T-Test: Independent Samples

Data Analysis for Psychology in R 1

Semester 2, Week 7

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Learning Objectives

- Understand when to use an independent samples t-test
- Understand the null hypothesis for an independent sample t-test
- Understand how to calculate the test statistic
- Know how to conduct the test in R

Topics for Today

- Conceptual background and introduction to our example
- Calculations and R-functions
- Assumptions and effect size

Independent T-Test Purpose & Data

- The independent t-test is used when we want to test the difference in mean between two measured groups.
- The groups must be independent:
  - No person can be in both groups.
- Examples:
  - Treatment versus control group in an experimental study
  - Married versus not married
- Data Requirements:
  - A continuously measured variable
  - A binary variable denoting groups

t-statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{SE_{(\bar{x}_1 - \bar{x}_2)}}$$

- Where
  - $\bar{x}_1$ and $\bar{x}_2$ are the sample means in each group
  - δ_0 is the hypothesised population difference in means in the null hypothesis $(\mu_1 - \mu_2)$
  - $SE_{(\bar{x}_1 - \bar{x}_2)}$ is standard error of the difference
- Sampling distribution is a t-distribution with $n - 2$ degrees of freedom, where n = $n_1 + n_2$.

Standard Error Difference

- First calculate the pooled standard deviation.

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

- Then use this to calculate the SE of the difference.

$$SE_{(\bar{x}_1 - \bar{x}_2)} = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Hypotheses

- Two-tailed:

$$
\begin{aligned}
H_0 &: \mu_1 = \mu_2 \\
H_1 &: \mu_1 \neq \mu_2
\end{aligned}
$$

- One-tailed:

$$
\begin{aligned}
H_0 &: \mu_1 = \mu_2 \\
H_1 &: \mu_1 < \mu_2 \\
H_1 &: \mu_1 > \mu_2
\end{aligned}
$$

- Two-tailed:

$$
\begin{aligned}
H_0 &: \mu_1 - \mu_2 = 0 \\
H_1 &: \mu_1 - \mu_2 \neq 0
\end{aligned}
$$

- One-tailed:

$$
\begin{aligned}
H_0 &: \mu_1 - \mu_2 = 0 \\
H_1 &: \mu_1 - \mu_2 < 0 \\
H_1 &: \mu_1 - \mu_2 > 0
\end{aligned}
$$

Questions?

Example

- Example taken from Howell, D.C. (2010). *Statistical Methods for Psychology, 7th Edition*. Belmont, CA: Wadsworth Cengage Learning.

- Data from Aronson, Lustina , Good, Keough , Steele and Brown (1998). Experiment on stereotype threat.

    - Two independent groups college students (n=12 control; n=11 threat condition).
    - Both samples excel in maths.
    - Threat group told certain students usually do better in the test

Data

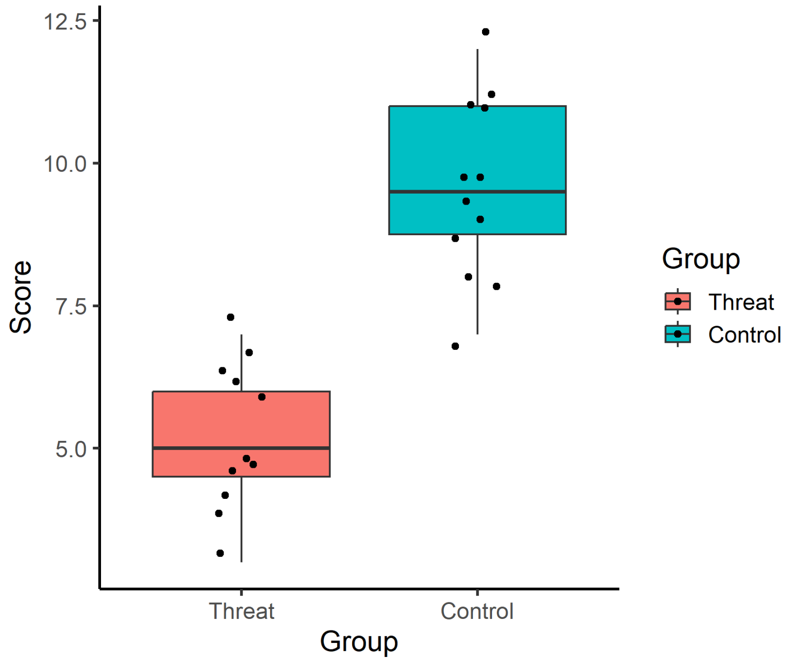
```
## # A tibble: 23 × 2
##    Group  Score
##    <fct>  <dbl>
##  1 Threat     7
##  2 Threat     5
##  3 Threat     6
##  4 Threat     5
##  5 Threat     6
##  6 Threat     5
##  7 Threat     4
##  8 Threat     7
##  9 Threat     4
## 10 Threat     3
## # … with 13 more rows
```

