REGRESSION WITH A SINGLE FEATURE

The Business Problem

- CEO of a restaurant chain
- Thinking about how to expand the business
- Have data on current restaurants
    - population of town in which restaurant is located
    - profit from restaurant per quarter
- Where should the next 5 restaurants be opened to maximize profit growth?

Regression w/ Single Feature ②

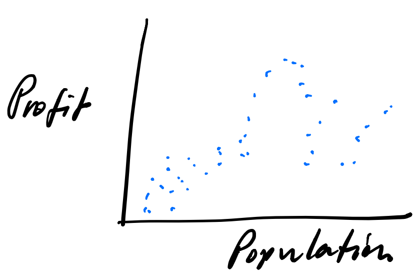
The Data

Population (10,000)	Profit ($100,000)
6.11	17.59
5.53	9.13
8.52	13.67
⋮	⋮
12.68	15.34

Structure of the data

- food-truck-profits.txt
- 97 rows
- numerical information (not categorical)
- No missing information

Step 1



Scatterplot of the dataset to visualize it

Regression w/ Single Feature 3

STEP 2 Define the task
(w.r.t. the dataset)

Input $\rightarrow$ the population size

Output $\rightarrow$ the profit

Task $\rightarrow$ Given a population size,
predict the profit.

STEP 3 Define the Model

Feature = input = population size

(later we'll see how features can differ from inputs)

Transform the feature into an output

$$(w_0 \times x_0^{(1)}) + (w_1 \times x_1^{(1)}) = \hat{y}^{(1)}$$

Parameter 1 — Constant feature — Parameter 2 — Feature 1 — Predicted output — Actual output

w_0	x_0	w_1	x_1	$\hat{y}$	y
w_0	1	w_1	$x_1^{(1)}$	$\hat{y}^{(1)}$	$y^{(1)}$
w_0	1	w_1	$x_1^{(2)}$	$\hat{y}^{(2)}$	$y^{(2)}$
⋮	⋮	⋮	⋮	⋮	⋮
w_0	1	w_1	$x_1^{(97)}$	$\hat{y}^{(97)}$	$y^{(97)}$

m rows

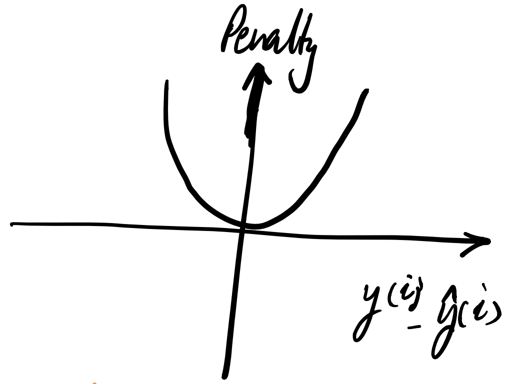
$m = 97$

These values don't change (w_0, 1, w_1)

These values change from row to row (x_1, $\hat{y}$, y)

$\boxed{\text{STEP 4}}$ Define the penalty for getting it wrong.

Penalty for each row $= \left(y^{(i)} - \hat{y}^{(i)}\right)^2$



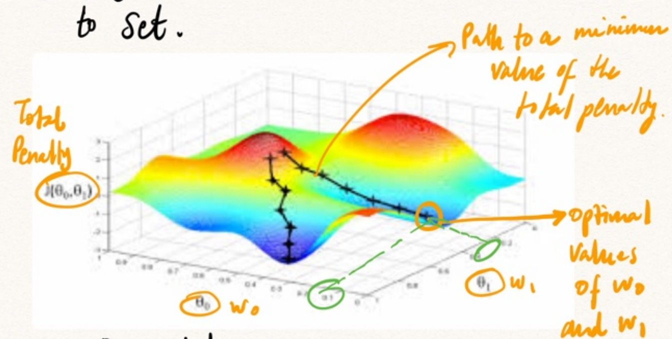
The more we get it wrong, the more we're penalized.

Total Penalty for the entire data set = Sum of penalty of each row.

= cost

$\boxed{\text{QUESTION}}$ What values of w_0 and w_1 will minimize the total penalty?

