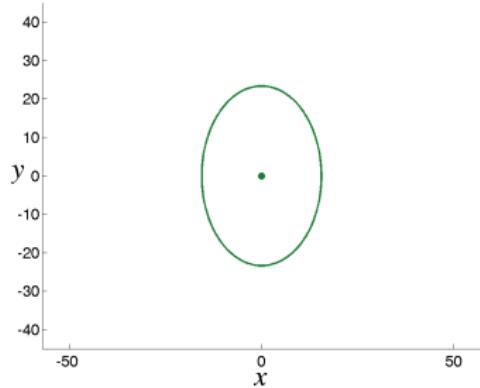
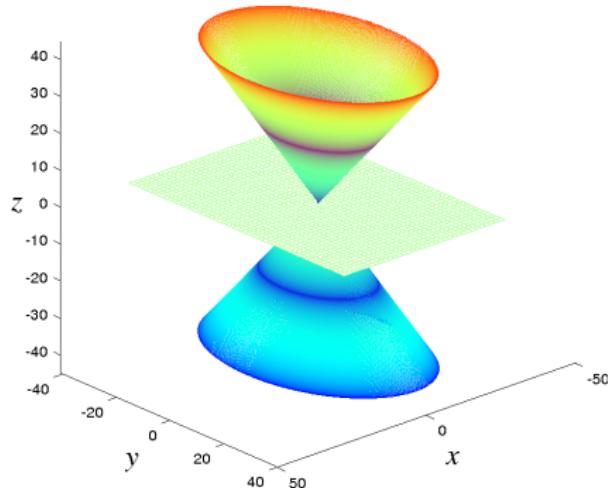
- Análogo ao caso anterior ($x \leftrightarrow y, a \leftrightarrow b$).

Interseção com planos horizontais: $z = k$



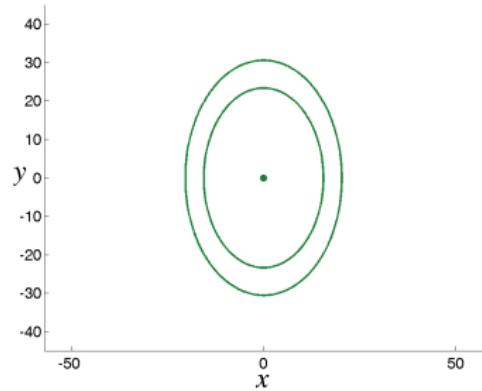
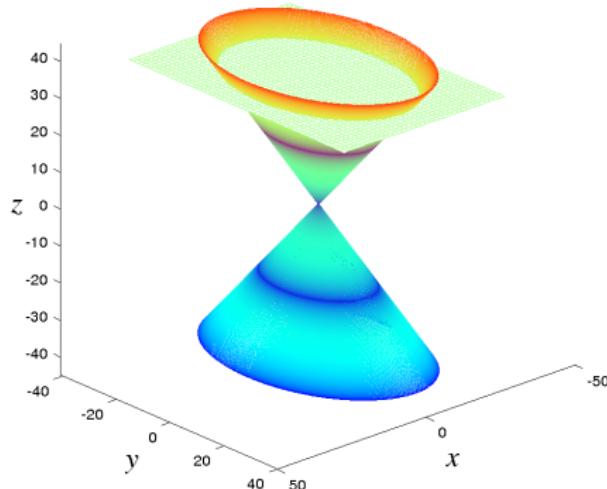
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{k^2}{c^2} \\ z = k \end{array} : \text{ elipses } \forall k \neq 0. \right.$$

Interseção com planos horizontais: $z = k$



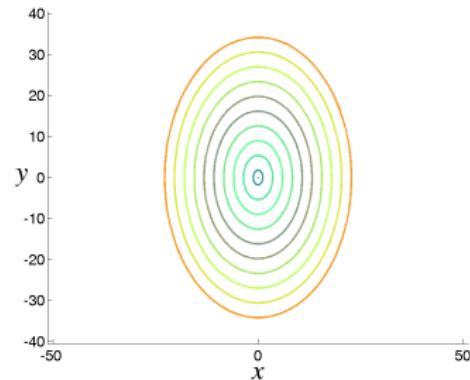
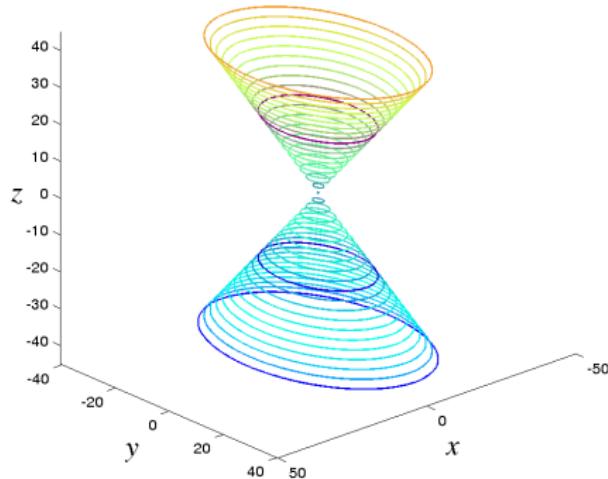
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 0 \\ z = 0 \end{array} \right. : \text{ um ponto } (k = 0).$$

Interseção com planos horizontais: $z = k$



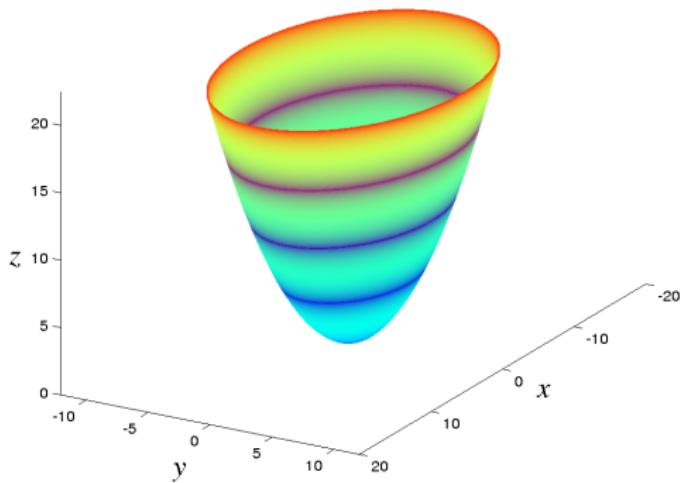
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{k^2}{c^2} \\ z = k \end{array} : \text{ elipses } \forall k \neq 0. \right.$$

Curvas de nível em \mathbb{R}^3 e em \mathbb{R}^2



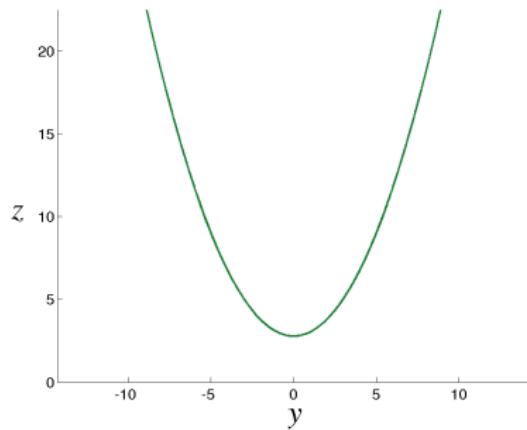
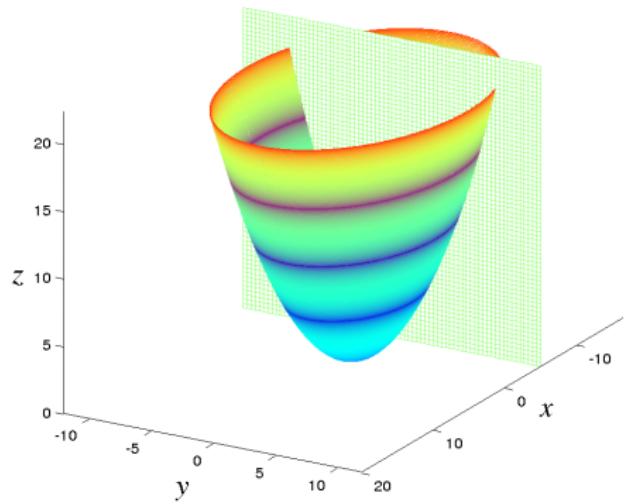
- As curvas de nível são elipses para todo $k \neq 0$ e um ponto para $k = 0$.

Parabolóide elíptico: gráfico e equação



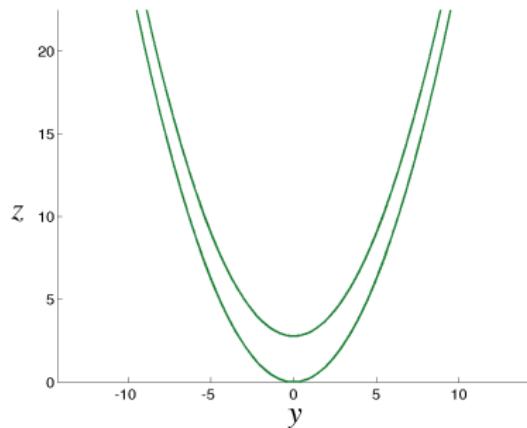
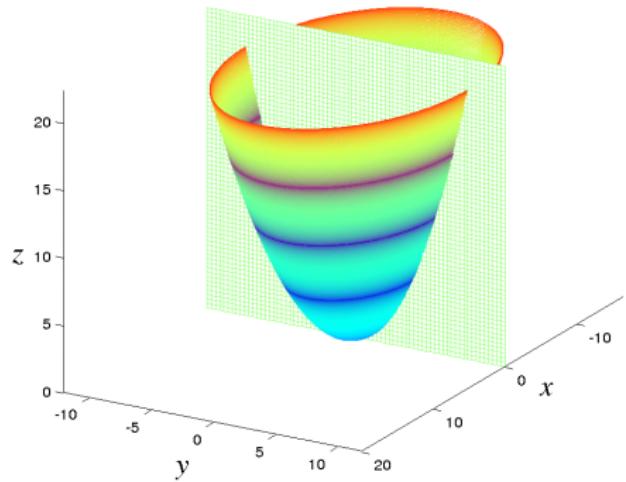
$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Interseção com planos verticais: $x = k$



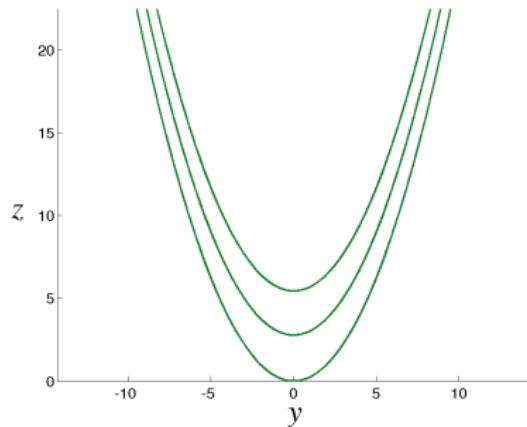
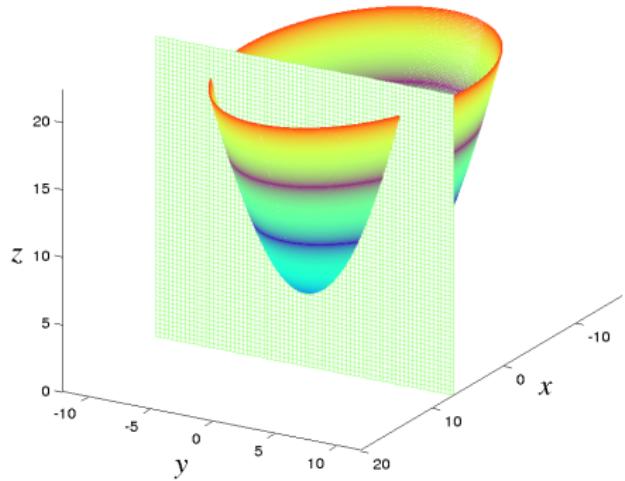
$$\left\{ \begin{array}{l} z = \frac{k^2}{a^2} + \frac{y^2}{b^2} \\ x = k \end{array} \right. : \text{parábolas } \forall k \in \mathbb{R}.$$

Interseção com planos verticais: $x = k$



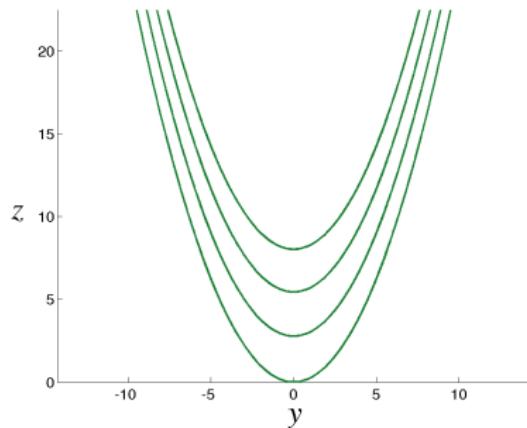
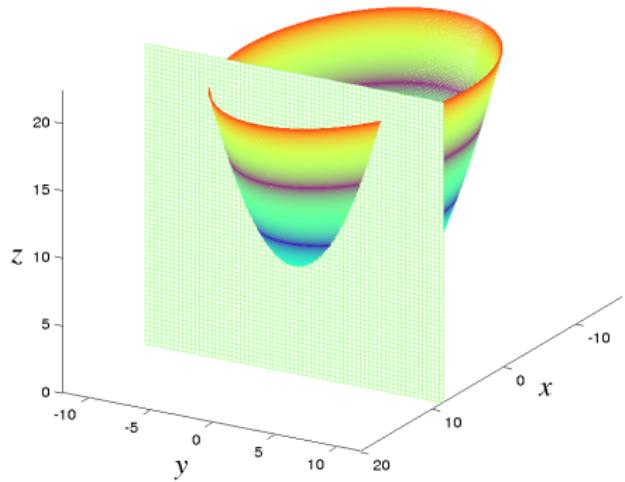
$$\begin{cases} z = \frac{k^2}{a^2} + \frac{y^2}{b^2} \\ x = k \end{cases} : \text{parábolas } \forall k \in \mathbb{R}.$$

