SSY345 Sensor fusion and non linear filtering

HA2 Analysis

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Discussions

I have discussed the assignment with Osvald Lindholm who usually is my LAB partner in other courses.

Code

The used matlab code for solving the problems can be found in the submitted files on Canvas.

1 - A first Kalman filter and its properties

(a)

Inspecting the plot in figure (1) we can see that the measurements tend to follow the state. When the state value is decreasing, so are the measurements.

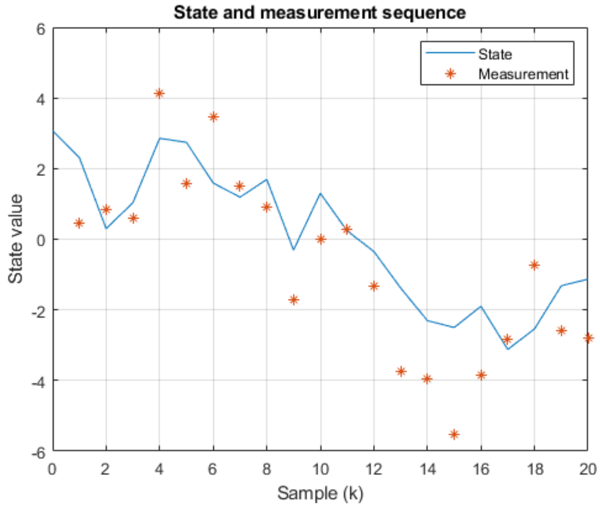

Figure 1: State values decreasing and also measurements.

(b)

The Kalman filter seems to track the real state sequence quite well and the true state never goes outside the 3σ level curves which is a good indication of the error covariance representing the uncertainty well. By further inspection of figure (2) we can see that the measurements have a significant impact on how the measurement update "pulls" the state to the measurements. An example of this is at k = 15 where the the measurement pulls the estimated state downwards, towards the measurement. Since the process noise (Q = 1.5) and measurement noise (R=2.5) have the same order of magnitude we would expect the measurement to have a noticeable impact.

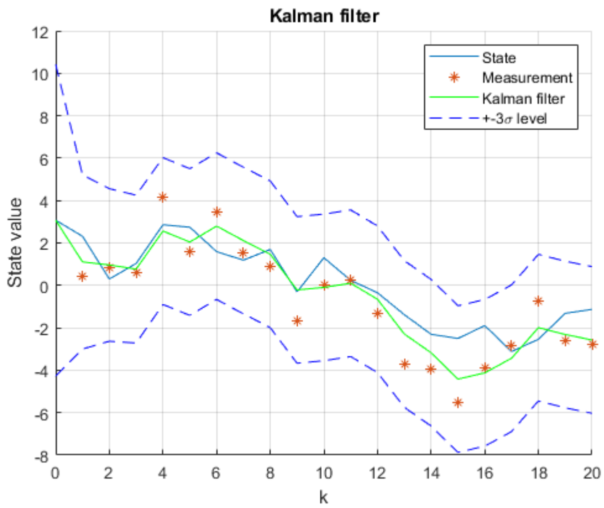

Figure 2: Comparison of Kalman filter vs true state and it's confidence bounds $\pm 3\sigma$

If we examine some of the posterior densities with the true state and measurement in figure 3 and 4 we can also here see that the true state is inside the $\pm 3\sigma$ bounds and that the measurement has affected the posterior density. I personally think that the true state should be replaced by the predicted state to show this more clearly since the true state is never known to the kalman filter, only the prediction.