Imagine we only had w_0 and w_1 to set.



$\theta_0 = w_0$

$\theta_1 = w_1$

NOTE The shape of this surface depends on the penalty function we choose.

Use gradient descent to solve the numerical optimization problem.

- Start at an arbitrary point on the surface – pick arbitrary values for w_0 and w_1
- From this point, take a step in the direction where the slope is descending most steeply.
- Do this again and again until you reach a minimum (the "valley floor")

Recap

Model – transforms the input(s) into the output.

Parameters – these are the fixed values (e.g., w_0, w_1) of the model.

Penalty – For each row of the dataset, what's the penalty/cost/price for being wrong?

Recap (Contd.)

Total Penalty – Sum of the penalty of each row of the dataset.

Problem – Find the parameter values of the model (w_0 and w_1 values) that **minimize** the total penalty.

Approach to Solving the Problem – Gradient Descent.

Hyper-Parameters

Gradient descent can produce different results based on how you set the values of the following parameters.

- Learning Rate. The size of the jump taken at each step.
- The total number of jumps taken.

Every machine learning problem has hyperparameters.

Why? Because every machine learning problem is a giant optimization problem – find the optimal values of the model parameters that minimizes the total penalty (aka cost).

To find these optimal values of w_0 and w_1 (for example), we use a learning algorithm like gradient descent.

Finding the right hyperparameters is a separate quest – one that must be solved every time you apply machine learning to solve a problem.

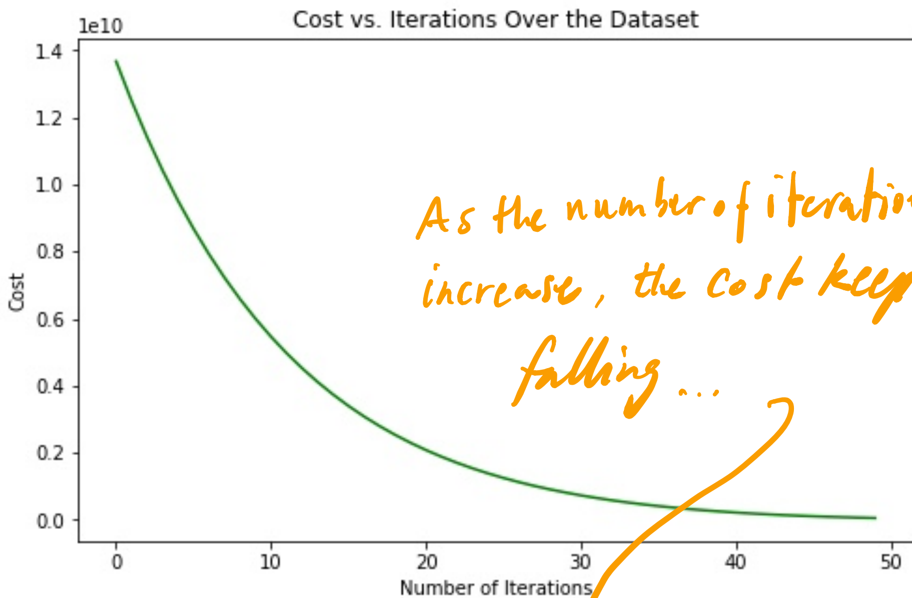
In a few sessions we'll see how this problem is tackled systematically.

