# Graph Search – A* algorithm

This document will guide you through the practical work related to path planning algorithms for searching a graph efficiently. In particular, this practical exercise consists on programming the A* algorithm to find the optimal path in a *visibility graph*. Additionally, you can try to apply the same algorithm to a discrete environment. The first part is mandatory and represents 80% of the mark. The second part is optional but allows you to reach the 100% of the mark. All the code has to be programmed in Python.

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## Visibility Graph

This programming exercise is the continuation of the previous laboratory in which an environment was defined using a CSV file where:

- The first column is the poligon id.
- The second column is the x coordinate of a vertex.
- The third column is the y coordinate of a vertex.
- Each row represents a vertex.
- The first row will ALLWAYS represent the START vertex.
- The last row will ALLWAYS represent the GOAL vertex.
- Vertices from the same object will be inserted sequentially. The object can be plotted by joining with a straight segment each vertex with the next one, and closing at the end the object from the last vertex till the first one.

The goal for the previous lab was to obtain the visibility graph of an environment as a list of edges. Here, we provide you the visibility graph for each environment in a second CSV file where:

- Each row represents a different edge of the visibility graph. The same edge will not be repeated by changing the order of the vertices. There is no rule for the order of the edges in the list.
- For each row, the first element indicates the index of one of the vertices (defined in the previous CSV file) and the second column indicates the index of the other vertex for this edge.

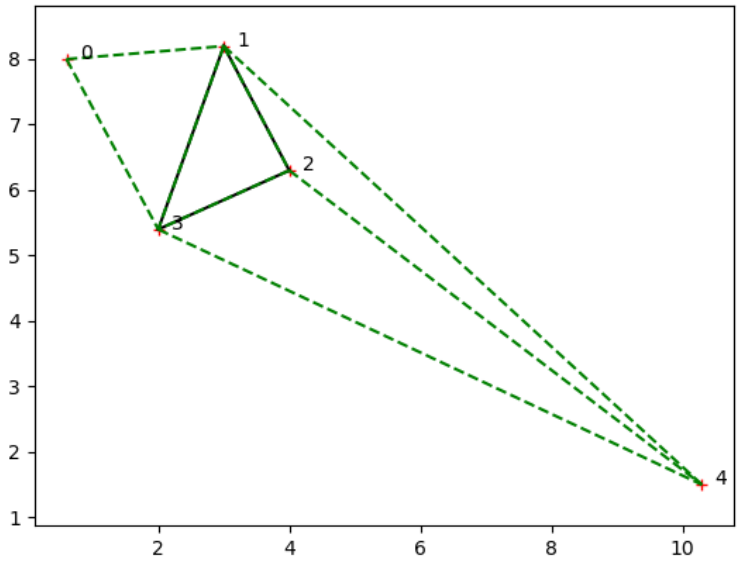
Here you can see an example of a visibility graph represented by the two CSV files and plotted in the following figure:

**Environment**

| object | x | y |
|---|---|---|
| 0 | 0.6 | 8 |
| 1 | 3 | 8.2 |
| 1 | 4 | 6.3 |
| 1 | 2 | 5.4 |
| 2 | 10.3 | 1.5 |

**Visibility Graph**

| vertex 1 | vertex 2 |
|---|---|
| 0 | 1 |
| 0 | 3 |
| 1 | 3 |
| 1 | 2 |
| 1 | 4 |
| 2 | 3 |
| 2 | 4 |
| 3 | 4 |




