$$\langle \mathbf{Ax}, \mathbf{x} \rangle = \langle \mathbf{x}, \mathbf{Ax} \rangle = \mathbf{x}^T \mathbf{Ax} =$$

$$= \begin{pmatrix} x_1 & x_2 & ... & x_n \end{pmatrix} \begin{pmatrix} a_{11}x_1 & a_{12}x_2 & \cdots & a_{1n}x_n \\ a_{21}x_1 & a_{22}x_2 & \cdots & a_{2n}x_n \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}x_1 & a_{n2}x_2 & \cdots & a_{nn}x_n \end{pmatrix} =$$

$$= x_1(a_{11}x_1 + \cdots + a_{1n}x_n) + x_2(a_{21}x_1 + \cdots + a_{2n}x_n) + \cdots + x_n(a_{n1}x_1 + \cdots + a_{nn}x_n) =$$

$$= \sum_{j=1}^{n} (x_j \sum_{k=1}^{n} a_{jk}x_k)$$

Atvasinām šo izteiksi pa saskaitāmajiem pēc x_i

$$\frac{\partial \langle \mathbf{Ax}, \mathbf{x} \rangle}{\partial x_i} =$$

$$= x_1 a_{1i} + x_2 a_{2i} + \cdots + x_{i-1} a_{(i-1)i} +$$
$$+ x_i'(a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n) + x_i(a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n)' +$$
$$+ x_{i+1} a_{(i+1)i} + \cdots + x_n a_{ni} =$$

$$= x_1 a_{1i} + x_2 a_{2i} + \cdots + x_{i-1} a_{(i-1)i} +$$
$$+ (a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n) + x_i a_{ii} +$$
$$+ x_{i+1} a_{(i+1)i} + \cdots + x_n a_{ni} =$$

$$= x_1 a_{1i} + x_2 a_{2i} + \cdots + x_{i-1} a_{(i-1)i} + x_i a_{ii} + x_{i+1} a_{(i+1)i} + \cdots + x_n a_{ni} +$$
$$+ (a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n) =$$

$$= \sum_{j=1}^{n} a_{ji}x_j + \sum_{k=1}^{n} a_{ik}x_k =$$

$$= \begin{pmatrix} a_{1i} & a_{2i} & \cdots & a_{ni} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} =$$

$$= \left(\begin{pmatrix} a_{1i} & a_{2i} & \cdots & a_{ni} \end{pmatrix} + \begin{pmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \end{pmatrix} \right) \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} =$$

$$= (\mathbf{A}_i^T + \mathbf{A}_i)\mathbf{x}$$