Bak-Sneppen

Group 0

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Research question:

How do the dynamics change in the Bak-Sneppen model, when applying a 1D and 2D framework?

Hypotheses:

1 mutation will have a bigger effect for the 2D model compared to the 1D model. If this increase is sufficiently large, the model could break as it is pushed out of its criticality state. This would happen when the power law exponent becomes > 2.

Self-Organized Criticality

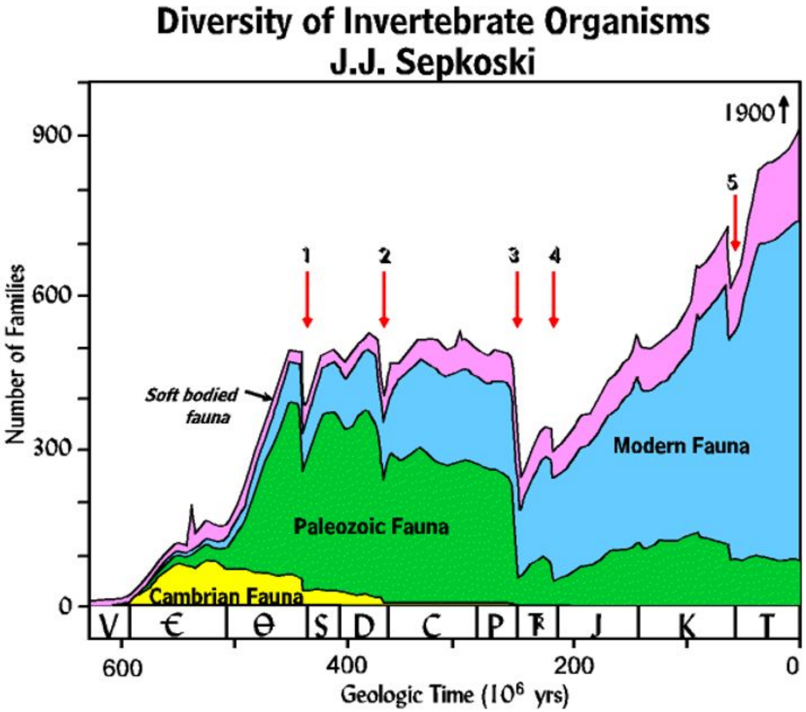


*Figure from reference 4

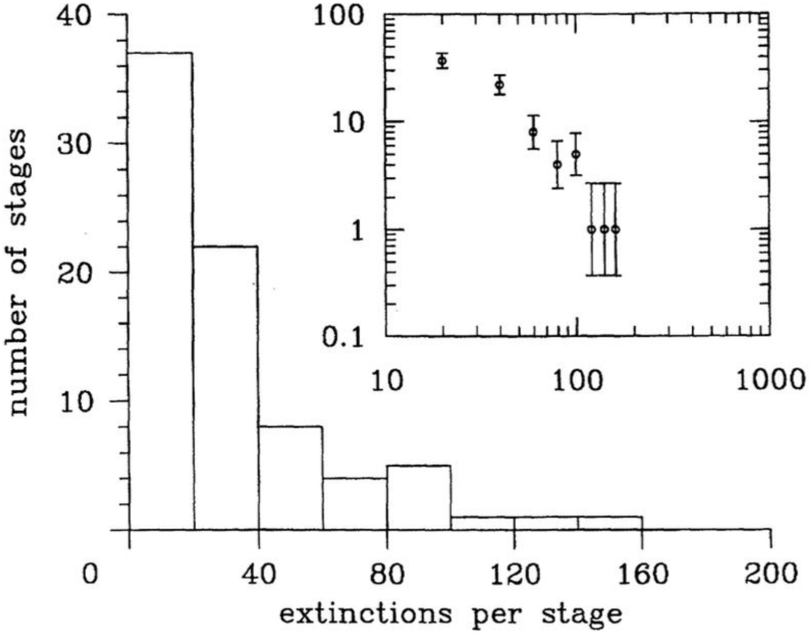
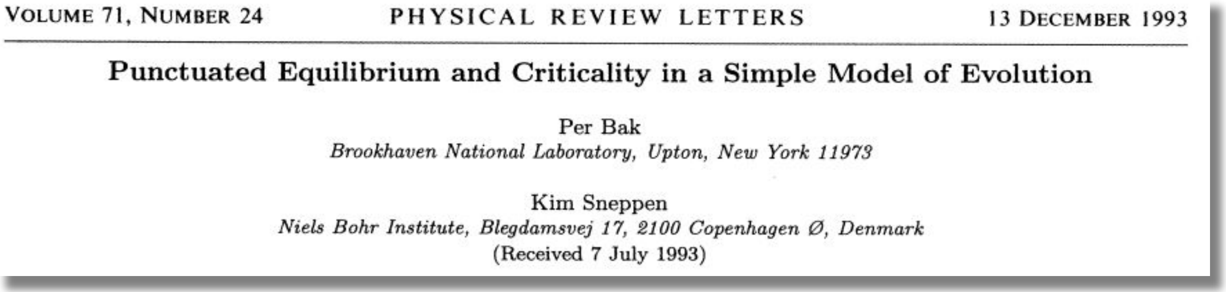


