Multitaper.jl: A Julia package for frequency domain analysis of time series

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Software

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Summary

Spectral analysis is widely used to infer physical mechanisms for underlying process dynamics from a realization of a stationary time series. The multitaper method is a nonparametric technique for estimating the power spectrum of a discrete time series that simultaneously controls bias and variance of the estimator by premultiplying the data by a set of orthogonal sequences—discrete prolate spheroidal sequences that are optimally concentrated in both time and frequency. These have the effect of stabilizing the bias and variance of the spectrum estimator (Thomson, 1982). While multitaper codes have been introduced in multiple languages, including Julia, the `Multitaper.jl` package that we present offers functionality beyond univariate and bivariate time series analysis and provides routines for a number of (selected) research-level topics not found in other packages.

Statement of need

`Multitaper.jl` is a Julia package for spectrum analysis of time series, multivariate time series, and spatial or space-time processes. The high-level character of Julia allows for widely readable and extendible codes, while the low-level functionality provides speed and efficiency. The `Multitaper.jl` package provides a user-friendly implementation of many of the basic concepts such as spectrum analysis, F-testing for harmonic analysis (Thomson, 1982), coherence and phase, jackknifed variance estimates (Thomson & Chave, 1991), and complex demodulation (Thomson, Lanzerotti, Vernon, III, Lessard, & Smith, 2007); more advanced techniques such as dual-frequency spectra, cepstrum, multitaper for time series containing gaps (Chave, 2019), T^2 tests for multiple line components (Thomson, 2011) implementations of higher-dimensional Slepian tapers on Cartesian domains (Simons & Wang, 2011) (Geoga, Haley, Siegel, & Anitescu, 2018); and others (Thomson & Haley, 2014) (Haley & Anitescu, 2017). In addition, we provide tutorial-style notebooks to allow accessibility to those new to these concepts or to Julia in general.

`Multitaper.jl` was designed to be useful to researchears in diverse fields, including geophysics (climate, seismology, limnology, and stratigraphy), cognitive radio, space science (solar physics), speech processing, astronomy, and biomedicine. It has been used in graduate courses to provide fast spectrum estimates of unequally sampled time series. Early versions of this code have also been used to compute figures for research publications.

Other software

Multitaper spectrum analysis is implemented in Julia in the `DSP.jl` package, but it is limited to estimation of the spectrum. In the R programming language the `R multitaper` package gives an R-wrapped Fortran 77 implementation, which provides fast spectrum estimates, F-tests, jackknifing, coherences, and complex demdodulation (Rahim & Burr, 2013). A C-subroutine for multitaper methods was introduced in (Lees & Park, 1995), and the derivative Python implementation `pymutt` (Smith, 2011) is a wrapped version of the former. The Fortran 90 library of multitaper methods (Prieto, Parker, & Vernon III, 2009) provides for spectrum analysis, F-testing, spectral reshaping, coherences, and quadratic inverse spectrum estimation. While wrapped versions of low-level codes run rapidly, they can be more difficult to extend by the research community. The Matlab Signal Processing Toolbox implements discrete prolate spheroidal sequences and multitaper spectrum estimation, while standalone contributions on the Matlab file exchange describe the extension to multitaper methods with gaps (Chave, 2019), higher-dimensional spectrum estimation on Cartesian domains (Simons & Wang, 2011) and the sphere (Simons, Dahlen, & Wieczorek, 2006) and the freely available `jlab` codes (Lilly, 2019).

To contribute

We welcome contributions of any kind via bitbucket issues or by personal communication.

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