Bak-Sneppen

Group 0

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Research question:

How do the dynamics change in the Bak-Sneppen model, when applying a 1D and 2D framework?

Hypotheses:

1 mutation will have a bigger effect for the 2D model compared to the 1D model. If this increase is sufficiently large, the model could break as it is pushed out of its criticality state. This would happen when the power law exponent becomes > 2.

Self-Organized Criticality

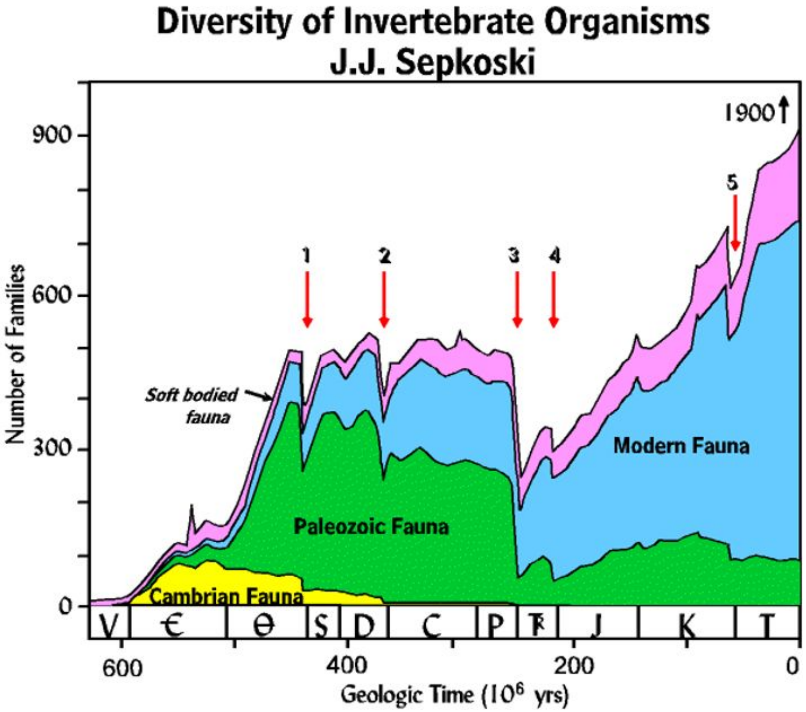

*Figure from reference 4

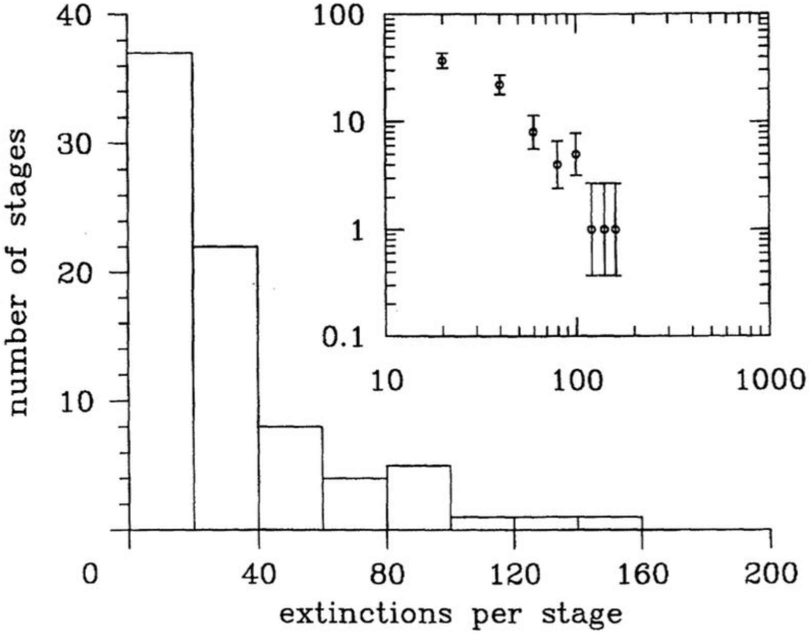

*Figure from reference 7

Bak-Sneppen Model

Simulation:

- N species, 1D line with periodic boundary conditions.
- Random fitness (0 to 1) for each species from uniform distribution.
- At each time step, lowest fitness is mutated to a random fitness.
- Then mutating the fitness of the two neighbors, randomly as well.