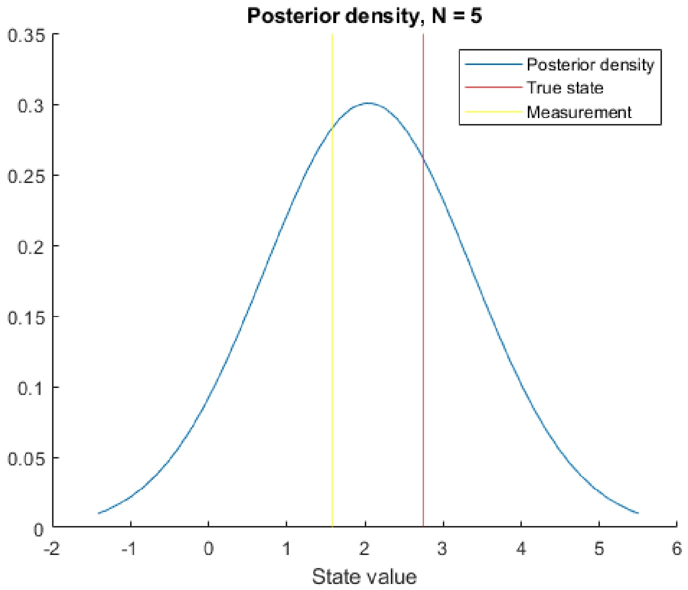

Figure 3: The posterior density is between the true state and the measurement.

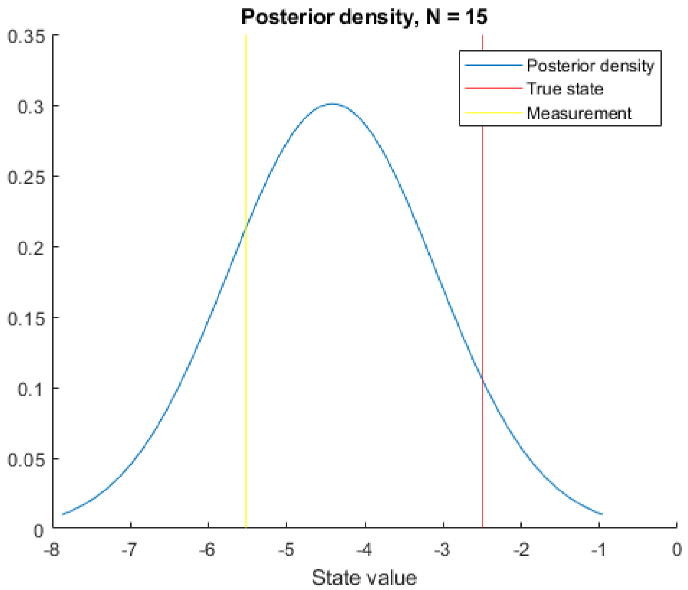

Figure 4: The posterior density is closer to the measurement than the true state.

(c)

Comparing the previous kalman estimate $p(x_{k-1}|y_{1:k-1})$ to the prediction $p(x_k|y_{1:k-1})$ where $k = 15$ we can verify our intuition that the uncertainty should grow when predicting, $P_{k|k-1} = A \cdot P_{k-1|k-1} \cdot A^T + Q$ as seen in figure 5. We can also see that the mean doesn't change in the prediction since our process model is $x_k = 1 \cdot x_{k-1} + q_{k-1}$. When we make the measurement update our mean changes and the uncertainty reduces as one would expect, $P_{k|k} = P_{k|k-1} - K \cdot S \cdot K^T$.