*Figure from reference 7

Bak-Sneppen Model

Simulation:

- N species, 1D line with periodic boundary conditions.
- Random fitness (0 to 1) for each species from uniform distribution.
- At each time step, lowest fitness is mutated to a random fitness.
- Then mutating the fitness of the two neighbors, randomly as well.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Punctuated Equilibrium and Criticality in a Simple Model of Evolution

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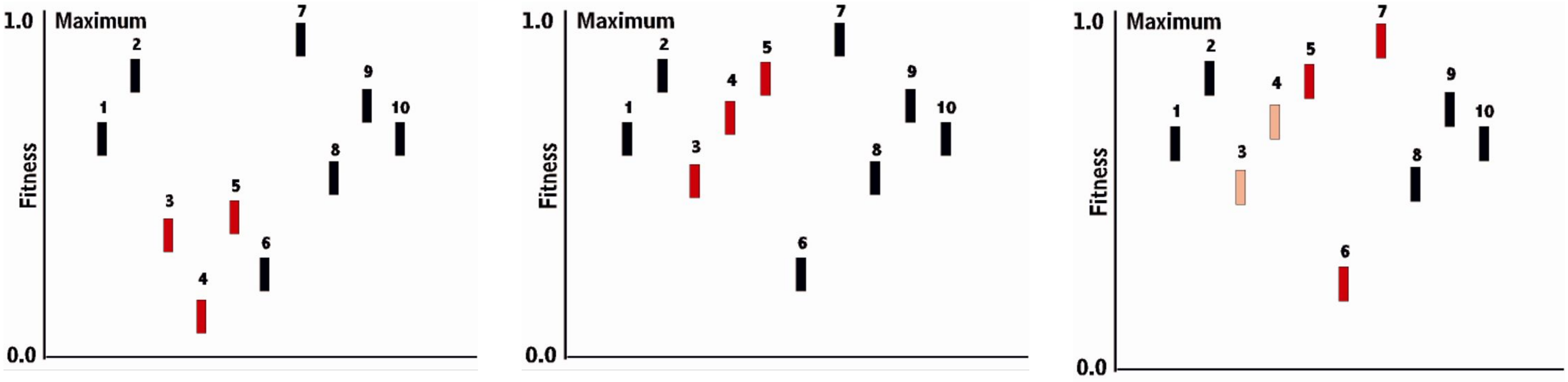
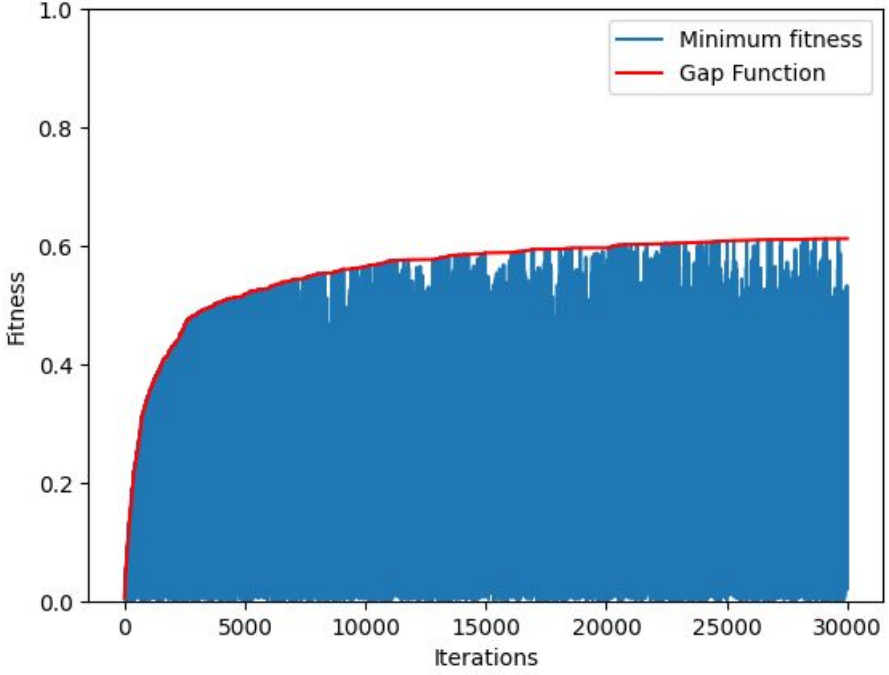
Bak-Sneppen Model