# The A* algorithm

Program the A* algorithm to solve the visibility graph, i.e., going from the start vertex to the goal vertex following the minimum path. You can use the following pseudocode (source).

```
function reconstruct_path(cameFrom, current)
    total_path := {current}
    while current in cameFrom.Keys:
        current := cameFrom[current]
        total_path.prepend(current)
    return total_path

% A* finds a path from start to goal.
% h is the heuristic function. h(n) estimates the cost to reach goal from n.
function A_Star(start, goal, h)
    % The set of discovered nodes that may need to be (re-)expanded.
```

```
    % Initially, only the start node is known.
    % This is usually implemented as a min-heap or priority queue
    % rather than a hash-set.
    openSet := {start}

    % For node n, cameFrom[n] is the node immediately preceding it on the
    % cheapest path from start to n currently known.
    cameFrom := an empty map

    % For node n, gScore[n] is the cost of the cheapest path from start
    % to n currently known.
    gScore := map with default value of Infinity
    gScore[start] := 0

    % For node n, fScore[n] := gScore[n] + h(n). fScore[n] represents our
    % current best guess as to how short a path from start to finish can
    % be if it goes through n.
    fScore := map with default value of Infinity
    fScore[start] := gScore[start] + h(start)

    while openSet is not empty
        % This operation can occur in O(1) time if openSet is a min-heap
        % or a priority queue
        current := the node in openSet having the lowest fScore[] value
        if current = goal
            return reconstruct_path(cameFrom, current)

        openSet.Remove(current)
        for each neighbor of current
            % d(current,neighbour) is the weight of the edge from current to
            % neighbour tentative_gScore is the distance from start to the
            % neighbour through current
            tentative_gScore := gScore[current] + d(current, neighbour)
            if tentative_gScore < gScore[neighbour]
                % This path to neighbour is better than any previous one.
                cameFrom[neighbor] := current
                gScore[neighbour] := tentative_gScore
                fScore[neighbour] := gScore[neighbour] + h(neighbour)
                if neighbor not in openSet
                    openSet.add(neighbour)

    % Open set is empty but goal was never reached
    return failure
```

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## Exercise

Program the `A*` algorithm using the Euclidean distance as a heuristic function. Use the following interface:

- Input:
  - path to csv environment file
  - path to csv visibility graph file for the environment
- Output:

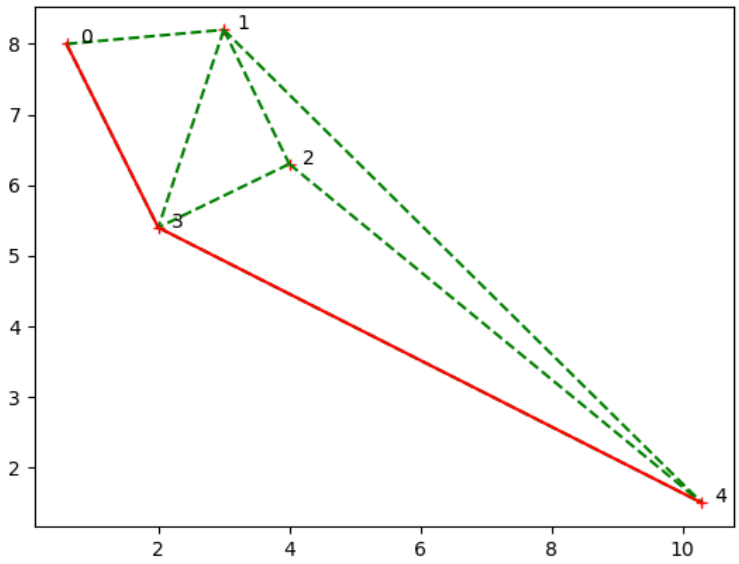
- path: a list of vertices representing the optimal path
- cost: the length of the optimal path

Given the path of the 2 CSV files it returns the minimum `path` (list of vertex) and the total `cost` (i.e., Euclidean path length).

The `path` variable is an array that contains the ordered list of vertices that are part of the minimum path. Since the first vertex of the *environment* file is the *start* vertex and the last vertex is the *goal* vertex, the `path` will always be `path = [0, ..., NumberRowVertices - 1]`. The `cost` variable is a scalar value that indicates the length of the optimal path.

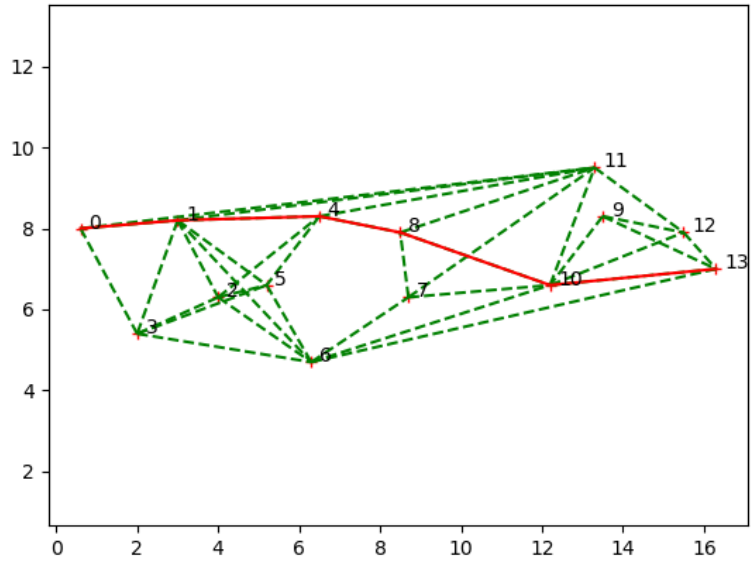
When the `A*` algorithm is called with the previous input files (i.e., `env_0.csv` and `visibility_graph_0.csv`), the next results should be obtained:

```
Path: [0, 3, 4]
Distance: 12.12356982653498
```