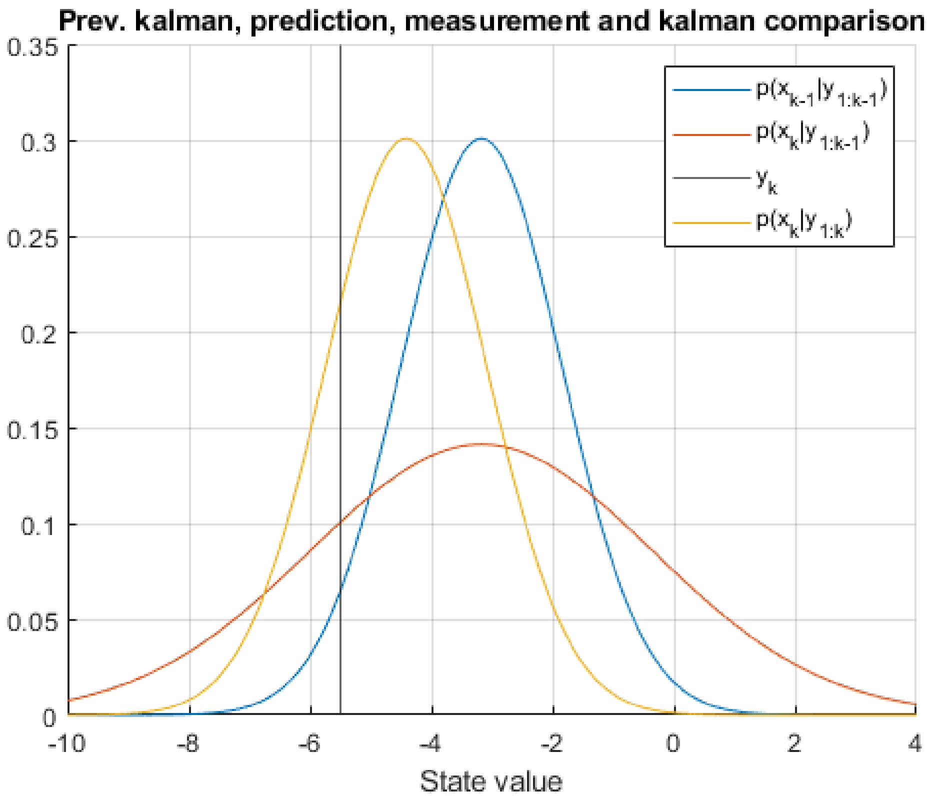

Figure 5: The density grows in uncertainty during the prediction and then gets better as we make a measurement update.

(d)

The histogram pillars in figure 6 shows us how often our estimates are off by a certain range and that it follows the general shape of $N(x; 0; P_{N|N}$. The variance converges to $P \approx 1.3$ already after 6 samples and then describes the true variance of our system when we use both the motion model and the process model. This new variance is lower than both the motion model ($Q = 1, 5$) and the measurement model ($R = 2.5$) which we would expect since we want to do a better job than one or the other.

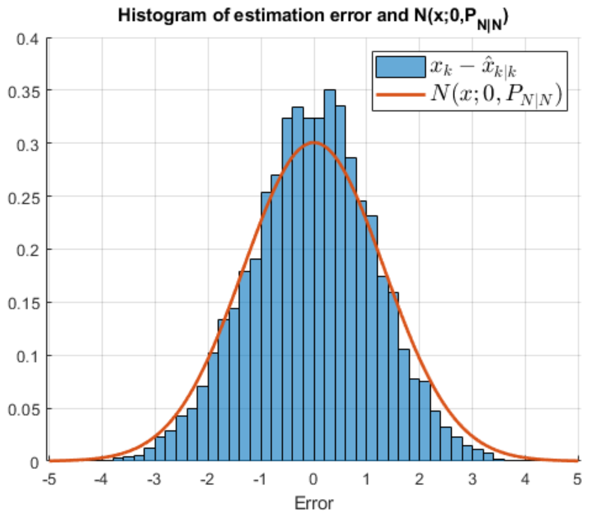

Figure 6: The histogram of the estimation error fits well with the pdf.

The estimated mean of the innovation which was calculated to $V_{mean} = 0.033$ for $N = 5000$ and the auto correlation function is seen if figure 7. The auto correlation plot tells us about how correlated one time instant is of a time delayed version of itself. If we would see a pattern or the red dots would be outside the blue lines we would not have white Gaussian noise. It shows that our assumption of the noise being white Gaussian is correct and that we didn't miss a fundamental underlying component like a sine in our modeling.