Interseção com planos verticais: $x = k$



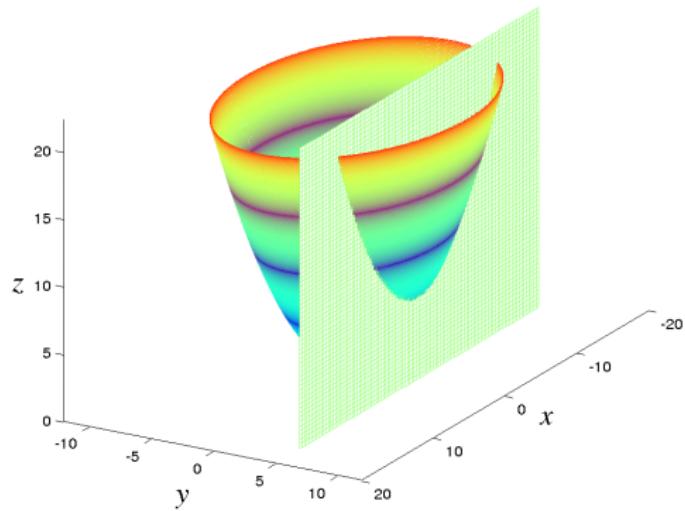
$$\begin{cases} z = \frac{k^2}{a^2} + \frac{y^2}{b^2} \\ x = k \end{cases} : \text{parábolas } \forall k \in \mathbb{R}.$$

Interseção com planos verticais: $x = k$



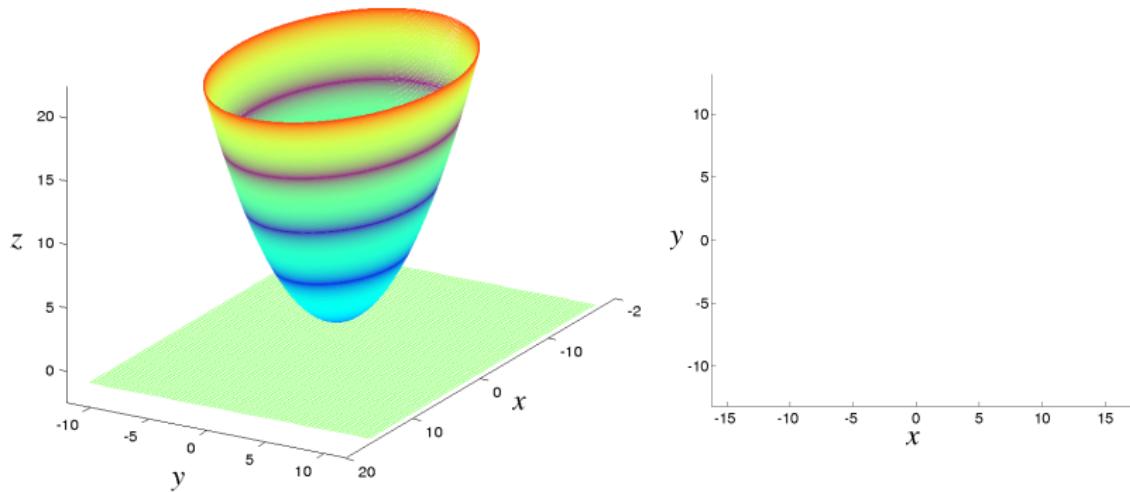
$$\begin{cases} z = \frac{k^2}{a^2} + \frac{y^2}{b^2} \\ x = k \end{cases} : \text{parábolas } \forall k \in \mathbb{R}.$$

Interseção com planos verticais: $y = k$



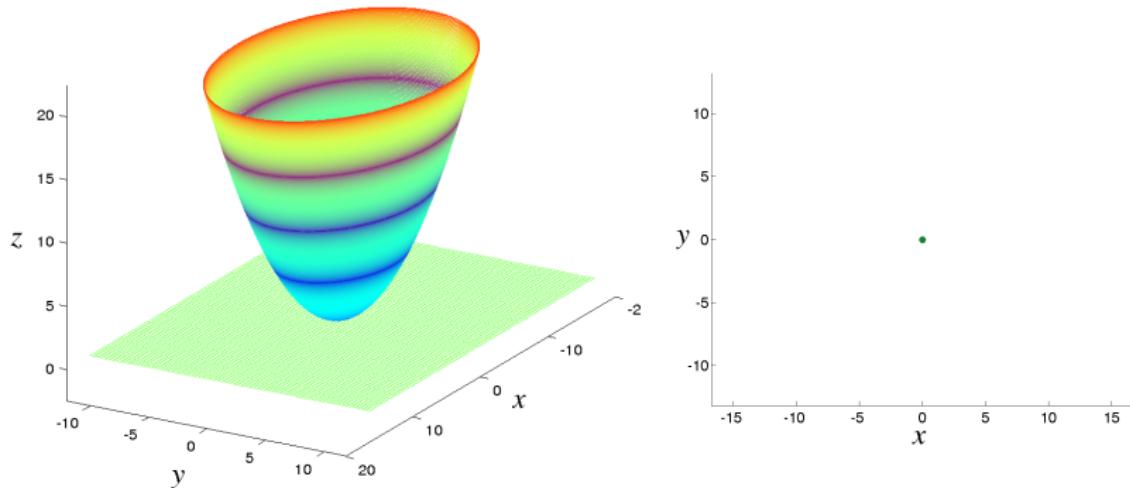
- Análogo ao caso anterior ($x \leftrightarrow y, a \leftrightarrow b$).

Interseção com planos horizontais: $z = k$



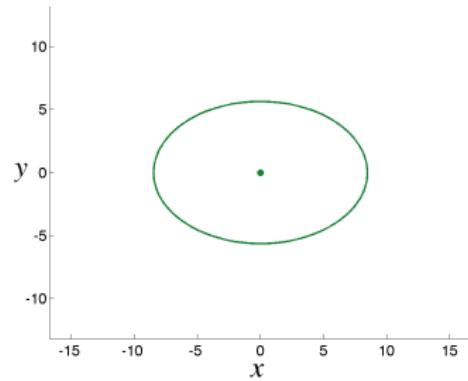
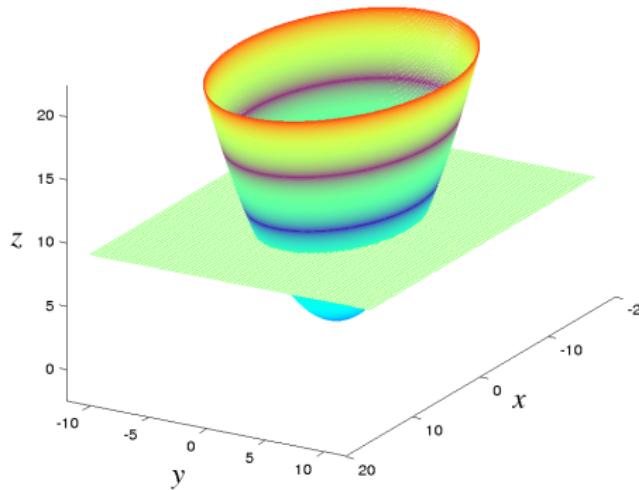
- O conjunto vazio quando $k < 0$.

Interseção com planos horizontais: $z = k$



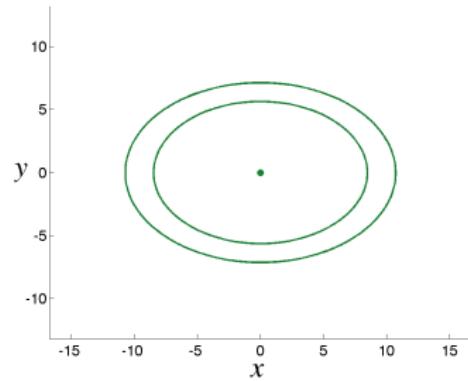
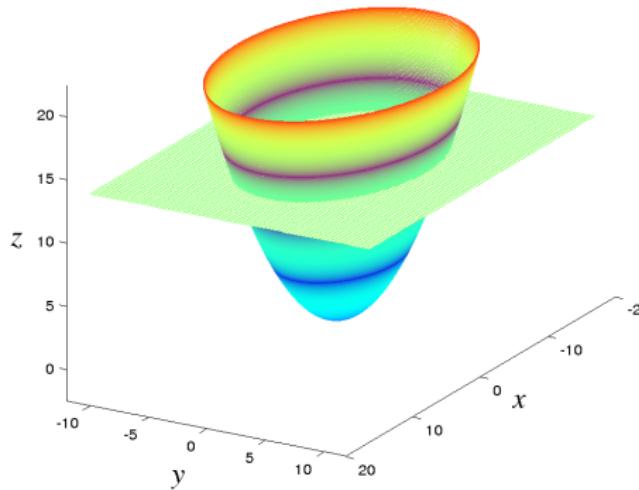
$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 0 \\ z = 0 \end{cases} : \text{ um ponto } (k = 0).$$

Interseção com planos horizontais: $z = k$



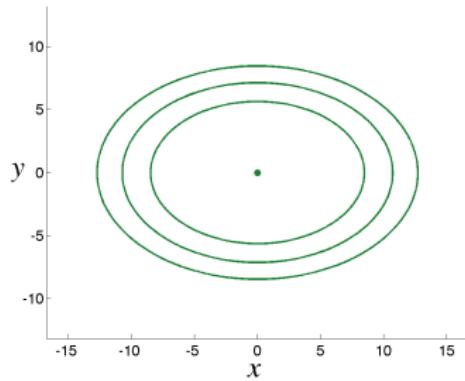
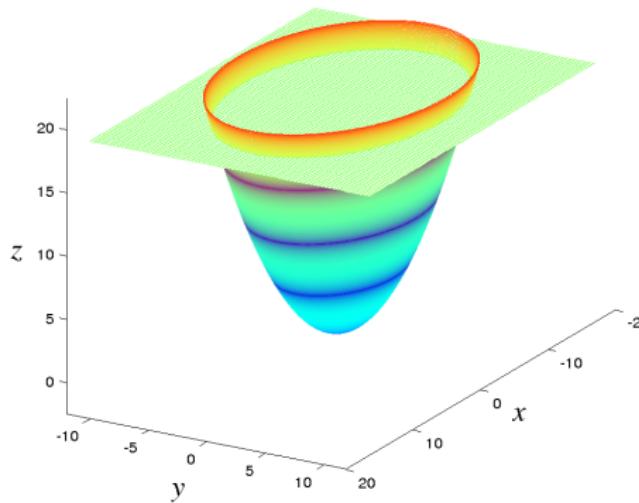
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} + \frac{y^2}{b^2} = k \\ z = k \end{array} : \text{ elipses } (k > 0). \right.$$

Interseção com planos horizontais: $z = k$



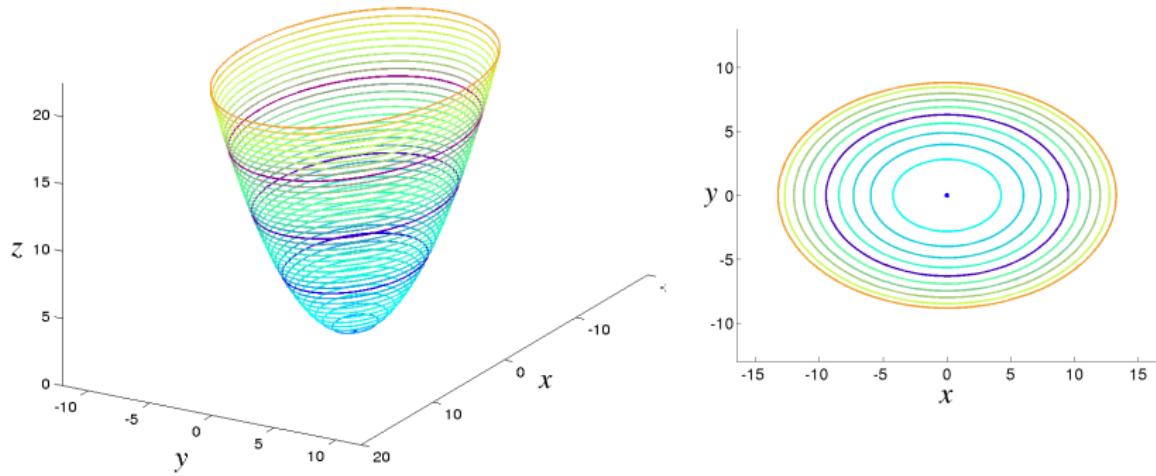
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} + \frac{y^2}{b^2} = k \\ z = k \end{array} : \text{ elipses } (k > 0). \right.$$