For now, let's explore the 2 main hyperparameters of gradient descent

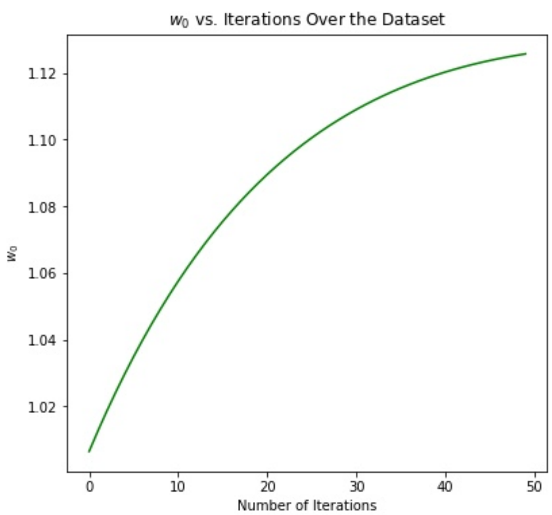
- Learning rate
- Number of iterations

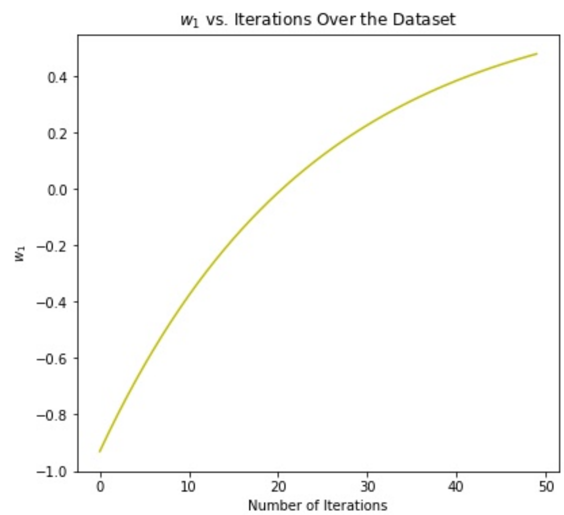
Learning rate = the size of the step taken in gradient descent

Number of iterations = the number of steps taken in gradient descent.



As the number of iterations increase, the cost keeps falling ...