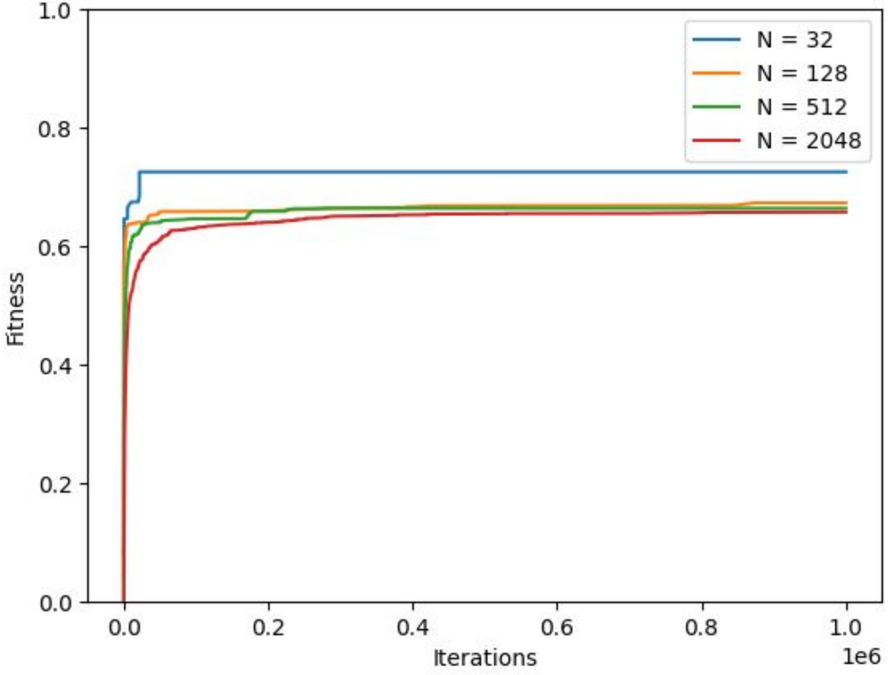


*Figures from reference 4

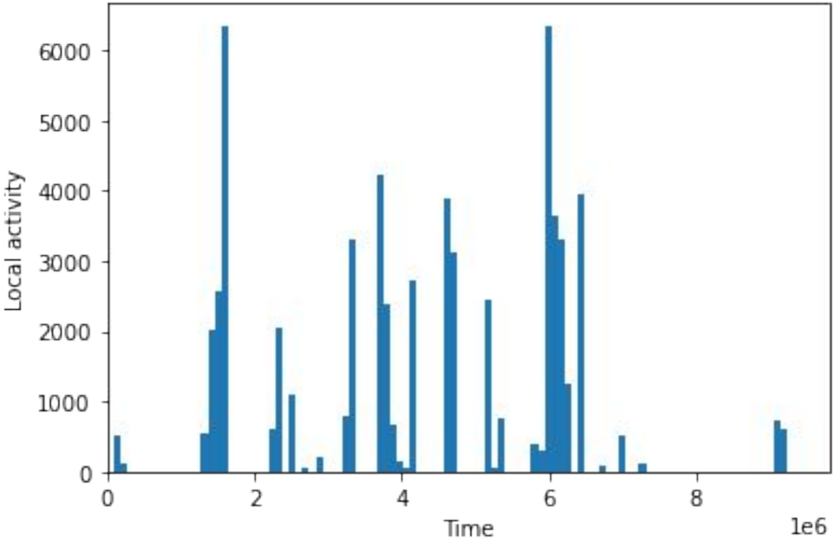
Gap function 1D

$f_c = 0.667$





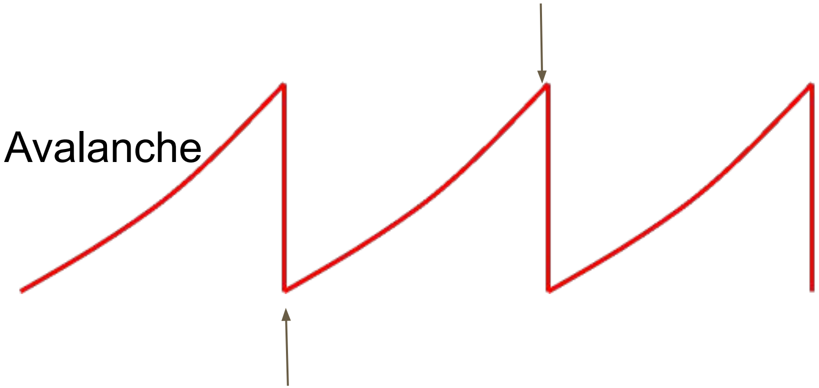
