

# Week 11: Samples, Statistics & Sampling Distributions

Data Analysis for Psychology in R 1

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# Course Overview

<b>Exploratory Data Analysis</b>	Research design and data
	Describing categorical data
	Describing continuous data
	Describing relationships
	Functions
<b>Probability</b>	Probability theory
	Probability rules
	Random variables (discrete)
	Random variables (continuous)
	<b>Sampling</b>

<b>Foundations of inference</b>	Confidence intervals
	Hypothesis testing (p-values)
	Hypothesis testing (critical values)
	Hypothesis testing and confidence intervals
	Errors, power, effect size, assumptions
<b>Common hypothesis tests</b>	One sample t-test
	Independent samples t-test
	Paired samples t-test
	Chi-square tests
	Correlation

# This Week's Learning Objectives

1. Understand the difference between a population parameter and a sample statistic
2. Understand the concept and construction of sampling distributions
3. Understand the effect of sample size on the sampling distribution
4. Understand how to quantify the variability of a sample statistic and sampling distribution (standard error)

# Concepts to carry forward

- Data can be of different types
- We can assign probabilities to outcomes of random experiments
- We can define a probability distribution that describes the probability of all possible events
- Dependent on type (continuous vs. discrete), we can visualise and describe the distribution of data differently

# Why are these concepts relevant to psych stats?

- In psychology, we design a study, measure variables, and use these measurements to calculate a value that carries some meaning
  - E.g. the difference in reaction times between groups
- Given it has meaning based on the study design, we want to know something about the value:
  - Is it unusual or not?
  - This is the focus throughout the next semester
- **Today:**
  - We will talk about populations, samples, and sampling
  - Basic concepts of sampling may seem simple and intuitive
  - These concepts will be very useful when we start talking about *statistical inference*, or how we make decisions about data

# Populations vs Samples

- In statistics, we often refer to populations and samples
  - **Population:** The entire group of people about whom you'd like to make inferences
  - **Sample:** The subset of the population from whom you will collect data to make these inferences
- To get the most accurate measure of our variable, it would be ideal to collect data from the entire population; however, this is not feasible
- In almost all cases, researchers need to collect data from samples and use these results to make inferences about the population
  - The population value of the variable of interest is known as a **population parameter**
  - The sample value of the variable of interest is known as a **sample statistic**, or **point-estimate**

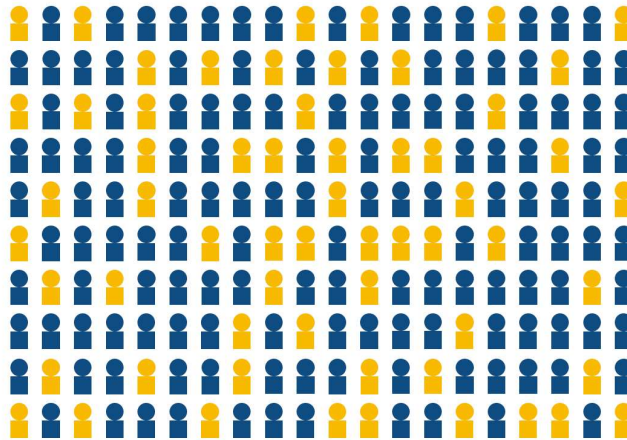
# Populations vs Samples - Notation

- It's important to know that although you may have seen these different types of notation used interchangeably in the past, they are actually slightly different when one is referring to a *population* versus a *sample*:

Population	Parameter	Sample
$\mu$	Mean	$\bar{x}$
$\sigma$	Standard Deviation	$s$
$N$	Size	$n$

# Populations vs Samples - Example

- Suppose I wanted to know the proportion of UG students at the University of Edinburgh who are permanent Scottish residents



**Test your Understanding:** What is the population in this example?

What is the variable of interest?

What is the parameter?

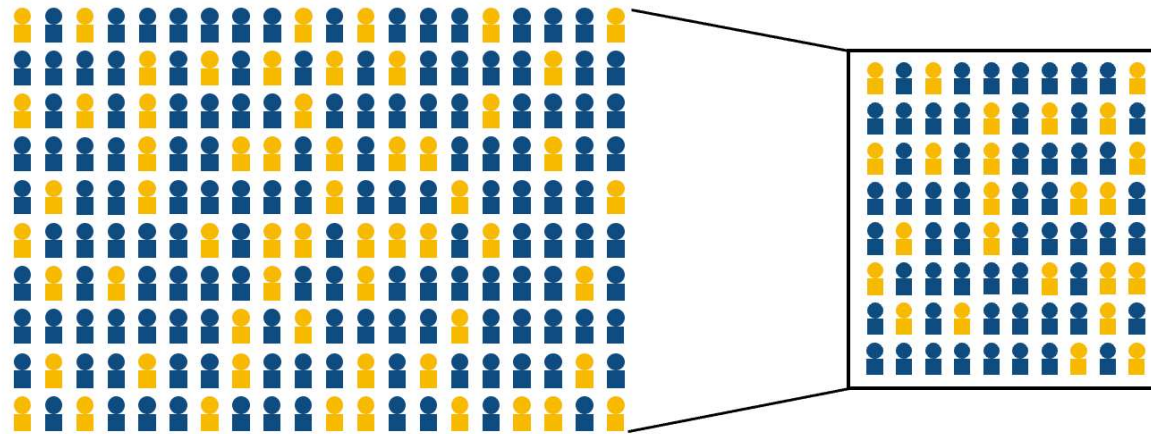


# Populations vs Samples - Example

- How can we collect this information?
  - We could send out an email requesting all students to provide their permanent residence...but it's not likely that all students will respond
  - We could ask instructors to collect this data from students in their classes, but not every student will attend each class, and not every instructor will comply
- Even with this relatively small, accessible population, it's unlikely we can collect information from every single member

