REGRESSION WITH A SINGLE FEATURE

The Business Problem

- CEO of a restaurant chain
- Thinking about how to expand the business
- Have data on current restaurants
  - population of town in which restaurant is located
  - profit from restaurant per quarter
- Where should the next 5 restaurants be opened to maximize profit growth?

Regression w/ Single Feature ②

The Data

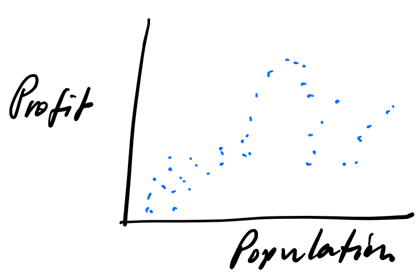
Population (10,000)	Profit ($100,000)
6.11	17.59
5.53	9.13
8.52	13.67
⋮	⋮
12.68	15.34

Structure of the data

- food-truck-profits.txt
- 97 rows
- numerical information (not Categorical)
- No missing information

$\boxed{\text{Step 1}}$

Scatterplot of the dataset to visualize it



$\boxed{\text{STEP 2}}$ Define the task (w.r.t. the dataset)

Input → the population size

Output → the profit

Task → Given a population size, predict the profit.

Regression w/ Single Feature (4)

$\boxed{\text{STEP 3}}$ Define the Model

Feature = input = population size

(later we'll see how features can differ from inputs)

Transform the feature into an output

$$(w_0 \times x_0^{(1)}) + (w_1 \times x_1^{(1)}) = \hat{y}^{(1)}$$

Parameter 1 — Constant feature — Parameter 2 — Feature 1 — Predicted output — Actual Output

w_0	x_0	w_1	x_1	$\hat{y}$	y
w_0	1	w_1	$x_1^{(1)}$	$\hat{y}^{(1)}$	$y^{(1)}$
w_0	1	w_1	$x_1^{(2)}$	$\hat{y}^{(2)}$	$y^{(2)}$
⋮	⋮	⋮	⋮	⋮	⋮
w_0	1	w_1	$x_1^{(97)}$	$\hat{y}^{(97)}$	$y^{(97)}$

m rows

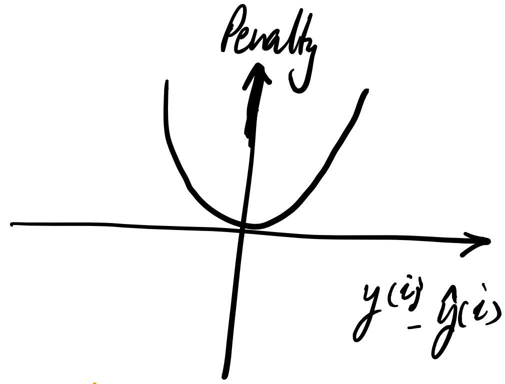
m = 97

These values don't change (w_0, 1, w_1)

These values change from row to row ($x_1^{(97)}$, $\hat{y}^{(97)}$, $y^{(97)}$)

STEP 4 Define the penalty for getting it wrong.

Penalty for each row $= \left(y^{(i)} - \hat{y}^{(i)}\right)^2$

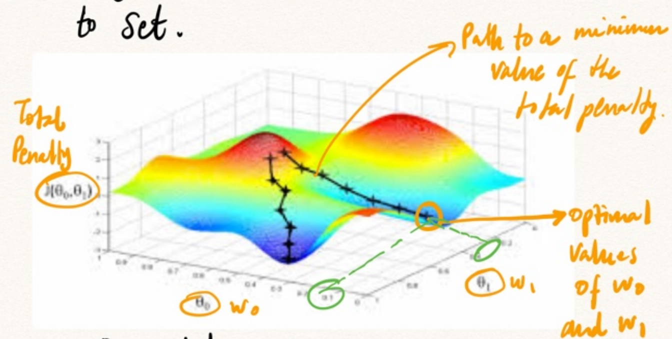


The more we get it wrong, the more we're penalized.

Total Penalty for the entire data set = Sum of penalty of each row.

QUESTION What values of w_0 and w_1 will minimize the total penalty?

Imagine we only had w_0 and w_1 to set.



$\theta_0 = W_0$

$\theta_1 = W_1$

NOTE The shape of this surface depends on the penalty function we choose.

Use gradient descent to solve the numerical optimization problem.

- Start at an arbitrary point on the surface – pick arbitrary values for w_0 and w_1
- From this point, take a step in the direction where the slope is descending most steeply.
- Do this again and again until you reach a minimum (the "valley floor")

Recap

Model – transforms the input(s) into the output.

Parameters – these are the fixed values (e.g., w_0, w_1) of the model.

Penalty – For each row of the dataset, what's the penalty/cost/price for being wrong?

Recap (Contd.)

Total Penalty - Sum of the penalty of each row of the dataset.

Problem - Find the parameter values of the model (w_0 and w_1 values) that **minimize** the total penalty.

Approach to Solving the Problem - Gradient Descent.

Hyper-Parameters

Gradient descent can produce different results based on how you set the values of the following parameters.

- Learning Rate. The size of the jump taken at each step.
- The total number of jumps taken.

Every machine learning problem has hyperparameters.

Why? Because every machine learning problem is a giant optimization problem – find the optimal values of the model parameters that minimizes the total penalty.

To find these optimal values of w_0 and w_1 (for example), we use a learning algorithm like gradient descent.

Finding the right <u>hyperparameters</u> is a separate quest – one that must be solved every time you apply machine learning to solve a problem.

In a few sessions we'll see how this problem is tackled.