Visualizing data

- We spoke earlier in the course about the importance of visualizing our data.
- Here, we want to show the mean and distribution of scores by group.
- So we want a.....

Visualizing data

```
ggplot(data = threat,
       aes(x = Group,  y = Score, fill = Group)
  geom_boxplot() +
  geom_jitter(width = 0.1)
```



Hypotheses

- My hypothesis is that the threat group will perform worse than the control group.
    - This is a one-tailed hypothesis.
- And I will use an $\alpha = .05$

Questions?

Calculation

- Steps in my calculations:
  - Calculate the sample mean in both groups $\bar{x}_1$ and $\bar{x}_2$.
  - Calculate the pooled SD (s_p).
  - Check I know my n.
  - Calculate the standard error (SE).
- Use all this to calculate t.

Calculation

```
threat %>%
  group_by(Group) %>%
  summarise(
    Mean = round(mean(Score),2),
    SD = round(sd(Score),2),
    n = n()
  ) %>%
  kable(digits = 2) %>%
  kable_styling(full_width = FALSE)
```

Group	Mean	SD	n
Threat	5.27	1.27	11
Control	9.58	1.51	12

Calculation

Group	Mean	SD	n
Threat	5.27	1.27	11
Control	9.58	1.51	12

- Calculate pooled standard deviation

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(11 - 1) * 1.27^2 + (12 - 1) * 1.51^2}{11 + 12 - 2}} = \sqrt{\frac{10 * 1.27^2 + 11 * 1.51^2}{11 + 12 - 2}} = \sqrt{\frac{41.21}{21}} = 1.401$$

- Calculate the standard error.

$$SE_{(\bar{x}_1 - \bar{x}_2)} = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 1.401 \sqrt{\frac{1}{11} + \frac{1}{12}} = 1.401 * 0.417 = 0.584$$

Calculation

- Steps in my calculations:
  - Calculate the sample mean in both groups - Threat $(\bar{x}_1 = 5.27)$, Control $(\bar{x}_2 = 9.58)$.
  - Calculate the pooled SD $(s_p = 1.401)$.
  - Check I know my n - Threat $(n_1 = 11)$ and Control $(n_2 = 12)$ - $n = 23$.
  - Calculate the standard error $(SE = 0.584)$.
- Use all this to calculate t.