# Populations vs Samples - Example

- Instead, we have to use the data from students who *do* respond to make inferences about the overall student population

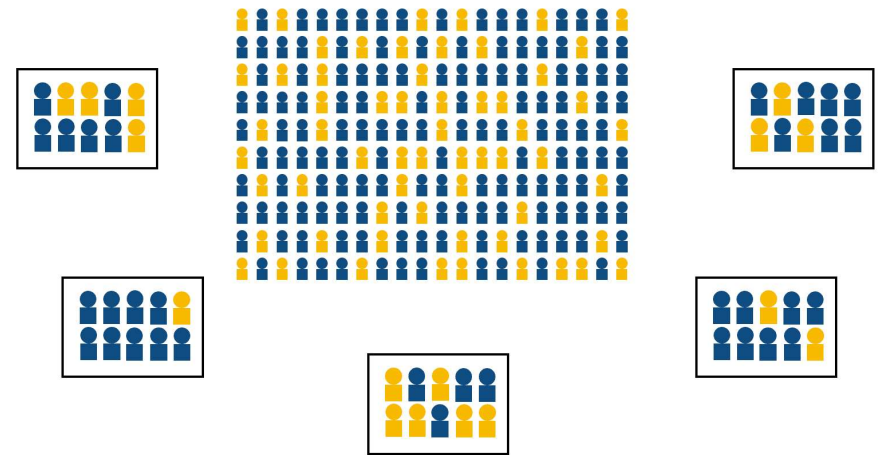


**Test your Understanding:** What is the sample?

What is the statistic, or the point-estimate?

# Parameters, point-estimates, and sampling distributions

- It is the population parameter (proportion of UoE students who are permanent Scottish residents) we are interested in. This is a *true* value of the world.
- We can draw a sample, and calculate this proportion in the sample
  - The point-estimate from the sample is our best guess at the population parameter
- If we draw multiple samples, we can produce a **sampling distribution**, which is a probability distribution of some statistic obtained from repeatedly sampling the population



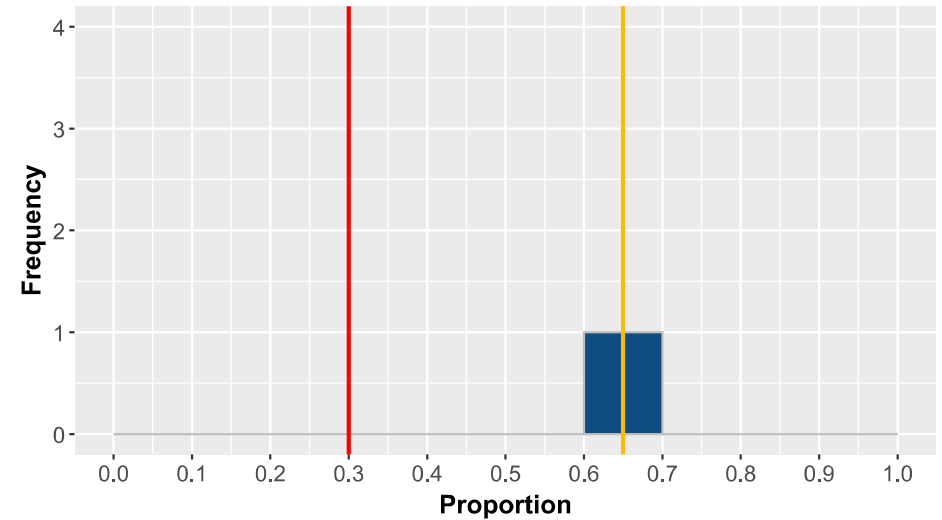
# 2021/22 actual proportion

- Let's use real data from 21/22 to demonstrate this concept
  - These data represent the entire student body from 21/22
- Using these data, we can:
  - 1) Simulate gathering multiple samples of UoE students
  - 2) Calculate the proportion of permanent Scottish residents in each sample
  - 3) Produce a frequency distribution of each sample's proportions

