

Recurrence relations Lab 6
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Recurrence relation: insertion sort

Base Case: $P(1)$ = the time to sort an array containing 1 element == constant:

$$P(1) = O(1) = 1$$

$$P(N) = P(N-1) + N-1$$

$$P(N-1) = P(N-2) + N-2$$

$$P(N-2) = P(N-3) + N-3$$

$$P(2) = P(1) + 1$$

$$P(N) + P(N-1) + P(N-2) + P(N-3) + \dots + P(3) + P(2) =$$

$$P(N-1) + P(N-2) + P(N-3) + \dots + P(3) + P(2) + P(1) + (N-1) + (N-2) + (N-3) + \dots + 3 + 2 + 1$$

$$P(N) = P(1) + N*(N-1)/2 + 1 + N*(N-1)/2$$

$$P(N) = 1 + N*(N-1)/2$$

$$\text{Therefore } P(N) = O(1 + N*(N-1)/2) = O(N^2)$$

Recurrence relation: quick sort

$$P(N) = P(N-1) + cN, \text{ where } N \geq 2$$

$$P(N-1) = P(N-2) + c(N-1)$$

$$P(N-2) = P(N-3) + c(N-2)$$

$$P(N-3) = P(N-4) + c(N-3)$$

$$P(2) = P(1) + c*2$$

$$P(N) + P(N-1) + P(N-2) + \dots + P(2) =$$

$$P(N-1) + P(N-2) + \dots + P(2) + P(1) + c(N) + c(N-1) + c(N-2) =$$

$$P(N) = P(1) + c(2 + 3 + \dots + N) =$$

$$P(N) = 1 + c(N(N+1)/2 - 1) =$$

$$P(N) = O(N^2)$$

Recurrence relation: merge sort

$$P(N) = 2*P(N/2) + N =$$

$$2*(2P(N/4) + N/2) + N =$$

$$4P(N/4) + N + N =$$

$$4(2P(N/8) + N/4) + N + N =$$

$$8P(N/8) + N + N + N =$$

$$N*P(N/N) + N + \dots + N + N + N =$$

$$N + N + \dots + N + N + N =$$

$$P(N) = O(N \lg N)$$