1D Bak-Sneppen Avalanches



Only 2% of species
System already in Critical state

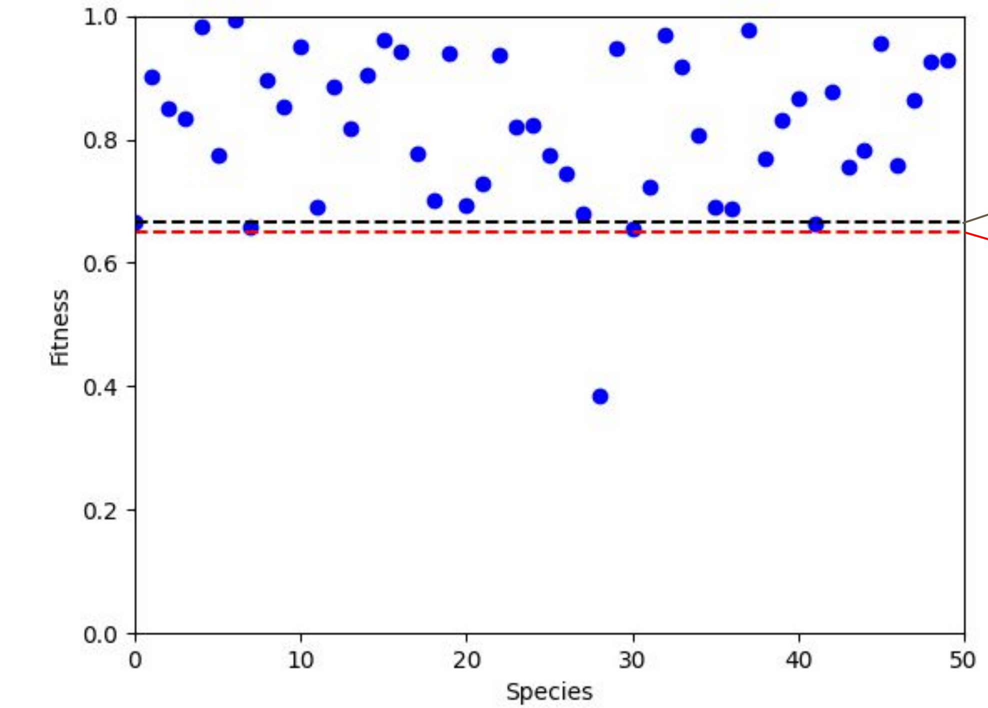
Minimum fitness
above the threshold

Avalanche