Interseção com planos horizontais: $z = k$



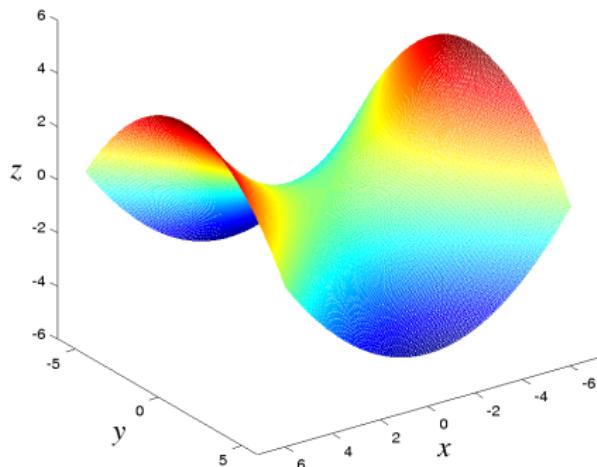
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} + \frac{y^2}{b^2} = k \\ z = k \end{array} : \text{ elipses } (k > 0). \right.$$

Curvas de nível em \mathbb{R}^3 e em \mathbb{R}^2



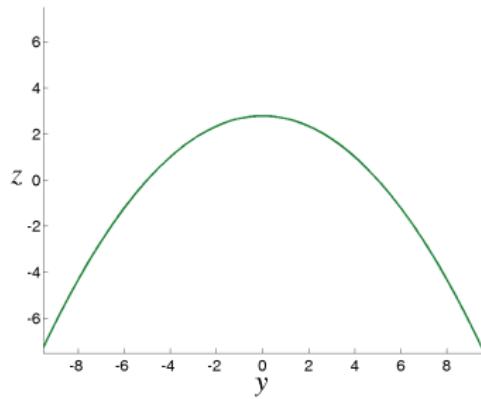
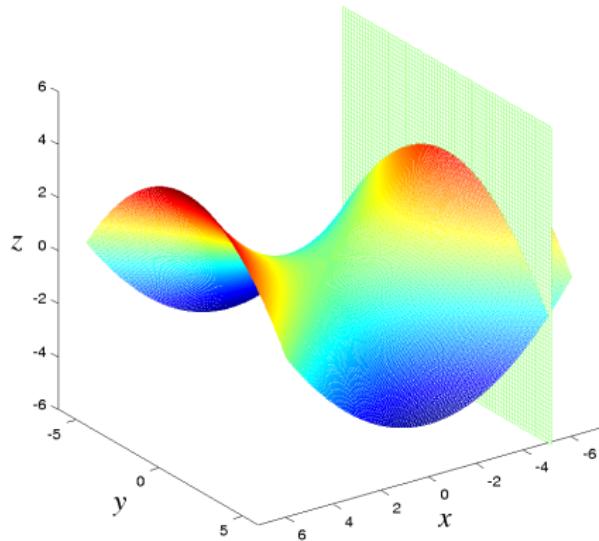
- As curvas de nível são elipses para todo $k > 0$, um ponto para $k = 0$ e o conjunto vazio para $k < 0$.

Parabolóide hiperbólico (sela): gráfico e equação



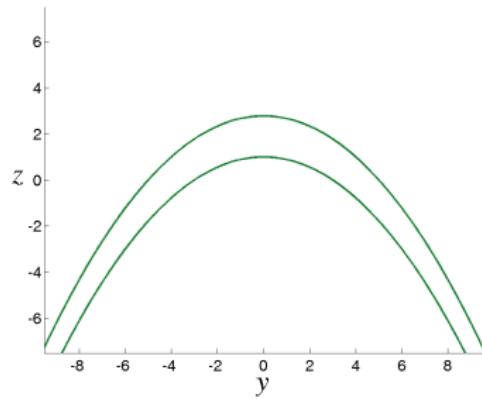
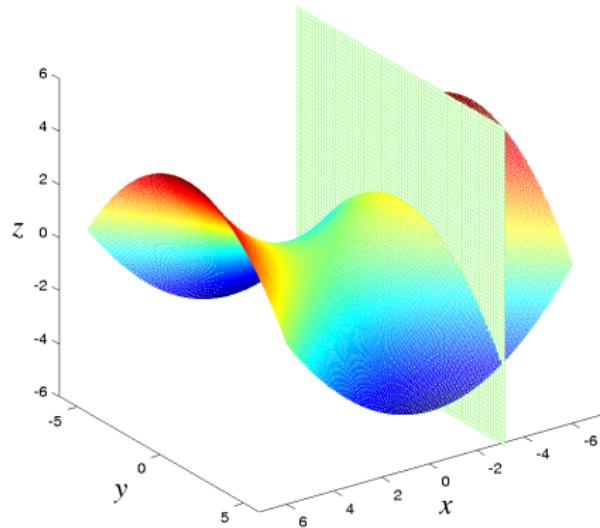
$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

Interseção com planos verticais: $x = k$



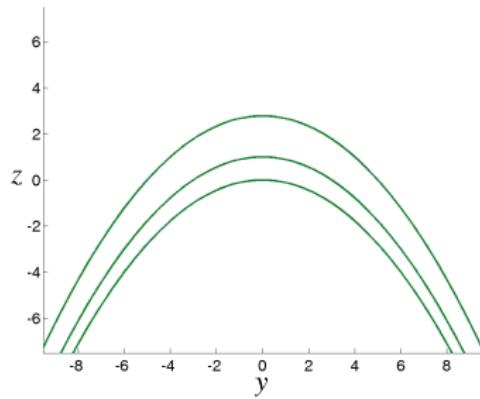
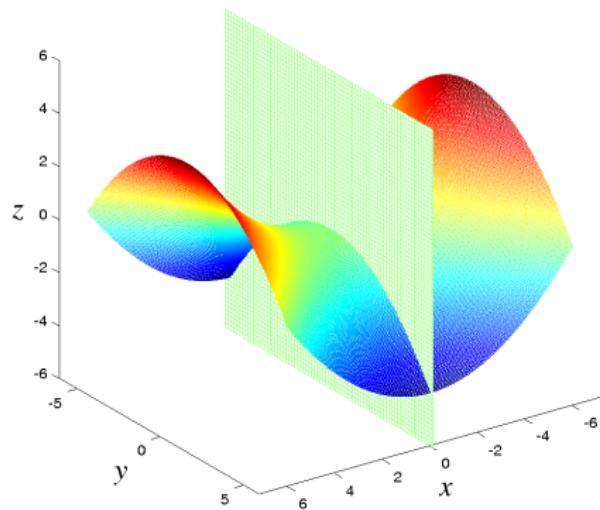
$$\left\{ \begin{array}{l} z = \frac{k^2}{a^2} - \frac{y^2}{b^2} \\ x = k \end{array} \right. : \text{ parábolas côncavas } \forall k \in \mathbb{R}.$$

Interseção com planos verticais: $x = k$



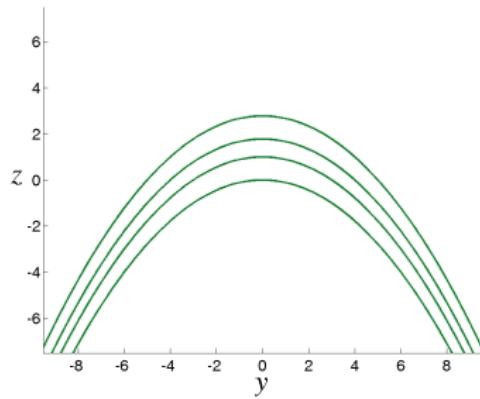
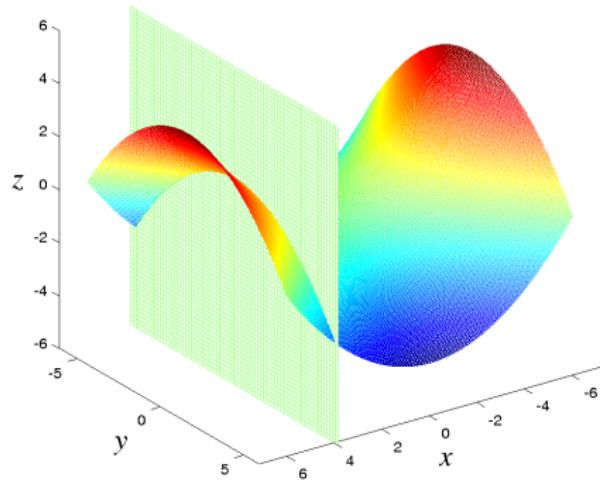
$$\left\{ \begin{array}{l} z = \frac{k^2}{a^2} - \frac{y^2}{b^2} \\ x = k \end{array} \right. : \text{ parábolas côncavas } \forall k \in \mathbb{R}.$$

Interseção com planos verticais: $x = k$



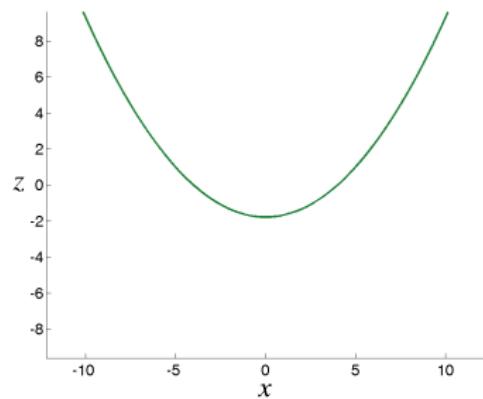
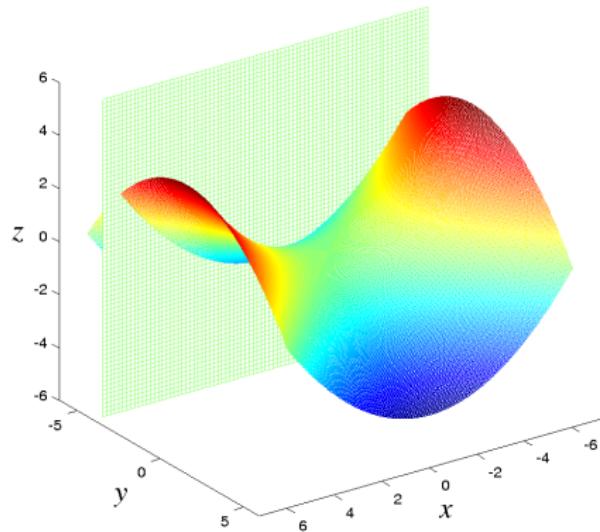
$$\left\{ \begin{array}{l} z = \frac{k^2}{a^2} - \frac{y^2}{b^2} \\ x = k \end{array} \right. : \text{ parábolas côncavas } \forall k \in \mathbb{R}.$$

Interseção com planos verticais: $x = k$



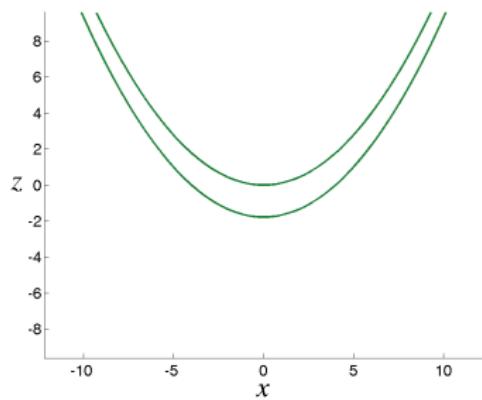
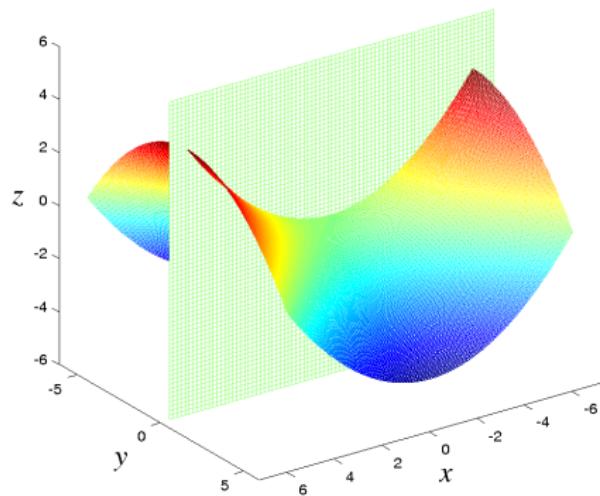
$$\left\{ \begin{array}{l} z = \frac{k^2}{a^2} - \frac{y^2}{b^2} \\ x = k \end{array} \right. : \text{ paráboras côncavas } \forall k \in \mathbb{R}.$$

