WEEK 2
Path Mediation

Data Analysis for Psychology in R 3

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Learning Objectives

1. Understand the purpose of mediation models and the conceptual challenges
2. Be able to describe direct, indirect and total effects in a mediation model.
3. Estimate and interpret a mediation model using `lavaan`

Part 1: Introduction to mediation

Part 2: Direct, indirect and total effects

Part 3: Estimating mediation in $lavaan$

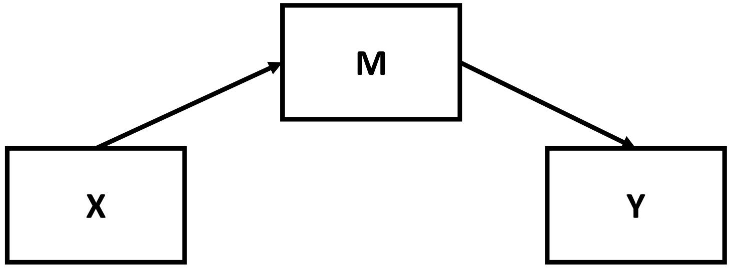
Part 4: Reporting

Mediation

- Is when a predictor X, has an effect on an outcome Y, via a mediating variable M
- The mediator **transmits** the effect of X to Y
- Examples of mediation hypotheses:
    - Conscientiousness (X) affects health (Y) via health behaviours (M)
    - Conduct problems (X) increase the risk of depression (Y) via peer problems (M)
    - Attitudes to smoking (X) predict intentions to smoke (M) which in turn predicts smoking behaviour (Y)
    - An intervention (X) to reduce youth crime (Y) works by increasing youth self-contol (M)

Visualising a mediation model

- In a SEM diagram we can represent mediation as:



Mediation...not to be confused with moderation

- Mediation is commonly confused with **moderation**
- Moderation is when a moderator z modifies the effect of X on Y
  - e.g., the effect of X on Y is stronger at higher levels of Z
  - Also known as an **interaction** between X and Z
- Examples of moderation could be:
  - An intervention (X) works better to reduce bullying (Y) at older ages (Z) of school pupil
  - The relation between stress (X) and depression (Y) is lower for those scoring higher on spirituality (Z)

End of Part 1

Part 1: Introduction to mediation

Part 2: Direct, indirect and total effects

Part 3: Estimating mediation in *lavaan*

Part 4: Reporting

Direct and indirect effects in mediation

- We seldom hypothesise that a mediator completely explains the relation between X and Y
- More commonly, we expect both **indirect effects** and **direct effects** of X on Y
    - The indirect effects of X on Y are those transmitted via the mediator
    - The direct effect of X on Y is the remaining effect of X on Y

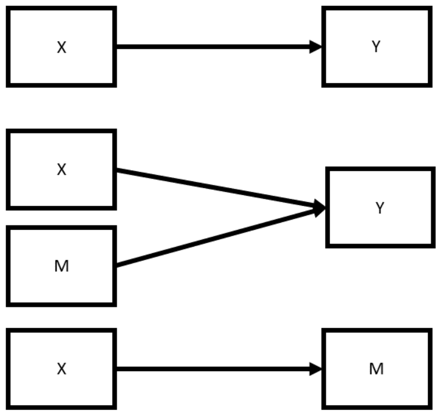
Visualing direct and indirect effects in mediation

Testing mediation

- Traditionally, mediation was tested using a series of separate linear models:

    1. Y~X
    2. Y~X+M
    3. M~X

- May see this referred to as th Baron and Kenny approach.



Traditional methods for mediation

- The three regression models:
  1. Y~X
  2. Y~X+M
  3. M~X

- Model 1 estimates the overall effect of X on Y
- Model 2 estimates the partial effects of X and M on Y

- Model 3 estimates the effect of X on M

- If the following conditions were met, mediation was assumed to hold:

  - The effect of X on Y (eq.1) is significant
  - The effect of X on M (eq.3) is significant
  - The effect of X on Y becomes reduced when M is added into the model (eq.2)

Limitations of traditional methods for mediation

- Low power
- Very cumbersome for multiple mediators, predictors, or outcomes
- You don't get an estimate of the magnitude of the indirect effect
- Much better way: **path mediation model**

BREAK QUIZ

- Quiz question:
    - Which of these hypotheses is a mediation hypothesis?
        - 1) Vocabulary development in childhood follows a non-linear trajecrtory
        - 2) The effects of conscientiousness on academic achievement are stronger at low levels of cognitive ability
        - 3) Poverty affects child behaviour problems through increasing parental stress
        - 4) Earlier pubertal onset increases the risk of antisocial behaviour only in girls and not boys