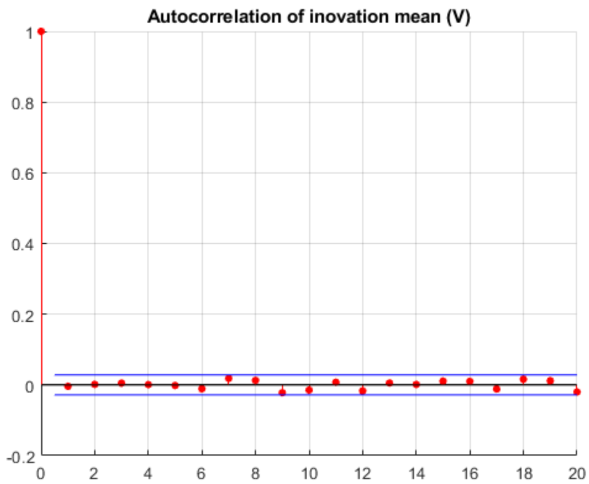

Figure 7: No red dots outside the blue lines indicate no correlation and hence showing that the noise indeed is white Gaussian.

(f)

Even though we start the Kalman filter with a wrong initial state we can see that it quickly converges (5-6 samples in this case). The reason for it converging is that the as soon as we start doing measurements updates the innovation will redirect the wrong prediction at each update. After a couple of updates the innovation has removed most of the impact of the wrong initial guess. In our case the motion noise and measurement noise have the same order of magnitude but if the measurement noise would be a lot bigger it would take a longer time for the filter to converge with a wrong initial state since it won't trust the measurements as much.

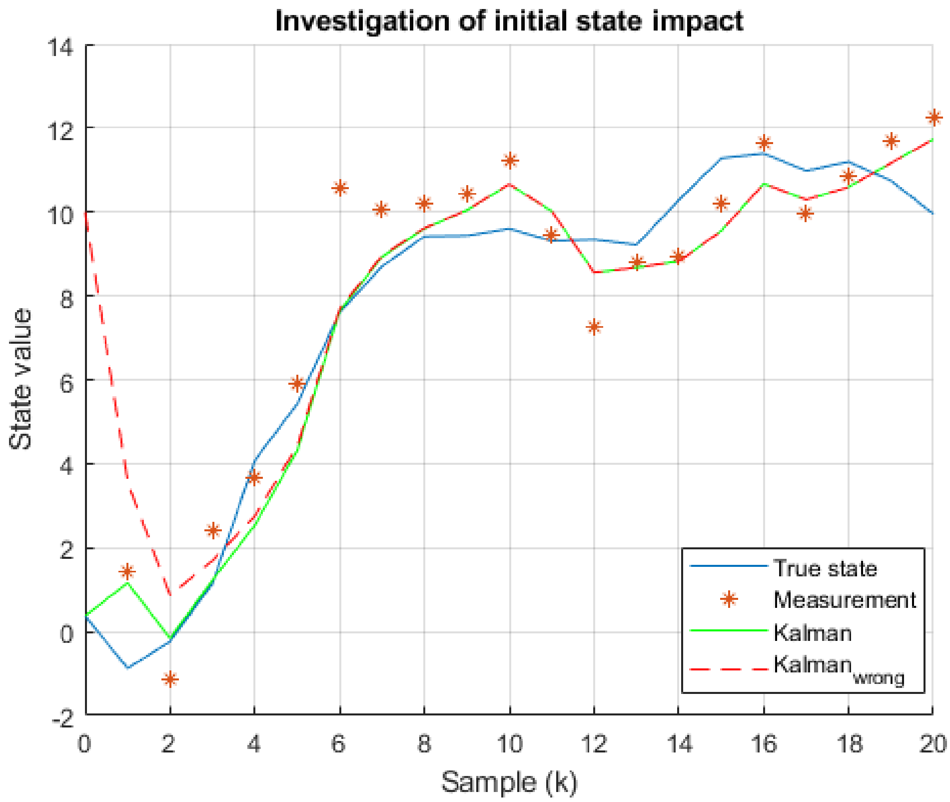

Figure 8: Comparison of convergence with wrong initial state for Kalman

2 - Kalman filter and its tuning

(a)

The motion model is a constant velocity model and is modeled by a deterministic constant velocity with an added noise. The generated state sequence of the position and measurements of the position can be seen in figure 9. To decide if it looks reasonable we can look at the velocity in figure 10 and see that the speed is positive during $t \in [0; 0.3]$ which explains the increase in position during that time and the position is decreasing during $t \in [0.35; 0.4]$ because the velocity is negative. The measurements is a bit harder to reason about since they have quite big variance ($\sigma^2 = 2$) but by visual inspection they seem to inscrease up until $t = 0.35s$ which seems reasonable since the state also is.

