Worked Example of 3x3

In this document I will show the analysis of the 3x3 design with dummy variables. For each parameter I will show the cells in the mean classification table used in the calculation, the relation to the dummy variable coding, and the working to solve for the beta coefficients from the cell means.

The workings follow the 2x2 case, in that we are looking at the differences between cell means for values coded 1 against the reference group, and for the interactions, we are identifying 4 cells, and considering the difference in differences.

Full equation:

$$Y = b_0 + b_1D_1 + b_2D_2 + b_3D_3 + b_4D_4 + b_5D_1D_3 + b_6D_1D_4 + b_7D_2D_3 + b_8D_2D_4$$

INTERCEPT: b_0

Mean of reference group

Dummy Variable Coding

Row Levels	Column Levels	Row/Column Dummies				Interactions Dummies			
		D_1 (b_1)	D_2 (b_2)	D_3 (b_3)	D_4 (b_4)	D_1D_3 (b_5)	D_1D_4 (b_6)	D_2D_3 (b_7)	D_2D_4 (b_8)
A	A	0	0	0	0	0	0	0	0
A	B	0	0	1	0	0	0	0	0
A	C	0	0	0	1	0	0	0	0
B	A	1	0	0	0	0	0	0	0
B	B	1	0	1	0	1	0	0	0
B	C	1	0	0	1	0	1	0	0
C	A	0	1	0	0	0	0	0	0
C	B	0	1	1	0	0	0	1	0
C	C	0	1	0	1	0	0	0	1

Classification of means

	A	B	C
A	12.4	12.7	12.9
B	11.5	11.1	13.4
C	10.7	16.1	35.4

	A	B	C
A	AA (1,1)	AB (1,2)	AC (1,2)
B	BA (2,1)	BB (2,2)	BC (2,3)
C	CA (3,1)	CB (3,2)	CC (3,3)

Solved

$b_0 = \mu_{11}$

$b_0 = 12.4$

CONDITIONAL MAIN EFFECTS: b_1

Difference between row level A and row level B.

Dummy Variable Coding

Row Levels	Column Levels	Row/Column Dummies				Interactions Dummies			
		D_1 (b_1)	D_2 (b_2)	D_3 (b_3)	D_4 (b_4)	D_1D_3 (b_5)	D_1D_4 (b_6)	D_2D_3 (b_7)	D_2D_4 (b_8)
A	A	0	0	0	0	0	0	0	0
A	B	0	0	1	0	0	0	0	0
A	C	0	0	0	1	0	0	0	0
B	A	1	0	0	0	0	0	0	0
B	B	1	0	1	0	1	0	0	0
B	C	1	0	0	1	0	1	0	0
C	A	0	1	0	0	0	0	0	0
C	B	0	1	1	0	0	0	1	0
C	C	0	1	0	1	0	0	0	1

Classification of means

	A	B	C
A	12.4	12.7	12.9
B	11.5	11.1	13.4
C	10.7	16.1	35.4

	A	B	C
A	AA (1,1)	AB (1,2)	AC (1,2)
B	BA (2,1)	BB (2,2)	BC (2,3)
C	CA (3,1)	CB (3,2)	CC (3,3)

Solved

$b_1 = \mu_{21} - \mu_{11}$

$b_1 = 11.5 - 12.4$

$b_1 = -0.9$

CONDITIONAL MAIN EFFECTS: b_2

Difference between row level A and row level C

Dummy Variable Coding

Row Levels	Column Levels	Row/Column Dummies				Interactions Dummies			
		D_1 (b_1)	D_2 (b_2)	D_3 (b_3)	D_4 (b_4)	D_1D_3 (b_5)	D_1D_4 (b_6)	D_2D_3 (b_7)	D_2D_4 (b_8)
A	A	0	0	0	0	0	0	0	0
A	B	0	0	1	0	0	0	0	0
A	C	0	0	0	1	0	0	0	0
B	A	1	0	0	0	0	0	0	0
B	B	1	0	1	0	1	0	0	0
B	C	1	0	0	1	0	1	0	0
C	A	0	1	0	0	0	0	0	0
C	B	0	1	1	0	0	0	1	0
C	C	0	1	0	1	0	0	0	1

Classification of means

	A	B	C
A	12.4	12.7	12.9
B	11.5	11.1	13.4
C	10.7	16.1	35.4

	A	B	C
A	AA (1,1)	AB (1,2)	AC (1,2)
B	BA (2,1)	BB (2,2)	BC (2,3)
C	CA (3,1)	CB (3,2)	CC (3,3)