Interseção com planos verticais: $y = k$



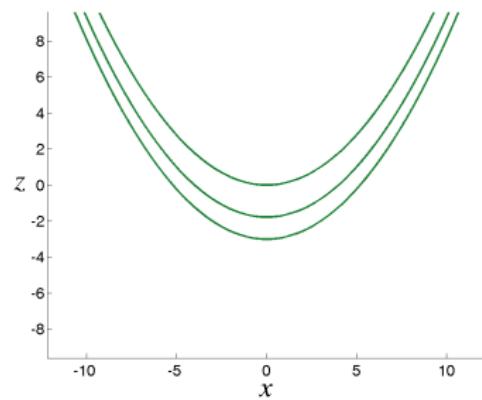
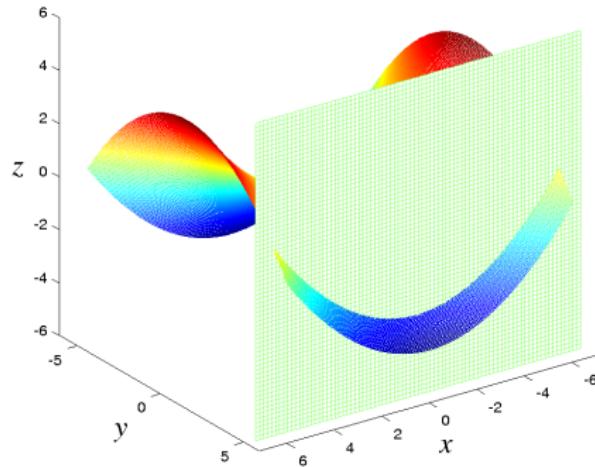
$$\left\{ \begin{array}{l} z = \frac{x^2}{a^2} - \frac{k^2}{b^2} \\ y = k \end{array} \right. : \text{paráboras convexas } \forall k \in \mathbb{R}.$$

Interseção com planos verticais: $y = k$



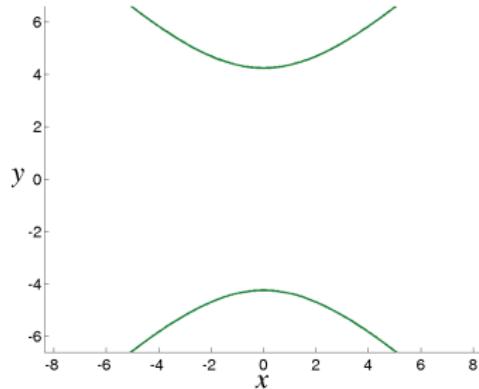
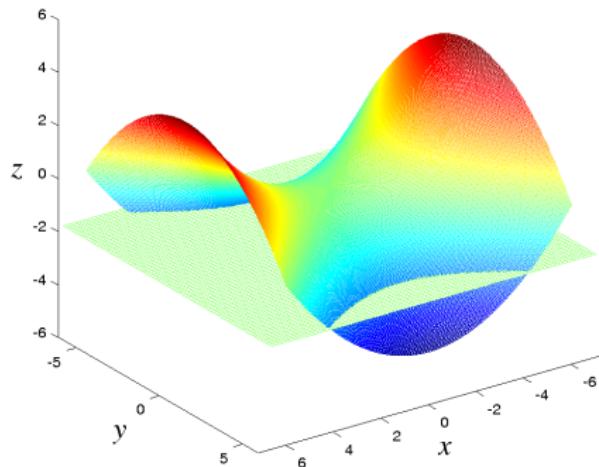
$$\left\{ \begin{array}{l} z = \frac{x^2}{a^2} - \frac{k^2}{b^2} \\ y = k \end{array} \right. : \text{ paráboras convexas } \forall k \in \mathbb{R}.$$

Interseção com planos verticais: $y = k$



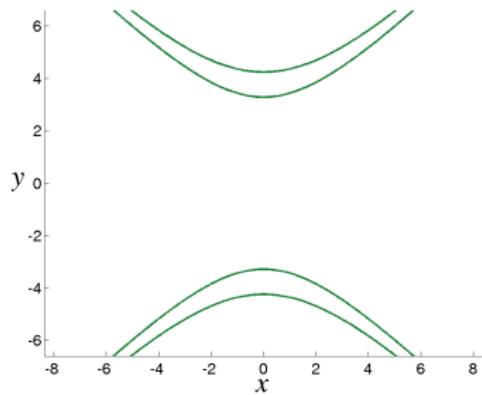
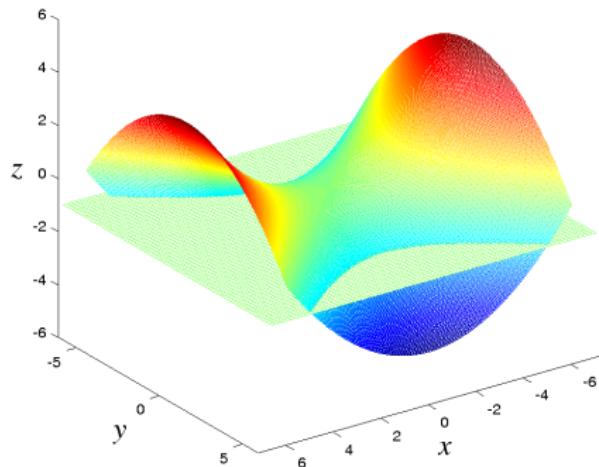
$$\left\{ \begin{array}{l} z = \frac{x^2}{a^2} - \frac{k^2}{b^2} \\ y = k \end{array} \right. : \text{ parábolas convexas } \forall k \in \mathbb{R}.$$

Interseção com planos horizontais: $z = k$



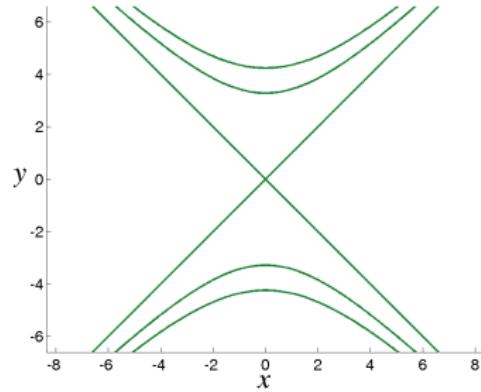
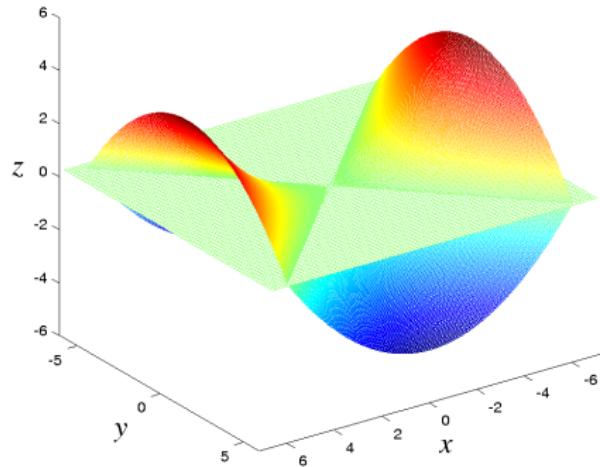
$$\left\{ \begin{array}{l} -\frac{x^2}{a^2} + \frac{y^2}{b^2} = -k \\ z = k \end{array} \right. : \text{ hipérboles na vertical } \forall k < 0.$$

Interseção com planos horizontais: $z = k$



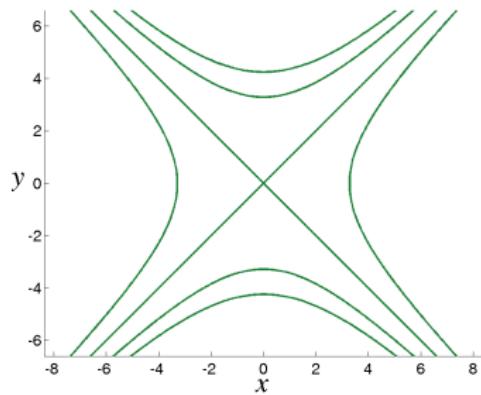
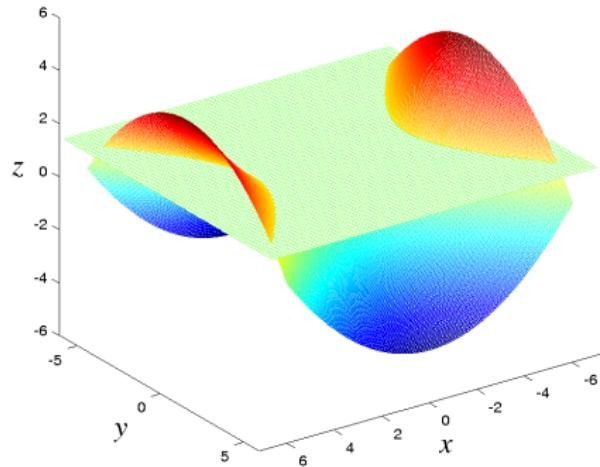
$$\left\{ \begin{array}{l} -\frac{x^2}{a^2} + \frac{y^2}{b^2} = -k \\ z = k \end{array} \right. : \text{ hipérboles na vertical } \forall k < 0.$$

Interseção com planos horizontais: $z = k$



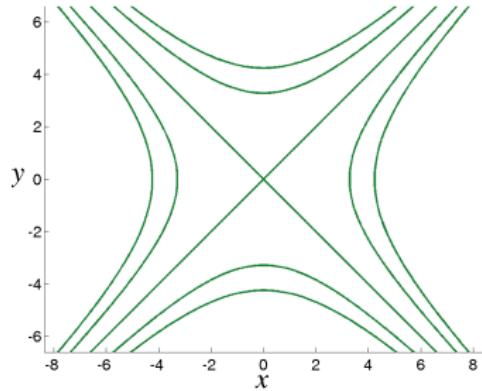
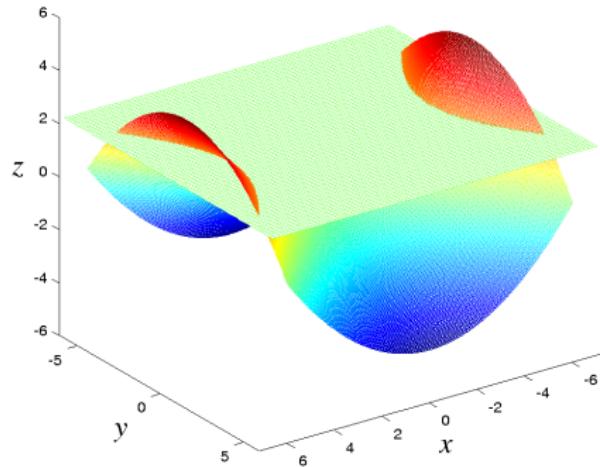
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0 \\ z = 0 \end{array} \right. : \text{ duas retas concorrentes } (k = 0).$$

Interseção com planos horizontais: $z = k$



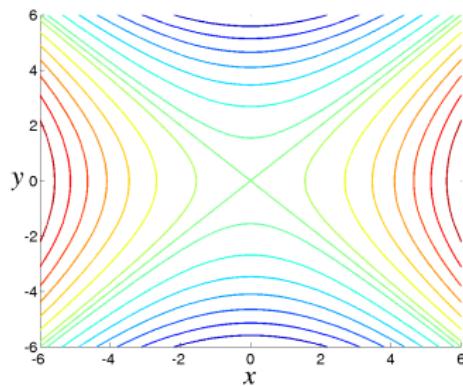
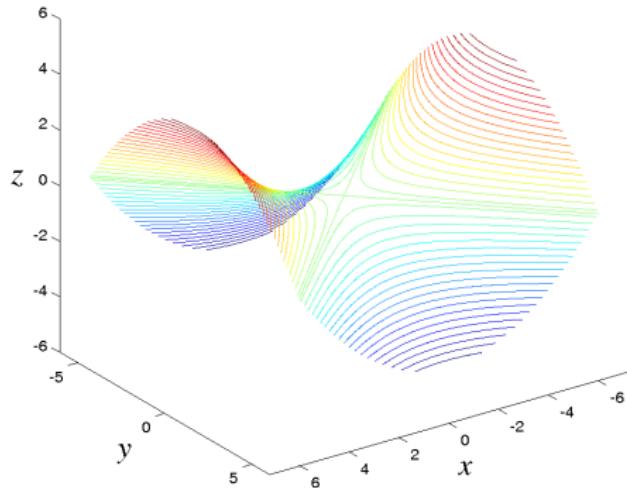
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} - \frac{y^2}{b^2} = k \\ z = k \end{array} \right. : \text{ hipérboles na horizontal } \forall k > 0.$$

Interseção com planos horizontais: $z = k$



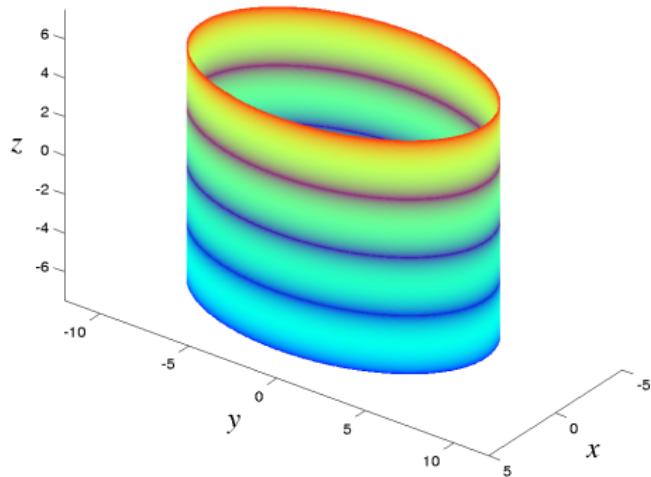
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} - \frac{y^2}{b^2} = k \\ z = k \end{array} \right. : \text{ hipérboles na horizontal } \forall k > 0.$$