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Punctuated Equilibrium and Criticality in a Simple Model of Evolution

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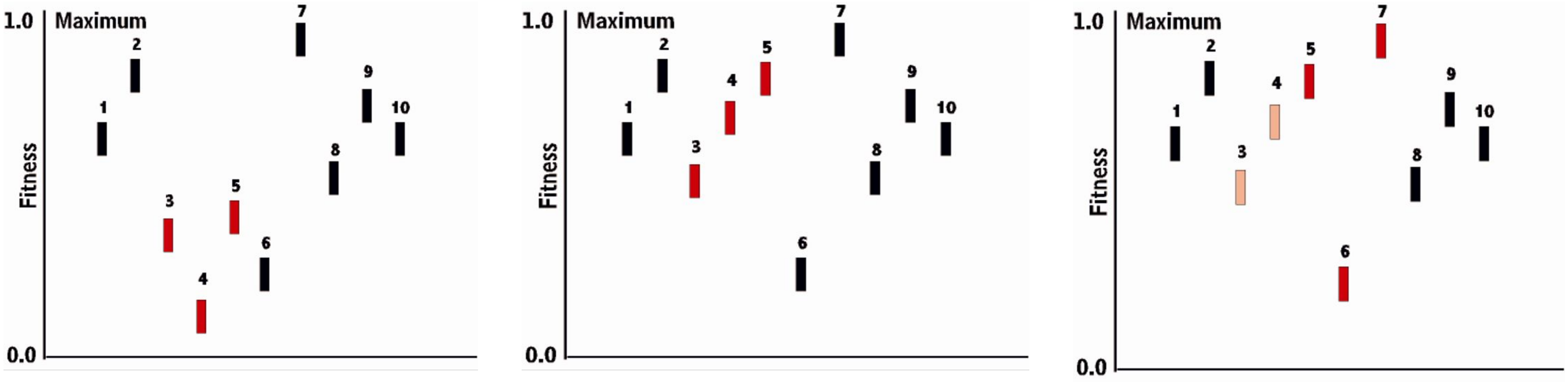
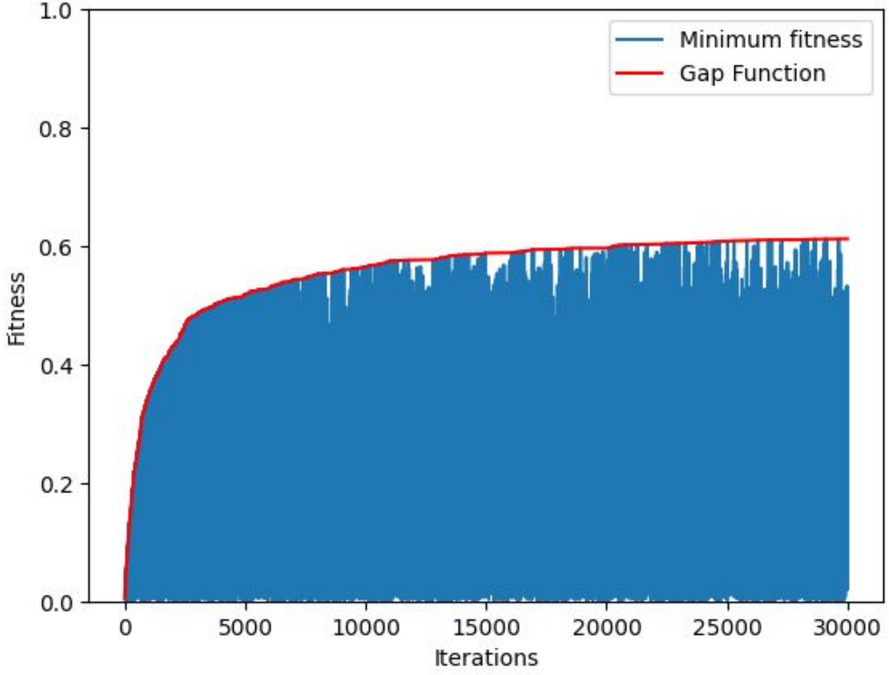
Bak-Sneppen Model