Scottish	n	Freq
No	20090	0.7
Yes	8665	0.3

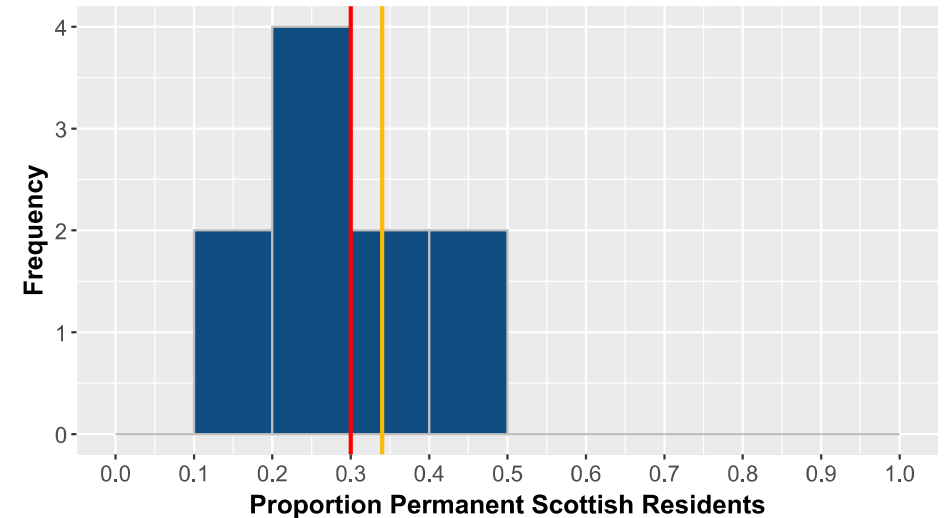
# Visualising sampling distributions

- Imagine we took a single sample of 10 students
- This demonstrates how a statistic from a single small sample may or may not capture the population parameter



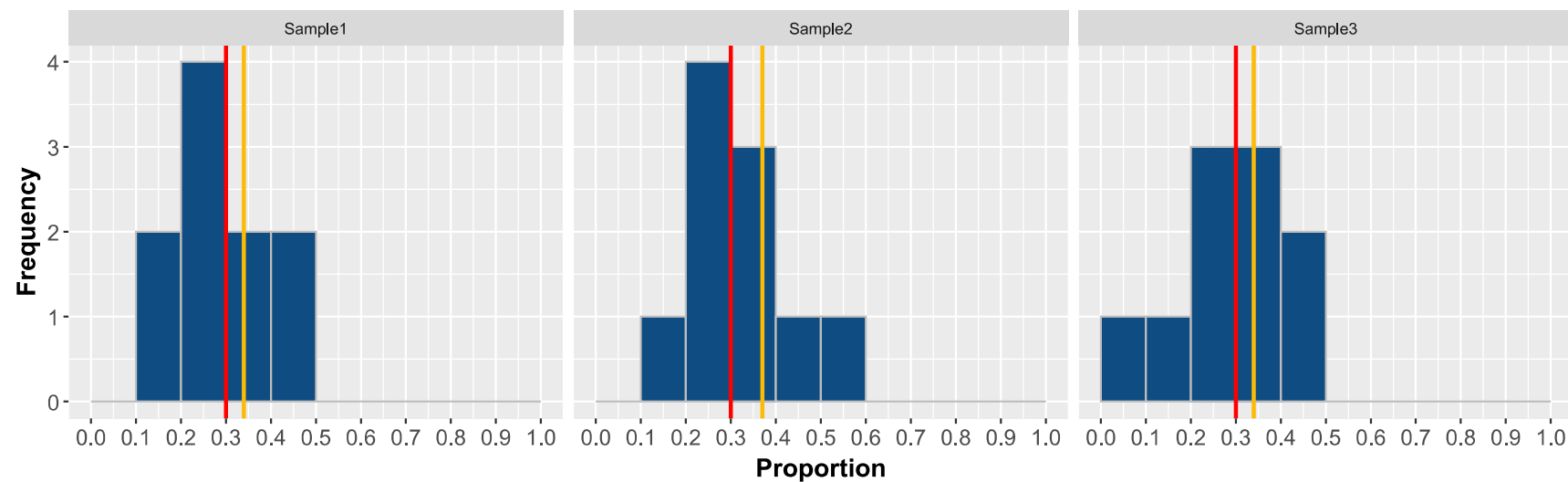
# Visualising sampling distributions

- Imagine that we instead took 10 samples of 10 students each
- What happens to the difference between the mean sampling statistic and the population parameter?