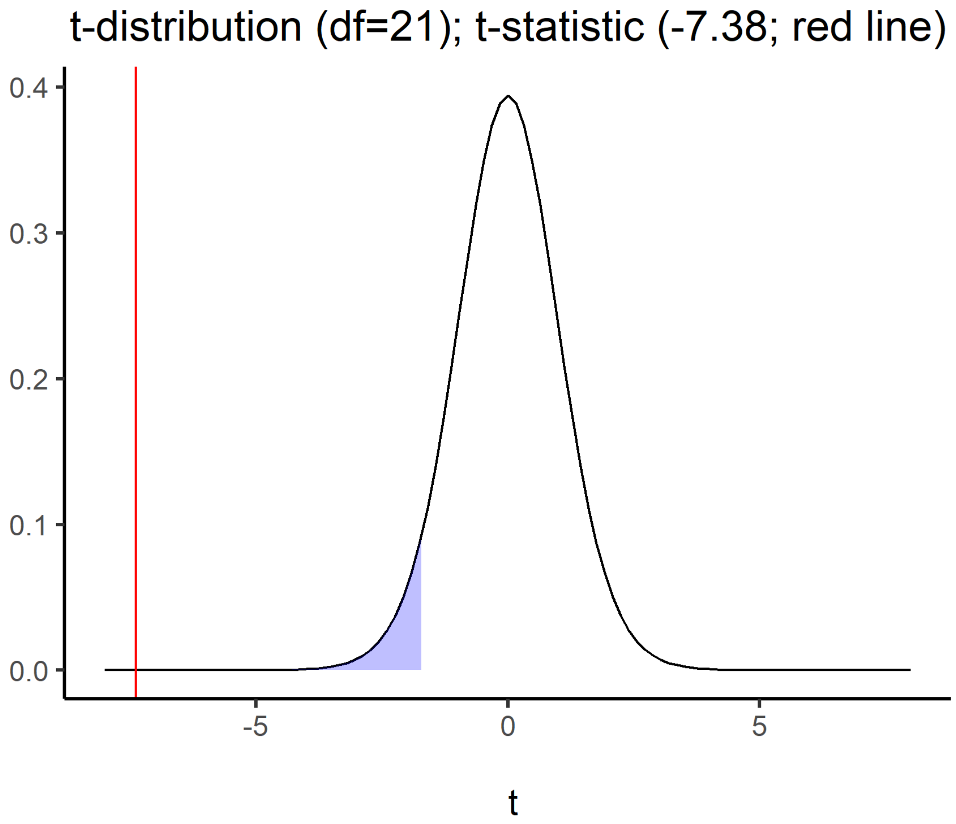
$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{SE_{(\bar{x}_1 - \bar{x}_2)}} = \frac{5.27 - 9.58}{0.584} = -7.38$$

- So in our example $t = -7.38$
- Note: When doing hand calculations there might be a small amount of rounding error when we compare to t calculated in R.

Is our test significant?

- We have all the pieces we need:
  - Degrees of freedom = $n - 2 = (12 + 11) - 2 = 23 - 2 = 21$
  - We have our t-statistic (-7.38)
  - Hypothesis to test (one-tailed)
  - α level (.05).
- So now all we need is the critical value from the associated t-distribution in order to make our decision.

Is our test significant?




```r
tibble(
  LowerCrit = round(qt(0.05, 21),2),
  Exactp = 1-pt(7.3817, 21)
)
```

```
## # A tibble: 1 × 2
##   LowerCrit      Exactp
##       <dbl>       <dbl>
## 1     -1.72 0.000000146
```

Is my test significant?

- So our critical value is -1.72
  - Our t-statistic (-7.38) is larger than this
  - So we reject the null hypothesis
- $t(21) = -7.38, p < .05, one - tailed.$

Our Test: In R

```r
res <- t.test(threat$Score ~ threat$Group,
       alternative = "less",
       mu = 0,
       var.equal = TRUE)
res
```

```
##
##     Two Sample t-test
##
## data:  threat$Score by threat$Group
## t = -7.4, df = 21, p-value = 1e-07
## alternative hypothesis: true difference in means between group Threat and group Control is less than 0
## 95 percent confidence interval:
##    -Inf -3.306
## sample estimates:
##  mean in group Threat mean in group Control
##                 5.273                 9.583
```

Write up

An independent sample t-test was used to assess whether the maths score mean of the control group $(n = 12)$ was higher than that of the stereotype threat group $(n = 11)$. There was a significant difference in test score between the control $(Mean = 9.58; SD = 1.51)$ and threat $(Mean = 5.27; SD = 1.27)$ groups, where the scores were significantly higher in the control group (t(21)=-7.38, $p < .05, one - tailed)$. Therefore, we can reject the null hypothesis. The direction of difference supports our directional hypothesis and indicates that the threat group performed more poorly than the control group.