The output for `env_2.csv` and `visibility_graph_2.csv` should be:

```
Path: [0, 1, 4, 8, 10, 13]
Distance: 15.990555296232605
```

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# Optional part (20% of the mark)

This part is optional but without it the maximum mark that can be achieved is 8/10.
This part consist on using the same A* algorithm to find the optimal path on a grid map like environment.

## Grid map environment

We are going to use grayscale images to define our grid map environments. In Python, the `PIL` library can be used to load a map image and the `matplotlib` library can be used to show it. Transform the image to a 2D `numpy` array to simplify its manipulation. Note that when a grayscale image is used as an environment, 0 is black and it use to represent an obstacle while 1 (or 255) is white and it is normally used to represent the free space. Therefore, we need to binarize the loaded image to ensure that there are no intermediate values as well as to invert the values to have 0 as free space and 1 as obstacles, that is the *standard* definition of *free* and *occupied* space in motion planning.

```
import numpy as np
from matplotlib import pyplot as plt
from PIL import Image

# Load grid map
image = Image.open('map0.png').convert('L')
grid_map = np.array(image.getdata()).reshape(image.size[0], image.size[1])/255
# binarize the image
grid_map[grid_map > 0.5] = 1
grid_map[grid_map <= 0.5] = 0
# Invert colors to make 0 -> free and 1 -> occupied
grid_map = (grid_map * -1) + 1
# Show grid map
plt.matshow(grid_map)
plt.colorbar()
plt.show()
```

The following maps with the proposed *start* and *goal* locations are provided:

| grid map name | start | goal |
|---|---|---|
| map0.png | (10, 10) | (90, 70) |
| map1.png | (60, 60) | (90, 60) |
| map2.png | (8, 31) | (139, 38) |
| map3.png | (50, 90) | (375, 375) |

## Discrete A* algorithm

The same pseudocode can be used to implement the discrete version of the A*. The Euclidean distance can be used here too as heuristic function. For the *neighbours* of the current position, use any surronding cell (i.e., connect-4 or connect-8) that is not occupied. E.g., Cell $C$ at position [1, 1] has 8 neighbours if connect-8 is used. However, only [0, 1], [0, 2], [1, 2] and [2, 2] are valid neigbours, since the other cells are occupied:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & C & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

# Exercise

Program the `A*` algorithm using the following interface:

```
$ ./a_star_discrete_YOUR_NAME.py path_to_grid_map_image start_x start_y goal_x
goal_y
```

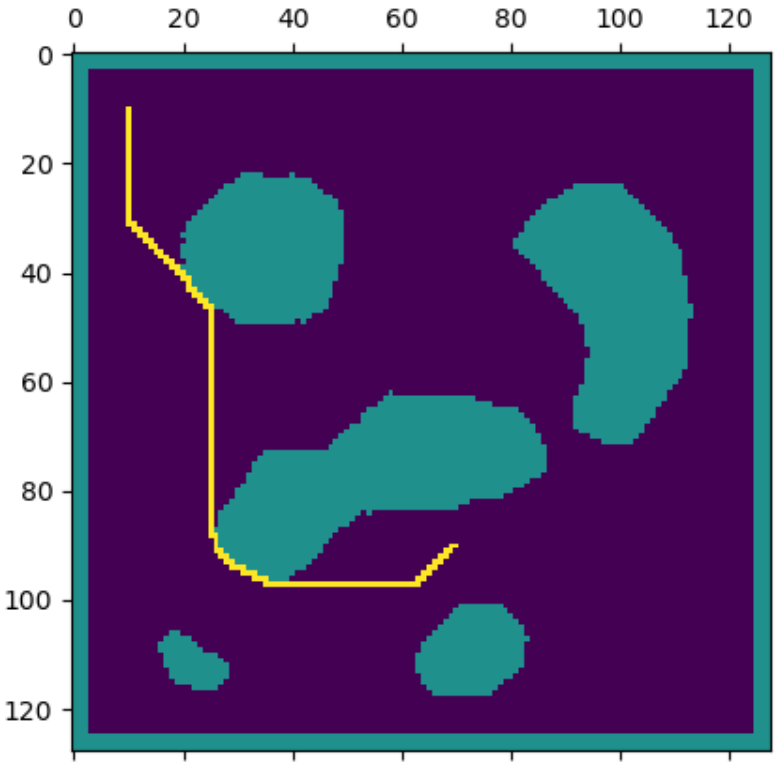
Given the path to the grid map image as well as the `start` and `goal` location in pixels, the script must return the minimum `path` (i.e., list of `(X, Y)` positions in pixels) and the total `cost` (i.e., Euclidean path length). An image illustrating the resulting path on the original map must also be shown.