# Visualising sampling distributions

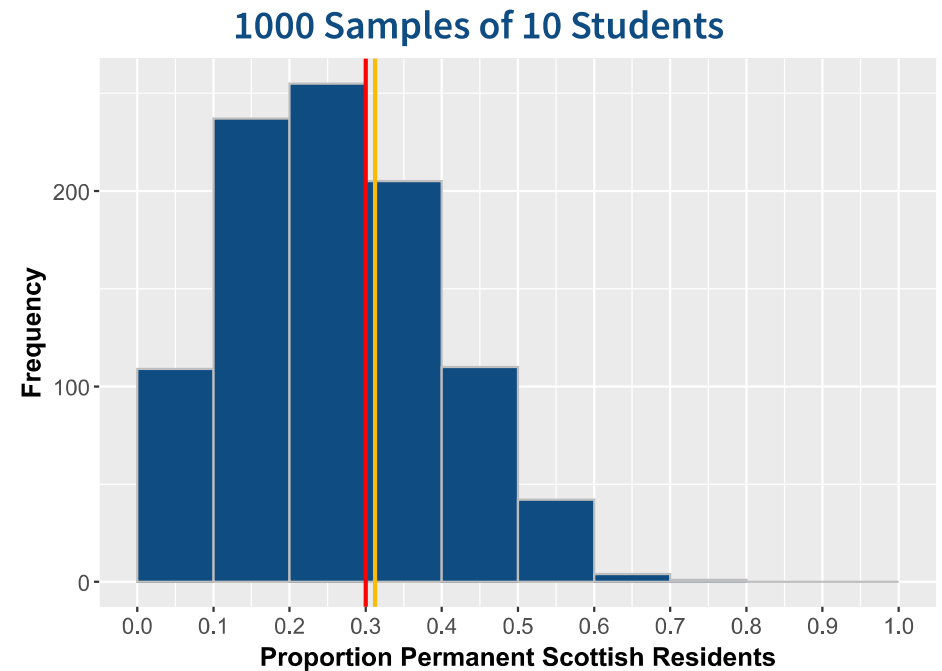
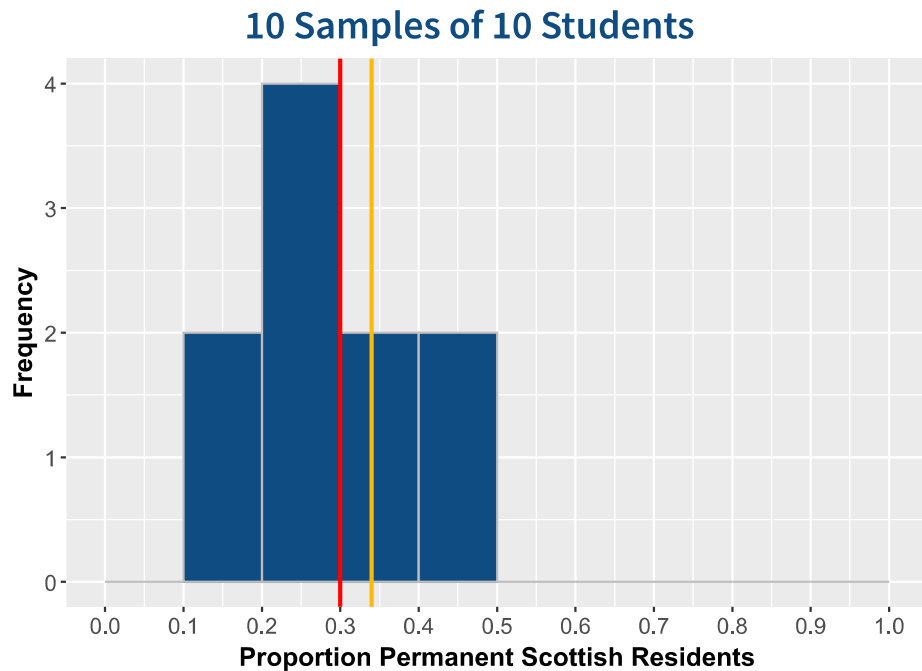
- If we were to repeat this process 2 more times, we can create three sampling distributions, each of which look different.



- Each sampling distribution is characterising the *sampling variability* in our estimate of the parameter of interest
- **Do samples with values close to the population value tend to be more or less likely?**

# Taking more samples

- So far we have taken 10 samples... what if we took more?
- Let's imagine we sampled 10 students 100 times

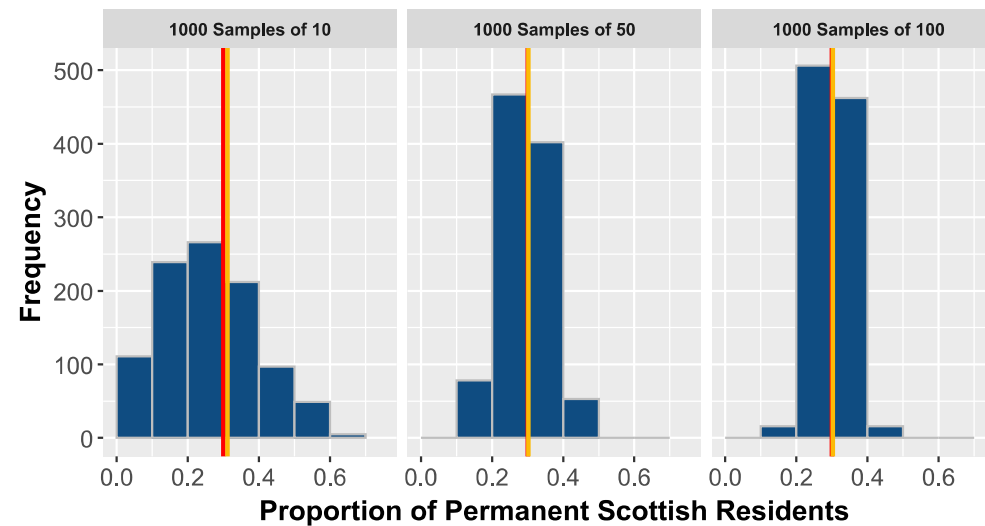


- What differences do you notice between these two sampling distributions?



# Bigger samples

- We've been taking samples of 10 students. Let's see what happens when we increase our sample size to  $n = 50$ , and then  $n = 100$ .



- What changes as we increase sample size?