End of Part 2

Part 1: Introduction to mediation

Part 2: Direct, indirect and total effects

Part 3: Estimating mediation in *lavaan*

Part 4: Reporting

WELCOME BACK

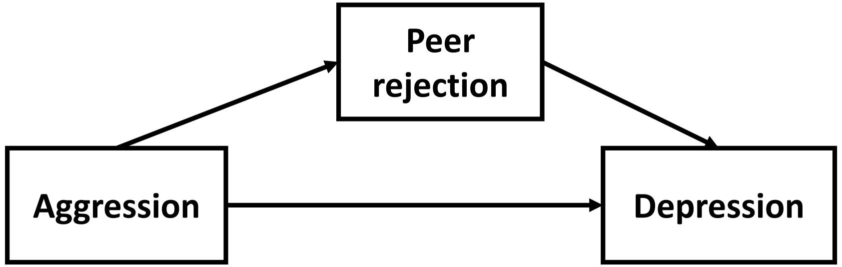
- Welcome back!
- The answer to the quiz question is...
  - Which of these hypotheses is a mediation hypothesis?
    - 1) Vocabulary development in childhood follows a non-linear trajecrtory
    - 2) The effects of conscientiousness on academic achievement are stronger at low levels of cognitive ability
    - 3) **Poverty affects child behaviour problems through increasing parental stress**
    - 4) Earlier pubertal onset increases the risk of antisocial behaviour only in girls and not boys

Testing a path mediation model in lavaan

- Specification
    - Create a lavaan syntax object
- Estimation
    - Estimate the model using e.g., maximum likelihood estimation
- Evaluation/interpretation
    - Inspect the model to judge how good it is
    - Interpret the parameter estimates

Example

- Does peer rejection mediate the association between aggression and depression?



The data

```
slice(agg.data2, 1:10)
```

```
##         Dep       PR      Agg
## 1   1.8730  0.93073  3.36176
## 2   0.5729  0.03075  0.87809
## 3   1.3726  0.54845 -0.23319
## 4  -0.0117  0.29221 -0.60339
## 5  -0.3733  0.65174 -0.05626
## 6  -0.5427 -0.02317 -0.73165
## 7   0.4146  0.64086  0.14327
## 8  -1.0071  0.18180 -0.25944
## 9   1.0511 -0.43144 -0.59311
## 10 -3.2104 -1.93771 -0.51254
```

Mediation Example

- Does peer rejection mediate the association between aggression and depression?

```
model1<-'Dep ~ PR       # Depression predicted by peer rejection
         Dep ~ Agg      # Depression predicted by aggression (the direct effect)
         PR ~ Agg       # Peer rejection predicted by aggression
'
```

- Estimate the model

```
model1.est<-sem(model1, data=agg.data2)
```

The model output

```r
summary(model1.est, fit.measures=T)
```

```
## lavaan 0.6-12 ended normally after 1 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                         5
##
##   Number of observations                           500
##
## Model Test User Model:
##
##   Test statistic                                 0.000
##   Degrees of freedom                                 0
##
## Model Test Baseline Model:
##
##   Test statistic                               219.872
##   Degrees of freedom                                 3
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    1.000
##   Tucker-Lewis Index (TLI)                       1.000
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -1294.513
##   Loglikelihood unrestricted model (H1)      -1294.513
```

Things to note from the model output

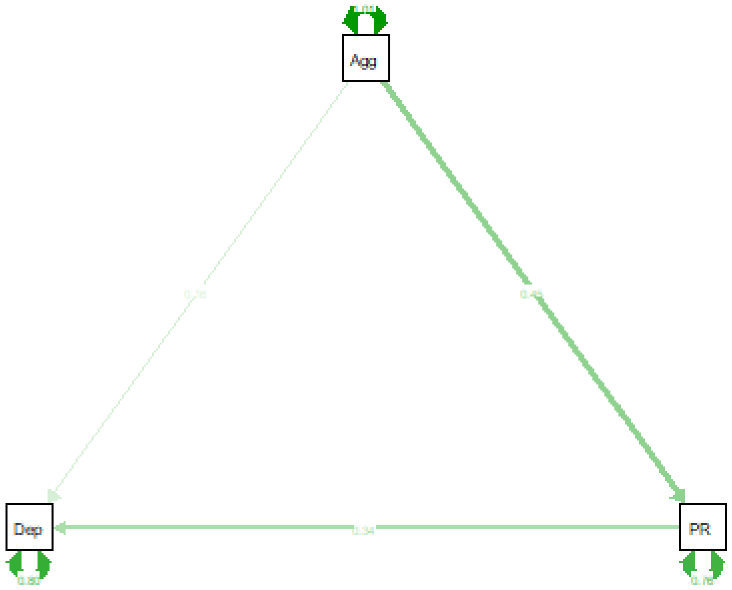
- All three regressions paths are statistically significant
- The model is **just-identified**
    - The degrees of freedom are equal to 0
    - The model fit cannot be tested
    - The model fit statistics (TLI, CFI, RMSEA, SRMR) all suggest perfect fit but this is meaningless