Curvas de nível em \mathbb{R}^3 e em \mathbb{R}^2



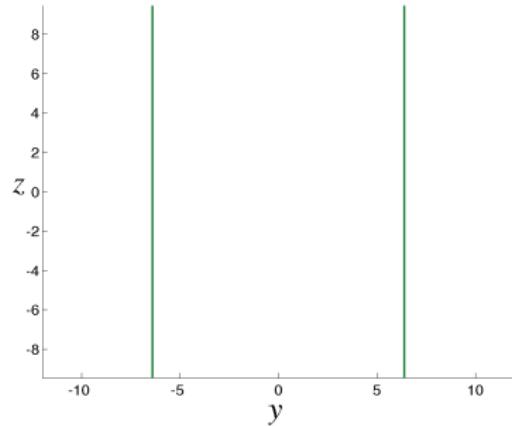
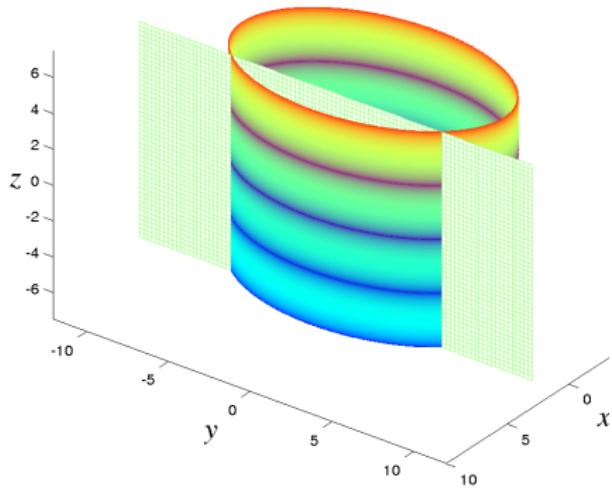
- As curvas de nível são hipérboles para todo $k \neq 0$ e duas retas concorrentes para $k = 0$.

Cilindro elíptico: gráfico e equação



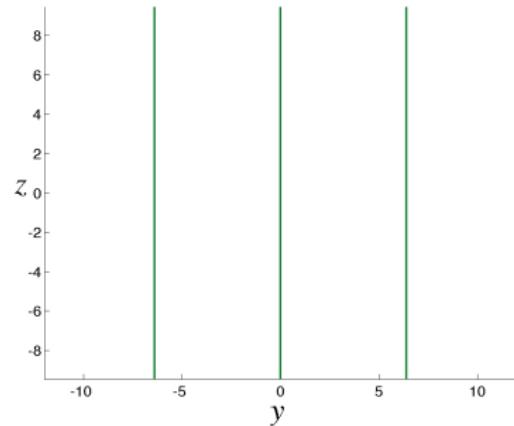
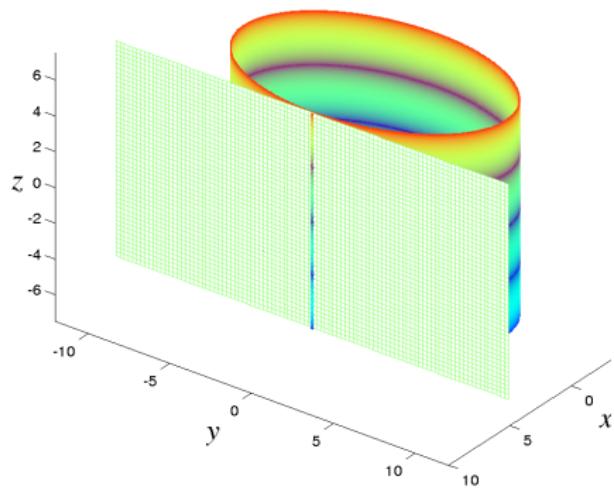
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Interseção com planos verticais: $x = k$



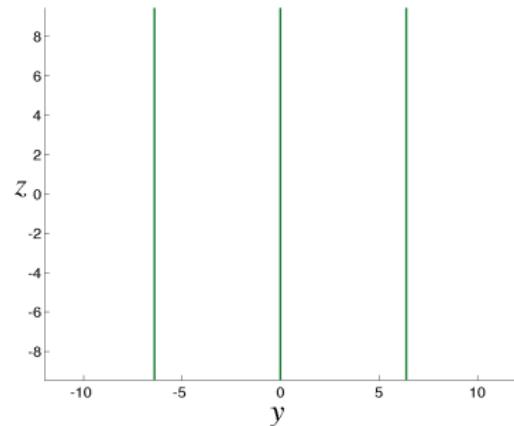
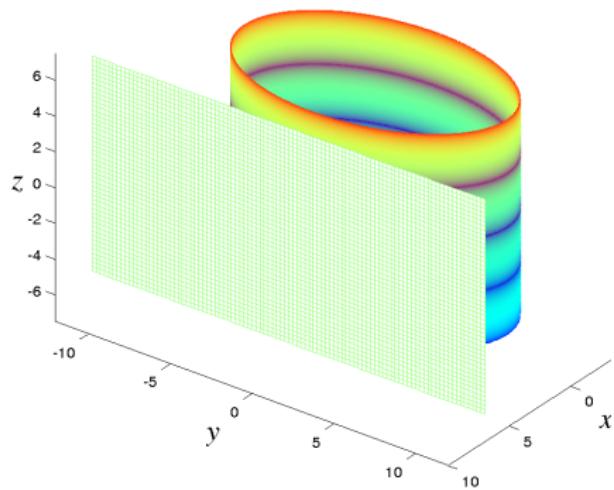
$$\left\{ \begin{array}{l} \frac{y^2}{b^2} = 1 - \frac{k^2}{a^2} \\ x = k \end{array} \right. : \text{duas retas paralelas } (-a < k < a).$$

Interseção com planos verticais: $x = k$



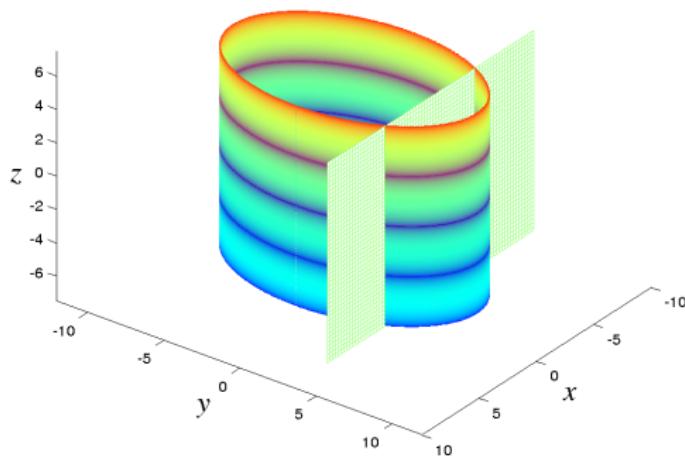
$$\begin{cases} y = 0 \\ x = k \end{cases} : \text{ uma única reta } (k = -a \text{ ou } k = a).$$

Interseção com planos verticais: $x = k$



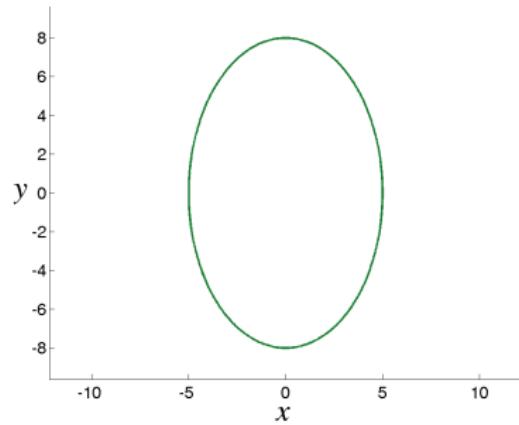
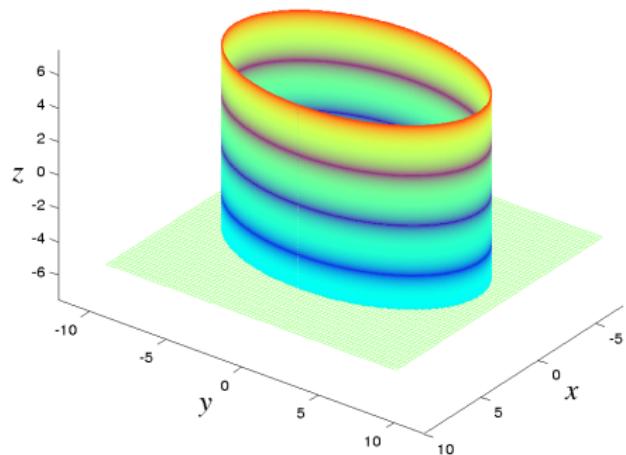
- O conjunto vazio quando $k < -a$ ou $k > a$.

Interseção com planos verticais: $y = k$



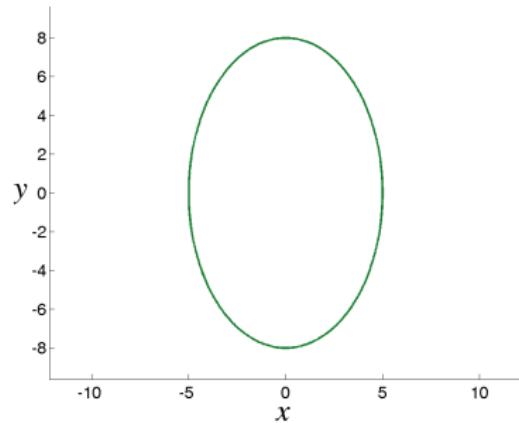
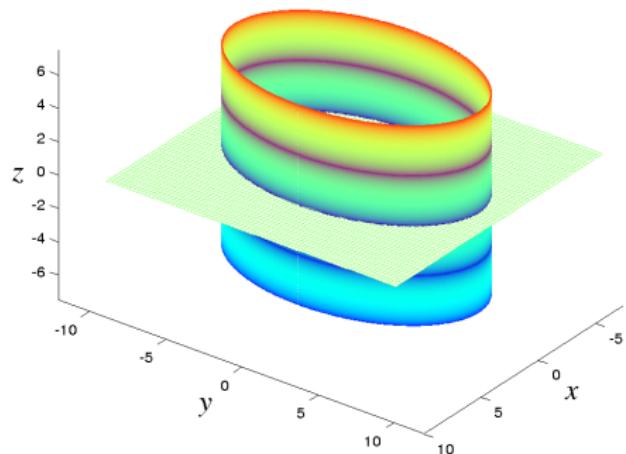
- Análogo ao caso anterior ($x \leftrightarrow y, a \leftrightarrow b$).

Interseção com planos horizontais: $z = k$



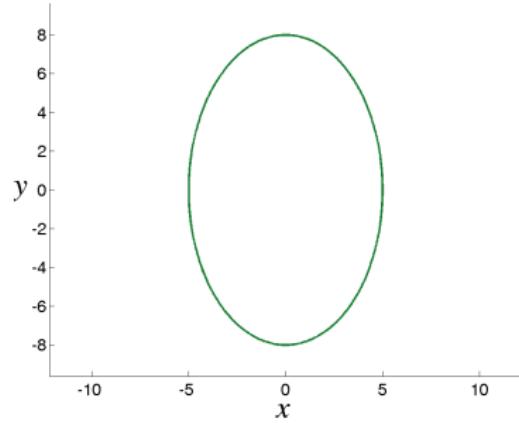
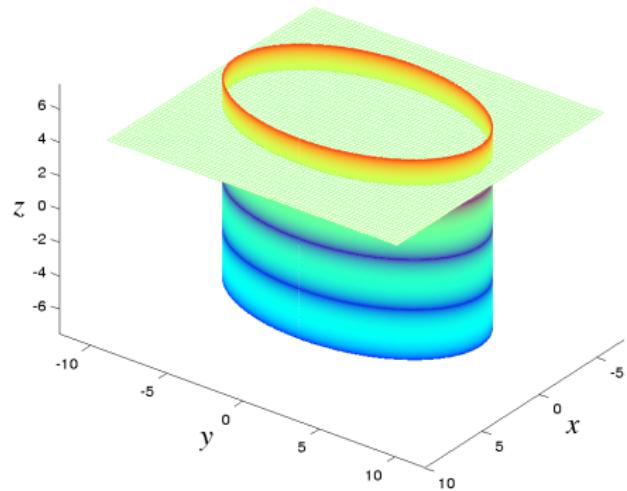
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \\ z = k \end{array} \right. : \text{ elipses iguais } \forall k \in \mathbb{R}.$$