Minimum fitness
below the threshold

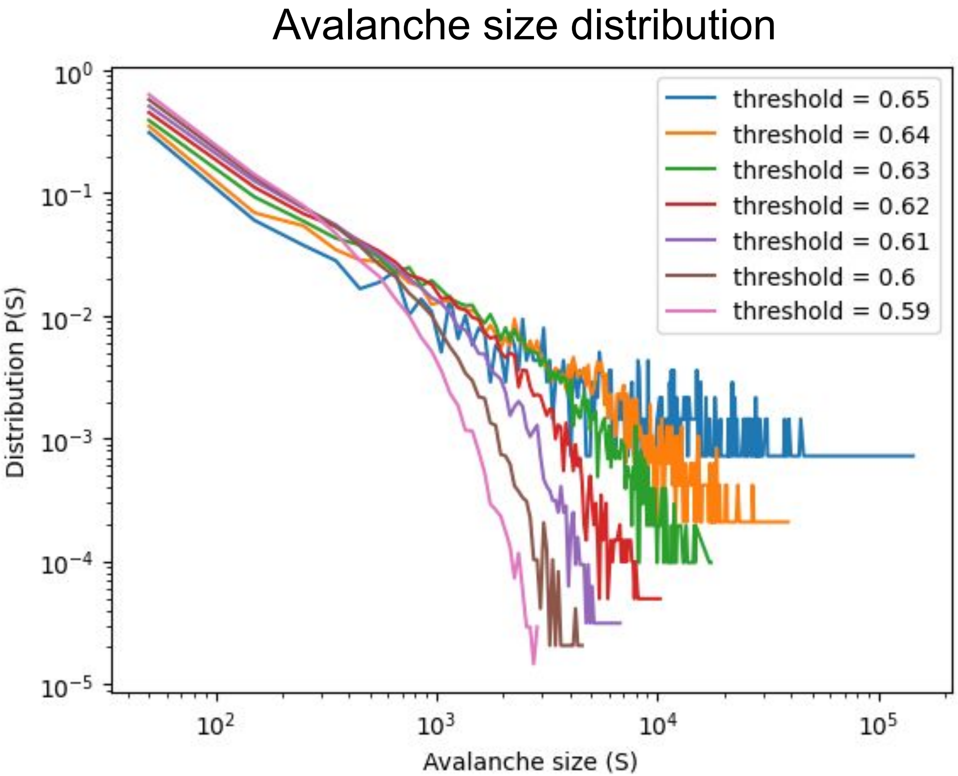
Fitness changes in 1D model- animation



Self-Organized Critical threshold (f_c)

Avalanche threshold (f_0)