# Properties of sampling distributions

- Sampling distributions are characterising the variability in sample estimates
  - Variability can be thought of as the spread in data/plots
- So as we increase  $n$ , we get less variable samples (the distribution of sample statistics is more tightly clustered around the population parameter)
  - Harder to get an unrepresentative sample as your  $n$  increases
- Let's put this phenomenon in the language of probability:
  - As sample  $n$  increases, the probability of observing a point-estimate that is a long way from the population parameter (here 0.30) decreases (becomes less probable)
- So when we have large samples, the point-estimates from those samples are likely to be closer to the population value

# Standard error

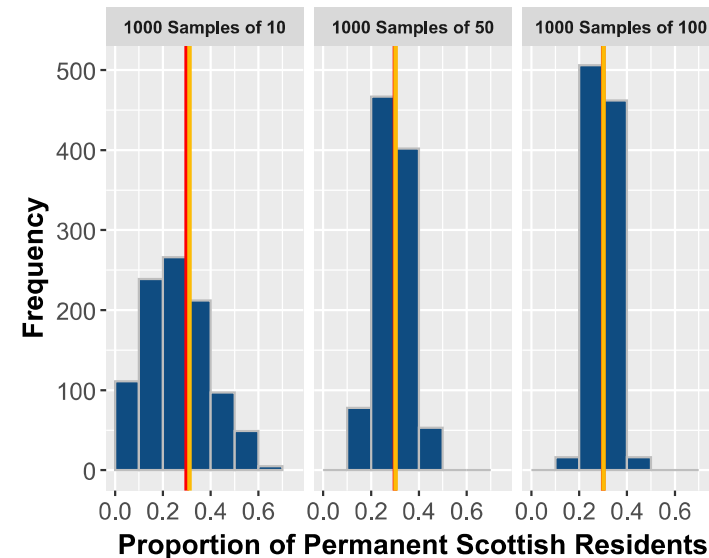
- We can formally calculate the variability of a sampling distribution, or the **standard error**

$$SE = \frac{\sigma}{\sqrt{n}}$$

- This is essentially calculating the standard deviation of the sampling distribution, with a key difference:
  - The standard deviation describes the variability *within* one sample
  - The standard error describes variability *across* multiple samples
- With continuous data, the standard error gives you a sense of how different  $\bar{x}$  is likely to be from  $\mu$
- In this example, we're working with binomial data (Scottish Residency = Yes or No), so the standard error indicates how greatly a particular sample proportion is likely to differ from the proportion in the population

# Properties of sampling distributions

- Mean of the sampling distribution is close to  $\mu$ , even with a small number of samples
- As the number of samples increases:
  - The sampling distribution approaches a normal distribution
  - Sample  $\bar{x}$ s pile up around  $\mu$
- As  $n$  per sample increases, the SE of the sampling distribution decreases (becomes narrower)
  - With large  $n$ , all our point-estimates are closer to the population parameter

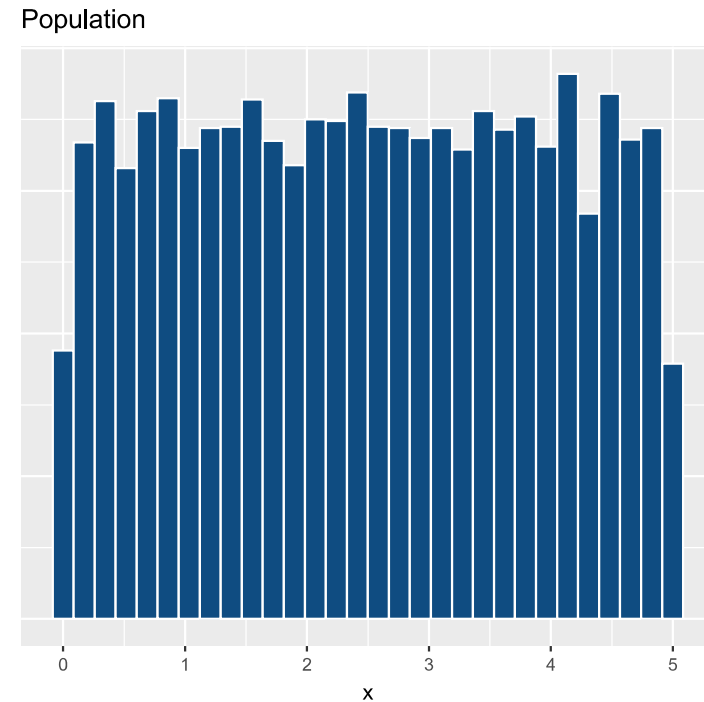


# Two Related Concepts

- These properties illustrate two important concepts:
- **The Law of Large Numbers:** As  $n$  increases,  $\bar{x}$  approaches  $\mu$
- **Central Limit Theorem:** When estimates of  $\bar{x}$  are based on increasingly large samples (  $n$  ), the sampling distribution of  $\bar{x}$  becomes more normal (symmetric), and narrower (quantified by the standard error)
- These concepts hold regardless of the underlying shape of the distribution
- To demonstrate this, let's explore some different distributions

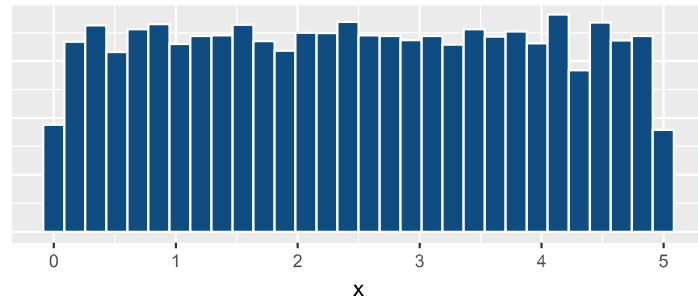
# Uniform distribution

- Continuous probability distribution
- There is an equal probability for all values within a given range