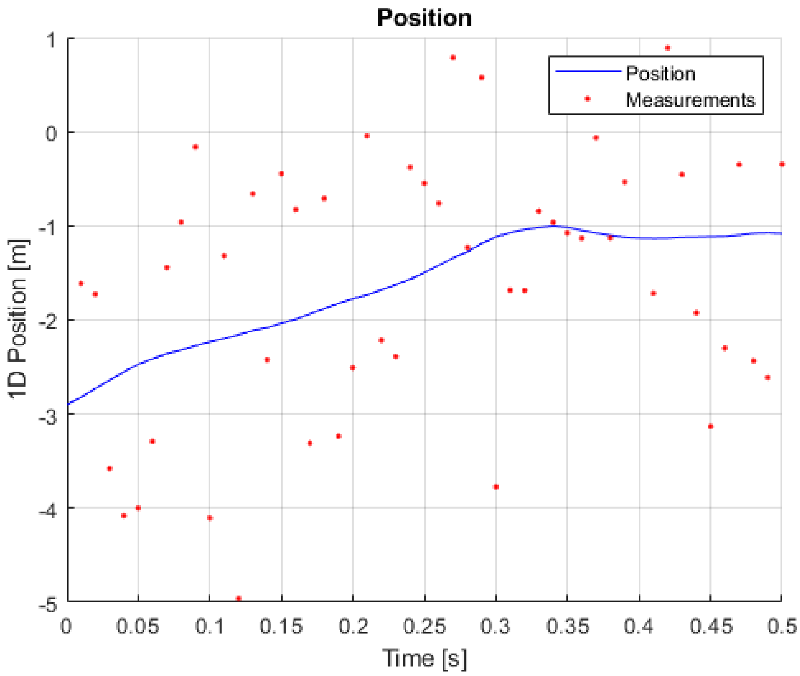

Figure 9: Position sequence shown in blue and the measurements shown in red.

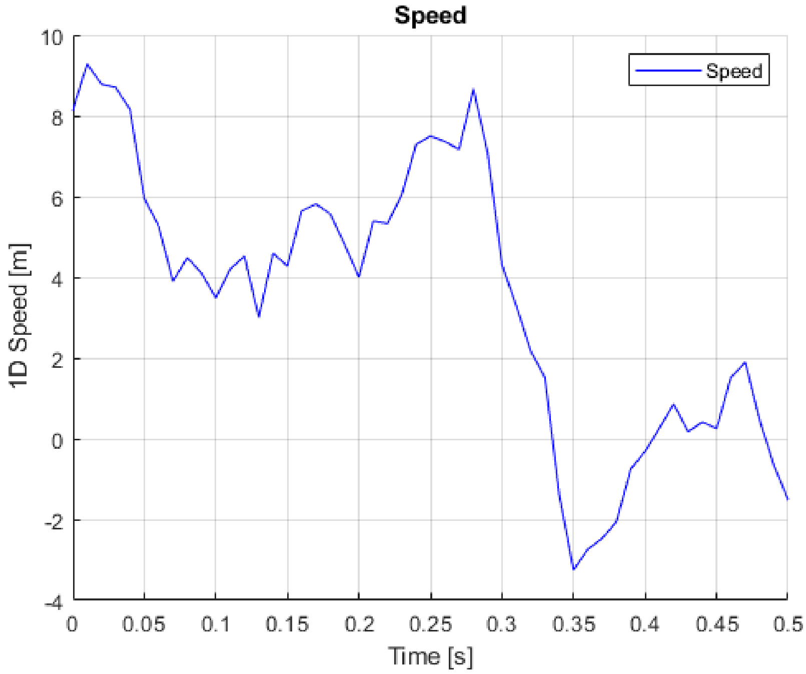

Figure 10: Velocity sequence, positive up until around 0.35s, then negative to 0.4s.