Interseção com planos horizontais: $z = k$



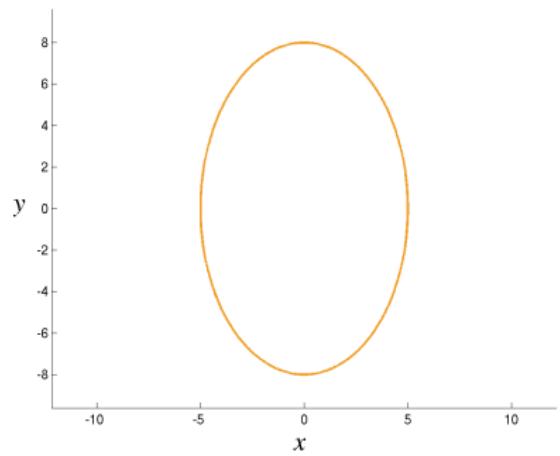
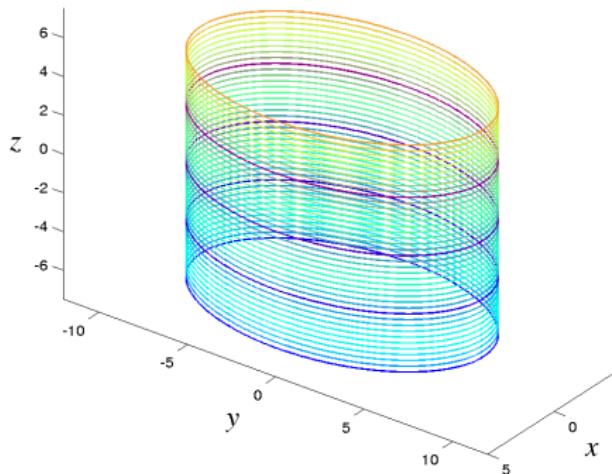
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \\ z = k \end{array} \right. : \text{ elipses iguais } \forall k \in \mathbb{R}.$$

Interseção com planos horizontais: $z = k$



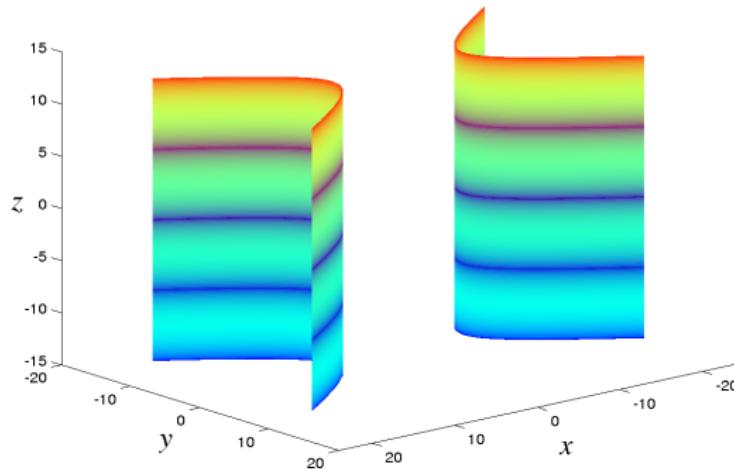
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \\ z = k \end{array} \right. : \text{ elipses iguais } \forall k \in \mathbb{R}.$$

Curvas de nível em \mathbb{R}^3 e em \mathbb{R}^2



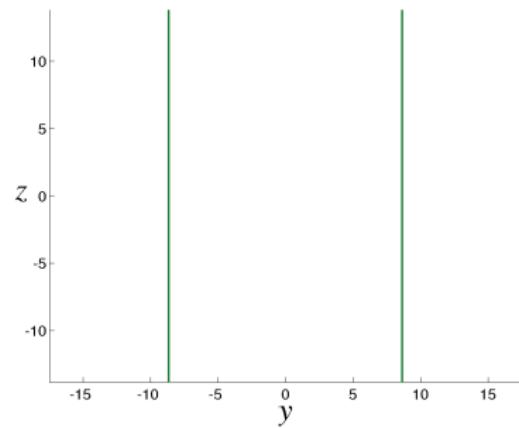
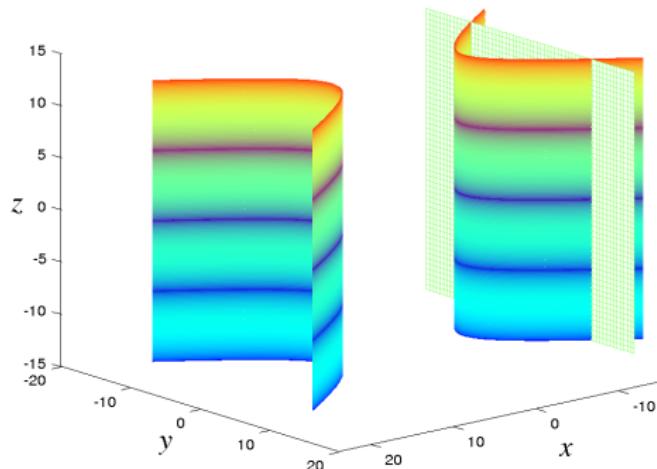
- As curvas de nível são elipses iguais para todo $k \in \mathbb{R}$.

Cilindro hiperbólico: gráfico e equação



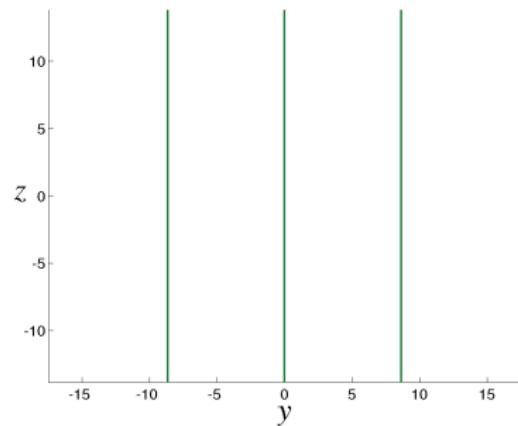
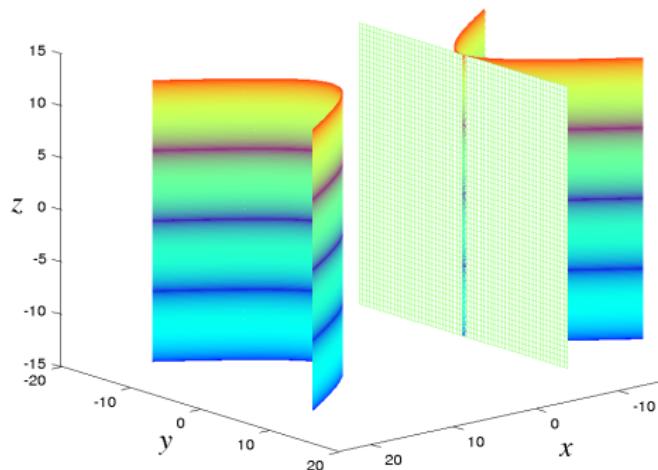
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Interseção com planos verticais: $x = k$



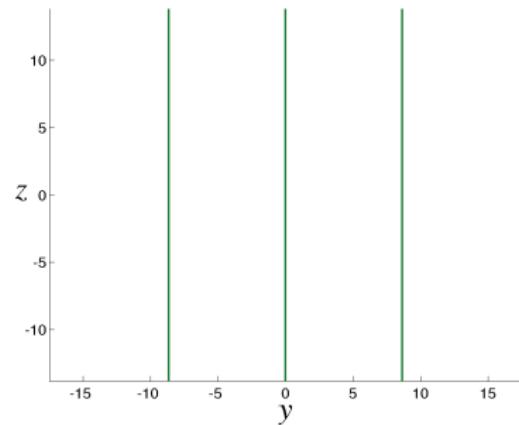
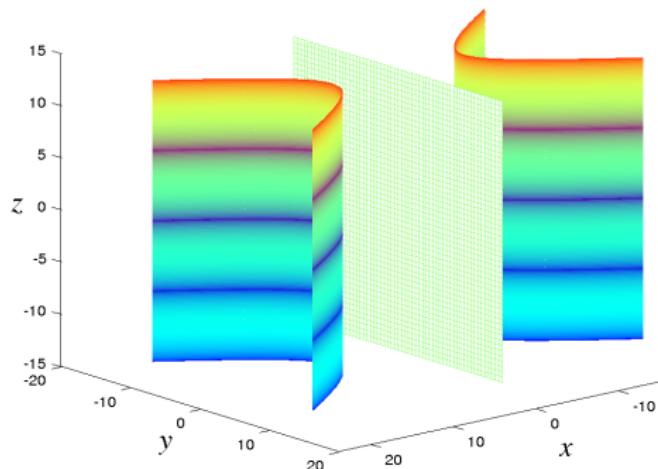
$$\left\{ \begin{array}{l} \frac{y^2}{b^2} = \frac{k^2}{a^2} - 1 \\ x = k \end{array} \right. : \text{ duas retas paralelas } (k < -a \text{ ou } k > a).$$

Interseção com planos verticais: $x = k$



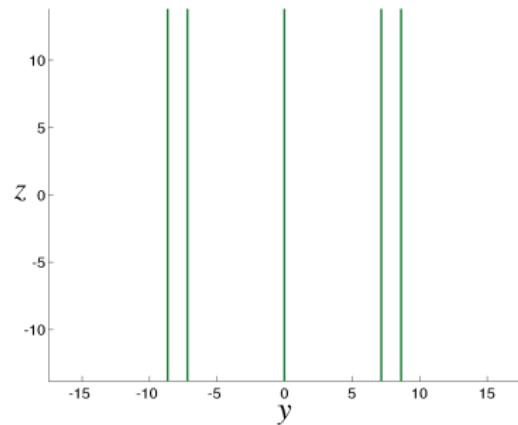
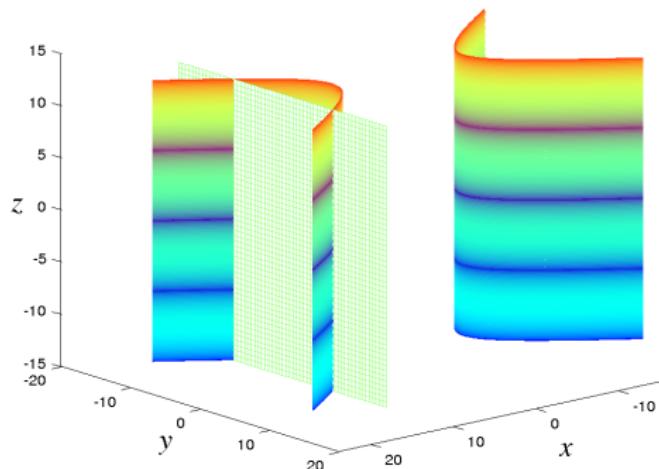
$$\begin{cases} y = 0 \\ x = k \end{cases} : \text{ uma única reta } (k = -a \text{ ou } k = a).$$

Interseção com planos verticais: $x = k$



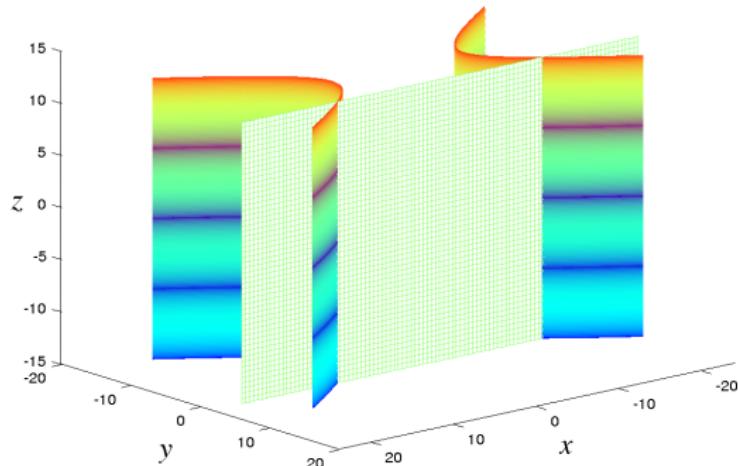
- O conjunto vazio quando $-a < k < a$.

