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papaya2: 2D Irreducible Minkowski Tensor computation

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Summary

A common challenge in scientific and technical domains is the quantitative description of geometries and shapes, e.g. in the analysis of microscope imagery or astronomical observation data. Frequently, it is desirable to go beyond scalar shape metrics such as porosity and surface to volume ratios because the samples are anisotropic or because direction-dependent quantities such as conductances or elasticity are of interest. Popular analysis software such as ImageJ and SExtractor provide only limited tooling for higher-order anisotropy characterization; usually only the tensor of inertia (rank 2) is available.

Minkowski Tensors are a systematic family of versatile and robust higher-order shape descriptors, originating in integral geometry, see (Schröder-Turk et al., 2011) for an introduction and detailed references. They allow for shape characterization to arbitrary order and promise a path to systematic structure-function relationships for direction-dependent properties. Minkowski Tensors have previously been applied to data as diverse as ice grain microstructure (G. E. Schröder-Turk et al., 2010), granular packing geometries (Schaller et al., 2015; G. E. Schröder-Turk, Mickel, et al., 2010), astronomical data (Joby, Chingangbam, Ghosh, Ganesan, & Ravikumar, 2019; Kerscher et al., 2001; Klatt & Mecke, 2020), neuronal data (Beisbart, Barbosa, Wagner, & Costa, 2006), foams (Evans, Schröder-Turk, & Kraynik, 2017; Saadatfar et al., 2012) and random sets, tessellations and point patterns (Kapfer et al., 2010; Klatt et al., 2017). An accessible introduction to Minkowski Tensors can be found on www.morphometry.org.

Here, we present `papaya2`, a C++ library which facilitates computation of irreducible Minkowski Tensors for two-dimensional geometries and shapes, including planar objects bounded by polygonal contours, collections of points (point patterns) and greyscale pixel data.

This library is accompanied by example programs and bindings for Matlab and the Python language. We also present the Morphometer, an interactive online resource for analyzing two-dimensional structures.

`Papaya2` is a rewrite of `papaya` with a library interface, support for irreducible Minkowski Tensors and interpolated marching squares, and extensions to Matlab and Python provided. While the tensor of inertia is computed by many tools, we are not aware of other open-source software which provides higher-rank shape characterization in 2D.

C++ library papaya2

The C++ 11 library papaya2 contains the core algorithms to compute irreducible Minkowski tensors of two-dimensional geometries. It processes both polygonal and 2D image input data.

papaya2 is a header-only template library designed to operate on user data structures. We bundle several example programs which can be adapted to user requirements, or employed directly for simple analyses (see section *Command-line tools*).

The main components of the library are defined in the header file `papaya2/papaya2.hpp`. Analysis results are returned in a `MinkowskiAccumulator` object, which offers accessors to retrieve common morphometric data, including the following:

- `area()` The 2D volume (area) enclosed by the geometry
- `perimeter()` The perimeter (boundary length) of the geometry
- `msm(s)` The s-th Minkowski structure metric q_s, see Morphometry page and (Mickel, Kapfer, Schröder-Turk, & Mecke, 2013) for details
- `imt(s)` The s-th irreducible Minkowski tensor Ψ_s, see Morphometry page for details

The library provides convenient wrapper functions which encapsulate common analysis tasks. In general, these functions are C++ function templates which operate on user data structures. User-supplied data structures need to include some required methods and operators as documented in the headers. The most important entrypoints are

- `papaya2::imt_polygon`: compute the irreducible Minkowski tensors of closed convex polygons, specified as a sequence of vertices in counterclockwise order.
- `papaya2::imt_interpolated_marching_squares`: computes the irreducible Minkowski tensors of an excursion set of a single channel of a raster graphics image (bitmap). An extended version of the classic Marching Squares algorithm is used which computes interpolated contours from 2x2 neighborhoods, see (Mantz, Jacobs, & Mecke, 2008) for details. The input data is passed to papaya2 by reference via a suitable adapter class to avoid copies. There are several examples of adapter classes provided, as well as a copying container (`BasicPhoto`).
- `papaya2::minkowski_map_interpolated_marching_squares`: implements the Minkowski map algorithm (Schröder-Turk et al., 2010) for a space-resolved anisotropy analysis.

The supplementary header `<papaya2/voronoi.hpp>` implements the Minkowski Tensor analysis of point patterns via the Voronoi tessellation approach (Kapfer et al., 2010). The command-line tool `ppanalysis` exemplifies how to use this header file. For computing the Voronoi diagram, the CGAL library is required.

Application: Morphometer

We use the papaya2 library in our interactive analysis tool Morphometer. It provides rapid analysis of small amounts of data (up to 500 points, or 500x500 pixels). For routine analysis we recommend using the command-line tools or Python/Matlab bindings.