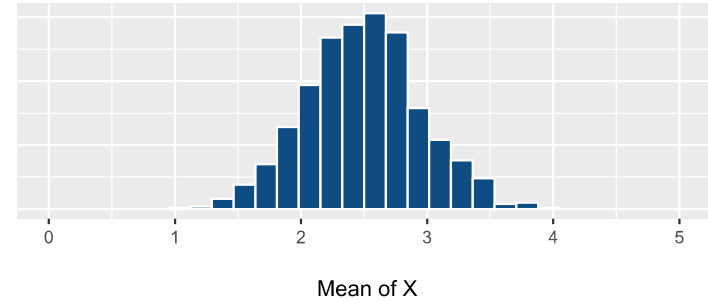


# Uniform distribution

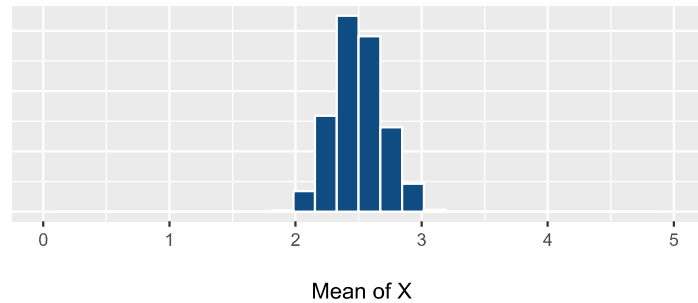
Population



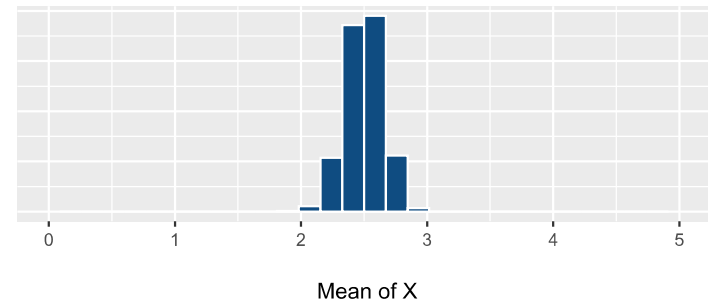
N = 10



N = 50

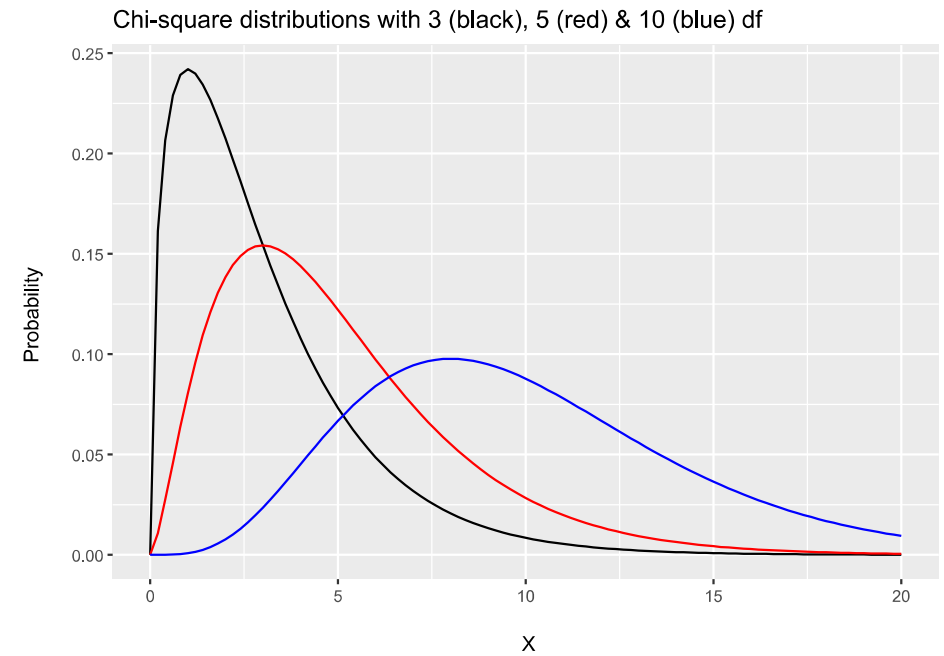


N = 100



# $\chi$ -square distribution

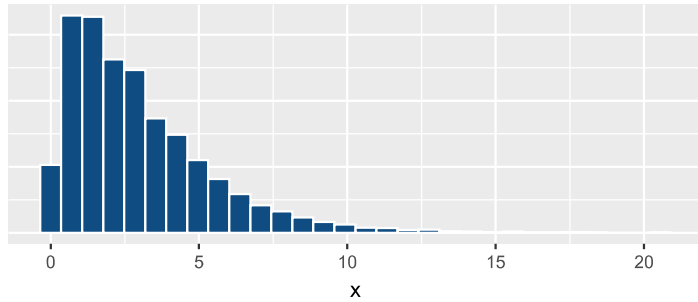
- Continuous probability distribution
- Non-symmetric



