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DETERMINANTE DE MATRIZ 4X4

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1) Deduza A o determinante 4x4 usando a fórmula: $\det(A) = \sum_{\sigma \in S_n} (\mathrm{sgn}(\sigma)) \prod_{i=1}^{n} a_{i,\sigma(i)}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \tag{1}$$

Resolução:

$$\det(A) = \sum_{\sigma \in S_4} (\mathrm{sgn}(\sigma)) \prod_{i=1}^{4} a_{i,\sigma(i)} =$$

$$\begin{aligned}
&+ (\mathrm{sgn}(1,2,3,4)) \cdot a_{1,1} \cdot a_{2,2} \cdot a_{3,3} \cdot a_{4,4} \quad +(\mathrm{sgn}(1,2,4,3)) \cdot a_{1,1} \cdot a_{2,2} \cdot a_{3,4} \cdot a_{4,3} \\
&+ (\mathrm{sgn}(1,3,2,4)) \cdot a_{1,1} \cdot a_{2,3} \cdot a_{3,2} \cdot a_{4,4} \quad +(\mathrm{sgn}(1,3,4,2)) \cdot a_{1,1} \cdot a_{2,3} \cdot a_{3,4} \cdot a_{4,2} \\
&+ (\mathrm{sgn}(1,4,2,3)) \cdot a_{1,1} \cdot a_{2,4} \cdot a_{3,2} \cdot a_{4,3} \quad +(\mathrm{sgn}(1,4,3,2)) \cdot a_{1,1} \cdot a_{2,4} \cdot a_{3,3} \cdot a_{4,2} \\
&+ (\mathrm{sgn}(2,1,3,4)) \cdot a_{1,2} \cdot a_{2,1} \cdot a_{3,3} \cdot a_{4,4} \quad +(\mathrm{sgn}(2,1,4,3)) \cdot a_{1,2} \cdot a_{2,1} \cdot a_{3,4} \cdot a_{4,3} \\
&+ (\mathrm{sgn}(2,3,1,4)) \cdot a_{1,2} \cdot a_{2,3} \cdot a_{3,1} \cdot a_{4,4} \quad +(\mathrm{sgn}(2,3,4,1)) \cdot a_{1,2} \cdot a_{2,3} \cdot a_{3,4} \cdot a_{4,1} \\
&+ (\mathrm{sgn}(2,4,1,3)) \cdot a_{1,2} \cdot a_{2,4} \cdot a_{3,1} \cdot a_{4,3} \quad +(\mathrm{sgn}(2,4,3,1)) \cdot a_{1,2} \cdot a_{2,4} \cdot a_{3,3} \cdot a_{4,1} \\
&+ (\mathrm{sgn}(3,1,2,4)) \cdot a_{1,3} \cdot a_{2,1} \cdot a_{3,2} \cdot a_{4,4} \quad +(\mathrm{sgn}(3,1,4,2)) \cdot a_{1,3} \cdot a_{2,1} \cdot a_{3,4} \cdot a_{4,2} \\
&+ (\mathrm{sgn}(3,2,1,4)) \cdot a_{1,3} \cdot a_{2,2} \cdot a_{3,1} \cdot a_{4,4} \quad +(\mathrm{sgn}(3,2,4,1)) \cdot a_{1,3} \cdot a_{2,2} \cdot a_{3,4} \cdot a_{4,1} \\
&+ (\mathrm{sgn}(3,2,1,4)) \cdot a_{1,3} \cdot a_{2,2} \cdot a_{3,1} \cdot a_{4,4} \quad +(\mathrm{sgn}(3,2,4,1)) \cdot a_{1,3} \cdot a_{2,2} \cdot a_{3,4} \cdot a_{4,1} \\
&+ (\mathrm{sgn}(3,4,1,2)) \cdot a_{1,3} \cdot a_{2,4} \cdot a_{3,1} \cdot a_{4,2} \quad +(\mathrm{sgn}(3,4,2,1)) \cdot a_{1,3} \cdot a_{2,4} \cdot a_{3,2} \cdot a_{4,1} \\
&+ (\mathrm{sgn}(4,1,2,3)) \cdot a_{1,4} \cdot a_{2,1} \cdot a_{3,2} \cdot a_{4,3} \quad +(\mathrm{sgn}(4,1,3,2)) \cdot a_{1,4} \cdot a_{2,1} \cdot a_{3,3} \cdot a_{4,2} \\
&+ (\mathrm{sgn}(4,2,1,3)) \cdot a_{1,4} \cdot a_{2,2} \cdot a_{3,1} \cdot a_{4,3} \quad +(\mathrm{sgn}(4,2,3,1)) \cdot a_{1,4} \cdot a_{2,2} \cdot a_{3,3} \cdot a_{4,1} \\
&+ (\mathrm{sgn}(4,3,1,2)) \cdot a_{1,4} \cdot a_{2,3} \cdot a_{3,1} \cdot a_{4,2} \quad +(\mathrm{sgn}(4,3,2,1)) \cdot a_{1,4} \cdot a_{2,3} \cdot a_{3,2} \cdot a_{4,1}
\end{aligned}$$

