1. $L[\phi] = \sum\limits_{i=1}^{I} (\phi_0 + \phi_1 x_i - y_i)^2$

$\dfrac{\partial L}{\partial \phi_0} = \sum\limits_{i=1}^{I} 2(\phi_0 + \phi_1 x_i - y_i)$

$\dfrac{\partial L}{\partial \phi_1} = \sum\limits_{i=1}^{I} 2 x_i (\phi_0 + \phi_1 x_i - y_i)$

2. $\dfrac{\partial L}{\partial \phi_0} = 0 \rightarrow \sum\limits_{i=1}^{I} \phi_0 + \phi_1 x_i - y_i = 0$

$I\phi_0 + \phi_1 \underbrace{\sum\limits_{i=1}^{I} x_i}_{S_x} - \underbrace{\sum\limits_{i=1}^{I} y_i}_{S_y} = 0$

$\phi_0 = \dfrac{S_y - S_x \phi_1}{I}$

$\dfrac{\partial L}{\partial \phi_1} = 0 \rightarrow \phi_0 \sum\limits_{i=1}^{I} x_i + \phi_1 \underbrace{\sum\limits_{i=1}^{I} x_i^2}_{S_{xx}} - \underbrace{\sum\limits_{i=1}^{I} x_i y_i}_{S_{xy}}$

$\phi_0 S_x + \phi_1 S_{xx} - S_{xy} = 0$

$\dfrac{S_y - S_x \phi_1}{I} S_x + \phi_1 S_{xx} - S_{xy} = 0$

$\dfrac{S_y S_x}{I} + \phi_1 \left(S_{xx} - \dfrac{S_x^2}{I}\right) - S_{xy} = 0$

$\therefore \phi_1 = \dfrac{S_{xy} - S_y S_x / I}{S_{xx} - S_x^2 / I} = \dfrac{I S_{xy} - S_y S_x}{I S_{xx} - S_x^2}$

$\therefore \phi_0 = (S_y S_{xx} - S_x S_{xy}) / I(I S_{xx} - S_x^2)$

3. Generative. $x = g(y, \phi) = \phi_0 + \phi_1 y$

$$L(\phi) = \sum_{i=1}^{I} (x_i - g(y_i, \phi))^2$$

$$= \sum_{i=1}^{I} (x_i - (\phi_0 + \phi_1 y))^2 \quad \text{(cost function)}$$

$$y = g^{-1}(x_1, \phi)$$

$$x = \phi_0 + \phi_1 y \implies y = \frac{x - \phi_0}{\phi_1}$$

Generative $\quad y = \dfrac{x}{\phi_1} - \dfrac{\phi_0}{\phi_1}$

Discriminative $\quad y = \phi_1 x + \phi_0$

- These are not the same.
- The slopes & y-intercepts are not the same.