Interseção com planos verticais: $x = k$



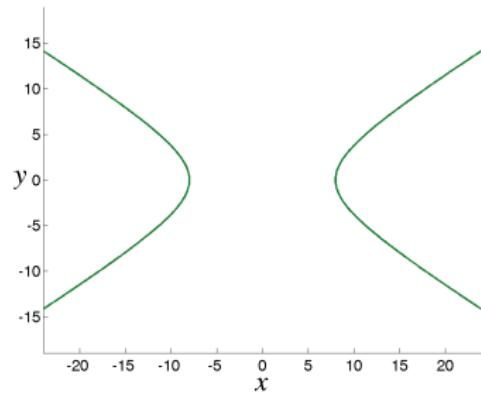
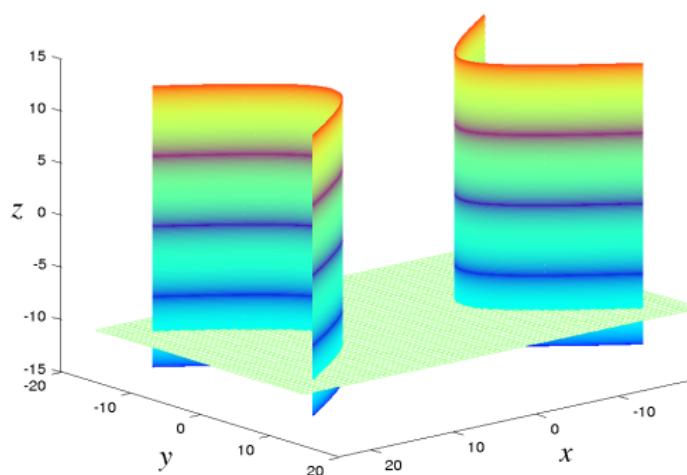
$$\left\{ \begin{array}{l} \frac{y^2}{b^2} = \frac{k^2}{a^2} - 1 \\ x = k \end{array} \right. : \text{ duas retas paralelas } (k < -a \text{ ou } k > a).$$

Interseção com planos verticais: $y = k$



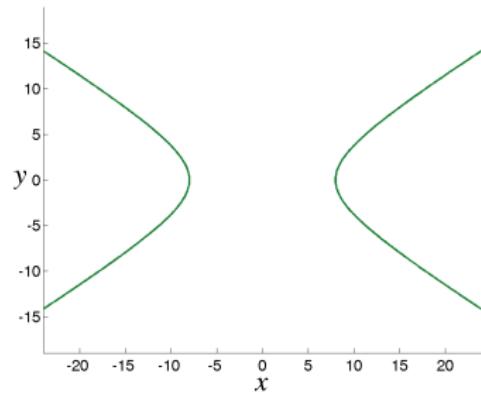
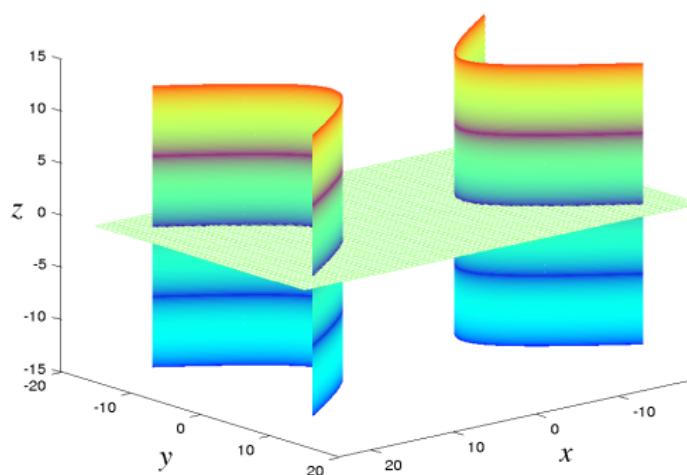
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} = 1 + \frac{k^2}{b^2} \\ y = k \end{array} \right. : \text{duas retas paralelas } \forall k \in \mathbb{R}.$$

Interseção com planos horizontais: $z = k$



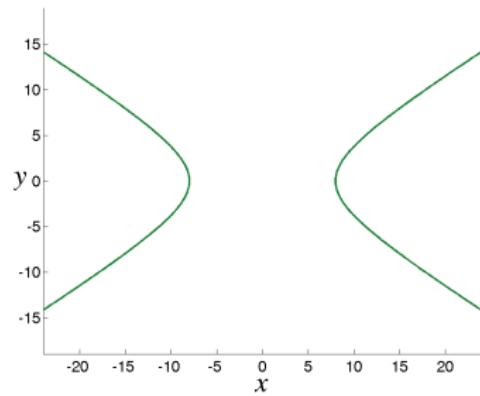
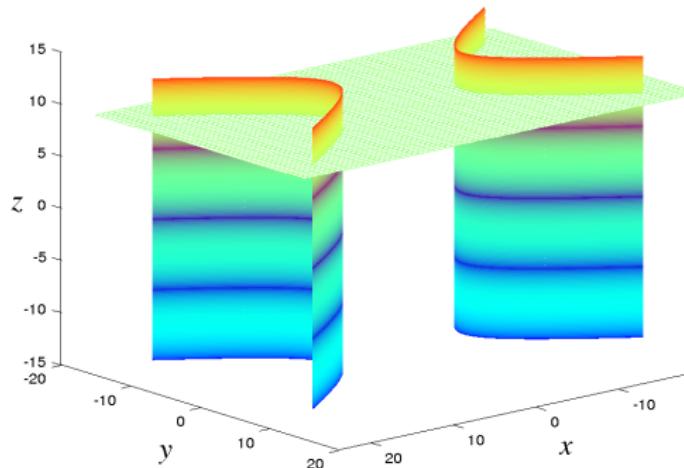
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \\ z = k \end{array} \right. : \text{ hipérboles iguais } \forall k \in \mathbb{R}.$$

Interseção com planos horizontais: $z = k$



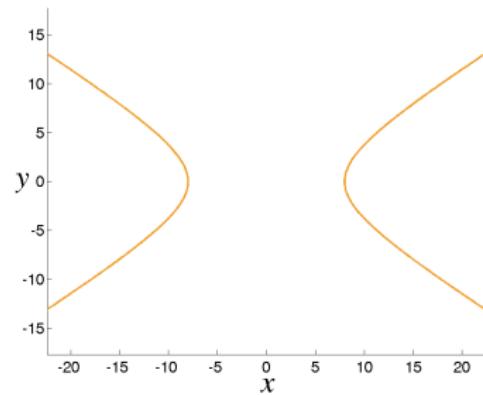
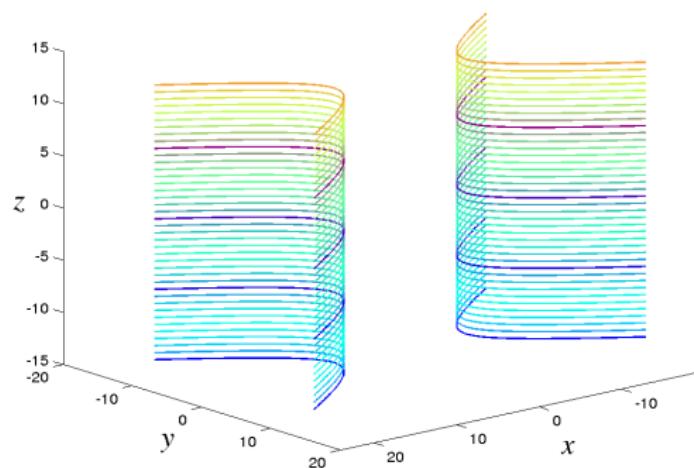
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \\ z = k \end{array} \right. : \text{ hipérboles iguais } \forall k \in \mathbb{R}.$$

Interseção com planos horizontais: $z = k$



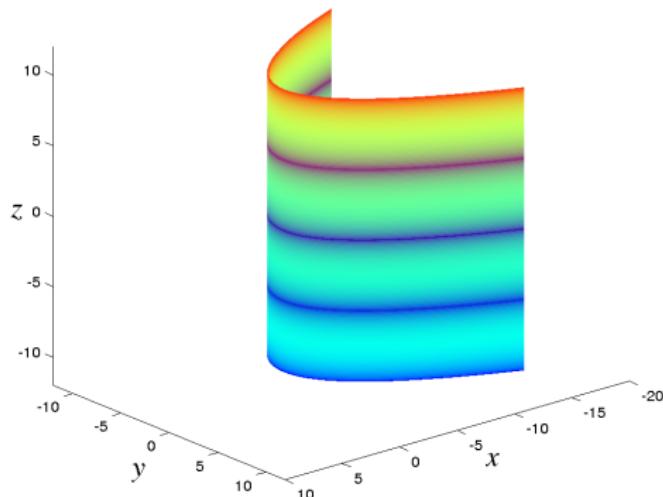
$$\left\{ \begin{array}{l} \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \\ z = k \end{array} \right. : \text{ hipérboles iguais } \forall k \in \mathbb{R}.$$

Curvas de nível em \mathbb{R}^3 e em \mathbb{R}^2



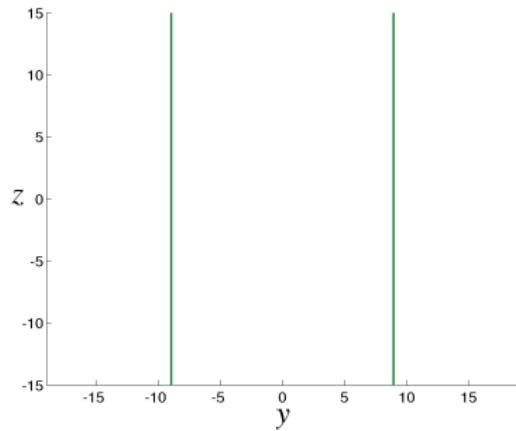
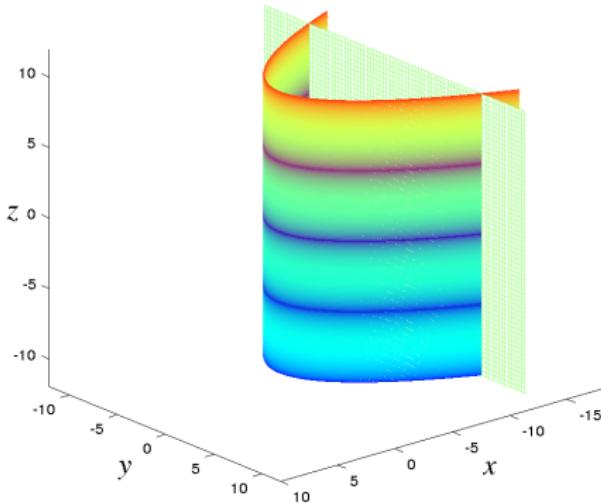
- As curvas de nível são hipérboles iguais para todo $k \in \mathbb{R}$.

Cilindro parabólico: gráfico e equação



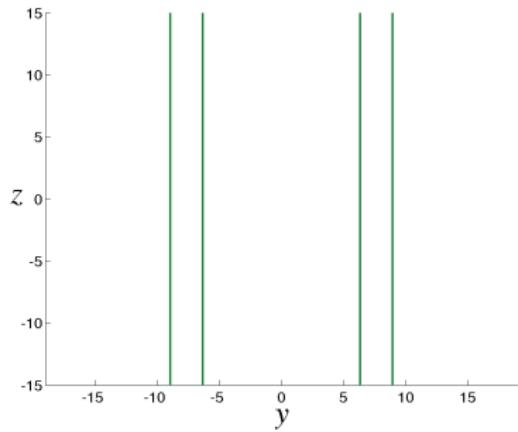
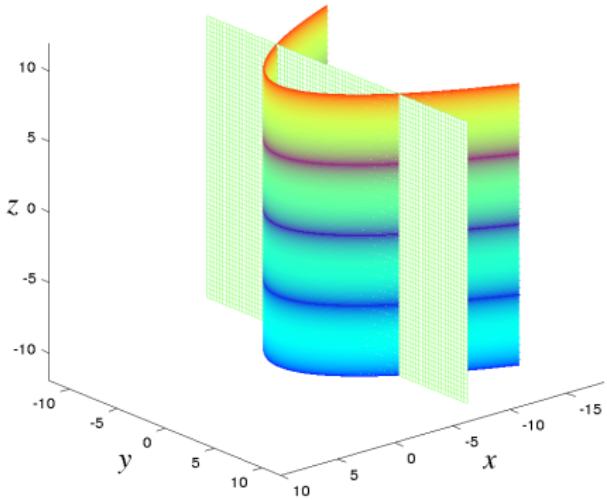
$$y^2 = -2px$$

Interseção com planos verticais: $x = k$



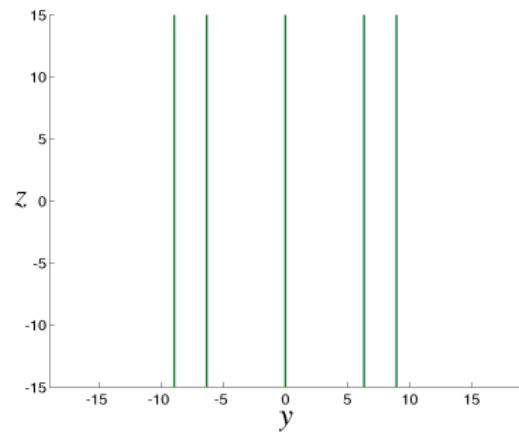
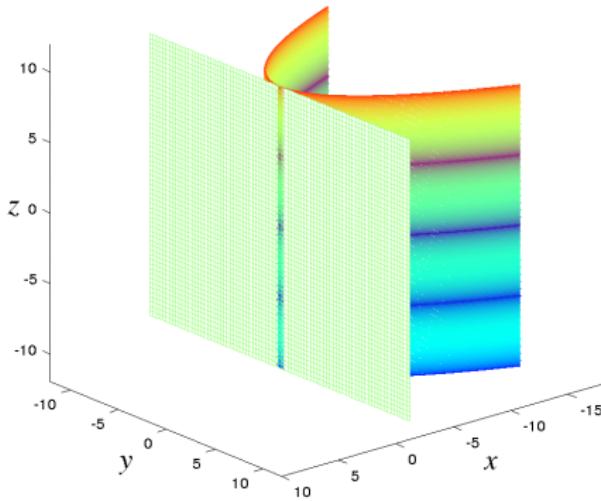
$$\begin{cases} y^2 = -2pk \\ x = k \end{cases} : \text{duas retas paralelas } (k < 0).$$

