Worked Example of 3x3 – Effects codes

Example 3x3 for effects coding.

Full equation:

$$Y = b_0 + b_1E_1 + b_2E_2 + b_3E_3 + b_4E_4 + b_5E_1E_3 + b_6E_1E_4 + b_7E_2E_3 + b_8E_2E_4$$

Note the use of E here, and D in the dummy code example is arbitrary.

INTERCEPT: b_0

Grand mean.

Effect Variable Coding

Row Levels	Column Levels	Row		Column		Interactions Dummies			
		E_1 (b_1)	E_2 (b_2)	E_3 (b_3)	E_4 (b_4)	E_1E_3 (b_5)	E_1E_4 (b_6)	E_2E_3 (b_7)	E_2E_4 (b_8)
A	A	1	0	1	0	1	0	0	0
A	B	1	0	0	1	0	1	0	0
A	C	1	0	-1	-1	-1	-1	0	0
B	A	0	1	1	0	0	0	1	0
B	B	0	1	0	1	0	0	0	1
B	C	0	1	-1	-1	0	0	-1	-1
C	A	-1	-1	1	0	-1	0	-1	0
C	B	-1	-1	0	1	0	-1	0	-1
C	C	-1	-1	-1	-1	1	1	1	1

Classification of means with marginal means (rounding to 2dp)

	A	B	C	
A	12.4	12.7	12.9	**12.67**
B	11.5	11.1	13.4	**12.00**
C	10.7	16.1	35.4	**20.73**
	11.53	**13.30**	**20.57**	**15.13**

	A	B	C	
A	AA (1,1)	AB (1,2)	AC (1,2)	$\mu_{1.}$
B	BA (2,1)	BB (2,2)	BC (2,3)	$\mu_{2.}$
C	CA (3,1)	CB (3,2)	CC (3,3)	$\mu_{3.}$
	$\mu_{.1}$	$\mu_{.2}$	$\mu_{.3}$	$\mu_{..}$

Solved

$b_0 = \mu_{..}$

$b_0 = 15.13$

SIMPLE MAIN EFFECTS (ROW): b_1

Effect of being in level A of row factor. Average over columns (i.e. row marginal). Difference between row marginal and grand mean.

Effect Variable Coding

Row Levels	Column Levels	Row		Column		Interactions Dummies			
		E_1 (b_1)	E_2 (b_2)	E_3 (b_3)	E_4 (b_4)	E_1E_3 (b_5)	E_1E_4 (b_6)	E_2E_3 (b_7)	E_2E_4 (b_8)
A	A	1	0	1	0	1	0	0	0
A	B	1	0	0	1	0	1	0	0
A	C	1	0	-1	-1	-1	-1	0	0
B	A	0	1	1	0	0	0	1	0
B	B	0	1	0	1	0	0	0	1
B	C	0	1	-1	-1	0	0	-1	-1
C	A	-1	-1	1	0	-1	0	-1	0
C	B	-1	-1	0	1	0	-1	0	-1
C	C	-1	-1	-1	-1	1	1	1	1

Classification of means with marginal means (rounding to 2dp)

	A	B	C	
A	12.4	12.7	12.9	**12.67**
B	11.5	11.1	13.4	**12.00**
C	10.7	16.1	35.4	**20.73**
	11.53	**13.30**	**20.57**	**15.13**

	A	B	C	
A	AA (1,1)	AB (1,2)	AC (1,2)	$\mu_{1.}$
B	BA (2,1)	BB (2,2)	BC (2,3)	$\mu_{2.}$
C	CA (3,1)	CB (3,2)	CC (3,3)	$\mu_{3.}$
	$\mu_{.1}$	$\mu_{.2}$	$\mu_{.3}$	$\mu_{..}$

Solved

$b_1 = \mu_{1.} - \mu_{..}$

$b_1 = 12.67 - 15.13$

$b_1 = -2.46$

SIMPLE MAIN EFFECTS (ROW): b_2

Effect of being in level B of row factor. Average over columns (i.e. row marginal). Difference between row marginal and grand mean.