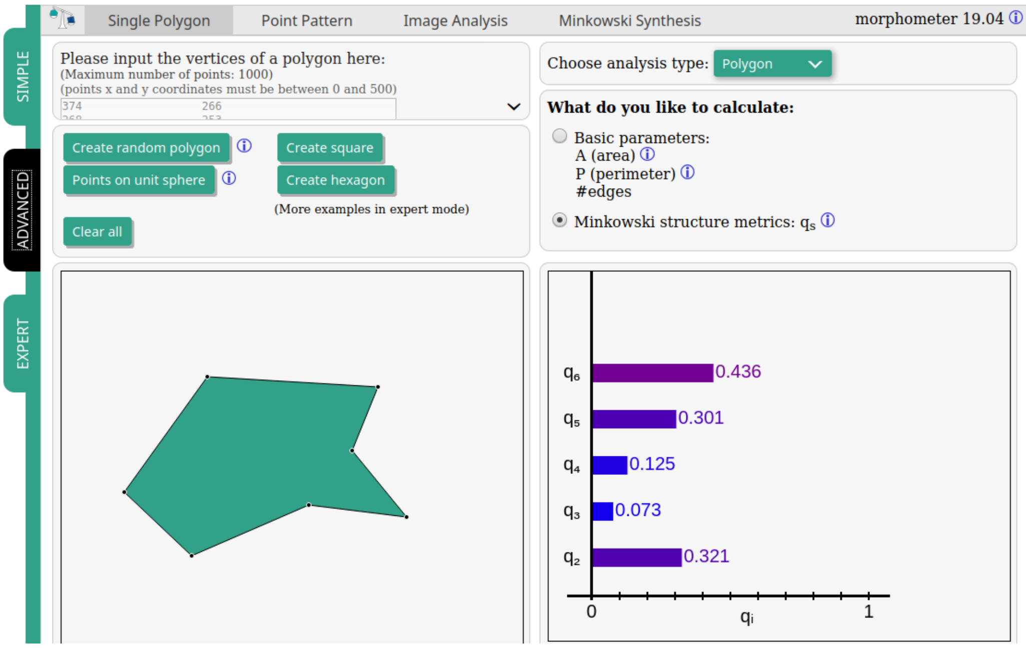

Figure 1: Minkowski Tensor analysis of a polygon in Morphometer.

Morphometer provides several advanced analysis modes:

- Single polygon mode: Figure 1 shows the user interface of Morphometer when analyzing a single polygon (left-hand side). On the right-hand side, the Minkowski structure metrics of the polygon are displayed.

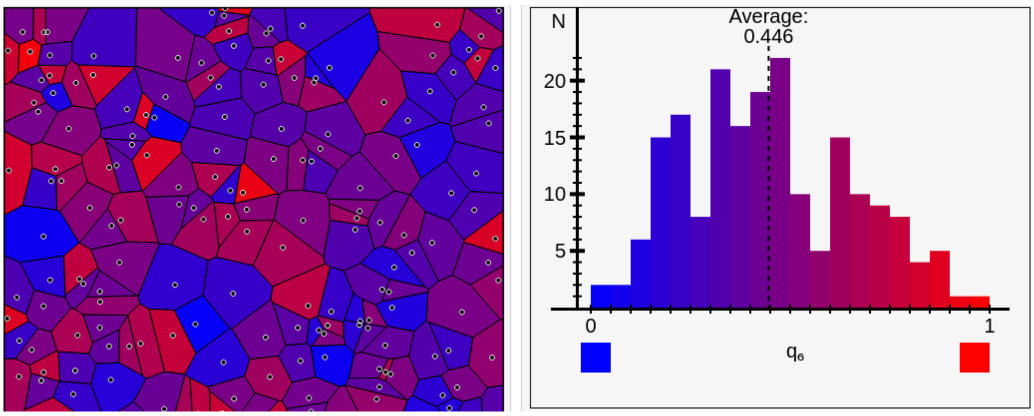

Figure 2: Minkowski Tensor analysis of a Poisson point pattern.

- Point pattern analysis mode can be used to analyze abstract point patterns and data of physical particle systems. For the analysis, a Voronoi tessellation of the points is constructed and Minkowski Tensors of the individual Voronoi cells are computed. Morphometer provides histograms and basic statistics of the morphometric data, see Figure 2.

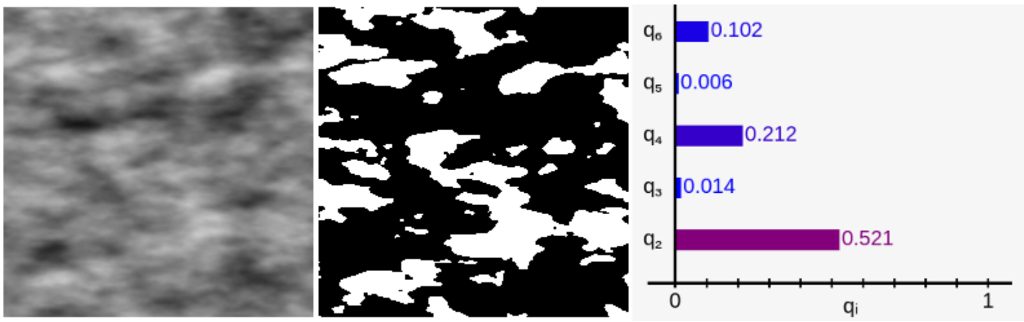

Figure 3: Minkowski Tensor analysis of a greyscale image: a Gaussian random field.

- Image analysis mode can be used to analyze single-channel (e.g. greyscale) pixel data. User-provided images are segmented using the Marching Squares algorithm, and a morphometric analysis using Minkowski Tensors is performed for either the complete image or a region of interest, see Figure 3.

Command-line tools and bindings

Bundled with the C++ library, we provide a number of example programs which use the library for data analysis. These are meant to be modified and adapted to user needs as required. For simple analyses, they can be used directly, see the documentation. The example programs can be built by typing `make`.

We also provide bindings of the library for Matlab and Python. For Matlab, example scripts are contained in `ppanalysis.m` and `imganalysis.m` in the `matlab` directory. The Python module can be built by typing `make pypaya2`. It currently has support for the point pattern analysis only.

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We use picopng for loading PNG images, emscripten for compiling to JavaScript, CGAL and d3-voronoi for Voronoi diagrams, and Catch2 for unit tests.

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