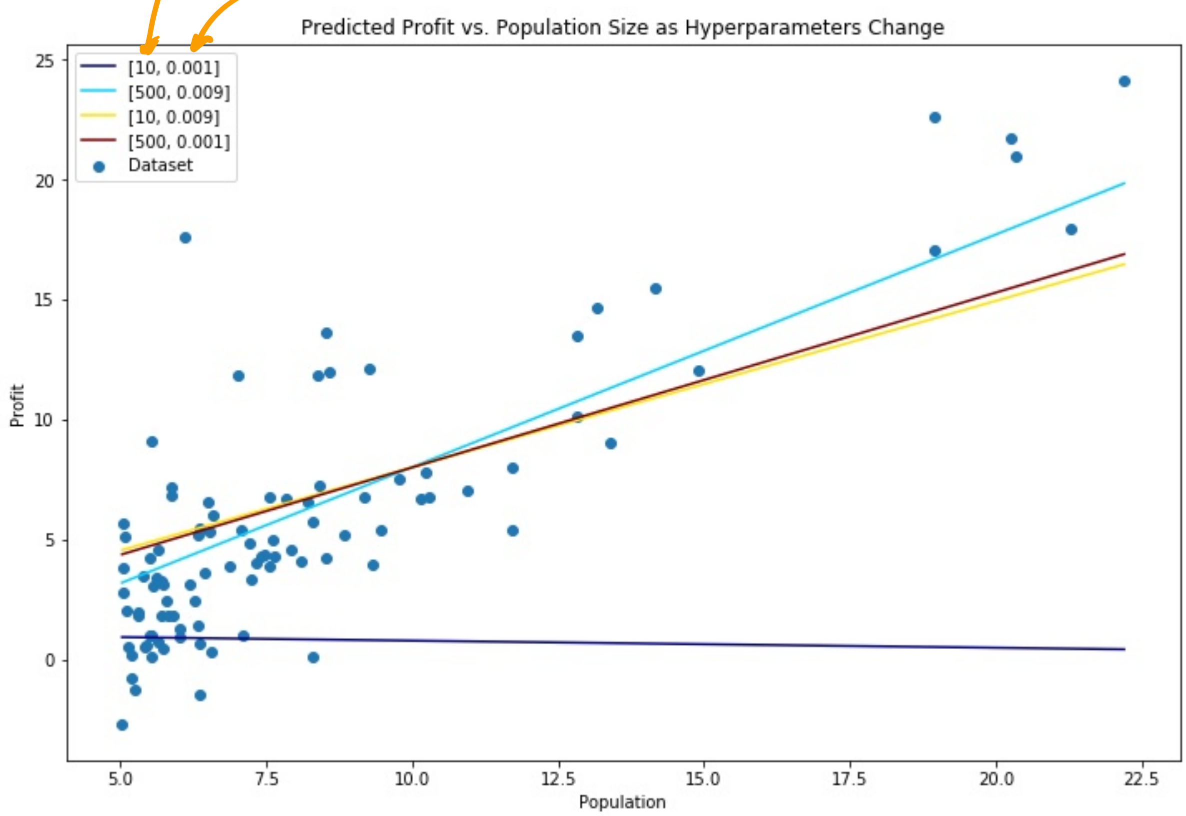


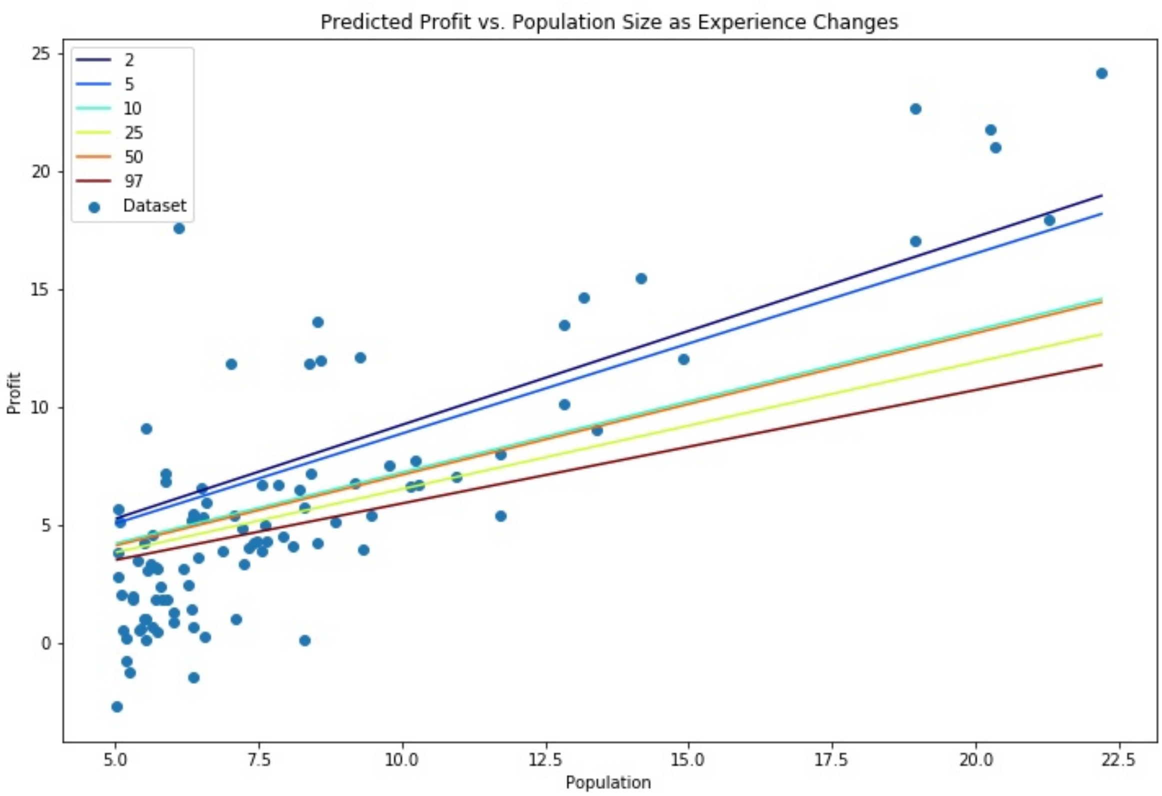


... and the parameters get to their stable

Number of steps or iterations

learning rate or the size of the jump





Predicted Profit vs. Population Size as Experience Changes
2
5
10
25
50
97
Dataset
Profit
Population
25
20
15
10
5
0
5.0
7.5
10.0
12.5
15.0
17.5
20.0
22.5

Step Size in Gradient Descent