Solved

$b_2 = \mu_{31} - \mu_{11}$

$b_2 = 10.7 - 12.4$

$b_2 = -1.7$

CONDITIONAL MAIN EFFECTS: b_3

Difference between column level A and column level B

Dummy Variable Coding

Row Levels	Column Levels	Row/Column Dummies				Interactions Dummies			
		D_1 (b_1)	D_2 (b_2)	D_3 (b_3)	D_4 (b_4)	D_1D_3 (b_5)	D_1D_4 (b_6)	D_2D_3 (b_7)	D_2D_4 (b_8)
A	A	0	0	0	0	0	0	0	0
A	B	0	0	1	0	0	0	0	0
A	C	0	0	0	1	0	0	0	0
B	A	1	0	0	0	0	0	0	0
B	B	1	0	1	0	1	0	0	0
B	C	1	0	0	1	0	1	0	0
C	A	0	1	0	0	0	0	0	0
C	B	0	1	1	0	0	0	1	0
C	C	0	1	0	1	0	0	0	1

Classification of means

	A	B	C
A	12.4	12.7	12.9
B	11.5	11.1	13.4
C	10.7	16.1	35.4

	A	B	C
A	AA (1,1)	AB (1,2)	AC (1,2)
B	BA (2,1)	BB (2,2)	BC (2,3)
C	CA (3,1)	CB (3,2)	CC (3,3)

Solved

$b_3 = \mu_{12} - \mu_{11}$

$b_3 = 12.7 - 12.4$

$b_3 = 0.3$

CONDITIONAL MAIN EFFECTS: b_4

Difference between column level A and column level C

Dummy Variable Coding

Row Levels	Column Levels	Row/Column Dummies				Interactions Dummies			
		D_1 (b_1)	D_2 (b_2)	D_3 (b_3)	D_4 (b_4)	D_1D_3 (b_5)	D_1D_4 (b_6)	D_2D_3 (b_7)	D_2D_4 (b_8)
A	A	0	0	0	0	0	0	0	0
A	B	0	0	1	0	0	0	0	0
A	C	0	0	0	1	0	0	0	0
B	A	1	0	0	0	0	0	0	0
B	B	1	0	1	0	1	0	0	0
B	C	1	0	0	1	0	1	0	0
C	A	0	1	0	0	0	0	0	0
C	B	0	1	1	0	0	0	1	0
C	C	0	1	0	1	0	0	0	1

Classification of means

	A	B	C
A	12.4	12.7	12.9
B	11.5	11.1	13.4
C	10.7	16.1	35.4

	A	B	C
A	AA (1,1)	AB (1,2)	AC (1,2)
B	BA (2,1)	BB (2,2)	BC (2,3)
C	CA (3,1)	CB (3,2)	CC (3,3)

Solved

$b_4 = \mu_{13} - \mu_{11}$

$b_4 = 12.9 - 12.4$

$b_4 = 0.5$

INTERACTIONS: b_5

Difference between row level A and row level B, is different across column level A and column level B

Dummy Variable Coding

Row Levels	Column Levels	Row/Column Dummies				Interactions Dummies			
		D_1 (b_1)	D_2 (b_2)	D_3 (b_3)	D_4 (b_4)	D_1D_3 (b_5)	D_1D_4 (b_6)	D_2D_3 (b_7)	D_2D_4 (b_8)
A	A	0	0	0	0	0	0	0	0
A	B	0	0	1	0	0	0	0	0
A	C	0	0	0	1	0	0	0	0
B	A	1	0	0	0	0	0	0	0
B	B	1	0	1	0	1	0	0	0
B	C	1	0	0	1	0	1	0	0
C	A	0	1	0	0	0	0	0	0
C	B	0	1	1	0	0	0	1	0
C	C	0	1	0	1	0	0	0	1

Classification of means

	A	B	C
A	12.4	12.7	12.9
B	11.5	11.1	13.4
C	10.7	16.1	35.4

	A	B	C
A	AA (1,1)	AB (1,2)	AC (1,2)
B	BA (2,1)	BB (2,2)	BC (2,3)
C	CA (3,1)	CB (3,2)	CC (3,3)