Interseção com planos verticais: $x = k$



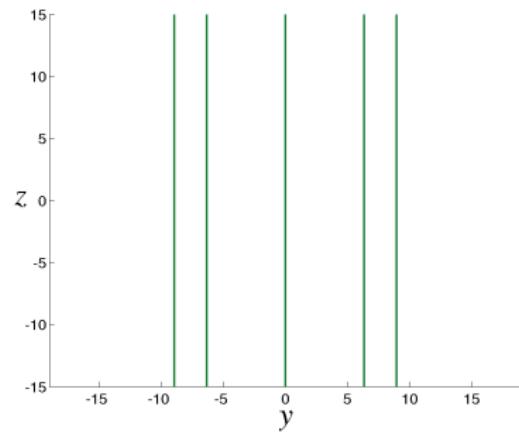
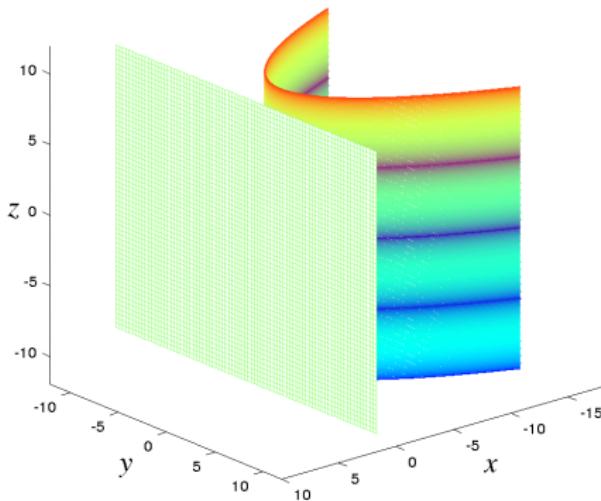
$$\begin{cases} y^2 = -2pk \\ x = k \end{cases} : \text{duas retas paralelas } (k < 0).$$

Interseção com planos verticais: $x = k$



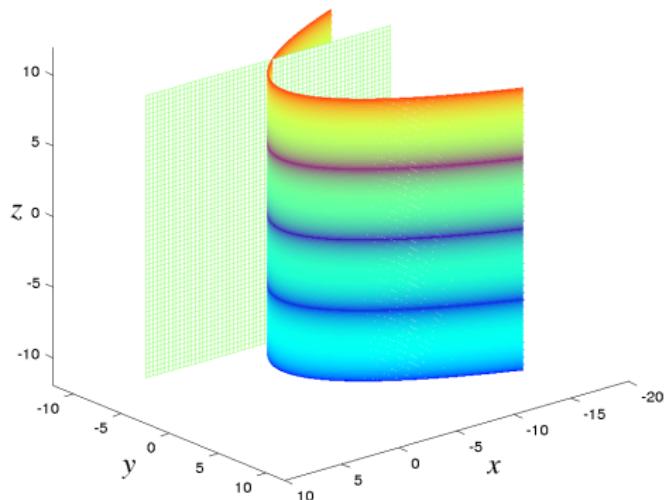
$$\begin{cases} y = 0 \\ x = k \end{cases} : \text{ uma única reta } (k = 0).$$

Interseção com planos verticais: $x = k$



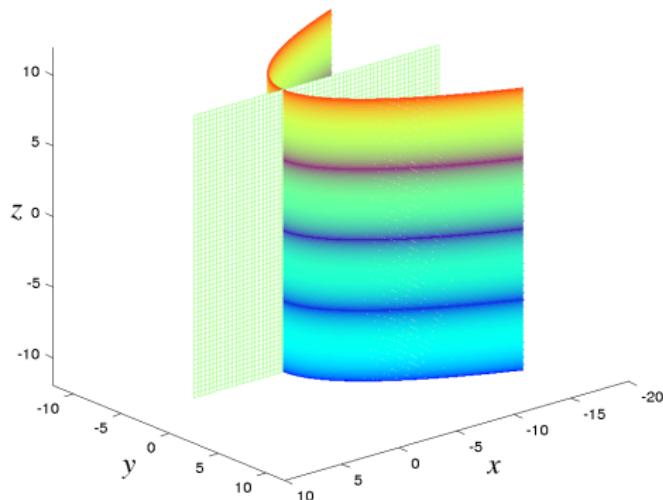
- O conjunto vazio quando $k > 0$.

Interseção com planos verticais: $y = k$



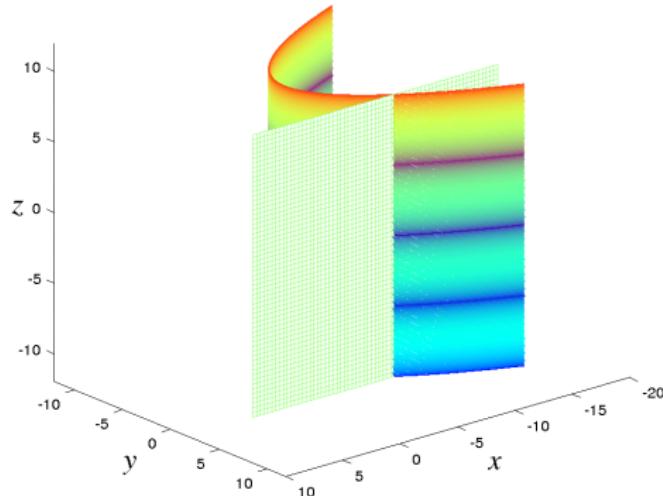
$$\begin{cases} -2px = k^2 \\ y = k \end{cases} : \text{ uma única reta } \forall k \in \mathbb{R}.$$

Interseção com planos verticais: $y = k$



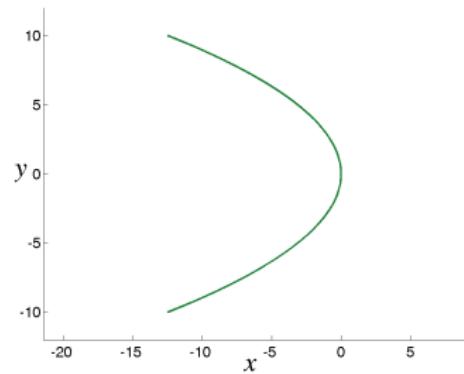
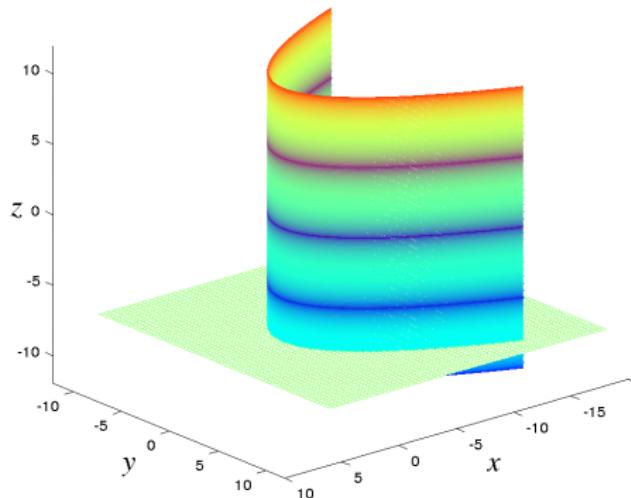
$$\begin{cases} -2px = k^2 \\ y = k \end{cases} : \text{ uma única reta } \forall k \in \mathbb{R}.$$

Interseção com planos verticais: $y = k$



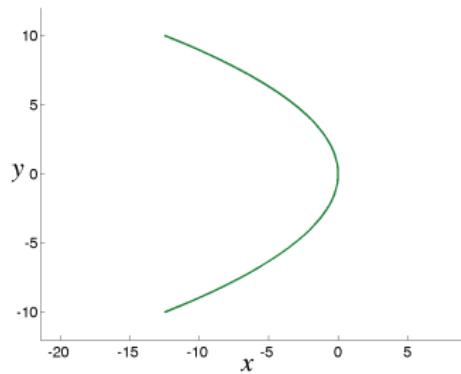
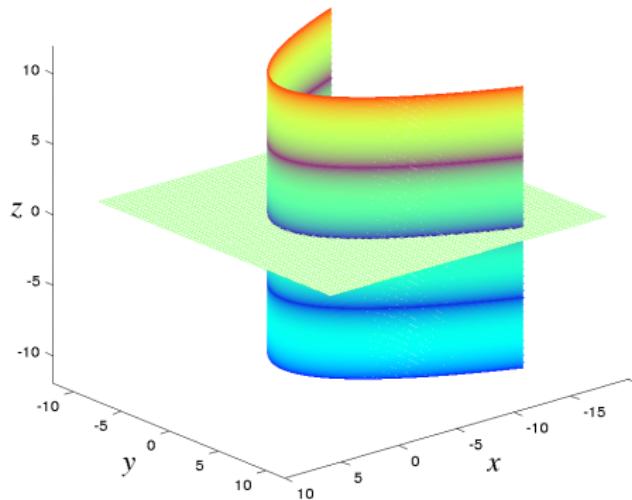
$$\begin{cases} -2px = k^2 \\ y = k \end{cases} : \text{ uma única reta } \forall k \in \mathbb{R}.$$

Interseção com planos horizontais: $z = k$



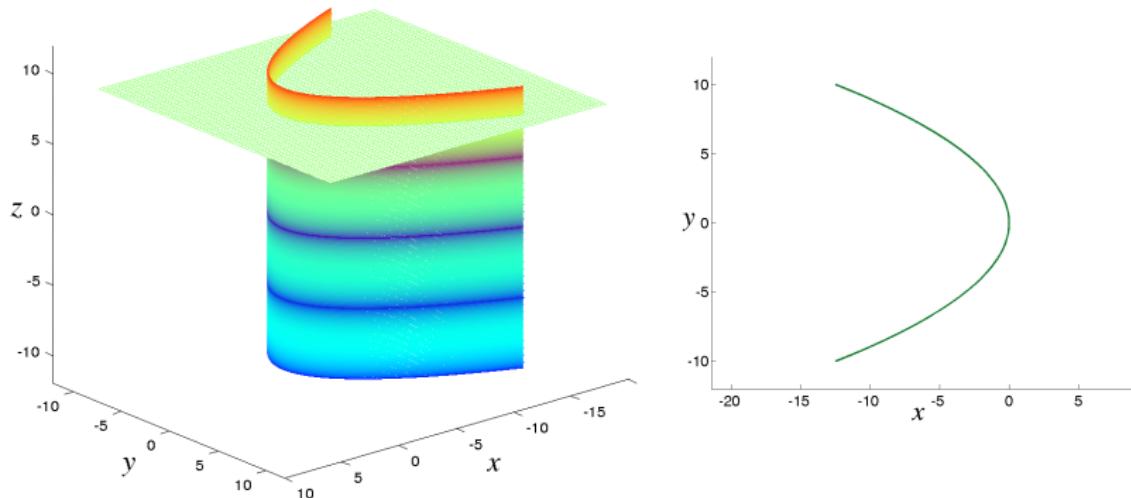
$$\begin{cases} y^2 = -2px \\ z = k \end{cases} : \text{parábolas iguais } \forall k \in \mathbb{R}.$$

Interseção com planos horizontais: $z = k$



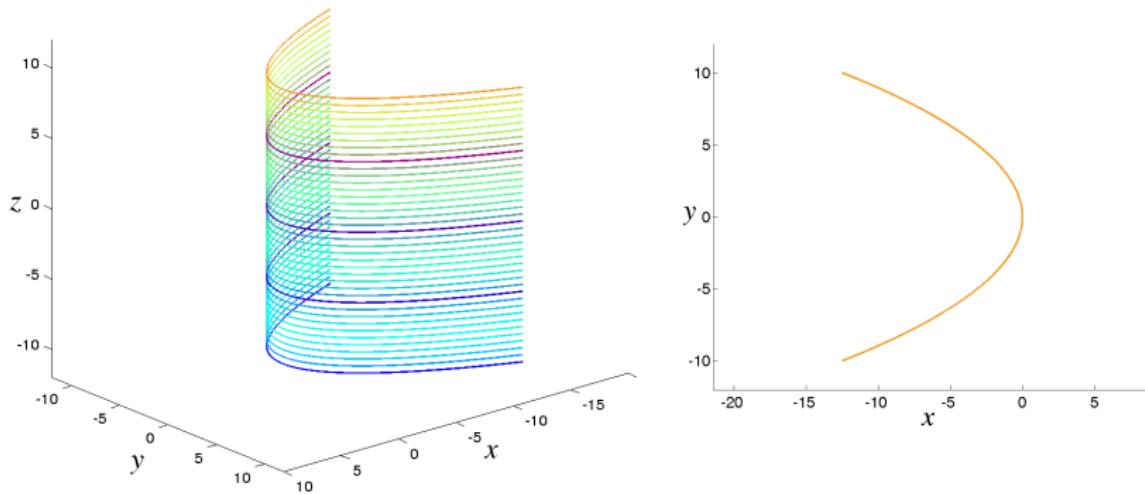
$$\begin{cases} y^2 = -2px \\ z = k \end{cases} : \text{parábolas iguais } \forall k \in \mathbb{R}.$$

Interseção com planos horizontais: $z = k$



$$\begin{cases} y^2 = -2px \\ z = k \end{cases} : \text{parábolas iguais } \forall k \in \mathbb{R}.$$

Curvas de nível em \mathbb{R}^3 e em \mathbb{R}^2



- As curvas de nível são parábolas iguais para todo $k \in \mathbb{R}$.