(b)

From figure 11 we see how the uncertainty of the Kalman filter (dark green) quickly reduces from the initial $p(x_0) = \mathbf{N}(x_0; [13]^T, 4\mathbf{I})$ to a much lower of around 0.67. (At t = 0.1s, $3\sigma \approx 2 \Leftrightarrow \sigma \approx 0.67$). It then stays there for the rest of the simulation indicating that we can't do better than this without decreasing the process or measurement noise.

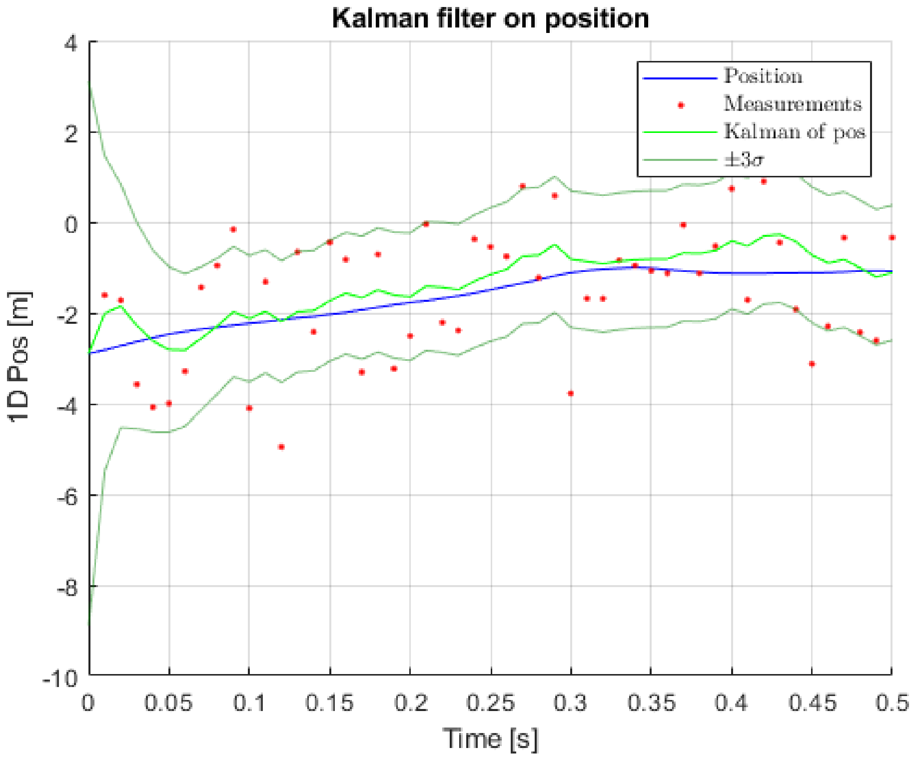

Figure 11: Kalman filter of the position quickly reducing the initial variance seen in dark green.

In the plot of the speed in figure 12 we can see that the uncertainty (dark green) is greatly increased compared to the position. This is because we do not get any measurements of the speed directly and only use measurements from the position. The biggest mismatch between the true speed and the kalman can also here bee seen between $t \in [0.35; 0.4]$ and can't be compensated since we do not measure the speed.