Visualising the model using `semPaths()`

- We can use semPaths() from the semPlot package to help us visualise the model
    - Shows the parameter estimates within an SEM diagram

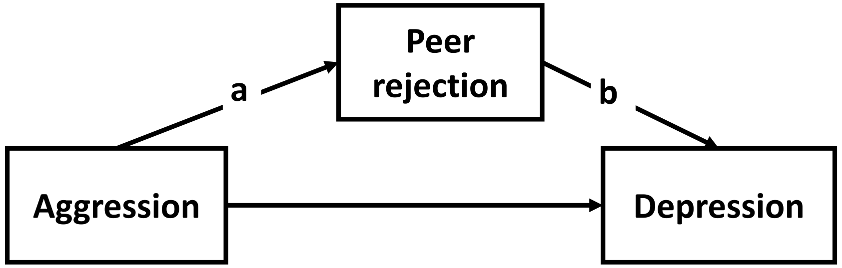
```
library(semPlot)
semPaths(model1.est, what='est')
```

Visualising the model using `semPaths()`



Calculating the indirect effects

- To calculate the indirect effect of X on Y in path mediation, we need to create some new parameters
- The indirect effect of X on Y via M is:
    - $a * b$
    - a = the regression coefficient for M~X
    - b = the regression coefficient for Y~M



Calculating indirect effects in lavaan

- To calculate the indirect effect of X on Y in lavaan we:
- Use parameter labels 'a' and 'b' to label the relevant paths
    - a is for the effect of X on M
    - b is for the effect of M on Y
- Use the ':=' operator to create a new parameter 'ind'
    - 'ind' represents our indirect effect

```
model1<-'Dep~b*PR
         Dep~Agg
         PR~a*Agg
         ind:=a*b
         '
```

Indirect effects in the output

```
model1.est<-sem(model1, data=agg.data2)
summary(model1.est)
```

```
## lavaan 0.6-12 ended normally after 1 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                         5
##
##   Number of observations                           500
##
## Model Test User Model:
##
##   Test statistic                                 0.000
##   Degrees of freedom                                 0
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   Dep ~
##     PR          (b)    0.336    0.046    7.308    0.000
##     Agg                0.159    0.045    3.560    0.000
##   PR ~
##     Agg         (a)    0.446    0.039   11.505    0.000
##
```

Statistical significance of the indirect effects

- Default method of assessing the statistical significance of indirect effects assume normal sampling distribution
- May not hold for indirect effects which are the product of regression coefficients
- Instead we can use **bootstrapping**
    - Allows 95% confidence intervals (CIs) to be computed
    - If 95% CI includes 0, the indirect effect is not significant at alpha=.05

Bootstapped CIs for indirect effect in lavaan

```
  model1<-'Dep~b*PR
           Dep~Agg
           PR~a*Agg
ind:=a*b'

model1.est<-sem(model1, data=agg.data2, se='bootstrap') #we add the argument se='bootstrap'
```

Output for bootstrapped CIs

```r
summary(model1.est, ci=T) # we add the argument ci=T to see the confidence intervals in the output
```

```
## lavaan 0.6-12 ended normally after 1 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                         5
##
##   Number of observations                           500
##
## Model Test User Model:
##
##   Test statistic                                 0.000
##   Degrees of freedom                                 0
##
## Parameter Estimates:
##
##   Standard errors                            Bootstrap
##   Number of requested bootstrap draws             1000
##   Number of successful bootstrap draws            1000
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|) ci.lower ci.upper
##   Dep ~
##     PR         (b)    0.336    0.046    7.366    0.000    0.247    0.424
##     Agg               0.159    0.050    3.160    0.002    0.063    0.262
##   PR ~
##     Agg        (a)    0.446    0.034   13.122    0.000    0.379    0.510
##
## Variances:
```