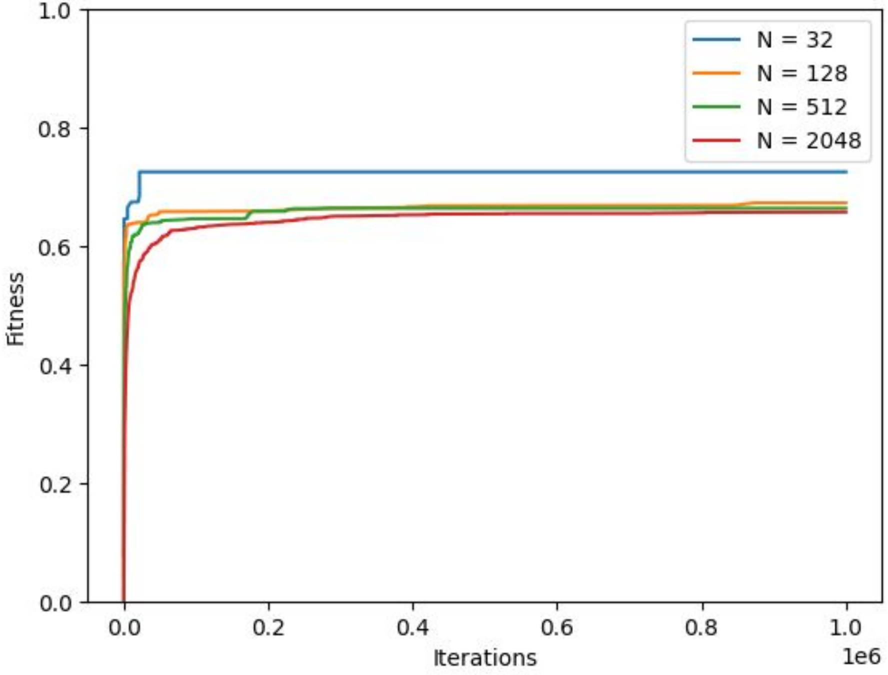


*Figures from reference 4

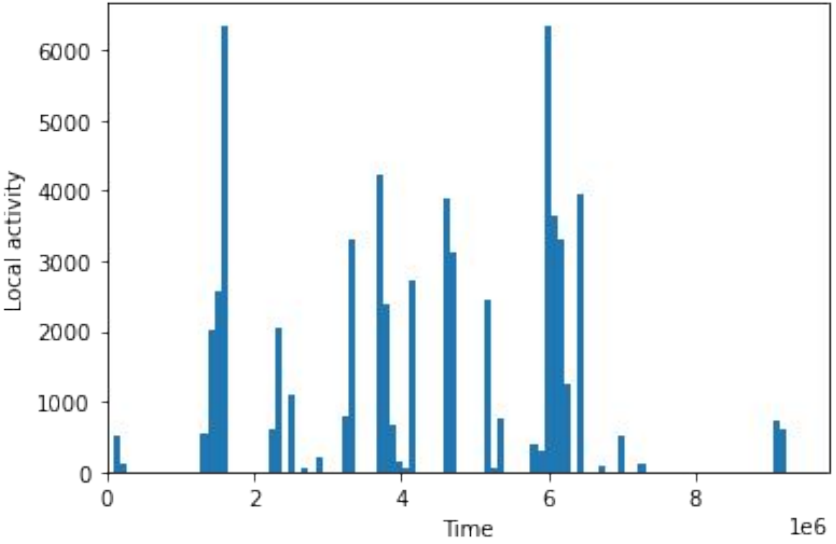
Gap function 1D

$f_c = 0.667$

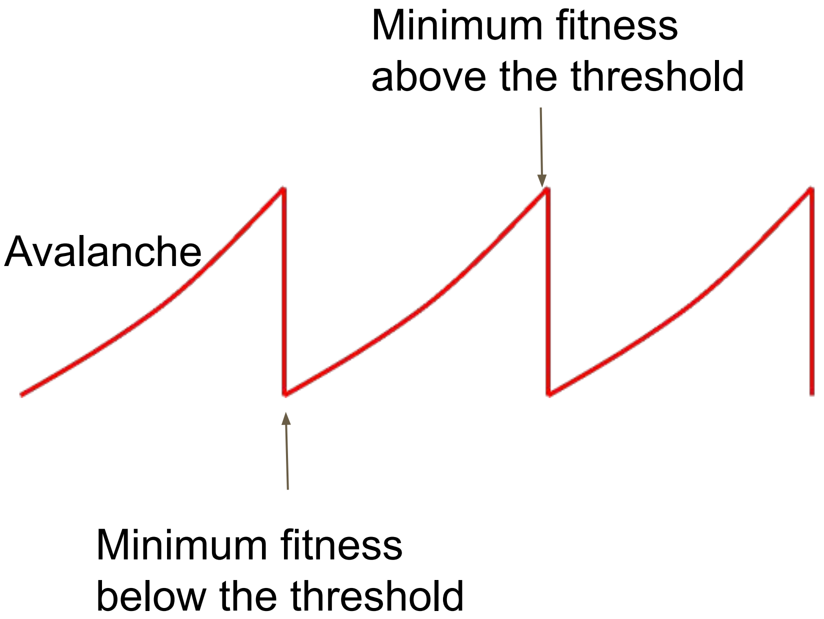




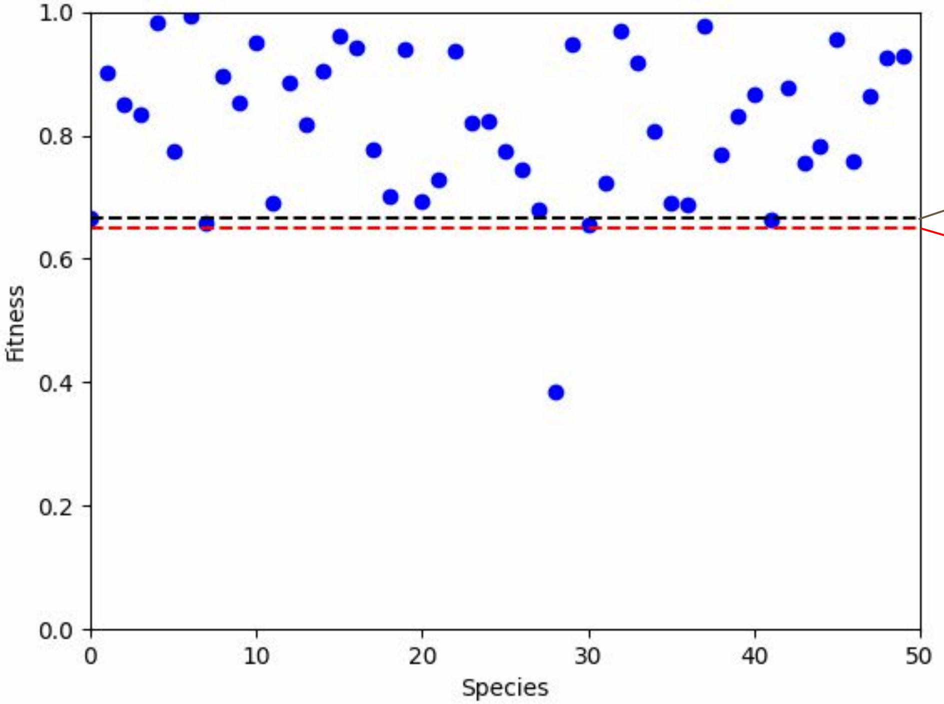
