Additional Detail to the 3x2 class example

This document is intended to fully run through the 3x2 example from the end of the lecture. The example concerned treatment schedules in two different hospitals. The group means are given below.

	Hospital 1	Hospital 2
Treatment A	10.80	7.87
Treatment B	9.43	13.11
Treatment C	10.11	7.98

We can also express this table in terms of the specific cell means:

	Hospital 1	Hospital 2
Treatment A	μ_{11}	μ_{12}
Treatment B	μ_{21}	μ_{22}
Treatment C	μ_{31}	μ_{32}

We can then create a table to show the dummy codes for this example. We will use T to denote the dummies for Treatment, and H for Hospital to more easily differentiate them. In total we will need:

- (r-1) dummy variables for treatments. 3-1 = 2
- (c-1) dummy variables for hospital. 2-1 = 1
- And we need (r-1)(c-1) for the interactions. (3-1)(2-1) = 2*1 = 2

We will set Treatment A and Hospital 1 as the references.

Row	**Column**	**T1**	**T2**	**H1**	**T1H1**	**T2H1**
Treat A	Hosp 1	0	0	0	0	0
Treat B	Hosp 1	1	0	0	0	0
Treat C	Hosp 1	0	1	0	0	0
Treat A	Hosp 2	0	0	1	0	0
Treat B	Hosp 2	1	0	1	1	0
Treat C	Hosp 2	0	1	1	0	1

The full linear model for this design is given by:

$$y = b_0 + b_1 T1 + b_2 T2 + b_3 H1 + b_4 T1H1 + b_5 T2H1$$

Using the references to cell means above, we can now insert values from the dummy table into this full equation and there be able to state the relation between the cell means and the coefficients in the model:

$$
\begin{aligned}
\mu_{11} &= E(Y|R=1, C=1) \\
&= b_0 + (b_1 T1 * 0) + (b_2 T2 * 0) + (b_3 H1 * 0) + (b_4 T1H1 * 0) + (b_5 T2H1 * 0) \\
&= b_0
\end{aligned}
$$

$$
\begin{aligned}
\mu_{21} &= E(Y|R=2, C=1) \\
&= b_0 + (b_1 T1 * 1) + (b_2 T2 * 0) + (b_3 H1 * 0) + (b_4 T1H1 * 0) + (b_5 T2H1 * 0) \\
&= b_0 + b_1
\end{aligned}
$$

$$
\begin{aligned}
\mu_{31} &= E(Y|R=3, C=1) \\
&= b_0 + (b_1 T1 * 0) + (b_2 T2 * 1) + (b_3 H1 * 0) + (b_4 T1H1 * 0) + (b_5 T2H1 * 0) \\
&= b_0 + b_2
\end{aligned}
$$

$$
\begin{aligned}
\mu_{12} &= E(Y|R=1, C=2) \\
&= b_0 + (b_1 T1 * 0) + (b_2 T2 * 0) + (b_3 H1 * 1) + (b_4 T1H1 * 0) + (b_5 T2H1 * 0) \\
&= b_0 + b_3
\end{aligned}
$$

$$
\begin{aligned}
\mu_{22} &= E(Y|R=2, C=2) \\
&= b_0 + (b_1 T1 * 1) + (b_2 T2 * 0) + (b_3 H1 * 1) + (b_4 T1H1 * 1) + (b_5 T2H1 * 0) \\
&= b_0 + b_1 + b_3 + b_4
\end{aligned}
$$

$$
\begin{aligned}
\mu_{32} &= E(Y|R=3, C=2) \\
&= b_0 + (b_1 T1 * 0) + (b_2 T2 * 1) + (b_3 H1 * 1) + (b_4 T1H1 * 0) + (b_5 T2H1 * 1) \\
&= b_0 + b_2 + b_3 + b_5
\end{aligned}
$$

From this point, our next step is to solve/rearrange the table above such that we represent the coefficients in terms of the cell means (thus taking us to the comparisons we were making in class.

B0 - intercept

$$\mu_{11} = b_0$$

B1 – conditional main effect

$$
\begin{aligned}
\mu_{21} &= b_0 + b_1 \\
&= \mu_{11} + b_1 \\
\mu_{21} - \mu_{11} &= b_1
\end{aligned}
$$

B2 – conditional main effect

$$
\begin{aligned}
\mu_{31} &= b_0 + b_2 \\
&= \mu_{11} + b_2 \\
\mu_{31} - \mu_{11} &= b_2
\end{aligned}
$$

B3 – conditional main effect

$$
\begin{aligned}
\mu_{12} &= b_0 + b_3 \\
&= \mu_{11} + b_3 \\
\mu_{12} - \mu_{11} &= b_3
\end{aligned}
$$

B4 - interaction

μ_{22}	=	$b_0 + b_1 + b_3 + b_4$
	=	$\mu_{11} + (\mu_{21} - \mu_{11}) + (\mu_{12} - \mu_{11}) + b_4$
b_4	=	$\mu_{22} - (\mu_{11} + (\mu_{21} - \mu_{11}) + (\mu_{12} - \mu_{11}))$
b_4		$\mu_{22} - \mu_{21} - \mu_{12} + \mu_{11}$

B5 - interaction

μ_{32}	=	$b_0 + b_2 + b_3 + b_5$
	=	$\mu_{11} + (\mu_{31} - \mu_{11}) + (\mu_{12} - \mu_{11}) + b_5$
b_5	=	$\mu_{32} - (\mu_{11} + (\mu_{31} - \mu_{11}) + (\mu_{12} - \mu_{11}))$
b_5		$\mu_{32} - \mu_{31} - \mu_{12} + \mu_{11}$

Finally, we can relate the formal statement of the null hypotheses for each of these coefficients to the solved equations. In each case, the null is saying that the particular difference between groups, or difference between differences, that a coefficient represents is equal to 0.

A:

Solved	**Null (H0)**
$b_0 = \mu_{11}$	$\mu_{11} = 0$
$b_1 = \mu_{21} - \mu_{11}$	$\mu_{21} - \mu_{11} = 0$
$b_2 = \mu_{31} - \mu_{11}$	$\mu_{31} - \mu_{11} = 0$
$b_3 = \mu_{12} - \mu_{11}$	$\mu_{12} - \mu_{11} = 0$
$b_4 = \mu_{22} - (\mu_{11} + (\mu_{21} - \mu_{11}) + (\mu_{12} - \mu_{11}))$	$(\mu_{21} - \mu_{11}) - (\mu_{22} - \mu_{12}) = 0$
$b_4 = \mu_{22} - \mu_{21} - \mu_{12} + \mu_{11}$	
$b_5 = \mu_{32} - (\mu_{11} + (\mu_{31} - \mu_{11}) + (\mu_{12} - \mu_{11}))$	$(\mu_{31} - \mu_{11}) - (\mu_{32} - \mu_{12}) = 0$
$b_5 = \mu_{32} - \mu_{31} - \mu_{12} + \mu_{11}$	

B:

	Null (H0) in words
b_0	Mean of treatment A hospital 1 is equal to 0.
b_1	Difference between Treatment B and Treatment A in Hospital 1 is 0.
b_2	Difference between Treatment C and Treatment A in Hospital 1 is 0.
b_3	Difference between Treatment A in Hospital 1 and Hospital 2 is 0.
b_4	Difference between Treatment A and Treatment B, is different in Hospital 1 and Hospital 2
b_5	Difference between Treatment A and Treatment C, is different in Hospital 1 and Hospital 2