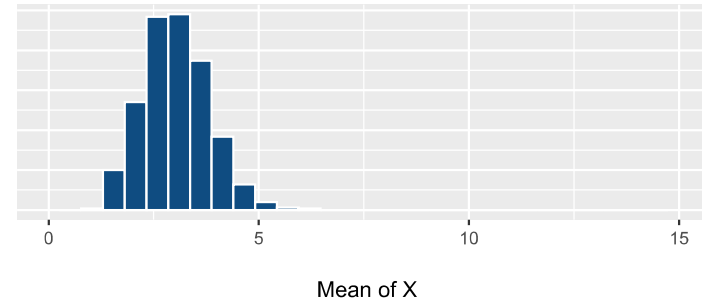


# $\chi$ -square distribution

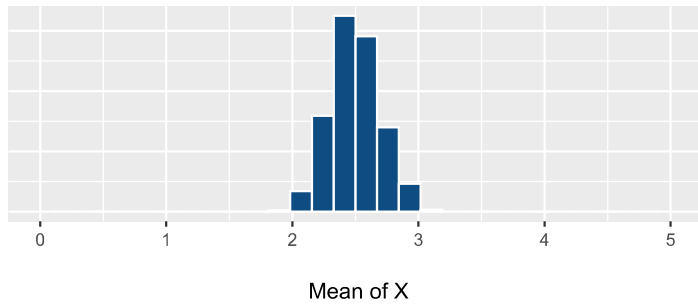
Population



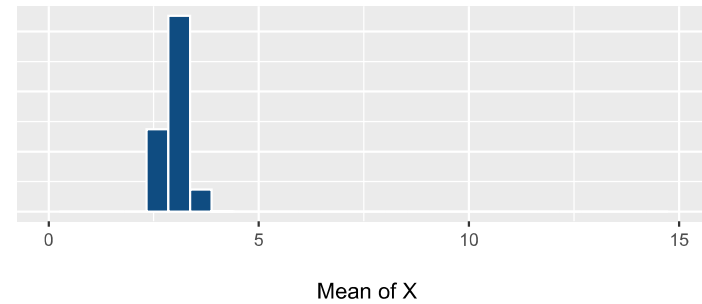
N = 10



N = 50

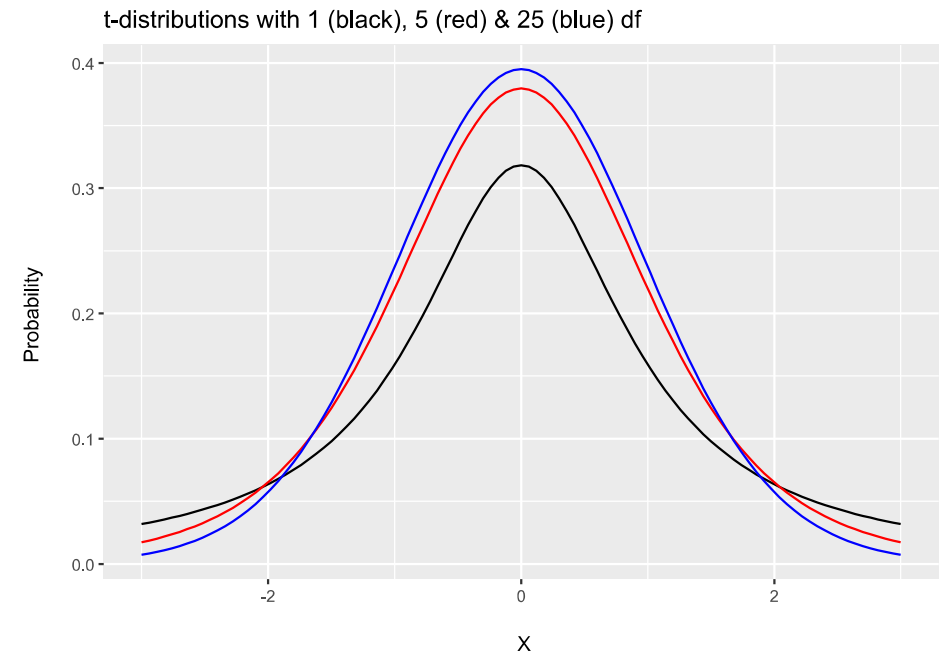


N = 100



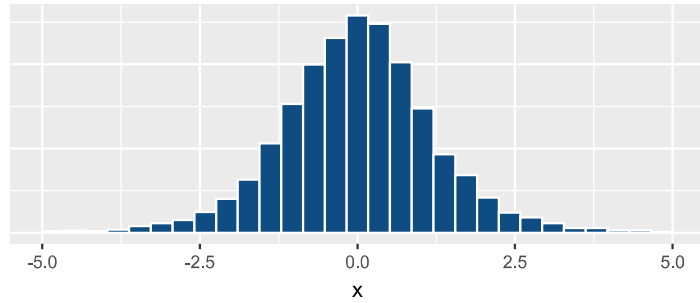
# $t$ -distribution

- Continuous probability distribution
- Symmetric and uni-modal (similar to the normal distribution)
  - "Heavier/fatter tails" = greater chance of observing a value further from the mean