1D Bak-Sneppen Avalanches



Only 2% of species
System already in Critical state



Fitness changes in 1D model- animation

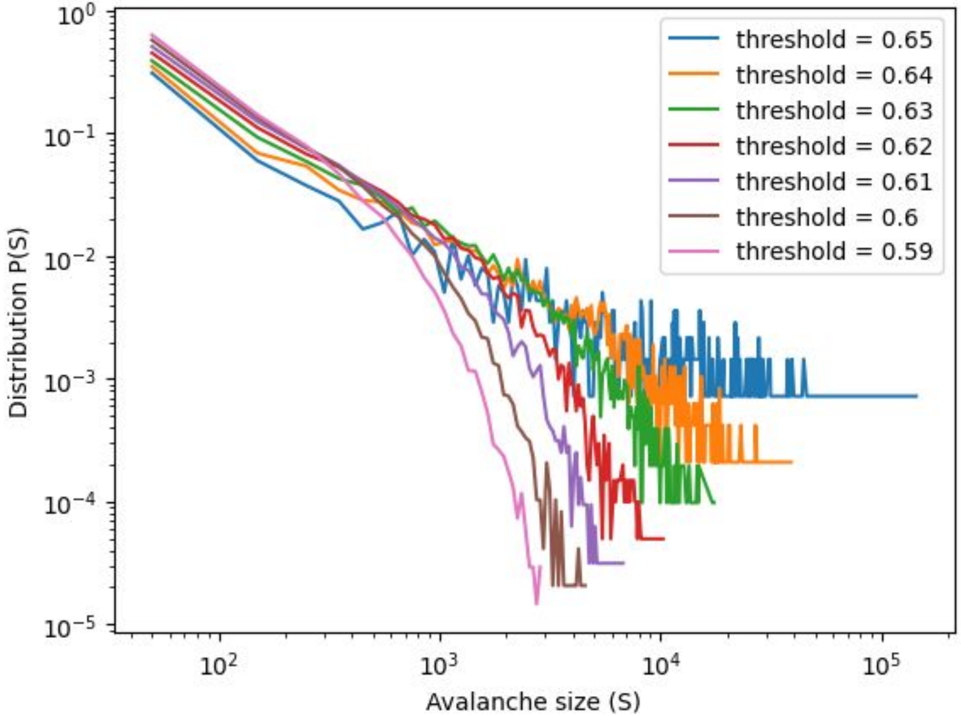


Self-Organized Critical threshold (f_c)