Effect Variable Coding

Row Levels	Column Levels	Row		Column		Interactions Dummies			
		E_1 (b_1)	E_2 (b_2)	E_3 (b_3)	E_4 (b_4)	E_1E_3 (b_5)	E_1E_4 (b_6)	E_2E_3 (b_7)	E_2E_4 (b_8)
A	A	1	0	1	0	1	0	0	0
A	B	1	0	0	1	0	1	0	0
A	C	1	0	-1	-1	-1	-1	0	0
B	A	0	1	1	0	0	0	1	0
B	B	0	1	0	1	0	0	0	1
B	C	0	1	-1	-1	0	0	-1	-1
C	A	-1	-1	1	0	-1	0	-1	0
C	B	-1	-1	0	1	0	-1	0	-1
C	C	-1	-1	-1	-1	1	1	1	1

Classification of means with marginal means (rounding to 2dp)

	A	B	C	
A	12.4	12.7	12.9	**12.67**
B	11.5	11.1	13.4	**12.00**
C	10.7	16.1	35.4	**20.73**
	11.53	**13.30**	**20.57**	**15.13**

	A	B	C	
A	AA (1,1)	AB (1,2)	AC (1,2)	$\mu_{1.}$
B	BA (2,1)	BB (2,2)	BC (2,3)	$\mu_{2.}$
C	CA (3,1)	CB (3,2)	CC (3,3)	$\mu_{3.}$
	$\mu_{.1}$	$\mu_{.2}$	$\mu_{.3}$	$\mu_{..}$

Solved

$b_2 = \mu_{2.} - \mu_{..}$

$b_2 = 12.00 - 15.13$

$b_2 = -3.13$

SIMPLE MAIN EFFECTS (COLUMN): b_3

Effect of being in level A of column factor. Average over rows (i.e. row marginal). Difference between column marginal and grand mean.

Effect Variable Coding

Row Levels	Column Levels	Row		Column		Interactions Dummies			
		E_1 (b_1)	E_2 (b_2)	E_3 (b_3)	E_4 (b_4)	E_1E_3 (b_5)	E_1E_4 (b_6)	E_2E_3 (b_7)	E_2E_4 (b_8)
A	A	1	0	1	0	1	0	0	0
A	B	1	0	0	1	0	1	0	0
A	C	1	0	-1	-1	-1	-1	0	0
B	A	0	1	1	0	0	0	1	0
B	B	0	1	0	1	0	0	0	1
B	C	0	1	-1	-1	0	0	-1	-1
C	A	-1	-1	1	0	-1	0	-1	0
C	B	-1	-1	0	1	0	-1	0	-1
C	C	-1	-1	-1	-1	1	1	1	1

Classification of means with marginal means (rounding to 2dp)

	A	B	C	
A	12.4	12.7	12.9	**12.67**
B	11.5	11.1	13.4	**12.00**
C	10.7	16.1	35.4	**20.73**
	11.53	**13.30**	**20.57**	**15.13**

	A	B	C	
A	AA (1,1)	AB (1,2)	AC (1,2)	$\mu_{1.}$
B	BA (2,1)	BB (2,2)	BC (2,3)	$\mu_{2.}$
C	CA (3,1)	CB (3,2)	CC (3,3)	$\mu_{3.}$
	$\mu_{.1}$	$\mu_{.2}$	$\mu_{.3}$	$\mu_{..}$

Solved

$b_3 = \mu_{.1} - \mu_{..}$

$b_3 = 11.53 - 15.13$

$b_3 = -3.6$

SIMPLE MAIN EFFECTS (COLUMN): b_4

Effect of being in level A of column factor. Average over rows (i.e. row marginal). Difference between column marginal and grand mean.

Effect Variable Coding

Row Levels	Column Levels	Row		Column		Interactions Dummies			
		E_1 (b_1)	E_2 (b_2)	E_3 (b_3)	E_4 (b_4)	E_1E_3 (b_5)	E_1E_4 (b_6)	E_2E_3 (b_7)	E_2E_4 (b_8)
A	A	1	0	1	0	1	0	0	0
A	B	1	0	0	1	0	1	0	0
A	C	1	0	-1	-1	-1	-1	0	0
B	A	0	1	1	0	0	0	1	0
B	B	0	1	0	1	0	0	0	1
B	C	0	1	-1	-1	0	0	-1	-1
C	A	-1	-1	1	0	-1	0	-1	0
C	B	-1	-1	0	1	0	-1	0	-1
C	C	-1	-1	-1	-1	1	1	1	1

Classification of means with marginal means (rounding to 2dp)

	A	B	C	
A	12.4	12.7	12.9	**12.67**
B	11.5	11.1	13.4	**12.00**
C	10.7	16.1	35.4	**20.73**
	11.53	**13.30**	**20.57**	**15.13**

	A	B	C	
A	AA (1,1)	AB (1,2)	AC (1,2)	$\mu_{1.}$
B	BA (2,1)	BB (2,2)	BC (2,3)	$\mu_{2.}$
C	CA (3,1)	CB (3,2)	CC (3,3)	$\mu_{3.}$
	$\mu_{.1}$	$\mu_{.2}$	$\mu_{.3}$	$\mu_{..}$