Solved

$b_5 = \mu_{22} - \mu_{21} - \mu_{12} + \mu_{11}$

$b_5 = 11.1 - 11.5 - 12.7 + 12.4$

$b_5 = -0.7$

INTERACTIONS: b_6

Difference between row level A and row level B, is different across column level A and column level C

Dummy Variable Coding

Row Levels	Column Levels	Row/Column Dummies				Interactions Dummies			
		D_1 (b_1)	D_2 (b_2)	D_3 (b_3)	D_4 (b_4)	D_1D_3 (b_5)	D_1D_4 (b_6)	D_2D_3 (b_7)	D_2D_4 (b_8)
A	A	0	0	0	0	0	0	0	0
A	B	0	0	1	0	0	0	0	0
A	C	0	0	0	1	0	0	0	0
B	A	1	0	0	0	0	0	0	0
B	B	1	0	1	0	1	0	0	0
B	C	1	0	0	1	0	1	0	0
C	A	0	1	0	0	0	0	0	0
C	B	0	1	1	0	0	0	1	0
C	C	0	1	0	1	0	0	0	1

Classification of means

	A	B	C
A	12.4	12.7	12.9
B	11.5	11.1	13.4
C	10.7	16.1	35.4

	A	B	C
A	AA (1,1)	AB (1,2)	AC (1,2)
B	BA (2,1)	BB (2,2)	BC (2,3)
C	CA (3,1)	CB (3,2)	CC (3,3)

Solved

$b_6 = \mu_{23} - \mu_{21} - \mu_{13} + \mu_{11}$

$b_6 = 13.4 - 11.5 - 12.9 + 12.4$

$b_6 = 1.4$

INTERACTIONS: b_7

Difference between row level A and row level C, is different across column level A and column level B

Dummy Variable Coding

Row Levels	Column Levels	Row/Column Dummies				Interactions Dummies			
		D_1 (b_1)	D_2 (b_2)	D_3 (b_3)	D_4 (b_4)	D_1D_3 (b_5)	D_1D_4 (b_6)	D_2D_3 (b_7)	D_2D_4 (b_8)
A	A	0	0	0	0	0	0	0	0
A	B	0	0	1	0	0	0	0	0
A	C	0	0	0	1	0	0	0	0
B	A	1	0	0	0	0	0	0	0
B	B	1	0	1	0	1	0	0	0
B	C	1	0	0	1	0	1	0	0
C	A	0	1	0	0	0	0	0	0
C	B	0	1	1	0	0	0	1	0
C	C	0	1	0	1	0	0	0	1

Classification of means

	A	B	C
A	12.4	12.7	12.9
B	11.5	11.1	13.4
C	10.7	16.1	35.4

	A	B	C
A	AA (1,1)	AB (1,2)	AC (1,2)
B	BA (2,1)	BB (2,2)	BC (2,3)
C	CA (3,1)	CB (3,2)	CC (3,3)

Solved

$b_7 = \mu_{32} - \mu_{31} - \mu_{12} + \mu_{11}$

$b_7 = 16.1 - 10.7 - 12.7 + 12.4$

$b_7 = 5.1$

INTERACTIONS: b_8

Difference between row level A and row level C, is different across column level A and column level C

Dummy Variable Coding

Row Levels	Column Levels	Row/Column Dummies				Interactions Dummies			
		D_1 (b_1)	D_2 (b_2)	D_3 (b_3)	D_4 (b_4)	D_1D_3 (b_5)	D_1D_4 (b_6)	D_2D_3 (b_7)	D_2D_4 (b_8)
A	A	0	0	0	0	0	0	0	0
A	B	0	0	1	0	0	0	0	0
A	C	0	0	0	1	0	0	0	0
B	A	1	0	0	0	0	0	0	0
B	B	1	0	1	0	1	0	0	0
B	C	1	0	0	1	0	1	0	0
C	A	0	1	0	0	0	0	0	0
C	B	0	1	1	0	0	0	1	0
C	C	0	1	0	1	0	0	0	1

Classification of means

	A	B	C
A	12.4	12.7	12.9
B	11.5	11.1	13.4
C	10.7	16.1	35.4

	A	B	C
A	AA (1,1)	AB (1,2)	AC (1,2)
B	BA (2,1)	BB (2,2)	BC (2,3)
C	CA (3,1)	CB (3,2)	CC (3,3)

Solved

$b_8 = \mu_{33} - \mu_{31} - \mu_{13} + \mu_{11}$

$b_8 = 35.4 - 10.7 - 12.9 + 12.4$

$b_8 = 24.2$