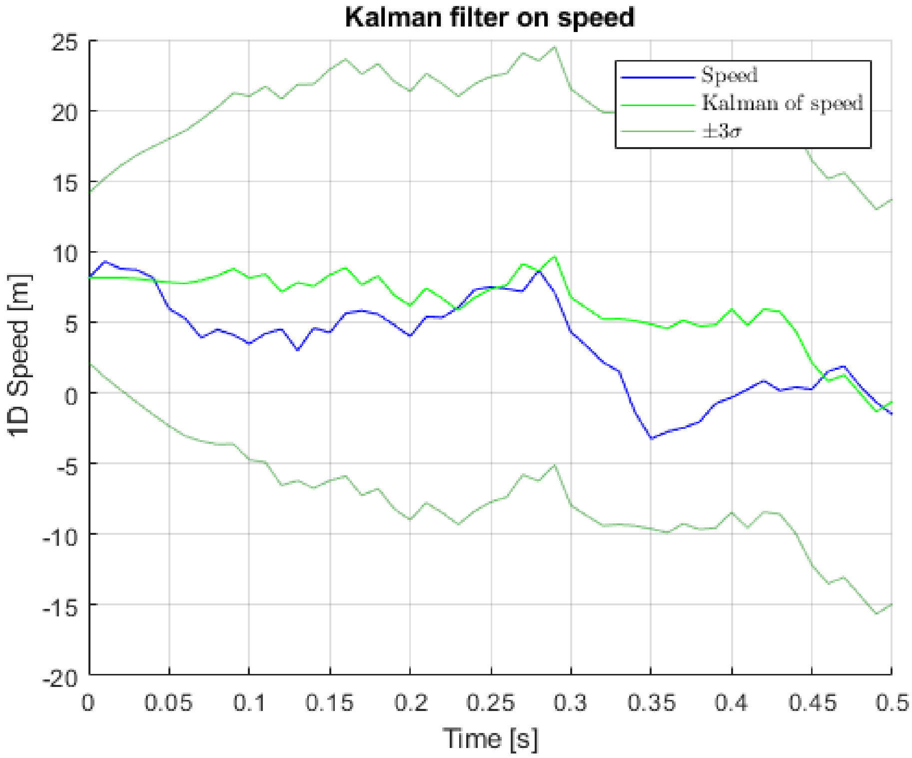

Figure 12: A small process noise Q means we will trust the process model alot more

(c)

If we imagine not knowing the motion noise variance and analysing the kalman filter for different Q we can try to tune the filter. When we look at $Q = 0.1$ in figure (13) we see (if we zoom in) that the measurements have a very low impact (smooth curve) since we will trust the process model more ($Q << R$). A big consequence of this can be seen at $t \in [0.35; 0.5]$ where the measurements can't seem to pull the filter towards the true states enough because of the heavy trust on the process model.

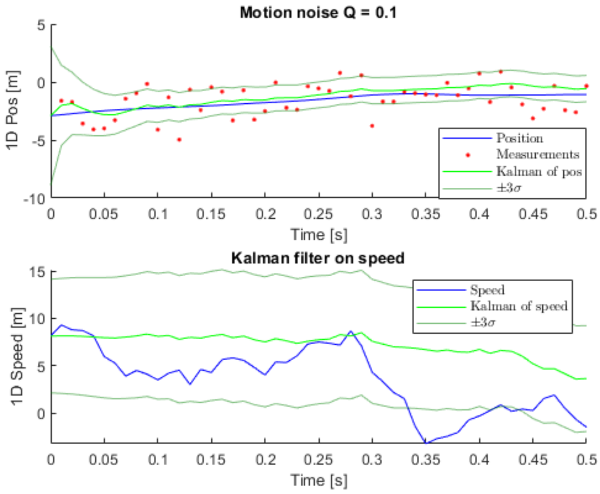

Figure 13: A small process noise Q means we will trust the process model alot more

If we instead look at the opposite scenario where $Q = 10$ in figure (14) where we will trust the measurements much more ($Q >> R$) and as a result have a more uneven/jagged plot because we will make much larger corrections in the update step in the kalman filter. We can also note the huge uncertainty in the speed estimate compared to $Q = 0.1$.