Impact thresholds for 1D

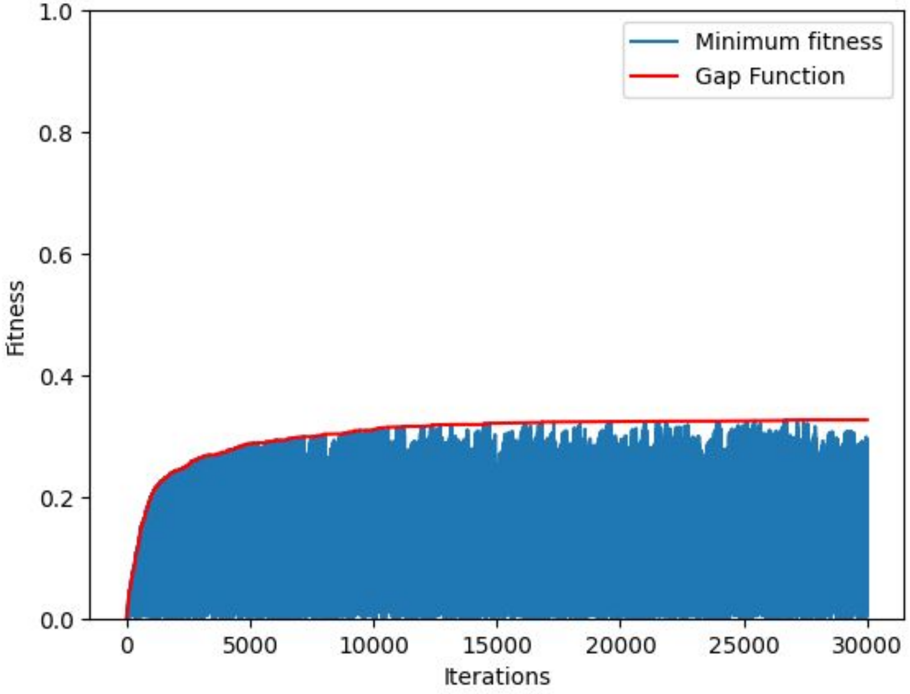


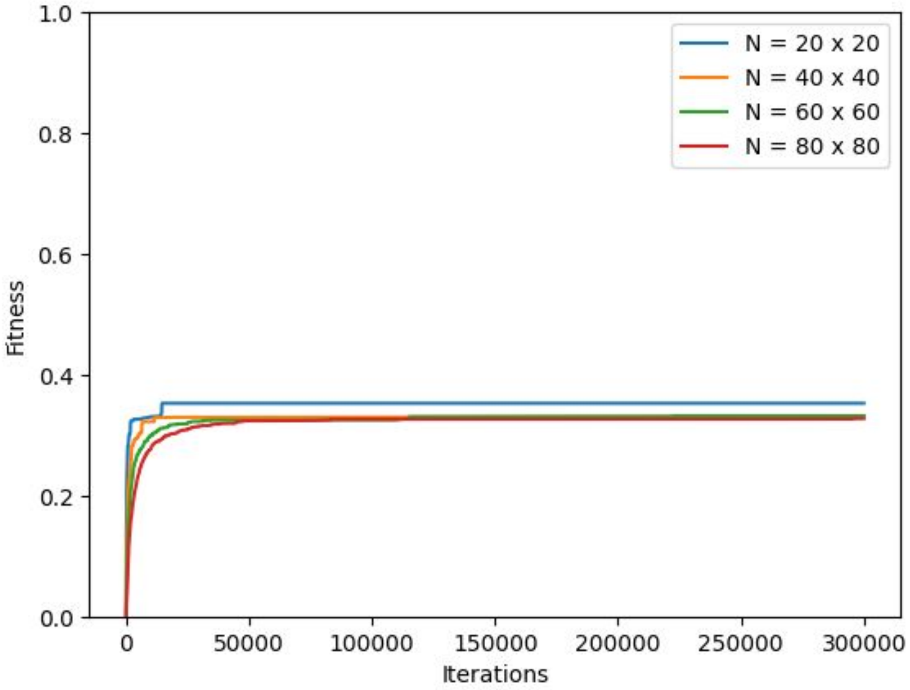
$$P(S, f_0) = S^{-\tau} g(S(f_c - f_0)^{1/\sigma})$$

