Multivariate: Linear Mixed Models

Lecture 5: Power and GLMM

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Lecture 5

Good morning

Course Announcements

- Coursework set at the end of this week.
- Revision/Q&A sessions likely to be in LG.07 lab, DHT, WB 8th April
    - TBC

Today

- Example: Trial by person
- Generalized linear mixed models (GLMM)
- Examples: Logisitic
- Power in LMM
- In closing. . . remembering why LMM are useful!

Questions up until now...

Example: Trial by person (off we go to Notebooks)

Generalized linear mixed models (GLMM)

Overview

- Linear model requires our outcome variable to continuous.
  - Often it is not.
  - Sometimes we ignore this
  - Sometimes we can not.
- When we can not, enter the generalized linear model.

Principle of GLM

- In a standard linear model, we assume:
  - The outcome is normally distributed
  - The predictors to have a linear relation to the expected value of the outcome
- But the normal (Gaussian) distribution is one of a large number of distributions our outcome could come from.
- In a generalized linear model we add a step.
  - This step states that the linear relation of the predictors to the outcome follows a particular functional form, that links to the underlying distribution of the outcome.
  - Hence the term "link functions"

Some common link functions

Outcome	**Distribution**	**Link**
Continuous	Gaussian	Identity
Binary	Binomial	Logit or Probit
Count	Poisson	Log

GLMM

- In the context of LMM, we can apply the same steps as we would for a LM in order to fit GLM.
- In which case, we are fitting a **G**eneralized **L**inear **M**ixed **M**odel, or GLMM.
- To fit these models we use the `glmer()` function instead of `lmer()`

Example: Logistic model (off we go to Notebooks)

Power in LMM

Single level power

- 4 components comprise "standard" analytic power calculations:
  - Type I error rate
  - Sample size
  - Effect size
  - Power level
- If we know 3 elements, we can work out the final one.

Power for *t*-tests: Analytic (`pwr`)

```
pwr.t.test(n = 20, d = 0.5, sig.level = .05,
           type= "two.sample",
           alternative = "two.sided")
```

Power for *t*-tests: Analytic

```
##
##      Two-sample t test power calculation
##
##               n = 20
##               d = 0.5
##       sig.level = 0.05
##           power = 0.337939
##     alternative = two.sided
##
## NOTE: n is number in *each* group
```

Power for *t*-tests: Analytic

```
pwr.t.test(power = 0.95, d = 0.5, sig.level = .05,
           type= "two.sample", alternative = "two.sided")
```

Power for t-tests: Analytic

```
##
##      Two-sample t test power calculation
##
##               n = 104.9279
##               d = 0.5
##       sig.level = 0.05
##           power = 0.95
##     alternative = two.sided
##
## NOTE: n is number in *each* group
```

Power via simulation (`paramtest`)

- Based on choosing a hypothesised model and true parameter values.
- Then draws a large number of samples and estimates the model in each sample.
- Then the parameter values and standard errors are averaged over the samples.
- The relevant outcomes are parameter estimate bias, standard error bias, coverage and power.

Power for *t*-test: Simulation

```r
t_func <- function(simNum, N, d) {
    x1 <- rnorm(N, 0, 1)
    x2 <- rnorm(N, d, 1)

# runs t-tests on the simulated datasets
    t <- t.test(x1, x2, var.equal=TRUE)

# extracts t-values from the t-tests
    stat <- t$statistic

# extracts p-values from the t-tests
    p <- t$p.value

# returns a named vector with the results we want to keep
    return(c(t = stat, p = p, sig = (p < .05)))
}
```

Power for *t*-test: Simulation

```r
head(results(power_ttest <- run_test(
  t_func, n.iter = 1000, output = 'data.frame',
  N = 20, d = 0.5)))
```

```
## Running 1,000 tests...
```

```
##   iter        t.t           p sig
## 1    1 -2.7431742 0.009232470   1
## 2    2 -2.5346744 0.015495088   1
## 3    3 -0.8661068 0.391867338   0
## 4    4 -1.4839723 0.146064309   0
## 5    5 -2.0288295 0.049524830   1
## 6    6 -2.8443590 0.007128224   1
```

Power for *t*-test: Simulation

```r
table(power_ttest$results$sig)
```

```
##
##   0   1
## 654 346
```

Power for LMM

- As has generally been the case, power in the LMM context is more complex.
- The factors effecting power are similar:
  - Number of clusters (Level 2 sample size)
  - Cluster size (Level 1 sample size)
  - ICC (relation between level 1 and 2 variance)
  - Effect size

Sample Size

- Fixed effects and their SE:
  - When N of groups is below 50, begin to see some bias in the estimation of SE.
- Random effects and SE:
  - Somewhat dependent on estimator
  - REML provides better estimates with fewer groups than ML

Sample Size

- Rules of thumb are hard to set.
- Power is based on a combination of the N at different levels.
- Dependent on interest, various suggestions have been made for the N's at L2/L1
    - General rule - 30/30
    - Cross-level interactions - 50/20
    - Variance-covariance - 100/10

Sample Size

- Gellman & Hill: Group N $<$ 5, not enough information to estimate group-level variation.
  - Unlikely that MLM improves over no pooling models -However, same authors argue that even with as few as 2 observations per group we can use LMM.
  - Group level intercepts will be poorly estimated, but still contribute to the estimation of individual level effects.

Effect sizes

- We have already spoken about some of the difficulties with quantities such as R-square.
- We also spoke about PRV as a local measure of effect size.
- With standardized variables we can also consider the size of the coefficients we would expect to see.
- The issue with all of the above is that LMM are large, multivariable models, and so there are lots of parameters to consider plausible values for.

Practical approaches to power for LMM

- We could write a function to simulate data from an LMM and use `paramtest`
  - Lisa DeBruine @Glasgow Uni has done the first step for us.
  - https://debruine.github.io/posts/
- `SIMR` package
  - Arguably most flexible package.
  - Takes some getting used to!
- Anoter option is WebPower
  - https://webpower.psychstat.org/wiki/

Why LMM are useful!

The take homes

1. Single framework for modelling longitudinal, cross-sectional, experimental with a variety of dependent variable types.
2. Explicitly model variation at multiple levels of nested and crossed structure.
3. Include predictors to explain that variance.
4. These predictors can be continuous or categorical without the need for additional assumptions (e.g. ANCOVA)
5. Handles unbalanced data better than ANOVA.
6. Fewer and less restrictive assumptions than ANOVA.

That's all for THE COURSE! :-(