Avalanche threshold (f_0)

Impact thresholds for 1D

Avalanche size distribution

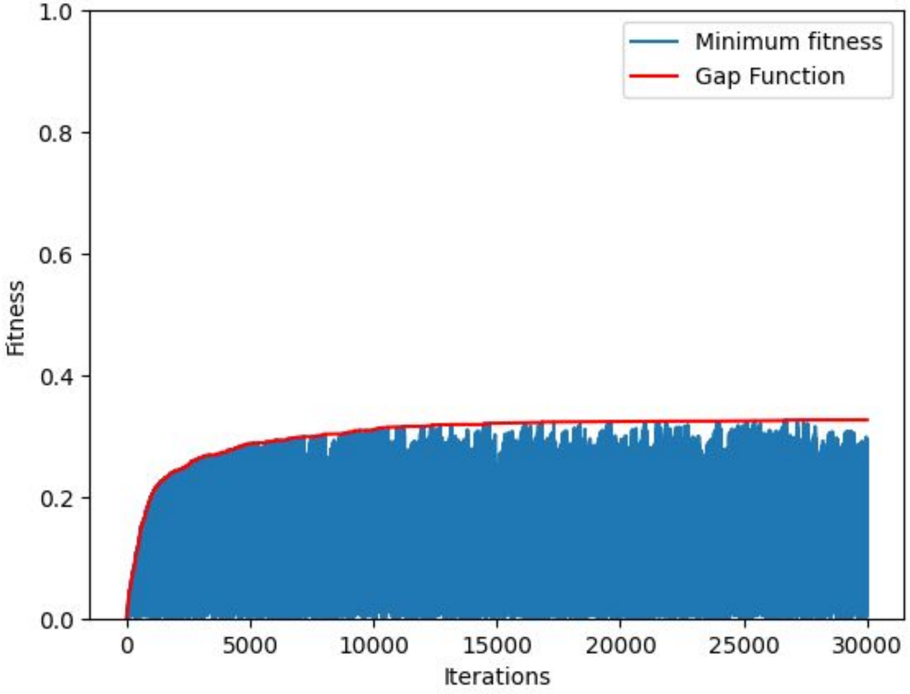


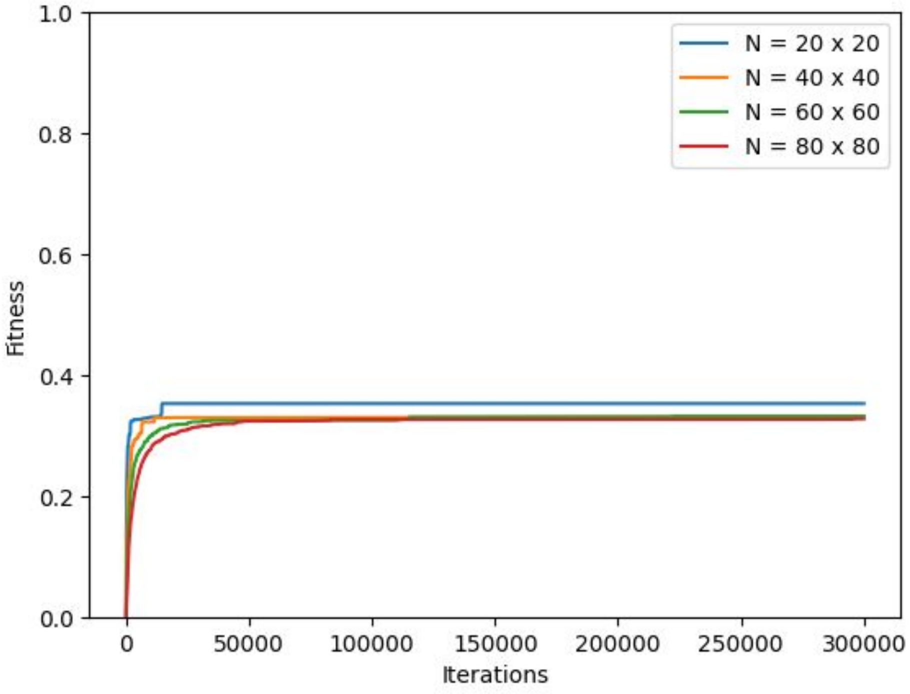
$$P(S, f_0) = S^{-\tau} g(S(f_c - f_0)^{1/\sigma})$$