Questions?

Assumption checks summary

	Description	One-Sample t-test	Independent Sample t-test	Paired Sample t-test
Normality	Continuous variable (and difference) is normally distributed.	Yes (Population)	Yes (Both groups/ Difference)	Yes (Both groups/ Difference)
Tests:	Descriptive Statistics; Shapiro-Wilks Test; QQ-plot			
Independence	Observations are sampled independently.	Yes	Yes (within and across groups)	Yes (within groups)
Tests:	None. Design issue.			
Homogeneity of variance	Population level standard deviation is the same in both groups.	NA	Yes	Yes
Tests:	F-test			
Matched Pairs in data	For paired sample, each observation must have matched pair.	NA	NA	Yes
Tests:	None. Data structure issue.			

Assumptions

- The independent sample t-test has the following assumptions:
  - Independence of observations within and across groups.
  - Continuous variable is approximately normally distribution **within both groups**.
    - Equivalently, that the difference in means is normally distributed.
  - Homogeneity of variance across groups.

Assumption checks: Normality

- Descriptive statistics:
  - Skew:
    - Below are some rough guidelines on how to interpret skew.
    - No strict cuts for skew - these are loose guidelines.

Verbal label	Magnitude of skew in absolute value
Generally not problematic	$\lvert \text{Skew} \rvert < 1$
Slight concern	$1 > \lvert \text{Skew} \rvert < 2$
Investigate impact	$\lvert \text{Skew} \rvert > 2$

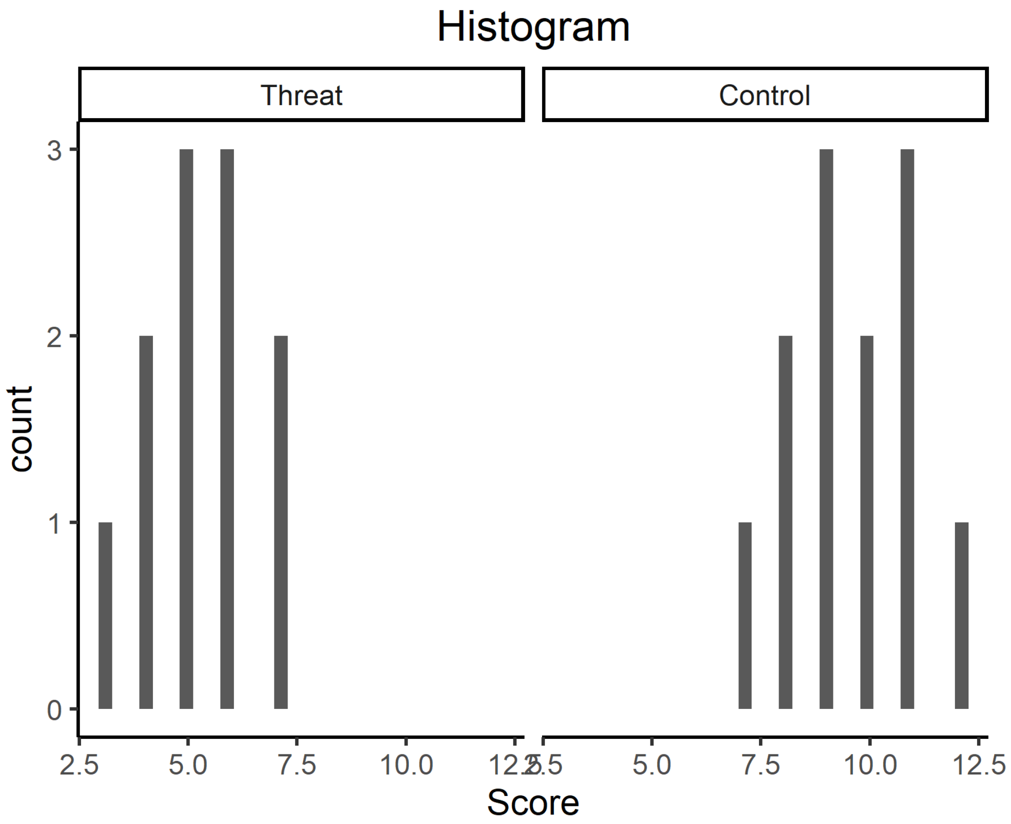
Skew

```
library(psych)
threat %>%
  group_by(Group) %>%
    summarise(
      skew = round(skew(Score),2)
  )
```

```
## # A tibble: 2 × 2
##   Group    skew
##   <fct>   <dbl>
## 1 Threat  -0.2
## 2 Control -0.07
```

