hw2_part1

October 14, 2023

[ ]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import sympy as sy


import seaborn as sns
from sklearn.metrics import confusion_matrix

from IPython.display import display, Math, Latex

import nn
```

1 Part 1: One Layer NN with One Output

1.1 Problem 2

1.1.1 Create Dataset

The data is meant to be a clustering problem in 3 dimensions. Anything close to the origin has a value of 0 and anything further has a value of 1.

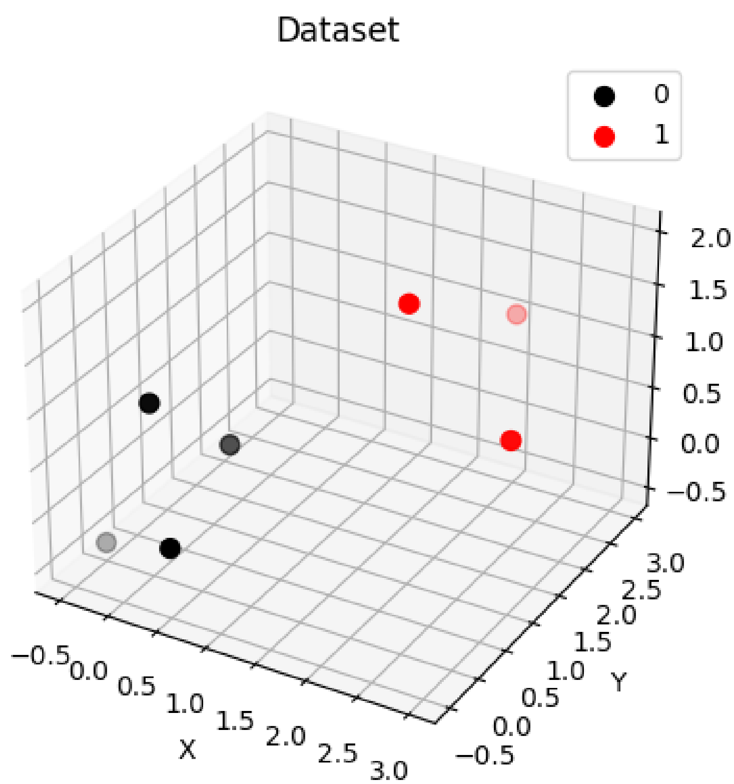
[ ]:
```
data = pd.read_csv("A2_Data_EliWeissler.csv")
data
```

[ ]:

	X	Y	Z	LABEL
0	-0.5	0.0	-0.5	0
1	0.5	-0.5	0.0	0
2	0.0	0.0	1.0	0
3	0.5	0.5	0.5	0
4	2.0	1.0	2.0	1
5	3.0	1.0	1.0	1
6	2.0	3.0	1.0	1

[ ]:
```
fig = plt.figure()
ax = plt.axes(projection='3d')

# plotting
```

```
colors = ["k", "r"]
for i in [0, 1]:
    data_subset = data[data["LABEL"] == i]
    ax.scatter(data_subset["X"], data_subset["Y"], data_subset["Z"],
               s=50, c=colors[i], label=data_subset["LABEL"].iloc[0])
ax.set_title('Dataset')
ax.set_xlabel("X")
ax.set_ylabel("Y")
ax.set_zlabel("Z")
ax.legend()
plt.show()
```



1.1.2 Create X and Y

[ ]:
```
X, Y = nn.normalize_data(data)
Math("X = " + sy.latex(sy.Matrix(np.round(X, 2))) +\
     "\quad Y = " + sy.latex(sy.Matrix(Y)))
```

[ ]:

$$X = \begin{bmatrix} 0 & 0.14 & 0 \\ 0.29 & 0 & 0.2 \\ 0.14 & 0.14 & 0.6 \\ 0.29 & 0.29 & 0.4 \\ 0.71 & 0.43 & 1.0 \\ 1.0 & 0.43 & 0.6 \\ 0.71 & 1.0 & 0.6 \end{bmatrix} \quad Y = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

1.1.3 Do Feed Forward to H1

[ ]:
```
# Define weights and biases
W1 = np.ones((3, 4))
W2 = 2*np.ones((4, 1))
B = np.zeros((1, 4))
C = 0