Total effects in path mediation

- As well as the direct and indirect effect, it is often of interest to know the **total** effect of X on Y

$$Total = Indirect + Direct$$

Total effects in path mediation

$$Total = a * b + c$$

Total effect in lavaan

```
  model1<-'Dep~b*PR
           Dep~c*Agg          # we add the label c for our direct effect
           PR~a*Agg
ind:=a*b
total:=a*b+c                  # we add a new parameter for the total effect'

model1.est<-sem(model1, data=agg.data2, se='bootstrap') #we add the argument se='bootstrap'
```

Total effect in lavaan output

```r
summary(model1.est, ci=T)
```

```
## lavaan 0.6-12 ended normally after 1 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                         5
##
##   Number of observations                           500
##
## Model Test User Model:
##
##   Test statistic                                 0.000
##   Degrees of freedom                                 0
##
## Parameter Estimates:
##
##   Standard errors                            Bootstrap
##   Number of requested bootstrap draws             1000
##   Number of successful bootstrap draws            1000
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|) ci.lower ci.upper
##   Dep ~
##     PR         (b)    0.336    0.045    7.471    0.000    0.246    0.422
##     Agg        (c)    0.159    0.047    3.361    0.001    0.069    0.251
##   PR ~
##     Agg        (a)    0.446    0.034   13.127    0.000    0.378    0.513
##
## Variances:
```

Why code the total effect in lavaan?

- We could have just added up the coefficients for the direct and indirect effects
- By coding it in lavaan, however, we can assess the statistical significance of the total effect
- Useful because sometimes the direct and indirect effects are not individually significant but the total effect is
    - May be especially relevant in cases where there are many mediators of small effect

Interpreting the total, direct, and indirect effect coefficients

- The total effect can be interpreted as the **unit increase in Y expected to occur when X increases by one unit**
- The indirect effect can be interpreted as the **unit increase in Y expected to occur via M when X increases by one unit**
- The direct effect can be interpreted as the **unit increase in Y expected to occur with a unit increase in X over and above the increase transmitted by M**
- **Note**: 'direct' effect may not actually be direct - it may be acting via other mediators not included in our model

Standardised parameters

- As with CFA models, standardised parameters can be obtained using:

```
summary(model1.est, ci=T, std=T)
```

Standardised parameters

```
## lavaan 0.6-12 ended normally after 1 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                         5
##
##   Number of observations                           500
##
## Model Test User Model:
##
##   Test statistic                                 0.000
##   Degrees of freedom                                 0
##
## Parameter Estimates:
##
##   Standard errors                            Bootstrap
##   Number of requested bootstrap draws             1000
##   Number of successful bootstrap draws            1000
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|) ci.lower ci.upper
##   Dep ~
##     PR          (b)    0.336    0.045    7.471    0.000    0.246    0.422
##     Agg         (c)    0.159    0.047    3.361    0.001    0.069    0.251
##   PR ~
##     Agg         (a)    0.446    0.034   13.127    0.000    0.378    0.513
##    Std.lv  Std.all
##
##     0.336    0.332
##     0.159    0.162
##
```

BREAK QUIZ

- Time for a pause
- Quiz question
    - If the effect of X on M is b=.30 and the effect of M on Y is b=.10, what is the indirect effect of X on Y?
        - 1) b=.40
        - 2) b=.03
        - 3) b=.30
        - 4) b=.10

End of Part 3

Part 1: Introduction to mediation

Part 2: Direct, indirect and total effects

Part 3: Estimating mediation in *lavaan*

Part 4: Reporting

Welcome back

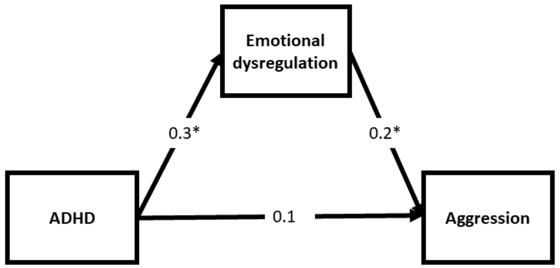
- The answer to the quiz question is...
- Quiz question
    - If the effect of X on M is b=.30 and the effect of M on Y is b=.10, what is the indirect effect of X on Y?
        - 1) b=.40
        - 2) **b=.03**
        - 3) b=.30
        - 4) b=.10

