

1.

$$\begin{aligned}
 L[\phi] &= \sum_{i=1}^I \ell_i = \sum_{i=1}^I (f[x_i, \phi] - y_i)^2 \\
 &= \sum_{i=1}^I (\phi_0 + \phi_1 x_i - y_i)^2,
 \end{aligned} \tag{6.5}$$

$$\ell_i = (\phi_0 + \phi_1 x_i - y_i)^2$$

$$\frac{\partial \ell_i}{\partial \phi_1} = 2 x_i (\phi_0 + \phi_1 x_i - y_i)$$

$$\frac{\partial \ell_i}{\partial \phi_0} = 2 (\phi_0 + \phi_1 x_i - y_i)$$

$$\therefore \frac{\partial \ell_i}{\partial \phi} = \begin{pmatrix} \frac{\partial \ell_i}{\partial \phi_0} \\ \frac{\partial \ell_i}{\partial \phi_1} \end{pmatrix} = \begin{pmatrix} 2(\phi_0 + \phi_1 x_i - y_i) \\ 2x_i(\phi_0 + \phi_1 x_i - y_i) \end{pmatrix}$$

3.

$$f[x, \phi] = \sin[\phi_0 + 0.06 \cdot \phi_1 x] \cdot \exp\left(-\frac{(\phi_0 + 0.06 \cdot \phi_1 x)^2}{32.0}\right). \tag{6.8}$$

$$L[\phi] = \sum_i \ell_i = \sum_i (f(x_i, \phi) - y_i)^2$$

$$\ell_i = \left(\sin(\phi_0 + 0.06 \cdot \phi_1 x) e^{-\frac{(\phi_0 + 0.06 \phi_1 x)^2}{32}} - y_i \right)^2$$

$$\frac{\partial \ell_i}{\partial \phi_0} = 2 \left(\sin(\phi_0 + 0.06 \phi_1 x_i) e^{-\frac{1}{32}(\phi_0 + 0.06 \phi_1 x_i)^2} - \frac{1}{16}(\phi_0 + 0.06 \phi_1 x_i) e^{-\frac{1}{32}(\phi_0 + 0.06 \phi_1 x_i)^2} \sin(\phi_0 + 0.06 \phi_1 x_i) \right)$$

$$\left(\sin(\phi_0 + 0.06 \phi_1 x_i) e^{-\frac{(\phi_0 + 0.06 \phi_1 x_i)^2}{32}} - y_i \right) \cdot \frac{1}{16}(\phi_0 + 0.06 \phi_1 x_i) e^{-\frac{1}{32}(\phi_0 + 0.06 \phi_1 x_i)^2}$$

$$\frac{\partial \ell_i}{\partial \phi_1} = 2 \left(0.06 x_i \sin(\phi_0 + 0.06 \phi_1 x_i) e^{-\frac{1}{32}(\phi_0 + 0.06 \phi_1 x_i)^2} - \frac{0.06}{16} x_i \frac{1}{e^{32}} (\phi_0 + 0.06 \phi_1 x_i)^2 \sin(\phi_0 + 0.06 \phi_1 x_i) \right) \cdot \left(\sin(\phi_0 + 0.06 \phi_1 x_i) e^{-\frac{1}{32}(\phi_0 + 0.06 \phi_1 x_i)^2} - y_i \right)$$