# First hidden layer
Z1 = X@W1 + B
H1 = nn.sigmoid(Z1)

# Show H1
np.round(H1, 2)
```

[ ]:
```
array([[0.54, 0.54, 0.54, 0.54],
       [0.62, 0.62, 0.62, 0.62],
       [0.71, 0.71, 0.71, 0.71],
       [0.73, 0.73, 0.73, 0.73],
       [0.89, 0.89, 0.89, 0.89],
       [0.88, 0.88, 0.88, 0.88],
       [0.91, 0.91, 0.91, 0.91]])
```

1.1.4 Calculate a Loss Function

[ ]:
```
# Output layer
Z2 = H1@W2 + C
yhat = nn.sigmoid(Z2)
loss = nn.loss_MSE(Y, yhat)
loss
```

[ ]:
```
array([[4.86508842e-01],
       [4.93010336e-01],
       [4.96549485e-01],
       [4.96995824e-01],
       [3.01441059e-07],
       [3.60751217e-07],
       [2.36959392e-07]])
```

1.2 Problem 3

1.2.1 Print out all the Matrices for Checking:

```
[ ]: display(Math("X = " + sy.latex(sy.Matrix(np.round(X, 2)))))
display(Math("Y = " + sy.latex(sy.Matrix(Y))))
display(Math("W^{(1)} = " + sy.latex(sy.Matrix(W1))))
display(Math("B = " + sy.latex(sy.Matrix(B))))
display(Math("Z^{(1)} = " + sy.latex(sy.Matrix(np.round(Z1,2)))))
display(Math("H = " + sy.latex(sy.Matrix(np.round(H1,2)))))
display(Math("W^{(2)} = " + sy.latex(sy.Matrix(W2))))
display(Math("Z^{(2)} = " + sy.latex(sy.Matrix(np.round(Z2,2)))))
display(Math("C = " + str(np.round(C, 2))))
display(Math("\hat{y} = " + sy.latex(sy.Matrix(np.round(yhat,3)))))
display(Math("\hat{y}-Y = " + sy.latex(sy.Matrix(np.round(yhat-Y,5)))))
display(Math("L_{contributions} = " + sy.latex(sy.Matrix(loss**2))))
display(Math("L_{MSE} = " + str(np.round(np.mean(loss), 4))))
```

