Understanding Derivatives

Applied Calculus Final Portfolio

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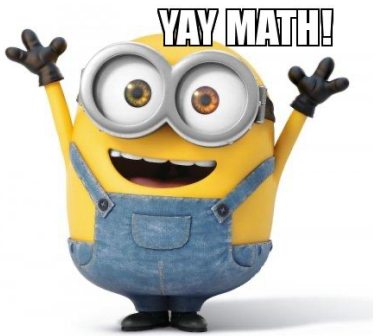

What is a Derivative?

- - A derivative measures how a function changes at a single point.
- - It’s like finding the slope of a curve at that exact point.
- Notation: f'(x) or dy/dx
- - Derivatives show rates of change, like speed or growth.

Key Derivative Rules

Derivative Rules: Building Blocks

In what follows, f and g are differentiable functions of x.

(a) **Constant Multiple Rule:** $\frac{d}{dx}(kf) = kf'$

(b) **Sum (or Difference) Rule:** $\frac{d}{dx}(f+g) = f'+g'$ **(or** $\frac{d}{dx}(f-g) = f'-g'$**)**

(c) **Power Rule:** $\frac{d}{dx}(x^n) = nx^{n-1}$

Special cases: $\frac{d}{dx}(k) = 0$ (because $k = kx^0$)

$\frac{d}{dx}(x) = 1$ (because $x = x^1$)

(d) **Exponential Functions:** $\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(a^x) = \ln a \cdot a^x$

(e) **Natural Logarithm:** $\frac{d}{dx}(\ln x) = \frac{1}{x}$

Example 1 – Power Rule

- Problem:
- y = 8x^{1/2}

- Solution:
- Bring the exponent down and subtract 1:
- y' = 8 * (1/2) x^{-1/2}
- y' = 4 / √(x)

- This one shows how to handle fractional exponents.

Example 2 – Polynomial Derivative

- Problem:
- f(x) = 17x^{10} + 13x^{8} - 1.8x + 1003

- Solution:
- Apply the power rule to each term:
- f'(x) = 17*10 x^9 + 13*8 x^7 - 1.8 + 0
- f'(x) = 170x^9 + 104x^7 - 1.8

- This is a good example of how the derivative of a constant is always zero.

Example 3 – Roots, Negative Exponents, and Exponentials

- Problem:
- f(x) = 3√(x) - 4/x^3 + 5e^x

- Rewrite:
- f(x) = 3x^{1/2} - 4x^{-3} + 5e^x

- Solution:
- Differentiate term by term:
- f'(x) = (3/2)x^{-1/2} + 12x^{-4} + 5e^x
- f'(x) = 3/(2sqrt(x)) + 12/x^4 + 5e^x

- This example shows how to handle square roots and negative exponents.

Example 4 – Exponentials and Logs

- Problem:
- y = 3e^x - 2 ln x

- Solution:
- Derivative of e^x is itself.
- Derivative of ln x is 1/x.
- y' = 3e^x - 2/x

- This combines two important derivative rules.

Putting it All Together

- We break down the function and find the derivative of each part.
- The rules we just practiced are the main tools we use.
- Understanding these rules makes derivatives much easier!

Practice Problems – Let's Try Together!

1) $f(x)=5/x^{2} + 3\sqrt{x}$

2) $f(x)=7e^{x} - 4x^{3}$

3) $f(x)=2/x + \ln|x|$

Solutions – Let's Check!

- 1) Rewrite:
- f(x) = 5x^{-2} + 3x^{1/2}
- Derivative:
- f'(x) = -10x^{-3} + (3/2)x^{-1/2} = -10/x^3 + 3/(2√(x))

- 2) Derivative:
- f'(x) = 7e^x - 12x^2

- 3) Rewrite:
- f(x) = 2x^{-1} + ln x
- Derivative:
- f'(x) = -2x^{-2} + 1/x = -2/x^2 + 1/x

- These were a bit trickier, but great practice!

Thank You!

- Thanks so much for listening to my presentation! I hope these examples made derivatives feel a bit more clear and approachable. Let me know if you have any questions!