Reporting path mediation models

- Methods/ Analysis Strategy
    - The model being tested
    - e.g. 'Y was regressed on both X and M and M was regressed on X'
    - The estimator used (e.g., maximum likelihood estimation)
    - The method used to test the significance of indirect effects ('bootstrapped 95% confidence intervals')
- Results
    - Model fit (for over-identified models)
    - The parameter estimates for the path mediation and their statistical significance
    - Can be useful to present these in a SEM diagram
    - The diagrams from R not considered 'publication quality' draw in powerpoint or similar

Reporting path mediation models - example of SEM diagram with results

- Include the key parameter estimates
- Indicate statistically significant paths (e.g. with an '*')
- Include a figure note that explains how statistically significant paths (and at what level) are signified



Reporting path mediation models - the indirect effects

- Results
  - The coefficient for the indirect effect and the bootstrapped 95% confidence intervals
  - Common to also report **proportion mediation**:

$$\frac{indirect}{total}$$

- However, important to be aware of limitations:
  - Big proportion mediation possible when total effect is small - makes effect seem more impressive
  - Small proportion mediation even when total effect is big - can underplay importance of effect
  - Should be interpreted in context of total effect
- Tricky interpretation if there are a mix of negative and positive effects involved

Extensions of path mediation models

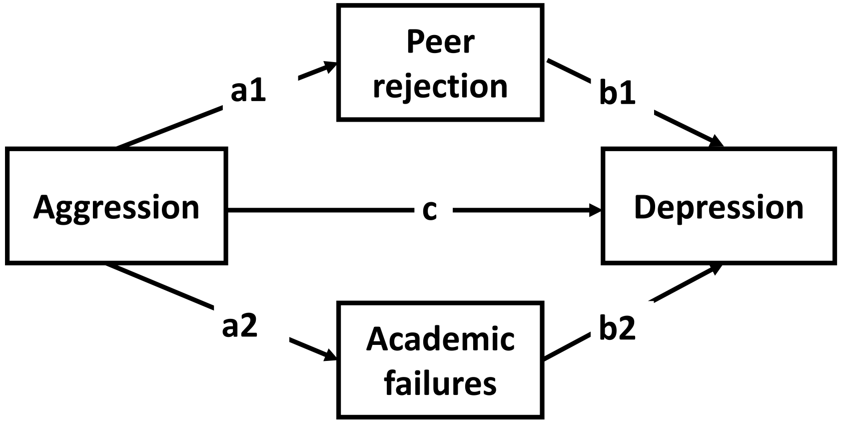
- We can extend our path mediation model in various ways:
  - Several mediators in sequence or parallel
  - Multiple outcomes
  - Multiple predictors
  - Multiple groups (e.g., comparing direct and indirect effects across males and females)
  - Add covariates to adjust for potential confounders

Example: Multiple mediation model

```
model2<-'Dep~b2*Aca
         Aca~a2*Agg
         Dep~b1*PR
         PR~a1*Agg
         Dep~c*Agg

        ind1:=a1*b1
        ind2:=a2*b2

        total=a1*b1+a2*b2+c
'
```



Other path analysis models

- Path mediation models are a common application of path models
    - But they are just one example
- Anything that can be expressed in terms of regressions between observed variables can be tested as a path model
    - Can include ordinal or binary variables
    - Can include moderation
- Other common path analysis models include:
    - Autoregressive models for longitudinal data
    - Cross-lagged panel models for longitudinal data

Making model modifications

- You may want to make some modifications to your initially hypothesised model
    - non-significant paths that you want to trim
    - include some additional paths not initially included
- Remember that this now moves us into exploratory territory where:
    - Model modifications should be substantively as well as statistically justifiable
    - You must be aware of the possibility that you are capitalising on chance
    - You should aim to replicate the modifications in independent data

Cautions regarding path analysis models

- Assumption that the paths represent causal effects is only an assumption
  - Especially if using cross-sectional data
  - Mediation models should ideally be estimated on longitudinal data.
  - X time 1
  - M time 2
  - Y time 3
- The parameters are only accurate if the model is correctly specified

Cautions: Indistinguishable models

Measurement error in path analysis

- Path analysis models use observed variables
    - Assumes no measurement error in these variables
- Path coefficients likely to be attenuated due to unmodelled measurement error
- Structural equation models solve this issue
    - They are path analysis models where the paths are between latent rather than observed variables
    - ...very brief comment on this in the final week

Path analysis summary

- Path analysis can be used to fit sets of regression models
    - Common path analysis model is the path mediation model
    - But very flexible huge range of models that can be tested
- In R, path analysis can be done using the `sem()` function in `lavaan`
- Need to be aware that we aren't *testing* causality but assuming it

End