The `path` variable is an array that contains the ordered list of pixel positions that belong to the minimum path. The firts pixel must always be the *start* position and the last one the *goal* (e.g., `path = [(start_x, start_y), ..., (goal_x, goal_y)]`). The `cost` is a scalar value that indicates the length of the optimal path.

When the `A*` algorithm is called with the following input variables: `map0.png`, `start=(10, 10)` and `goal=(90, 70)`, the following result should be obtained:

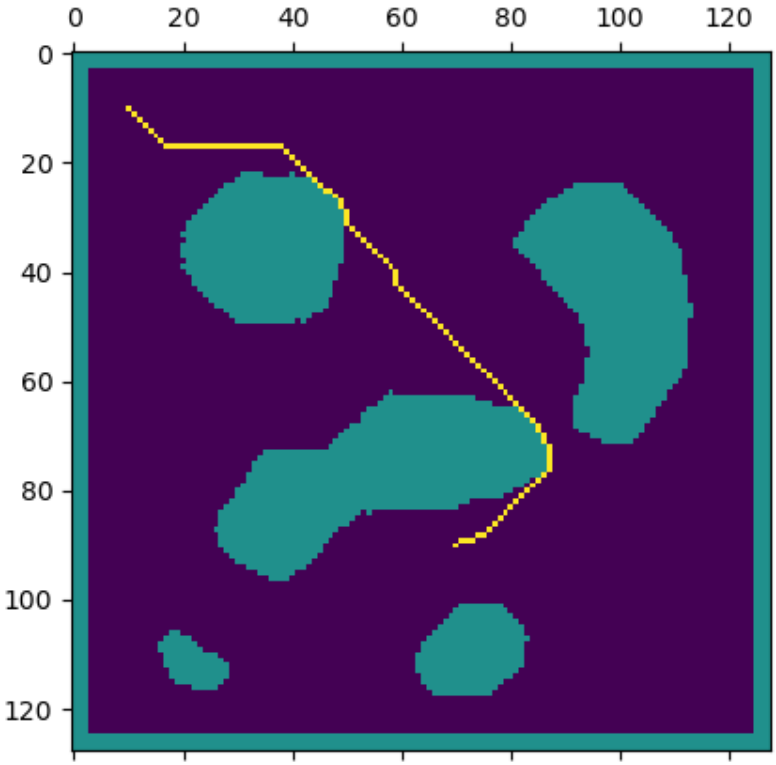
**Using connectivity 4**:

```
Path cost: 154.0
Path: [(10, 10), (11, 10), (12, 10), (13, 10), (14, 10), (15, 10), (16, 10), (17,
10), (18, 10), (19, 10), (20, 10), (21, 10), (22, 10), (23, 10), (24, 10), (25,
10), (26, 10), (27, 10), (28, 10), (29, 10), (30, 10), (31, 10), (31, 11), (32,
11), (32, 12), (33, 12), (33, 13), (34, 13), (34, 14), (35, 14), (35, 15), (36,
15), (36, 16), (37, 16), (37, 17), (38, 17), (38, 18), (39, 18), (39, 19), (40,
19), (40, 20), (41, 20), (41, 21), (42, 21), (43, 21), (43, 22), (44, 22), (44,
23), (45, 23), (45, 24), (46, 24), (46, 25), (47, 25), (48, 25), (49, 25), (50,
25), (51, 25), (52, 25), (53, 25), (54, 25), (55, 25), (56, 25), (57, 25), (58,
25), (59, 25), (60, 25), (61, 25), (62, 25), (63, 25), (64, 25), (65, 25), (66,
25), (67, 25), (68, 25), (69, 25), (70, 25), (71, 25), (72, 25), (73, 25), (74,
25), (75, 25), (76, 25), (77, 25), (78, 25), (79, 25), (80, 25), (81, 25), (82,
25), (83, 25), (84, 25), (85, 25), (86, 25), (87, 25), (88, 25), (88, 26), (89,
26), (90, 26), (91, 26), (91, 27), (92, 27), (92, 28), (93, 28), (93, 29), (94,
29), (94, 30), (94, 31), (95, 31), (95, 32), (95, 33), (96, 33), (96, 34), (96,
35), (97, 35), (97, 36), (97, 37), (97, 38), (97, 39), (97, 40), (97, 41), (97,
42), (97, 43), (97, 44), (97, 45), (97, 46), (97, 47), (97, 48), (97, 49), (97,
50), (97, 51), (97, 52), (97, 53), (97, 54), (97, 55), (97, 56), (97, 57), (97,
58), (97, 59), (97, 60), (97, 61), (97, 62), (97, 63), (96, 63), (96, 64), (95,
64), (95, 65), (94, 65), (94, 66), (93, 66), (93, 67), (92, 67), (92, 68), (91,
68), (91, 69), (90, 69), (90, 70)]
```