$$\text{cutoff } S_{co} = (f_c - f_0)^{-1/\sigma}$$

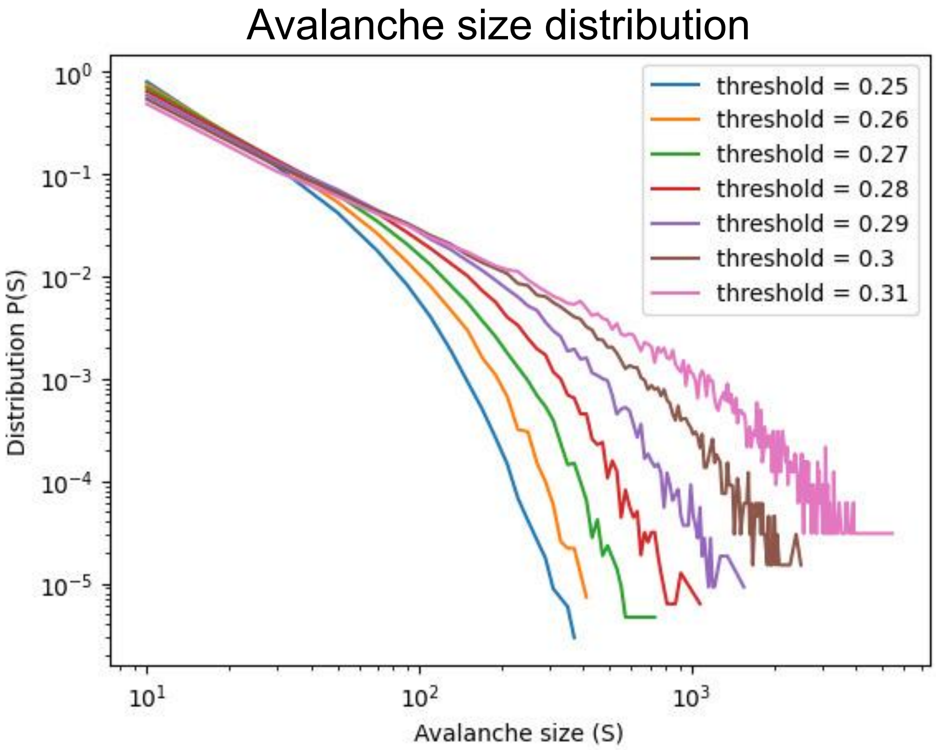
Gap function 2D

$f_c = 0.329$





Impact thresholds for 2D

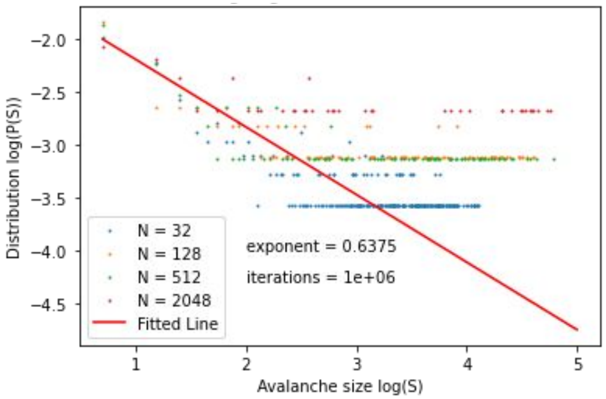


$$P(S, f_0) = S^{-\tau} g(S(f_c - f_0)^{1/\sigma})$$

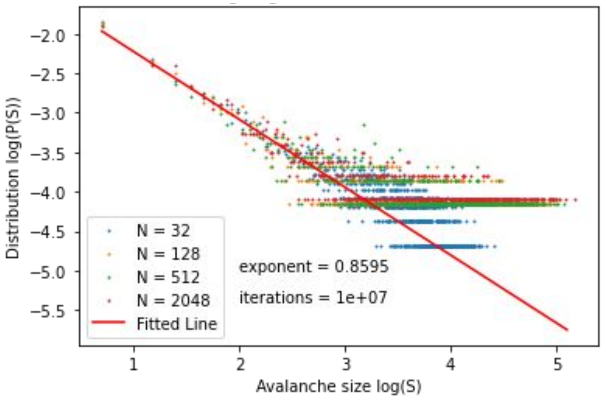
$$\text{cutoff } S_{co} = (f_c - f_0)^{-1/\sigma}$$

Power law fit 1D

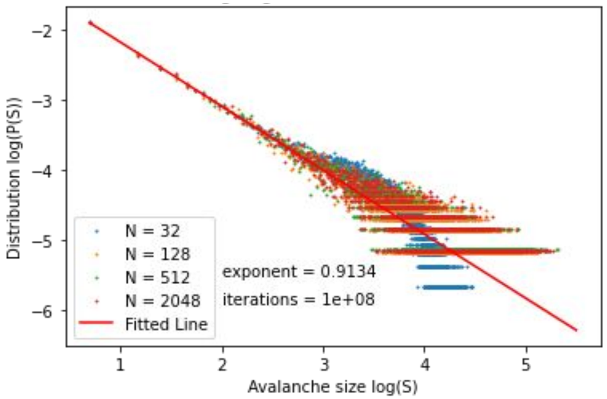
- Simulation avalanche threshold $f_0 = 0.65$
- Critical threshold $f_c = 0.667$
- Fitted exponent = 0.91
- Critical exponent $\tau = 1.07$