$$\text{cutoff } S_{co} = (f_c - f_0)^{-1/\sigma}$$

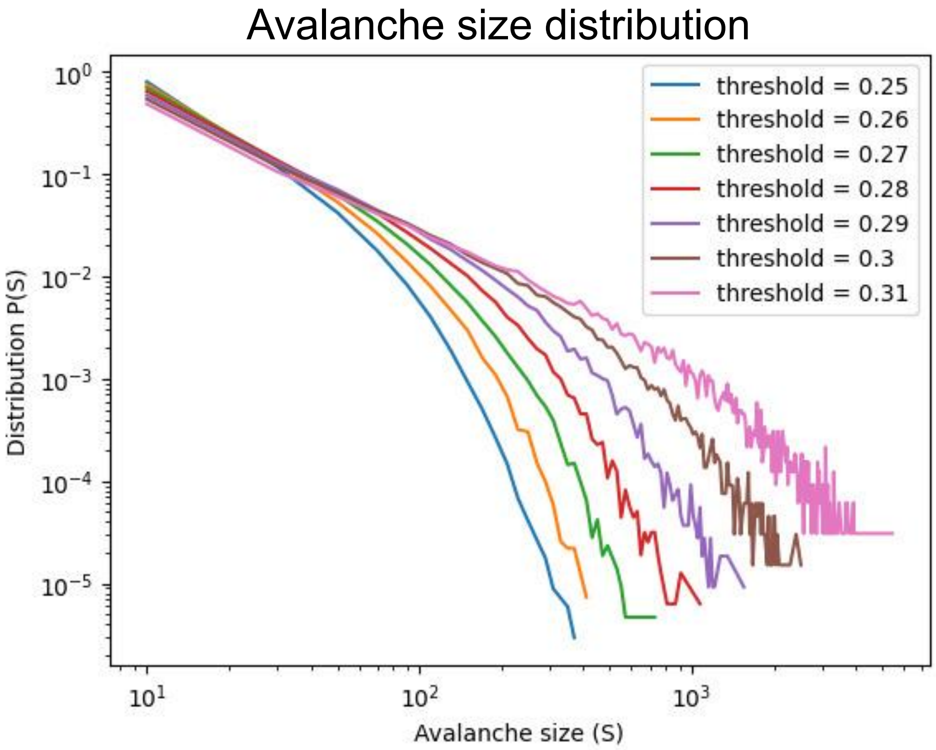
Gap function 2D

$f_c = 0.329$





Impact thresholds for 2D

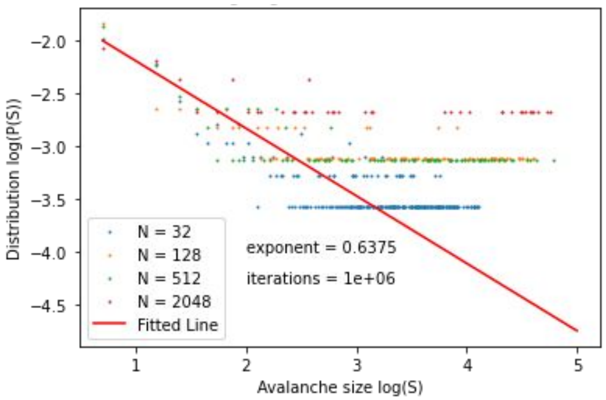


$$P(S, f_0) = S^{-\tau} g(S(f_c - f_0)^{1/\sigma})$$

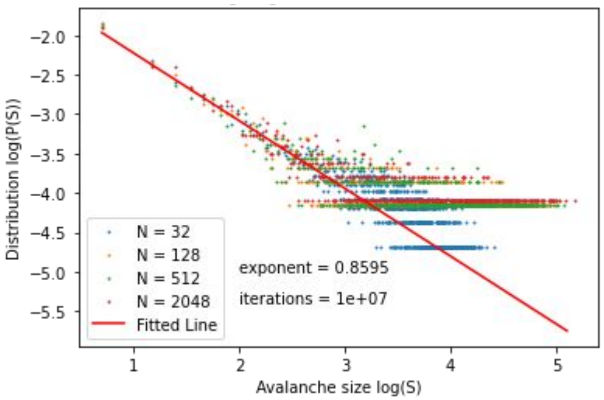
cutoff $S_{co} = (f_c - f_0)^{-1/\sigma}$

Power law fit 1D

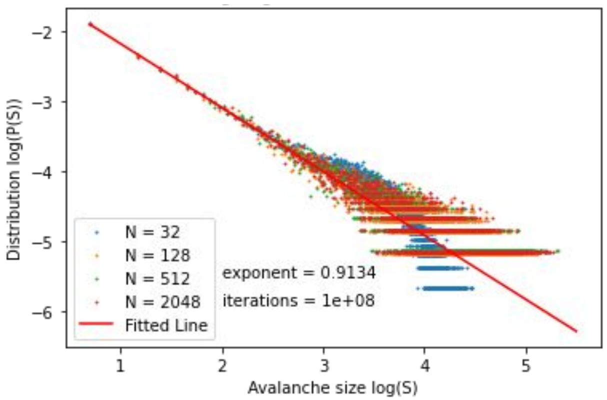