$= \mathrm{a}_{1,1}a_{2,2}a_{3,3}a_{4,4} + a_{1,2}a_{2,1}a_{3,4}a_{4,3} + a_{1,2}a_{2,3}a_{3,1}a_{4,4} + a_{1,2}a_{2,4}a_{3,3}a_{4,1} + a_{1,3}a_{2,1}a_{3,2}a_{4,4} + a_{1,1}a_{2,3}a_{3,4}a_{4,2} + a_{1,1}a_{2,4}a_{3,2}a_{4,3} + a_{1,4}a_{2,1}a_{3,3}a_{4,2} + a_{1,4}a_{2,2}a_{3,1}a_{4,3} + a_{1,3}a_{2,2}a_{3,4}a_{4,1} + a_{1,3}a_{2,4}a_{3,1}a_{4,2} + a_{1,4}a_{2,3}a_{3,2}a_{4,1} - a_{1,1}a_{2,2}a_{3,4}a_{4,3} - a_{1,1}a_{2,3}a_{3,2}a_{4,4} - a_{1,1}a_{2,4}a_{3,3}a_{4,2} - a_{1,2}a_{2,1}a_{3,3}a_{4,4} - a_{1,2}a_{2,3}a_{3,4}a_{4,1} - a_{1,2}a_{2,4}a_{3,1}a_{4,3} - a_{1,3}a_{2,1}a_{3,4}a_{4,2} - a_{1,3}a_{2,2}a_{3,1}a_{4,4} - a_{1,3}a_{2,4}a_{3,2}a_{4,1} - a_{1,4}a_{2,1}a_{3,2}a_{4,3} - a_{1,4}a_{2,2}a_{3,3}a_{4,1} - a_{1,4}a_{2,3}a_{3,1}a_{4,2}$

2) Calcule o determinante, usando o que foi deduzido, de duas matrizes definidas pelo autor. Considere uma matriz A cujo det(A) = 0 e outra matriz B cujo det(B) $\neq 0$.

Resolução:

Matriz de det(A) = 0

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{2}$$

$$\begin{aligned} \det(A) &= (1)(1)(1)(1) + (1)(1)(1)(1) + (1)(1)(1)(1) + (1)(1)(1)(1) + (1)(1)(1)(1) \\ &+ (1)(1)(1)(1) + (1)(1)(1)(1) + (1)(1)(1)(1) + (1)(1)(1)(1) + (1)(1)(1)(1) \\ &+ (1)(1)(1)(1) + (1)(1)(1)(1) - (1)(1)(1)(1) - (1)(1)(1)(1) - (1)(1)(1)(1) \\ &- (1)(1)(1)(1) - (1)(1)(1)(1) - (1)(1)(1)(1) - (1)(1)(1)(1) - (1)(1)(1)(1) \\ &- (1)(1)(1)(1) - (1)(1)(1)(1) - (1)(1)(1)(1) - (1)(1)(1)(1) \\ &= 0 \end{aligned}$$