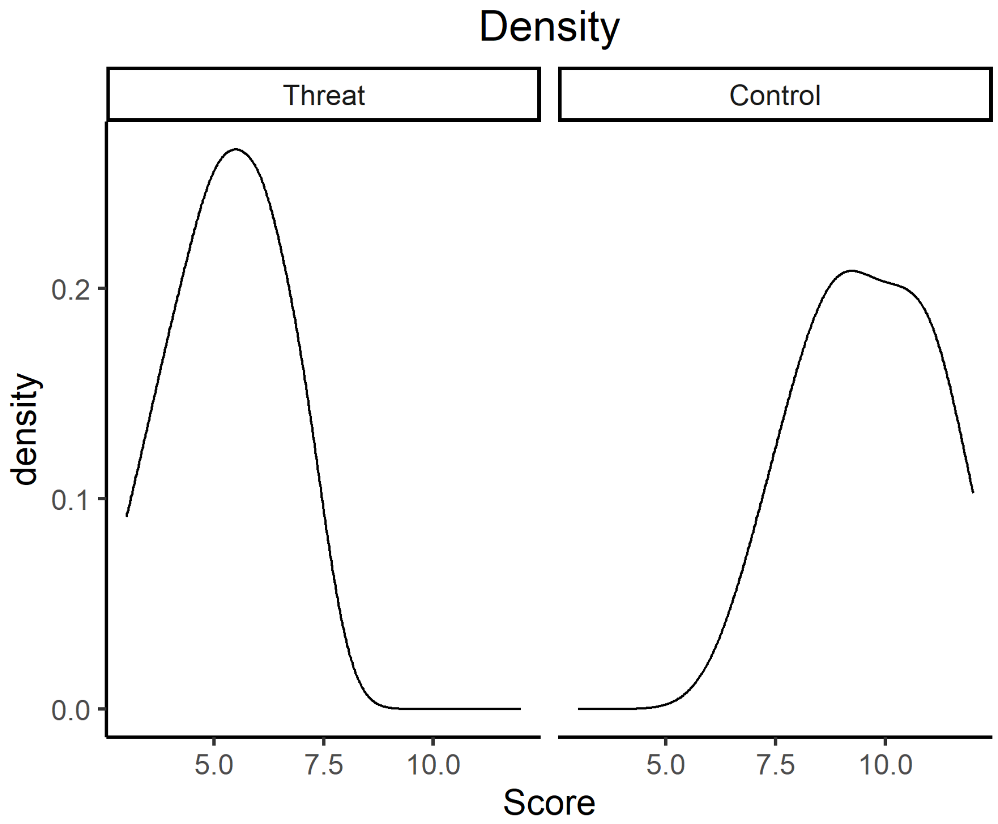
Histograms

```
ggplot(threat, aes(x=Score)) +
  geom_histogram() +
  facet_wrap(~ Group)  +
  labs(title = "Histogram")
```



Density

```
ggplot(threat, aes(x=Score)) +
  geom_density() +
  facet_wrap(~ Group) +
  labs(title = "Density")
```