**Using connectivity 8**:

```
Path cost: 133.58073580374347
Path: [(10, 10), (11, 11), (12, 12), (13, 13), (14, 14), (15, 15), (16, 16), (17,
17), (17, 18), (17, 19), (17, 20), (17, 21), (17, 22), (17, 23), (17, 24), (17,
25), (17, 26), (17, 27), (17, 28), (17, 29), (17, 30), (17, 31), (17, 32), (17,
33), (17, 34), (17, 35), (17, 36), (17, 37), (17, 38), (18, 39), (19, 40), (20,
41), (21, 42), (22, 43), (23, 44), (24, 45), (25, 46), (25, 47), (26, 48), (27,
49), (28, 49), (29, 50), (30, 50), (31, 50), (32, 51), (33, 52), (34, 53), (35,
54), (36, 55), (37, 56), (38, 57), (39, 58), (40, 59), (41, 59), (42, 59), (43,
60), (44, 61), (45, 62), (46, 63), (47, 64), (48, 65), (49, 66), (50, 67), (51,
68), (52, 69), (53, 70), (54, 71), (55, 72), (56, 73), (57, 74), (58, 75), (59,
76), (60, 77), (61, 78), (62, 79), (63, 80), (64, 81), (65, 82), (66, 83), (67,
84), (68, 85), (69, 85), (70, 86), (71, 86), (72, 87), (73, 87), (74, 87), (75,
87), (76, 87), (77, 86), (78, 85), (79, 84), (80, 83), (81, 82), (82, 81), (83,
80), (84, 79), (85, 78), (86, 77), (87, 76), (88, 75), (88, 74), (89, 73), (89,
72), (89, 71), (90, 70)]
```




## Submission

Submit a report in PDF and one or two Python scripts (`a_star_YOUR_NAME.py` and optionally `a_star_discrete_YOUR_NAME.py`). In the report, explain in detail the work done. For each proposed environment show the shortest path graphically. Explain also the problems or dificulties faced while implementing the scripts. The algorithm may be evaluated with other environments than the ones provided here. **BE SURE** that your algorithm is able to correctly load any valid CSV file and any valid grid map environment!

## WARNING:

We encourage you to help or ask your classmates for help, but the direct copy of a lab will result in a failure (with a grade of 0) for all the students involved.

It is possible to use functions or parts of code found on the internet only if they are limited to a few lines and correctly cited (a comment with a link to where the code was taken from must be included).

**Deliberately copying entire or almost entire works will not only result in the failure of the laboratory but may lead to a failure of the entire course or even to disciplinary action such as temporal or permanent expulsion of the university.** Rules of the evaluation and grading process for UdG students.

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Narcís Palomeras

Last review May 2022.