Matriz de det(B)$\neq 0$

$$B = \begin{bmatrix} 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$\begin{aligned} \det(B) &= (3)(1)(2)(1) + (3)(0)(1)(0) + (3)(0)(1)(0) + (1)(0)(2)(0) + (1)(0)(0)(1) \\ &+ (1)(0)(2)(0) + (0)(0)(1)(1) + (0)(1)(1)(0) + (0)(0)(0)(0) + (0)(0)(2)(0) \\ &+ (0)(1)(0)(0) + (0)(0)(1)(0) - (3)(1)(1)(0) - (3)(0)(1)(1) - (3)(0)(2)(0) \\ &- (1)(0)(2)(1) - (1)(0)(1)(0) - (1)(0)(0)(0) - (0)(0)(1)(0) - (0)(1)(0)(1) \\ &- (0)(0)(1)(0) - (0)(0)(1)(0) - (0)(1)(2)(0) - (0)(0)(0)(0) \\ &= 6. \end{aligned}$$

3. Método em Python

```python
from functools import reduce
from itertools import permutations
from typing import List

SAMPLE_MATRIX_A: List[List[int]] = [
    [1, 1, 1, 1],
    [1, 1, 1, 1],
    [1, 1, 1, 1],
    [1, 1, 1, 1]
]
SAMPLE_MATRIX_B: List[List[int]] = [
    [3, 1, 0, 0],
    [0, 1, 0, 0],
    [0, 1, 2, 1],
    [0, 0, 0, 1]
]


1 usage   Elizabete
def to_permutation_list(matrix_lenght) -> list:
    return list(permutations(range(matrix_lenght)))


1 usage   Elizabete
def get_sign(matrix_list) -> int:
    count = 0
    for i in range(len(matrix_list)):
        for j in range(i + 1, len(matrix_list)):
            if matrix_list[i] > matrix_list[j]:
                count += 1
    return 1 if (count % 2) == 0 else -1
```

Figura 1: Parte 1

```python
4 usages  Elizabete
def calculate_determinant(matrix) -> int:
    ''' Get the determinant of the matrix.

    :param list[list[int]] matrix: Matrix to operate on.
    :return: An integer determinant.

    Note: This only works for square matrices.
    '''

    if len(matrix) != len(matrix[0]):
        raise Exception('Must be square')

    determinant = 0
    matrix_lenght = len(matrix)
    permutation_list = to_permutation_list(matrix_lenght)

    for value in permutation_list:
        signal = get_sign(value)
        product_diagonal_elements = reduce(
            lambda a1, a2: a1 * a2,
            [matrix[i][value[i]] for i in range(matrix_lenght)]
        )
        determinant += signal * product_diagonal_elements

    return determinant


if __name__ == '__main__':
    print(
        f'Determinante da matriz A {SAMPLE_MATRIX_A} = \
        {calculate_determinant(SAMPLE_MATRIX_A)}'
    )
    print(f'Determinante da matriz B {SAMPLE_MATRIX_B} = \
        {calculate_determinant(SAMPLE_MATRIX_B)}'
```

Figura 2: Parte 2

```
Python Console
>>> runfile('/home/bete/python_elizabete/fatec/determinante/determinant.py', wdir='/home/bete/python_elizabete/fatec/determinante')
Determinante da matriz A [[1, 1, 1, 1], [1, 1, 1, 1], [1, 1, 1, 1], [1, 1, 1, 1]] =         0
Determinante da matriz B [[3, 1, 0, 0], [0, 1, 0, 0], [0, 1, 2, 1], [0, 0, 0, 1]] =         6
```

Figura 3: Console