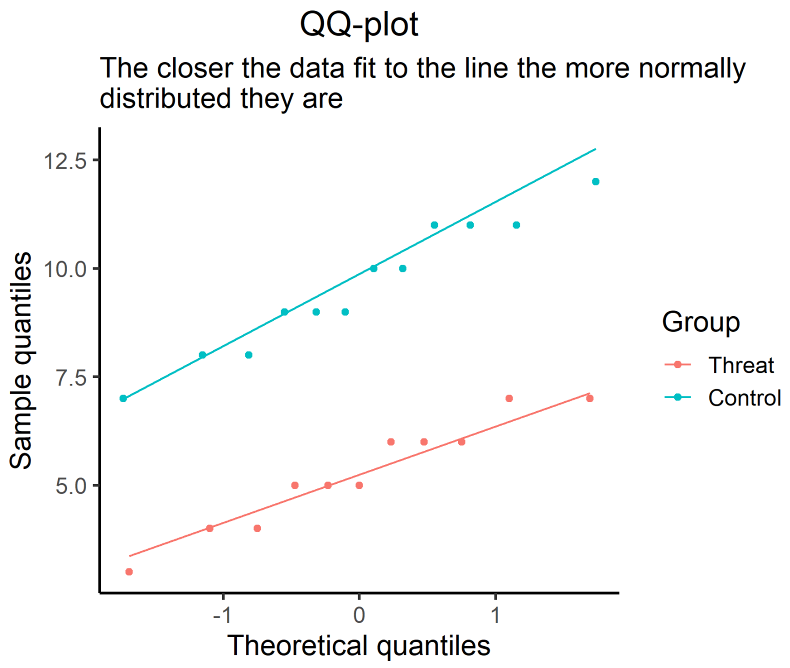


Assumption checks: Normality

- QQ-plots:
  - Plots the sorted quantiles of one data set (distribution) against sorted quantiles of data set (distribution).
  - Quantile = the percent of points falling below a given value.
  - For a normality check, we can compare our own data to data drawn from a normal distribution

QQ-plots

```
ggplot(data = threat,
       aes(sample = Score, colour = Group)) +
  geom_qq() +
  geom_qq_line() +
      labs(title="QQ-plot",
       subtitle="The closer the data fit to the
       x = "Theoretical quantiles",
       y = "Sample quantiles")
```



- This looks reasonable in both groups

Assumption checks: Normality

- Shapiro-Wilks test:
  - Checks properties of the observed data against properties we would expected from normally distributed data.
  - Statistical test of normality.
  - H_0: data = a normal distribution.
  - p-value $< \alpha$ = reject the null, data are not normal.
    - Sensitive to n as all p-values will be.
    - In very large n, normality should also be checked with QQ-plots alongside statistical test.

Shapiro-Wilks R

```r
threat %>%
  filter(Group == "Control") %>%
  pull(Score) %>%
shapiro.test()
```

```
##
##      Shapiro-Wilk normality test
##
## data:  .
## W = 0.96, p-value = 0.7
```

$W = 0.96, p = .70$

```r
thr <- threat %>%
  filter(Group == "Threat") %>%
  select(Score)
shapiro.test(thr$Score)
```

```
##
##      Shapiro-Wilk normality test
##
## data:  thr$Score
## W = 0.94, p-value = 0.5
```

$W = 0.94, p = .50$

Assumption checks: Homogeneity of variance

- The F-test is a test that compares the variances of two groups.
  - This test is preferable for t-test.
  - H_0: Population variances are equal.
  - p-value $< \alpha$ = reject the null, the variances differ across groups.

F-test R

```r
var.test(threat$Score ~ threat$Group, ratio = 1, conf.level = 0.95)
```

```
##
##      F test to compare two variances
##
## data:  threat$Score by threat$Group
## F = 0.71, num df = 10, denom df = 11, p-value = 0.6
## alternative hypothesis: true ratio of variances is not equal to 1
## 95 percent confidence interval:
##  0.2026 2.6181
## sample estimates:
## ratio of variances
##             0.7144
```

- Why `ratio = 1`?