- Simulation avalanche threshold f_0 = 0.65
- Critical threshold f_c = 0.667
- Fitted exponent = 0.91
- Critical exponent τ = 1.07



10^6 iterations



10^7 iterations

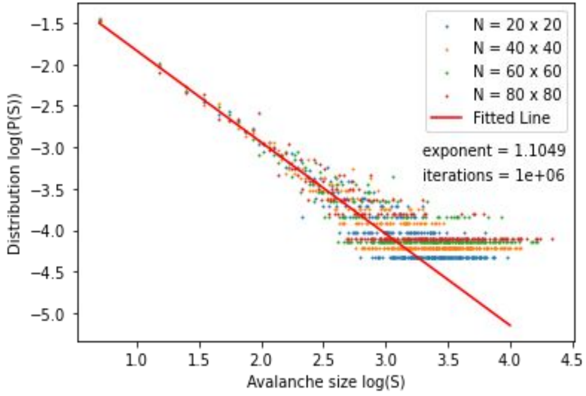


10^8 iterations

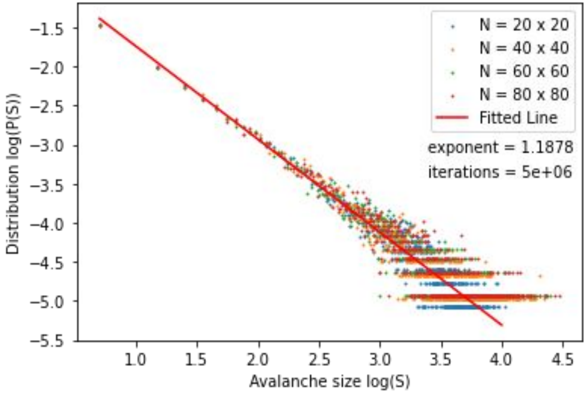
Power law fit 2D

- Simulation threshold $f_0 = 0.32$
- Critical threshold $f_c = 0.329$
- Fitted exponent = 1.247
- Critical exponent $\tau = 1.245$