$$X = \begin{bmatrix} 0 & 0.14 & 0 \\ 0.29 & 0 & 0.2 \\ 0.14 & 0.14 & 0.6 \\ 0.29 & 0.29 & 0.4 \\ 0.71 & 0.43 & 1.0 \\ 1.0 & 0.43 & 0.6 \\ 0.71 & 1.0 & 0.6 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$W^{(1)} = \begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Z^{(1)} = \begin{bmatrix} 0.14 & 0.14 & 0.14 & 0.14 \\ 0.49 & 0.49 & 0.49 & 0.49 \\ 0.89 & 0.89 & 0.89 & 0.89 \\ 0.97 & 0.97 & 0.97 & 0.97 \\ 2.14 & 2.14 & 2.14 & 2.14 \\ 2.03 & 2.03 & 2.03 & 2.03 \\ 2.31 & 2.31 & 2.31 & 2.31 \end{bmatrix}$$

$$H = \begin{bmatrix} 0.54 & 0.54 & 0.54 & 0.54 \\ 0.62 & 0.62 & 0.62 & 0.62 \\ 0.71 & 0.71 & 0.71 & 0.71 \\ 0.73 & 0.73 & 0.73 & 0.73 \\ 0.89 & 0.89 & 0.89 & 0.89 \\ 0.88 & 0.88 & 0.88 & 0.88 \\ 0.91 & 0.91 & 0.91 & 0.91 \end{bmatrix}$$

$$W^{(2)} = \begin{bmatrix} 2.0 \\ 2.0 \\ 2.0 \\ 2.0 \end{bmatrix}$$

$$Z^{(2)} = \begin{bmatrix} 4.29 \\ 4.95 \\ 5.66 \\ 5.8 \\ 7.16 \\ 7.07 \\ 7.28 \end{bmatrix}$$

$$C = 0$$

$$\hat{y} = \begin{bmatrix} 0.986 \\ 0.993 \\ 0.997 \\ 0.997 \\ 0.999 \\ 0.999 \\ 0.999 \end{bmatrix}$$

$$\hat{y} - Y = \begin{bmatrix} 0.98642 \\ 0.99299 \\ 0.99654 \\ 0.99699 \\ -0.00078 \\ -0.00085 \\ -0.00069 \end{bmatrix}$$

$$L_{contributions} = \begin{bmatrix} 0.236690853107355 \\ 0.24305919110782 \\ 0.246561391447607 \\ 0.247004848981784 \\ 9.08667119272342 \cdot 10^{-14} \\ 1.30141440748151 \cdot 10^{-13} \\ 5.6149753579297 \cdot 10^{-14} \end{bmatrix}$$

$$L_{MSE} = 0.2819$$

1.3 Problem 5/6

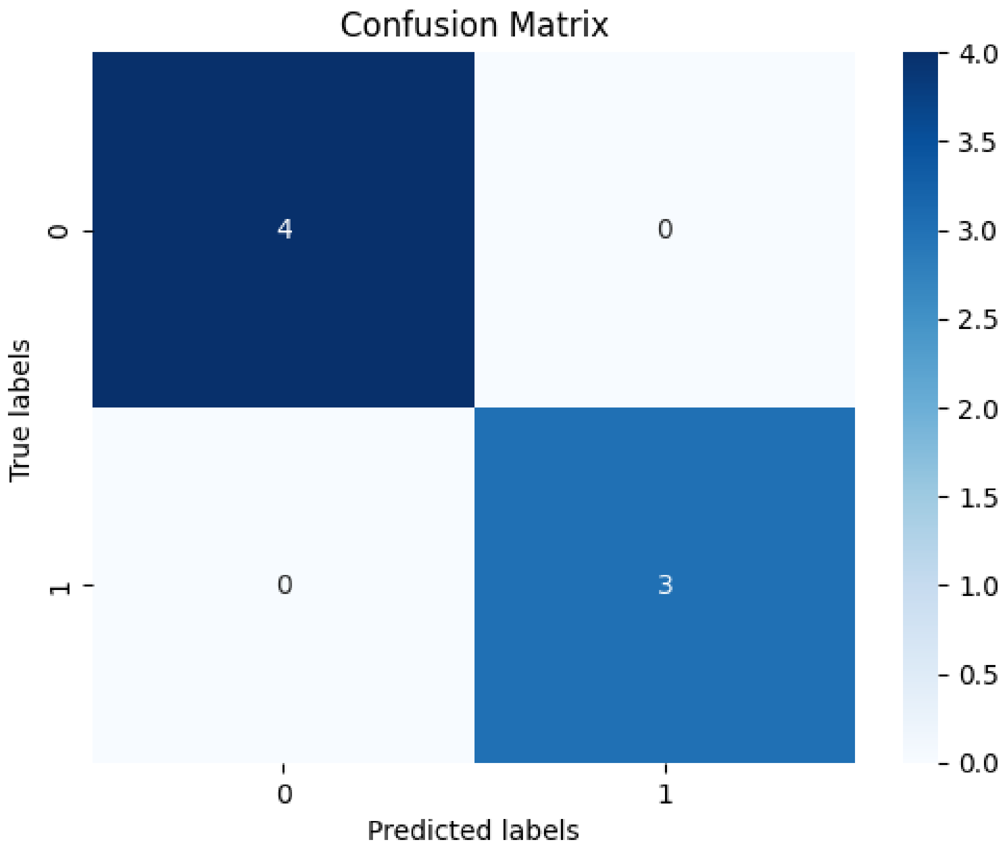
1.3.1 Try training the network

[ ]:
```python
# Initialize network and load data
data = pd.read_csv("A2_Data_EliWeissler.csv")
X, Y = nn.normalize_data(data)