- $H_0 : \sigma_1^2 = \sigma_2^2$
  - $H_1 : \sigma_1^2 \neq \sigma_2^2$

- $H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$
  - $H_1 : \frac{\sigma_1^2}{\sigma_2^2} \neq 1$

Violation of homogeneity of variance

- If the variances differ, we can use a Welch test.
- Conceptually very similar, but we do not use a pooled standard deviation.
    - As such our estimate of the SE of the difference changes
    - As do our degrees of freedom

Welch test

- If the variances differ, we can use a Welch test.
- Test statistic = same:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{SE_{(\bar{x}_1 - \bar{x}_2)}}$$

- SE calculation:

$$SE_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

- And degrees of freedom (don't worry, not tested)

$$df = \frac{(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2})^2}{\frac{(\frac{s_1^2}{n_1})^2}{n_1 - 1} + \frac{(\frac{s_2^2}{n_2})^2}{n_2 - 1}}$$

Welch: In R

```r
t.test(Score ~ Group,
       var.equal = FALSE, #default, only here to highlight difference
       alternative = "less",
       data = threat)
```

```
##
##      Welch Two Sample t-test
##
## data:  Score by Group
## t = -7.4, df = 21, p-value = 1e-07
## alternative hypothesis: true difference in means between group Threat and group Control is less than 0
## 95 percent confidence interval:
##     -Inf -3.313
## sample estimates:
##  mean in group Threat mean in group Control
##                 5.273                 9.583
```

Cohen's D: Independent samples t-test

- Independent-sample t-test (if you do have equality of variances):

$$D = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{s_p}$$

- $\bar{x}_1$ = mean group 1
- $\bar{x}_2$ = mean group 2
- δ_0 is the hypothesised population difference in means in the null hypothesis $(\mu_1 - \mu_2)$
- s_p = pooled standard deviation

- Independent-sample t-test (if you do not have equality of variances):
    - Calculate via `cohens_d()` function from `effectsize` package in `R` - do not calculate by hand.

- Recall the common "cut-offs" for D-scores:

Verbal label	Magnitude of D in absolute value
Small (or weak)	≤ 0.20
Medium (or moderate)	≈ 0.50
Large (or strong)	≥ 0.80

Cohen's D in R

```r
library(effectsize)
cohens_d(threat$Score ~ threat$Group, mu = 0, alternative = "less", var.equal = TRUE)
```

```
## Cohen's d |         95% CI
## -------------------------
## -3.08     | [-Inf, -2.02]
##
## - Estimated using pooled SD.
## - One-sided CIs: lower bound fixed at [-Inf].
```

```r
#to get upper and lower CIs - recall this is now a two-sided test:
cohens_d(threat$Score ~ threat$Group, mu = 0, var.equal = TRUE)
```

```
## Cohen's d |          95% CI
## --------------------------
## -3.08     | [-4.30, -1.83]
##
## - Estimated using pooled SD.
```

Write up: Assumptions

The DV of our study, Score, was measured on a continuous scale, and data were not independent (participants belonged to one of two groups - Control or Threat). The assumption of normality was visually assessed (via histograms, density plots, and a QQplot) as well as statistically via a Shapiro-Wilks test. The QQplots did not show much deviation from the diagonal line in either group, and the Shapiro-Wilks test for both the Control $(W = 0.96, p = .70)$ and Threat $(W = 0.94, p = .50)$ conditions suggested that the samples came from a population that was normally distributed. This was inline with the histogram and density plots for each group, which suggested that Score was normally distributed (and where $skew < 1$). Based on the results of our F-test, there was no significant difference between the two population variances $(F(10, 11) = 0.71, p = .60)$. The size of the effect was found to be large $D = -3.08[-4.30, -1.83]$.

Summary

- Today we have covered:
  - Basic structure of the independent-sample t-test
  - Calculations
  - Interpretation
  - Assumption checks
  - Effect size measures