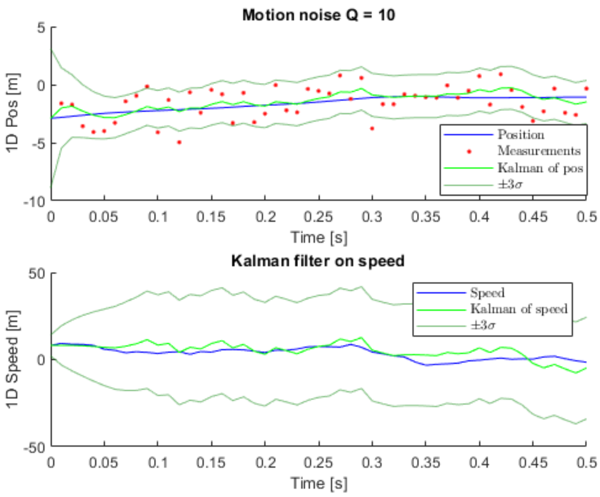

Figure 14: A big process noise Q means we will trust the measurement alot

When $Q = 1$ *or* $Q = 1.5$ in figure (15) and (16) we trust both the process model and the measurement model, Q is only a little big smaller than R ($Q = 1$ *or* $Q = 1.5 < R = 2.5$). This means the measurements will have a significant impact if e.g multiple measurements in a row are larger/smaller than the prediction. Both $Q = 1$ and $Q = 1.5$ create plots which are hard to distinguish from each other and I would therefor argue that the exact value for Q is not of the greatest importance, only the order of magnitudeis. That is when $Q = 0.1$ we had to little impact of the measurements and the speed did not track well while when we had $Q = 10$ we got too large uncertainty in the speed (At $t = 0.3$ in figure (14), $3\sigma \approx 30 \Leftrightarrow \sigma \approx 10$). In the end when $Q = 1$ and $Q = 1.5$ we saw the smallest uncertainty in the position and speed and hence these were the best but deciding between them I was not able to do.

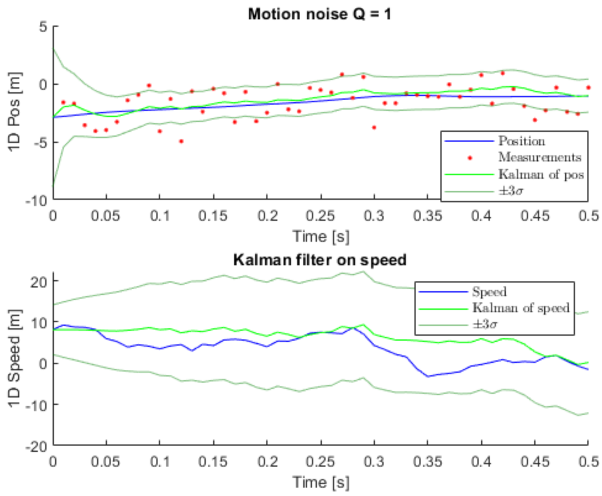

Figure 15: Q and R is balanced to eachother and both have a significant impact.

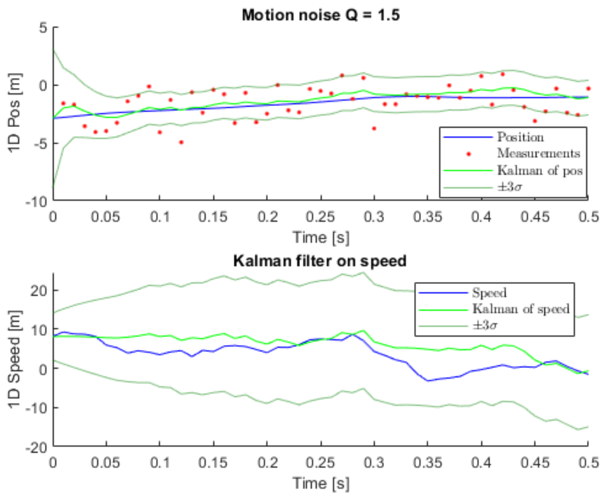

Figure 16: The true process noise $Q = 1.5$