input_size = 3
hidden_layers = [4]
output_size = 1
activation_fns = [nn.sigmoid, nn.sigmoid]
loss_fn = nn.loss_MSE
random_initialize = False
network = nn.NeuralNetwork(input_size, output_size, hidden_layers,
                           activation_fns, loss_fn,␣
 ↪random_initialize=random_initialize)
```

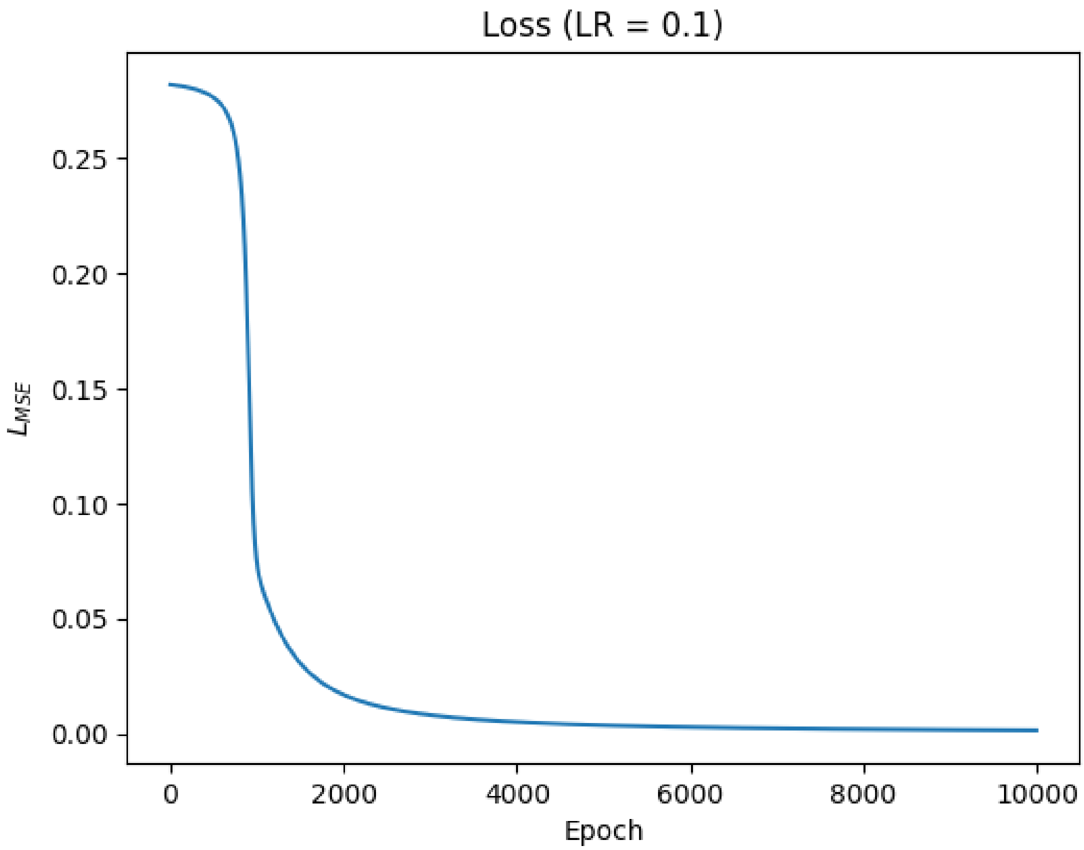
[ ]:
```python
# Train network
epochs = 10000
lr = 0.1
batch_size = 7
loss = network.train(X, Y, X, Y, epochs=epochs, lr=lr, batch_size=batch_size)
```

```
Epoch 0 (out of 10000) -- Loss: 0.2819
Epoch 1000 (out of 10000) -- Loss: 0.0737
Epoch 2000 (out of 10000) -- Loss: 0.0169
Epoch 3000 (out of 10000) -- Loss: 0.0081
Epoch 4000 (out of 10000) -- Loss: 0.0051
Epoch 5000 (out of 10000) -- Loss: 0.0036
Epoch 6000 (out of 10000) -- Loss: 0.0028
Epoch 7000 (out of 10000) -- Loss: 0.0023
Epoch 8000 (out of 10000) -- Loss: 0.0019
Epoch 9000 (out of 10000) -- Loss: 0.0017
```

[ ]:
```python
# Predict and plot
pred = network.feed_forward(X)

nn.plot_confusion_matrix(Y, pred)
nn.plot_loss(loss, lr)
```



Confusion Matrix
4
0
0
3
True labels
0
1
0
1
Predicted labels
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0


Loss (LR = 0.1)
L_{MSE}
0.25
0.20
0.15
0.10
0.05
0.00
0
2000
4000
6000
8000
10000
Epoch