Solved

$b_4 = \mu_{.2} - \mu_{..}$

$b_4 = 13.30 - 15.13$

$b_4 = -1.83$

INTERACTIONS GENERAL:

From the quiz recall solving for beta for interactions:

$$(\mu_{ij} - \mu_{..}) - (\mu_{i.} - \mu_{..}) - (\mu_{.j} - \mu_{..})$$

Or alternatively (use both to see for yourself they get to the same answer):

$$\mu_{ij} - \mu_{i.} - \mu_{.j} + \mu_{..}$$

If you look back to calculating the interaction beta's in dummy coding, you will see the above has exactly the same form as that calculation, only concerning row and column marginal, and the grand mean.

INTERACTIONS: b_5

Testing whether the effect of being in row factor level A is the same/different dependent on levels of column factor level A (or vice versa – can be written either way).

Effect Variable Coding

Row Levels	Column Levels	Row		Column		Interactions Dummies			
		E_1 (b_1)	E_2 (b_2)	E_3 (b_3)	E_4 (b_4)	E_1E_3 (b_5)	E_1E_4 (b_6)	E_2E_3 (b_7)	E_2E_4 (b_8)
A	A	1	0	1	0	1	0	0	0
A	B	1	0	0	1	0	1	0	0
A	C	1	0	-1	-1	-1	-1	0	0
B	A	0	1	1	0	0	0	1	0
B	B	0	1	0	1	0	0	0	1
B	C	0	1	-1	-1	0	0	-1	-1
C	A	-1	-1	1	0	-1	0	-1	0
C	B	-1	-1	0	1	0	-1	0	-1
C	C	-1	-1	-1	-1	1	1	1	1

Classification of means with marginal means (rounding to 2dp)

	A	B	C	
A	12.4	12.7	12.9	**12.67**
B	11.5	11.1	13.4	**12.00**
C	10.7	16.1	35.4	**20.73**
	11.53	**13.30**	**20.57**	**15.13**

	A	B	C	
A	AA (1,1)	AB (1,2)	AC (1,2)	$\mu_{1.}$
B	BA (2,1)	BB (2,2)	BC (2,3)	$\mu_{2.}$
C	CA (3,1)	CB (3,2)	CC (3,3)	$\mu_{3.}$
	$\mu_{.1}$	$\mu_{.2}$	$\mu_{.3}$	$\mu_{..}$

Solved

$b_5 = (\mu_{11} - \mu_{..}) - (\mu_{1.} - \mu_{..}) - (\mu_{.1} - \mu_{..})$

$b_5 = \mu_{11} - \mu_{1.} - \mu_{.1} + \mu_{..}$

$b_5 = 12.4 - 12.67 - 11.53 + 15.13$

$b_5 = 3.33$

INTERACTIONS: b_6

Testing whether the effect of being in row factor level A is the same/different dependent on levels of column factor level B (or vice versa – can be written either way).

Effect Variable Coding

Row Levels	Column Levels	Row		Column		Interactions Dummies			
		E_1 (b_1)	E_2 (b_2)	E_3 (b_3)	E_4 (b_4)	E_1E_3 (b_5)	E_1E_4 (b_6)	E_2E_3 (b_7)	E_2E_4 (b_8)
A	A	1	0	1	0	1	0	0	0
A	B	1	0	0	1	0	1	0	0
A	C	1	0	-1	-1	-1	-1	0	0
B	A	0	1	1	0	0	0	1	0
B	B	0	1	0	1	0	0	0	1
B	C	0	1	-1	-1	0	0	-1	-1
C	A	-1	-1	1	0	-1	0	-1	0
C	B	-1	-1	0	1	0	-1	0	-1
C	C	-1	-1	-1	-1	1	1	1	1

Classification of means with marginal means (rounding to 2dp)

	A	B	C	
A	12.4	12.7	12.9	**12.67**
B	11.5	11.1	13.4	**12.00**
C	10.7	16.1	35.4	**20.73**
	11.53	**13.30**	**20.57**	**15.13**

	A	B	C	
A	AA (1,1)	AB (1,2)	AC (1,2)	$\mu_{1.}$
B	BA (2,1)	BB (2,2)	BC (2,3)	$\mu_{2.}$
C	CA (3,1)	CB (3,2)	CC (3,3)	$\mu_{3.}$
	$\mu_{.1}$	$\mu_{.2}$	$\mu_{.3}$	$\mu_{..}$

Solved

$$b_6 = (\mu_{12} - \mu_{..}) - (\mu_{1.} - \mu_{..}) - (\mu_{.2} - \mu_{..})$$

$$b_6 = \mu_{12} - \mu_{1.} - \mu_{.2} + \mu_{..}$$

$$b_6 = 12.7 - 12.67 - 13.30 + 15.13$$

$$b_6 = 1.86$$

INTERACTIONS: b_7

Testing whether the effect of being in row factor level B is the same/different dependent on levels of column factor level A (or vice versa – can be written either way).