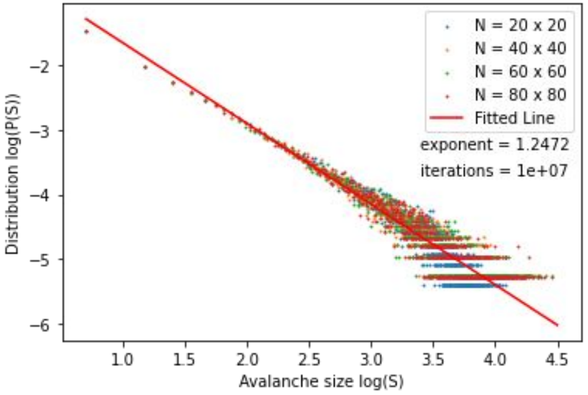
10^6 iterations



5×10^6 iterations



10^7 iterations



Results: Comparison of 1D and 2D

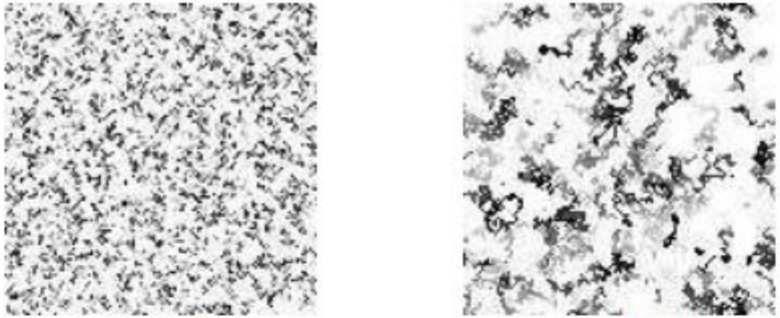
Hypotheses:

1 mutation will have a bigger effect for the 2D model compared to the 1D model. If this increase is sufficiently large, the model could break as it is pushed out of its criticality state. This would happen when the power law exponent becomes > 2.

	Simulation Threshold	**Critical Threshold**	**Simulation Exponent**	**Critical Exponent**
1D	0.65	0.667	0.913	1.07
2D	0.32	0.329	1.247	1.245

Growth Opportunities

- Implementing Migration
- Networks
- Increasing connections → when no longer SOC?



$t = 10$ $t = 10000$

Figure 3. Distribution of fitness values on the grid. Lighter grey areas correspond to particles with lower fitness. $X \times Y$: 60×60; $n = 1200$.

*Figures from reference 3.

Questions?

References

1. Bak, Per, and Kim Sneppen. "Punctuated equilibrium and criticality in a simple model of evolution." *Physical review letters* 71.24 (1993): 4083.
2. Boettcher, Stefan, and Maya Paczuski. "Exact results for spatiotemporal correlations in a self-organized critical model of punctuated equilibrium." *Physical Review Letters* 76.3 (1996): 348.
3. Fernandes, Carlos, Juan Laredo, J. J. Merelo, Carlos Cotta, and Agostinho Rosa. "Adapting the Bak-Sneppen Model to a Dynamic and Partially Connected Grid of Hierarchical Species." In *ECAL 2013: The Twelfth European Conference on Artificial Life*, pp. 523-530. MIT Press, 2013.
4. Fitcher, Lynn S., Pyle, E.L., and Whitmeyer, S.J. "Strategies and Rubrics for Teaching Chaos and Complex Systems Theories as Elaborating, Self-Organizing, and Fractionating Evolutionary Systems." Presentation at James Madison University, 2010. PowerPoint presentation.
5. Han, Jihui, Wei Li, and Weibing Deng. "Critical behavior of a generalized Bak-Sneppen model." In *Journal of Physics: Conference Series*, vol. 1113, no. 1, p. 012011. IOP Publishing, 2018.
6. Murase, Yohsuke, and Per Arne Rikvold. "Conservation of population size is required for self-organized criticality in evolution models." *New Journal of Physics* 20, no. 8 (2018): 083023.
7. Newman, M. E. J. "Self-organized criticality, evolution and the fossil extinction record." *Proceedings of the Royal Society of London. Series B: Biological Sciences* 263, no. 1376 (1996): 1605-1610.
8. Paczuski, Maya, Sergei Maslov, and Per Bak. "Avalanche dynamics in evolution, growth, and depinning models." *Physical Review E* 53, no. 1 (1996): 414.
9. Wikipedia contributors. “Bak–Sneppen Model.” *Wikipedia*, May 2023, en.wikipedia.org/wiki/Bak%E2%80%93Sneppen_model.

Appendix

Adapting Bak-Sneppen to 2D Migration

Additions to Simulation:

- N species, 2D system with periodic boundary conditions.
- Von Neumann neighborhood mutations
- Moore neighborhood movement (migration)
- Leaves mark at previous location (migration)

Adapting the Bak-Sneppen Model to a Dynamic and Partially Connected Grid of Hierarchical Species

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Migration

- Implementing migration should result in clustering throughout the system
- We were not quite able to get full clustering
- Small clusters did form, but only briefly so it’s mostly random
- This could be because we could not get the periodic grid to work