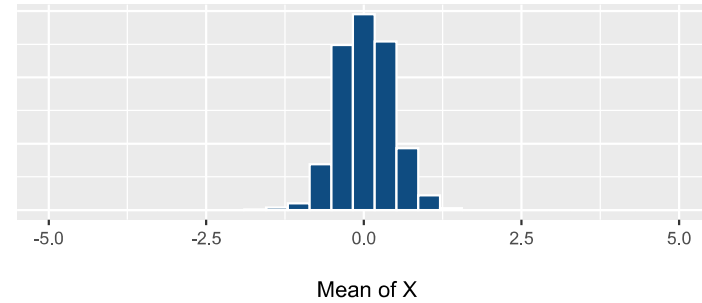


# $t$ -distribution

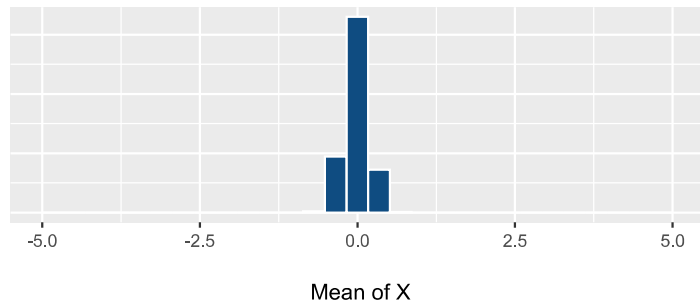
Population



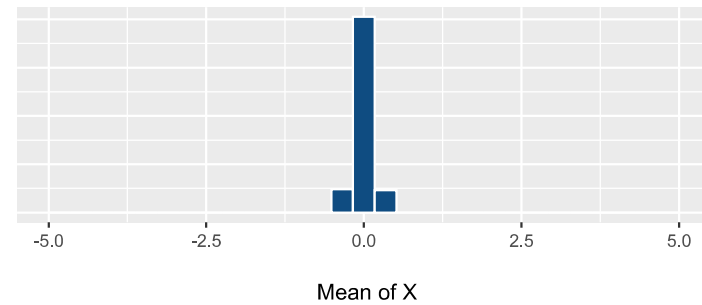
N = 10



N = 50



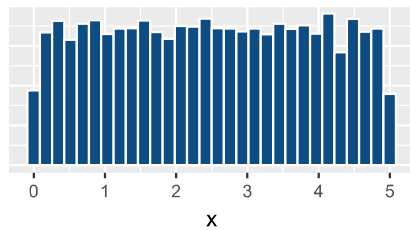
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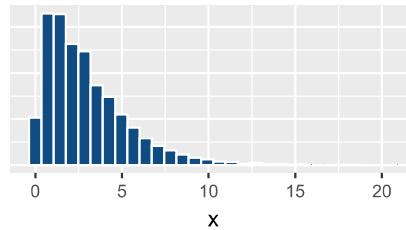
# Central Limit Theorem

- These examples all demonstrate the Central Limit Theorem
- When  $n$  is large enough,  $\bar{x}$ 's approximate a normal distribution around  $\mu$ , regardless of the underlying population distribution

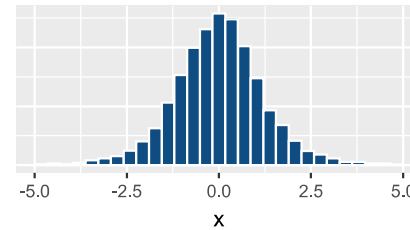
Population



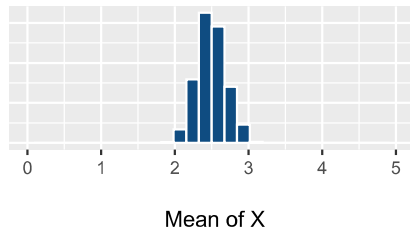
Population



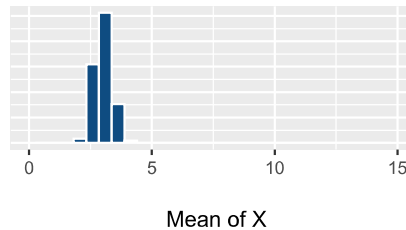
Population



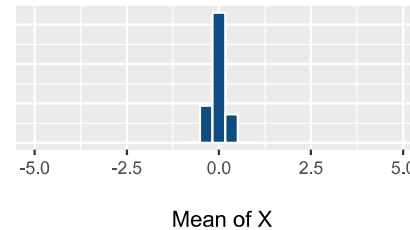
N = 50



N = 50



N = 50



# Features of samples

- Is our sample...
  - Biased?
  - Representative?
  - Random?
- If a sample of  $n$  is drawn at random, it is likely to be unbiased and representative of  $N$
- Our point estimates from such samples will be good guesses at the population parameter

# Summary of today

- Samples are used to estimate the population
- Samples provide point estimates of population parameters
- Properties of samples and sampling distributions
- Properties of good samples

# This week

## Tasks

- Attend both lectures
- Attend your lab and work together on the lab tasks
- Complete the weekly quiz
  - Opened Monday at 9am
  - Closes Sunday at 5pm
- Submit Formative Report B by 12 noon on Friday the 29th of November 2024

## Support

- **Office hours:** for one-to-one support on course materials or assessments (see LEARN > Course information > Course contacts)
  - Note: No office hours between 2 Dec and 10 Jan
- **Piazza:** help each other on this peer-to-peer discussion forum
- **Student Adviser:** for general support while you are at university (find your student adviser on MyEd/Euclid)