Effect Variable Coding

Row Levels	Column Levels	Row		Column		Interactions Dummies			
		E_1 (b_1)	E_2 (b_2)	E_3 (b_3)	E_4 (b_4)	E_1E_3 (b_5)	E_1E_4 (b_6)	E_2E_3 (b_7)	E_2E_4 (b_8)
A	A	1	0	1	0	1	0	0	0
A	B	1	0	0	1	0	1	0	0
A	C	1	0	-1	-1	-1	-1	0	0
B	A	0	1	1	0	0	0	1	0
B	B	0	1	0	1	0	0	0	1
B	C	0	1	-1	-1	0	0	-1	-1
C	A	-1	-1	1	0	-1	0	-1	0
C	B	-1	-1	0	1	0	-1	0	-1
C	C	-1	-1	-1	-1	1	1	1	1

Classification of means with marginal means (rounding to 2dp)

	A	B	C	
A	12.4	12.7	12.9	**12.67**
B	11.5	11.1	13.4	**12.00**
C	10.7	16.1	35.4	**20.73**
	11.53	**13.30**	**20.57**	**15.13**

	A	B	C	
A	AA (1,1)	AB (1,2)	AC (1,2)	$\mu_{1.}$
B	BA (2,1)	BB (2,2)	BC (2,3)	$\mu_{2.}$
C	CA (3,1)	CB (3,2)	CC (3,3)	$\mu_{3.}$
	$\mu_{.1}$	$\mu_{.2}$	$\mu_{.3}$	$\mu_{..}$

Solved

$b_7 = (\mu_{21} - \mu_{..}) - (\mu_{2.} - \mu_{..}) - (\mu_{.1} - \mu_{..})$

$b_7 = \mu_{21} - \mu_{2.} - \mu_{.1} + \mu_{..}$

$b_7 = 11.50 - 12.00 - 11.53 + 15.13$

$b_7 = 3.1$

INTERACTIONS: b_8

Testing whether the effect of being in row factor level B is the same/different dependent on levels of column factor level B (or vice versa – can be written either way).

Effect Variable Coding

Row Levels	Column Levels	Row		Column		Interactions Dummies			
		E_1 (b_1)	E_2 (b_2)	E_3 (b_3)	E_4 (b_4)	E_1E_3 (b_5)	E_1E_4 (b_6)	E_2E_3 (b_7)	E_2E_4 (b_8)
A	A	1	0	1	0	1	0	0	0
A	B	1	0	0	1	0	1	0	0
A	C	1	0	-1	-1	-1	-1	0	0
B	A	0	1	1	0	0	0	1	0
B	B	0	1	0	1	0	0	0	1
B	C	0	1	-1	-1	0	0	-1	-1
C	A	-1	-1	1	0	-1	0	-1	0
C	B	-1	-1	0	1	0	-1	0	-1
C	C	-1	-1	-1	-1	1	1	1	1

Classification of means with marginal means (rounding to 2dp)

	A	B	C	
A	12.4	12.7	12.9	**12.67**
B	11.5	11.1	13.4	**12.00**
C	10.7	16.1	35.4	**20.73**
	11.53	**13.30**	**20.57**	**15.13**

	A	B	C	
A	AA (1,1)	AB (1,2)	AC (1,2)	$\mu_{1.}$
B	BA (2,1)	BB (2,2)	BC (2,3)	$\mu_{2.}$
C	CA (3,1)	CB (3,2)	CC (3,3)	$\mu_{3.}$
	$\mu_{.1}$	$\mu_{.2}$	$\mu_{.3}$	$\mu_{..}$

Solved

$$b_8 = (\mu_{22} - \mu_{..}) - (\mu_{2.} - \mu_{..}) - (\mu_{.2} - \mu_{..})$$

$$b_8 = \mu_{22} - \mu_{2.} - \mu_{.2} + \mu_{..}$$

$$b_8 = 11.10 - 12.00 - 13.30 + 15.13$$

$$b_8 = 0.93$$