10^6 iterations



10^7 iterations

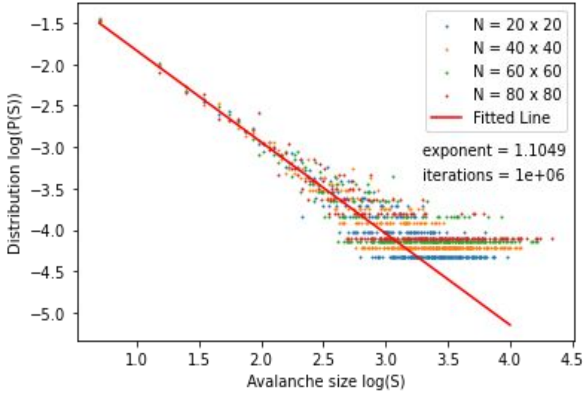


10^8 iterations

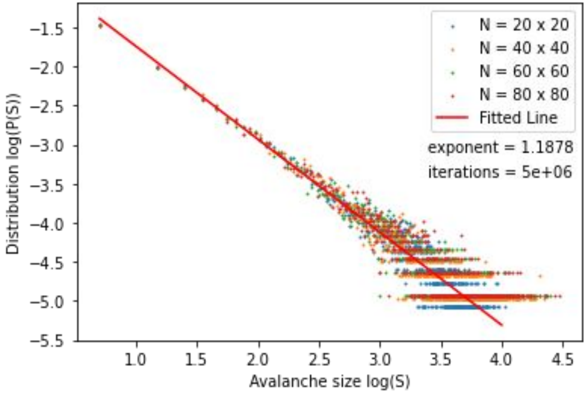
Power law fit 2D

- Simulation threshold $f_0 = 0.32$
- Critical threshold $f_c = 0.329$
- Fitted exponent = 1.247
- Critical exponent τ = 1.245

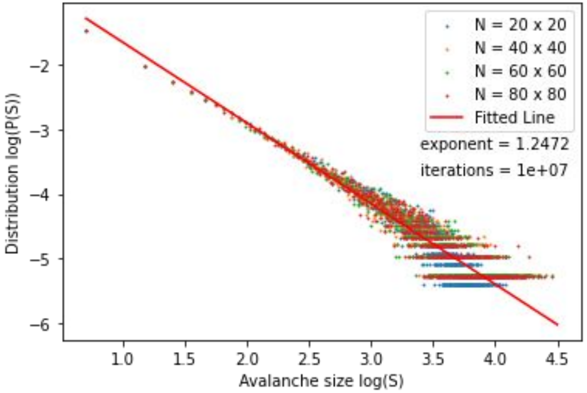
10^6 iterations



$5x10^6$ iterations



10^7 iterations



Results: Comparison of 1D and 2D

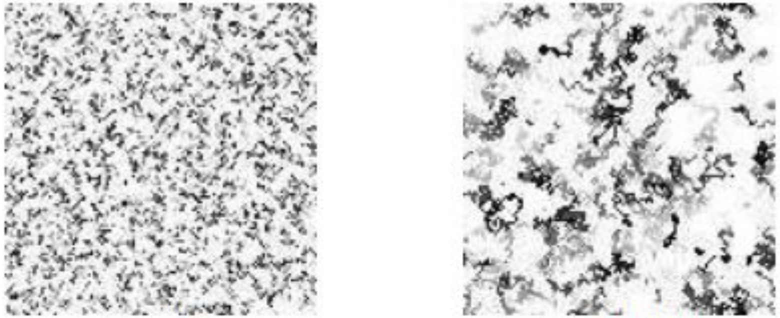
Hypotheses:

1 mutation will have a bigger effect for the 2D model compared to the 1D model. If this increase is sufficiently large, the model could break as it is pushed out of its criticality state. This would happen when the power law exponent becomes > 2.

	Simulation Threshold	Critical Threshold	Simulation Exponent	Critical Exponent
1D	0.65	0.667	0.913	1.07
2D	0.32	0.329	1.247	1.245

Growth Opportunities

- Implementing Migration
- Networks
- Increasing connections → when no longer SOC?



$t = 10$ $\qquad$ $t = 10000$

Figure 3. Distribution of fitness values on the grid. Lighter grey areas correspond to particles with lower fitness. $X \times Y$: 60×60; $n = 1200$.

*Figures from reference 3.

Questions?

References

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Appendix

Adapting Bak-Sneppen to 2D Migration

Additions to Simulation:

- N species, 2D system with periodic boundary conditions.
- Von Neumann neighborhood mutations
- Moore neighborhood movement (migration)
- Leaves mark at previous location (migration)

Adapting the Bak-Sneppen Model to a Dynamic and Partially Connected Grid of Hierarchical Species

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Migration

- Implementing migration should result in clustering throughout the system
- We were not quite able to get full clustering
- This comes from not being able to find out the exact order in which to move the cells
- Small clusters did form, but only briefly so it’s mostly random
- This could be because we could not get the periodic grid to work